Chapter

7

Properties of 2-D Figures

GOALS

You will be able to

- Determine and apply properties of exterior and interior angles of polygons
- Determine and apply properties of the diagonals and the shapes formed by joining the midpoints of polygons
- Make and test conjectures based on investigations of geometric properties

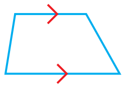
? Examine the quadrilaterals in the picture.
What is the least number of angles you would have to measure to determine all the angles in each quadrilateral?

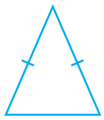
7 Getting Started

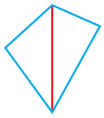
WORDS YOU NEED to Know

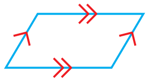
1. Match each word with the picture that best represents it.

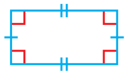
a) parallelogram
b) rhombus
c) trapezoid
d) rectangle
e) diagonal
f) midpoint
g) isosceles triangle
h) equilateral triangle

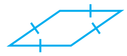
i) 

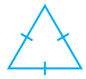
ii) 

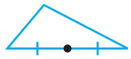
iii) 

iv) 

v) 

vi) 

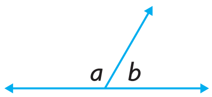
vii) 

viii) 

SKILLS AND CONCEPTS You Need

Straight Angles

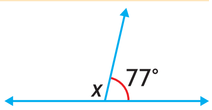
The sum of angles that form a straight angle is 180°.
$\angle a + \angle b = 180°$



Study *Aid*
- For more help and practice, see Appendix A-16.

EXAMPLE

Determine the value of the unknown angle.

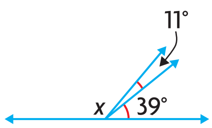


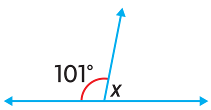
Solution

Since $\angle x$ and 77° form a straight angle, their sum is 180°.

$77° + \angle x = 180°$
$\angle x = 180° - 77°$
$\angle x = 103°$

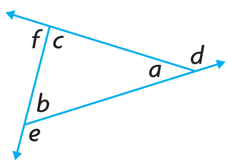
2. Determine each unknown angle.

a) 

b) 

Interior and Exterior Angles of a Triangle

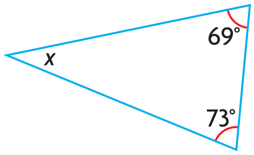
The sum of the interior angles in a triangle is 180°.
$\angle a + \angle b + \angle c = 180°$
Each exterior angle equals the sum of the two interior angles opposite it.



$\angle d = \angle b + \angle c$	$\angle e = \angle a + \angle c$	$\angle f = \angle a + \angle b$

Example

Determine the value for the unknown angle.



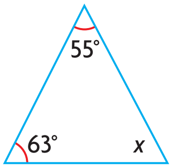
Solution

The sum of the interior angles in a triangle is 180°.

$$\angle x + 73° + 69° = 180°$$
$$\angle x = 180° - 69° - 73°$$
$$\angle x = 38°$$

3. Determine the value of each unknown angle.

a)



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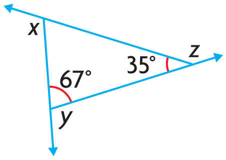
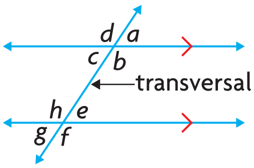


Angle Properties of Parallel Lines

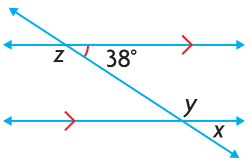


When a transversal crosses 2 parallel lines:

- Corresponding angles are equal.
  $\angle a = \angle e$ $\quad$ $\angle c = \angle g$
  $\angle b = \angle f$ $\quad$ $\angle d = \angle h$
- Alternate angles are equal.
  $\angle b = \angle h$ $\quad$ $\angle c = \angle e$
- The sum of the interior angles on the same side of the transversal is 180°.
  $\angle b + \angle e = 180°$ $\quad$ $\angle c + \angle h = 180°$

Example

Determine the values of the unknown angles.
Explain your solution.

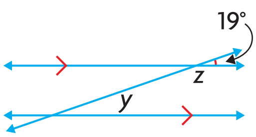


Solution

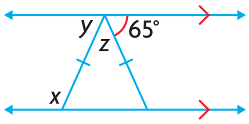
- $\angle x = 38°$ since the angles are corresponding angles.
- The sum of the interior angles on the same side of the transversal is 180°. This means that $\angle y = 180° - 38° = 142°$.
- $\angle z = \angle y = 142°$ since the two angles are alternate angles.

4. Determine the values of the unknown angles. Explain your solution.

a)



b)



Study | *Aid*

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
5 and 7	A-16
6, 8, and 9	A-17

PRACTICE

5. Match each property to its corresponding word.

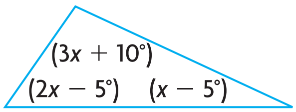
a) two lines at right angles to each other
b) two angles or sides next to each other
c) two straight lines that do not intersect
d) identical in size and shape
e) two angles whose sum is 180°
f) a line that intersects two or more other lines
g) an angle of 180°
h) a polygon with equal sides and angles

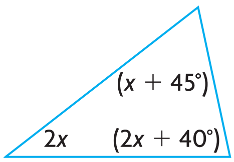
i) parallel
ii) regular
iii) transversal
iv) perpendicular
v) congruent
vi) adjacent
vii) straight angle
viii) supplementary

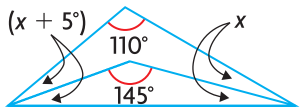
6. Describe a difference and a similarity for each pair of shapes.

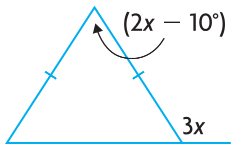
a) a square and a rhombus
b) a rectangle and a parallelogram
c) a rhombus and a parallelogram
d) an equilateral triangle and an isosceles triangle

7. Find each missing value.

a)


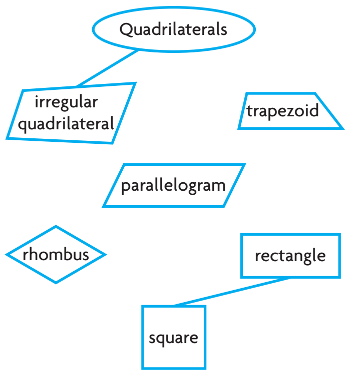
b)


c)


d)


8. This web diagram classifies quadrilaterals.

a) Copy the diagram into your notebook and draw any missing lines.
b) Explain why these lines are needed.



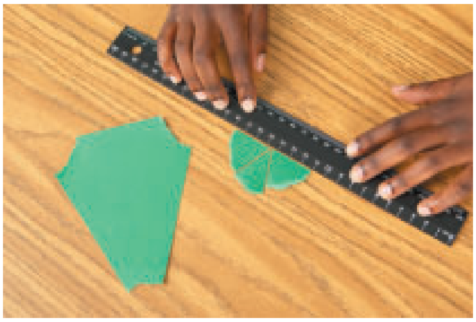
9. Name a quadrilateral with each property using the web diagram from question 8.

a) four congruent sides
b) four different angles
c) two pairs of congruent sides
d) two pairs of congruent angles
e) only two right angles
f) two acute angles and two obtuse angles

APPLYING What You Know

Triangle Tearing

Alyssa tore the corners off a triangular card. She noticed that when she put the three corners together, they seemed to form a straight line.



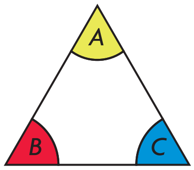
YOU WILL NEED

- scissors
- ruler
- compass

? Will the three corners of every triangle form a straight line?

A. Draw two copies of a large **equilateral** triangle.

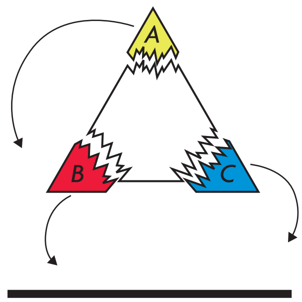
Colour the corners and label them *A*, *B*, and *C*.



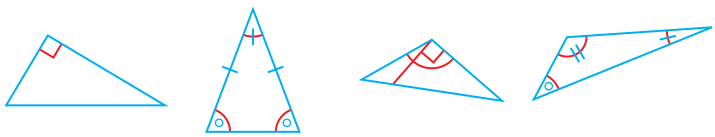
B. Cut out one copy.

C. Tear off the corners *A*, *B*, and *C*.

D. Fit the three corners together. Do they form a form a straight line?



E. Repeat parts A to D for triangles like these.



F. Go back to the uncut copy of each triangle.
Extend one side at each vertex. You have formed three new angles.

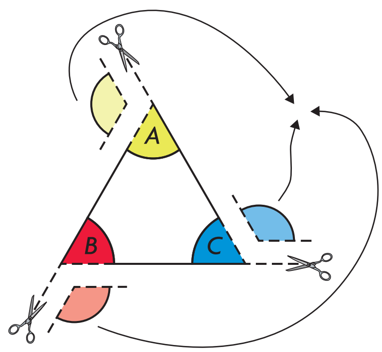
What do you think the sum of these angles is for each triangle?

G. For each triangle, cut out and then fit the three new angles together and measure their sum.

What do you notice?

H. You formed straight lines with the three corners of each triangle. What does this tell you about the sum of the angles of a triangle?

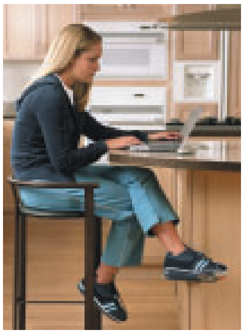
You formed circles with the outside angles of each triangle. What does this tell you about the sum of the outside angles of a triangle?



7.1 Exploring Interior Angles of Polygons

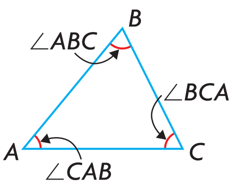
YOU WILL NEED

- grid paper
- protractor
- dynamic geometry software (optional)



interior angle

the angle formed inside each vertex of a polygon (e.g., ΔABC has three interior angles: $\angle ABC$, $\angle BCA$, and $\angle CAB$)



Communication | *Tip*

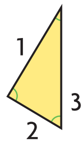
An *n*-sided polygon is often called an *n*-gon. So, a 20-sided polygon is called a 20-gon.

GOAL

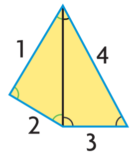
Investigate the sum of the interior angles of polygons.

EXPLORE the Math

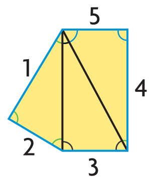
Denise created a triangle on the computer.



She began a pattern of polygons by adding non-overlapping right triangles.



Denise thought, "I know the sum of the **interior angles** of a triangle is 180°. I wonder if I can determine the sum of the angles of any polygon using non-overlapping triangles."



? How can you determine the sum of the interior angles of a 20-gon?

A. Draw a quadrilateral.

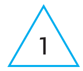
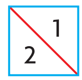
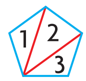
B. Estimate the sum of the interior angles and confirm it by measuring.

C. Draw as many non-overlapping diagonals as you can inside the figure.

D. Calculate the sum of the angles of all the triangles.

Compare to your answer from part A.

E. Repeat parts A to D for each polygon in the table on the next page.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angles	Sketch of Polygon
triangle	3	1	180° (180° × 1)	
quadrilateral	4	2	360° (180° × 2)	
pentagon	5	3	540° (180° × 3)	
hexagon	6			
heptagon	7			
octagon	8			
n-gon	*n*			

Tech Support

For help on constructing a line segment, a triangle, or a polygon; measuring interior angles; or performing a calculation in *The Geometer's Sketchpad*, see Appendix B-16, B-20, B-21, and B-23.

F. Complete the table on the right.

Graph the ordered pairs in it.

x: Sides	*y*: Sum of Interior Angles
3	180°
4	
5	

G. What relationship do you see in your graph?

Explain whether you would join the points.

H. Write the equation of the line.

What is the slope?

What is the *y*-intercept for the linear relationship?

I. Determine the sum of the interior angles of any 20-gon using your equation from part H.

Reflecting

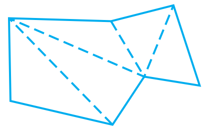
J. Why must the triangles in parts C and D not overlap?

K. What is the formula for the sum of the interior angles of any polygon? Write the formula two ways.

In Summary

Key Ideas

- You can draw non-intersecting diagonals to divide the interior of an n-gon into $n - 2$ non-overlapping triangles.
- The sum of the interior angles of an n-gon is $(n - 2) \times 180°$.

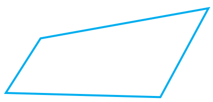


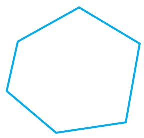
Need to Know

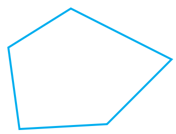
- The sum of the interior angles of a triangle is 180°.
- The sum of the interior angles of a quadrilateral is 360°.

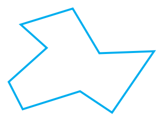
FURTHER Your Understanding

1. Copy the following polygons. Draw as many non-intersecting diagonals as possible to create non-overlapping triangles. What is the sum of the interior angles in each case?

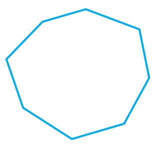
a) 

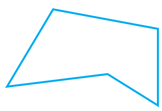
b) 

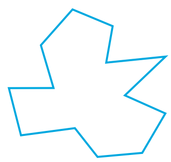
c) 

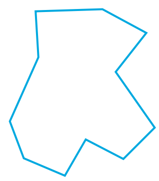
d) 

2. Calculate the sum of the interior angles of each polygon.

a) 

b) 

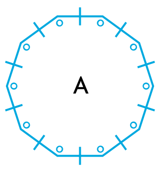
c) 

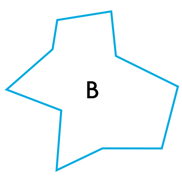
d) 

Communication *Tip*

Regular polygons have equal sides and equal interior angles. Irregular polygons do not.

3. Polygon A is a regular 10-gon and polygon B is an irregular 10-gon. Are the sums of their interior angles equal? Explain.





4. What is the measure of each interior angle of a regular 14-gon?

5. The sum of the interior angles in a polygon is 1440°. How many sides does the polygon have?

7.2 Angle Properties of Polygons

GOAL

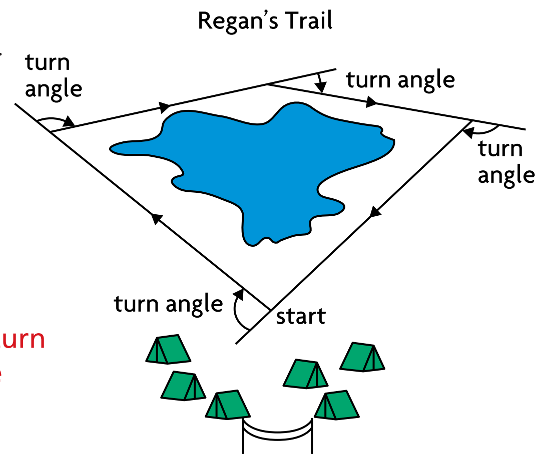
Apply the exterior and interior angle properties of polygons.

YOU WILL NEED

- protractor
- scissors
- dynamic geometry software (optional)

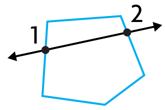
INVESTIGATE the Math

Regan set up an orienteering trail around the lake at summer camp. Her trail formed a **convex polygon**, instead of a **concave polygon**. She measured the turn angles, or **exterior angles**, on her trail.



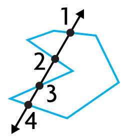
convex polygon

a polygon with every interior angle less than 180°; any straight line through it crosses, at most, two sides



concave polygon

a polygon with at least one interior angle greater than 180°; a straight line through it may cross more than two sides



? What is the sum of the turn angles for one complete tour of Regan's trail?

A. Draw a rectangle.

B. Measure its interior and exterior angles.

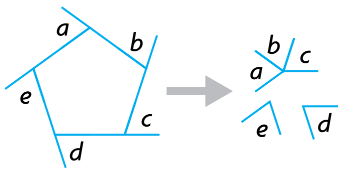
Determine the sum of the exterior angles.

C. Repeat parts A and B for several different convex quadrilaterals.

D. Repeat parts A and B for several different convex polygons with five sides or more.

E. Cut out the exterior angles of each polygon.

Place the angles together so the vertices all touch.

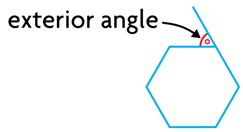


F. What do you notice?

What does this tell you about the sum of the turn angles on Regan's trail?

exterior angle

the angle formed by extending a side of a convex polygon; the angle between any extended side and its adjacent side



Reflecting

G. How are the exterior and interior angles at each vertex of a convex polygon related?

H. What conclusions can you draw about the sum of the exterior angles of any convex polygon?

Tech *Support*

For help on constructing and measuring exterior angles in *The Geometer's Sketchpad*, see Appendix B-22.

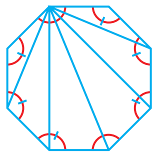
APPLY the Math

EXAMPLE 1 Connecting exterior angle sums to interior angles

What is the sum of the exterior angles in a regular octagon?

Jordan's Solution

$$\text{sum of interior angles} = 180° \times (n - 2)$$
$$= 180° \times (8 - 2)$$
$$= 180° \times 6$$
$$= 1080°$$



The sum was 1080°.

There were 8 sides, so there were $8 - 2 = 6$ triangles.

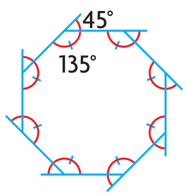
I calculated the sum of the interior angles using the formula $180° \times (n - 2)$ where n is the number of sides.

$$\text{measure of one angle} = \frac{1080°}{8}$$
$$= 135°$$

The interior angles are equal. So, I divided by 8 to determine their measure.

$$\text{exterior angle} + \text{interior angle} = 180°$$
$$\text{Therefore, one exterior angle} = 180° - \text{interior angle}$$
$$= 180° - 135°$$
$$= 45°$$

Each exterior angle and adjacent interior angle add to 180°. So, the measure of each exterior angle was 45°.



$$\text{sum of exterior angles} = 8 \times 45°$$
$$= 360°$$

The sum of the exterior angles was 360°.

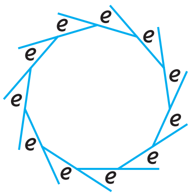
There were 8 exterior angles, so I multiplied by 8.

In any regular n-gon the exterior angles are equal. This helps you determine their measure if you know the value of n.

EXAMPLE 2 Determining exterior angles using reasoning

Determine the measure of each exterior angle in a regular 11-gon.

Lakmini's Solution



$$e = \frac{360°}{n}$$
$$= \frac{360°}{11}$$
$$\doteq 33°$$

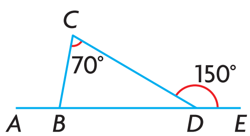
I knew the exterior angles are equal and add to 360° in any regular convex polygon.

I divided 360° by the number of exterior angles.

Each exterior angle is about 33°.

EXAMPLE 3 Solving a problem using angle properties

Determine the measure of $\angle CBA$.



Regan's Solution

$$\angle CDB = 180° - \angle CDE$$
$$= 180° - 150°$$
$$= 30°$$

I saw that $\angle CDB$ and $\angle CDE$ were the interior and exterior angles at vertex D. So, they add to 180°.

$$\angle CBD + \angle CDB + \angle BCD = 180°$$
$$\angle CBD + 30° + 70° = 180°$$
$$\angle CBD + 100° = 180°$$
$$\angle CBD = 80°$$

I knew the sum of the interior angles in a triangle is 180°.

$$\angle CBA = 180° - \angle CBD$$
$$= 180° - 80°$$
$$= 100°$$

$\angle CBA$ and $\angle CBD$ add to 180°. This is because they were the interior and exterior angles at vertex B.

In Summary

Key Ideas

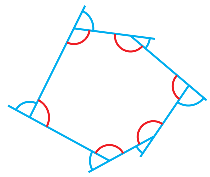
- You can determine unknown angles in polygons using angle properties.
- The sum of the exterior angles of a convex polygon is 360°.

Need to Know

- You can form an exterior angle for a convex polygon by extending a side past its endpoint.
- An exterior angle and its adjacent interior angle are supplementary; they add to 180°.

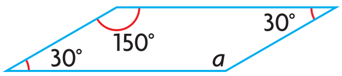
CHECK Your Understanding

1. What is the relationship between the interior angle and the exterior angle at each vertex of a polygon?

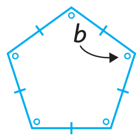


2. Determine the measure of each missing angle.

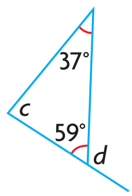
a)



b)



c)



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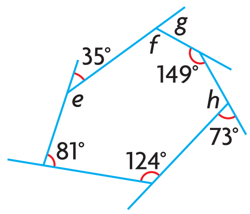
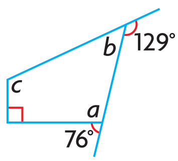


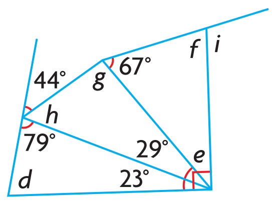
PRACTISING

3. Determine the measure of each missing angle.

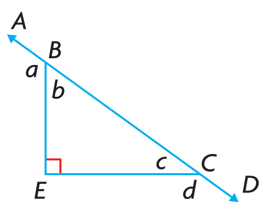
a)



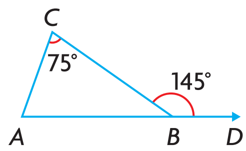
b)



4. In this diagram, $\angle E$ in ΔBEC is a right angle. What is the sum of angles a and d?



5. What is the measure of $\angle CAB$ in this diagram?

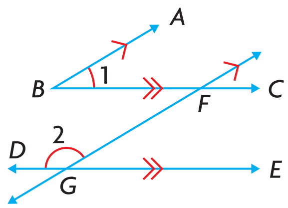


6. In the diagram, AB is parallel to FG and BC is parallel to DE.

a) What is the relationship between $\angle 1$ and $\angle 2$?

b) Use *The Geometer's Sketchpad* or several examples to support your answer in part a).

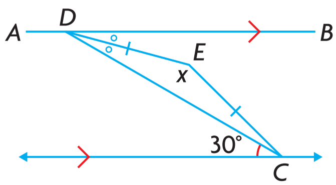
c) Write an expression for your answer.



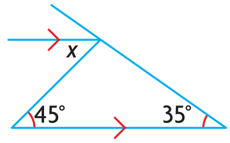
7. Determine the measure of each missing angle.

K

a)



c)



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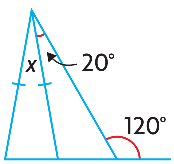

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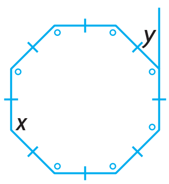
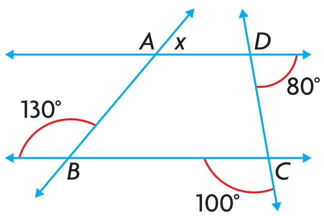


8. For each diagram, state the equation that expresses the relationship needed to solve the problem. Then, determine the measure of each variable. Show the steps in the solution.

C

a)



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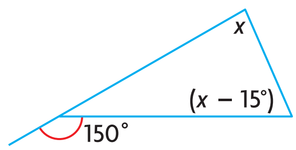

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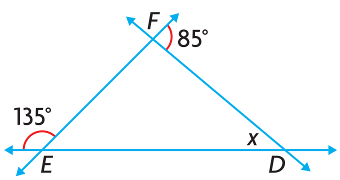

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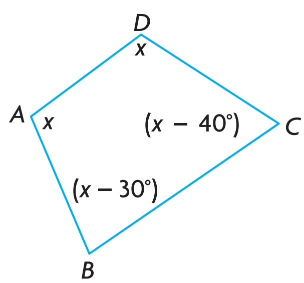


9. An interior angle of a parallelogram is the measure of the exterior angle adjacent to it multiplied by 4. Determine the measure of each interior angle. Draw the parallelogram.

10. In ΔABC, the measure of $\angle B$ is 21° less than the measure of $\angle A$ multiplied by 4. The measure of $\angle C$ is 1° more than the measure of $\angle A$ multiplied by 5. Determine the measure of each interior angle and each exterior angle of ΔABC.

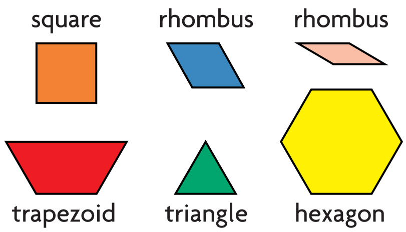
11. In a regular polygon, the ratio of the measure of the exterior angle to the measure of its adjacent interior angle is 1 to 4. How many sides does the polygon have?

12. For any regular *n*-gon, develop a formula for calculating the measure of each interior angle.

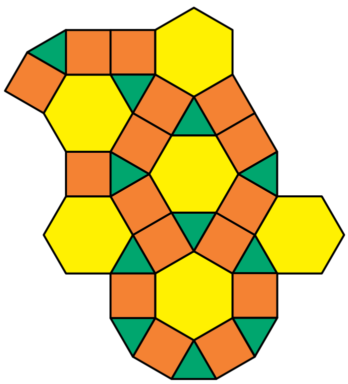
13. Why is the sum of the interior angles of a convex polygon usually greater than the sum of its exterior angles? Explain with an example.

Extending

14. When pattern blocks are used to tile a surface, they have to fit together to join along sides and vertices.

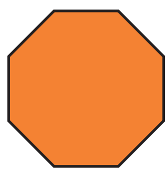


Pentagons were not included in the set of pattern blocks. Explain why pentagons cannot be used to tile a surface.



15. a) Suppose you are going to tile a floor with tiles shaped like an octagon and one other shape. What other shape can you use?

b) Determine two other tile shapes you can use to tile a floor.



7 Mid-Chapter Review

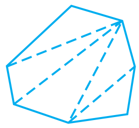
FREQUENTLY ASKED Questions

Q: What is the formula for the sum of the interior angles of a polygon?

A: The sum of the interior angles of a polygon with n sides is $(n - 2) \times 180°$. This is because the polygon can be divided into $n - 2$ non-overlapping triangles, and the sum of the interior angles of each triangle is 180°.

EXAMPLE

Determine the sum of the interior angles of this heptagon.

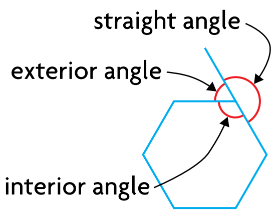


Solution

A heptagon has 7 sides and can be divided into 5 triangles. So, the sum of its interior angles is $(7 - 2) \times 180° = 900°$.

Q: What is the relationship between the interior and exterior angles of a convex polygon?

A: The interior and exterior angles at any vertex of a convex polygon form a straight angle, or 180°. That means that the measure of any exterior angle equals 180° minus the measure of its adjacent interior angle. They are supplementary angles.



EXAMPLE

Calculate the measure of each interior angle in a regular hexagon.

Solution

In a regular hexagon, each exterior angle is 360° divided by 6 angles, or 60°. So, each interior angle is $180° - 60° = 120°$.

Q: How can you calculate the sum of the exterior angles of a convex polygon?

A: The sum of the exterior angles of any convex polygon is 360°. You can calculate this by measuring, by determining each exterior angle from its adjacent interior angle, or by using reasoning if the polygon is regular.

Study *Aid*

- See Lesson 7.1.
- Try Mid-Chapter Review Questions 1 and 2.

Study *Aid*

- See Lesson 7.2, Example 2.
- Try Mid-Chapter Review Questions 4, 5, and 6.

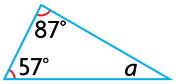
Study *Aid*

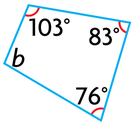
- See Lesson 7.2, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 4, 5, and 6.

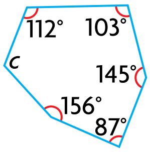
PRACTICE Questions

Lesson 7.1

1. Determine the measure of each missing interior angle.

a)


b)


c)


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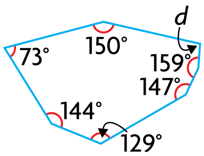
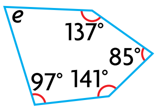
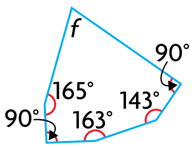


e)


f)


2. Determine the measure of the interior angles of each figure.

a) a regular 12-gon

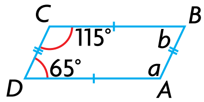
b) a regular 15-gon

c) a regular 20-gon

Lesson 7.2

3. Each interior angle a in a regular n-gon has a measure of $a = 20n$. How many sides does the polygon have?

4. Determine the measure of each missing angle. Support your answer with mathematical reasoning.

a)


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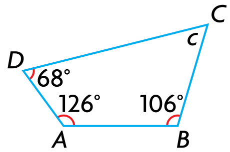
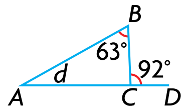


c)


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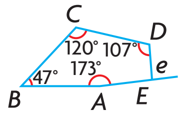
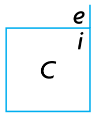
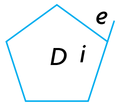
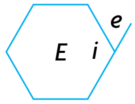
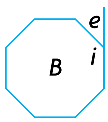


5. In a regular polygon, the ratio of the measure of the exterior angle to the measure of the adjacent interior angle is 2 to 3. How many sides does the polygon have?

6. Complete the table for each regular polygon.

Figure	Measure of Each Interior Angle	Measure of Each Exterior Angle	Sum of Interior Angles	Sum of Exterior Angles
				
				
				
				

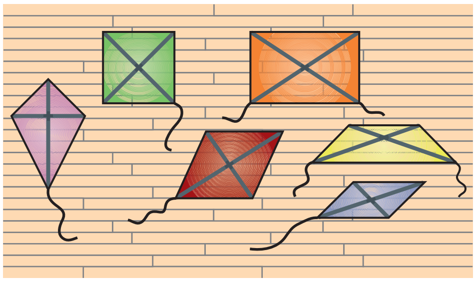
7.3 Exploring Quadrilateral Diagonal Properties

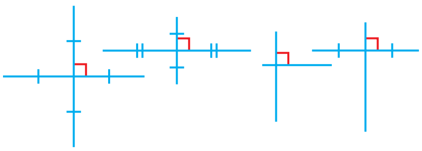
GOAL

Create and test conjectures about properties of quadrilaterals.

EXPLORE the Math

Santos was making flying **kites** of different shapes with two cross pieces. He made a **conjecture** that the shape of each kite depended on how he arranged the diagonals. He started with perpendicular diagonals.





? How can Santos predict the shape of a quadrilateral by using line and angle properties of the diagonals?

A. Draw two intersecting perpendicular line segments of any length. An example is shown to the right.

B. Create a shape using the endpoints of the segments as vertices and the segments as diagonals. An example is shown to the right.

C. Describe the quadrilateral you created.

D. Form a conjecture about what types of quadrilaterals you can construct with perpendicular diagonals.

E. Sketch and label an example of each type of quadrilateral.

F. Draw two intersecting non-perpendicular line segments of any length.

G. Create a quadrilateral using the segments as diagonals.

H. What types of quadrilaterals can you construct?

I. Sketch and label an example of each type.

YOU WILL NEED

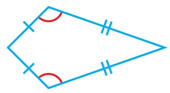
- grid paper
- dynamic geometry software (optional)

Tech *Support*

For help on constructing a midpoint or a segment perpendicular to another segment in *The Geometer's Sketchpad*, see Appendix B-25 and B-26.

kite

a quadrilateral that has two pairs of equal sides with no sides parallel

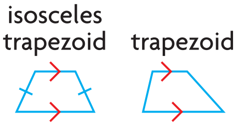


conjecture

a guess or prediction based on limited evidence

Communication | Tip

"Isosceles" means having two equal sides. Triangles can be isosceles, and so can trapezoids.

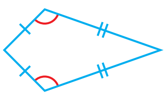


Communication | Tip

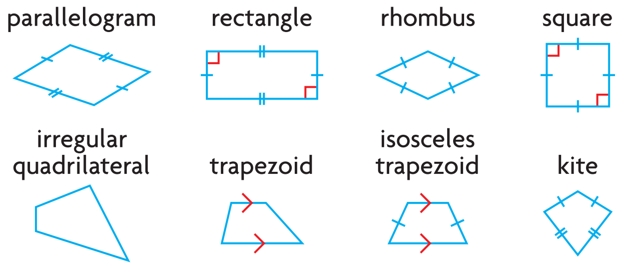
The word "kite" has several meanings, including a flying toy that can be any shape, or a geometric figure.

A flying kite

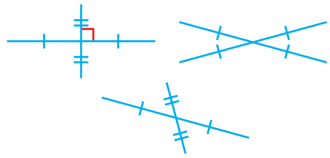
A geometric kite



J. What is the arrangement of the diagonals for each shape?



Reflecting



K. Could you form a square, a rectangle, a rhombus, and a parallelogram using these diagonals? Explain how you know.

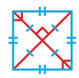
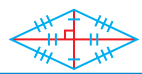
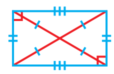
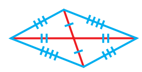
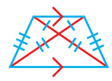
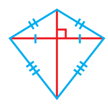
L. Explain why a square is always a rhombus but a rhombus is not always a square. Refer to diagonals in your answer.

M. How do the relationships between the diagonals help you predict the shape of a quadrilateral?

In Summary

Key Idea

- The diagonals of certain quadrilaterals have special properties:

Type of Quadrilateral	The diagonals...	The diagonals form angles that are...	Diagram
square	are equal and bisect each other.	all 90°.	
rhombus (not a square)	are not equal and bisect each other.	all 90°.	
rectangle (not a square)	are equal and bisect each other.	equal when opposite and supplementary when adjacent.	
parallelogram (not a rectangle or rhombus)	are not equal and bisect each other.	equal when opposite and supplementary when adjacent.	
isosceles trapezoid (not a rectangle or rhombus)	are equal and intersect to form two pairs of equal line segments.	equal when opposite and supplementary when adjacent.	
kite	may or may not be equal and only one is bisected by the other.	all 90°.	

Need to Know

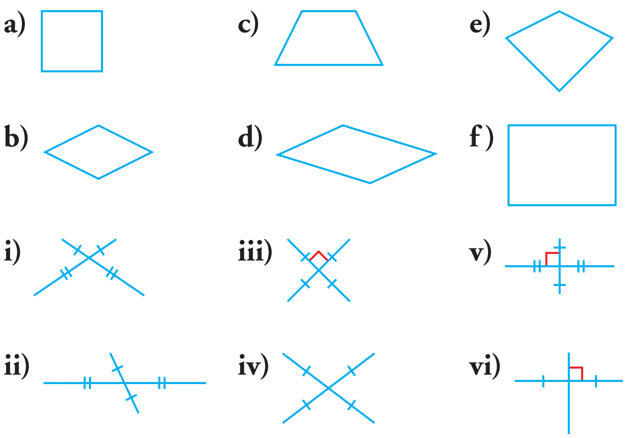
- You can identify the type of quadrilateral by using its diagonal properties.

FURTHER Your Understanding

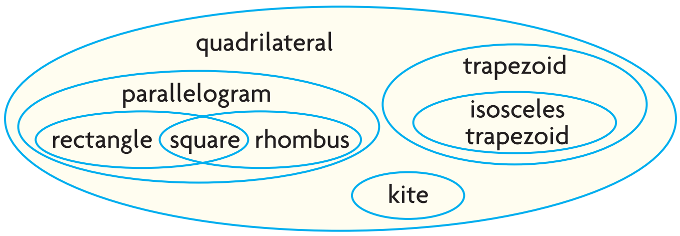
1. Each quadrilateral ABCD below has these three vertices: $A(0, 0)$, $B(3, 4)$, and $C(8, 4)$. Use diagonal properties to identify the coordinates of the fourth vertex D in each case. Explain your method.

a) rhombus
b) isosceles trapezoid
c) kite

2. Match each pair of diagonals with its quadrilateral. Explain your reasoning.



3. Explain why the quadrilaterals are in different parts of the Venn diagram. Refer to the properties of sides, angles, and diagonals of quadrilaterals.



4. The diagonals and the sides of a quadrilateral form four triangles. Complete the table for the triangles formed by these quadrilaterals. Draw diagrams to support your answers.

Quadrilateral	Number of Congruent Triangles
square	
rhombus	
rectangle	
parallelogram	
isosceles trapezoid	
kite	

7.4 Reasoning About Triangle and Quadrilateral Properties

YOU WILL NEED

- protractor
- dynamic geometry software (optional)

GOAL

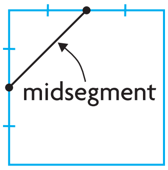
Form and test conjectures about properties of quadrilaterals.

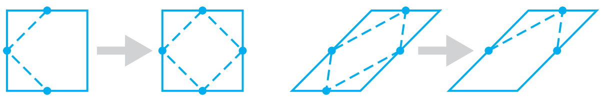
LEARN ABOUT the Math

Jafar created square and parallelogram display boards for an art gallery. He made a border for the text area of each board by joining the **midpoints** with string to create the **midsegments.**
The shape inside the square board looked like a square and the shape inside the parallelogram looked like a parallelogram.

midsegment

a line segment connecting the midpoints of two adjacent sides of a polygon





? What figure is formed by the midsegments of a quadrilateral?

EXAMPLE 1 Forming and testing a conjecture

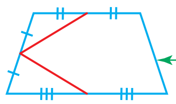
Jafar, Maria, and Elani had different conjectures. They tested them in different ways.

Reasoning | *Checklist*

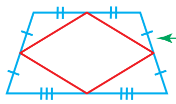
- ✔ Did you explain your reasoning clearly?
- ✔ Are your conclusions reasonable?
- ✔ Did you justify your conclusions?

Jafar's Solution: Rejecting a conjecture

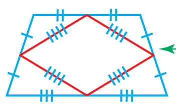
Conjecture: The shape formed by the midsegments of a quadrilateral has the same shape as the original quadrilateral.



I tried an isosceles trapezoid. I thought I would get a trapezoid. I joined the midpoints of the first two sides.



I joined the midpoints of the last two sides. The shape in the middle was a rhombus, not a trapezoid.



I found a **counterexample** to my conjecture. So, I would have to test other quadrilaterals, and then form a new conjecture.

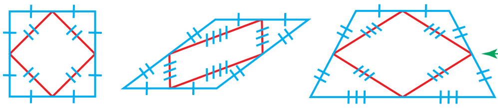
My conjecture was incorrect.

counterexample

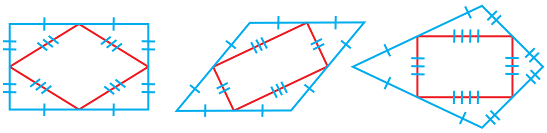
an example that proves that a hypothesis or conjecture is false

Maria's Solution: Revising a conjecture

I will test some quadrilaterals to revise Jafar's conjecture.



I joined the midpoints of some quadrilaterals. I always got a parallelogram. I thought this might happen for any quadrilateral.



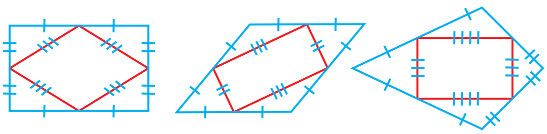
I checked more quadrilaterals. The shape in the middle was always a parallelogram.

Conjecture: All the interior shapes formed by the midsegments are parallelograms.

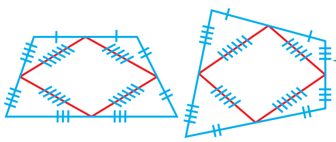
I still had to determine if this held for any quadrilateral. I could create more examples, but I could not be fully sure.

Elani's Solution: Supporting a conjecture

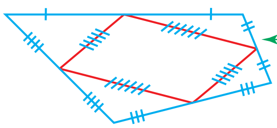
Conjecture: The shape formed by the midsegments of a quadrilateral is a parallelogram.



I joined the midpoints of these quadrilaterals. They confirmed my conjecture.



I checked more quadrilaterals. Each time, the shape in the middle was a parallelogram.



I joined the midpoints of another quadrilateral. The new shape was a parallelogram.

My examples support my conjecture.

Each interior shape formed by the midsegments is a parallelogram.

I reasoned my conjecture was very likely true. However, I could not be fully sure. There might be a counterexample.

Reflecting

A. Explain how Jafar determined that his conjecture was incorrect.

B. Should Maria have tested other quadrilaterals? Explain.

C. Explain how Elani's examples supported the conjecture she tested but did not prove it.

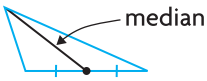
APPLY the Math

EXAMPLE 2 Confirming or denying a conjecture

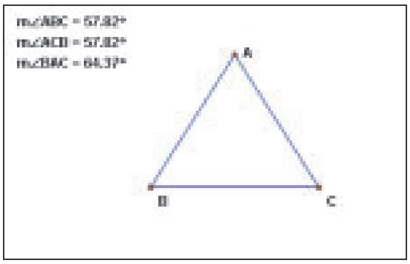
Sven's sister saw him doing geometry homework on isosceles triangles. All isosceles triangles have two equal angles and a third angle. She told him that the **median** through the third angle is never perpendicular to the base. Sven decided to test her conjecture.

median

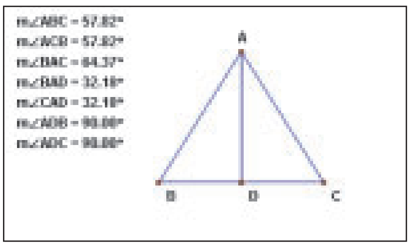
the line drawn from a vertex of a triangle to the midpoint of the opposite side



Sven's Solution



I constructed an isosceles triangle using *The Geometer's Sketchpad*.

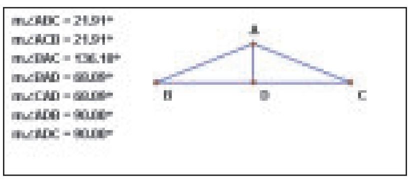


I drew the median through the third angle. I measured the angle at the base. It was 90°, so the median and base were perpendicular.

Tech Support

For help on constructing and labelling a triangle, measuring an angle, or constructing a midpoint in *The Geometer's Sketchpad*, see Appendix B-20, B-21, and B-25.

I reasoned that this triangle was a counterexample to my sister's conjecture.



To check my counterexample, I dragged the vertices of the triangle to form different isosceles triangles. The median was always perpendicular to the base.

My sister's conjecture was incorrect.

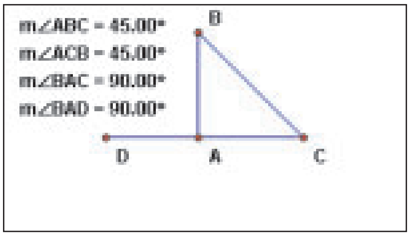
EXAMPLE 3 | Testing and revising a conjecture

Make a conjecture about the relationship between the exterior angle of a triangle and the two interior angles opposite it. Then test your conjecture.

Aisha's Solution

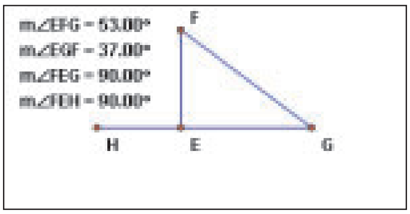
Conjecture: The sum of an exterior angle of a triangle and the two interior angles opposite it is 180°.

I noticed that the sum of the exterior angle of a triangle and the two interior angles opposite it was sometimes 180°.



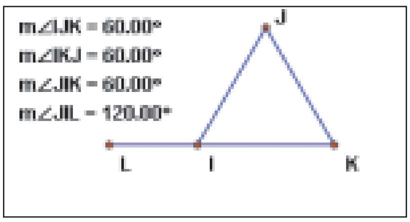
I tested my conjecture with an exterior angle for a right isosceles triangle using *The Geometer's Sketchpad*.

The example confirmed my conjecture.



I tested an exterior angle for a triangle with interior angles of 90°, 37°, and 53°. The sum of ∠*FEH*, ∠*EFG*, and ∠*EGF* was 180°.

The example confirmed my conjecture.



I tested an equilateral triangle. The sum of ∠*JIL*, ∠*IJK*, and ∠*IKJ* was 240°.

My conjecture was false. This was a counterexample to my conjecture.

New conjecture: The exterior angle of a triangle is equal to the sum of the two interior angles opposite to it.

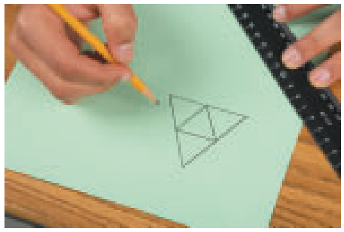
In each example, the exterior angle was equal to the sum of the two interior angles opposite it. I revised my conjecture. The new conjecture needed testing.

Tech | *Support*

For help on measuring an exterior angle in *The Geometer's Sketchpad*, see Appendix B-22.

EXAMPLE 4 Testing then confirming a conjecture

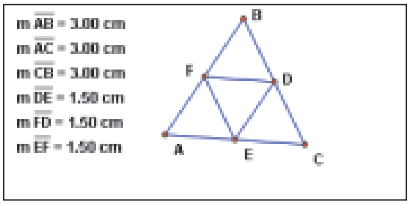
Jeff was making designs. He drew triangles and then formed the midsegments. He thought he saw a relationship between each midsegment and its opposite side.



Jeff's Solution

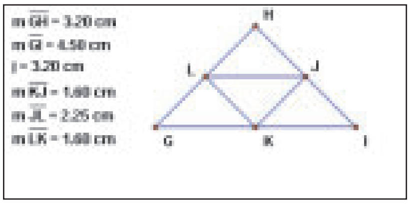
Conjecture: A midsegment of a triangle is half as long as its opposite side.

I made a conjecture. I tested it by constructing examples.



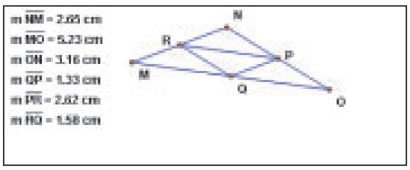
I tested an equilateral triangle. I determined the lengths of the sides and midsegments using *The Geometer's Sketchpad*.

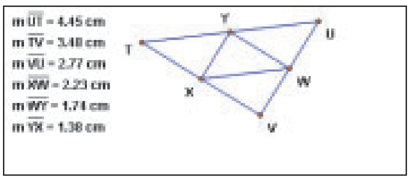
The lengths in the example supported my conjecture.



I tested an isosceles triangle.

The lengths in the example supported my conjecture.





I tested two scalene triangles.

The lengths in the examples supported my conjecture.

My examples confirm my conjecture. The length of the midsegment is half the length of the opposite side.

I reasoned my conjecture was very likely correct. I could not be fully sure; a counterexample might still exist.

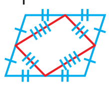
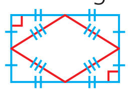
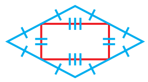
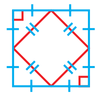
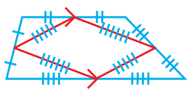
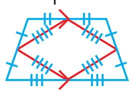
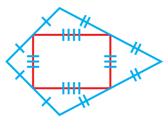
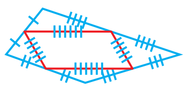
In Summary

Key Ideas

- Examples can support a conjecture about a geometric relationship, but do not prove it.
- You only need one counterexample to disprove a conjecture about a geometric relationship.

Need to Know

- The midsegments of any quadrilateral form a parallelogram:

Midsegments Form a Parallelogram	Midsegments Form a Rhombus	Midsegments Form a Rectangle	Midsegments Form a Square
• parallelogram 	• rectangle 	• rhombus 	• square 
• trapezoid 	• isosceles trapezoid 	• kite 	
• irregular quadrilateral 			

- The median through the angle formed by the two equal sides of an isosceles triangle is perpendicular to the third side.
- The exterior angle at a vertex of a triangle equals the sum of the two interior angles opposite it.
- The length of a midsegment in a triangle equals half the length of the side opposite it.

CHECK Your Understanding

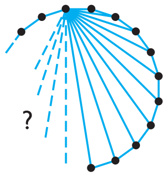
1. Test this conjecture: "Midsegments in a triangle are always parallel to the side opposite to them." Support your reasoning with examples.

2. Test this conjecture: "If a quadrilateral has perpendicular diagonals, then it is a square." Support your reasoning with examples.

PRACTISING

3. Predict whether a polygon's sides are all equal if its interior angles are all equal. Support your conjecture with examples or disprove it with a counterexample.

4. Predict whether a polygon's interior angles are all equal if its sides are all equal. Support your conjecture with examples or disprove it with a counterexample.

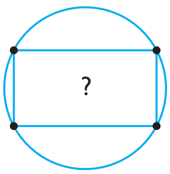


5. Create a conjecture to predict the number of diagonals from any one vertex of a convex polygon with *n* sides. Support your conjecture with examples or disprove it with a counterexample.

6. Test this conjecture: "If the midsegments of a quadrilateral form a square, then the quadrilateral is itself a square."

7. Test this conjecture: "The medians of a triangle always intersect at exactly one point."

8. Create a conjecture to predict the ratio of the area of a triangle to the area of the shape formed by its midsegments. Support your conjecture with examples or disprove it with a counterexample.



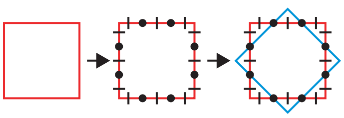
9. Test this conjecture: "It is always possible to draw a circle through all four vertices in a rectangle."

10. Geometric relationships and properties are often discovered using conjectures and counterexamples. Describe the process you use to solve geometric problems using words or diagrams, such as a flow chart.

Extending

11. Create a conjecture to predict when the midsegments of a pentagon form a regular pentagon. Support your conjecture with examples or disprove it with a counterexample.

12. Draw the inner quadrilateral of a square using its midsegments. Then, draw a new inner quadrilateral inside that one, and then another inside the second. Do the same for a rectangle, a rhombus, a parallelogram, a kite, and a trapezoid. Do you notice any patterns? Begin with a conjecture, then either support it with examples or disprove it with a counterexample.

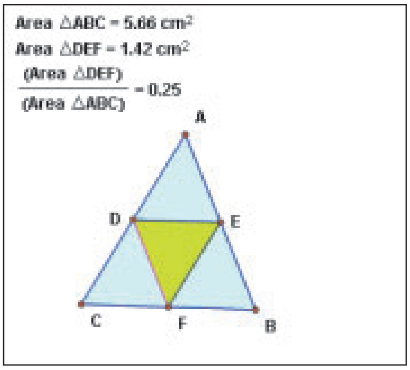


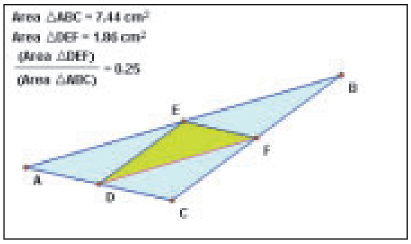
13. To trisect a line segment, divide it into three equal parts. Musim trisected each side of the red square. Then he drew lines to create the blue quadrilateral. What do you notice about it? Form a conjecture about the quadrilateral that is created when you trisect the sides of other quadrilaterals. Support your conjecture with examples or disprove it with a counterexample.

Curious Math

Meeting of Midpoints

The area of the triangle formed by the midsegments of a triangle is always 25% of the area of the original triangle. Do you think the same relationship is true for other polygons?





1. Draw several quadrilaterals either on grid paper or using dynamic geometry software.

Calculate or estimate their areas.

2. Mark midpoints on each side.

Then draw the midsegments.

3. Calculate the areas of the shapes formed by the midsegments.

4. What are the ratios of the inner quadrilateral areas to the outer quadrilateral areas?

5. Predict the result if you were to change the number of sides from 4 to 5.

Repeat parts 1 and 2 for pentagons to test your conjecture.

6. Do you think there is a relationship between the area of a polygon and the area of the shape formed by its midsegments?

Form a conjecture, and then test it with hexagons, heptagons, and octagons.

Tech | *Support*

For help on measuring the area of a polygon in *The Geometer's Sketchpad*, see Appendix B-28.

7.5 Reasoning About Properties of Polygons

YOU WILL NEED

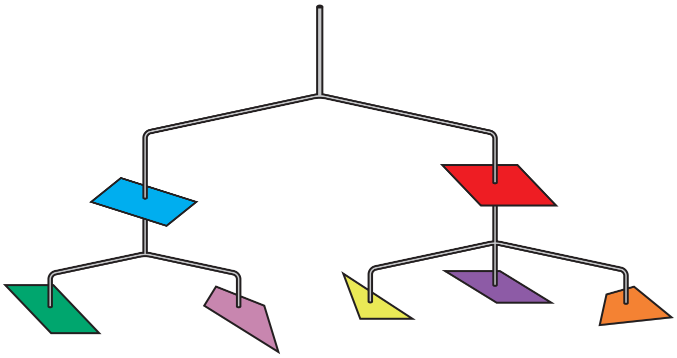
- construction paper
- ruler
- protractor
- scissors
- dynamic geometry software (optional)

GOAL

Apply properties of triangles and quadrilaterals.

INVESTIGATE the Math

In art class, Daniel created a mobile using shapes with three and four sides. He wanted to attach each shape to the string so that it would not hang crookedly. Daniel knew that every 2-D shape has a **centroid,** or centre of gravity.



? How can Daniel determine the centroid of each shape using triangle and quadrilateral properties?

A. Cut a triangle out of construction paper.

How might you find its centre of gravity?

B. State a conjecture about the triangle's centroid based on the intersection of lines such as angle bisectors, perpendicular bisectors, or medians.

C. Construct the centroid using your conjecture.

Describe your construction method. What lines did you draw?

D. Test by placing the centroid on the eraser end of a pencil.

Does the triangle balance? Move the pencil until it does, if needed.

E. Cut out a square, a parallelogram, a trapezoid, and a kite.

F. State a conjecture about the centroid in each shape. Base your conjecture on the intersection of lines, such as diagonals, angle bisectors, perpendicular bisectors, or **bimedians.**

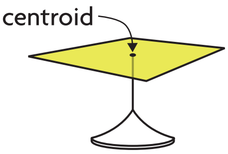
Then repeat parts B through D.

G. Draw medians for the triangle and bimedians for the square, parallelogram, trapezoid, and kite.

Try balancing each. What do you notice?

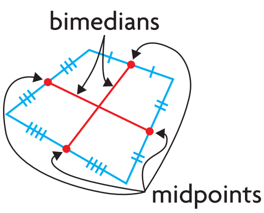
centroid

the centre of an object's mass; the point at which it balances; also known as the centre of gravity



bimedian

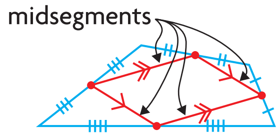
the line joining the midpoints of two opposite sides in a quadrilateral



Reflecting

H. Where is the centroid located in a quadrilateral? in a triangle?

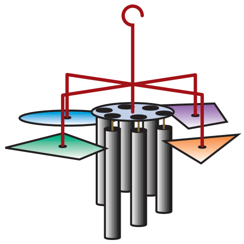
I. Describe how to determine the centroid of a quadrilateral using its midsegments.



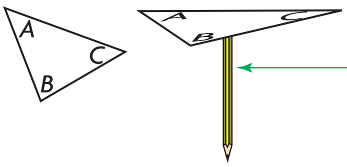
APPLY the Math

EXAMPLE 1 Forming and testing a conjecture about the centroid of a triangle

Daniel wanted to make a wind chime using geometric shapes. He needed to find the centroid of each shape to drill the hole it would hang from.

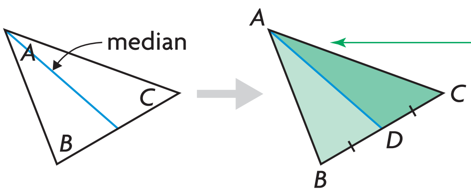


Daniel's Solution

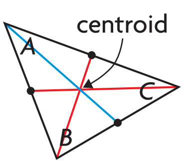


I drew the triangle on cardboard. I cut it out and tried to balance it on a pencil eraser.

It seemed the centroid was on the median through vertex *A*. I thought each half of the triangle might balance the other that way.



I drew the median through vertex *A*. I placed the median on the edge of a ruler. Each half of the triangle balanced out the other.



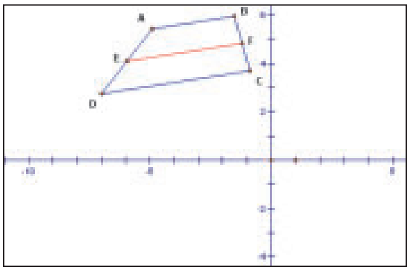
I drew the other medians. I verified that the centroid was on each by balancing on the edge of a ruler.

I realized that the centroid must be the intersection of the medians. I tested by hanging the triangle with string through it.

EXAMPLE 2 Forming and testing a conjecture about bimedians

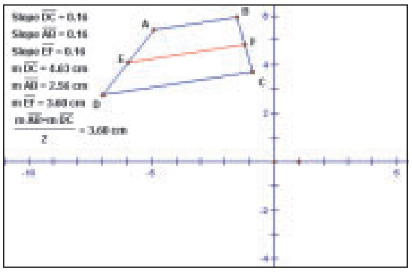
Mingmei drew the bimedian connecting the two non-base sides of a trapezoid. She noticed that it seemed parallel to the base sides. She decided to investigate the properties of the bimedian.

Mingmei's Solution



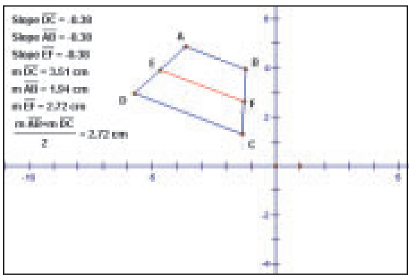
I drew a trapezoid in *The Geometer's Sketchpad* to determine its centroid.

I plotted the midpoints *E* and *F* of the non-base sides. Then I joined them to form the bimedian.



I calculated the slopes of the bimedian and the base sides. The three lines were parallel.

I determined the lengths of the bimedian and the base sides. The length of the bimedian was half the sum of the lengths of the base sides.



I tested other trapezoids by dragging vertices. Each time, the bimedian was parallel to the base sides. Its length was always the mean of their lengths.

I reasoned that these properties made sense. Triangle midsegments are parallel to the opposite side and half its length. Trapezoids are triangles with the top cut off parallel to the bottom side. So, the bimedian is like a triangle midsegment.

The bimedian between the non-base sides of each trapezoid I tested is half the length of the base sides and parallel to them.

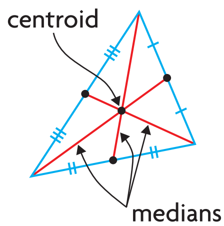
Tech Support

For help measuring the length of a segment in *The Geometer's Sketchpad*, see Appendix B-24.

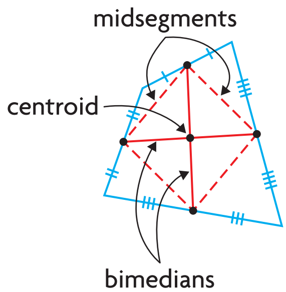
In Summary

Key Ideas

- The centroid of a triangle is located at the intersection of its medians. Medians are lines from each vertex of a triangle to the midpoint of the opposite side.

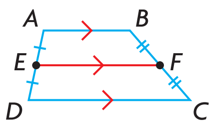


- The centroid of a quadrilateral is located at the intersection of its bimedians. Bimedians are lines joining the midpoints of opposite sides. They are the diagonals of the parallelogram formed by the midsegments of the quadrilateral.



Need to Know

- You can determine the centroid of some quadrilaterals by locating the point of intersection of their diagonals. This works for squares, rectangles, rhombuses, and parallelograms, but not for trapezoids and kites.
- The bimedian of the non-base sides of a trapezoid is parallel to its bases. Its length is the mean of their lengths.



$$EF = \frac{AB + CD}{2}$$

CHECK Your Understanding

1. Choose three figures from this list: triangle, square, rhombus, rectangle, parallelogram, kite, isosceles trapezoid, non-isosceles trapezoid, irregular quadrilateral. Determine the centroid of each figure you chose.

2. Answer each question using mathematical terminology.

a) What quadrilateral is formed by the midsegments of a rectangle?

b) Where is the centroid of a parallelogram located?

c) What quadrilateral is formed by the midsegments of a trapezoid?

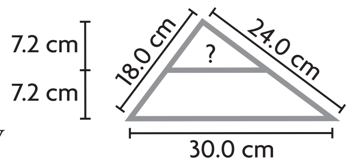
d) The bimedians of any quadrilateral form the diagonals of a smaller quadrilateral. What are the sides of this smaller quadrilateral called?

PRACTISING

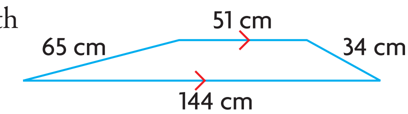
3. Complete the table.
K

Quadrilateral	Centroid Construction Method	Bimedian Geometric Properties
square	intersection of bimedians or diagonals	• bisect each other • equal length • intersect at right angles • split square into four smaller congruent squares
rhombus		
rectangle		
parallelogram		
kite		
isosceles trapezoid		
non-isosceles trapezoid		
irregular quadrilateral		

4. Remi is building a triangular wooden shelving unit. The base measures 30 cm and the slant sides measure 18 cm and 24 cm. He wants a horizontal shelf halfway between the base and the top. What length of wood should he cut for the shelf?
A



5. A trapezoid has parallel sides of length 51 cm and 144 cm. Its other sides measure 34 cm and 65 cm.



a) Determine the length of the bimedian joining the two non-parallel sides.

b) The distance between the parallel sides is 8 cm. How far is the bimedian from each?

6. Martin is making a stencil in the shape of an isosceles triangle. The median to the base side is 12 cm long. The midsegment parallel to the base is 5 cm long.

a) What is the length of the base?

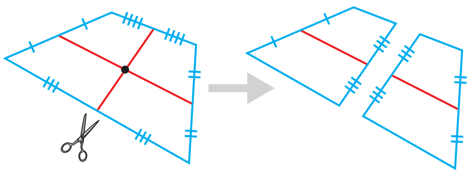
b) What is the length of the slant sides?

7. Jerry is making a kite in the shape of an isosceles triangle. Two sides are
T 41 cm and the other side is 18 cm. Jerry wants to attach a plastic support along the median meeting the midpoint of the shortest side.

   a) Determine the length of plastic Jerry must cut for the support.

   b) Jerry wants to place another plastic support along the midsegment opposite the 18 cm side. Determine the length of this plastic support.

8. Serwa constructed a quadrilateral for an art project. She located its
C centroid by drawing the bimedians. Then she decided to cut the quadrilateral into two pieces along one bimedian. Explain how to determine the centroids for her new quadrilaterals using only the remaining bimedian.



9. Explain how diagonal properties help determine the centroid in different quadrilaterals.

Extending

10. Draw any irregular quadrilateral, then connect its diagonals. Locate the midpoints of both diagonals, then join them.

    a) Determine the midpoint of the segment you just found. What special point have you constructed?

    b) Explain how you determined your answer.

11. Describe how to divide any quadrilateral into four equal regions without using the midpoints of its sides.

7 Chapter Review

FREQUENTLY ASKED Questions

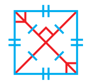
Study | *Aid*

- See Lesson 7.3.
- Try Chapter Review Questions 7 and 8.

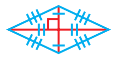
Q: How can you identify a quadrilateral using its diagonal properties and reasoning?

A: Each type of quadrilateral has special properties associated with the diagonals.

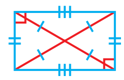
- The diagonals of a square are equal and bisect each other at right angles.



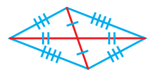
- The diagonals of a rhombus bisect each other at right angles.



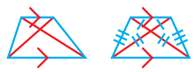
- The diagonals of a rectangle are equal and bisect each other.



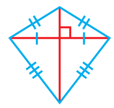
- The diagonals of a parallelogram bisect each other.



- The diagonals of a non-isosceles trapezoid have no special properties. The diagonals of an isosceles trapezoid are equal and intersect to form two different pairs of equal line segments.



- The diagonals of a kite are perpendicular to each other, and one of the diagonals is bisected by the other.



Study | *Aid*

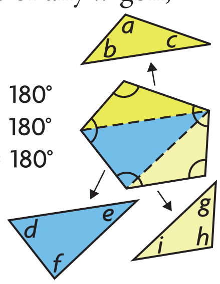
- See Lesson 7.4, Example 1.
- Try Chapter Review Questions 9, 10, 11, and 12.

Q: How can you use reasoning to make conjectures about angle or line properties?

A: First, start with information that you know. For example, to determine how to find the sum of the interior angles of any n-gon, start with what you know:

- every polygon with n sides can be divided into $n - 2$ non-overlapping triangles and
- the sum of the interior angles of each triangle is 180°.

$a + b + c = 180°$
$d + e + f = 180°$
$g + h + i = 180°$



EXAMPLE

Determine the sum of the interior angles in a pentagon.

Solution

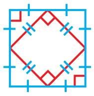
A pentagon has five sides, so it is made up of $5 - 2 = 3$ triangles.
Therefore, the sum of the interior angles of a pentagon is $3 \times 180° = 540°$.

EXAMPLE

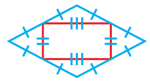
Form your conjecture based on the facts you know. It seems as if the sum of the angles of a polygon is always the sum of the angles in each triangle. There are always $n - 2$ triangles in each n-gon, so the sum will always be $(n - 2) \times 180°$.

Q: What properties do the midpoints of a quadrilateral have?

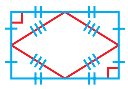
A:
- The midsegments of a square form a square.



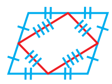
- The midsegments of a rhombus form a rectangle.



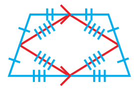
- The midsegments of a rectangle form a rhombus.



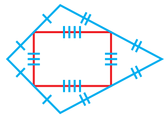
- The midsegments of a parallelogram form a parallelogram.



- The midsegments of an isosceles trapezoid form a rhombus.



- The midsegments of a kite form a rectangle.



Study Aid

- See Lesson 7.4, Example 1.
- Try Chapter Review Question 13.

Q: How can you solve problems involving polygons?

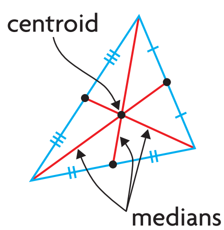
A: Look for a property of the polygon that seems related to the problem. Then, try to apply to the problem.

Study Aid

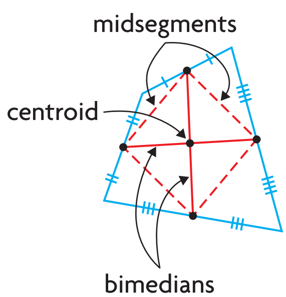
- See Lesson 7.5, Examples 1 and 2.
- Try Chapter Review Questions 13, 14, and 15.

EXAMPLE

To locate the centroid of a triangle, draw the medians. These are lines from each vertex to the midpoint of the opposite side. The triangle balances on each median. So, the centroid lies on their intersection.



To locate the centroid of a quadrilateral, draw the bimedians. These are lines between midpoints of opposite sides. The quadrilateral balances on each bimedian. So, the centroid lies at their intersection.

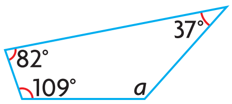


PRACTICE Questions

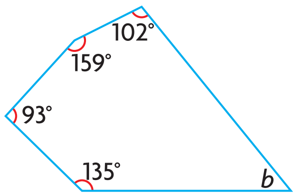
Lesson 7.1

1. Calculate the missing angle in each case.

a)



b)



2. Bob claims that the sum of the interior angles of a regular octagon is $900°$. Is he correct? Justify your decision.

3. The formula for calculating the sum of the interior angles of any n-gon is $(n - 2) \times 180°$.

a) Explain why 2 is subtracted from n.

b) Explain why $(n - 2)$ is multiplied by $180°$.

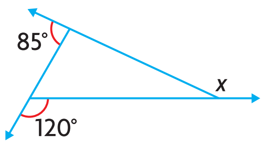
Lesson 7.2

4. a) Calculate the measure of each interior angle of a regular 25-gon.

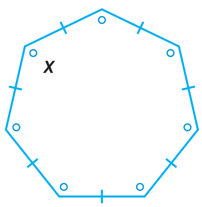
b) What is the measure of each exterior angle?

5. Find the value of each unknown.

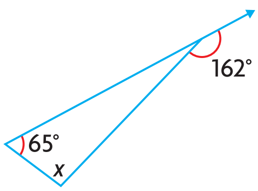
a)



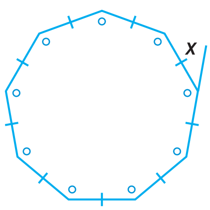
c)



b)

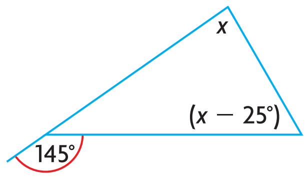


d)

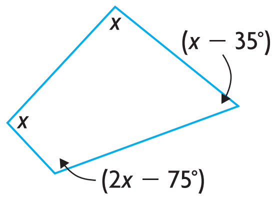


6. Calculate the value of x in each case.

a)



b)



Lesson 7.3

7. Complete each sentence using "always," "never," or "sometimes."

a) A square is ■ a rhombus.

b) The diagonals of a parallelogram ■ bisect its angles.

c) A quadrilateral with one pair of congruent sides and one pair of parallel sides is ■ a parallelogram.

d) The diagonals of a rhombus are ■ congruent.

e) The consecutive sides of a rectangle are ■ congruent.

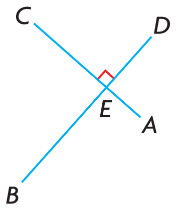
f) The diagonals of a rectangle are ■ perpendicular to each other.

g) The diagonals of a rhombus ■ bisect each other.

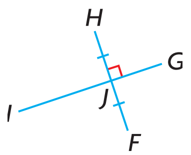
h) The diagonals of a parallelogram are ■ perpendicular bisectors of each other.

8. Describe the possible type(s) of quadrilateral that could be made with each set of diagonals. Justify your answers.

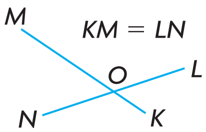
a)



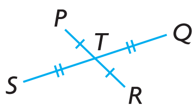
b)



c)



d)



Lesson 7.4

9. Karim wanted to determine all the types of quadrilaterals whose midsegments form either a square or a rectangle. Create a conjecture for Karim's problem, then either support it with examples or disprove it with a counterexample.

10. Test this conjecture: "At least one median in every triangle is the angle bisector at the vertex it intersects."

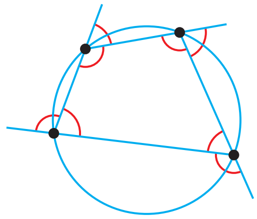
11. Test this conjecture. "The median from the vertex joining the two equal sides of an isosceles triangle always bisects the angle formed by the two equal sides." Support your reasoning with examples.

12. Test this conjecture. "A median of a triangle always divides the area of the triangle in half." Support your reasoning with examples.

Lesson 7.5

13. What kind of figure is formed by joining the midpoints of the sides of a polygon? Explain your answer.

14. Draw a circle and plot four points on it. Join adjacent points to form a quadrilateral. Extend a side at each vertex.

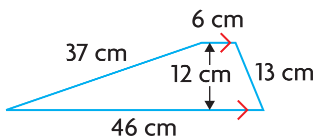


a) Compare each interior angle with the exterior angle at the opposite vertex. What do you notice?

b) Repeat your angle measurements for a quadrilateral whose vertices cannot all be plotted on one circle.

c) Form a conjecture from your observations, and explain how to test it.

15. A trapezoid has parallel sides 6 cm and 46 cm. The other sides measure 37 cm and 13 cm.



a) Determine the length of the bimedian joining the two non-parallel sides.

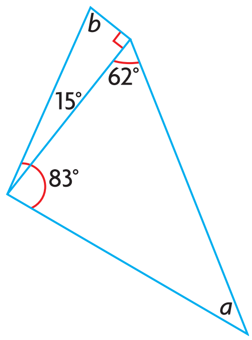
b) The distance between the parallel sides is 12 cm. How far is the bimedian from each?

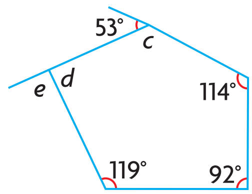
7 Chapter Self-Test

1. What is the relationship between the number of sides of a polygon and the number of diagonals that can be drawn from one vertex?

2. Asad designed a tabletop in the shape of a regular pentagon. His teacher suggested he redesign it as a regular hexagon. By how much would each interior angle change?
   A. 12° **B.** 30° **C.** 36° **D.** 180°

3. Determine the missing angles in each case.

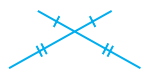
   a)


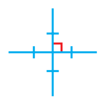
   b)


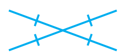
4. Use words or pictures to explain why the sum of the exterior angles of a convex pentagon is 360°.

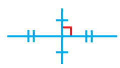
5. Hannah cut a quadrilateral from a piece of cardboard. The diagonals were congruent, perpendicular, and bisected each other. Which type of quadrilateral did Hannah cut out?
   A. kite **B.** rectangle **C.** rhombus **D.** square

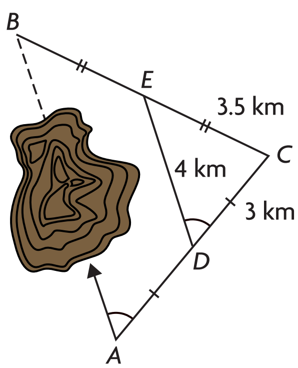
6. Match the quadrilateral to the picture of its diagonals.

   a) rectangle **i)** 

   b) isosceles trapezoid **ii)** 

   c) square **iii)** 

   d) rhombus **iv)** 



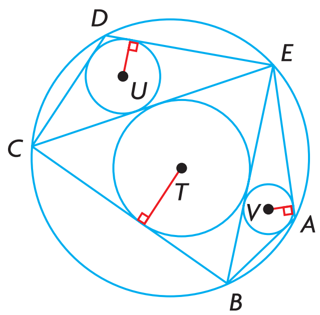
7. A straight train tunnel is being built through a mountain, as shown. The construction company needs to know the length of track needed from point A to point B. Determine the distance using the diagram. Explain your method.

8. Every median in a triangle can be divided into two parts: the length from the vertex to the centroid, and the length from the centroid to the midpoint of the opposite side. Test the conjecture that the longer segment is twice the length of the shorter segment, for the medians in any triangle.

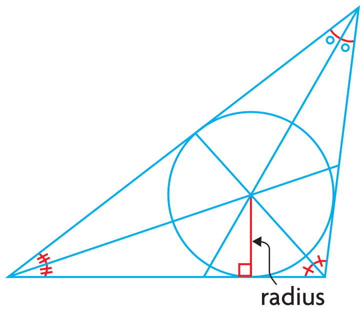
7 Chapter Task

Pentagonal Properties

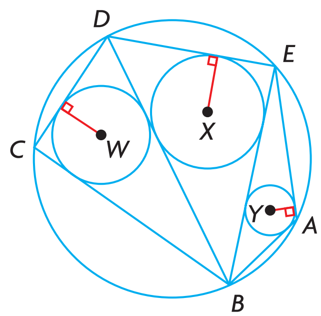
Denise is putting together a poster presentation for the regional Math Fair. She decides to research geometric properties of pentagons.



1. She draws a large circle and marks five points on its circumference.
2. She draws a pentagon using the points on the circle as vertices.
3. She divides the pentagon by drawing diagonals from one vertex to the two opposite it.
4. She draws an inscribed circle in each triangle. To do this, she determines the centre of the circle by locating the intersection of the lines that bisect the interior angles of the triangle. Then she calculates the radius by drawing a line from the centre of the circle perpendicular to any side of the triangle.



Denise divides the pentagon into triangles a different way. Then she measures the radii for the new set of inscribed circles. She discovers that the sums of the radii were equal. She thinks this must be true for all pentagons drawn on a circle.



? How can you confirm or deny Denise's conjecture?

A. Construct a pentagon using Denise's method.

B. Does the sum of the radii of the inscribed circles depend on how you divide the pentagon into triangles?

C. Will Denise's conjecture work for any pentagon? Support your answer with examples.

D. Determine whether a result similar to that in part B is true for all polygons inscribed in a circle.

Record your results in a table.

Task *Checklist*

- ✔ Did you check your geometric constructions for accuracy?
- ✔ Did you test the conjecture with several examples?
- ✔ Did you use a table to organize your results?
- ✔ Did you use appropriate math language?