The AIMMS Interface to Constraint Programming

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Outline

- Motivation
- Language Extensions
  - Basic CP Functionality
  - Global Constraints
  - Scheduling Problems
- Graphical Objects
- Search and Solver Interface
- Demo

Constraint Programming

- Origins in Artificial Intelligence, Logic Programming, and Operations Research
- Modern CP (1994-)
  - Modular approach to combinatorial problem solving
  - Problem structure is modeled through high-level constraints
  - Systematic search combined with inference methods

minimize	max(i, End(i))
s.t.	SequentialSchedule(i, Start(i), Dur(i), End(i))

(minimize makespan for disjunctive scheduling over activities i)

Why CP in AIMMS?

- CP has intuitive and expressive **modeling** syntax
  - arbitrary logical and algebraic expressions
  - constraints can be nonlinear, non-convex, non-continuous, …
  - special 'global constraints' such as alldifferent, scheduling, …
- CP has powerful **solving** methodology
  - particularly effective for highly combinatorial problems, e.g., rostering, scheduling, …
- Increased need for **integrated** methods (e.g., MIP+CP)
  - constraint-based column generation
  - logic-based Benders decomposition
  - large neighborhood search

CP modeling examples

- Variables range over a finite domain:
  $v \in \{a,b,c,d\}$, $start \in \{0,1,2,8,9,10\}$
- Algebraic expressions:
  $x^3(y^2 - z) \geq 25 + x^2 \cdot \max(x,y,z)$
- Variables as subscripts:
  $y = cost[x]$ (here y and x are variables, 'cost' is an array of parameters)
- Reasoning with meta-constraints:
  $\sum_i (x_i > T_i) \leq 5$
- Logical relations in which (meta-)constraints can be mixed:
  $((x < y) \text{ OR } (y < z)) \Rightarrow (c = \min(x,y))$
- Global constraints (a.k.a. symbolic constraints):
  $\mathit{AllDifferent}(x_1,x_2, \ldots,x_n)$

Why AIMMS for CP

- CP is popular among dedicated professionals, but
  - considerable learning curve
  - CP solvers typically have low-level interface (Prolog, C++, scripting)

- AIMMS offers more gentle access to CP technology
  - modeling style is similar to existing AIMMS syntax
  - AIMMS is solver (i.e., vendor) independent
  - easy integration of MIP, QP, NLP, and now CP solvers
  - AIMMS offers advanced graphical interface

Why CP for CAPD audience?

- Industrial design and operations problems often involve
  - planning
  - scheduling
  - optimization
- CP/MILP Decomposition methods can be particularly effective
  - column generation [Junker et al., 1999]
  - logic-based Benders [Jain & Grossmann, 2001] [Hooker, 2007]
  - multi-stage scheduling [Harjunkoski & Grossmann, 2002]
  - …and many more (see, e.g., CPAIOR conference)

- AIMMS naturally facilitates the implementation of such hybrid methods, now also including CP

Selected Related work

- Existing modeling languages from OR community
  - AMPL: extension with CP [Fourer & Gay, 2002] (not released though)
  - XPRESS Mosel: CP extension, but solver dependent (CP Kalis)

- Existing modeling languages/systems from CP community
  - OPL: great for professionals, but solver dependent (IBM-ILOG), and perhaps not as gentle as AIMMS
  - Comet: CP, MIP and LS technology, but solver dependent
  - Zinc: solver independent, no NLP, no graphical interface, academic release
  - SIMPL: mix between model and algorithm development, academic release

Language Extensions

- Basic CP functionality
- Global constraints
- Scheduling problems

Basic CP functionality in AIMMS: Variables

- Variable types
  - continuous: not available for CP
  - integer: available for MIP, MINLP, and CP
  - **element variables**: new, available only for CP

```
ELEMENT VARIABLE:
  identifier     : SupplyingWarehouse
  index domain   : f
  range          : Warehouses ;

VARIABLE:
  identifier     : SupplyCost
  definition     : sum(f, SupplyCost(f, SupplyingWarehouse(f))) ;
```

set-based type checking for element variables

Basic CP functionality in AIMMS: Constraints

- Existing AIMMS syntax
  - standard arithmetic, logical, and set related operators
  - for MIP, these can be applied to condition an index set
- CP extension
  - semantic change: allow variables to appear in these expressions
  - minimal change to the AIMMS user

```
CONSTRAINT:
  identifier : MyTableConstraint
  definition : (Var1, Var2, Var3) in MyThreeDimRelation ;
```

MyThreeDimRelation represents a set of allowed tuples
(this is called a 'Table' constraint)

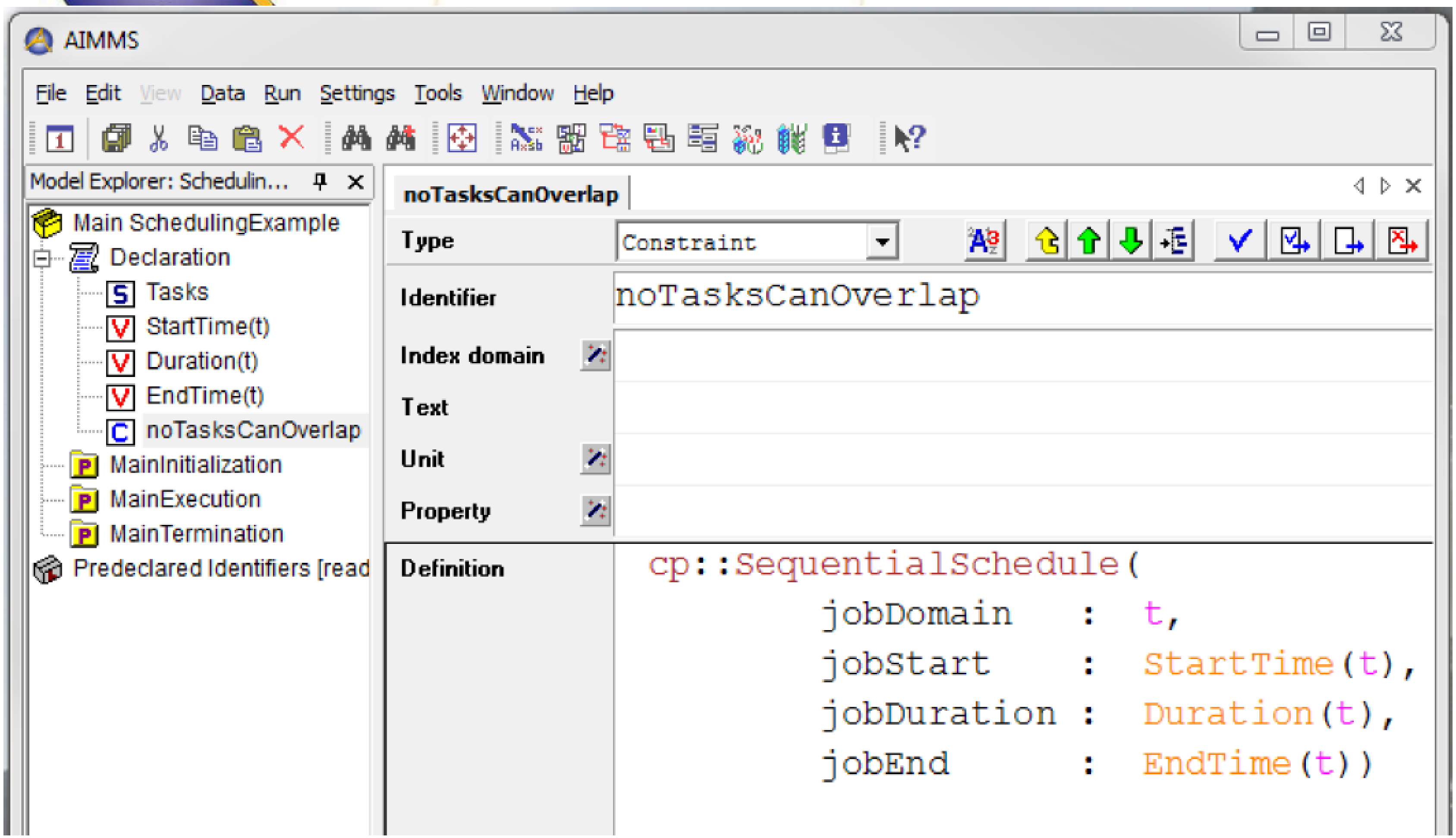
Global Constraints in AIMMS

- Global constraint catalog contains 354 global constraints from the literature…
- AIMMS offers all common global constraints (plus specific constraints for scheduling)

cp::AllDifferent	cp::Lexicographic
cp::BinPacking	cp::ParallelSchedule — *cumulative*
cp::Cardinality	cp::Sequence
cp::Channel	cp::SequentialSchedule — *unary*
cp::Count	

```
CONSTRAINT:
  identifier : DifferentColors
  definition : cp::AllDifferent(i, ColorOfItem(i)) ;
```

Representing Scheduling Problems

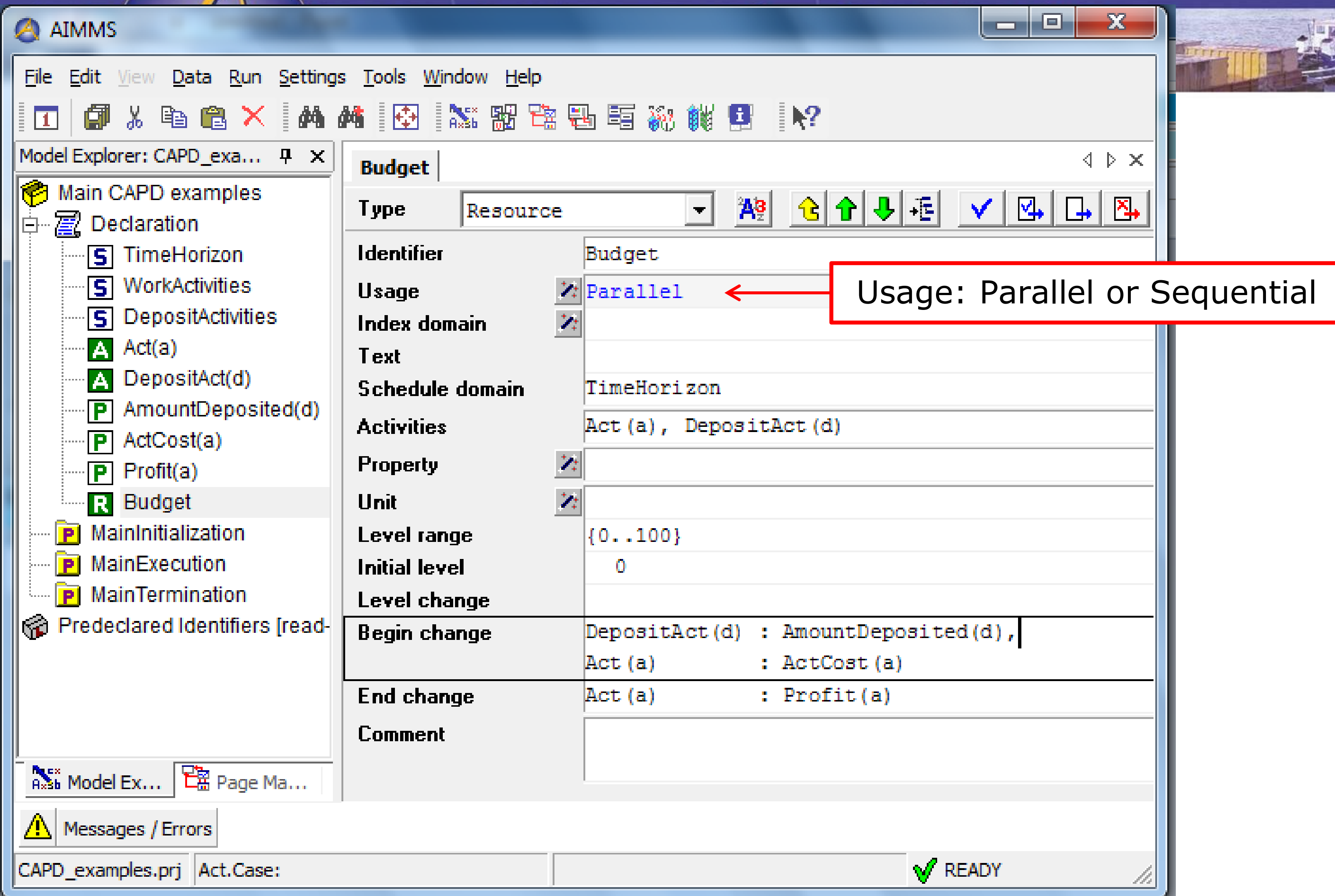


Representing Scheduling Problems: Advanced Models

- Advanced functionality
  - **Activities**: objects that must be scheduled
  - **Resources**: are impacted by the execution of activities

- Activities
  - Each activity `Act` has automatically associated variables
    - `Act.Begin`
    - `Act.End`
    - `Act.Length`
    - `Act.Size`
    - `Act.Present`
  - These can be used as normal variables elsewhere in the model

Resource Representation



Global Scheduling Constraints

- For advanced scheduling models, AIMMS offers the following global constraints

```
cp::BeginAtBegin         cp::EndBeforeBegin
cp::BeginAtEnd           cp::EndBeforeEnd
cp::BeginBeforeBegin     cp::Alternative
cp::BeginBeforeEnd       cp::Span
cp::EndAtBegin           cp::Synchronize
cp::EndAtEnd
```

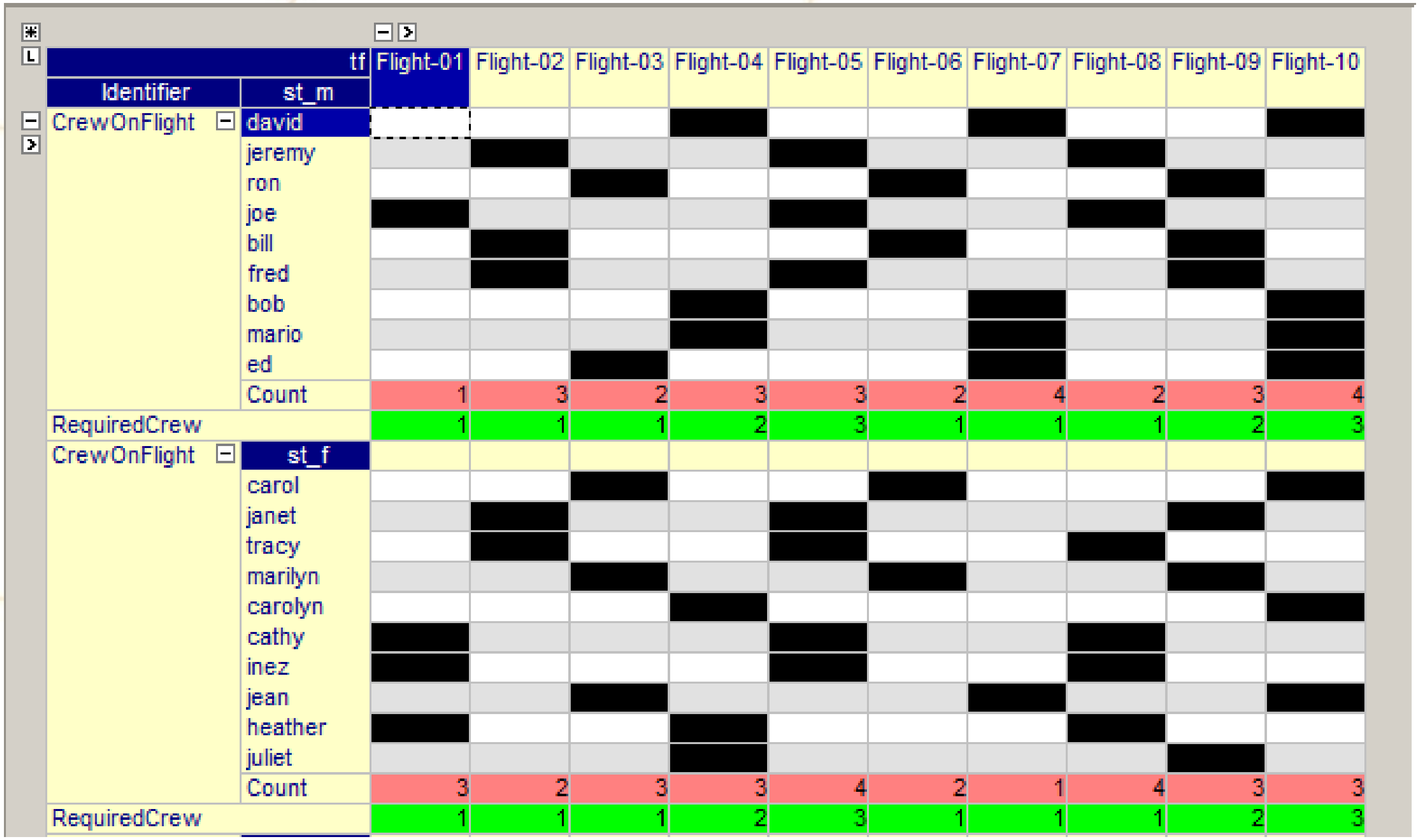
- Allows to build complex ‘hierarchical’ scheduling problems

Graphical Objects

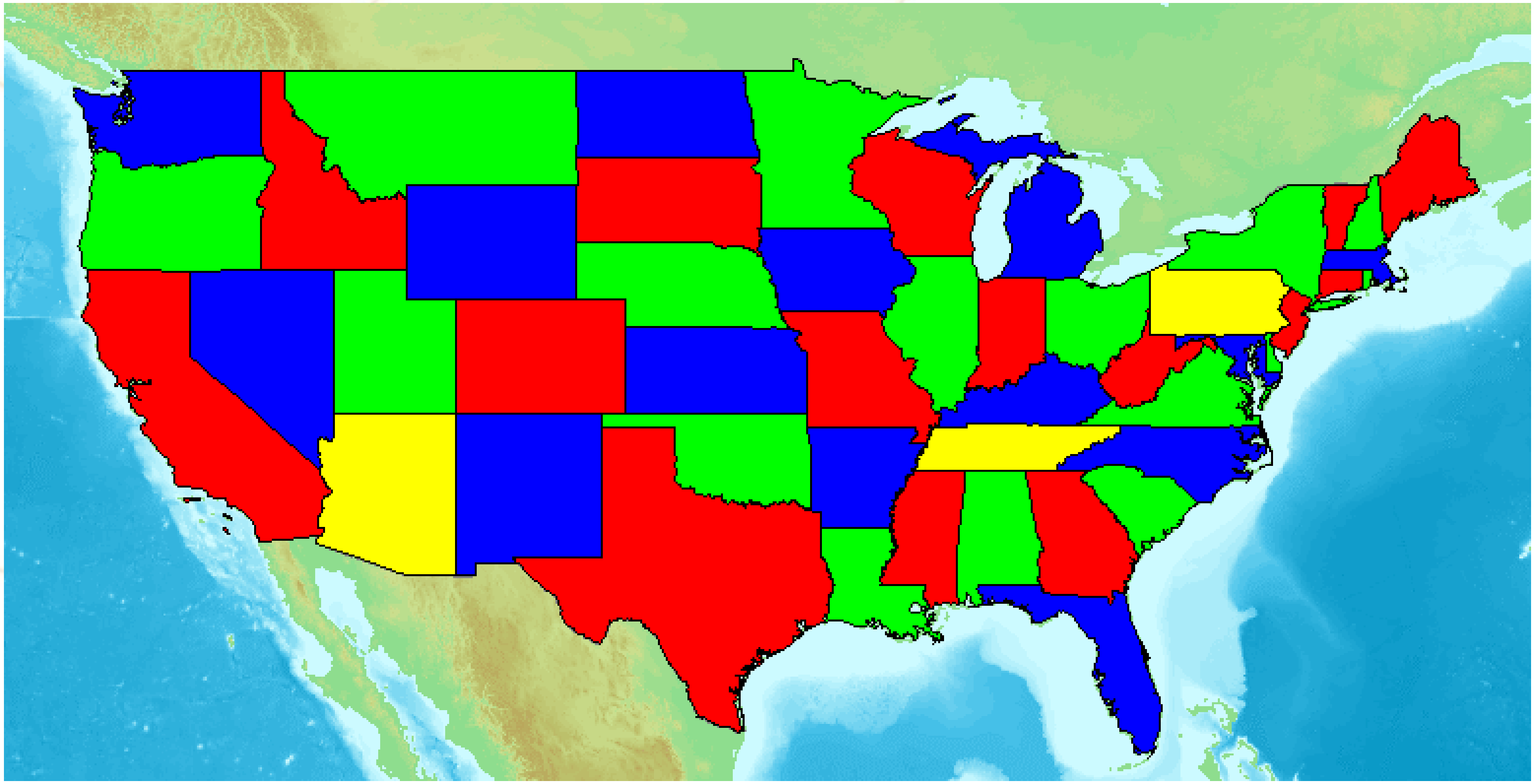
- AIMMS offer graphical user interface
  - display solution
  - what-if analysis (change data and resolve from output page)
  - end-user application (deployable)
- Also CP objects can be directly linked to graphical objects

- Examples:
  - Pivot tables
  - Network objects
  - Gantt charts
  - …

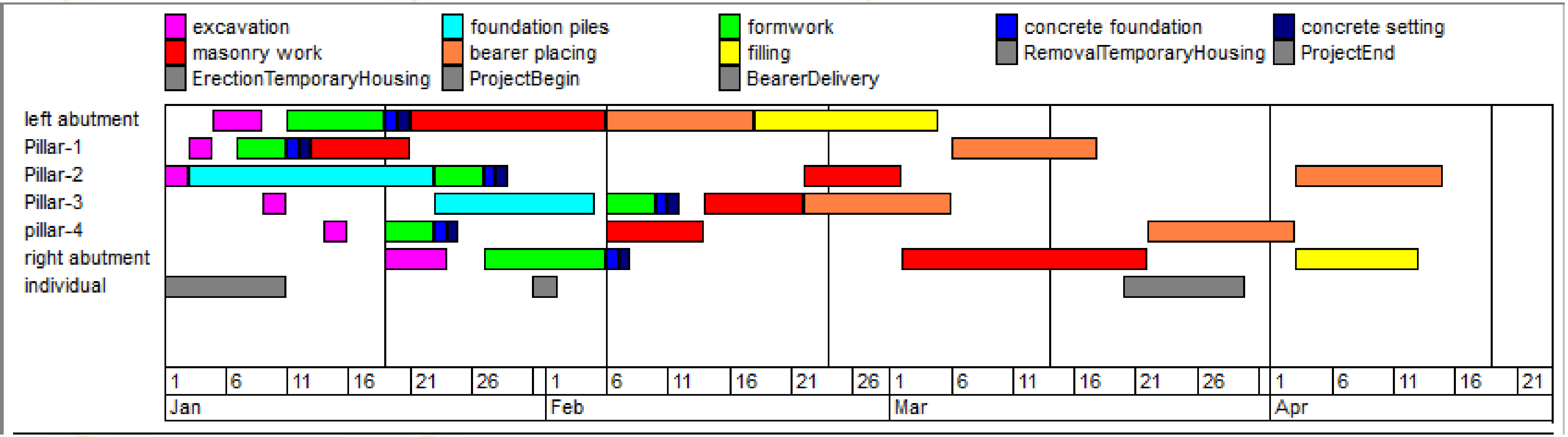
Pivot Table for Crew Scheduling



Network Object for Map Coloring



Gantt Chart for Construction Project Scheduling



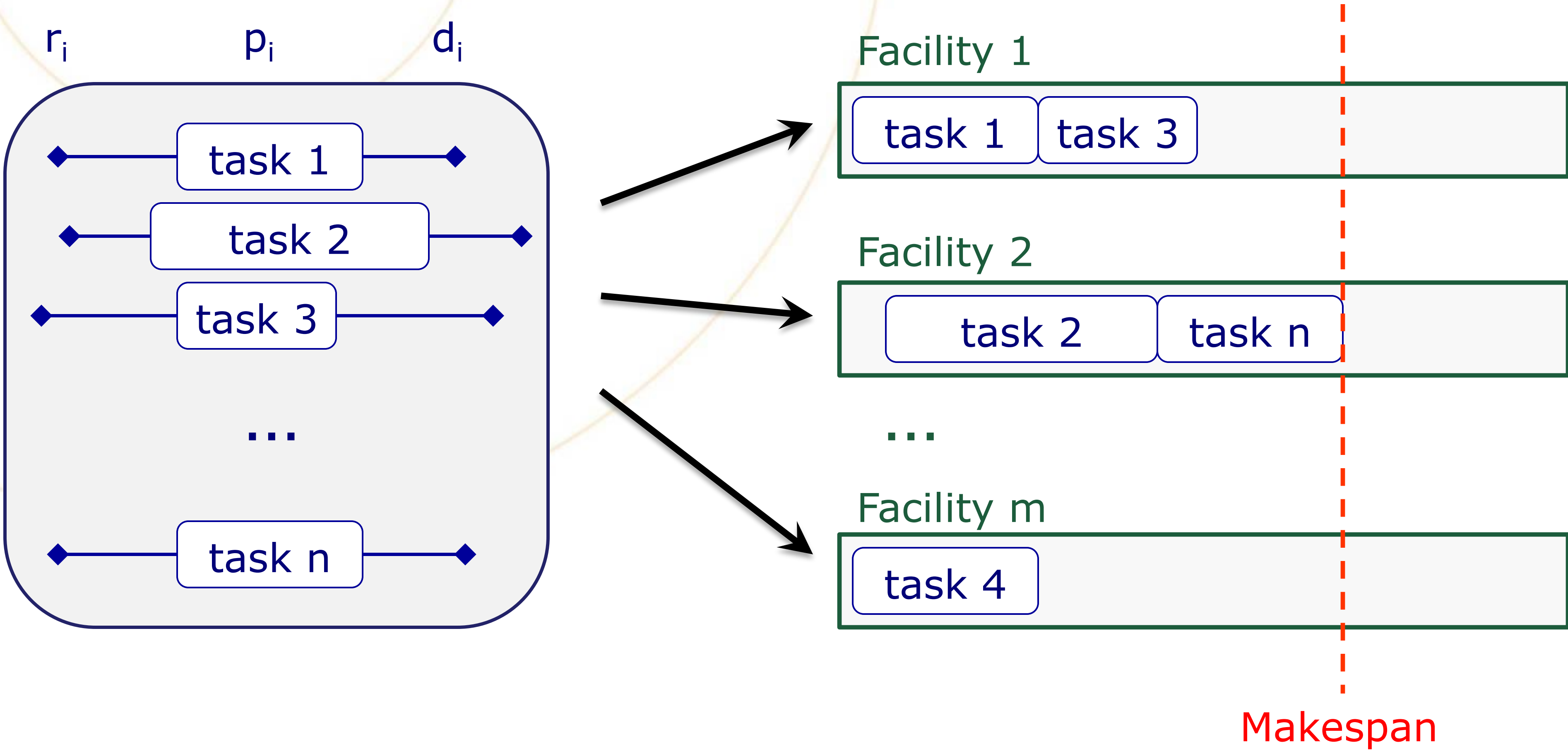
Search Strategies

- Several CP solvers offer the possibility to declare a search strategy in the model

- At the modeling level, AIMMS supports search phases
  - ‘priority’ field for a variable (similar to MIP)
  - first branch on group of variables with the first priority

- Furthermore, all common search strategies can be selected as ‘solver option’
  - depth-first, random restarts, impact-based, …

Demo: Logic-Based Benders for Task-Facility Allocation



Benders Decomposition

Assign tasks to facilities (MIP)

task assign-ments T(f) ↓ ↑ Benders cuts

Find schedule for each facility f (CP)

min Makespan

s.t. all tasks are assigned

Makespan ≥ totalLoad(f)/Capacity(f) for all f

Benders cuts; LBs and feasibility

min Max(EndOf(T(f)))

s.t. ParallelSchedule(T(f), Capacity(f))

[Hooker, 2007]

Summary

- The AIMMS interface to Constraint Programming offers gentle access to state-of-the-art CP technology

- Minimal changes to existing syntax
- Basic and extended CP functionality
  - global constraints, scheduling algorithms, …
- CP objects can be directly displayed in graphical interface
- Wide range of other solvers readily available
  - allows to easily integrate with MIP, NLP, …

- Free academic license available
- http://www.aimms.com/operations-research/mathematical-programming/constraint-programming