ENSC 861 – Source Coding in Digital Communications
Trellis Coded Quantization

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Outline

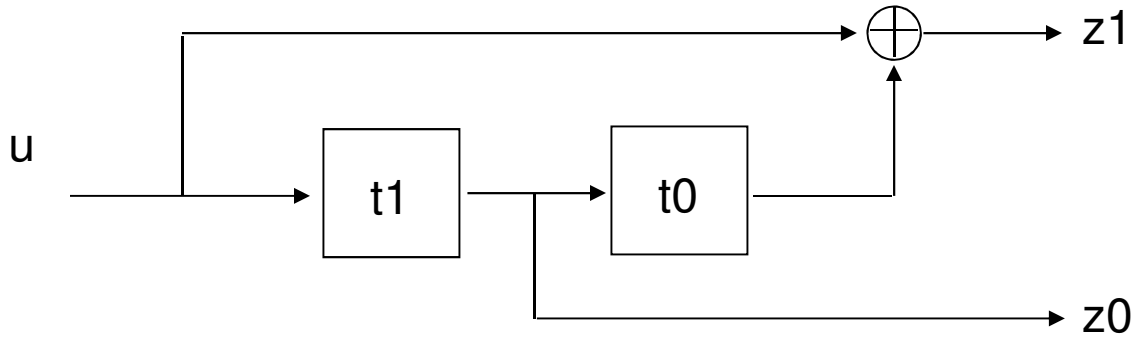
- General TCQ theory
- TCQ in JPEG 2000

Trellis Coded Quantization (TCQ)

- Computationally efficient form of vector quantization
- Theoretical advantage:
  - Entropy coded TCQ can achieve within 0.2dB of the D(R) function at all rates for any smooth pdf.
- Perceptual advantage is often higher
- Based on Trellis Coded Modulation (TCM)
- TCQ is used in JPEG 2000 Part II

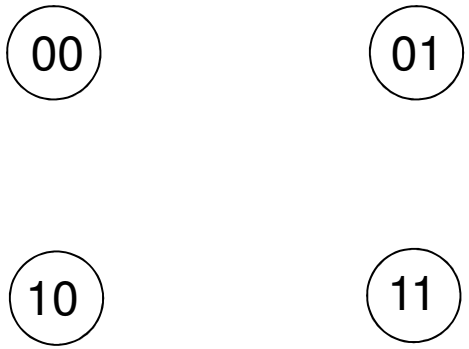
Trellis Coding

- A trellis is the evolution of a finite state machine over time.
- Example: a 4-state finite state machine with binary I/O



- t0 and t1 are shift registers:
  - The state is denoted as t1 t0 ➔ 4 states in this example.

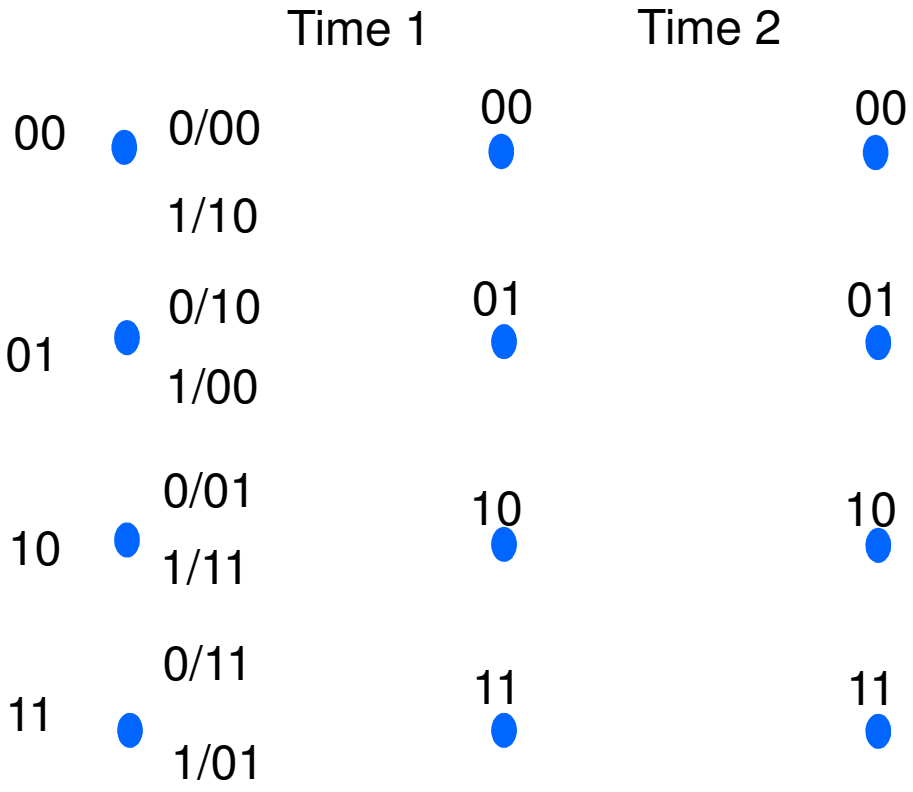
State Transition Diagram
(Finite State Machine)



Notation: u / z1 z0
(input / output)

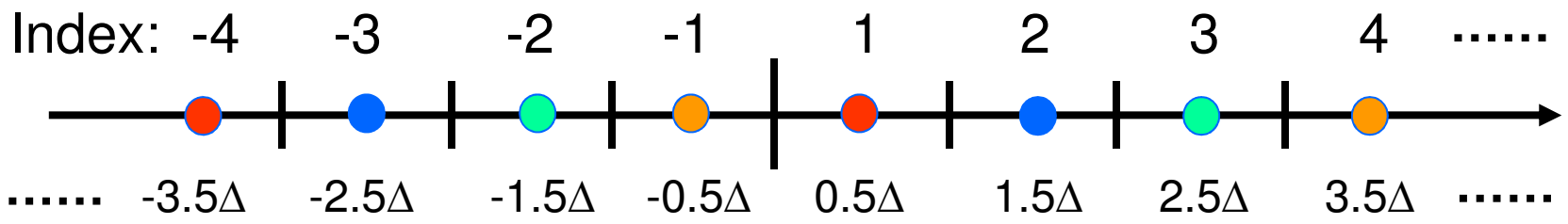
Trellis Coding

- Redraw the state transition diagram to incorporate time:



Trellis Coded Quantization (TCQ)

- To use TCQ:

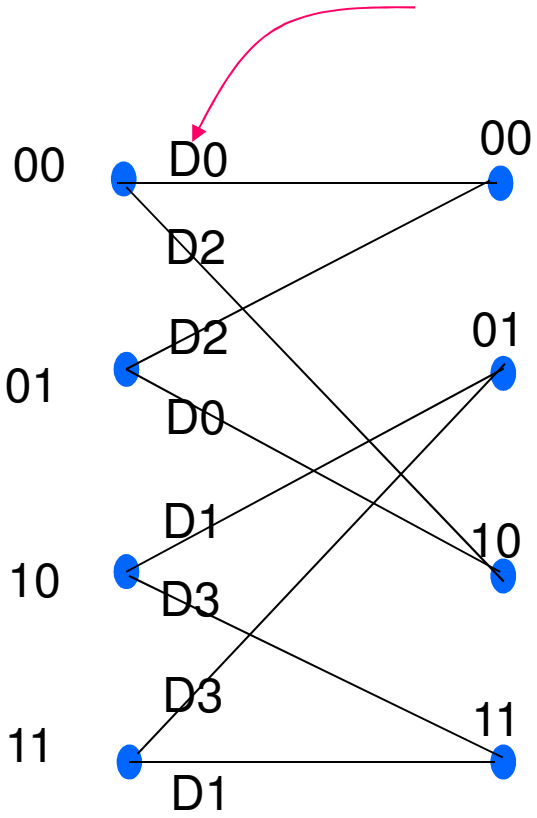


- To get quantization ratio of R bits / symbol, first create a (R+1) bits quantizer:
  - Partition all recon. levels into 4 subsets: D0, D1, D2, D3:
    - From left to right, assign to D0, D1, D2, D3, D0, D1, D2, D3, …
    - (The left-most codeword is assigned to D0)
  - Each subset has 2^{R-1} codewords, needs R-1 bits.

Trellis Coded Quantization (TCQ)

- Associate the 4 subsets to different branches of the trellis:
  - Note: the two branches leading to each state use different subsets.
- Each stage of the trellis quantizes one input symbol.

Di: The current input symbol will only be quantized by subset Di if we choose this branch ➔ R-1 bits.



Output of each stage of the trellis: R bits

$$B \underbrace{xx \ldots\ldots x}_{R-1 \text{ bits}}$$

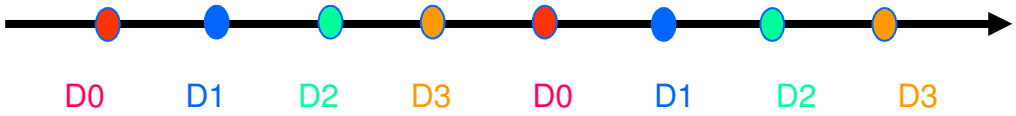
B:

0: use the first output branch of the starting state.

1: use the second branch of the starting state.

xxx ….x:

R-1 bit index of the best codewords of the allowed subset to encode the current symbol.

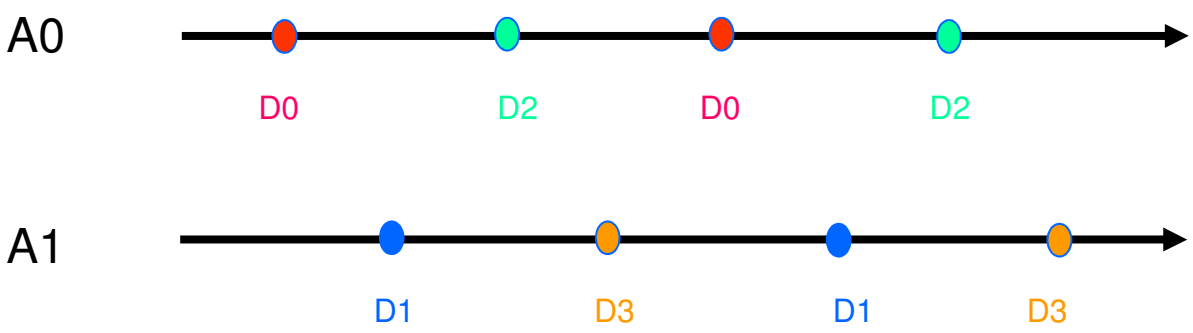


Trellis Coded Quantization (TCQ)

- Union codebooks:
  - Each state can use two subsets (in two branches):

$$A_0 = D_0 \cup D_2.$$

$$A_1 = D_1 \cup D_3.$$

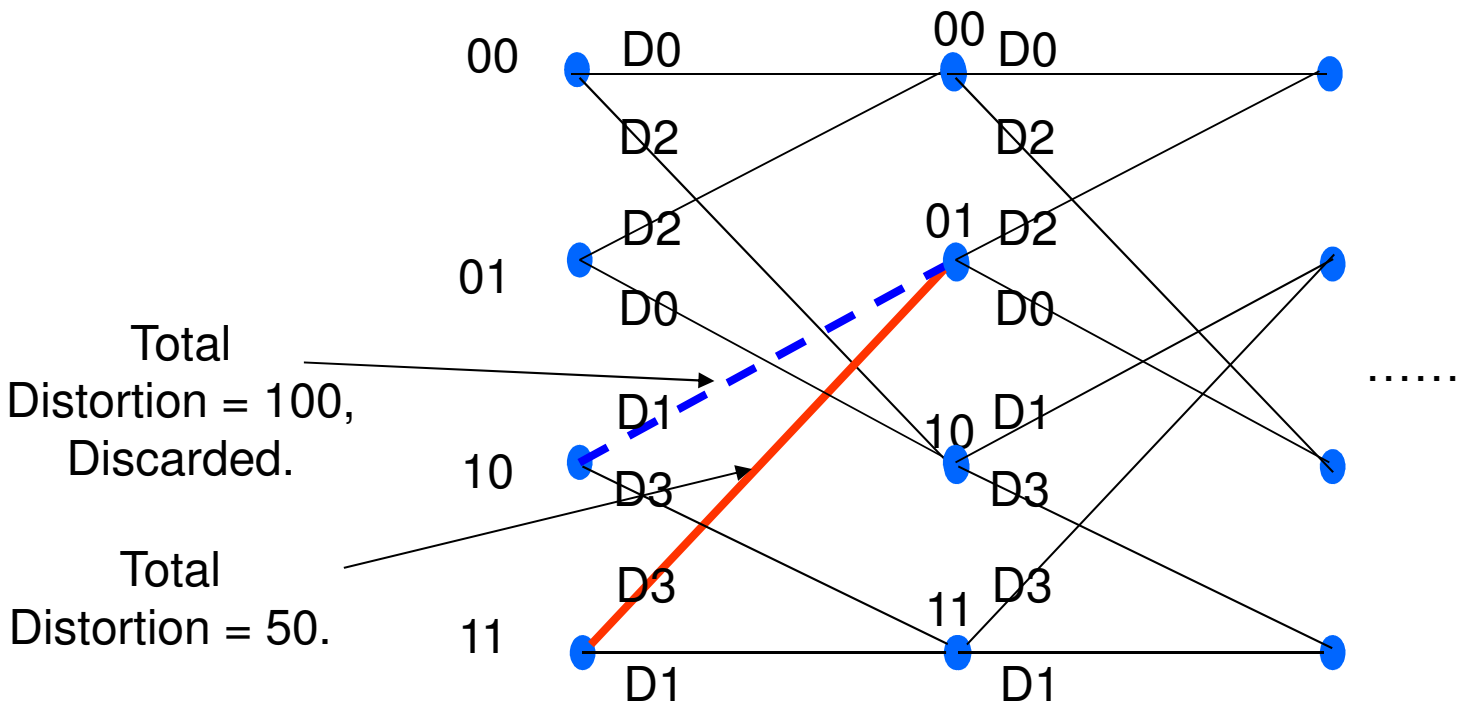


Trellis Coded Quantization (TCQ)

- Goal:
  - Try different quantizer choices for each sample.
  - Find a path through the trellis that minimizes the total distortion.
- Why TCQ is better?
  - The optimal quantization choice of each sample can be determined after the encoding of a sequence of samples.
  - Each individual choice may not be optimal, but the joint choice can achieve the minimal distortion:
    - TCQ is a vector quantization.
  - Two subsets for each state ➔ 2 x 2^{R-1} = 2^{R} possible choices for each state.
  - A path through an m-stage trellis can have 2^{mR} possibilities.
  - Viterbi algorithm can be used to reduce the search complexity, if symbol-by-symbol distortion is used:

$$d(\mathbf{x},\hat{\mathbf{x}}) = \sum_{i=1}^{m} d(x_i,\hat{x}_i).$$

The Viterbi Algorithm



Whenever two paths merge at a state, we can discard the one with higher distortion.

➔ Only need to keep track of N paths at any time. (N: # of states)

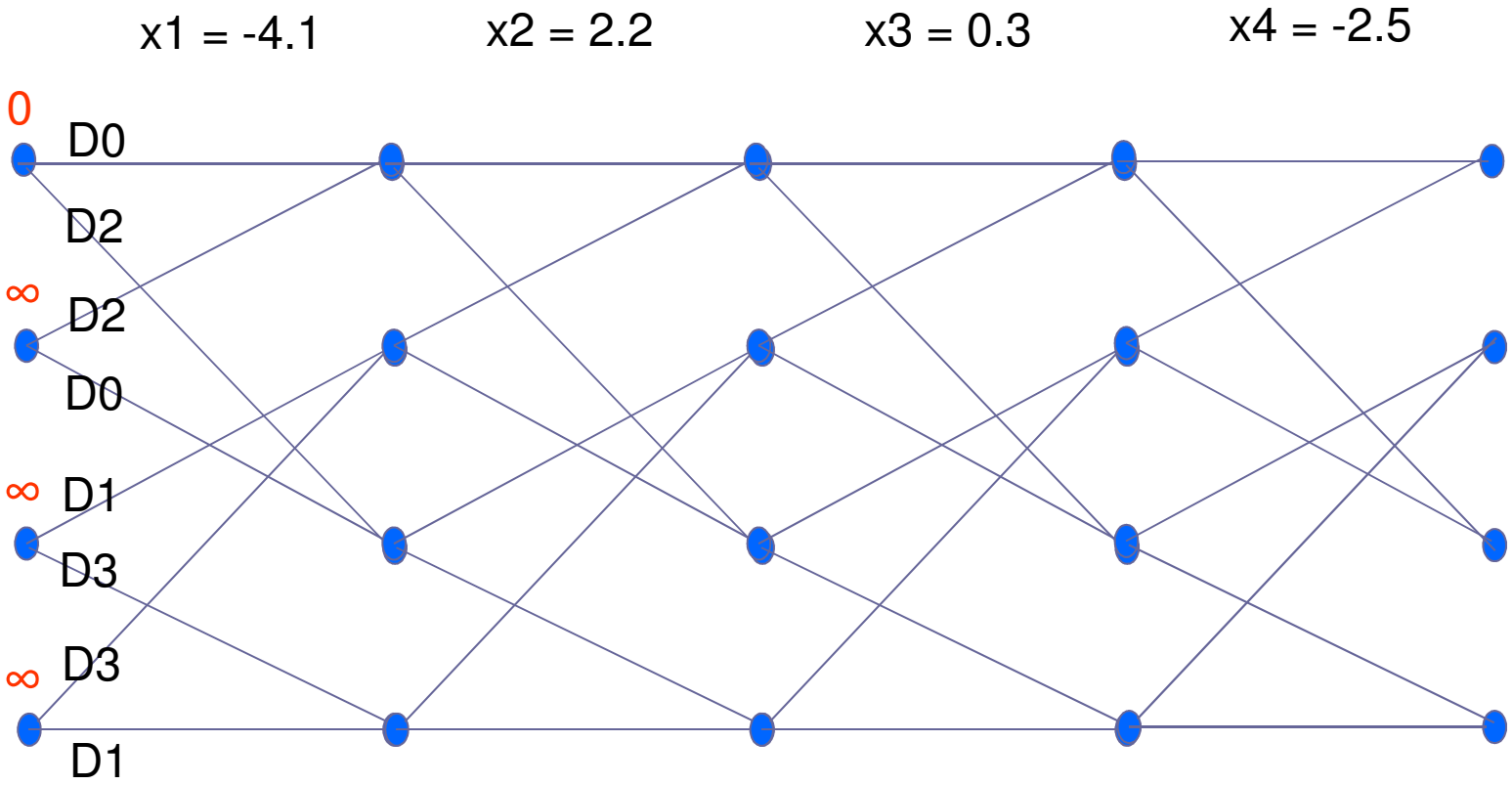
The final sequence can be obtained by tracing back after final decision.

Trellis Coded Quantization (TCQ)

- Example: xi = -4.1, 2.2, 0.3, -2.5.
- Codebook: -7, -5, -3, -1, 1, 3, 5, 7 ➔ R = 2 bits/sample
  D0: (-7, 1), D1: (-5, 3), D2: (-3, 5), D3: (-1, 7).
- Distortion: $|x - \hat{x}|$
- If D0 & D2 is used directly:
  - Codebook: -7, -3, 1, 5
  - -4.1 →
  - 2.2 →
  - 0.3 →
  - -2.5 →
- Total distortion:
- If D1 & D3 is used directly:
  - Codebook: -5, -1, 3, 7
  - -4.1 →
  - 2.2 →
  - 0.3 →
  - -2.5 →
- Total distortion: 4.5.

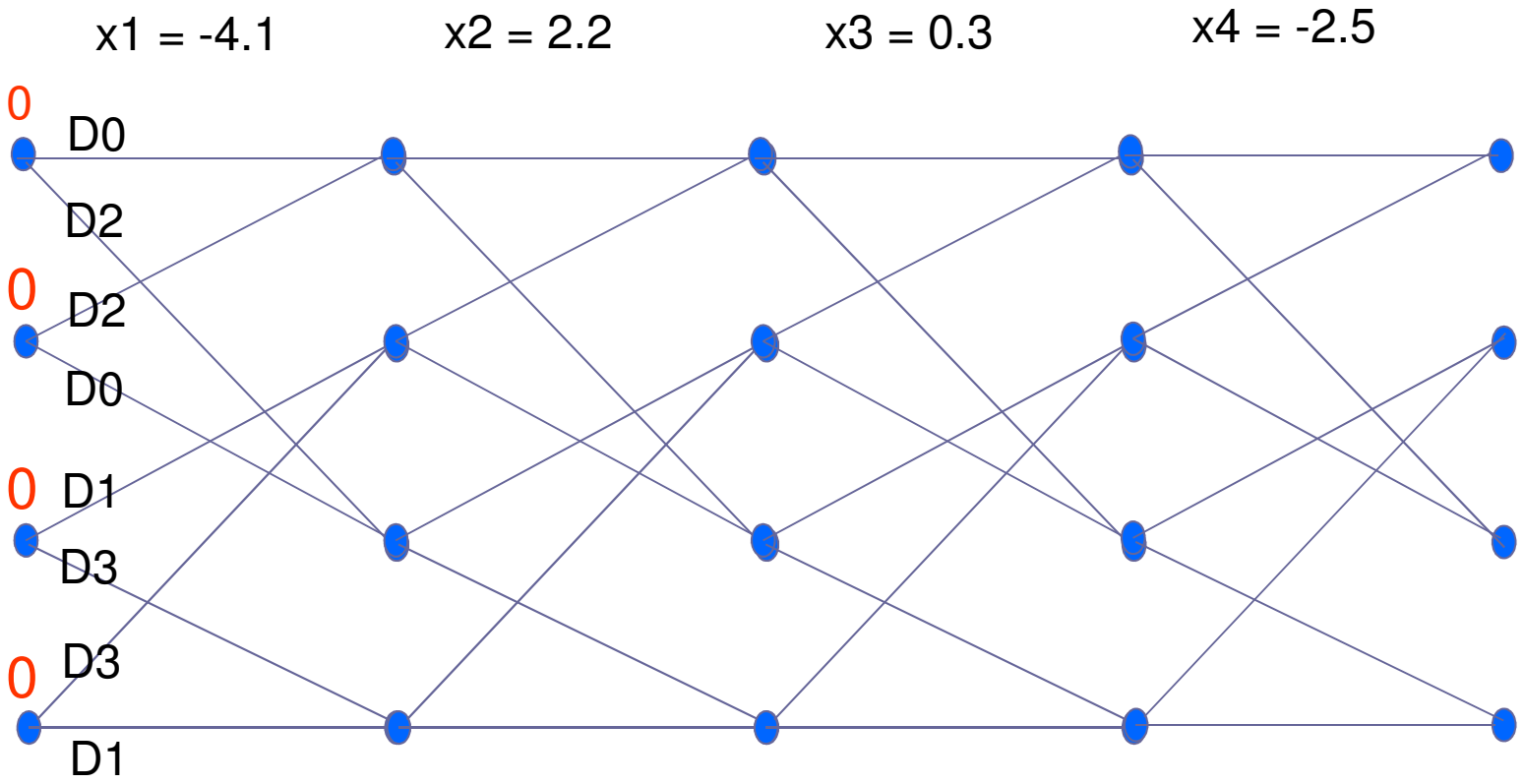
Trellis Coded Quantization (TCQ)

- Example: xi = -4.1, 2.2, 0.3, -2.5.
- Codewords: D0: (-7, 1), D1: (-5, 3), D2: (-3, 5), D3: (-1, 7).
- Distortion: $|x - \hat{x}|$
- Initialization: start from S_0, cost: (0, ∞, ∞, ∞).



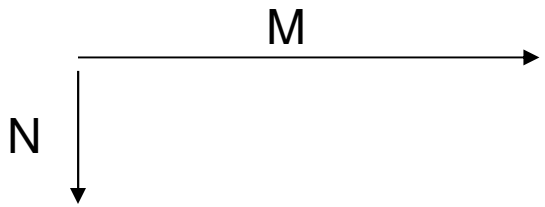
Trellis Coded Quantization (TCQ)

- The previous result is not optimal since we start from state 0. (The effect becomes negligible for long sequence).
- Alternatively, we can allow any starting state, but need to send the starting state to the decoder:



More flexibilities than the two scalar quantizers.

Memory Requirement of TCQ

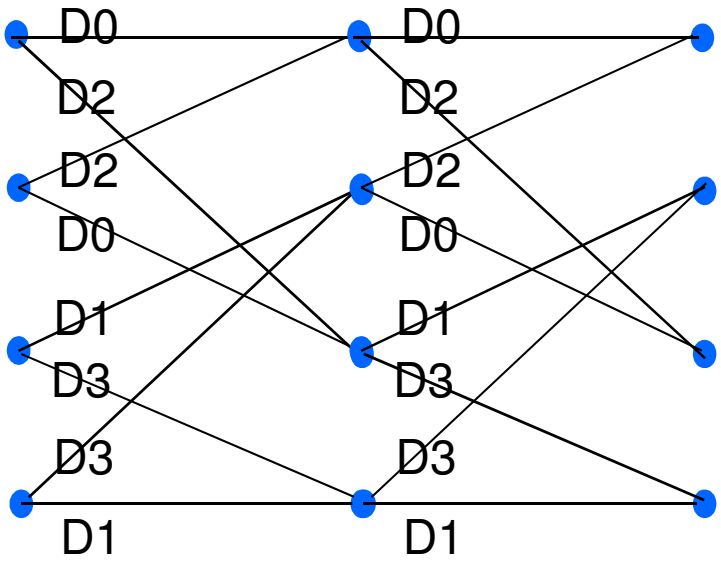


- N: number of states
- M: number of input symbols
- Each stage:
  - Perform 4 scalar quantizers with D0, D1, D2, D3.
- Memory requirement is proportional to MN:
  - Need to store all trellis stages for tracing back purpose.

- Memory can be reduced by limiting the maximal delay to be L ($L > 5 \log_2 N$ is enough):
  - Only keep the past L samples in the buffer.
  - Output the decision for x(n-L).
  - Discard "inconsistent" survivor paths at current stage:
    - Those do not merge with the best path when traced back.

TCQ is VQ

Consider two steps starting from State 0:

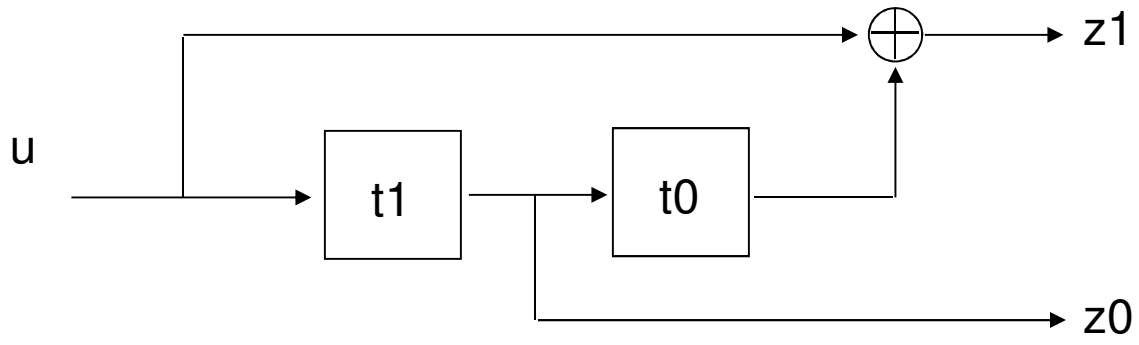


Codebook: -7, -5, -3, -1, 1, 3, 5, 7.
D0: (-7, 1), D1: (-5, 3),
D2: (-3, 5), D3: (-1, 7).

2-D Voronoi region is close to hexagon !
➔ TCQ is a low complexity vector Q.
TCQ can be within 0.2dB to the R-D bound.

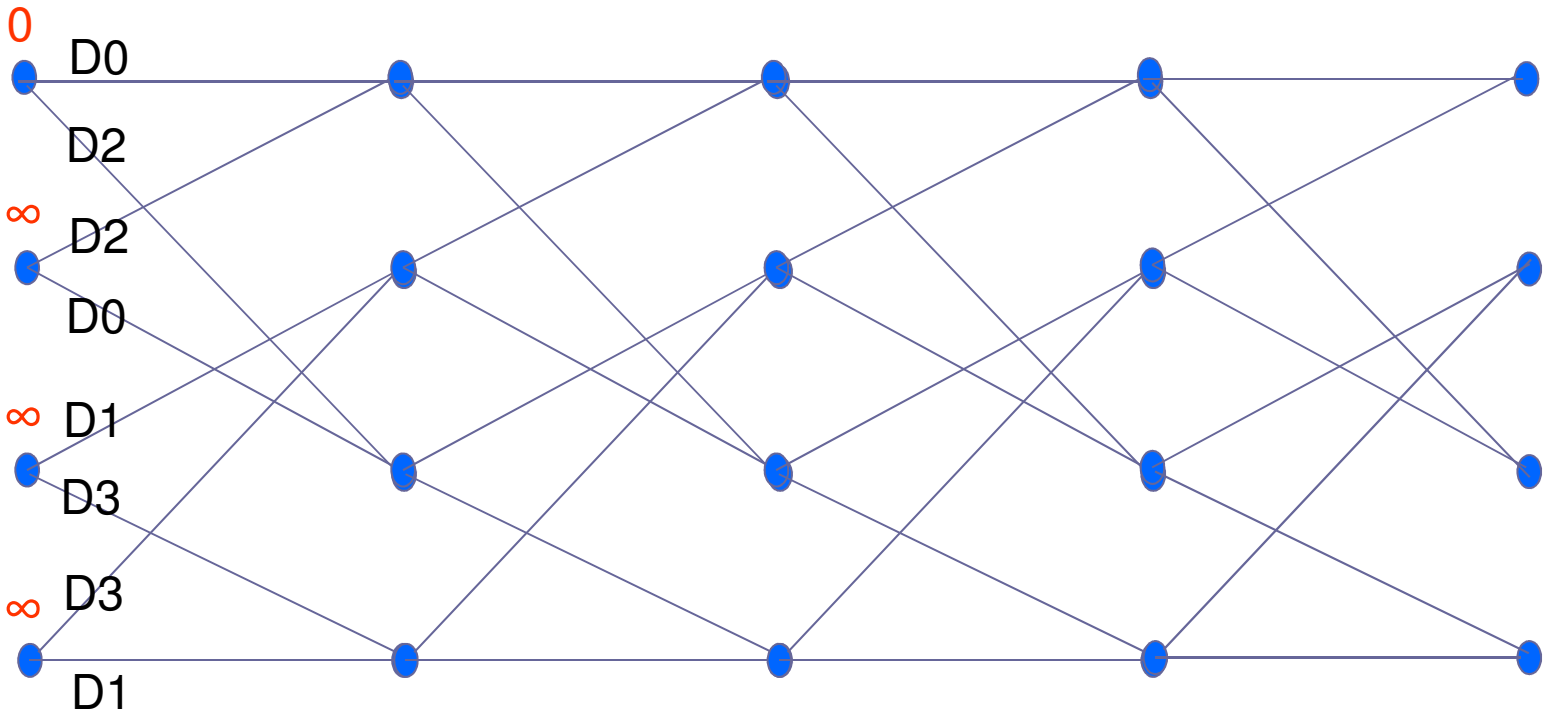
Error Propagation in TCQ

- Error propagation is not as serious as it appears in TCQ.
- If a bit error happens in the last R-1 bits in each stage, only one reconstruction point is affected:
  - Trellis path is still decoded correctly.
- If a bit error happens in the first bit in each stage:
  - Only ($\log_2 N + 1$) samples will be affected.
  - Because the state transition machine have no feedback.



Error Propagation in TCQ

- Example:
- Correct decoder input bits: 10 01 11 10
- Wrong decoder input bits: 00 01 11 10
- ➔ Only three decoding errors!



Outline

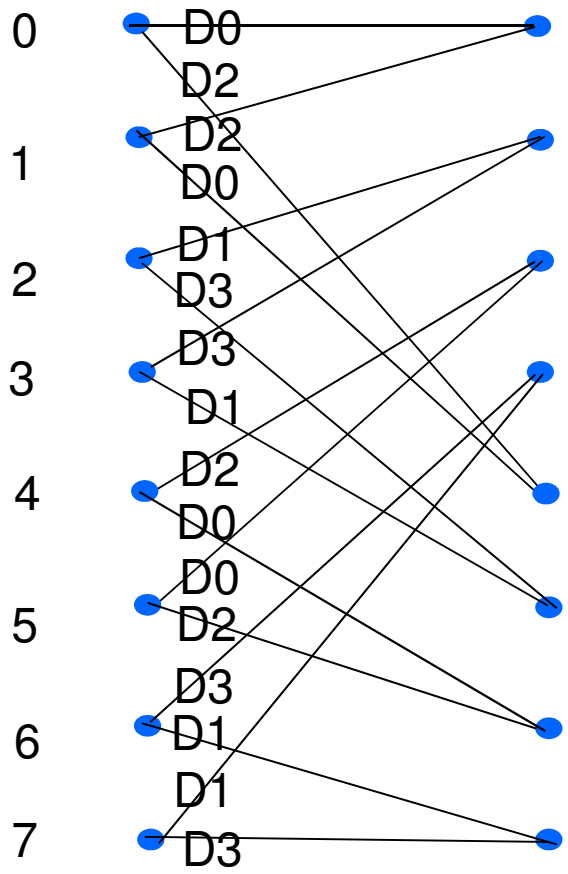
- General TCQ theory
- TCQ in JPEG 2000

TCQ in JPEG 2000

- Defined in JPEG 2000 Part II (not baseline)
- TCQ is applied after wavelet transform:
  - More small coefficients to encode.
- Entropy coding is applied after TCQ:
  - Performance is only limited by the shape of the Voronoi cell of the TCQ (granular error)
- Embedded decoding is possible.
- Can be decoded by scalar quantizer.

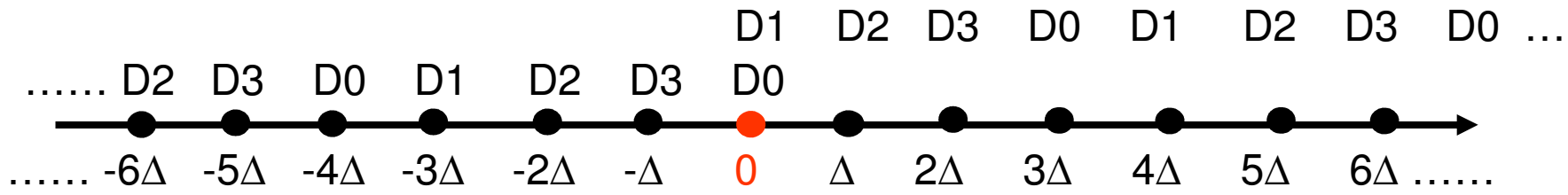
TCQ Trellis in JPEG 2000

- 8-state trellis

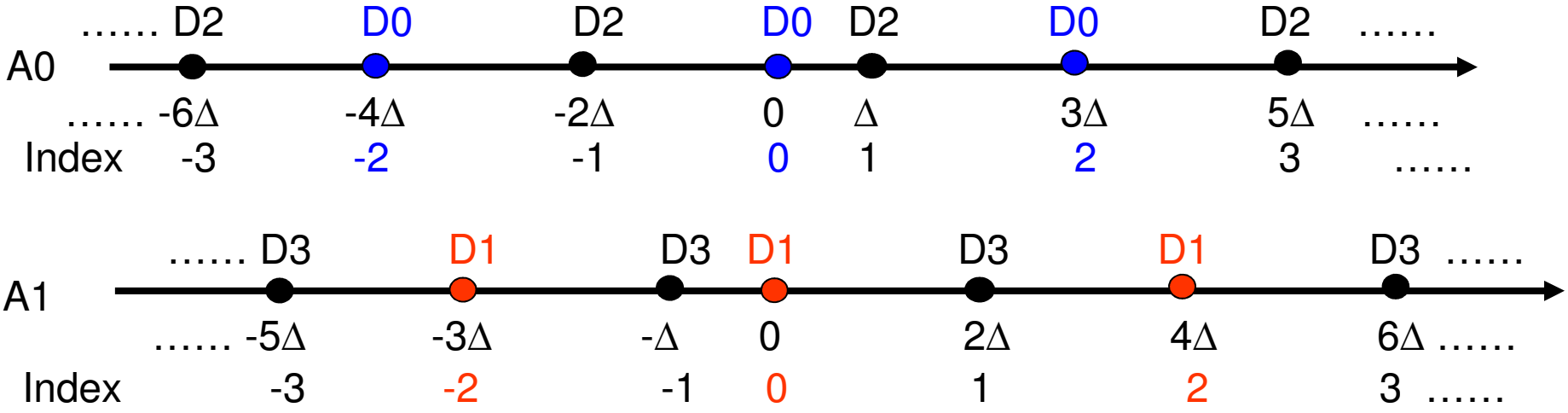


TCQ Codebooks in JPEG 2000

- Subsets (from a uniform midtread quantizer):



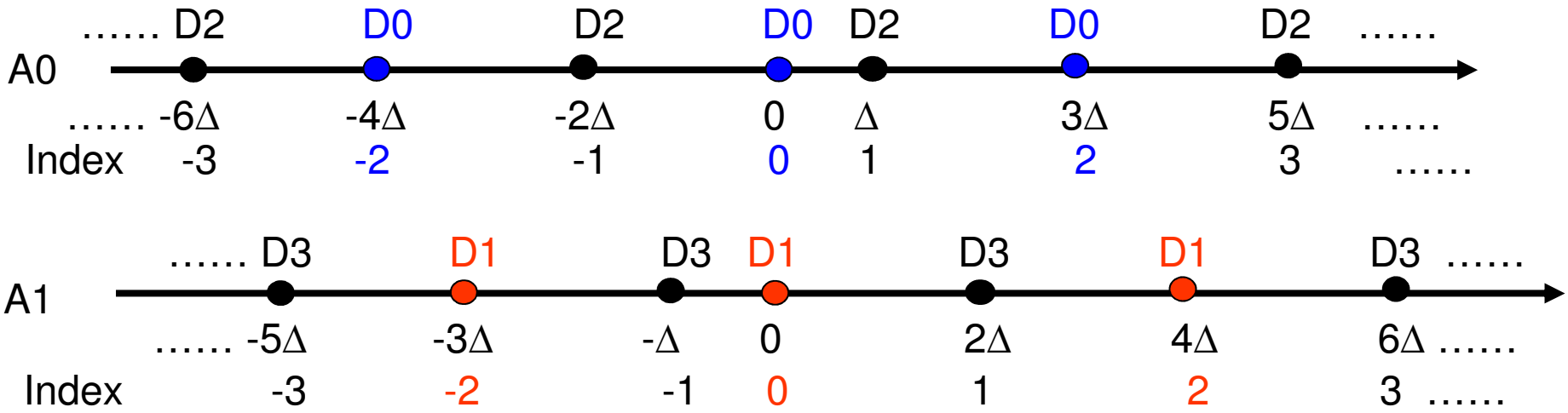
- Note: 0 are included in both D0 and D1.
- Union quantizers: A0: D0 & D2, A1: D1 & D3.



Note: this is not the final indices used in JPEG 2000, see sign flip slide later.

Signaling the Trellis Path

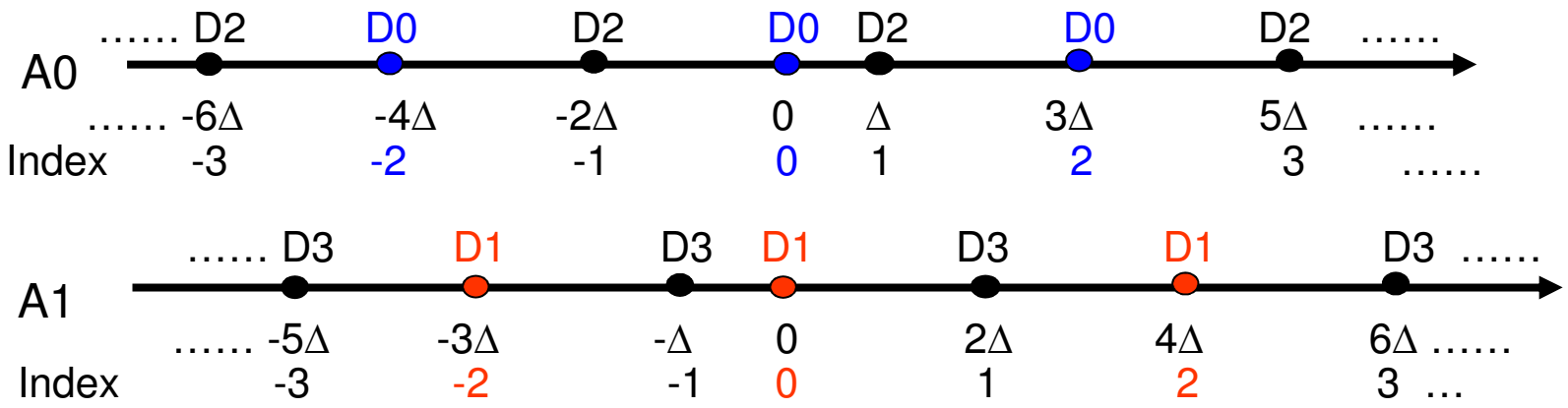
- Union quantizers: A0: D0 & D2, A1: D1 & D3.



- Observation: Indices of the two subsets within each union quantizer differs only in the LSB (Not true in previous def.)
- The index itself is sufficient to signal the trellis path:
  - No need to send 1 bit at the beginning of each stage to specify the branch:
  - ➔ Equivalent to send the 1 bit at the end of each stage.
  - Allow embedded coding.
  - Allow decoding by regular scalar quantizer.

Embedded TCQ

- The index assignment in JPEG 2000 allows embedded coding



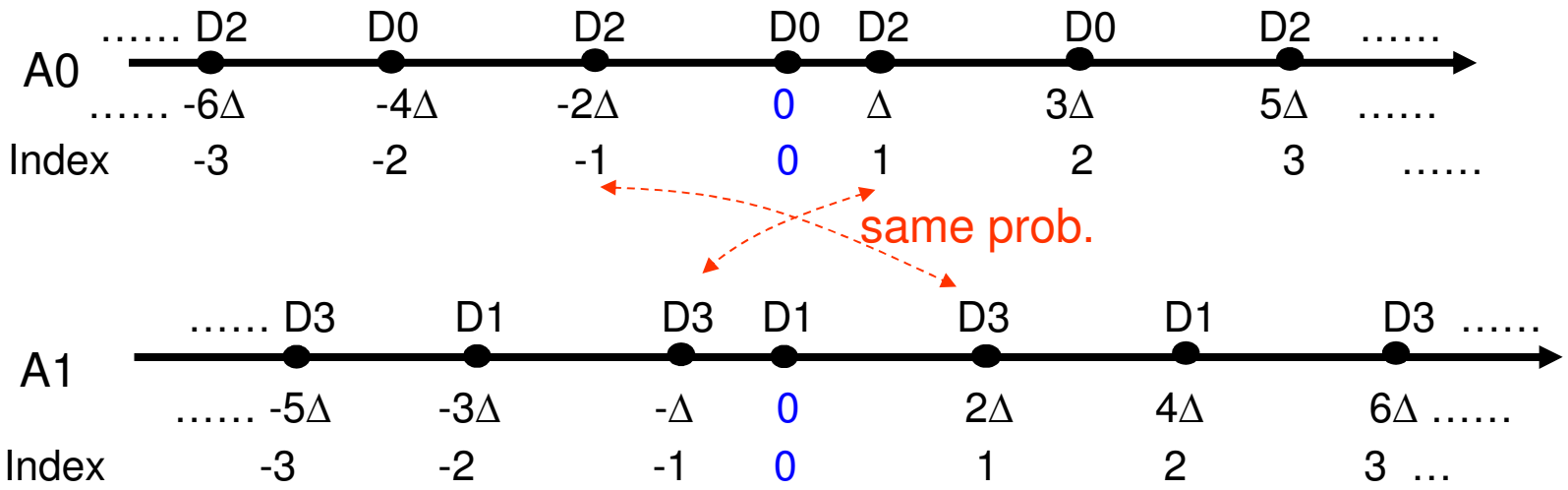
- If the LSB is not available:
  - Cannot differentiate the two subsets within each union quantizer.
  - ➔Cannot reconstruct the trellis path.
  - But there are only 4 possible choices.
- Example: received index is +1x

Embedded TCQ

- In general: If the last p LSB bits are missing
  1. Set the missing bits to 0 to get an index q_0.
  2. Choose the reconstruction level to be

$$\hat{x} = 2q_0\Delta.$$

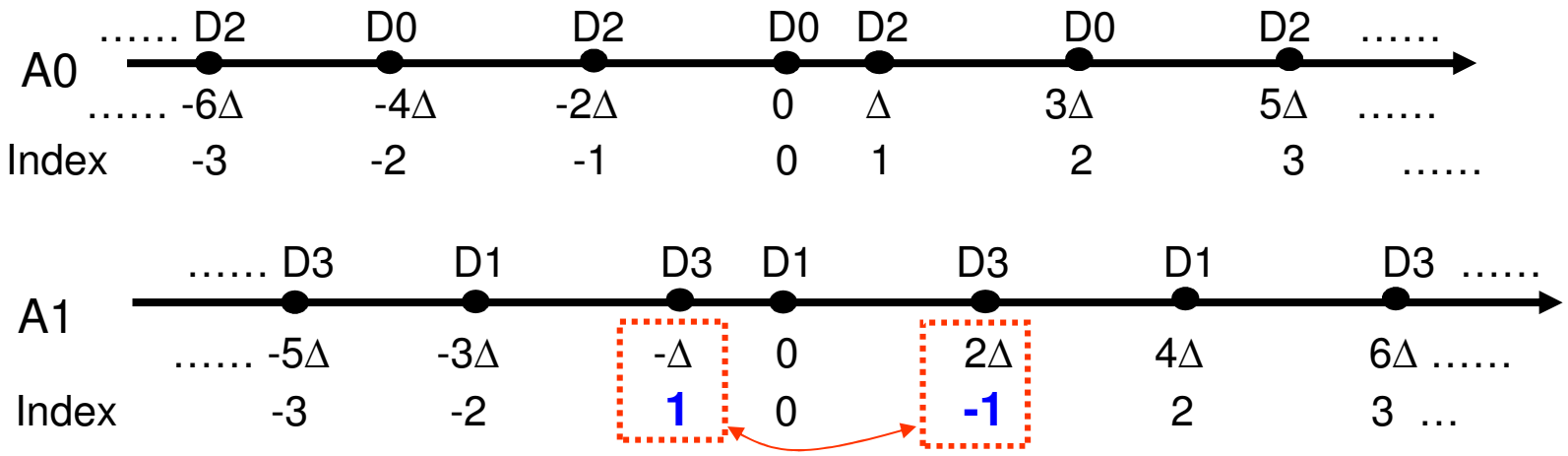
Index Sign Flip for Entropy Coding



- Observation: Index k in A0 has same prob as index -k in A1.
  - ➔ Switching the indices in A0 or A1 can simplify context modeling in entropy coding, because there is no need to consider trellis state
- But switch all indices prevent embedded decoding:
  - Example: received index is +1x

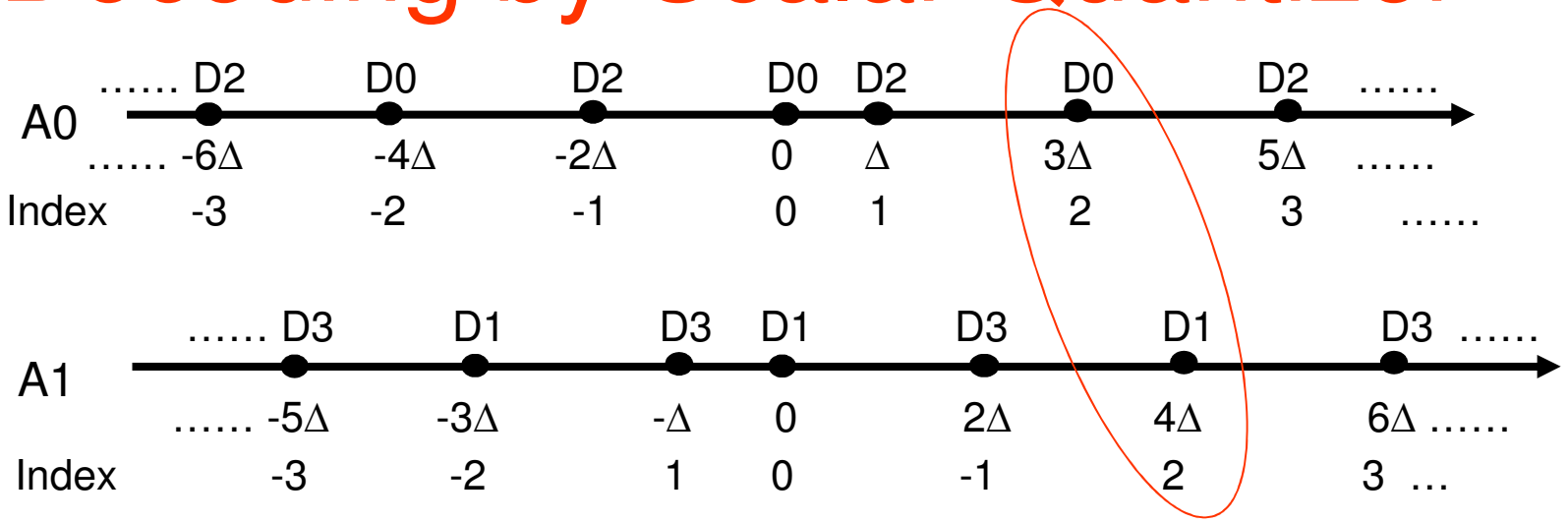
Index Sign Flip for Entropy Coding

- The final indices used in JPEG 2000:



- In JPEG 2000, only 1 and -1 in A1 are switched
  - Trellis state is not considered in entropy coding
  - pdf mismatch is not too much, because prob of 0, 1, and -1 are dominant.
- This has no impact on embedding, since 1 and -1 are only sent in the last bit plane. At this point the decoder can reconstruct the trellis path.

Decoding by Scalar Quantizer



- To the decoder, there are two possible codewords for each index generated by the TCQ:
  - One from A0 and one from A1.
  - The ambiguity can be resolved by TCQ through trellis.
- If decoded by a scalar quantizer:
  - Equivalent to a scalar quantization with stepsize 2Δ.

Reference

- Taubman JPEG 2000 book: Chap 3.5.
- M. W. Marcellin, M. A. Lepley, A. Bilgin, T. J. Flohr, T. T. Chinen, J. H. Kasner, ``An Overview of Quantization in JPEG-2000," *Signal Processing: Image Communications, Special Issue on JPEG-2000,* Vol.17/1, pp. 73-84, December 2001.

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- Ali Bilgin, Philip J. Sementilli, and Michael W. Marcellin, Progressive Image Coding Using Trellis Coded Quantization, IEEE Trans Image Processing, Vol. 8, No. 11, pp. 1638-1643, Nov. 1999.