Ecological Complexity 12 (2012) 70–74

Contents lists available at SciVerse ScienceDirect

Ecological Complexity

journal homepage: www.elsevier.com/locate/ecocom

Short Note

The Korcak-exponent: A non-fractal descriptor for landscape patchiness

Attila R. Imre [a,*], Josef Novotný [b], Duccio Rocchini [c]

[a] HAS Centre for Energy Research, H-1525, POB 49, Budapest, Hungary
[b] Department of Social Geography and Regional Development, Faculty of Science, Charles University, Albertov 6, 128 43 Praha 2, Czech Republic
[c] Fondazione E. Mach, Research and Innovation Centre, Dept. of Biodiversity and Molecular Ecology, GIS and Remote Sensing Unit, Via E. Mach 1, 38010 S. Michele all'Adige (TN), Italy

ARTICLE INFO

Article history:
Received 18 May 2012
Received in revised form 6 September 2012
Accepted 1 October 2012
Available online 30 October 2012

Keywords:
Korcak-plot
Scaling
Habitats
Power law

ABSTRACT

In this short paper we introduce a proper method to perform Korcak-analysis and obtain the correct Korcak-exponent on a set of patches, embedded into two-dimensions. Both artificial and natural data sets are used for the demonstration. The independence of the Korcak-exponent from the classical (Hausdorff) fractal dimension is also demonstrated.

1. Introduction

Analysis of size and shape of various patches embedded into two-dimensions is a common scientific problem in spatial ecology, since they represent ecological indicators of specific ecological processes acting in the landscape in space and time (Wu et al., 2000). The patches can be islands, lakes, forest fragments or other vegetation patches, soil pores, corrosion patches, etc. The common way to analyze them is the application of some computer-assisted image analysis and the common results of these kinds of analysis are some size-distribution function; very often a fractal dimension. Various fractal dimensions have been used to describe size- and shape-related properties of individual patches, as well as for sets of patches since the pioneering work of Mandelbrot (Mandelbrot, 1982; Martín et al., 2009). Although for a set of statistically similar patches, one can use perimeter–area analysis for fractal analysis; this method requires the accurate knowledge of the areas and perimeters of the patches composing the landscape mosaic. It has been known for some time, that perimeter values are problematic (Loehle, 2011); for example they can be measured with much bigger errors than the corresponding area values (Ken et al., 2008; Zunic and Martinez-Ortiz, 2009); this can be explained quite well with the recently described violation of translational and rotational invariances in digital geometry (Imre, 2006, 2007). Due to the error propagations, all descriptors calculated from the perimeters – like for example the fractal dimension calculated by the perimeter–area relation (Mandelbrot, 1982) – would have errors at least as big as the perimeter-measurement error. For this reason, the Korcak-method – which requires only the knowledge of the areas of the studied patches – seemed to be a good candidate to obtain the fractal dimension with satisfying accuracy of natural or artificial patch-sets (Hastings and Sugihara, 1993; Imre et al., 2011). The applicability of Korcak-method in ecology has been already proved in several times (Korčák, 1938; Sugihara and May, 1990; Imre et al., 2011); it is not our goal here to give just another exercise of this kind. Instead, we concentrate on the methodology behind the Korcak-analysis and on the deemed relationship between the Korcak-exponent and the classical Hausdorff fractal dimension.

As in the practical applications of other techniques used for the study of power law scaling of various phenomena, the use of Korcak-analysis often involves one or more subjective components associated with the data preparation and processing. The properties of the Korcak-method and of the Korcak-exponent are less-known than that of other fractal-methods (Russ, 1994; Imre et al., 2011), therefore this method is widely misused in various fields, including in ecology. The misuse can be realized in two ways. First, by choosing improper limiting sizes, one might obtain an incorrect Korcak-exponent (or one might even obtain an exponent in systems where the scaling – and the underlying statistical similarity – is not true). The second common mistake is the comparison of the obtained Korcak-exponent to fractal dimensions obtained by other methods which can lead to misleading results when studying ecosystems changes over time.

* Corresponding author.
E-mail address: imre.attila@energia.mta.hu (A.R. Imre).

1476-945X/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ecocom.2012.10.001

In this short methodology-paper we would like to show, how to properly use the Korcak-method and what can be the outcome of an improper analysis in terms of introducing a common error which might lead to incorrect Korcak-exponent. In addition, we show that the Korcak-exponent is not related to the Hausdorff fractal dimension as commonly assumed, following the original Mandelbrot's argument (Mandelbrot, 1982). Although the Korcak-exponent is a very good descriptor, any direct comparison with the results of fractal dimensions can be misleading.

2. Korcak-plot, Korcak-exponent and Korcak-dimension

The Korcak-exponent is named after the Czech geographer, Jaromír Korčák (Novotný and Nosek, 2009; Novotný, 2010). Drawing on the method used by his colleague for the determination of cartographic scales, Korčák noticed that the frequency distributions of occurrences of various geographical phenomena in maps typically reveal a highly right skewed distribution with a few large size observations and many small size observations. He documented this empirical regularity on an extensive body of material (Korčák, 1938, 1941).

Having a set of patches with an area-distribution, one can determine the number (N) of patches with area (A) bigger than a threshold area (A_0). By choosing a set of A_0s, one can make a double logarithmic plot; the points might be fitted by a line, satisfying the following equation:

$$N(A > A_0) = kA_0^{-K} \quad (1)$$

where k is a form-constant and K is the so-called Korcak-exponent, also called patchiness-index.

It should be known that for several real samples, the whole set can be fitted only in two (or sometimes more) distinct parts, giving more than one K values. An example is provided by (Imre et al., 2011), where for a woodlot a single K value was obtained for the whole size-range, while for open formations – like arable lands, meadows – two distinct K's were attained; 0.45 in the 100–4000 m^2 range and 1 in the 4000–100,000 m^2 range).

3. Artificial Korcak-exponent by improper choice of A_0

Korcak-plot – and Korcak-exponent – cannot be found in Korčák's original paper (Korčák, 1938). He had some tabulated data (probably the most famous was the island-size distribution of the Cyclades), showing the percentage of island in a size-range. The dividing sizes (3–10–25–50–100–200 km^2) were more or less logarithmically equidistant. Later, this kind of analysis was kept in Korcak-plots, i.e. the limiting sizes were more or less logarithmically equidistant.

Here we would like to demonstrate, how the choice of an improper set of limiting sizes might cause an artificial Korcak-exponent. This can happen, when the total number of patches is small as well as when $N(A > A_0)$ are small (i.e. with bigger island areas). In Fig. 1a, the Korcak-plot of a set (hundred pieces) of artificial patches can be seen. They were generated in a very simple way; for the n-th patch ($1 \leq n \leq 100$) the area had to satisfy the $\log(n) = -\log(A_n) + 4$ equation. After that (as it was proposed previously (Imre et al., 2011)) we prepared a Korcak-plot by using $A_n - \delta$ values as limiting A_0 values (δ is a small real number; in digital geometry when areas are given in pixels, it is usually 1). In that way, we had one patch with area bigger than the "(biggest area)-1", two patches with area bigger than the "(second biggest area)-1", etc. In that case, the slope obtained from our Korcak-plot was $K = 1$. In the next step, we prepared a second (third, etc.) Korcak-plot, where the i-th limiting size (A_{0i}) were chosen to satisfy the $A_i + \delta < A_{0i} < A_{i+1}$ relation. The corresponding N-values can be seen as small open circles. Graphically this means, that the original data-points (full squares) can shift to the right (parallel to the x-axis), all along the small circles, with unchanged y-values. While upward arrows show the data with properly chosen A_0 values, the downside-arrows shows the data with improperly chosen ones; the dotted line represents the fit with an artificial Korcak-exponent. In extreme cases, one can obtain a Korcak-plot where the limiting (gray dashed) line would be the virtually correct shape (linear), but utterly artificial Korcak-line. For small patch-size (where the Ns are high) one cannot see any characteristic difference, while at the "big-area" side (see the magnification in Fig. 1b) one can get $\log N - \log A_0$ plots with $K < 1$ (see the dashed limiting line, in this case it represents $K = 0.66$). Simply by the improper choice of the A_0 values, one might get an artificial K value, or – even as it can be seen in Fig. 1 – an artificial break and double K values. We propose that for correct Korcak-plots of Q patches with R different areas ($Q \geq R$, both are integers and they are equal when all patches have different sizes), limiting areas must be chosen to satisfy the following equation:

$$A_{0i} = A_i - \delta, \quad i = 1 \ldots R \quad (2)$$

where A_i is the i-th biggest area of the set, R is an integer, equal to the number of different areas in the set and δ is a small number; when the areas are given in pixel, $\delta = 1$, when they are given in other units, δ should be equal to the last significant digit of the given area-values (i.e. when $A = 3.765$ km^2, then $\delta = 0.001$ km^2, when $A = 3871$ m^2 then $\delta = 1$ m^2, etc.).

4. An ecological example: the lakes and reservoirs of the Isle of Man

For an ecological example for obtaining an improper Korcak-plot, we analyzed a set of lakes from the Isle of Man with surface areas between 1.1 and 28.7 ha. The area data were taken from the UK Lakes database (www.uklakes.net); some supporting

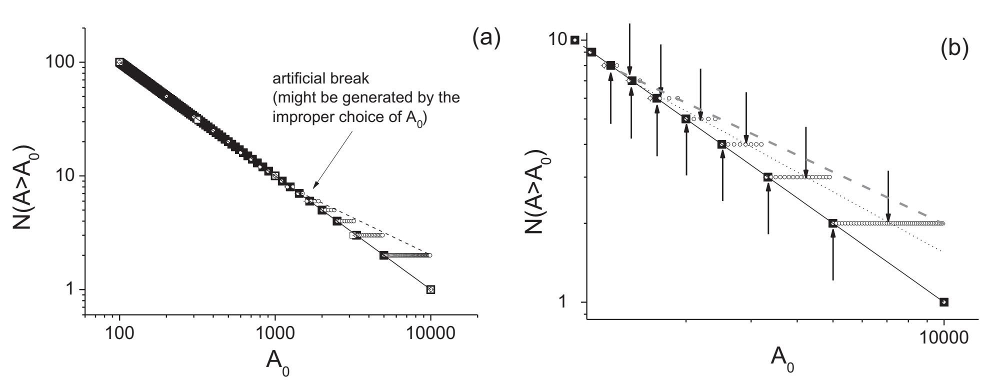

Fig. 1. (a) Korcak-plots with different choices of limiting areas (A_0). (b) Magnification of the high-A_0 part with the artificial fits.

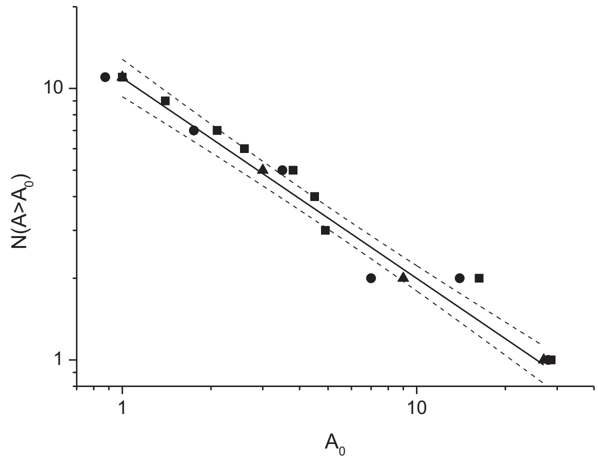

Fig. 2. Korcak-plot for the lakes and reservoirs located on the Isle of Man with three different choices of A_0-sets (circles: A_0-set constructed by dividing the previous limiting value by two, triangles: similar, but dividing by three, squares: A_0s are constructed as $A_i - \delta$). The solid line is a linear fit for the triangles, the dashed ones mark the 90% confidence band, showing that by the choice of the A_0-set by the observer can force the data to give some kind of Korcak-exponent.

information is available at the homepage of the Isle of Man Water and Sewerage Authority Website (http://www.gov.im/water/reservoirs). The reason of this choice is that we wished to analyze a well-defined, but sufficiently small set to demonstrate the effect of the choice of A_0. Most of the bodies of water are artificial reservoirs; the small ones are usually regularly shaped, while the two big ones have more complex shapes (except for the dam side), following the contour dictated by the neighboring hills and valleys. Surface areas are given in ha.

Three different choices for the A_0-set can be seen in Fig. 2. The first one (Option 1) is the one proposed by us in Eq. (2), when A_0s are chosen as $A_i - \delta$ (black squares), δ is 0.1 ha in this case, taken from the accuracy of the given data. For the next one (Option 2), we took an A_0-set as (28, 14, 7, 3.5, 1.75, 0.875), taking the first value slightly below the area of the biggest lake and dividing by two at each step (similar method used in box-counting, by measuring fractal dimension), similarly to a recent Korcak-exponent related paper (Erlandsson et al., 2011). This set is marked by dots in Fig. 2. Finally (Option 3), we also took the first value slightly below the biggest area and then at each step, we divided it by three, obtaining the (27, 9, 3, 1) A_0-set. These data are marked by triangles in Fig. 2. It should be noted here, that we – i.e. the observers – had two choices; one should choose the first limiting value (although the freedom is not full in this case, it should be "slightly" below the highest area) and then one can freely choose the method of reducing the A_0s.

It can be seen in Fig. 2 that with a proper initial choice (Option 3) one can obtain a fairly linear Korcak-plot with $K = 0.74 \pm 0.03$ slope ($R^2 = 0.99616$), represented by the solid line (90% confidence band are also marked by dashed lines). Choosing the second A_0-set (Option 2), an entirely different picture is attained; here the biggest,

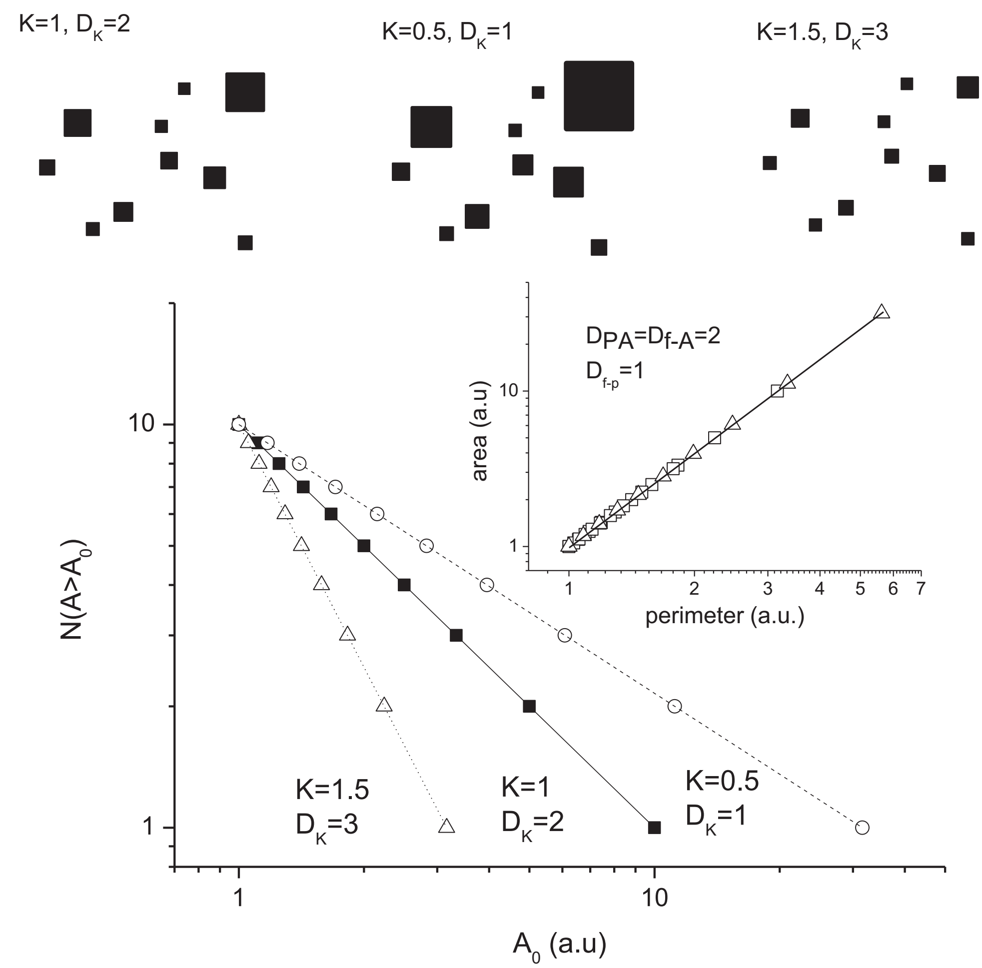

Fig. 3. Korcak-plot of a set of sample-patches (see the upper part of the figure); K and D_K values are given on the figure, area and perimeter values are given in arbitrary units (a.u.). Perimeter–area plot of the same sets can be seen in the insert; triangles: $K = 1$, squares: $K = 2$, circles: $K = 3$, showing that the perimeter–area fractal dimension of ALL the sets are identically equal to 2. This is also the value of the area Hausdorff dimension of the individual squares.

complex-shaped reservoirs can be clearly distinguished from the smaller ones. The same can be seen with our A_0-set (Option 1); the finer distribution is also revealed here. It is a basic criterion, that the result of a measurement should not be influenced by the observer (Rocchini and Neteler, 2012), neither during the actual measurement, nor during the data evaluation. Therefore, we argue that researchers undertaking a Korcak-analysis should avoid using A_0-sets that are independent from the data. The proposed alternative (Option 1) yields better results from both theoretical and applied viewpoints.

By this example we demonstrated that the method proposed here leads to better results not only from a merely theoretical but also from an applied ecological viewpoint.

5. The non-fractality of the Korcak-exponent

When Mandelbrot in his pioneering book (Mandelbrot, 1982) introduced the Korcak-plot, for a special case he deduced the following equality:

$$K = \frac{D_{f\text{-}p}}{2} \quad (3)$$

where $D_{f\text{-}p}$ is the fractal (Hausdorff) dimension of the perimeter. Later this equation was used as a general law (Othmani and Kaminsky, 1998; Kampichler, 1999; Peralta and Mather, 2000; Sasaki et al., 2006; Erlandsson et al., 2011 and others), extended to n-dimension by replacing the denominator with n. The notation K is used here for the Korcak-exponent, while D_K (the hypothetical Korcak-dimension) can be used as half of this value.

Here, we would like to demonstrate – by a simple example – that neither $D_{f\text{-}A}$, nor $D_{f\text{-}p}$ (area and perimeter Hausdorff dimensions) has any connection with the Korcak-exponent, not even in case of similar patches/islands. The relation given by Eq. (3) was never a theorem, only an assumption. When the experimental result was not supportive, usually the patch-similarity was blamed, rather than the validity of the assumption was questioned. Here we gave a counter-example to show the invalidity of this assumption, even when the patches are strictly similar.

In Fig. 3, one can see three different sets of square-shaped islands, with perimeter and area fractal dimensions of the individual patches are equal to 1 and 2, respectively. They are all similar, therefore one can apply perimeter–area analysis for all sets (Fig. 3, insert), showing that $D_{PA} = 1$ (where D_{PA} is the perimeter–area dimension). For similar patches, the perimeter–area dimension should be $2(D_{f\text{-}p}/D_{f\text{-}A})$, i.e. the two times the ratio of the perimeter and area dimensions; the later here is 2. On the lower part of Fig. 3, one can see the Korcak-plots of these sets, constructed with the previously proposed method. For one set, $K = 0.5$, for the other, $K = 1$, for the third, $K = 1.5$. According to Eq. (3), this would imply that their perimeter fractal dimensions would be 1, 2 and 3, respectively, but this is not true. The perimeter fractal dimension stays 1 for all sets; therefore the relationship hypothesized in Eq. (3) is not correct. In the same way one can construct patch-sets situated almost parallel with the x or y axes, hence giving $K = 0 + \varepsilon$ to $K = \infty - \varepsilon$ (ε is an arbitrarily small real number), while keeping the area fractal dimension and the perimeter fractal dimension as 2 and 1, respectively. It can be seen that these values does not have any connections with the Korcak-exponent.

Although Eq. (3) is the most used – or rather misused – relationship between K and the Hausdorff dimension, there are two other relationships which should be mentioned here. Russ proposed that D_K should be greater than or equal to the Hausdorff-dimension of the same set (Russ, 1994), but this is also not true, because one can easily construct a set with $K < 0.5$ (i.e. $D_K < 1$), where D_K is not greater than or equal to 1, which is the lower limit for the Hausdorff dimension for patches. Also, for a special case, Nikora et al. (1999) introduced a more complex relationship for D_K, using the linear combination of the various fractal dimensions (Nikora et al., 1999), including ones related not only to the individual patches but to the whole set. The study about the validity of Nikora's method for more general cases is still in progress.

6. Conclusions

In this short paper we introduced a proper method for Korcak-analysis to obtain the correct Korcak-exponent. The Korcak-exponent is a descriptor for the area distribution of a set of patches–embedded into 2-D – in a certain area-range. With improper analysis, one can easily obtain utterly artificial Korcak-exponents, leading to ecologically flawed results. For proper analysis, the limiting area (A_0) for Korcak-plot should be defined separately for each set as $A_{0i} = A_i - \delta$ to avoid artificial K-value at the high-area region. We can also retain more data points for fitting, which can help us to check the validity of the often mis-assumed linearity in double logarithmic plots as well as showing some finer details, overlooked when only a very few points are available (like in the log-equidistant case). It was also shown that the Korcak-exponent is not related to the Hausdorff fractal dimension of the individual patches, not even in case of statistically similar ones. Therefore this exponent should not be handled as $D_f/2$, it should be handled as a non-fractal descriptor.

After the pioneering papers by O'Neill et al. (1988) and Sugihara and May (1990), a number of papers have dealt with the use of fractals for explaining different ecological processes at different scales. With this manuscript we hope to stimulate discussion among ecologists about real fractal measures of landscape complexity, which is a long lasting theme in landscape ecology.

References

Erlandsson, J., McQuaid, C.D., Sköld, M., 2011. Patchiness and co-existence of indigenous and invasive mussels at small spatial scales: the interaction of facilitation and competition. PLoS ONE 6, e26958.
Hastings, H.M., Sugihara, G., 1993. Fractals: A User's Guide for the Natural Sciences. Oxford University Press, Oxford.
Imre, A.R., 2006. Artificial fractal dimension obtained by using perimeter – area relationship on digitalized images. Applied Mathematics and Computation 173, 443–449.
Imre, A.R., 2007. Systematic error in the determination of perimeter and area of off-lattice digitalized images. International Journal of Remote Sensing 28, 5071–5077.
Imre, A.R., Cseh, D., Neteler, M., Rocchini, D., 2011. Korcak dimension as a novel indicator of landscape fragmentation and re-forestation. Ecological Indicators 11, 1134–1138.
Kampichler, C., 1999. Fractal concepts in studies of soil fauna. Geoderma 88, 283–300.
Ken, C., Pan, Z., Rener, Y., 2008. A linear approach to metric circumference computation for digitized convex shapes. Journal of Electronics (China) 25, 572–575.
Korčak, J., 1938. Deux types of fondamentaux de distribution statistique. Bulletin De l'Institut International de Statistique III, 295–299.
Korčák, J., 1941. Přírodní dualita statistického rozložení. Statistický Obzor 22, 171–222.
Loehle, C., 2011. Complexity and the problem of ill-posed questions in ecology. Ecological Complexity 8, 60–67.
Martín, M.A., Pachepsky, Y.B., Perfect, E., Guber, A., 2009. Fractal modeling and scaling in natural systems. Ecological Complexity 6, 219–220.
Mandelbrot, B.B., 1982. The Fractal Geometry of Nature. Freeman, New York.
Nikora, V.I., Pearson, C.P., Shankar, U., 1999. Scaling properties in landscape patterns: New Zealand experience. Landscape Ecology 14, 17–33.
Novotný, J., Nosek, V., 2009. Nomothetic geography revisited: statistical distributions, their underlying principles, and inequality measures. Geografie 114, 282–297.
Novotný, J., 2010. Korčákův zákon aneb zajímavá historie přírodní duality statistického rozložení. Informace ČGS 29 (1), 1–10.
O'Neill, R.V., Krummel, J.R., Gardner, R.H., Sugihara, G., Jackson, B., DeAngelis, D.L., Milne, B.L., Turner, M.G., Zygmunt, B., Christensen, S.W., Dale, V.H., Graham, R.L., 1988. Indices of landscape pattern. Landscape Ecology 1, 153–162.
Othmani, A., Kaminsky, C., 1998. Three dimensional fractal analysis of sheet metal surfaces. Wear 214, 147–150.

Peralta, P., Mather, P., 2000. An analysis of deforestation patterns in the extractive reserves of Acre, Amazonia from satellite imagery: a landscape ecological approach. International Journal of Remote Sensing 21, 2555–2570.
Rocchini, D., Neteler, M., 2012. Let the four freedoms paradigm apply to ecology. Trends in Ecology & Evolution 27, 310–327.
Russ, J.C., 1994. Fractal Surfaces. Plenum Press, New York.
Sugihara, G., May, R.M., 1990. Applications of fractals in ecology. Trends in Ecology & Evolution 5, 79–86.
Wu, J., Jelinski, D.E., Luck, M., Tueller, P.T., 2000. Multiscale analysis of landscape heterogeneity: scale variance and pattern metrics. Geographic Information Sciences 6, 6–19.
Zunic, J., Martinez-Ortiz, C., 2009. Linearity measure for curve segments. Applied Mathematics and Computation 215, 3098–3105.