Earth-to-Moon Low Energy Transfers Targeting L_1 Hyperbolic Transit Orbit

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OUTLINE

- **Introduction:** State of the Art, Minimum Theoretical Δv
- **Dynamics:** R3BP Equations of Motion and Jacobi Constant
- **Linearization around L_1:** Linear Transit Orbits
- **Nonlinear Transit Orbits:** Moon resonances
- **Design Strategy:** L_1-Moon and L_1- Earth Legs
- **Patched Trajectory:** Earth-to-Moon Transfers
- **Results**
- **Final Remarks**

STATE OF THE ART

Problem: Find Δv-efficient Earth-to-Moon trajectories.

- **WSB:** Belbruno [1987], [1993] exploited the dynamics of the Sun-Earth-Moon system. The Moon is approached from the far side (L_2). Koon et al [2001] explained the capture mechanism by a dynamical system approach.

- $\boldsymbol{L_1}$ **:** Transfers through L_1 defined in the Earth-Moon R3BP.
  - Targeting: Bolt and Meiss [1995], Schroer and Ott [1997], Macau [1998], Ross [2003] using a sequence of small perturbations;
  - Numerical Search: Pernicka et al [1995], Yagasaki [2004], Mengali and Quarta [2005] patching trajectories at a mid-point or solving TPBVPs

$\Rightarrow$ The general idea is to exploit intrinsic features (chaotic dynamics, ballistic capture) of in n-body models in order to lower the cost of the transfers

MINIMUM THEORETICAL Δv

Sweetser [1991], in the frame of the R3BP, quantified the minimum theoretical Δv necessary to link a $h_E = 167\ km$ circular Earth orbit with a $h_M = 100\ km$ circular Moon orbit. The total cost is the sum of two maneuvers:

- $\Delta v_{th,e} = 3099\ m/s$ at Earth departure
- $\Delta v_{th,m} = 627\ m/s$ at Moon arrival
- $\Delta v_{th} = \Delta v_{th,e} + \Delta v_{th,m} = 3726\ m/s$

The two maneuvers must be carried out parallel to the velocity in the synodic frame in order to maximize the variation of the Jacobi constant. Unfortunately, the associated trajectory does not exist since it requires, theoretically, an infinite time to approach L_1 and to depart from it.

DYNAMICS

R3BP equations of motion:

$$\begin{cases} \ddot{X} - 2\dot{Y} & = & \Omega_X \\ \ddot{Y} + 2\dot{X} & = & \Omega_Y \\ \ddot{Z} & = & \Omega_Z \end{cases} \tag{1}$$

with $\Omega(X,Y,Z) = \frac{1}{2}(X^2+Y^2) + \frac{1-\mu}{R_1} + \frac{\mu}{R_2} + \frac{1}{2}\mu(1-\mu)$,

$R_1^2 = (X+\mu)^2 + Y^2 + Z^2$ and $R_2^2 = (X-1+\mu)^2 + Y^2 + Z^2$

Jacobi constant $$C = 2\Omega(X,Y,Z) - (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) \tag{2}$$

Conventions: sum of the Earth and Moon masses, Earth-Moon distance and Earth-Moon angular velocity set to 1 (the orbital period is 2π).

LINEARIZED DYNAMICS AROUND L_1

In a L_1 centered frame and scaled lengths, system 1 can be linearized as:

$$\begin{cases} \ddot{x} - 2\dot{y} - (1 + 2c_2)x & = & 0 \\ \ddot{y} + 2\dot{x} + (c_2 - 1)y & = & 0 \\ \ddot{z} + c_2 z & = & 0 \end{cases} \tag{3}$$

and its solution is:

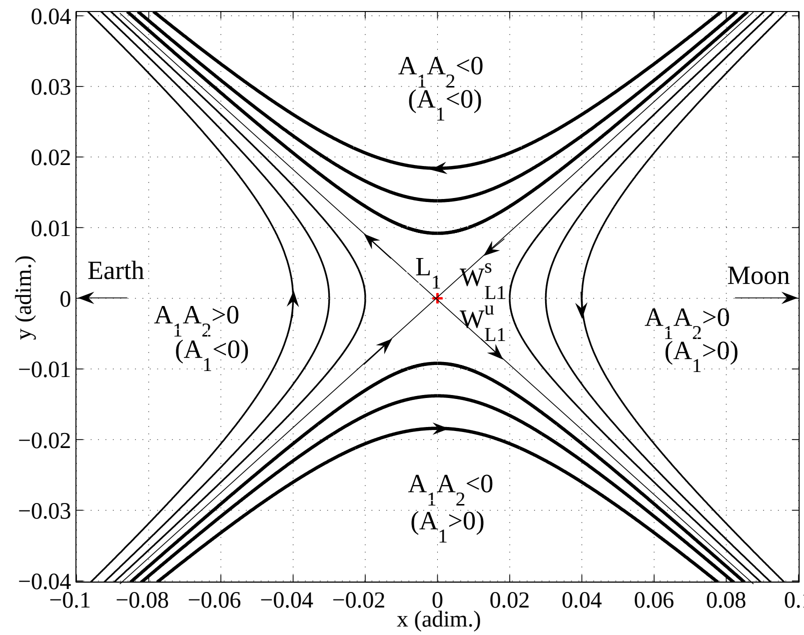
$$\begin{cases} x(t) & = & A_1 e^{\lambda t} + A_2 e^{-\lambda t} + A_3 \cos \omega t + A_4 \sin \omega t \\ y(t) & = & -k_1 A_1 e^{\lambda t} + k_1 A_2 e^{-\lambda t} - k_2 A_3 \sin \omega t + k_2 A_4 \cos \omega t \\ z(t) & = & A_5 \cos \nu t + A_6 \sin \nu t \end{cases}$$

with A_i $(i = 1, ..., 6)$ arbitrary amplitudes; constants depend just on μ.

LINEAR TRANSIT ORBITS

Linear transit orbits can be obtained by selecting non-trivial amplitudes A_1 and A_2 of the hyperbolic part. Conley [1968] showed that *transit* hyperbolic orbit are those orbits with $A_1 A_2 < 0$. Such orbits are the only ones able to link the Earth and Moon neighborhoods and *shadow* L_1 stable and unstable manifolds as A_1 or $A_2 \to 0$. Setting $x(0) = 0$ results in $A_2 = -A_1$ and so just A_1 is used to parameterize the transit orbits.

$$\begin{cases} x(t) = A_1 e^{\lambda t} + A_2 e^{-\lambda t} \\ y(t) = -k_1 A_1 e^{\lambda t} + k_1 A_2 e^{-\lambda t} \\ z(t) = 0 \end{cases}$$



NONLINEAR TRANSIT ORBITS

In analogy with the generation of the manifolds, linear transit orbits are used to supply an initial condition to flow under R3BP dynamics. The whole family of transit orbits is parameterized with the amplitude A_1 being such initial condition $\mathbf{X_0}(A_1) = d\mathbf{x_0}(A_1) + \{l_1, 0, 0, 0, 0, 0\}^T$ with d and l_1 constants.

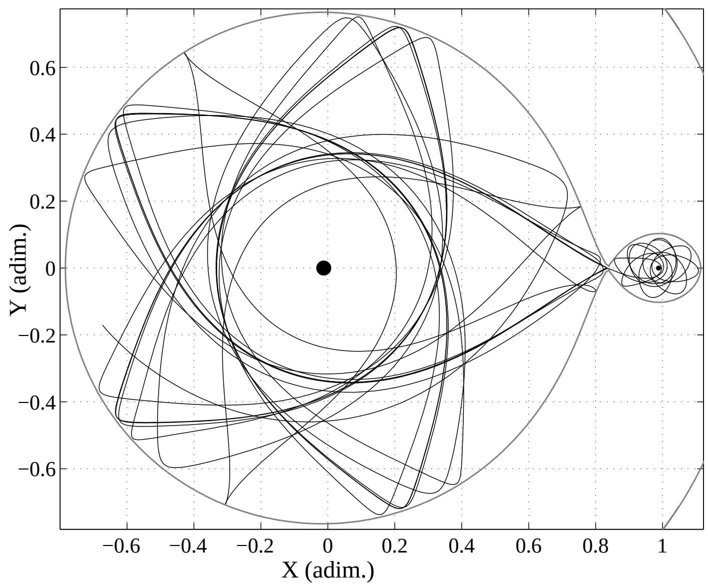


Figure 1: Synodic frame ($A_1 = 0.01$)

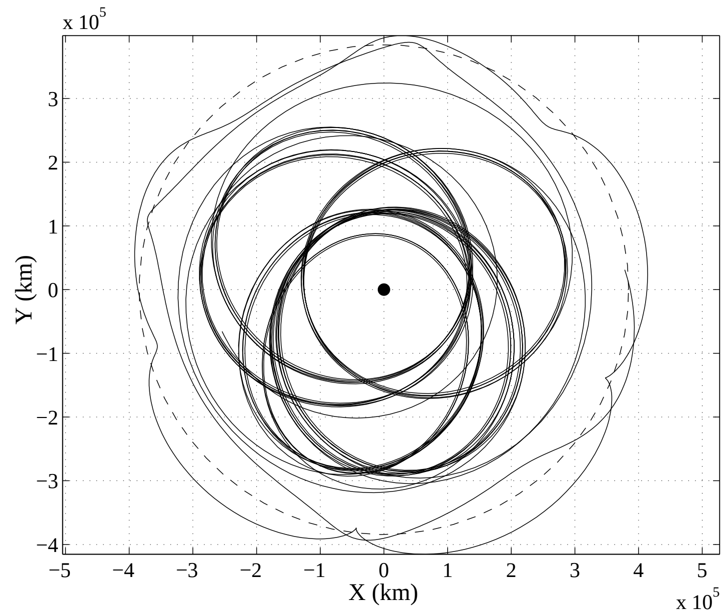


Figure 2: Earth-centered frame

MOON RESONANCES

Transit orbits are close to 5:2 resonant orbits with the Moon. Resonances occur roughly every 55 days when the line of apsides is close to the Earth-Moon line (X-axis in the synodic frame).

- osculating orbital elements swiftly change at each resonance (next slide)
- the s/c is "pumped-up" by the Moon's gravitational attraction

! in a short time (less than 400 days) transit orbits do not approach LEOs

! no direct injection (single burn) exists between LEOs and transit orbits

! no minimum theoretical Δv Earth-Moon transfers exist

! additional maneuvers are required to perform this link

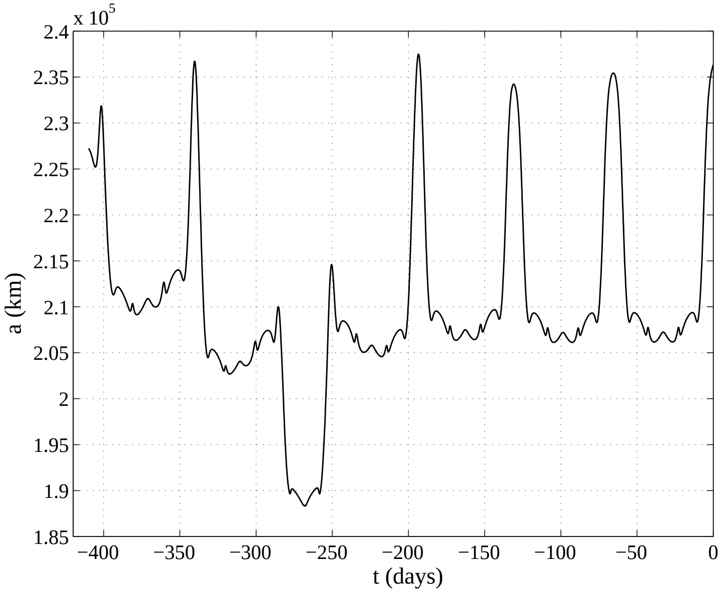


Figure 3: Semi-major axis

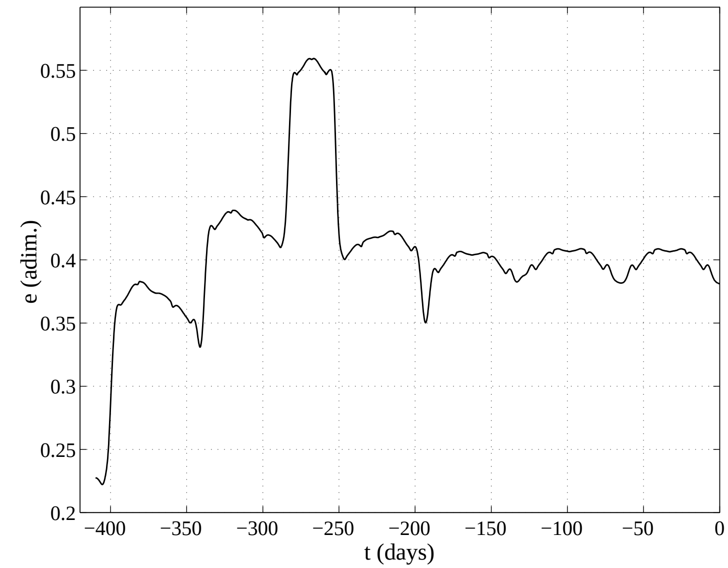


Figure 4: Eccentricity

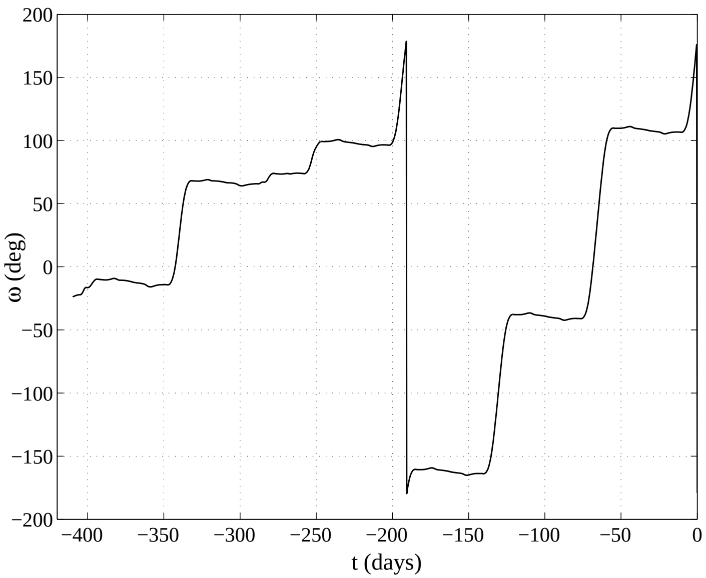


Figure 5: Pericenter anomaly

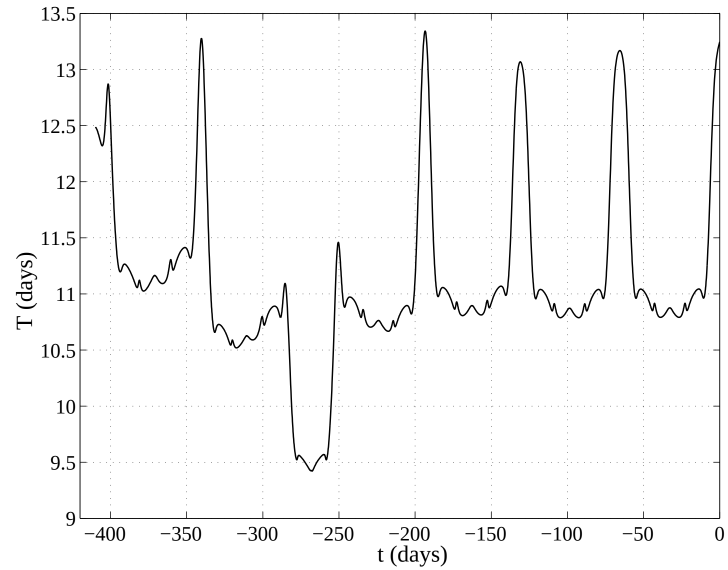
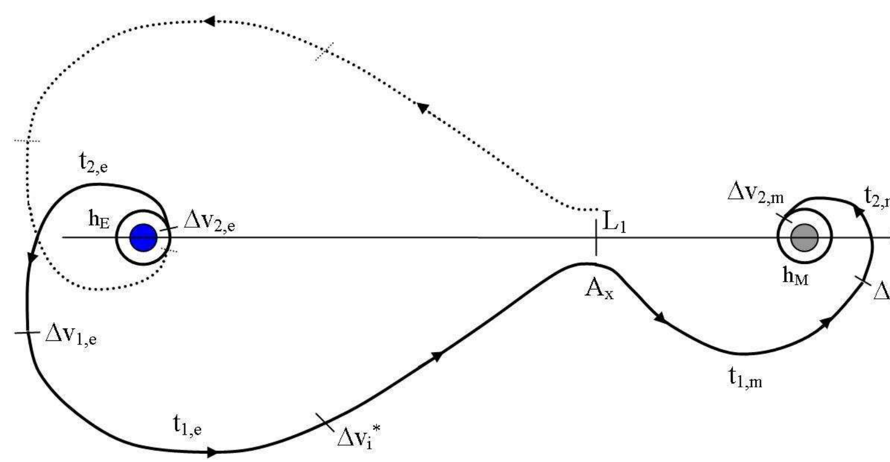


Figure 6: Period

DESIGN STRATEGY

- fix an initial amplitude A_1 associated to the linear transit orbit
- generate the nonlinear transit orbit flowing (fw / bk) $\mathbf{X_0}(A_1)$ under R3BP dynamics
- choose a h_E and h_M Earth and Moon circular orbits
- use a Lambert's three-body arc to perform the link between the transit orbit and the circular orbits

The Lambert's three-body arc is computed solving a TPBVP within R3BP dynamics. It links two given points in a given time.



L_1-Moon Leg

The cost and the time of flight of the L_1-Moon leg are:

$$\begin{aligned}\Delta v_m(t_{1,m}, t_{2,m}, \theta_m) &= \Delta v_{1,m} + \Delta v_{2,m} \\ \Delta t_m &= t_{1,m} + t_{2,m}\end{aligned}$$

- $\Delta v_{1,m}$: performs the passage from the transit orbits to the Lambert's arc
- $\Delta v_{2,m}$: needed to circularize the trajectory around the Moon
- $t_{1,m}$: time on the transit orbit
- $t_{2,m}$: time of the Lambert's arc
- θ_m: anomaly along the h_M circular orbit

L_1-Moon Leg

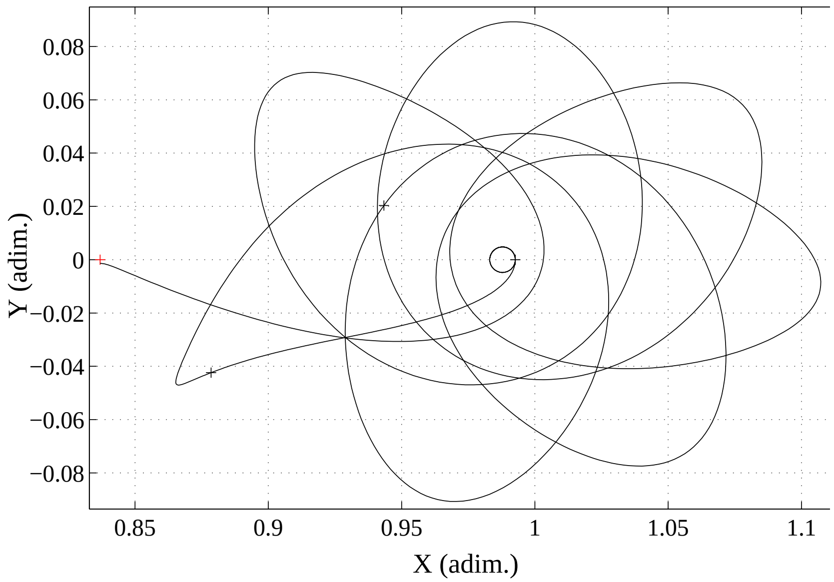


Figure 7: Synodic frame

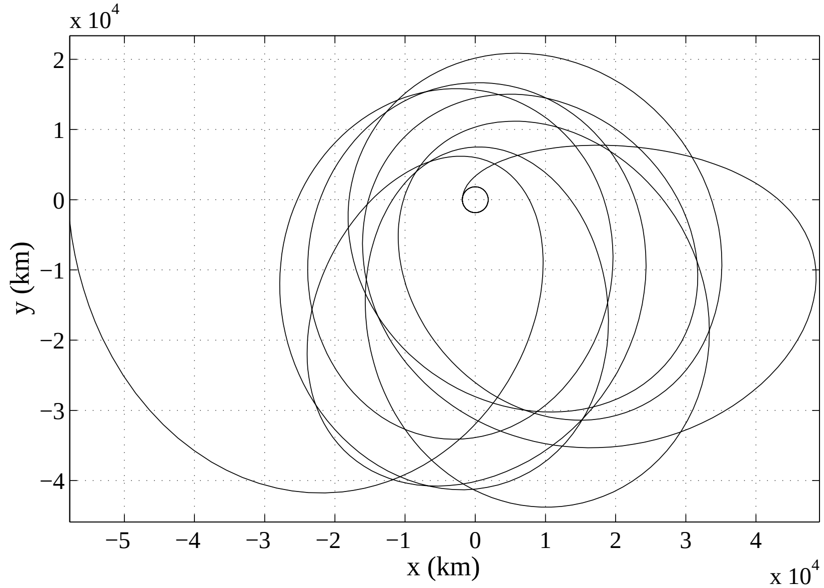


Figure 8: Moon-centered frame

- Sometimes a very-small Δv maneuver (first mark) is used to lower to total cost of the Lambert's arc

L_1-Earth Leg

The cost and the time of flight of the L_1-Earth leg are:

$$
\begin{aligned}
\Delta v_e(t_i^*, \Delta v_i^*, t_{1,e}, t_{2,e}, \theta_e) &= \textstyle\sum_{i=1}^n \Delta v_i^* + \Delta v_{1,e} + \Delta v_{2,e} \\
\Delta t_e &= \textstyle\sum_{i=1}^n t_i^* + t_{1,e} + t_{2,e}
\end{aligned}
$$

- $\Delta v_{1,e}$, $\Delta v_{2,e}$, $t_{1,e}$, $t_{2,e}$, θ_e: same meaning as in the L_1-Moon case
- Δv_i^* and t_i^*: cost and time of the i-th intermediate maneuver
- n: maximum number of allowed maneuvers (usually $n = 4$)

L_1-Earth Leg

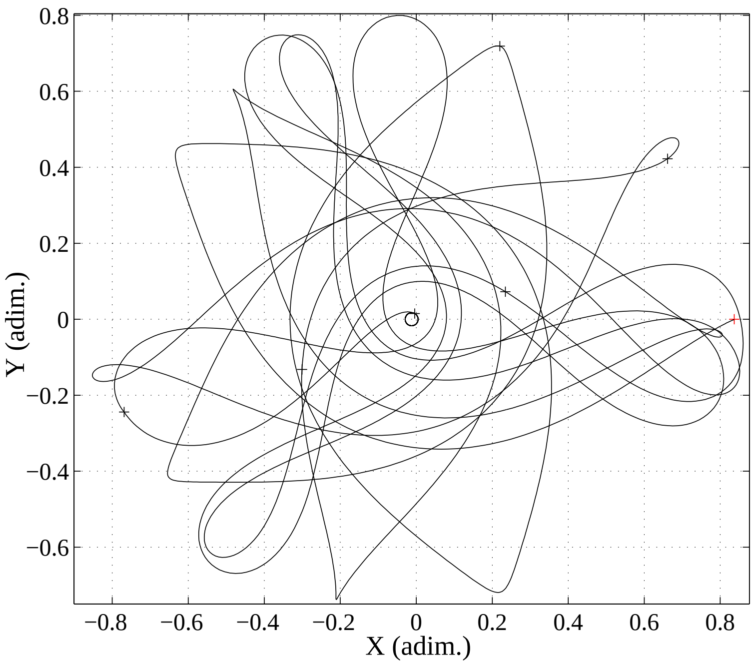


Figure 9: Synodic frame

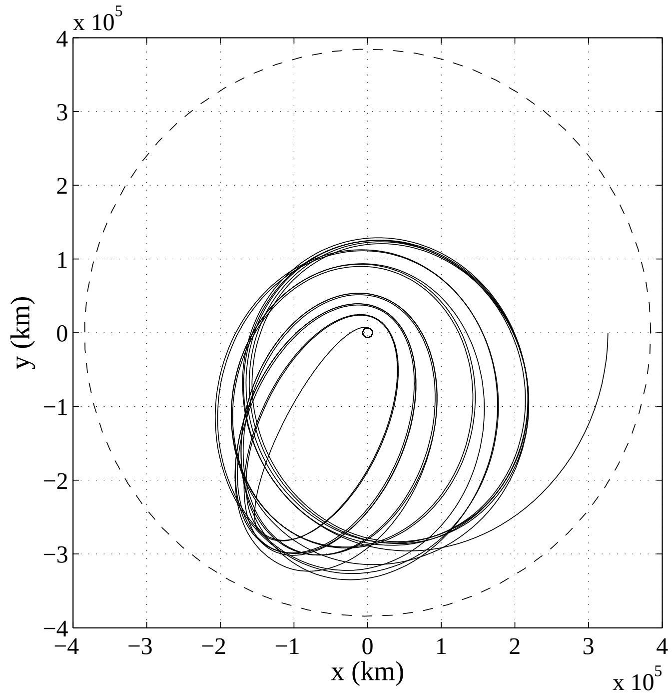


Figure 10: Earth-centered frame

- In this case finding an acceptable solution is much more difficult than in the L_1-Moon case. It is due to multiple revolutions, long times, higher number of variables.

Patched Trajectory

Once the two legs have been designed independently, the whole Earth-to-Moon trajectory can be obtained by patching them together. Just legs parameterized with the same A_1 amplitude can be patched together because only in this case the continuity is assured at the patching point $\mathbf{X_0}(A_1)$. The total cost and the time required to the Earth-Moon transfers are:

$$\begin{aligned} \Delta v &= \Delta v_m + \Delta v_e \\ \Delta t &= \Delta t_m + \Delta t_e \end{aligned} \tag{4}$$

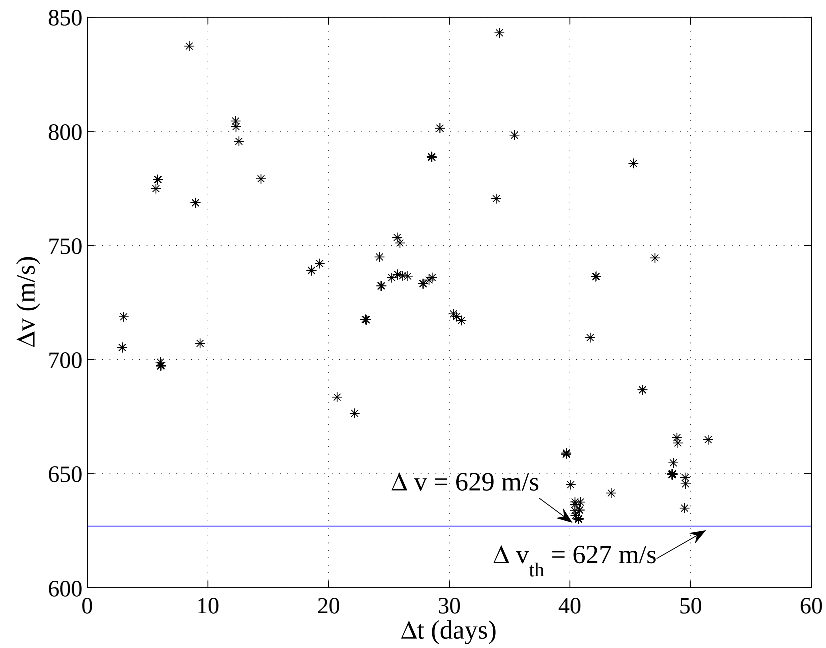
- The whole problem is split into two subproblems having a reduced number of variables. Hence, low energy solutions can be found more easily (especially for the L_1-Moon case)
- Altitudes of the departure and arrival orbits: $h_E = 167\ km$ and $h_M = 100\ km$
- Look for short-medium transfers: $\Delta t \leq 200\ days$

Results: L_1-Moon Leg

The minimum theoretical cost for the L_1-Moon leg is $\Delta v_{th,m} = 627\ m/s$

- solution 1 ($A_1 = 0.01$): $\Delta v_m = 629.9\ m/s$, $\Delta t_m = 40.7\ days$
- solution 2 ($A_1 = 0.1$): $\Delta v_m = 634.9\ m/s$, $\Delta t_m = 49.5\ days$

In the case of L_1-Moon leg the problem, as stated above, reveals very efficient because solutions very close to the minimum have been found. Best solutions are associated to $A_1 = 0.01$.

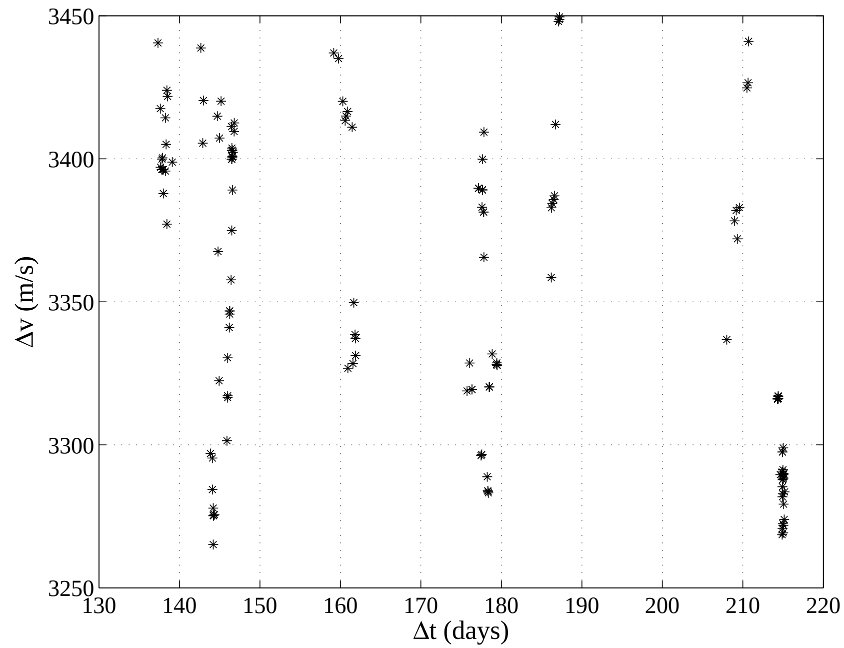


Results: L_1-Earth Leg

The minimum theoretical cost for the L_1-Earth leg is $\Delta v_{th,e} = 3100\ m/s$

- solution 1 ($A_1 = 0.01$): $\Delta v_e = 3301.4\ m/s$, $\Delta t_e = 145.9\ days$
- solution 2 ($A_1 = 0.1$): $\Delta v_e = 3265.1\ m/s$, $\Delta t_e = 144.2\ days$

In the case of L_1-Earth leg solutions close to the minimum have not been found. It is much more difficult wrt the previuos case because of the higher number of variables, long times of flight and difficulties in solving the 2PBVP between the transit and the circular Earth orbits.



Solution 2 ($A_1 = 0.01$)

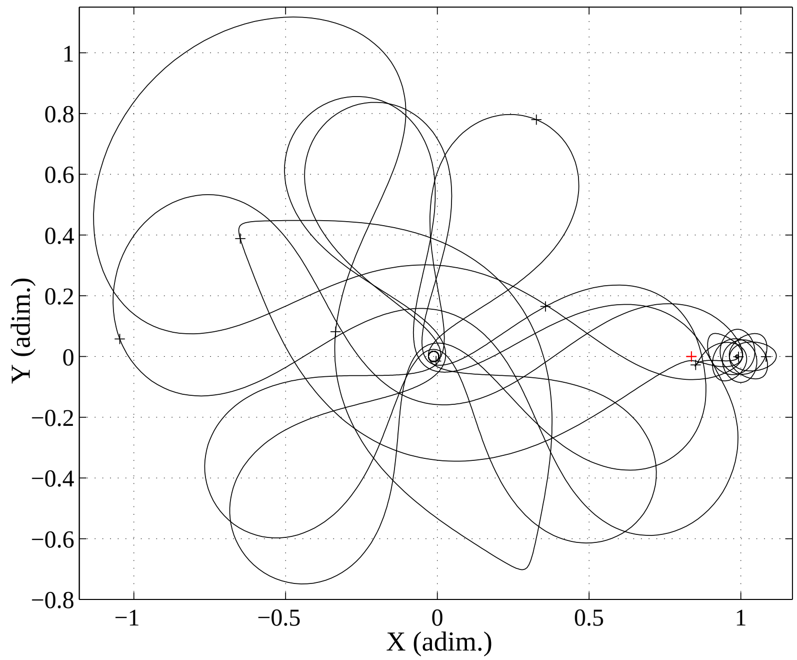


Figure 11: Synodic frame

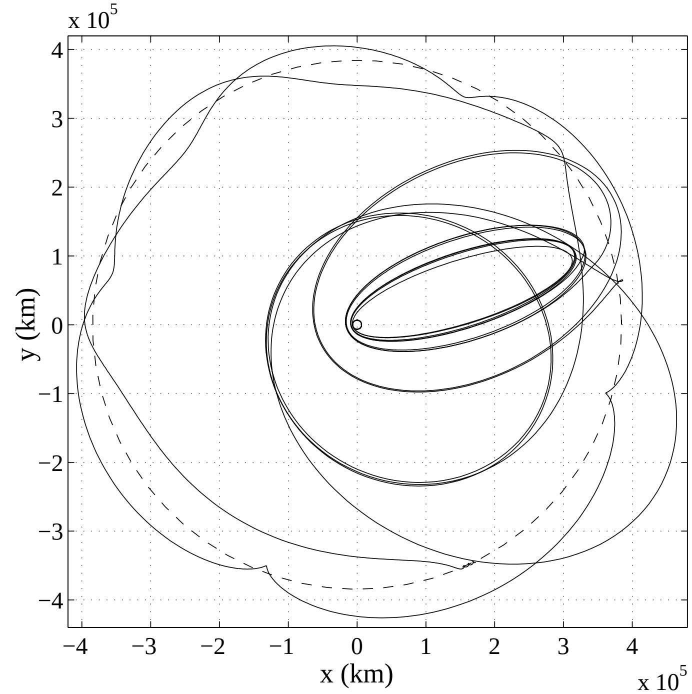


Figure 12: Earth-centered frame

- $\Delta v = 3900.0\ m/s$ and $\Delta t = 193.7\ days$
- several close-Earth passages, 3 Moon swing-by's, L_1 passage and Moon orbit insertion

Solution 1 ($A_1 = 0.1$)

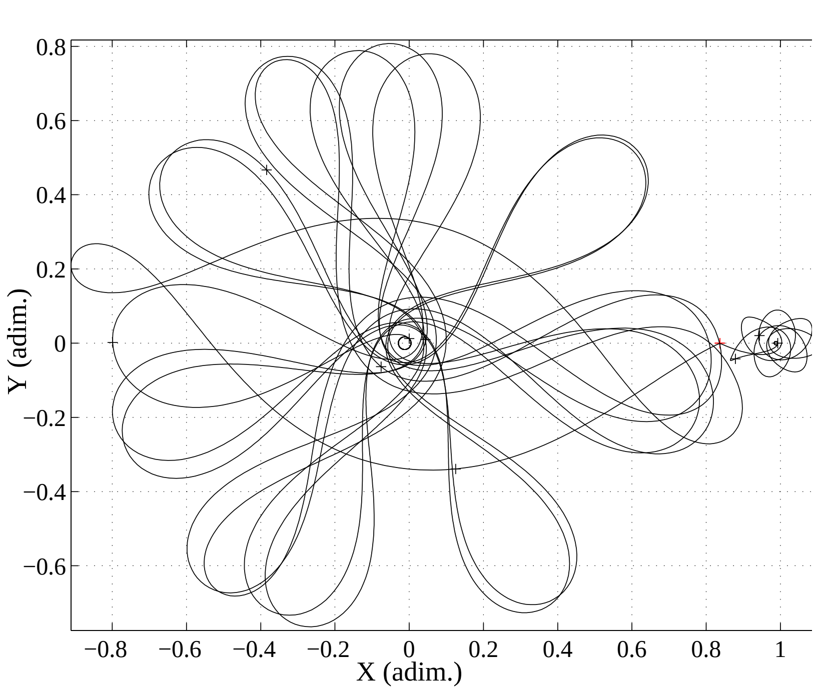


Figure 13: Synodic frame

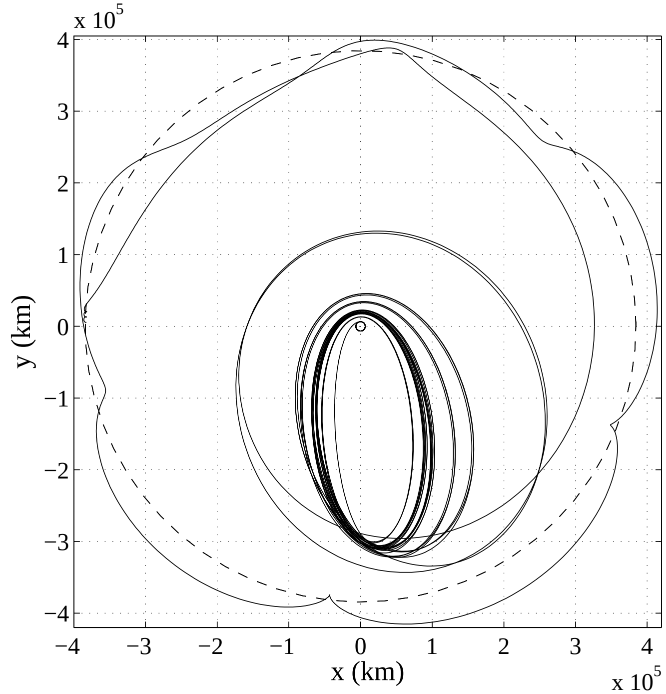


Figure 14: Earth-centered frame

- $\Delta v = 3931.3\ m/s$ and $\Delta t = 186.6\ days$
- spacecraft bounded within the Moon's orbit, Δv_i^* + Moon resonances raise the perigee and apogee until the transit occur

Final Remarks

- dynamical features of L_1 nonlinear transit orbit have been analyzed
- very cheap solutions found for the L_1-Moon leg (close to the theoretical minimum)
- L_1-Earth leg does not seem to be convenient as the L_1-Moon (solutions far from the minimum)
- approx $100\ m/s$ could be saved wrt Hohmann transfer

Future Works

- targeting the L_1 transit orbits with low thrust propulsion
- evaluation of the fourth-body perturbations (Sun) on the designed trajectories