Re-Match: A Two-stage Dynamic Scheduling Algorithm on Wireless Mesh Network

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Abstract-In highly dynamic wireless mesh networks, channel quality variations will affect network performance seriously. Channel assignment and scheduling algorithms have been applied in wireless mesh networks (WMN), so as to maximize network resource utilization. However, existing seminar works are mainly focusing on given set of channel quality values, which have not considered the channel quality variations in time scale. With highly dynamic channel quality, scheduling algorithm might possibly executed time to time, which will eventually deteriorate network performance. In this paper, we propose a stochastic programming model in order to reduce highly dynamic scheduling overhead and improve network utilization in heterogeneous wireless mesh networks. Since it has been proved that, achieving the optimal result is \mathcal{NP} -hard. A heuristic solution with two-stage maximum rematching algorithm is proposed, and simulation results show that, our two-stage dynamic scheduling algorithm is efficient as channels in network are highly dynamic on communication quality.

I. Introduction

The emergence of wireless mesh network (WMN) provides an effective solution in providing mobile terminals with versatile communication capability. Mesh routers for example, may take advantages of multiple radio interfaces, and achieve a relatively high throughput in dynamic wireless environment. With the increasing number of channels and radio interface, the designing complexity also increases accordingly. The first difficulty lies in heterogeneity, where in a heterogeneous WMN, transmission power, bandwidth, and available channels are all different to each node. Effective scheduling algorithm is hard to achieve due to complicated network structure and uncontrollable network factors such as power, bandwidth and available channels. The second difficulty lies in the channel variations. In wireless communications, the channel quality would possibly affected by geomagnetic conditions, signal propagation effects, and even weather conditions. During the transmission time, there would be a series of possible channel quality values, which are varying in different time span. All these values can be available by statistical method, as in a long period, channel quality value distribution is relatively steady.

Throughput optimization in MCMR WMN is very challenging and hard to tackle, because the highly dynamic network

scenario and very complicated network factors affect the network performance seriously. Many research works have been done on *MCMR* WMN scheduling algorithm [1] [9]. Some jointly study the channel assignment and routing algorithm in achieving throughput optimization algorithm [4]. Some of the research works study the channel assignment and link scheduling algorithm separately [3] [5]. Some jointly work on optimizations consider channel assignment, routing, power control and route selection [8] systematically and improve network performance dramatically. Some of the studies are concerning the bandwidth allocation in an optimized way, especially according to the traffic demand [8] [13].

In this paper, we make an investigation on the scheduling algorithm of *MCMR* WMN as dynamic channel states are presented. Contributions of this paper are listed as follows:

- 1) Re-match Method and Stochastic Programming Model: We proposed a stochastic programming model, and it is different to previous programming models. In our stochastic programming model, channel parameters are stochastic and the distributions of channel values are presented by a consequence of values. A two-stage matching algorithm is proposed in dealing with the stochastic programming difficulty. And our method is called "Re-match", since it is a two stage matching process in achieving a heuristic result.
- 2) Method on Achieving Channel Quality Values: In this paper, we propose a statistical method in achieving the channel quality values during a period of time. Our proposed method can adaptively measure the distribution of channel quality in an efficient way.
- 3) Distributed Matching Algorithm: In case of dynamic variations on available network resources, our proposed scheduling algorithm can be executed in a distributed way, which is important to WMN environment.

The remainders of this paper are organized as follows: In section II, related work is introduced. In section III, system model and problem formation are presented in description of our problem. In section IV, we make a thorough analysis and model our problem. Algorithm description is introduced in



section V. In section VI, we make performance evaluations by simulations. In section VII, we draw our conclusions and introduce the future work.

II. RELATED WORK

A. Multi-Channel and Multi-Radio Network

Capacity upper bound and lower bound for MCMR WMN are estimated according to the static and dynamic link channel assignment [3]. M. Alicherry, etc, [4] have proposed a joint channel assignment and routing optimization algorithm, unlike ours, it focuses on the infrastructure mesh networks. In [9] [10], Kyasanur and Vaidya present the capacity of multichannel wireless networks scale with respect to the number of orthogonal channels. Since the radio interface and channels are not available as theoretical results have assumed, it would not be applicable to MCMR WMN with limited number of channels. A distributed algorithm is proposed in [8] for spectrum allocation, power control, routing, and congestion control in wireless networks. Given a feasible spectrum allocation, the algorithm can effectively control the transmission powers. But they also assume that the spectrum can be divided into sufficient number of sub-channels. It is different to our models, because we don't assume every mesh router adopts optimal power transmission protocol and has sufficient channels. It is a more generalized and realistic model. An Interference-aware channel assignment is also in mesh routers and co-located wireless networks [5]. The proposed algorithm BFS-CA is a dynamic, interference-aware channel assignment algorithm. Scheduling of wireless data is considered in multi-carrier wireless data systems [6]. Different carriers have channel rates separately, and a scheduling algorithm is needed in order to enhance the performance of WiMax access point. Unlike ours, they only consider the one-hop scenario, and it is a data scheduling algorithm instead of a time-slot scheduling algorithm as [1] has proposed.

B. Opportunistic Network

Efforts are needed before finding an opportunistically optimal channel and time period to transmit packets. As mentioned in [16], it provides transmitters more channels, and software defined cognitive radio systems strengthen the multi-channel systems, which proves to be more intelligent by applying programmable paradigm embedded in hardware. Software radio system can intelligently tune the frequency bands and select among different modulation techniques, aiming at selecting the optimal frequency for transmissions [17]. Cognitive radio system, however, provides a newly built smart way to use radio frequencies. Due to the collaborative awareness on network environment, the system can make a better usage on network resources for its users [18].

Many seminar works have been proposed in order to make a better utilization on the time-varying channels [20] [21] [19]. Seminar work in [20] presents a pure threshold based algorithm on optimal stopping strategy of channel probing. It is a tradeoff between channel quality reward and probing cost. [21] propose a distributed scheme, which models the probing

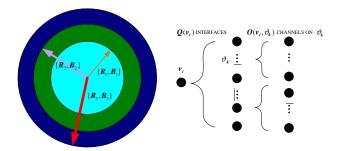


Fig. 1. Communication Resource Demonstration of MCMR Routers

procedure as a contention and threshold based process. Seminar work in [20] assumes that channel state is independent of the state of other channels during transmission. It is true if we take channel state as the only metric for opportunistic transmission. However, in real network deployment, if the channels are not enough, the links in network would congest for limited number of channels. Selecting one channel would face challenges, one is that the larger reward channels would possibly preferred by all channels and the correlation between channels and node deployment [20] would make the contention even worse. Another challenge is that, as the time progresses, more channels are probed, and the rest of the channels are unused by other transmissions are becoming less, which would the contention on channel selection.

III. SYSTEM MODEL

In this paper, we assume that there is a set V of mesh routers deployed in a plane. Each node $v_i \in V$ is a mesh router in network $G = \{V, E^T\}$, and we will illustrate the extended edge set E^T in the following paragraph. Let $Q(v_i)$ denote the number of radio interfaces on mesh router v_i . We assume that each node transmits at different power level on different wireless interfaces. Considering MIMO and OFDM technology, each router can effectively merge and split wireless bandwidth of each radio interface, and the number of orthogonal channels on interface ϑ_k of node v_i is $O(v_i, \vartheta_k)$. The notation $\sum_{1 \leq k \leq Q(v_i)} O(v_i, \vartheta_k)$ denotes the total number of

orthogonal channels of node v_i , while the orthogonal channel set of node v_i on interface ϑ_k can be denoted as $C(v_i,\vartheta_k,\lambda_j)$, where $1 \leq k \leq Q(v_i)$, and $1 \leq j \leq O(v_i,\vartheta_k)$. Each node v_i uses a transmission range $t(v_i,\vartheta_k)$ on interface ϑ_k to send data. As shown in Fig1, the bandwidth on interface ϑ_k can be denoted as $B_k = \{b_{1k}, b_{2k}, ..., b_{O(v_i,\vartheta_k),k}\}$, where the elements in the set b_{ik} are the values of bandwidth on channel i of interface ϑ_k . And $\|B_k\|$ denotes the total bandwidth on interface ϑ_k , where $\|B_k\| = \sum_{i=1}^{O(v_i,\vartheta_k)} b_{ik}$.

On node v_i , the transmission range $R_k \propto t(v_i, \vartheta_k)$, and $\{R_k, B_k\}$ characterize the heterogeneous network on radio interfaces. $e(v_i, v_j, \vartheta_k) \in E^T$ denotes extended undirected edge between node v_i and v_j on interface ϑ_k . If the following condition is satisfied, there exist an edge $e(v_i, v_j, \vartheta_k) \in E^T$

between v_i and v_j :

$$||v_i - v_j|| \le \min\{t(v_i, \vartheta_k), t(v_j, \vartheta_k)\}$$

The notation $\|v_i - v_j\|$ denotes the euclidian distance between node v_i and v_j . Note that, the number of radio interfaces would be identical to each other, which can be denoted as ϑ_k . Concurrent transmissions between neighbor nodes are available if and only if the following two conditions are satisfied:

- Two transmissions tⁱ_r and tⁱ_r are not using the same radio interfaces, which is denoted as (tⁱ_r) ≠ I(t^j_r).
- The second condition is that two transmissions belong to the same interface but they belong to different orthogonal wireless channels, that is, the radio interfaces of two transmissions are identical, $I(t_r^i) = I(t_r^j)$, and the orthogonal channels $OC(t_r^i) \neq OC(t_r^j)$.

Interference models on WMN generally fall into three types, which are proposed as the Protocol Interference Model (PrIM), Fixed Power Protocol Interferences Model, and RTS/CTS Model [1]. In this paper, we mainly focus on the RTS/CTS model, since it is widely applied in WMN. Our scheduling algorithm is relatively independent to interference models, and can be applicable to other interference models mentioned above with moderately modifications.

IV. PROBLEM FORMATION

On node v_i , the transmission power on interface ϑ_k is denoted as $t(v_i, \vartheta_k)$, which is determined by power control protocol. We rank the interfaces by transmission power on every node in network with descending order. It can be listed as $\vartheta^{(1)}$, $\vartheta^{(2)}$, ..., $\vartheta^{(Q)}$. Generally speaking, the number of power level is more than that of interfaces equipped on nodes in WMN. The interface with maximum power level is denoted as $\vartheta^{(1)}$, and the lowest power level is denoted as $\vartheta^{(2)}$, the ranking list can be denoted as $t(\cdot, \vartheta^{(1)}) \geq t(\cdot, \vartheta^{(2)}) \geq ... \geq t(\cdot, \vartheta^{(Q)})$.

We use the conflict graph F_G proposed in [1] to represent the interference in WMN graph. In our network model, each vertex of F_G corresponds to an edge $e(v_i,v_j,\vartheta_k)\in E^T$ in WMN, which is denoted by $l(v_i,v_j)$. Sub-graph $E^T(\vartheta^{(i)})$ is the network graph formed by edges of the same interface, such that $e(v_i,v_j,\vartheta^{(i)})\in E^T(\vartheta^{(i)})$, where $\vartheta^{(i)}$ is the i^{th} power level interface.

Given any two different level of interface $\vartheta^{(i)}$ and $\vartheta^{(i)}$, if i < j, and $e(v_i, v_j, \vartheta^{(j)}) \in E^T(\vartheta^{(j)})$, a link exists between node i and j on interface $\vartheta^{(i)}$, that is $e(v_i, v_j, \vartheta^{(i)}) \in E^T(\vartheta^{(i)})$. Channel assignment ς assigns each node v_i with a channel in the available orthogonal channels set. Let $\varsigma(v_i)$ denotes the set of different channels assigned to the node, which are selected among all the available channels of node , ranging from 1 to $Q(v_i)$.

The first goal of our scheduling algorithm is to propose a valid interference-free channel assignment and schedule among nodes in WMN. The second goal is to maximize the available bandwidth for each node. Since it is a WMN graph, there are two levels of research issues to consider in dealing with the two goals mentioned above. The first level is channel assignment, and the second level is link scheduling. In single channel wireless network, there is no need to do channel assignment. If the links on interference graph F^{G} are using the same channel, we have to schedule the link on the same channel in order to avoid the collision.

Given a network graph G=< V, E>, and weight of each edge is given, denoted by c_e , the optimal assignment is achieved as

$$\max\{\sum_{e \in E} c_e x_e \left| \sum_{e \in \delta(v)} x_e \le 1, \ \forall v \in V; x_e \in \{0, 1\}, \forall e \in E\}\right.$$

Consider stochastic programming cases, a first stage edge weight c_e , which is the probed channel quality value. And a second stage weight, which is a discretely distributed value, and each edge weight is ranging values in channel quality value set $d_e^1, d_e^2, ..., d_e^r$, with corresponding probabilities $p_1, p_2, ..., p_r$.

$$\max \sum_{e \in E} c_e x_e + \sum_{1}^{r} p_s \sum_{e \in E} d_e^s y_e^s$$

Subject to

$$\sum_{e \in \delta(v)} x_e + \sum_{e \in \delta(v)} y_e^s \le 1 \quad \forall v \in V, \ s = 1, 2, ..., r.$$

 $x_e \in \{0,1\}, y_e^s \in \{0,1\}, \forall e \in E, s = 1,2,...,r.$

Theorem 1: The two-stage stochastic matching is $\mathcal{NP}-complete$

Proof: The theorem has been proved in [22]

V. ALGORITHM DESCRIPTION

A. Achieving the Channel quality Distribution

At probing time point t, channel quality value is \mathcal{Q}_i . $P[a_i \leq \mathcal{Q}_i \leq a_{i+1}] = \frac{N^i}{\mathcal{N}}$, where \mathcal{N} is the total number of probing times and N^i is the total number of times that \mathcal{Q}_i falls into interval $[a_i, a_{i+1}]$. As we do not know channel quality distribution previously, the interval length and total number are difficult to set. If the interval length is too small, $P[a_i \leq \mathcal{Q}_i \leq a_{i+1}]$ would be accordingly small, and the total number of intervals are too large, which would lead to computational difficulty and the difference between channel quality is smoothed by so many interval steps. As the interval length is too large, it would lead to inaccurate statistical channel quality distribution.

In order to solve this problem, we propose an interval adaptation algorithm. The algorithm works as follows:

Assume that, there are totally \tilde{r} possible intervals for If $P[a_i \leq \mathcal{Q}_i \leq a_{i+1}] > \alpha \frac{1}{\tilde{r}}$, we should split the interval $[a_i, a_{i+1}]$ into two intervals with $[a_i, \frac{a_i + a_{i+1}}{2}]$ and $[\frac{a_i + a_{i+1}}{2}, a_{i+1}]$. If $P[a_i \leq \mathcal{Q}_i \leq \frac{a_i + a_{i+1}}{2}] > \alpha \frac{1}{\tilde{r}+1}$, we do the same interval-split process until $P[a_i \leq \mathcal{Q}_i \leq \frac{a_i + a_{i+1}}{2}] < \alpha \frac{1}{\tilde{r}+1}$.

Algorithm Two-stage Matching Algorithm

- 1: Compute First-stage Maximum Matching results, and the allocation factor \vec{x}
- 2: **for** i := 1 to s do **do**
- 3: Compute Second-stage Maximum Matching results, and $z^s = d^s y^s$
- 4: $\tilde{z} = maxz, \sum_{s=1}^{r} p_s z^s$
- 5: end for
- 6: if $\tilde{z} = z$ then
- 7: The first-stage result
- 8: else
- 9: The second-stage result
- 10: **end if**

Fig. 2. Two-Stage Matching Algorithm Description

B. Overview of Heuristic Re-Matching Algorithm

Our heuristic algorithm is based on the maximum bipartite matching algorithm. It constructs a conflict graph $F^G=G^\prime$, and makes a transformation, where the link set E^T and channel set I are two node sets on a bipartite graph. If a radio channel is available to link, there is an edge on bipartite graph. Neighboring nodes on conflict graph can not use the same radio interface. In our bipartite matching algorithm, only one channel of an interface in matching set is assigned to the link. The algorithm terminates if all the links in network are assigned a channel to transmit data and all the channels are assigned to links in network.

Let \vec{x} be a first-stage solution, and let $\vec{z} = \vec{c}\vec{x}$. For scenario s, let y^s be a second-stage myopic solution, and let $z^s = d^s y^s$. Let $\tilde{z} = maxz, \sum_{s=1}^r p_s z^s$. If $\tilde{z} = z$, then return and $(\vec{x}, \mathbf{0}, \mathbf{0}, ..., \mathbf{0})$; otherwise, return $(\mathbf{0}, y^1, ..., y^r)$ and $\sum_{s=1}^r p_s z^s$. Pseudo codes of two-stage matching algorithm are listed in Fig 2

C. The Distributed Matching Algorithm

Conversion process and tree decomposition algorithms are referred to similar works in [?] [?], which assume different ID should be assigned to each router. But in real network, especially WMN, it is very hard to make this kind of network management operation before network deployment in highly dynamic network. Ours need not assign each node with different ID. In order to void loop, modify the $\mathcal{N}(v)$ value with a moderately small factor ϵ , where $\epsilon \ll 1$.

Our tree decomposition algorithm is based on the number of neighbors and give a ranking among them; so that, each induced link is assigned a candidate "color" in partition the tree into set of trees from F_1 to F_{\triangle} , where \triangle is the maximum degree of nodes in network.

D. Maximum Weighted Matching

The distributed weighted maximum matching algorithm is executed based on the tree decomposition. Our distributed coloring algorithm can be obtained through coloring algorithm, and maximum weight matching is achieved in a heuristic way.

Algorithm Graph Conversion

- 1: Each node v Compute the number of neighbors $\mathcal{N}(v)$ and send message $\mathcal{N}(v)$ out
- 2: Listen and receive messages
- 3: if $\mathcal{N}(v) > \mathcal{N}(u)$ then
- 4: Direct the edge v to u
- 5: end if
- 6: if $\mathcal{N}(v) = \mathcal{N}(u)$ then
- 7: Random Select v or u and Direct the select one to the other
- 8: $\mathcal{N}(v) + \epsilon$
- 9: end if
- 10: if $\mathcal{N}(v) < \mathcal{N}(u)$ then
- 11: Direct edge u to v
- 12: end if

Fig. 3. Graph Conversion Pseudo-Code

Algorithm Network Partition Approach

- 1: Each node send its calculated number of neighbors $\mathcal{N}(v)$ to all neighbors.
- 2: Order the edges by decreasing $\mathcal{N}(v)$ value, which are the candidate proposed rank.
- 3: Listen and receive messages
- 4: The edge value of rank is determined by end node with higher $\mathcal{N}(v)$.
- 5: output the tree set with determined rank value i, which form the part of tree, F_i .

Fig. 4. Network Partition Algorithm Pseudo code

Li et al. [1], have proposed a heuristic coloring algorithm, which is suitable for distributed execution in WMN.

VI. PERFORMANCE EVALUATION

A. Simulation Configuration

Simulation parameters are listed in FigI. Our simulation environment is a network with many nodes randomly deployed. Each node is assigned a random number of available channels. Radio interfaces are randomly selected from $[1,R_{\rm max}]$. Number of channel is randomly selected from $[1,C_{\rm max}]$. If two nodes are closer, there would be more available channels and interfaces. And each node select the transmitting range according to the power level, which are listed in TABLE I. We compare our algorithm with the pure average value based one-stage matching algorithm. In this paper, we transform the extended graph into confliction graph separately, and use the heuristic algorithm based on coloring problem [14]. Simulations are done independently 50 times on each scenario, and the we compute the average result among them.

B. Simulation Results

As shown in Fig6, with increasing number of neighboring nodes, the throughput achieved by one-stage matching drops seriously, while the two-stage matching algorithm maintains a relatively high throughput. In order to , we change

Algorithm Maximum Weighted Matching Algorithm

- 1: Compute the $\triangle + 1$ -vertex coloring of each F_i .
- 2: Let c_i be the color of F_i
- 3: $\mathcal{M} = \emptyset$
- 4: **for** i := 1 to \triangle do **do**
- 5: **for** c := 1 to $\triangle + 1$ do **do**
- 6: every u such that $c_i(u) = c$ selects the most weight one of its outgoing edges; Let \mathcal{M}_c be the set of edges so selected
- 7: $\mathcal{M} = \mathcal{M} \cup \mathcal{M}_c$
- 8: Remove all vertices of M_c from the graph
- 9: end for
- 10: end for

Fig. 5. Weighted Maximum Matching Algorithm pseudo-code

TABLE I SIMULATION PARAMETERS

Parameters	Value
Avg. Neighbors	4, 8, 12, 16, 32
R_{max}	4
C_{max}	10
Transmitting Range (Meters)	50, 100, 150, 200
Channel quality values	0.4, 0.5, 0.6, 0.7, 0.8, 0.9
Probability values	0.1, 0.1, 0.1, 0.2, 0.5

our channel distribution listed in table I, and the probability distribution has a larger variance. And the values are 0.1, 0.15, 0.25, 0.3, 0.8, 0.9, and the probabilities are listed as 0.1, 0.3, 0.2.0.1, 0.3. As shown in Fig7 with increasing number of neighboring nodes, two-stage matching still maintains a relative high throughput with a relatively high stability factor. Our two stage scheduling algorithm effectively utilize network resources in highly dynamic environment and maintains a relatively high stability . Simulation results in Fig6 and Fig7 show that, ours is a stable and efficient algorithm in an unstable heterogeneous MCMR WMN.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we make an investigation on scheduling algorithm in heterogeneous MCMR WMN. In highly dynamic environment, achieving the optimal value, as proved according to stochastic programming model, is \mathcal{NP} -hard. In this paper Heuristic two-stage matching algorithm can effectively utilize network resources with given distributions. Simulation results show that, in highly dynamic environment, two-stage algorithm could achieve a relatively high network performance. More important, ours can be effectively working in distributed network environment, and it is more applicable in real network deployment. Future works include a better heuristic algorithm and a real distributed WMN deployment so as to verify our simulation results.

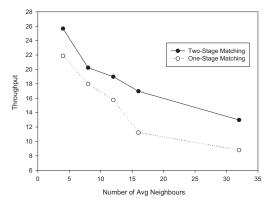


Fig. 6. Throughput with increasing No. of Neighbors

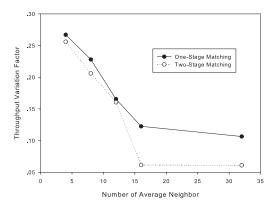


Fig. 7. Stability of Two-stage Matching with Increasing No. of Avg Neighbors

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