"Getting Around The Coriolis Force"

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Abstract

The Coriolis Force: most people know about it, but few understand it. A simple explanation not requiring an intuitive understanding of angular momentum is provided. Scales over which the Coriolis Force is relevant are also discussed.

1.0 Introduction and Motivation

At some point in their lives, most people hear about the Coriolis force, whether in reference to weather patterns, sea currents or, most prosaically, which way water flows down the sink. Unfortunately, while many have heard of it, few understand it well enough to explain it without resorting to vector equations.

Of course, most physics textbooks which deal with angular kinematics will have the following equation relating the Coriolis force to an object's mass (m), its velocity in a rotating frame (v_r) and the angular velocity of the rotating frame of reference (ω):

$$F_{Coriolis} = -2\ m\ (\omega \times v_r)$$

The text will then either explain the Coriolis force in terms of angular quantities such as conservation of angular momentum, or will use the Coriolis force to illustrate the angular kinematics. Unfortunately, most of us are not comfortable with angular mechanics. It would not be an exaggeration to say that some students dread it. Nor can we expect students to enter the classroom understanding the Coriolis force. Hence, whether using physics to explain the phenomenon or using the phenomenon to explain the physics, students are shaky on both sides of this relationship.

So, what to do? This article intends to develop a means of explaining the Coriolis force to people who haven't yet grasped angular mechanics. The explanation relies on linear quantities and uses rotational concepts infrequently.

2.0. The Basic Premises

The following principles are needed before starting the body of the explanation:

1. Newton's First Law in component form - Objects in motion tend to stay in motion unless acted on by an unbalanced force. A vector component of velocity will not be changed by a force perpendicular to that component.
2. Spherical Geometry of the Earth - X degrees of longitude gives you different distances between longitude lines (in miles or kilometers) at different latitudes, plus a few additional results of being on a sphere which will be detailed later.
3. Gravity - Objects under the influence of Earth's gravity will fall towards (and thus orbit) the center of mass of the Earth.

Premise 2 is probably the easiest for students to accept, since you can draw on a globe to demonstrate that an inch is 15 degrees of longitude at one latitude and 30 degrees at another. Having a ball or globe on hand for the explanation is generally helpful. Premises 1 and 3 require some science background, however, but should be acceptable to students in mechanics courses.

3.0. Explanation of the Coriolis Force

While all Coriolis-based deflection can be explained using rotational concepts, a linear explanation is simpler if you separate the effects into those in the north/south direction and those in the east/west direction. The deflection of objects moving north and south can be explained without invoking centripetal acceleration, as we see next.

3.1. I Feel the Earth Move under My Feet: North/South Motion

Note first that all points on the Earth have the same rotational velocity, ω (they go around once per day). Also, places at different latitudes have different linear speeds. A point near the equator may go around a thousand miles in an hour, while one near the North Pole could be moving only a few dozen miles in an hour.

Normally, objects in contact with the ground travel the same speed as the ground they stand on. As a result, the Coriolis force generally doesn't have a noticeable effect to people on the ground; the speed of the point you're standing on and the speed of the point you're stepping onto are too close for you to tell the difference. Or, looking back at the Coriolis Force equation above, if the velocity relative to the rotating frame (the Earth) is zero, so is the Coriolis force.

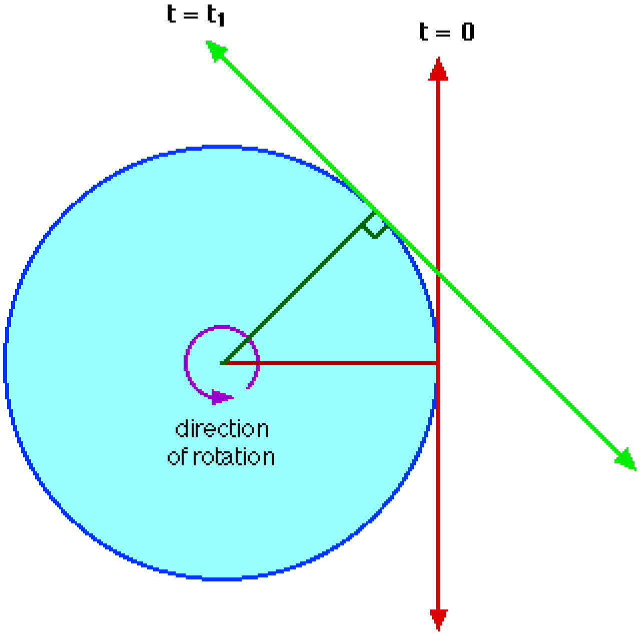

Figure 2: Change in the direction of "East" in a rotating system.

However, when an object moves north or south and is not firmly connected to the ground (air, artillery fire, etc), then it maintains its initial eastward speed as it moves. This is just an application of Newton's First Law. An object moving east continues going east at that speed (both direction and magnitude remain the same) until something exerts a force on it to change its velocity. Objects launched to the north from the equator retain the eastward component of velocity of other objects sitting at the equator. But if they travel far enough away from the equator, they will no longer be going east at the same speed as the ground beneath them.

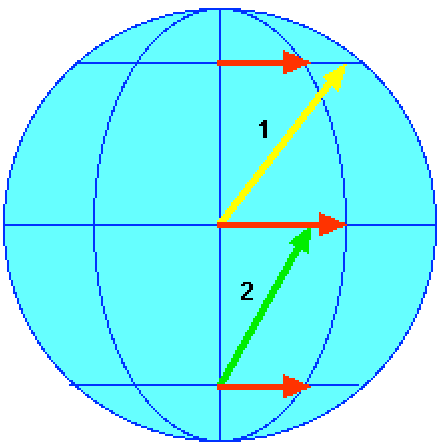

Figure 1: Apparent deflection of an object on a northerly trajectory. Solid arrows represent displacement of objects fixed to the surface, while gray arrows represent displacement during the same time interval of objects given initial northward velocities and not fixed to the surface.

The result is that an object traveling away from the equator will eventually be heading east faster than the ground below it and will seem to be forced east by some mysterious force. Objects traveling towards the equator will eventually be going more slowly than the ground beneath them and will seem to be forced west. In reality there is no actual force involved; the ground is simply moving at a different speed than its original "home ground" speed, which the object retains.

Consider Figure 1. Yellow arrow 1 represents an object sent north from the equator. By the time it reaches the labeled northern latitude, it has traveled farther east than a similar point on the ground at that latitude has, since it kept the eastward speed it had when it left the equator. Similarly, green arrow 2 started south of the equator at a slower eastward speed, and doesn't go as far east as the ground at the equator...seeming to deflect west from the point of view of the ground.

3.2. Well, It Used To Be East: East/West

Motion

In explaining how the Coriolis force affects objects moving to the east or west, it helps to turn off gravity for a moment. Don't worry, we'll turn it back on later, just be sure to put the lid back on your coffee.

Consider being on a rotating sphere with no gravity. An observer who is glued to the sphere throws a ball straight to the "east" on the globe, in the direction of rotation. Since there are no forces on the ball, it will travel in a straight line, the tangent line shown in Figure 2 at t=0.

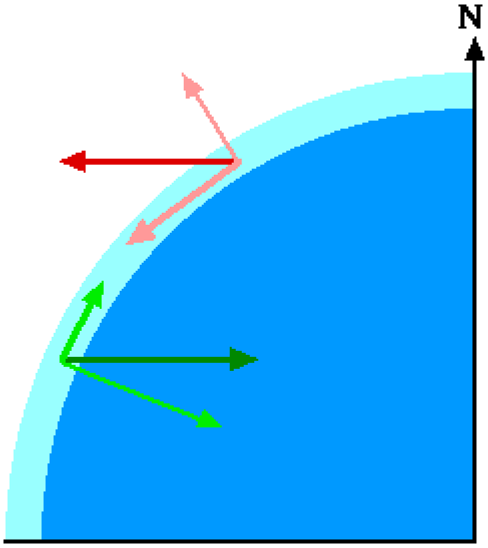

Figure 3: Coriolis deflection broken into radial and tangential (North/South) components. The view is of a cross-section of the Earth, with the thick black arrows pointing either towards or away from the axis.

Time passes, and the ball continues on its straight line. But the observer is attached to the globe and moves around to a new position. At this new position, the observer's definition of the "east" direction has changed, and is no longer the same as it was at time t=0. The ball is no longer traveling on the observer's "east" line, and, in fact, seems to have drifted off to one side. If the globe is spinning slowly enough that the observer can't feel the spin, then the natural conclusion would be that some mysterious force pushed the ball off course, sending it drifting away from the axis of rotation more quickly than it would go if it were still heading the "correct" easterly direction.

Similarly, if the observer throws a ball to the west at time t=0, it will seem to have been forced inward towards the axis of rotation because the "west" line has moved.

Now to turn gravity back on. Gravity pulls objects towards the center of mass of the Earth, which means it cannot change an object's velocity in the directions perpendicular to up and down. In other words, it won't change the east/west or north/south components of an object's velocity.

Figure 3 shows a slice through the Earth so that east points out of the page. The thick arrows show the directions that eastbound and westbound projectiles would seem to go as a result of the Coriolis force in the absence of gravity. The eastbound (red) projectile would seem to drift away from the axis, while the westbound (green) projectile would seem to drift towards the axis. Both of these lines have been split into components, with one component being "up/down" and the other being "north/south." Gravity will act against any "up" components, and the presence of the ground will act against any "down" components, so projectiles will stay within the light blue "atmosphere."

As a result of gravity pulling down on objects and the ground holding them up, the remaining effect of the Coriolis force on objects heading east or west is to deflect them to the north or south. In the northern hemisphere, objects heading east are deflected to the south, for example. The Coriolis force "pushes" them away from the axis, and gravity pulls the object back down to the ground so that the remaining effect is an apparent "push" to the south.

4.0. Putting It Together: Low Pressure Systems

Now we've explained how things moving towards the poles curve to the east, things moving away from the poles curve to the west, things moving east curve towards the equator and things moving west curve towards the poles. In other words, air (or anything else) moving freely in the northern hemisphere deflect to the right, air moving freely in the southern hemisphere deflect to the left. And this is what the result of the vector cross products in the Coriolis force equation says as well, in its mathematical shorthand.

What does this mean for, say, weather systems? Take, for example, a low pressure center, where there's less air than in the area around it. If there's less air in one place than in the surroundings, air will try to move in to balance things out.

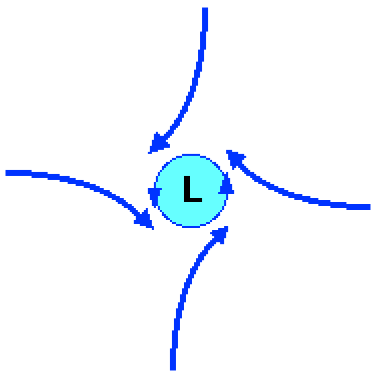

Figure 4: Vortex created in a low-pressure system.

Air starting at rest with respect to the ground will move towards a low pressure center. Such motion in the Northern Hemisphere will deflect to its right, as shown in Figure 4. However, the forces which got the air moving towards the low pressure center in the first place are still around, and the result will be a vortex of air spinning counter-clockwise. Air will try to turn to the right, the low pressure system will try to draw the air into itself, and the result is that air is held into a circle that actually turns to the left. Without the Coriolis force, fluid rushing in towards a point could still form a vortex, but the direction would either be random or depend solely on the initial conditions of the fluid.

The eye of a hurricane is a clear example of fast winds bent into a tight circle, moving so fast that they can't be "pulled in" to the center. The very low pressure at the center of the hurricane means that there is a strong force pulling air towards the center, but the high speed of the wind gives it enough Coriolis force that the forces reach a kind of balance. The net force on air at the eye wall is a centripetal force large enough to keep the air out at a given radius determined by its speed.

5.0. Other Results and Non-Results

"Fine," you may say, "that explains storms. But what about water going down the sink?" In fact, this question is a good "hook" for getting students interested in the Coriolis force in the first place.

Because the Earth's angular velocity is so small (360 degrees per day, or about 2×10^{-5} radians per second), the Coriolis force isn't really significant over small distances (As equation 1 shows, high velocity also can make a difference, but for the purposes of this paper small distance-high speed effects will not be considered). So, what things are likely to be affected by the Coriolis force in a large way?

5.1. Up In The Air

Just looking at a weather system on the nightly news gives one example that has already been addressed. Large weather systems feature masses of air and moisture that travel hundreds of miles and can have wind speeds reaching over a hundred miles an hour in the worst storms.

Another example of a quickly moving object in the sky which covers hundreds of miles is an airplane. All pilots need to have familiarity with the effects of the Coriolis force, since airplanes can reach speeds much higher than even the fastest hurricane winds. Over the course of a several hour

trip, an airplane could be deflected by a significant amount if the pilot didn't compensate for the Coriolis force.
Thirdly, in a more military vein, artillery shells and missiles fired over the horizon can miss by hundreds of meters if the Coriolis force is not taken into account. If you have a large rotating turntable available, you might have students fire spring-launched "shells" across the turntable and show how they're deflected.
So, fast things moving over great distances can be significantly affected by the Coriolis force. But what about the sink?

5.2. Water Going The Wrong Way Down The Sink

In a kitchen sink, of course, speeds and time scales are much smaller than hours and miles. Water rushing down a drain flows at speeds on the order of a meter per second in most sinks, which are themselves less than a meter wide. Qualitatively, there doesn't seem to be much chance for deflection. Quantitatively, putting these numbers into Equation 1 results in an estimated change in rotation of only a fraction of a degree per second, and a very small fraction at that...less than an arc-second (1/3600th of a degree) per second over the course of the entire draining of the sink, ignoring additional effects caused by conservation of angular momentum and the like. Under extremely controlled conditions, this can cause water to flow out of a container counter-clockwise in the northern hemisphere and clockwise in the southern hemisphere, but your kitchen sink is not so controlled. Things like leftover spin from filling the sink (even when the water looks still, it's rotating slowly for a long time after it seems to stop), irregularities in the construction of the basin, convection currents if the water is warmer or colder than the basin, and so forth, can affect the direction water goes down the sink. Any one of these factors is usually more than enough to overwhelm the small contribution of the Coriolis effect in your kitchen sink or bathtub. Research in the 1960s showed that if you do carefully eliminate these factors, the Coriolis force can be observed[1,2].

Water in the sink doesn't go far enough to trigger a noticeable north/south deflection. Most often, it simply spirals down the sink the way it went into the sink, and the same is true of things like the famous "demonstration" of the Coriolis force shown at tourist traps along the Equator. Maybe there's a conspiracy to manufacture right-handed sinks in the Northern Hemisphere and left-handed sinks in the Southern Hemisphere? In any case, don't blame it on the Coriolis force unless your sink is the size of a small ocean.

Acknowledgements

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Notes:

1. Shapiro, 1962, Bath Tub Vortex, Nature, v 196, pp 1080-81 (Northern Hemisphere)
2. Trefethen, et.al., 1965, The Bath Tub Vortex in the Southern Hemisphere, Nature, v 207, pp 1084-85