ALGORITHMS FOR CONSTRUCTING VORONOI DIAGRAMS

Vera Sacristán

Computational Geometry
Facultat d'Informàtica de Barcelona
Universitat Politècnica de Catalunya

Naive algorithm

Constructing Voronoi diagrams

NAIVE ALGORITHM

Constructing Voronoi diagrams

NAIVE ALGORITHM

For each p_i, construct its Voronoi region $Vor(p_i) = \bigcap_{j \neq i} H_{ij}$.

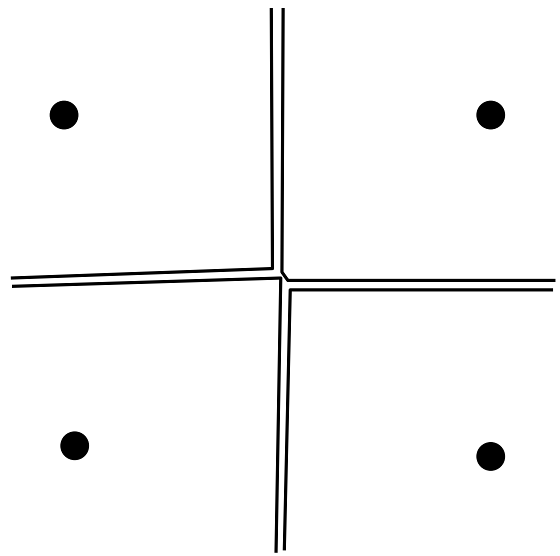
Constructing Voronoi diagrams

NAIVE ALGORITHM

For each p_i, construct its Voronoi region $Vor(p_i) = \bigcap_{j \neq i} H_{ij}$.

Inconvenients:

- It can cause inconsistency due to precision problems



Constructing Voronoi diagrams

NAIVE ALGORITHM

For each p_i, construct its Voronoi region $Vor(p_i) = \bigcap_{j \neq i} H_{ij}$.

Inconvenients:

- It can cause inconsistency due to precision problems
- It does not produce immediate neighborhood information

Constructing Voronoi diagrams

NAIVE ALGORITHM

For each p_i, construct its Voronoi region $Vor(p_i) = \bigcap_{j \neq i} H_{ij}$.

Inconvenients:

- It can cause inconsistency due to precision problems
- It does not produce immediate neighborhood information
- It runs in $O(n^2 \log n)$ time

Constructing Voronoi diagrams

NAIVE ALGORITHM

For each p_i, construct its Voronoi region $Vor(p_i) = \bigcap_{j \neq i} H_{ij}$.

Inconvenients:

- It can cause inconsistency due to precision problems
- It does not produce immediate neighborhood information
- It runs in $O(n^2 \log n)$ time

The fact that each Voronoi region, $Vor(p_i)$, is built in optimal $\Theta(n \log n)$ time does not implie that the construction of the entire diagram, $Vor(P)$, requires $\Omega(n^2 \log n)$ time, as we will see.

incremental algorithm

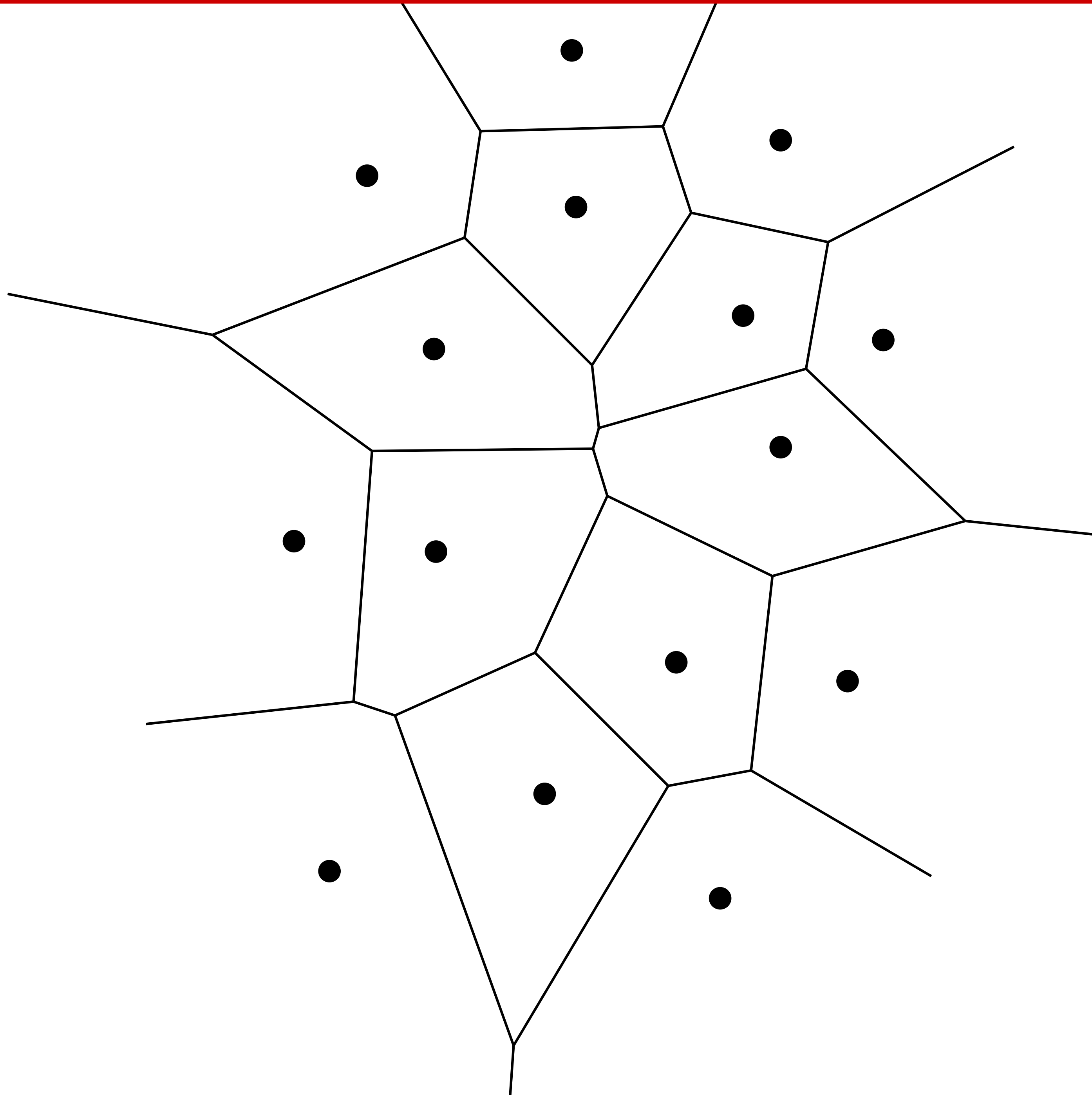
Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

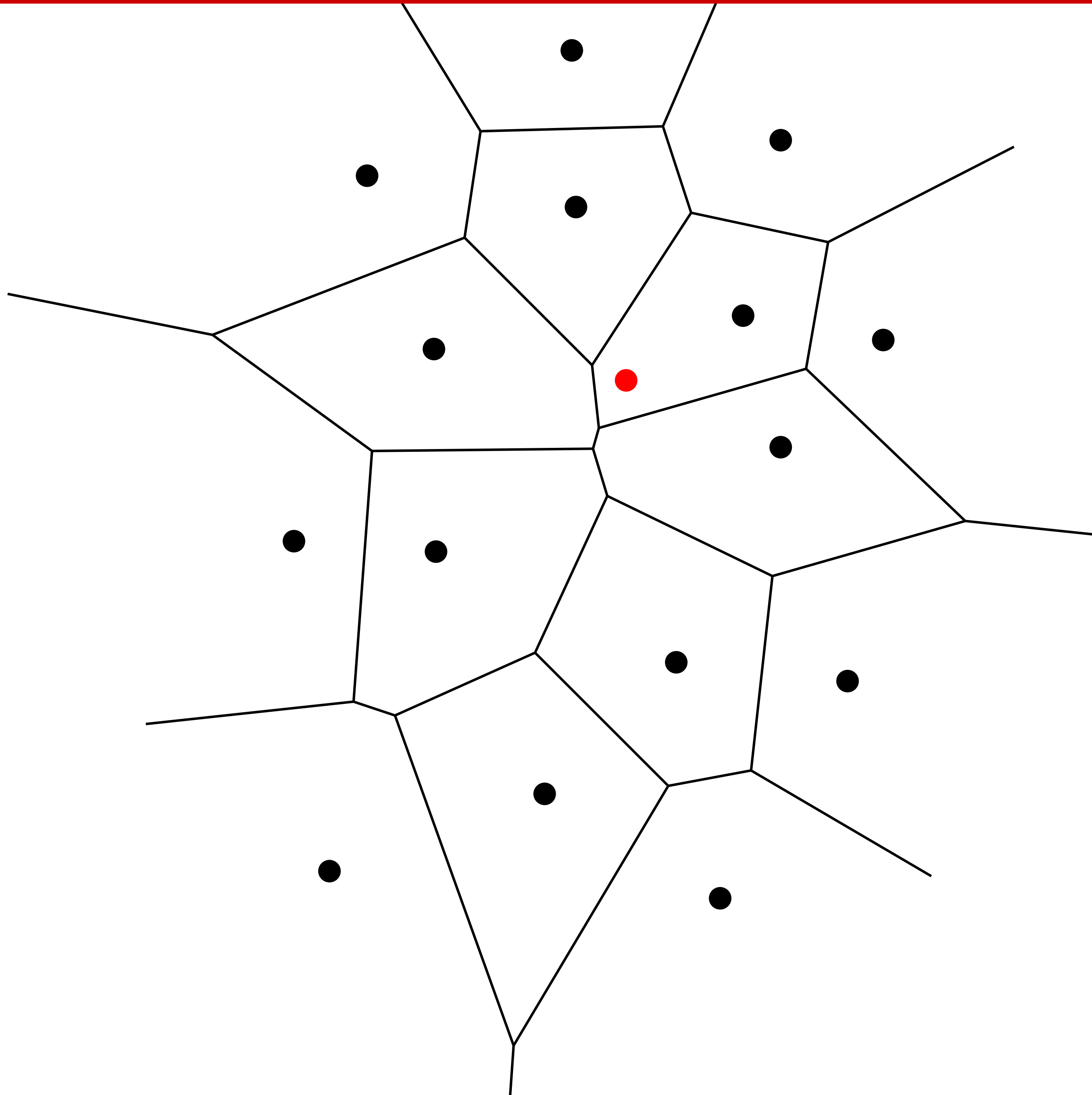


Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}



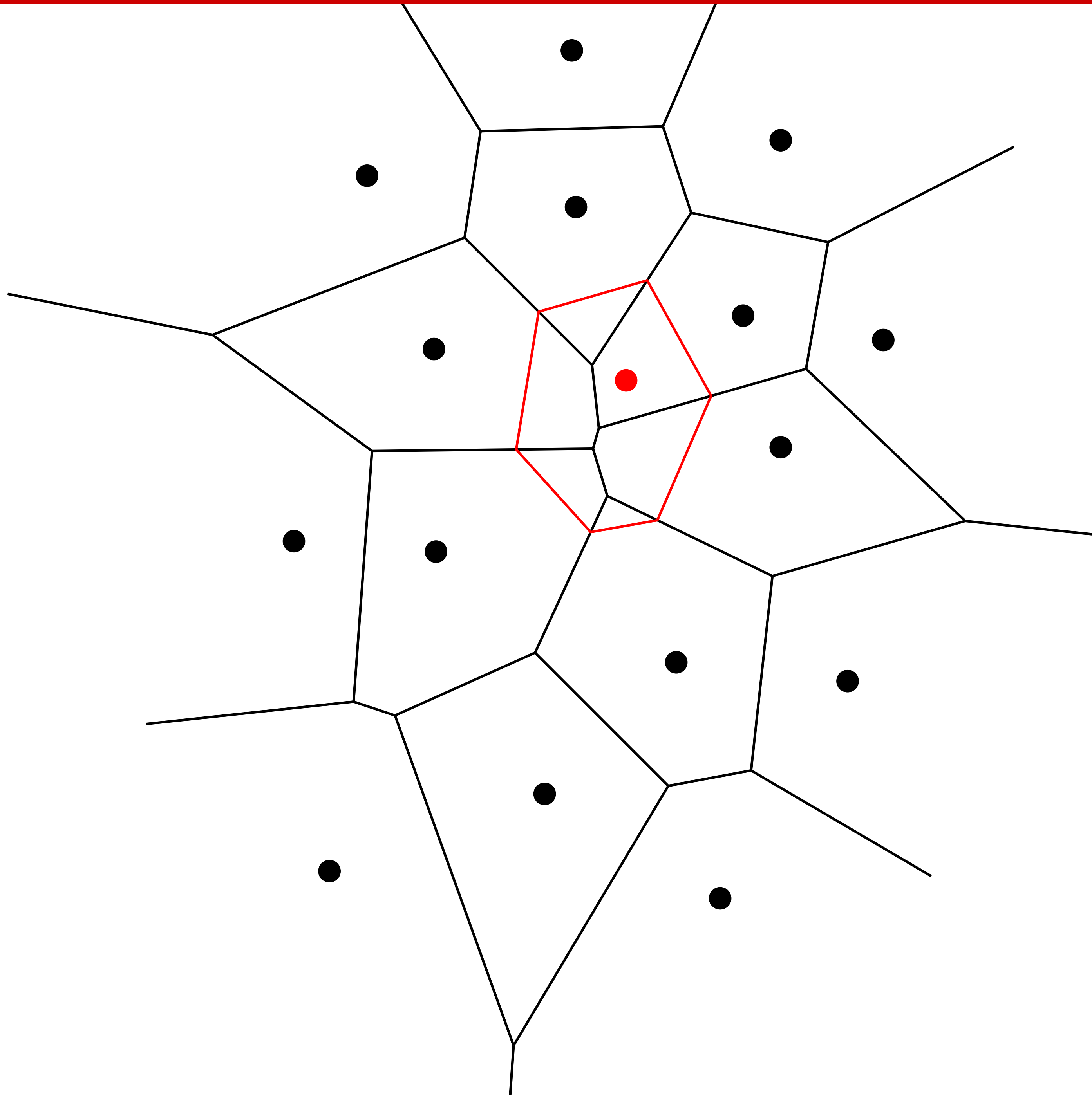
Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

... compute its region



Constructing Voronoi diagrams

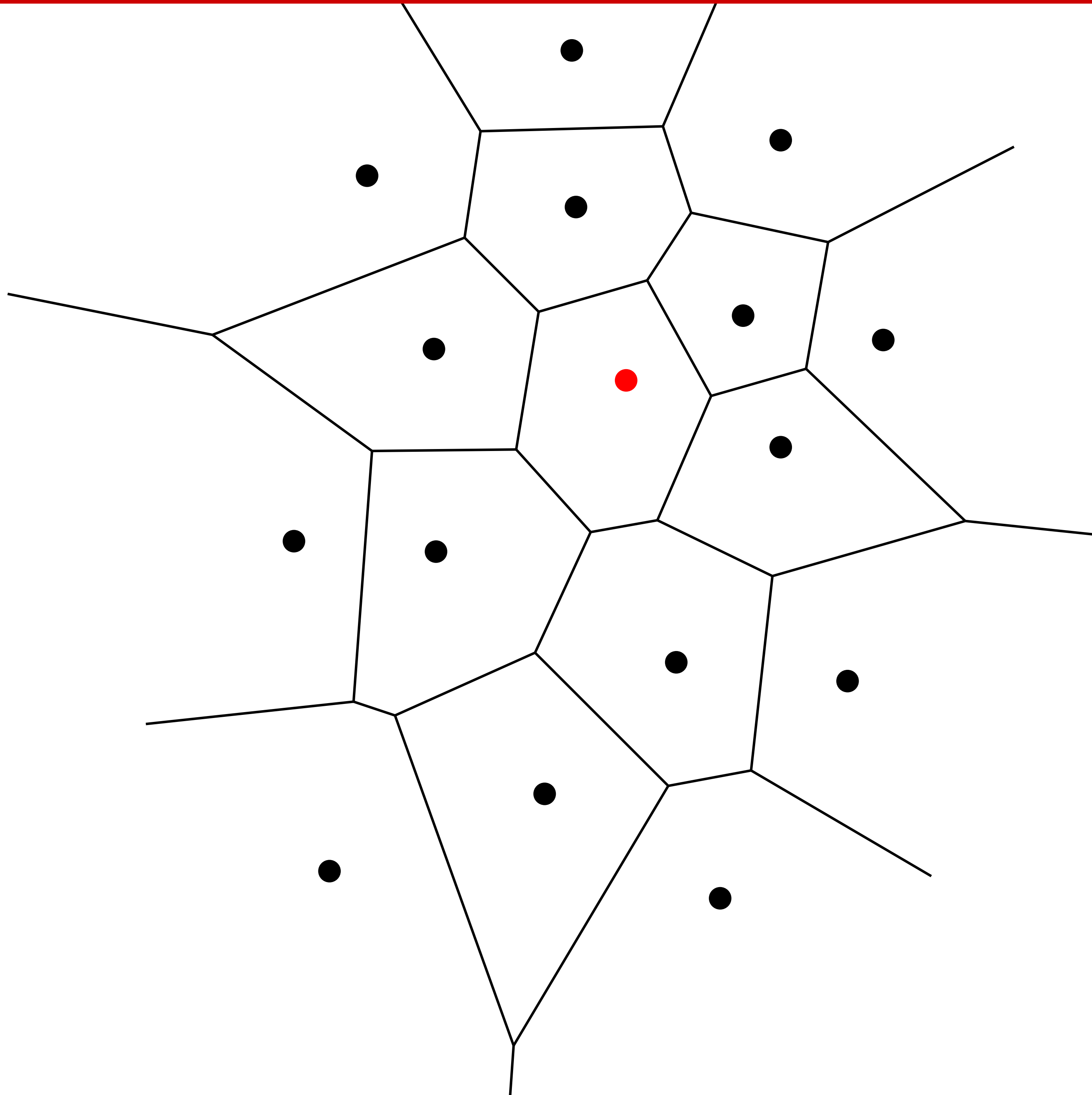
INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

... compute its region

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

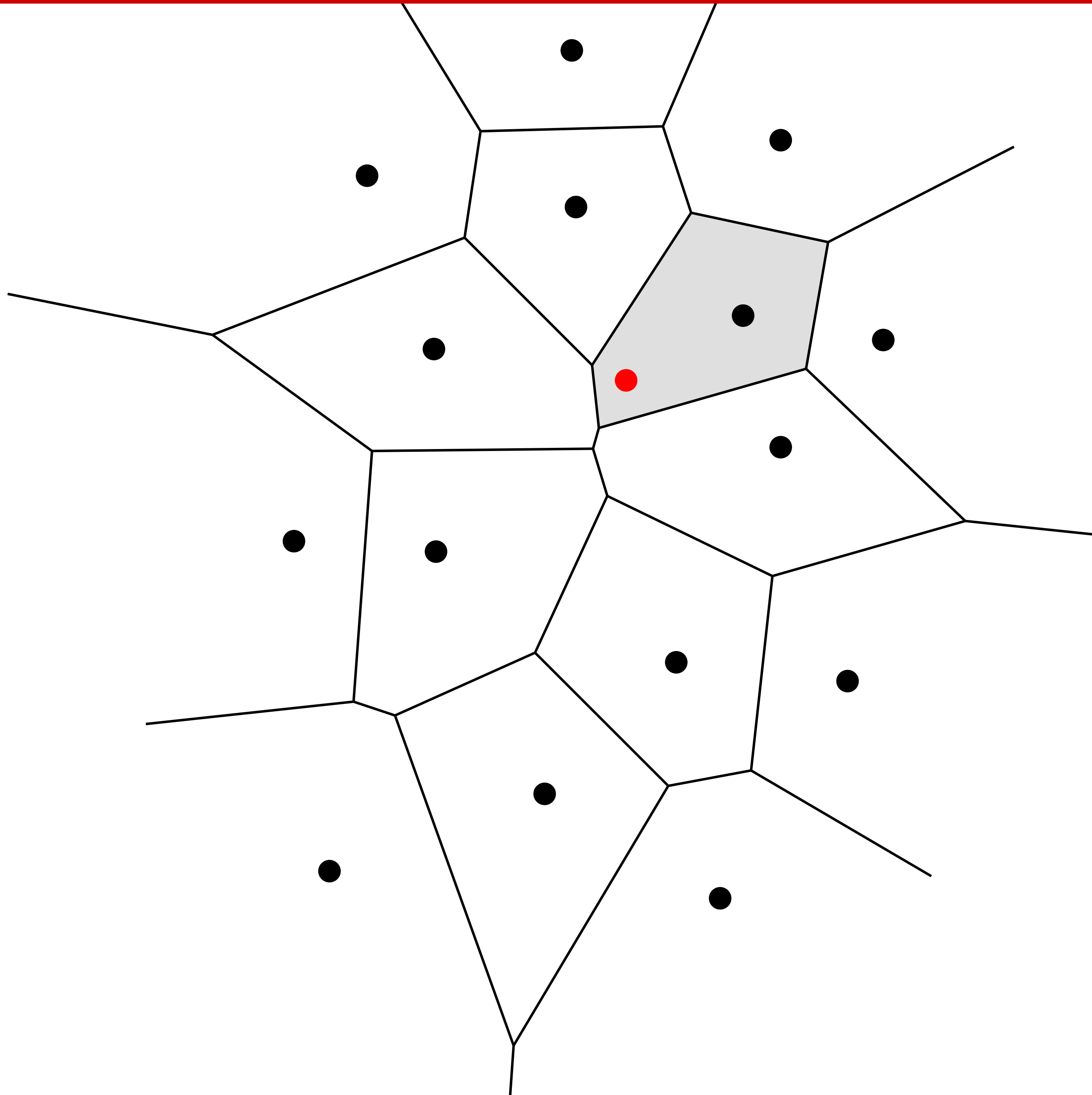
Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

> Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

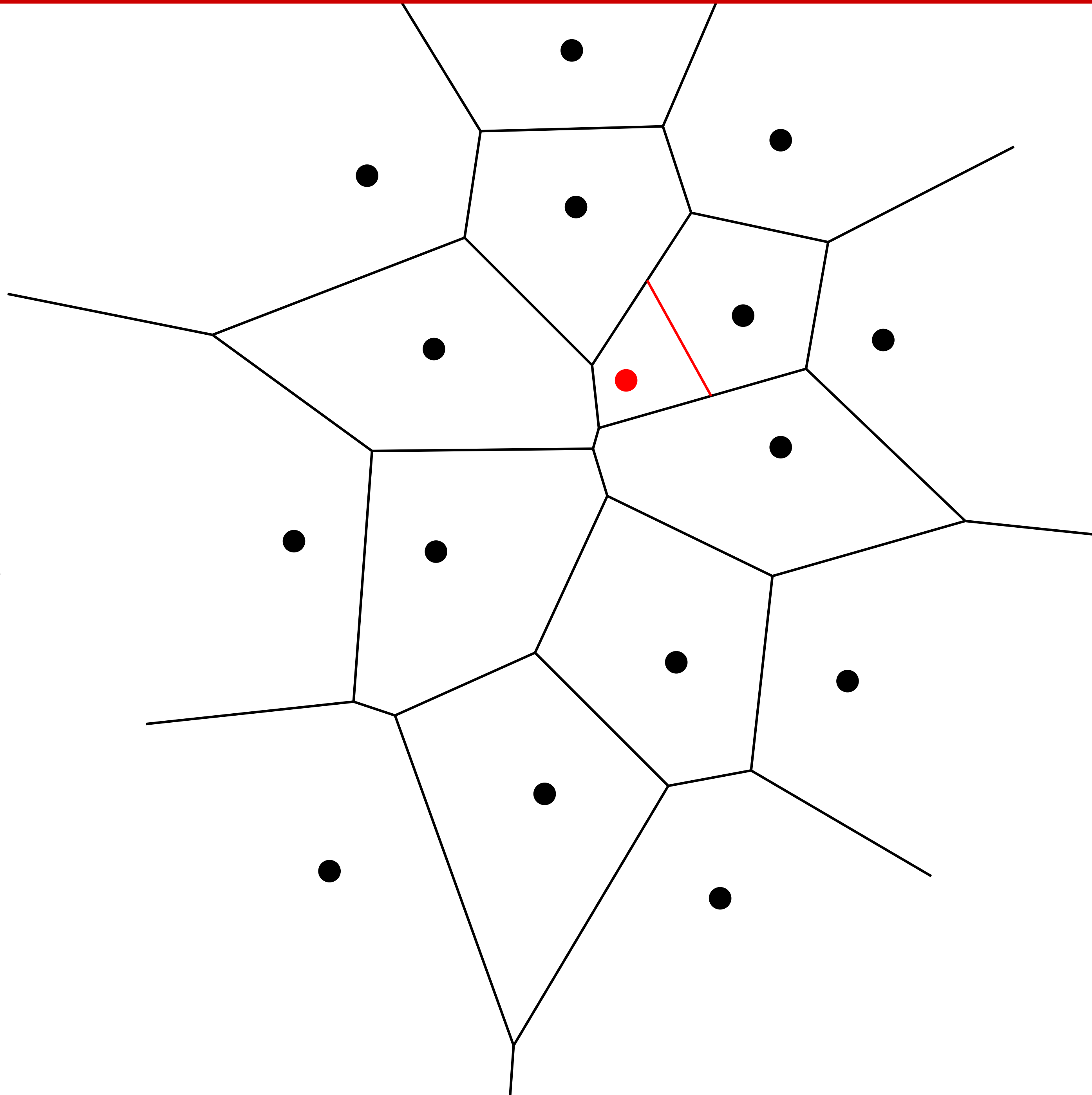
... add point p_{i+1}

Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

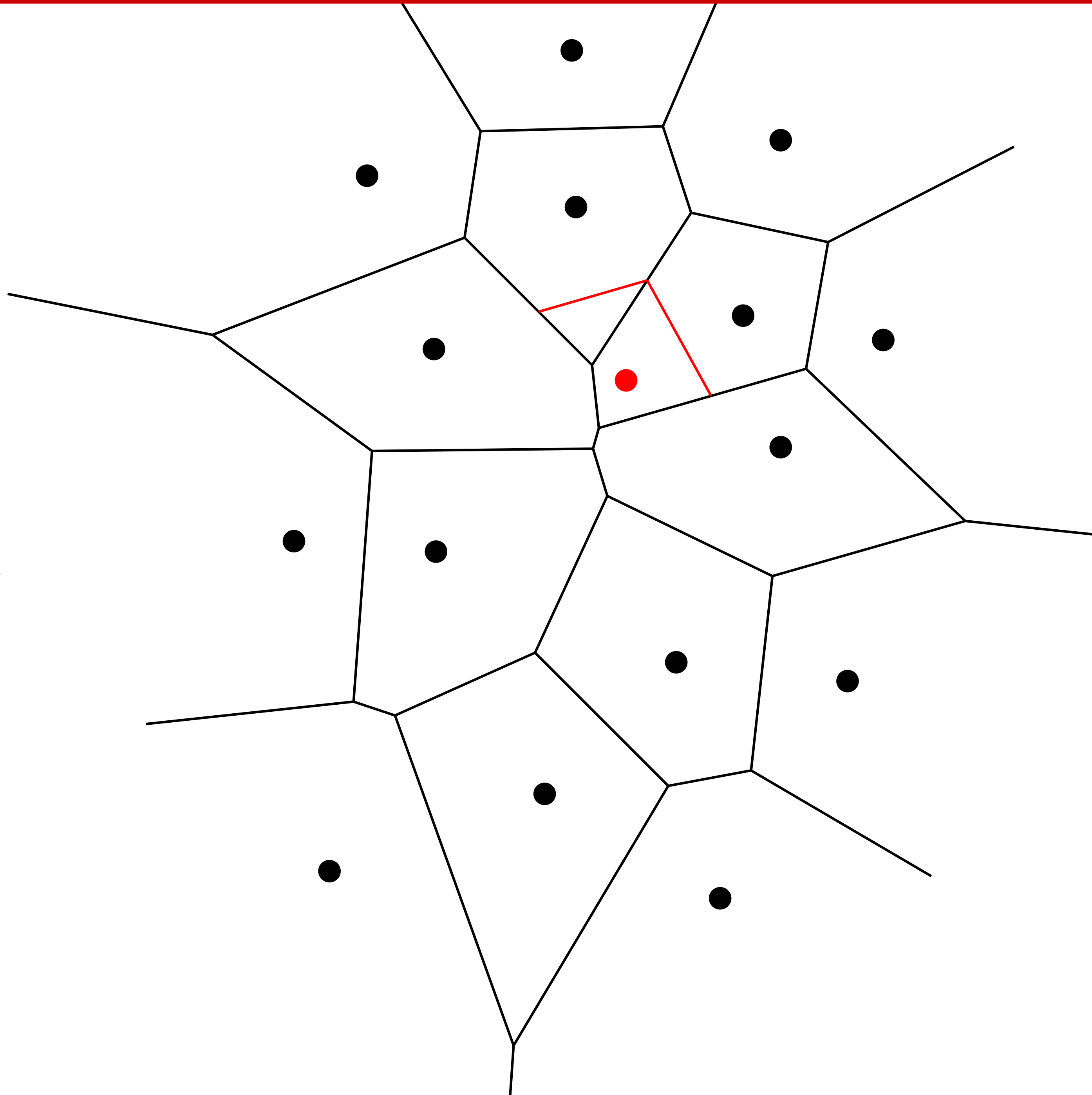
... add point p_{i+1}

> Explore all candidates to find the site p_j $(1 \le j \le i)$ closest to p_{i+1}.

... compute its region

> Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

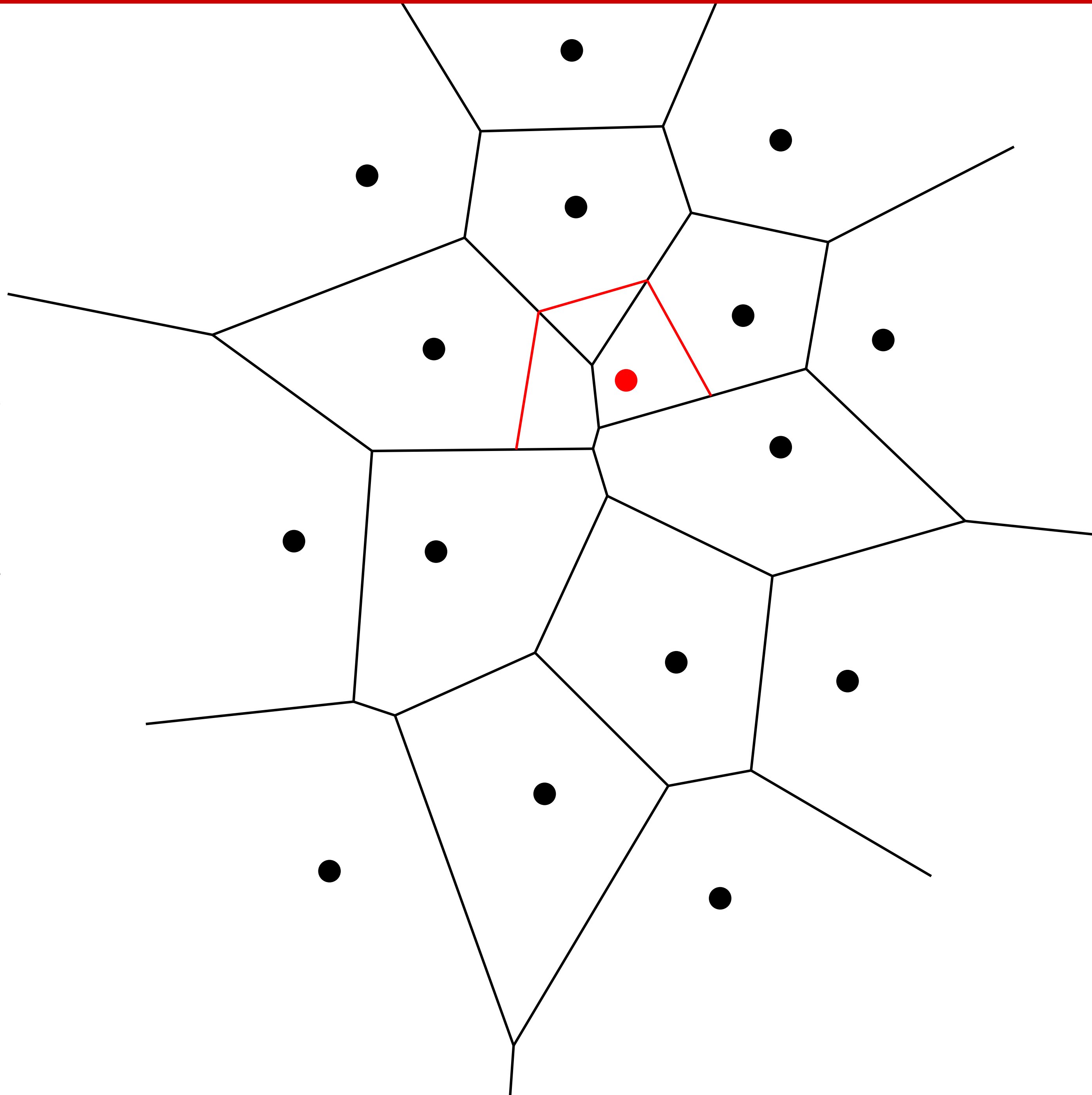
... add point p_{i+1}

Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \dots, p_i\}$...

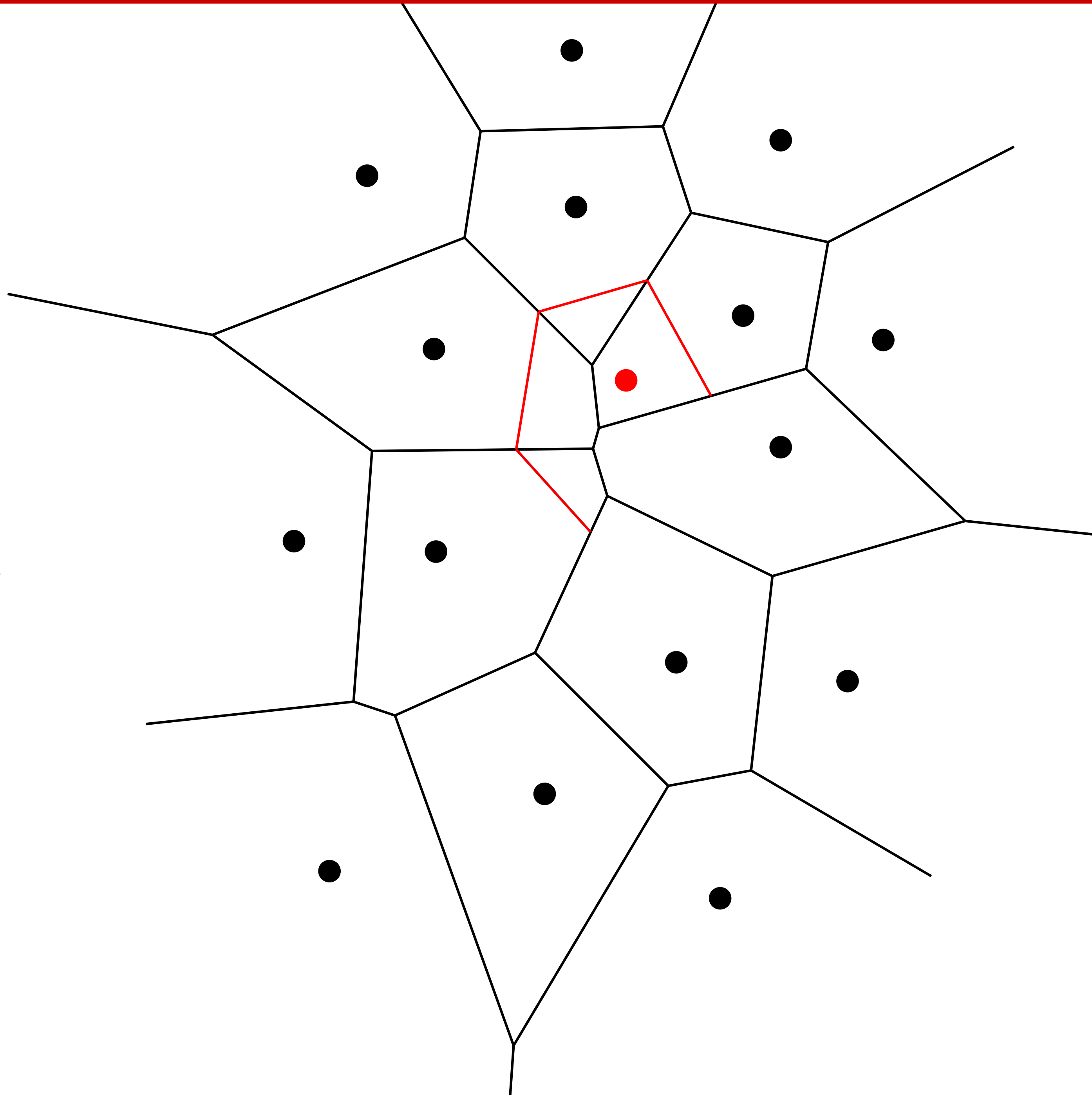
... add point p_{i+1}

Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

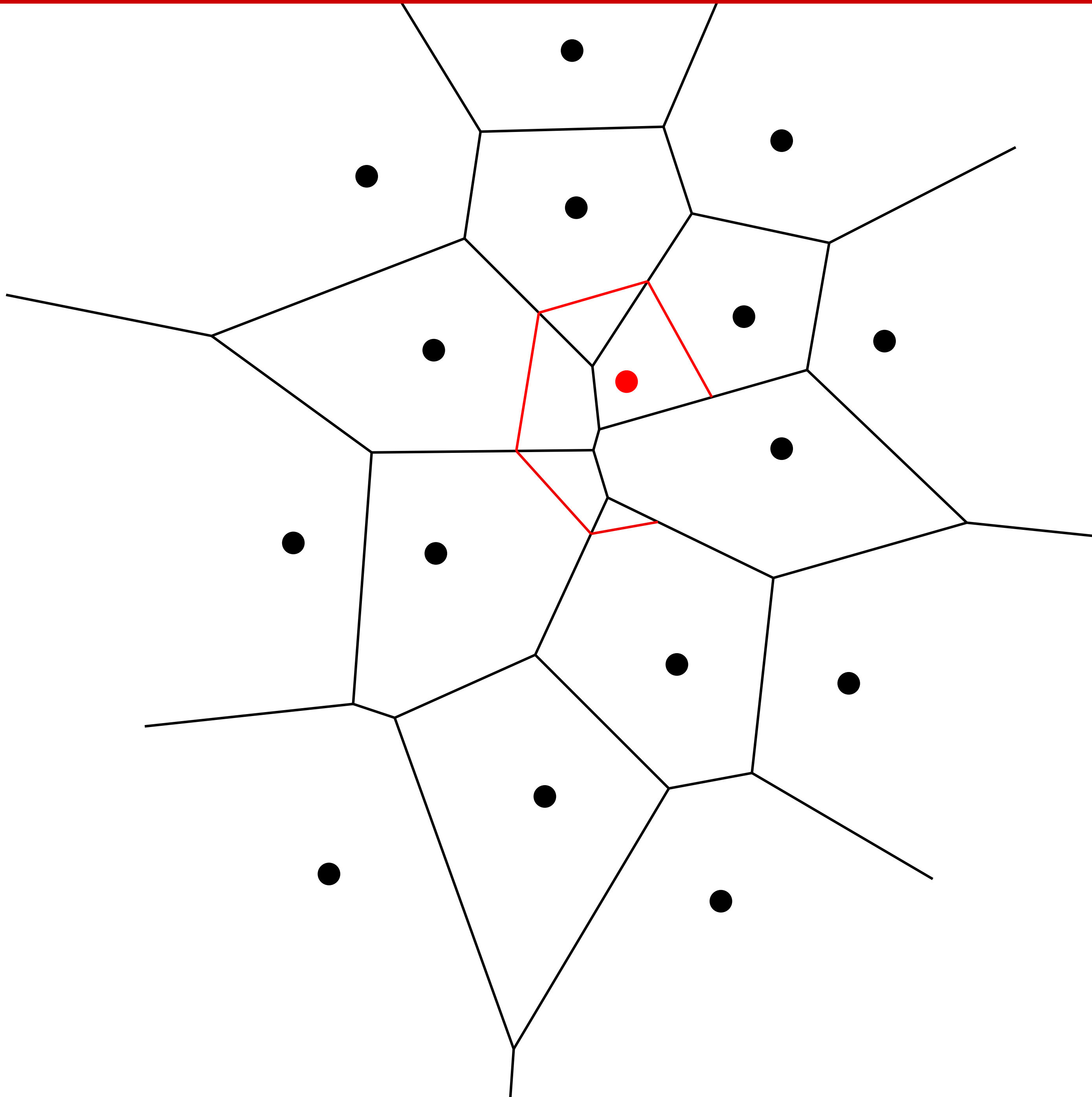
... add point p_{i+1}

> Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

> Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

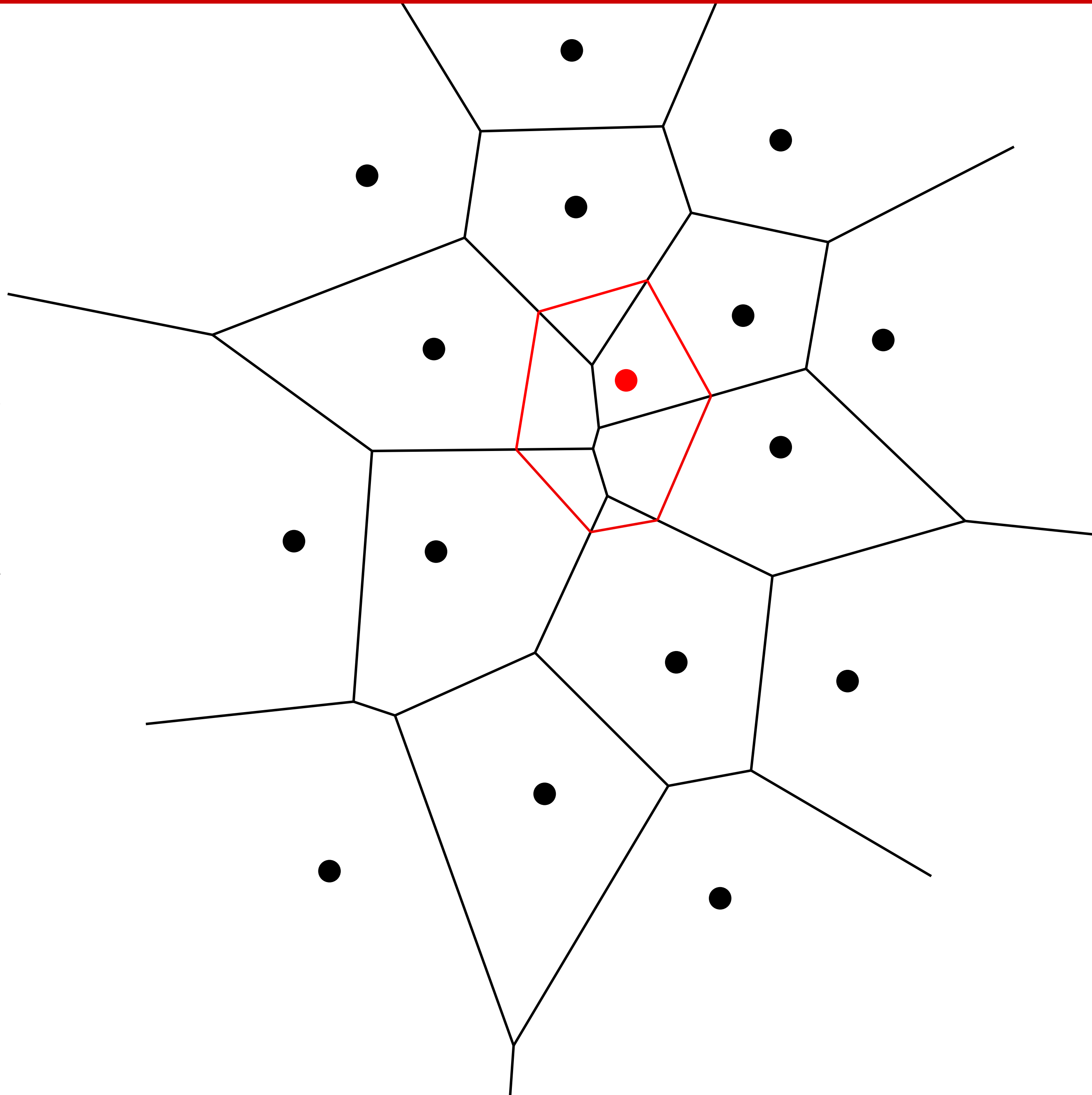
... add point p_{i+1}

> Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

> Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

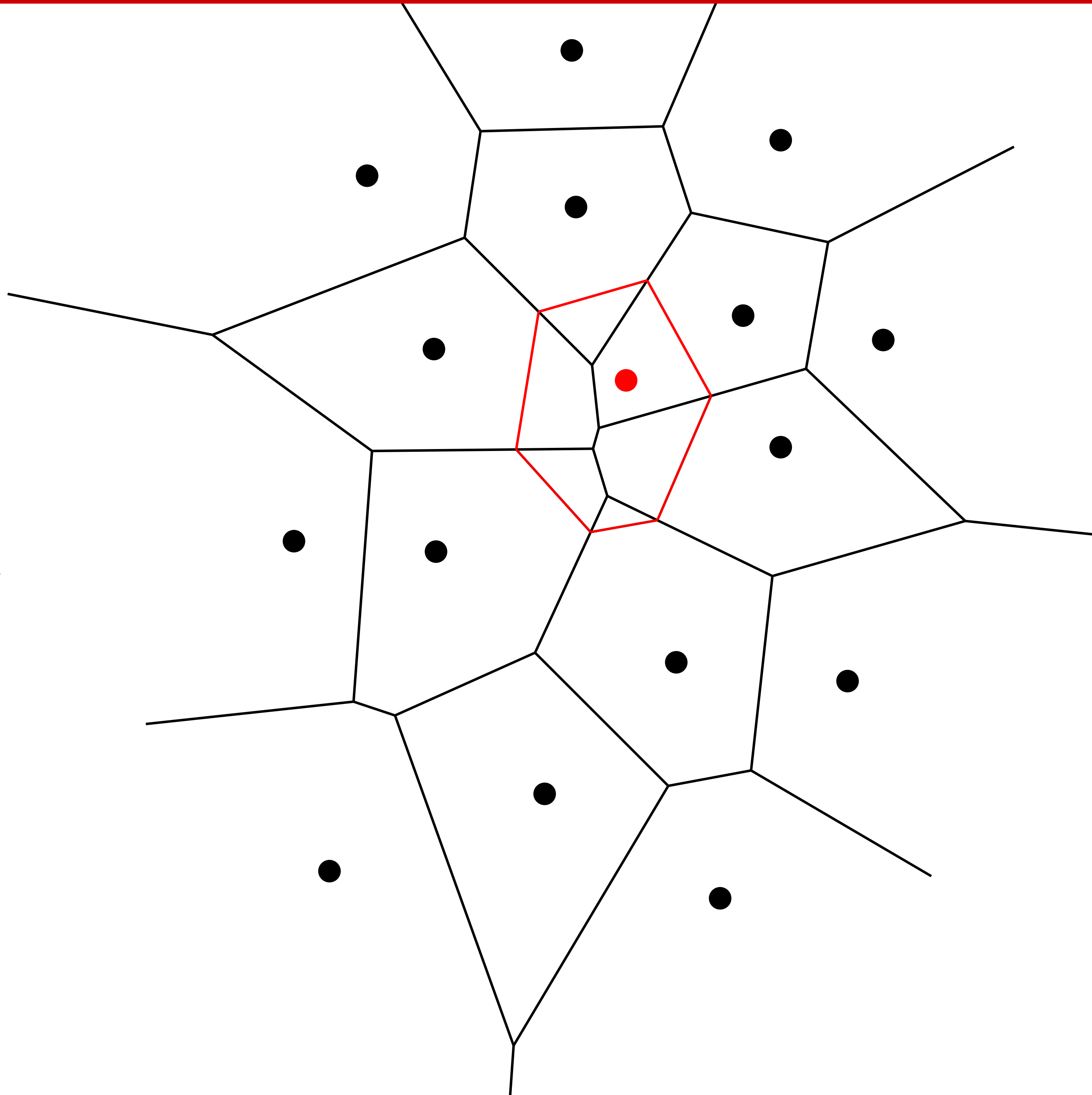
Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.

While building the Voronoi region of p_{i+1}, update the DCEL.



INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

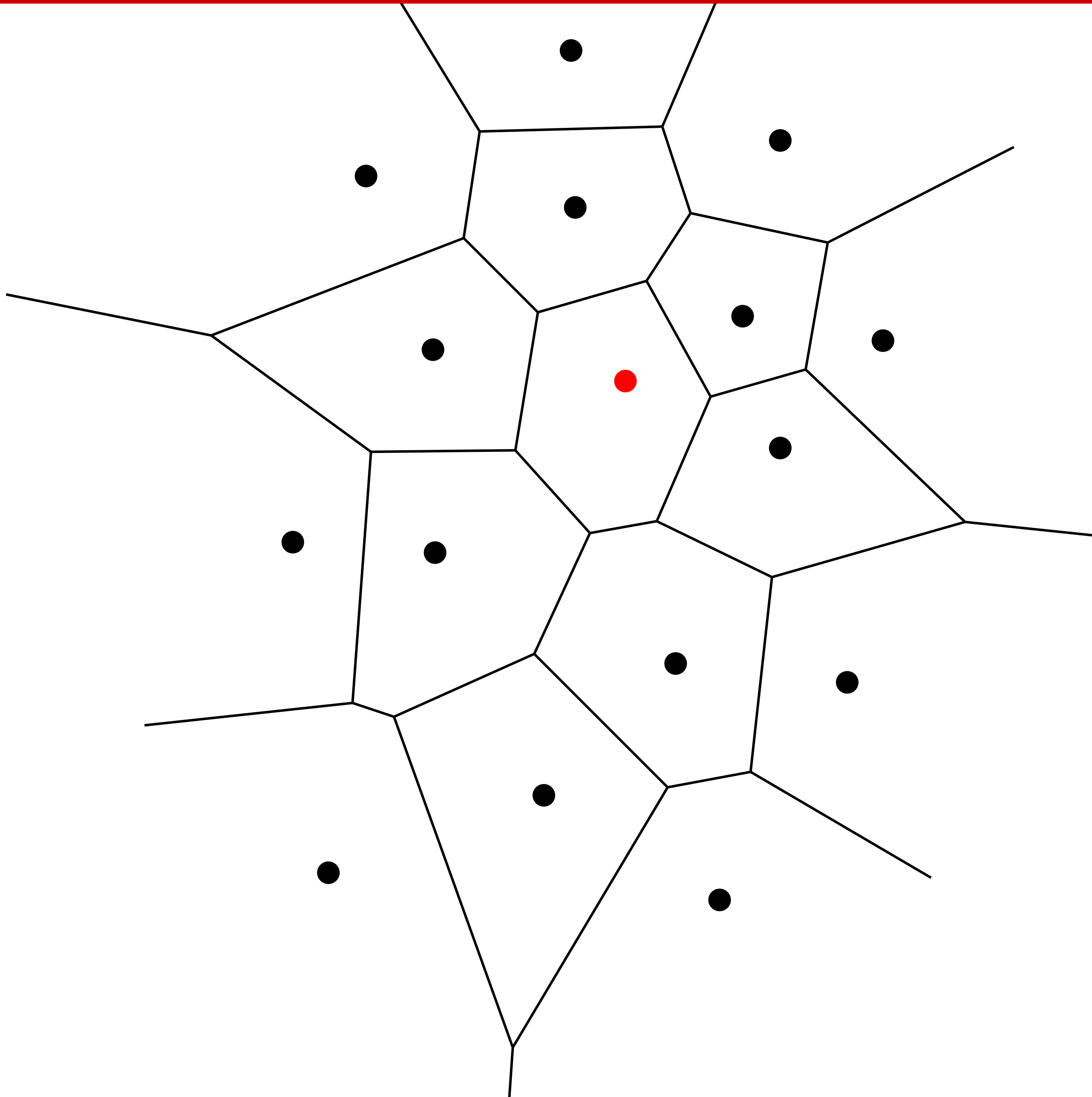
Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

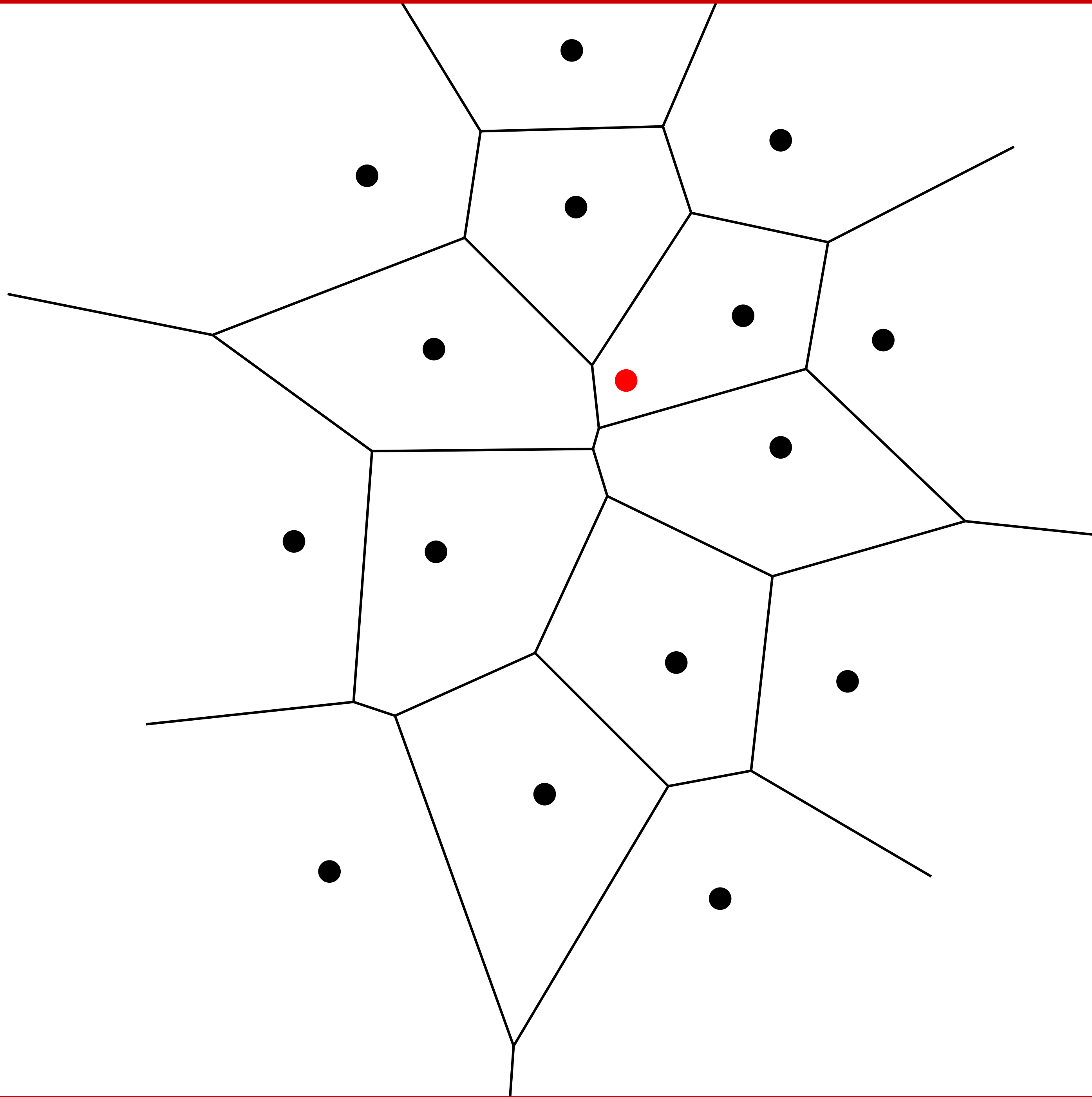
... and prune the initial diagram.

While building the Voronoi region of p_{i+1}, update the DCEL.



Constructing Voronoi diagrams

How to update the DCEL

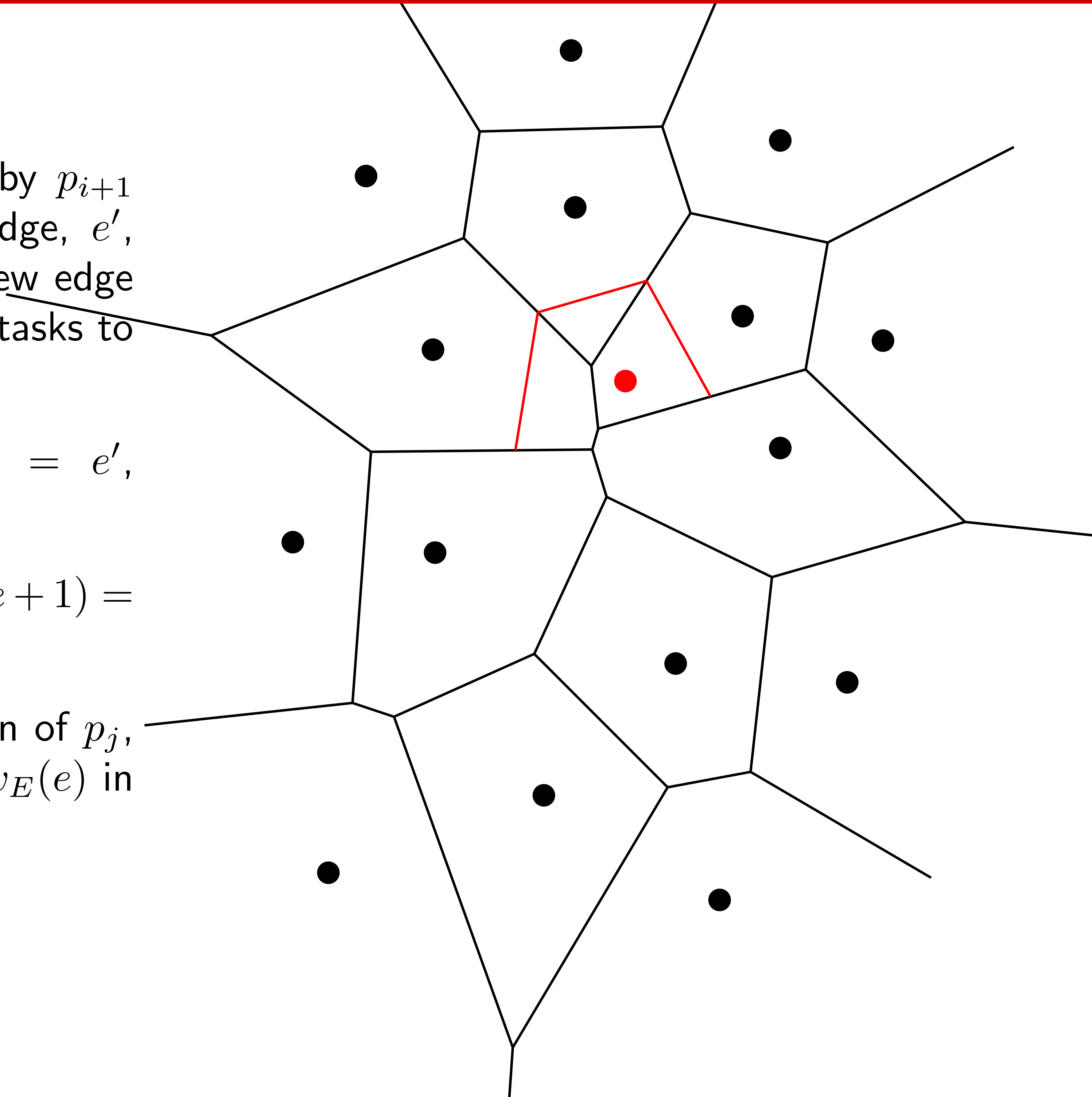


Constructing Voronoi diagrams

How to update the DCEL

Each time an edge e, generated by p_{i+1} and p_j, intersects a preexistent edge, e', a new vertex v is created and a new edge starts, $e+1$. Then, these are the tasks to perform:

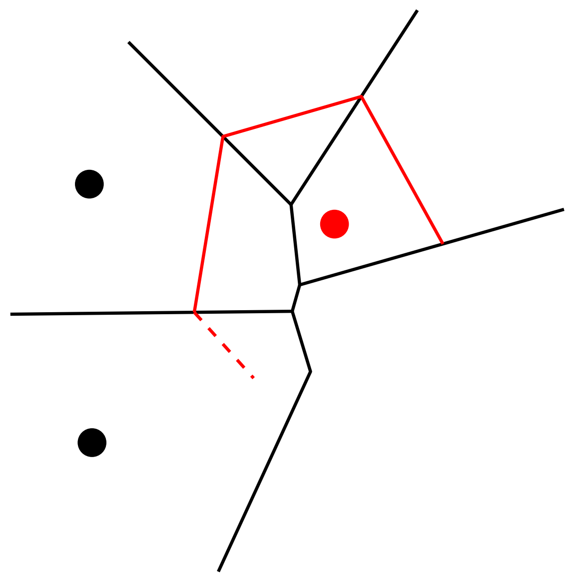
- Assign $v_E(e) = v$, $e_N(e) = e'$, $f_L(e) = i+1$, $f_R(e) = j$
- Create $e+1$ and assign $v_B(e+1) = v$, $e_P(e+1) = e$
- Delete all edges of the region of p_j, that lie between $v_B(e)$ and $v_E(e)$ in clockwise order
- Update $e(p_j) = e$
- Create v with $e(v) = e$



How to update the DCEL

Each time an edge e, generated by p_{i+1} and p_j, intersects a preexistent edge, e', a new vertex v is created and a new edge starts, $e+1$. Then, these are the tasks to perform:

- Assign $v_E(e) = v$, $e_N(e) = e'$, $f_L(e) = i+1$, $f_R(e) = j$
- Create $e+1$ and assign $v_B(e+1) = v$, $e_P(e+1) = e$
- Delete all edges of the region of p_j, that lie between $v_B(e)$ and $v_E(e)$ in clockwise order
- Update $e(p_j) = e$
- Create v with $e(v) = e$

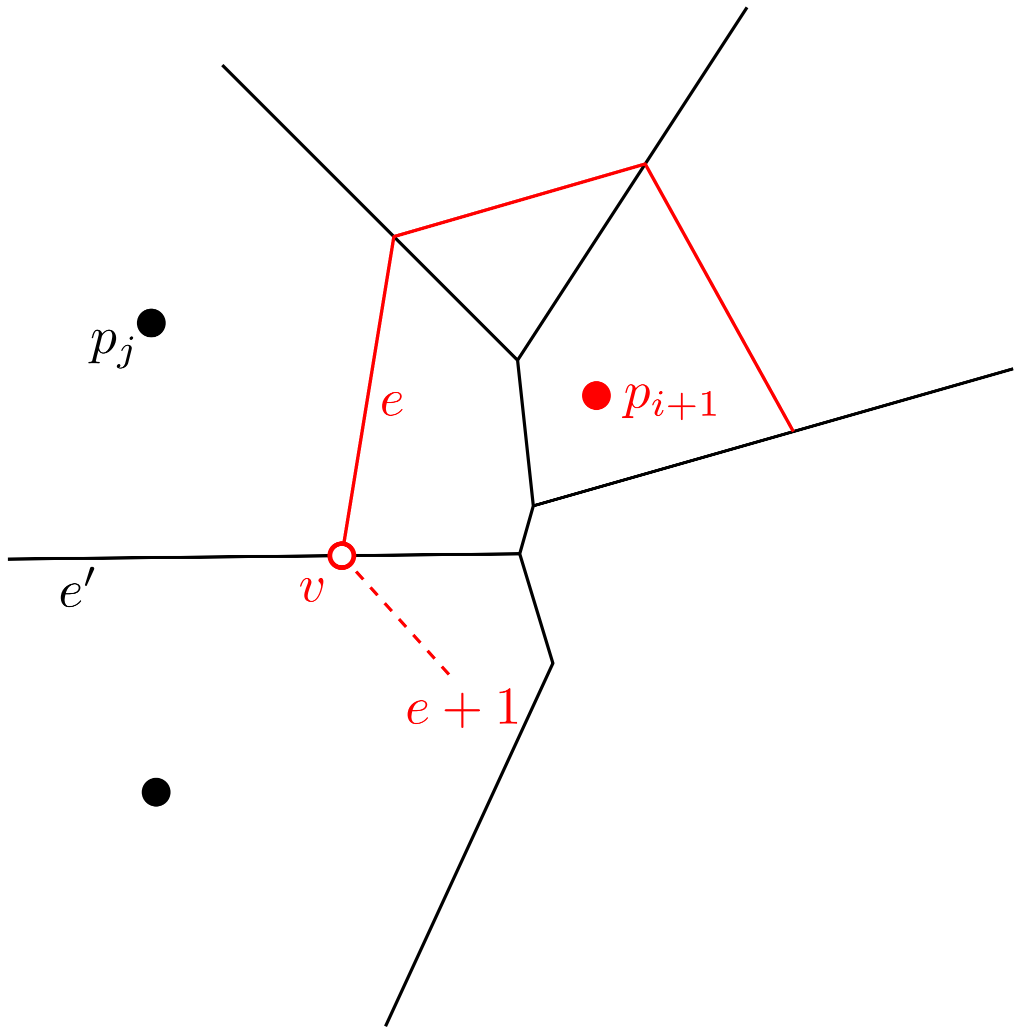


Constructing Voronoi diagrams

How to update the DCEL

Each time an edge e, generated by p_{i+1} and p_j, intersects a preexistent edge, e', a new vertex v is created and a new edge starts, $e+1$. Then, these are the tasks to perform:

- Assign $v_E(e) = v$, $e_N(e) = e'$, $f_L(e) = i+1$, $f_R(e) = j$
- Create $e+1$ and assign $v_B(e+1) = v$, $e_P(e+1) = e$
- Delete all edges of the region of p_j, that lie between $v_B(e)$ and $v_E(e)$ in clockwise order
- Update $e(p_j) = e$
- Create v with $e(v) = e$



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \ldots, p_i\}$...

... add point p_{i+1}

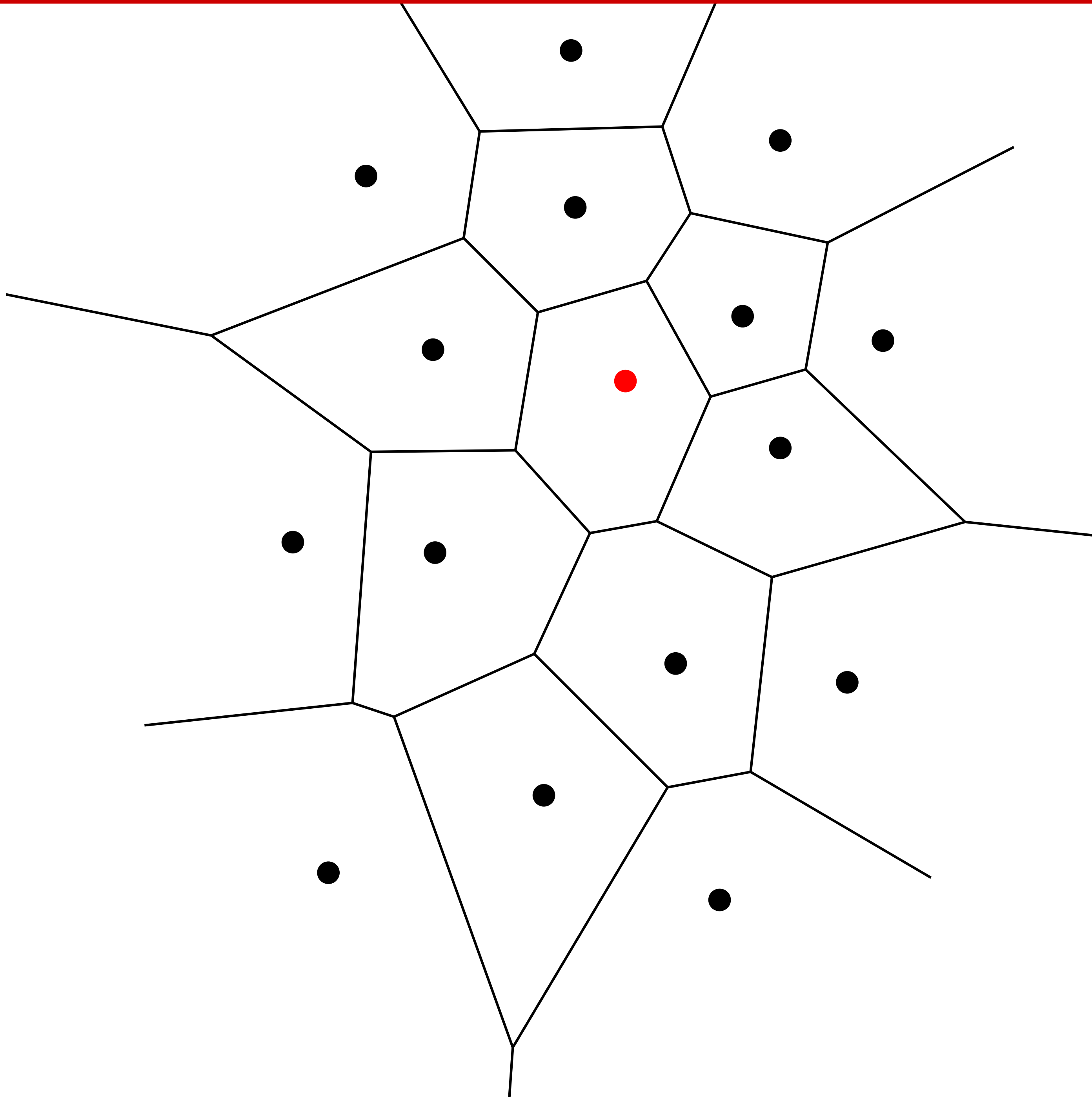
Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.

While building the Voronoi region of p_{i+1}, update the DCEL.



Constructing Voronoi diagrams

INCREMENTAL ALGORITHM

Starting with the Voronoi diagram of $\{p_1, \dots, p_i\}$...

... add point p_{i+1}

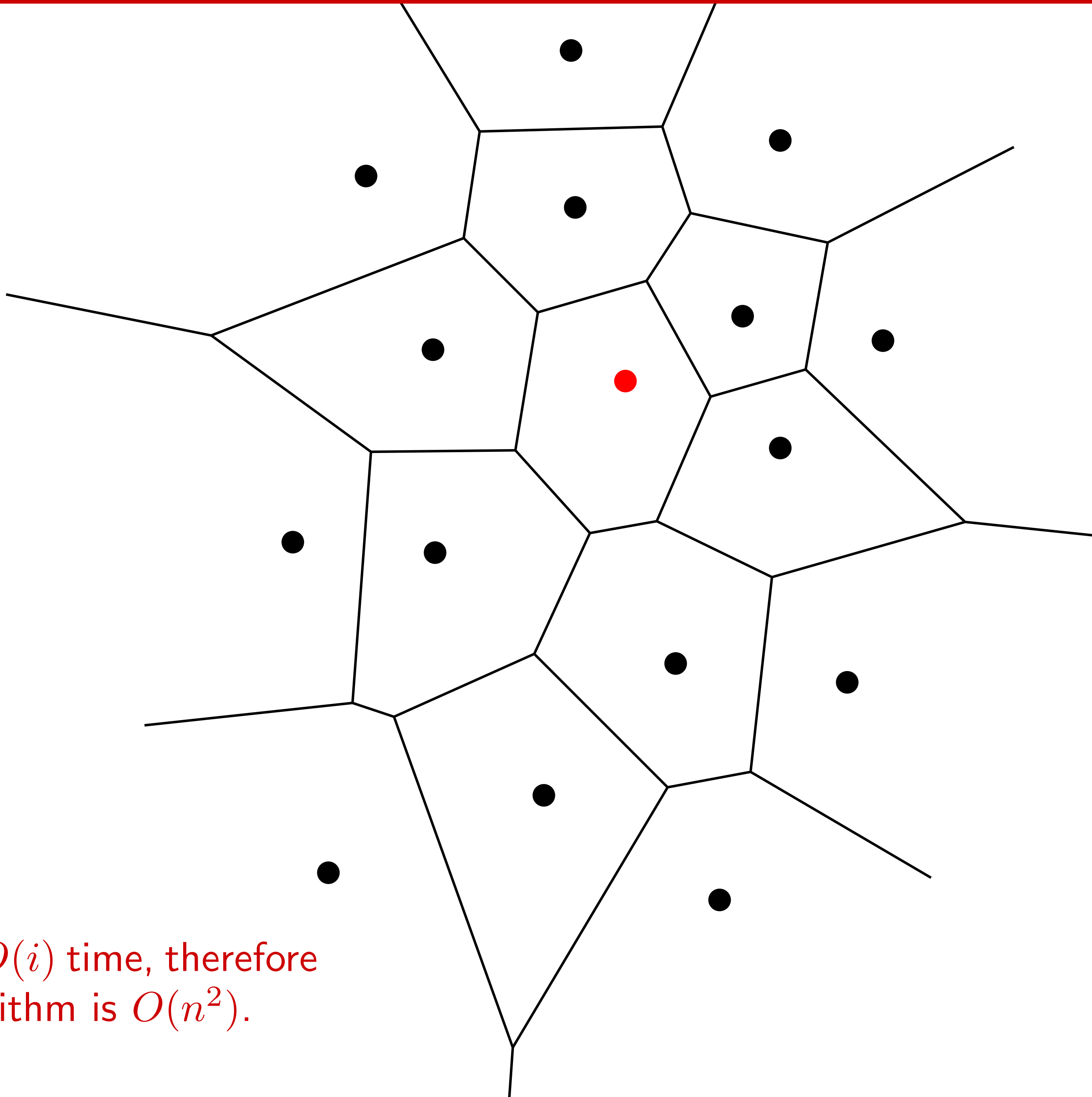
Explore all candidates to find the site p_j $(1 \leq j \leq i)$ closest to p_{i+1}.

... compute its region

Build its boundary starting from bisector $b_{i+1,j}$.

... and prune the initial diagram.

While building the Voronoi region of p_{i+1}, update the DCEL.



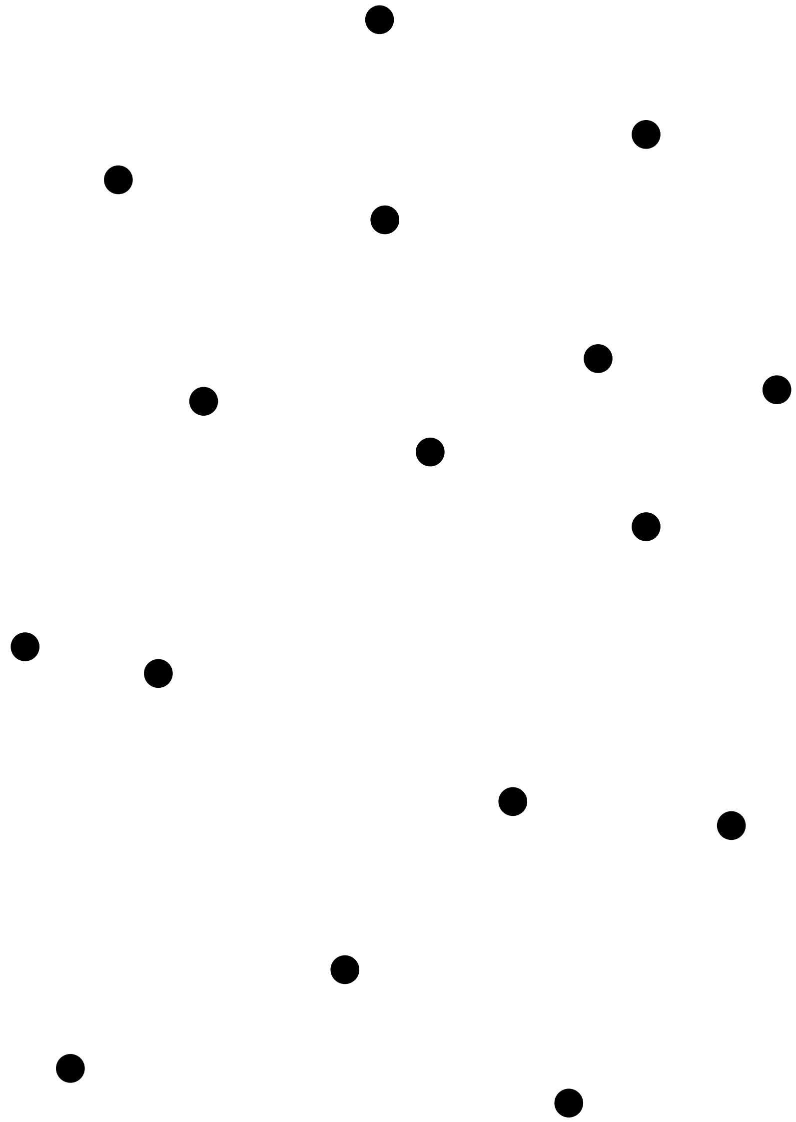
Running time: Each step runs in $O(i)$ time, therefore the total running time of the algorithm is $O(n^2)$.

divide and conquer algorithm

Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

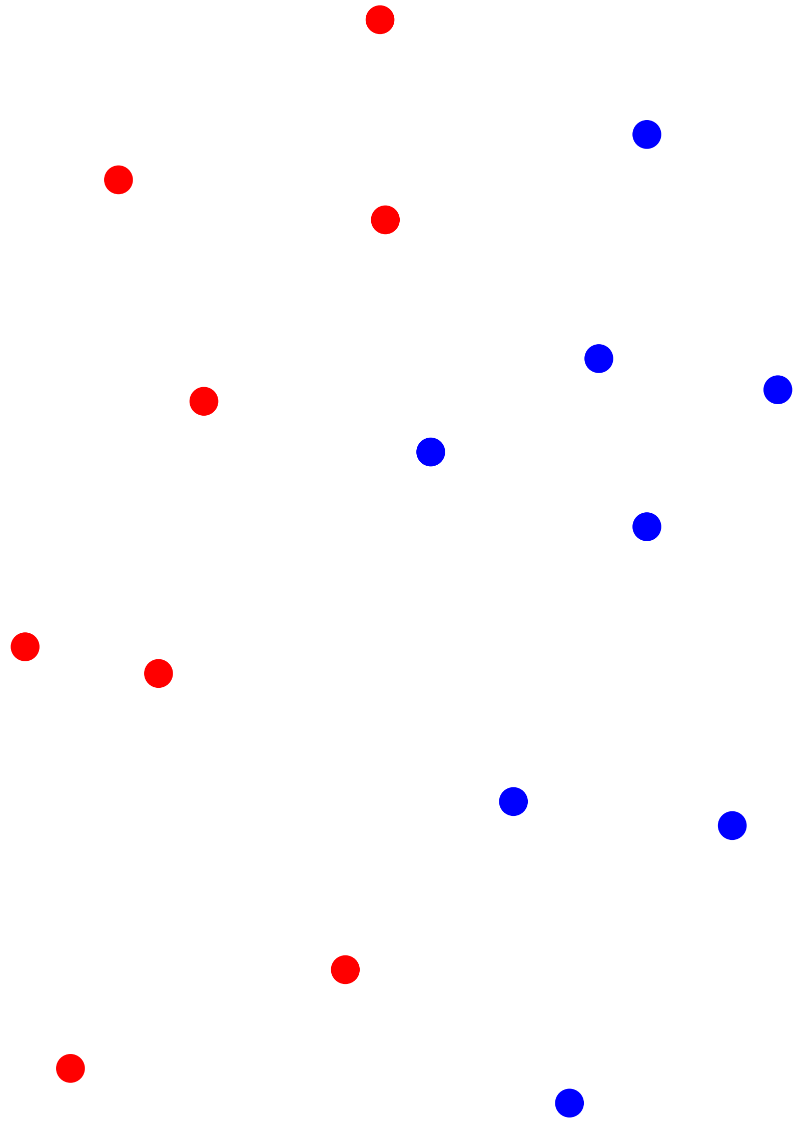


Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...



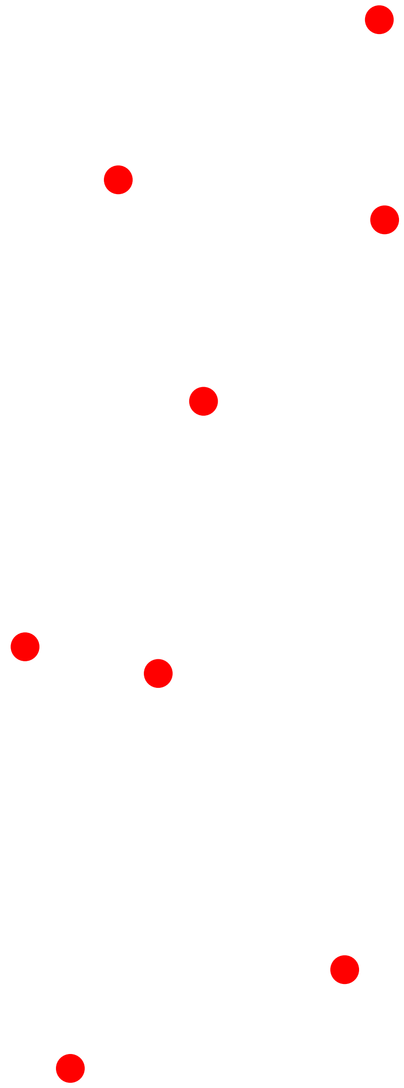
Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...



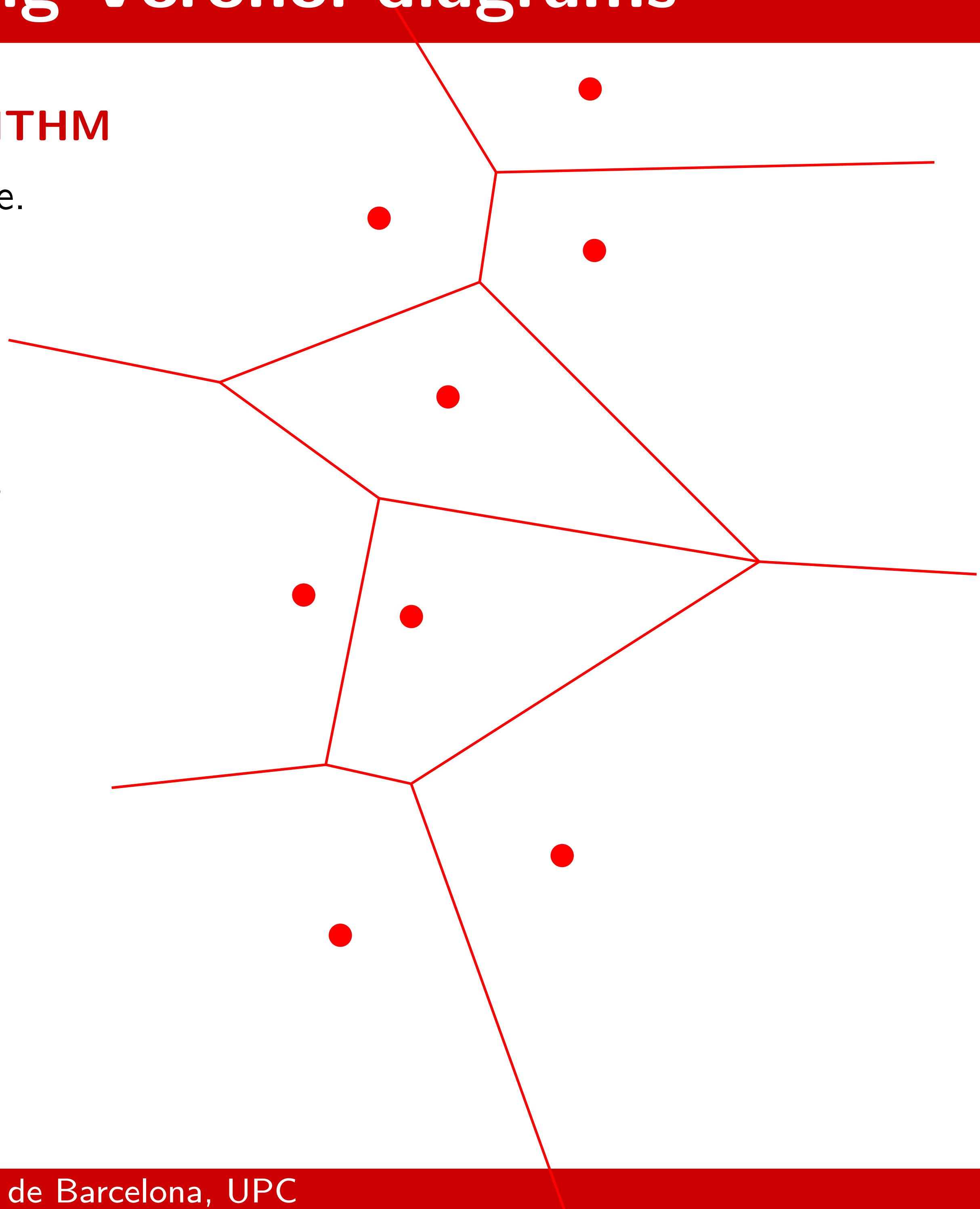
Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

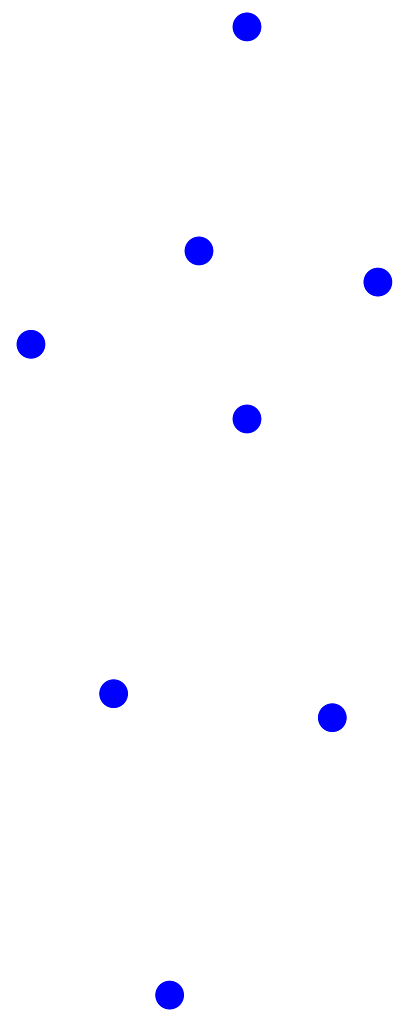
...consider the Voronoi diagram of the sets R and B...



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.



If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

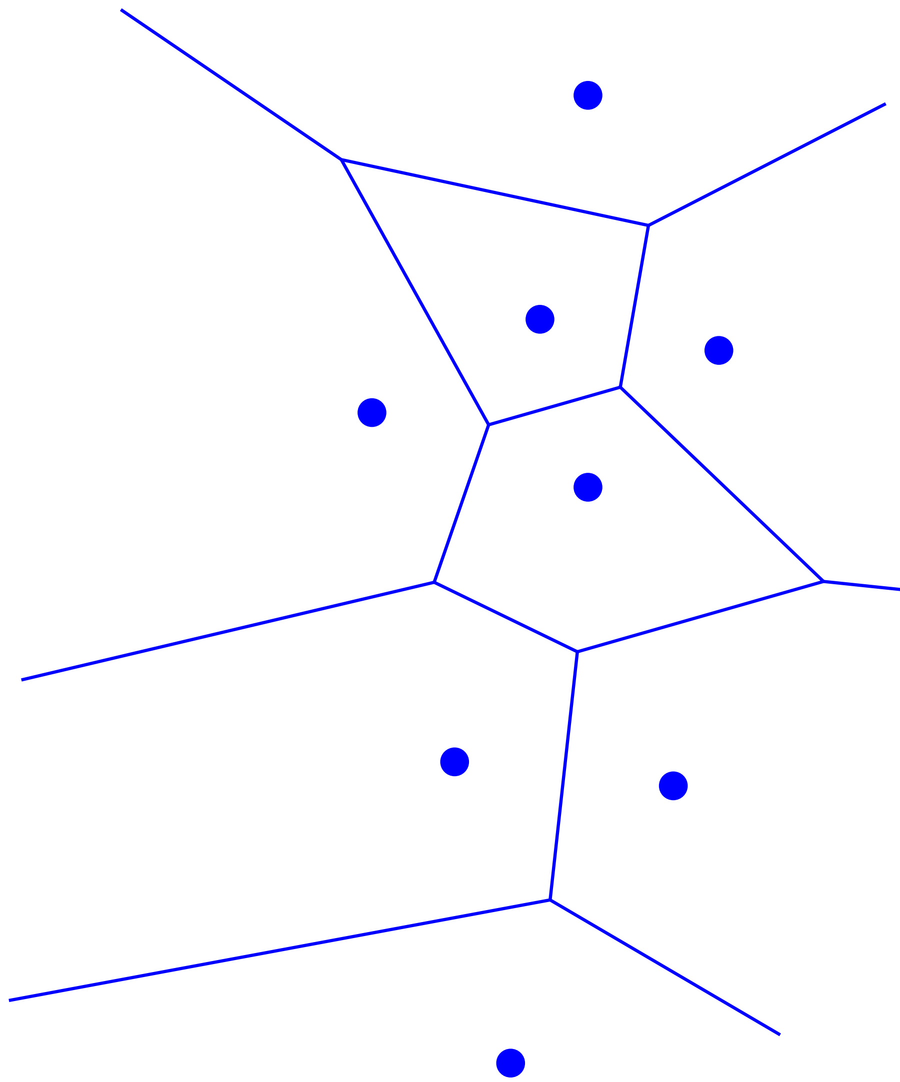
Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...



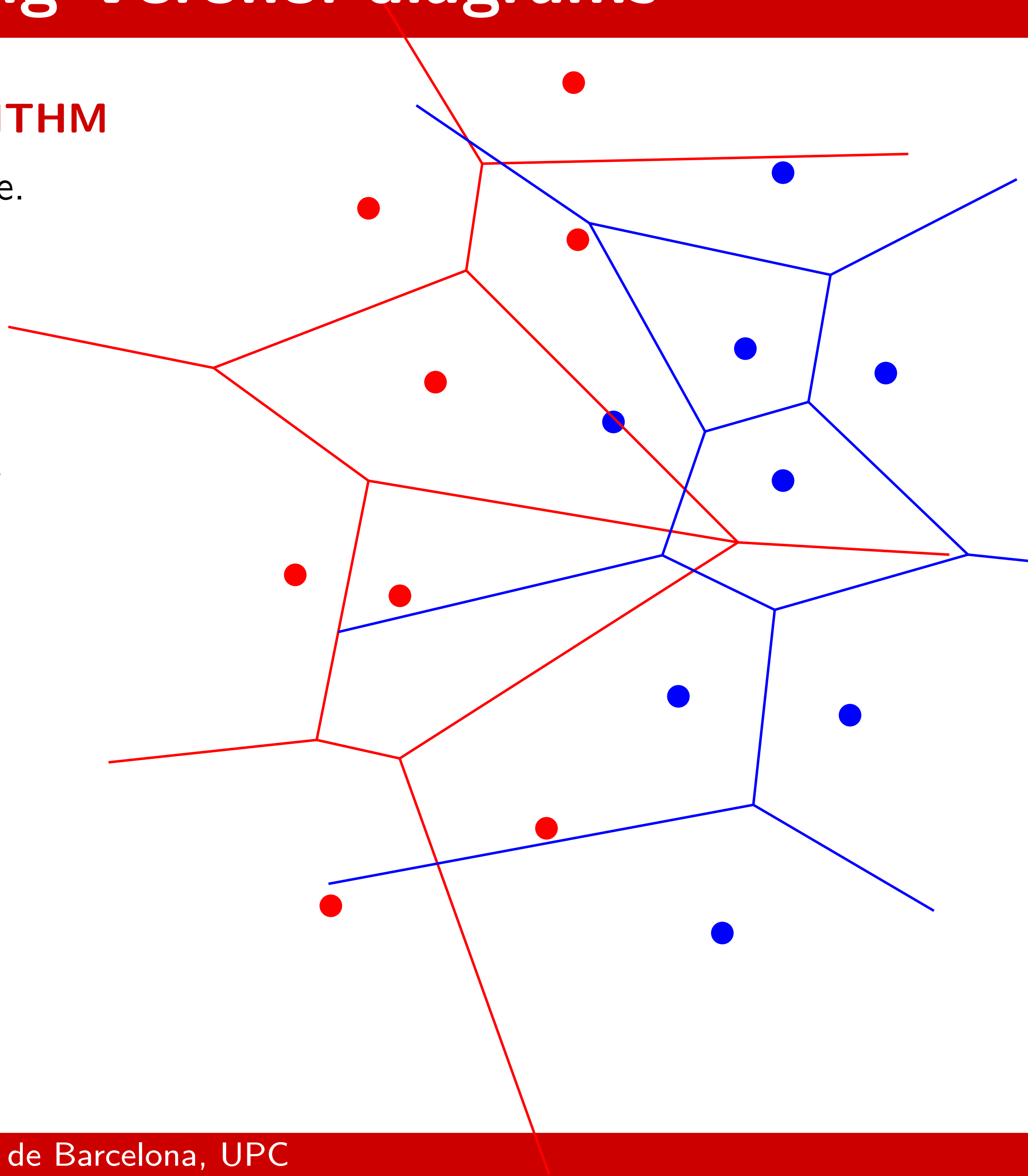
Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...



Constructing Voronoi diagrams

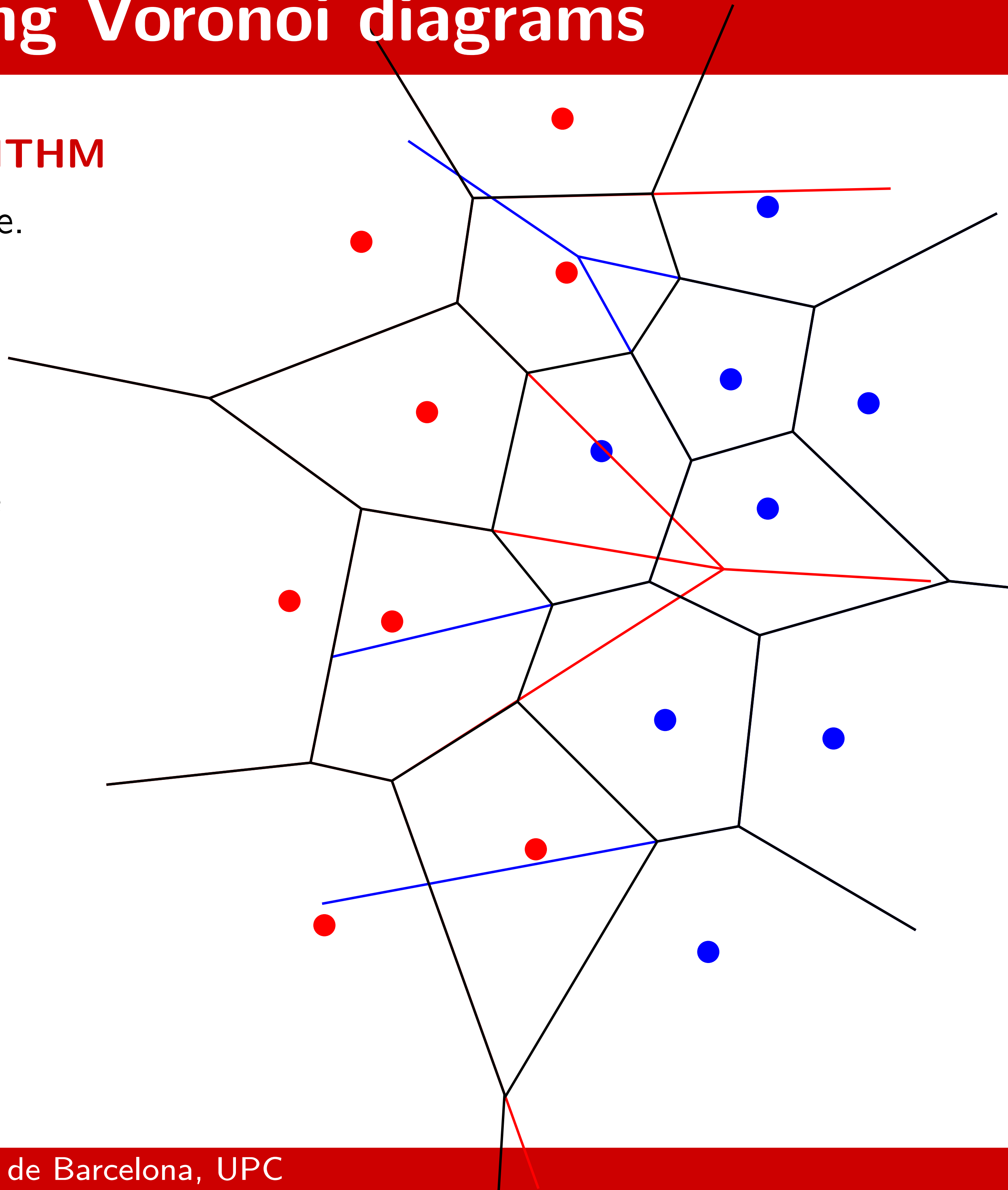
DIVIDE AND CONQUER ALGORITHM

Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

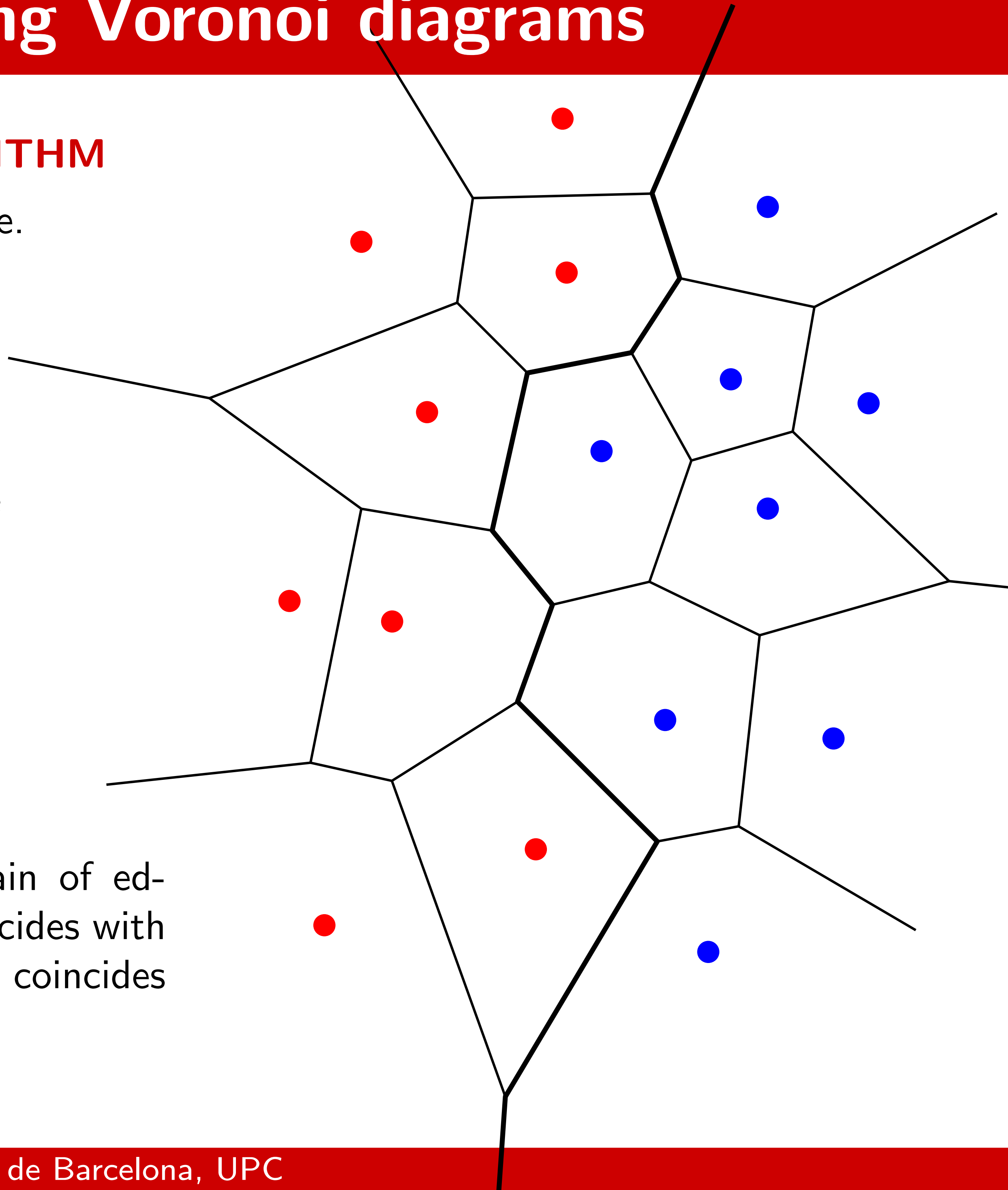
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



DIVIDE AND CONQUER ALGORITHM

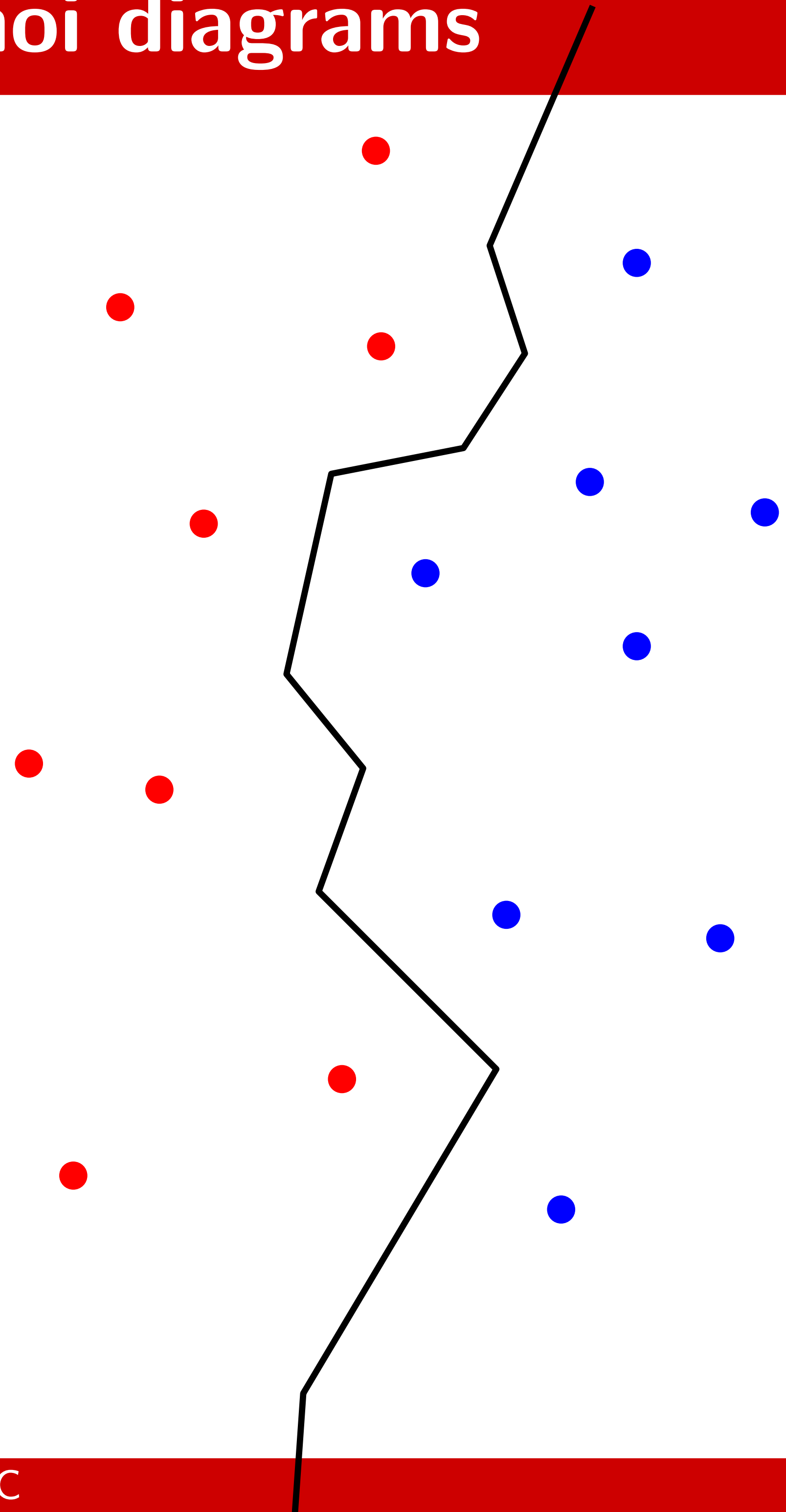
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

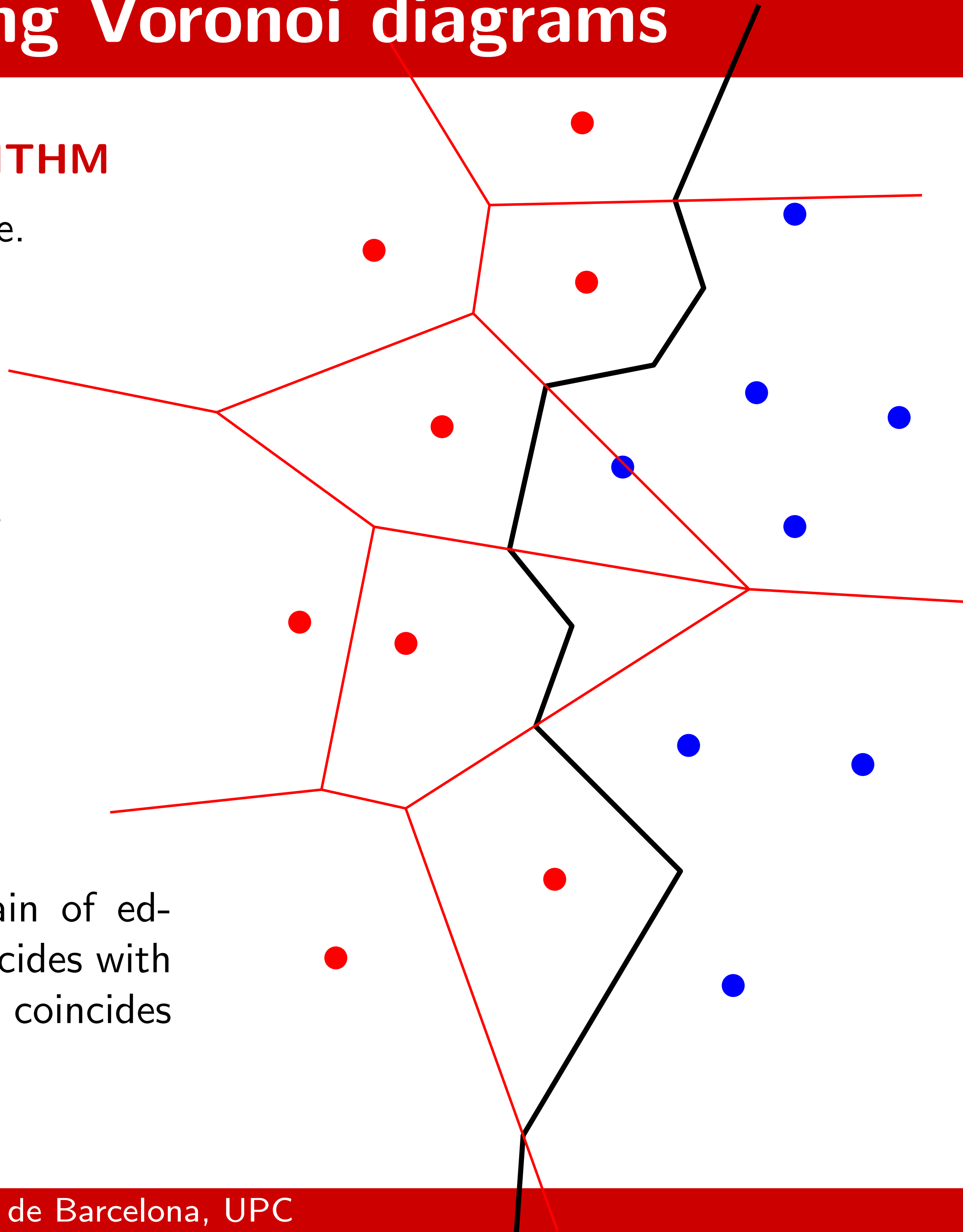
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

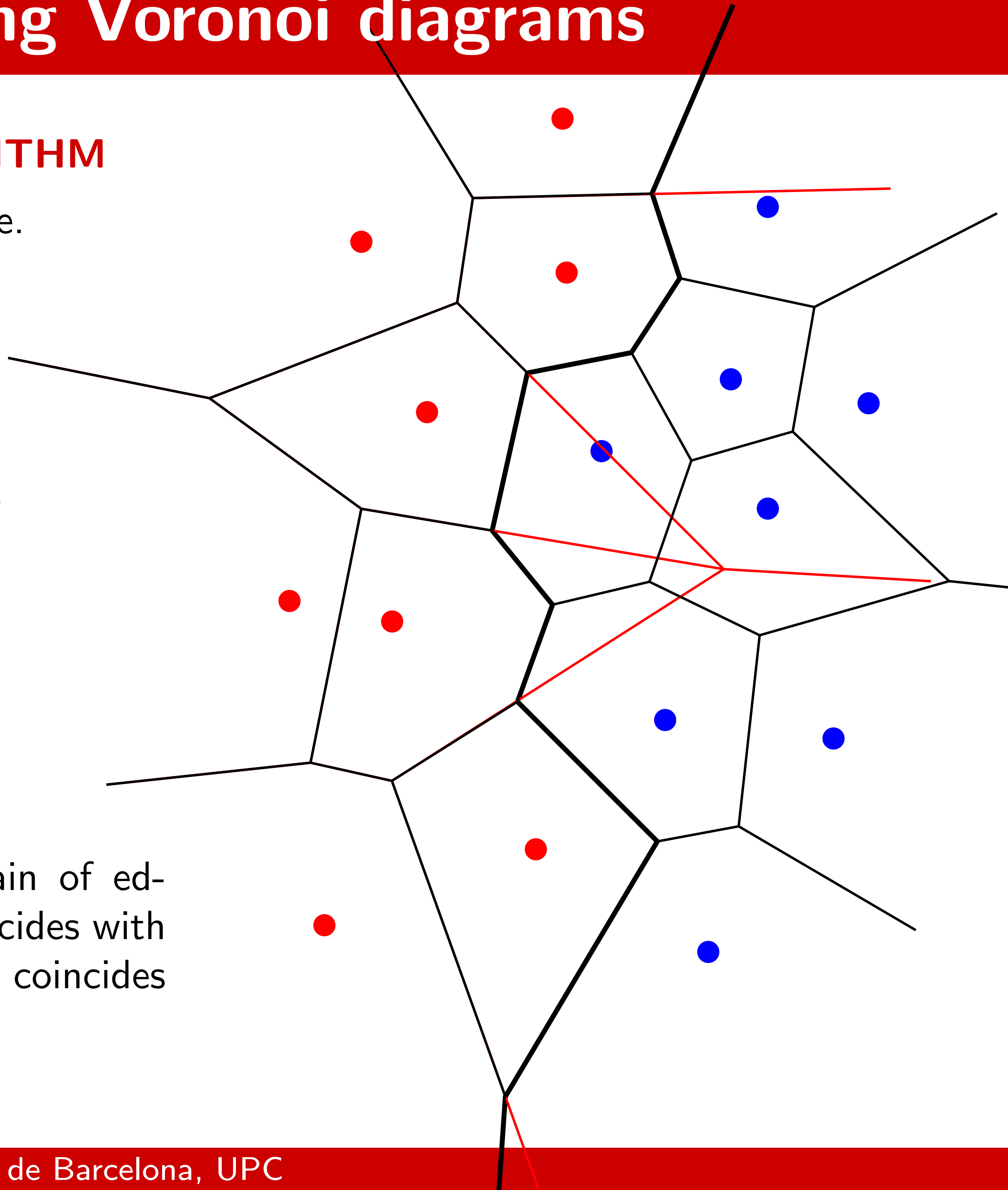
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

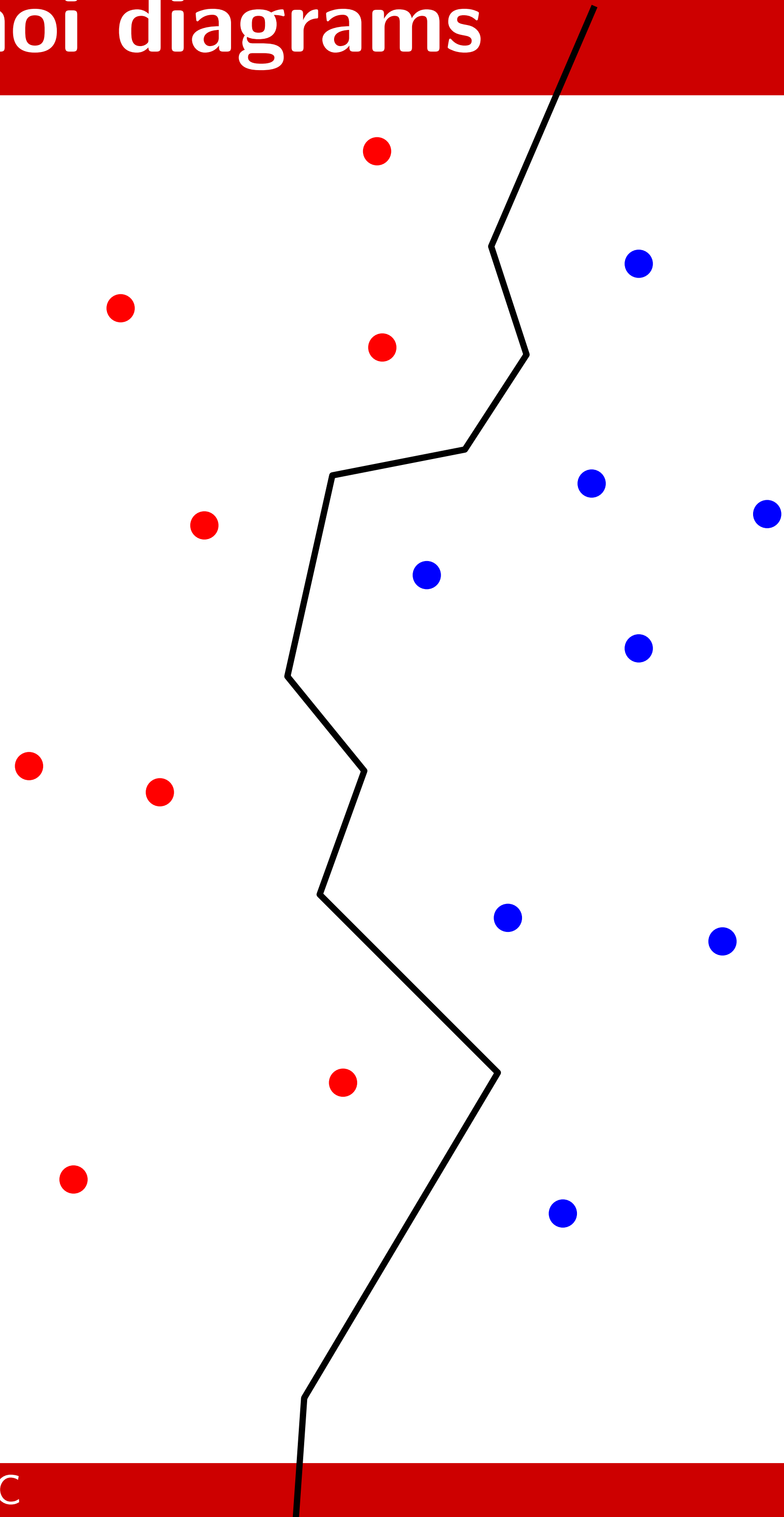
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

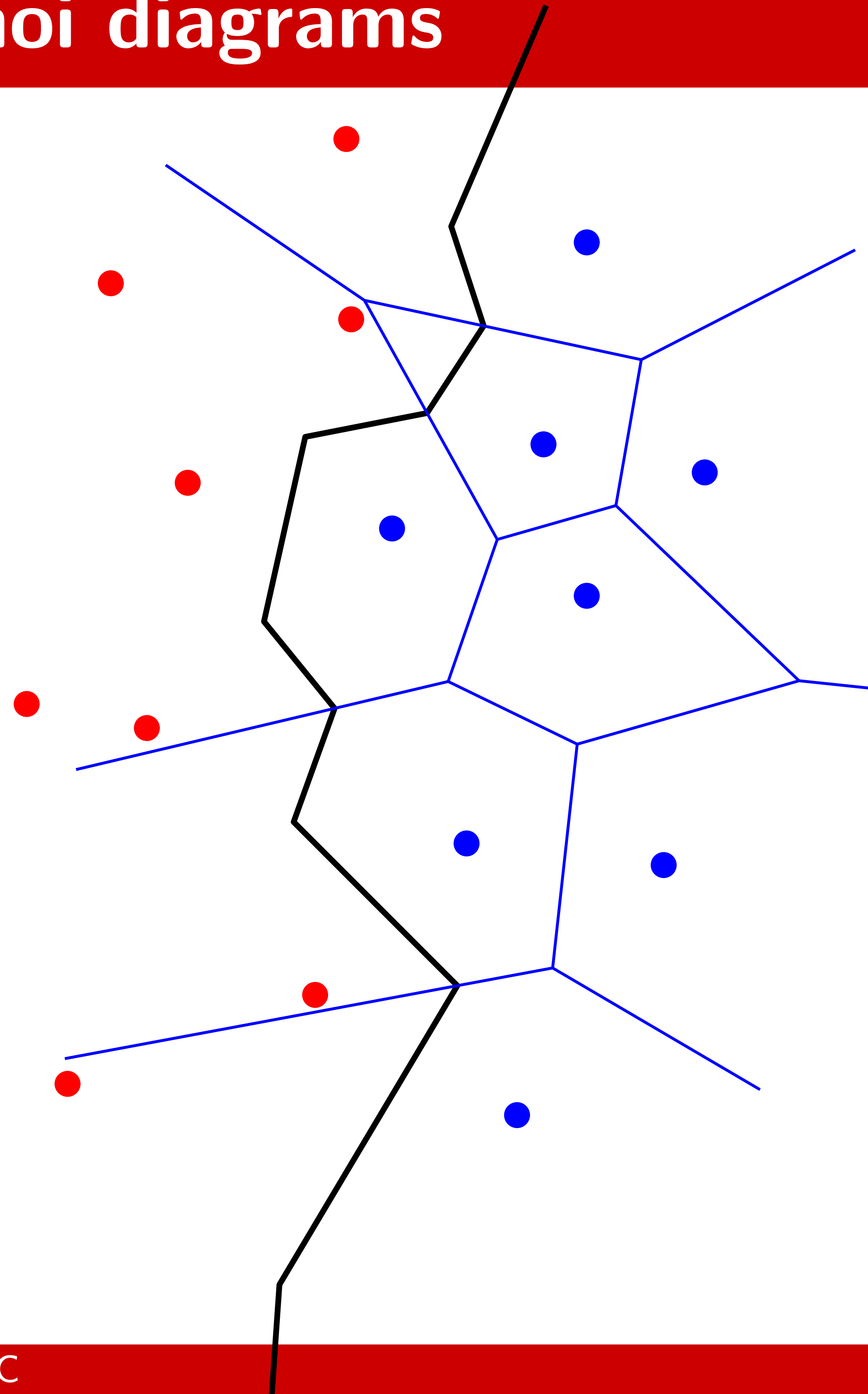
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

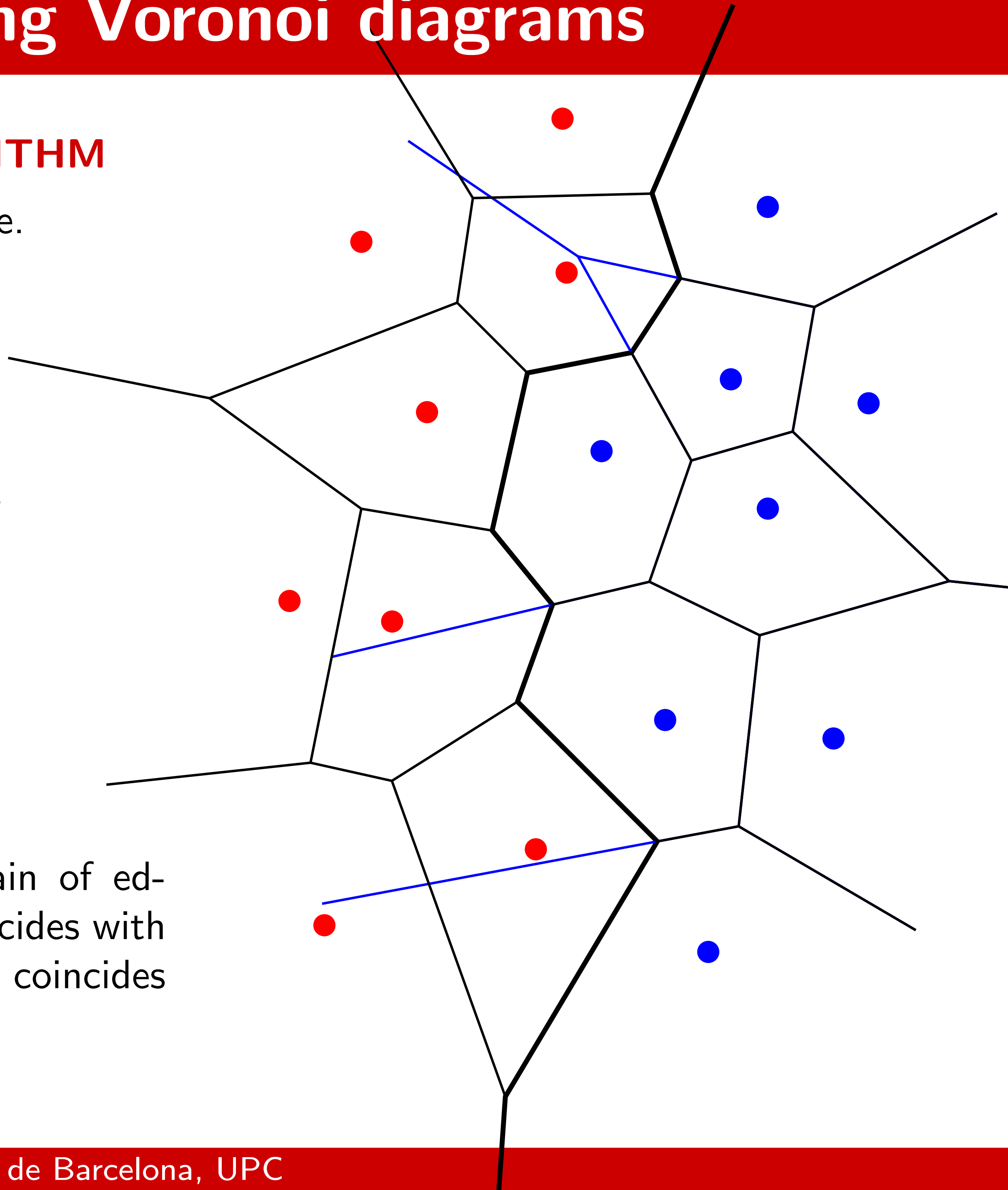
Let P be a set of n points in the plane.

If the points are vertically partitioned into two subsets R and B...

...consider the Voronoi diagram of the sets R and B...

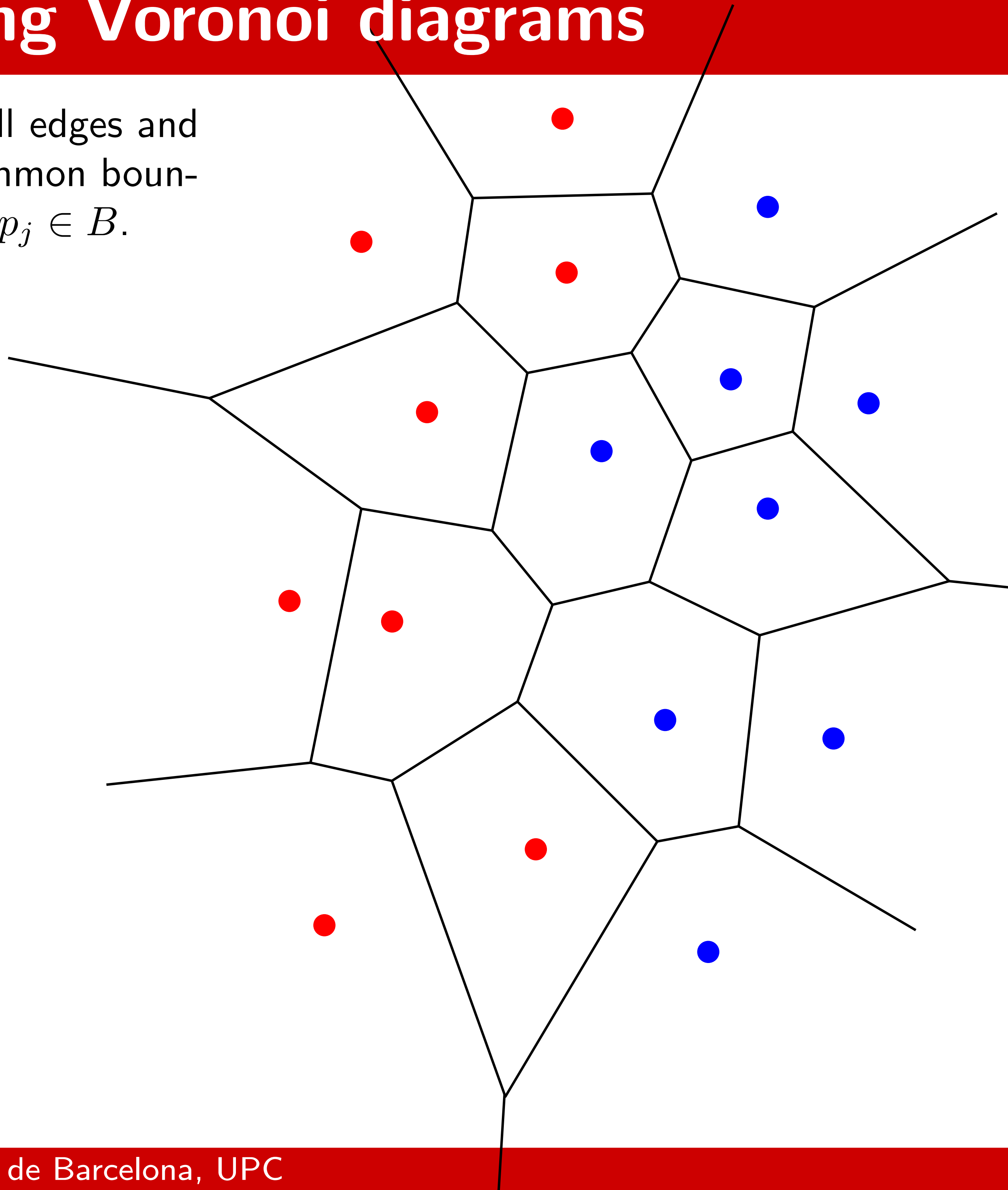
...then the Voronoi diagram of P substantially coincides with the Voronoi diagrams of R and B!

In fact, there exists a monotone chain of edges of $Vor(P)$ such that $Vor(P)$ coincides with $Vor(R)$ to the left of the chain, and it coincides with $Vor(B)$ to its right.



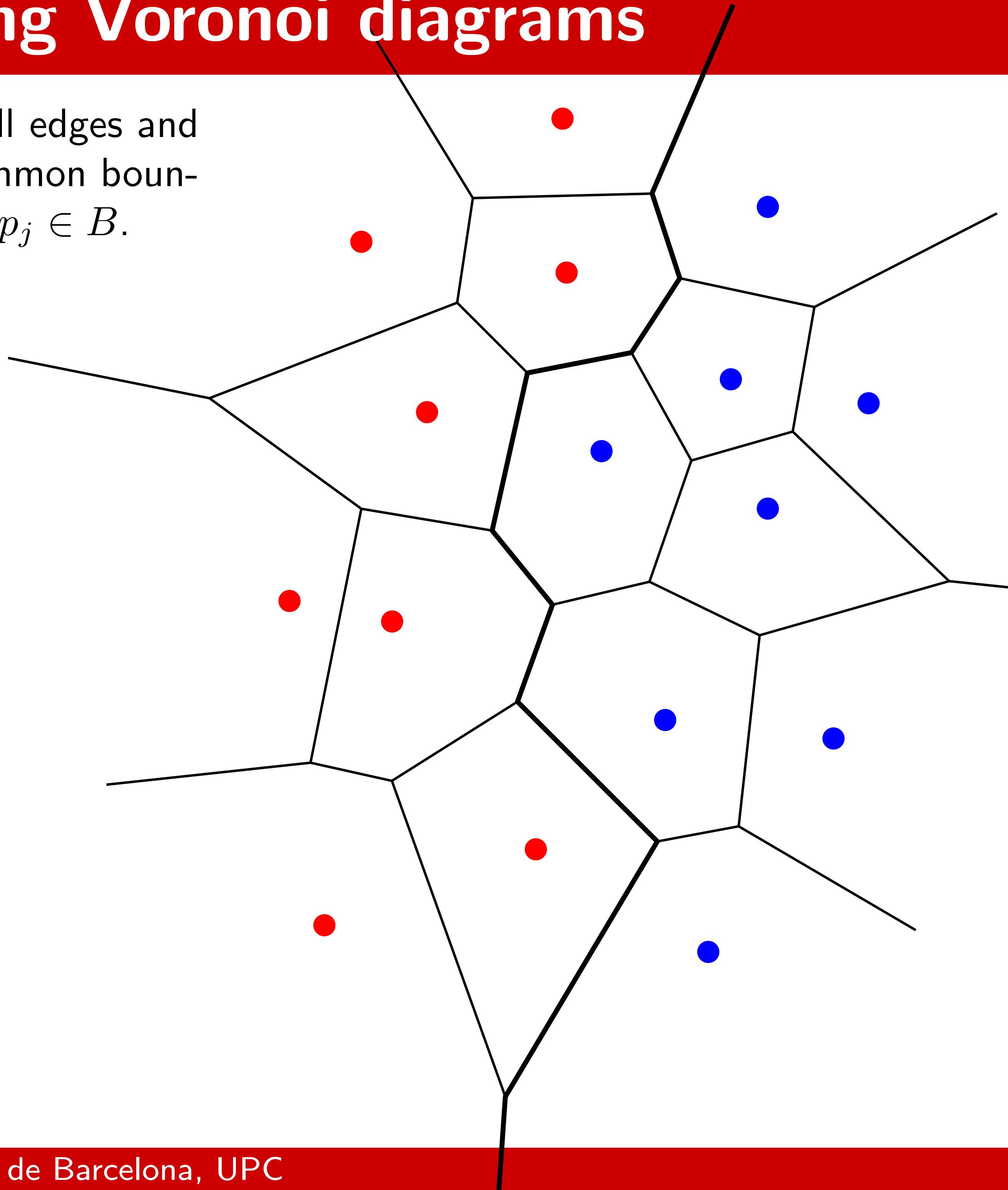
Constructing Voronoi diagrams

Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.



Constructing Voronoi diagrams

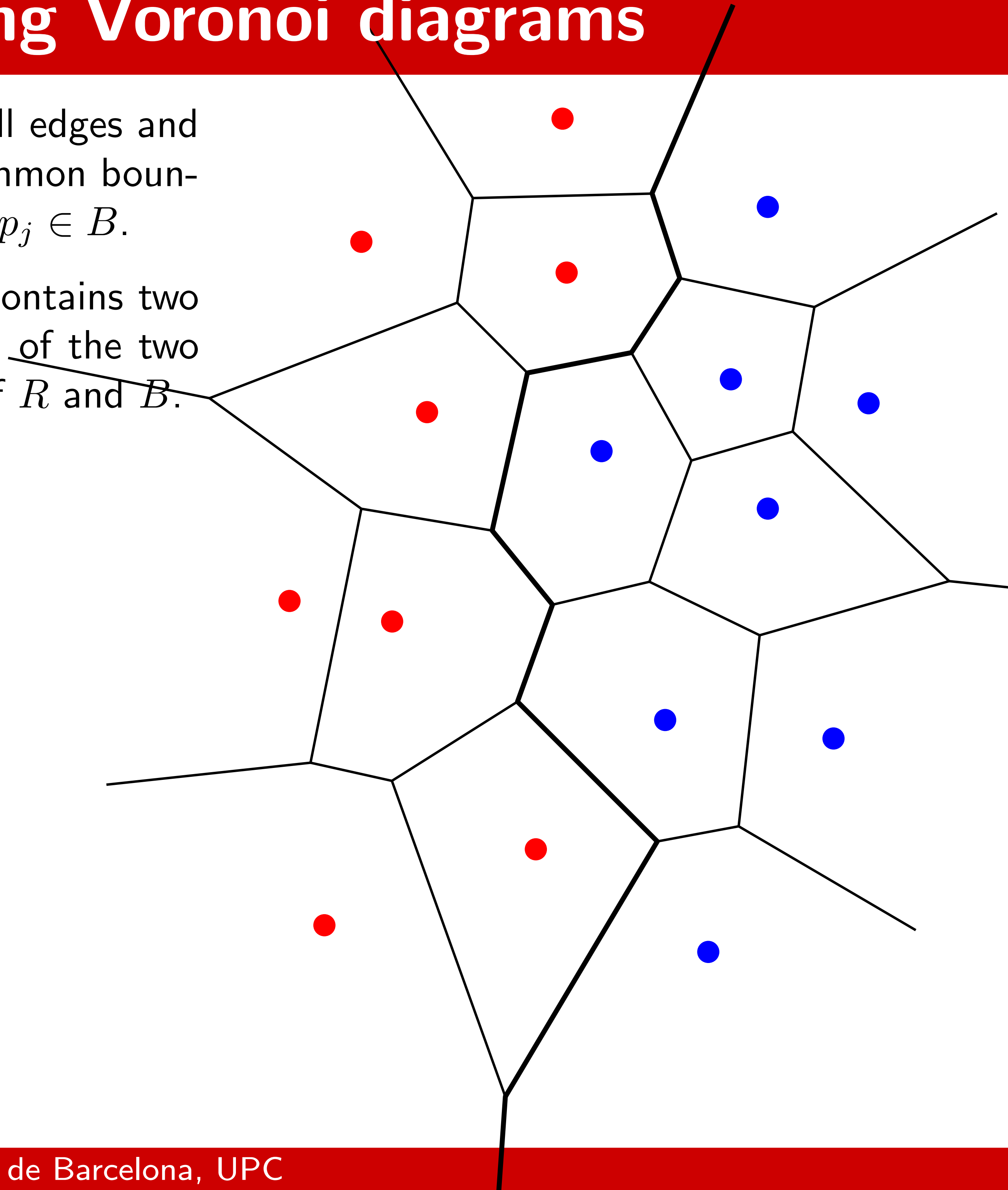
Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.



Constructing Voronoi diagrams

Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R, B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

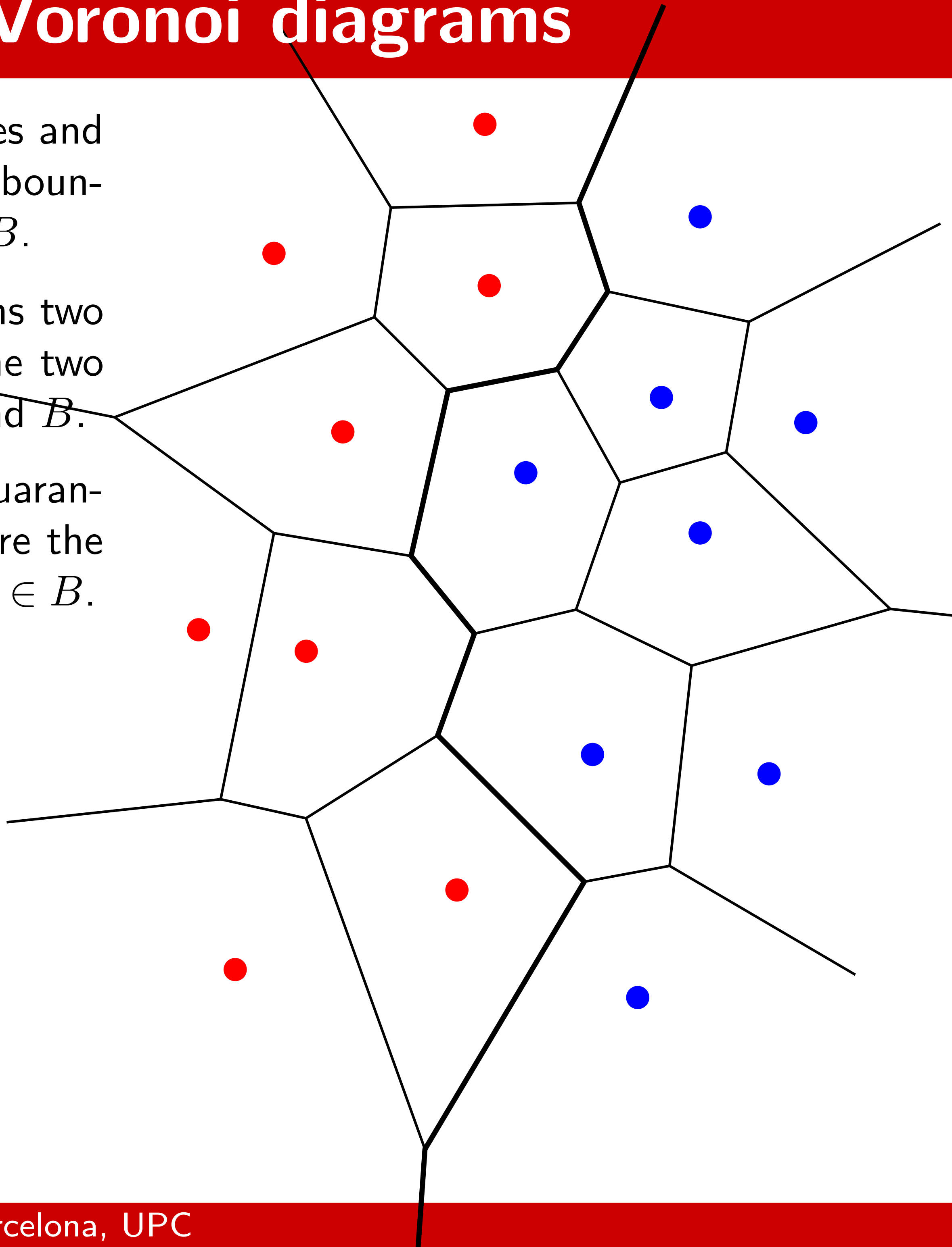


Constructing Voronoi diagrams

Definition. Let $b(R,B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R,B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

Proof. The vertical separation of R and B guarantees the existence of the "bridges", which are the edges of $ch(P)$ connecting a $p_i \in R$ to a $p_j \in B$.

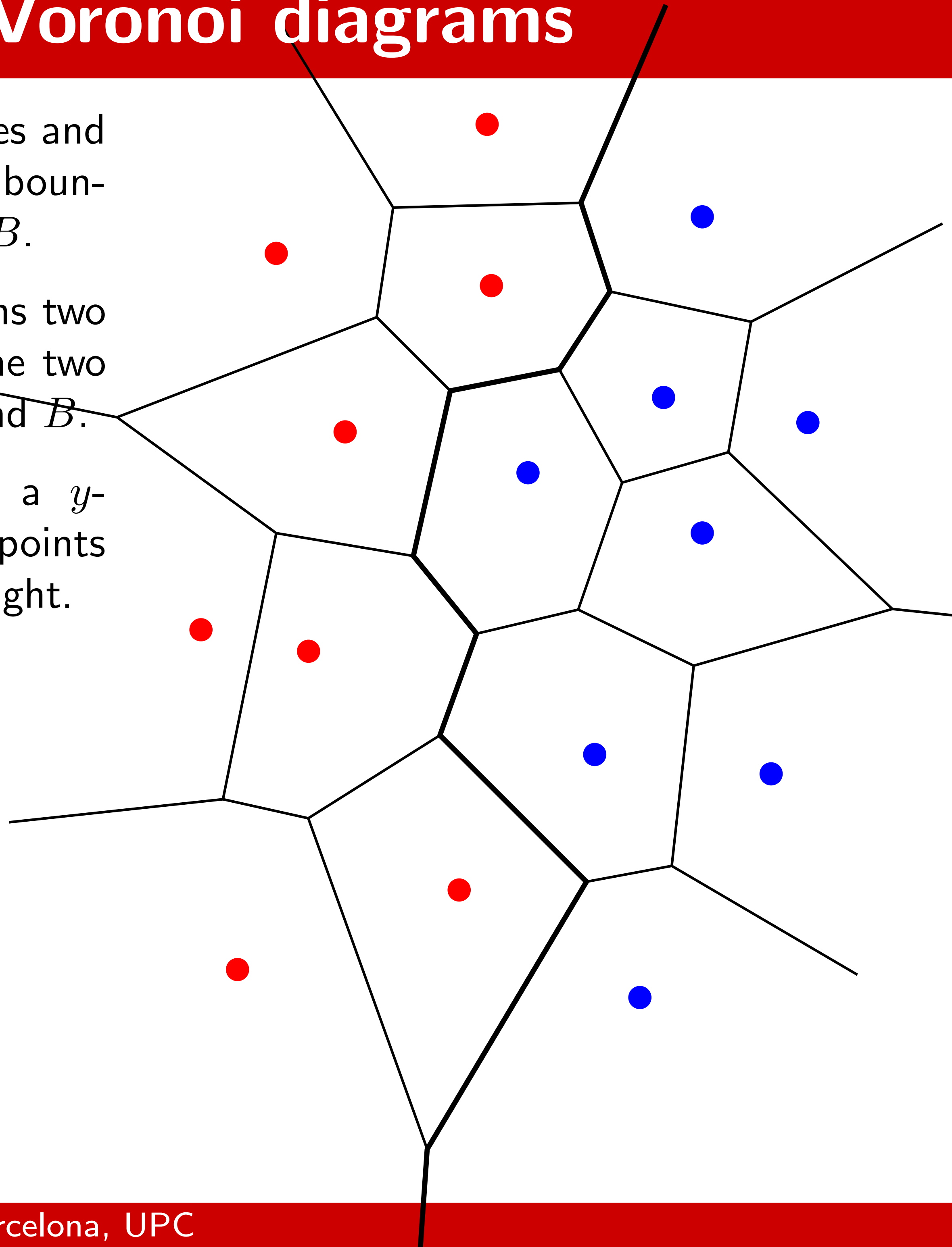


Constructing Voronoi diagrams

Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R, B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

Observation 2. The bisector $b(R, B)$ is a y-monotone chain leaving the regions of the points $p_i \in R$ to its left and those of $p_j \in B$ to its right.



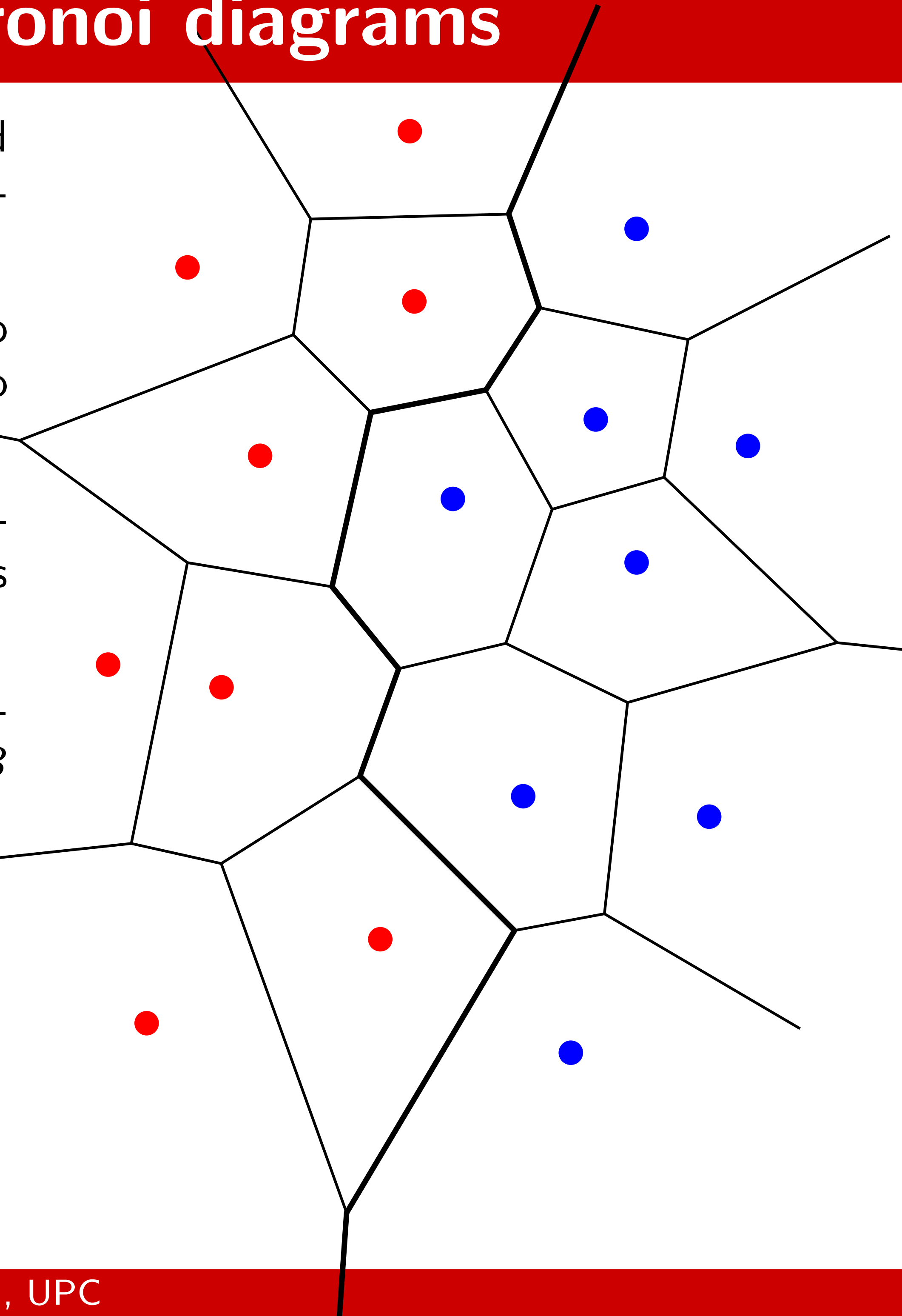
Constructing Voronoi diagrams

Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R, B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

Observation 2. The bisector $b(R, B)$ is a y-monotone chain leaving the regions of the points $p_i \in R$ to its left and those of $p_j \in B$ to its right.

Proof. Every edge e_{ij} of $b(R, B)$ must be non-horizontal, and leave $p_i \in R$ to its left and $p_j \in B$ to its right.



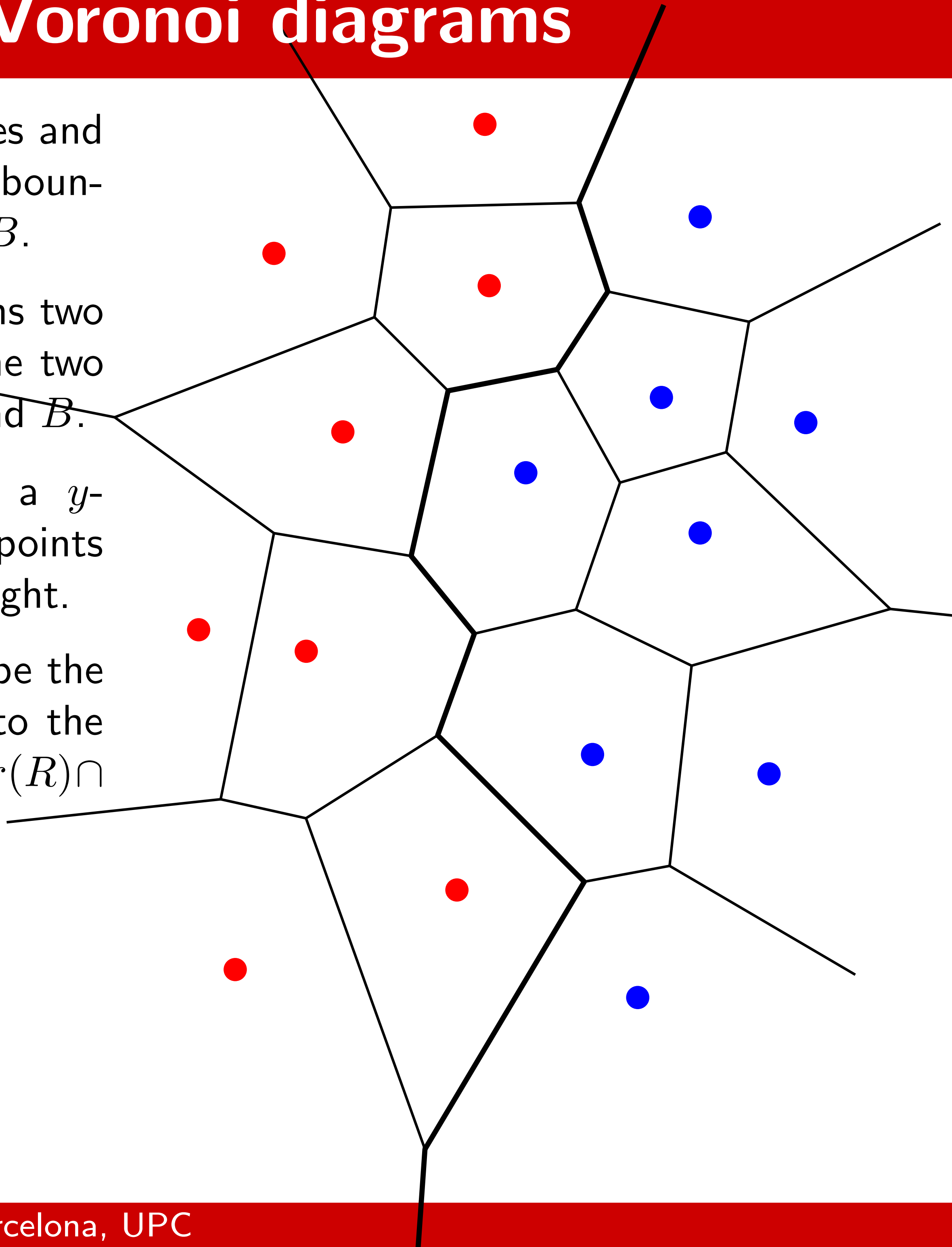
Constructing Voronoi diagrams

Definition. Let $b(R,B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R,B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

Observation 2. The bisector $b(R,B)$ is a y-monotone chain leaving the regions of the points $p_i \in R$ to its left and those of $p_j \in B$ to its right.

Observation 3. Let π_R and π_B respectively be the regions of the plane located to the left and to the right of $b(R,B)$. Then $Vor(P)$ consists of $Vor(R) \cap \pi_R$, $Vor(B) \cap \pi_B$ and $b(R,B)$.



Constructing Voronoi diagrams

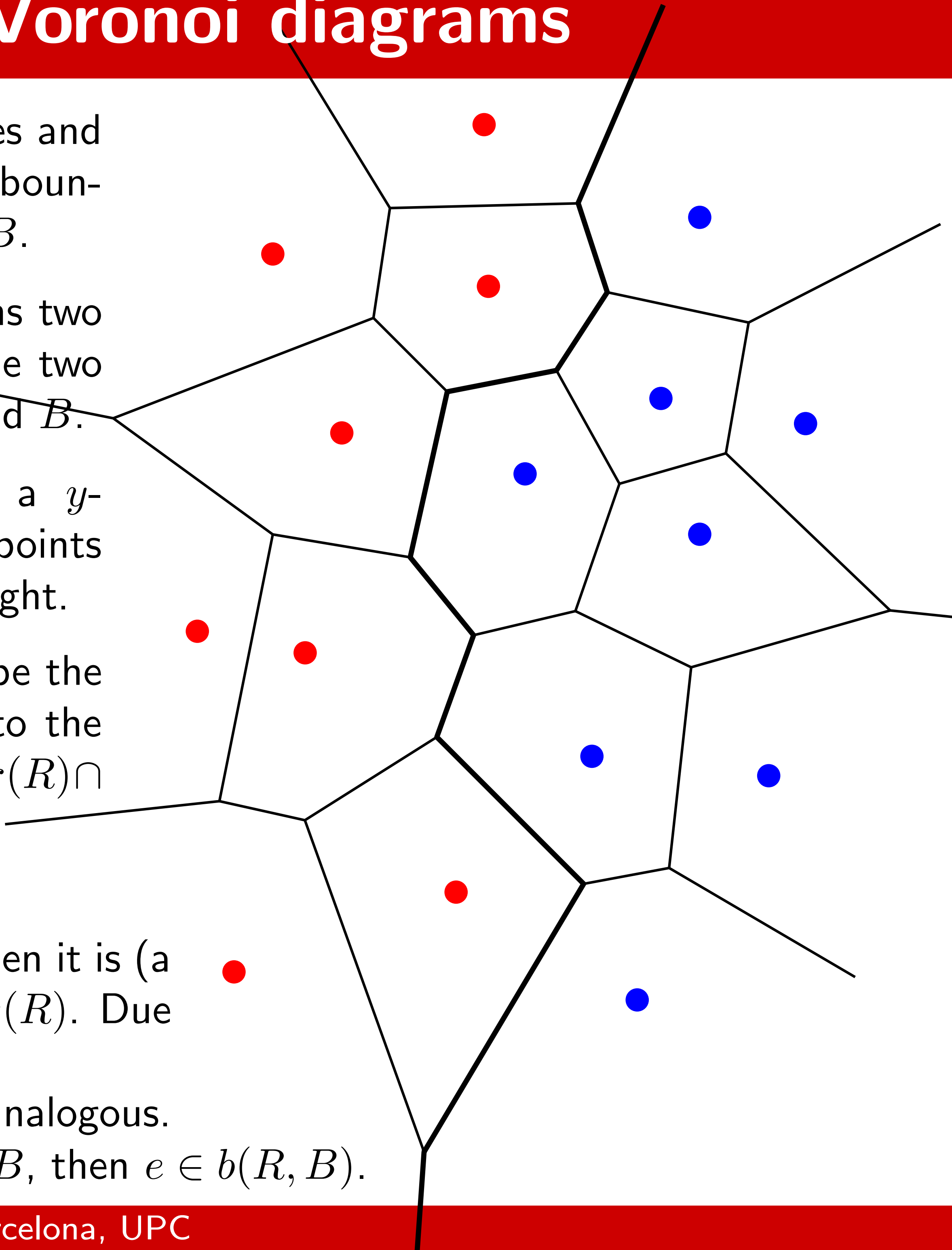
Definition. Let $b(R, B)$ be the set of all edges and vertices of $Vor(P)$ belonging to the common boundary of the regions of some $p_i \in R$ and $p_j \in B$.

Observation 1. The bisector $b(R, B)$ contains two half-lines, belonging to the bisectors b_{ij} of the two "bridges" connecting the convex hulls of R and B.

Observation 2. The bisector $b(R, B)$ is a y-monotone chain leaving the regions of the points $p_i \in R$ to its left and those of $p_j \in B$ to its right.

Observation 3. Let π_R and π_B respectively be the regions of the plane located to the left and to the right of $b(R, B)$. Then $Vor(P)$ consists of $Vor(R) \cap \pi_R$, $Vor(B) \cap \pi_B$ and $b(R, B)$.

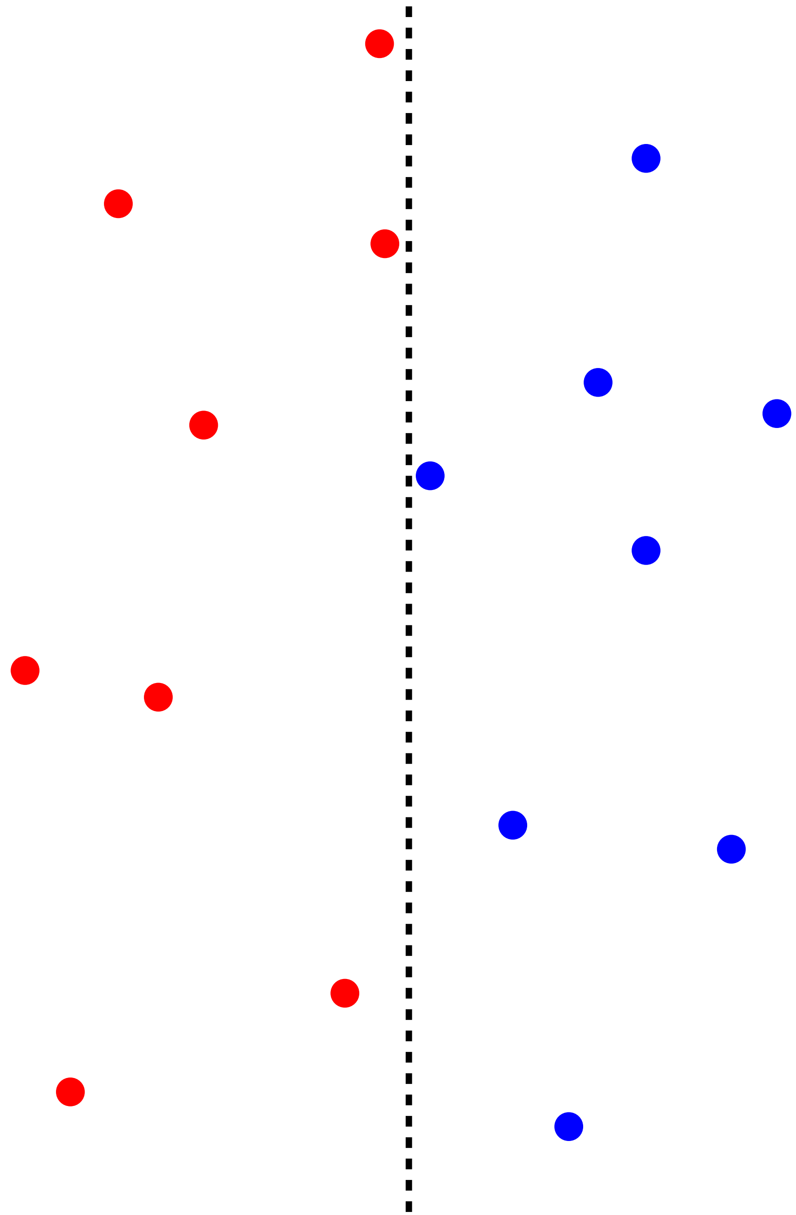
Proof. Let e be an edge of $Vor(P)$:
- If e separates two points of R in Vor(P), then it is (a portion of) the edge separating them in $Vor(R)$. Due to Obs. 2, e cannot belong to π_B.
- If e separates two points of B, the case is analogous.
- If e separates one point of R from one of B, then $e \in b(R, B)$.



Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

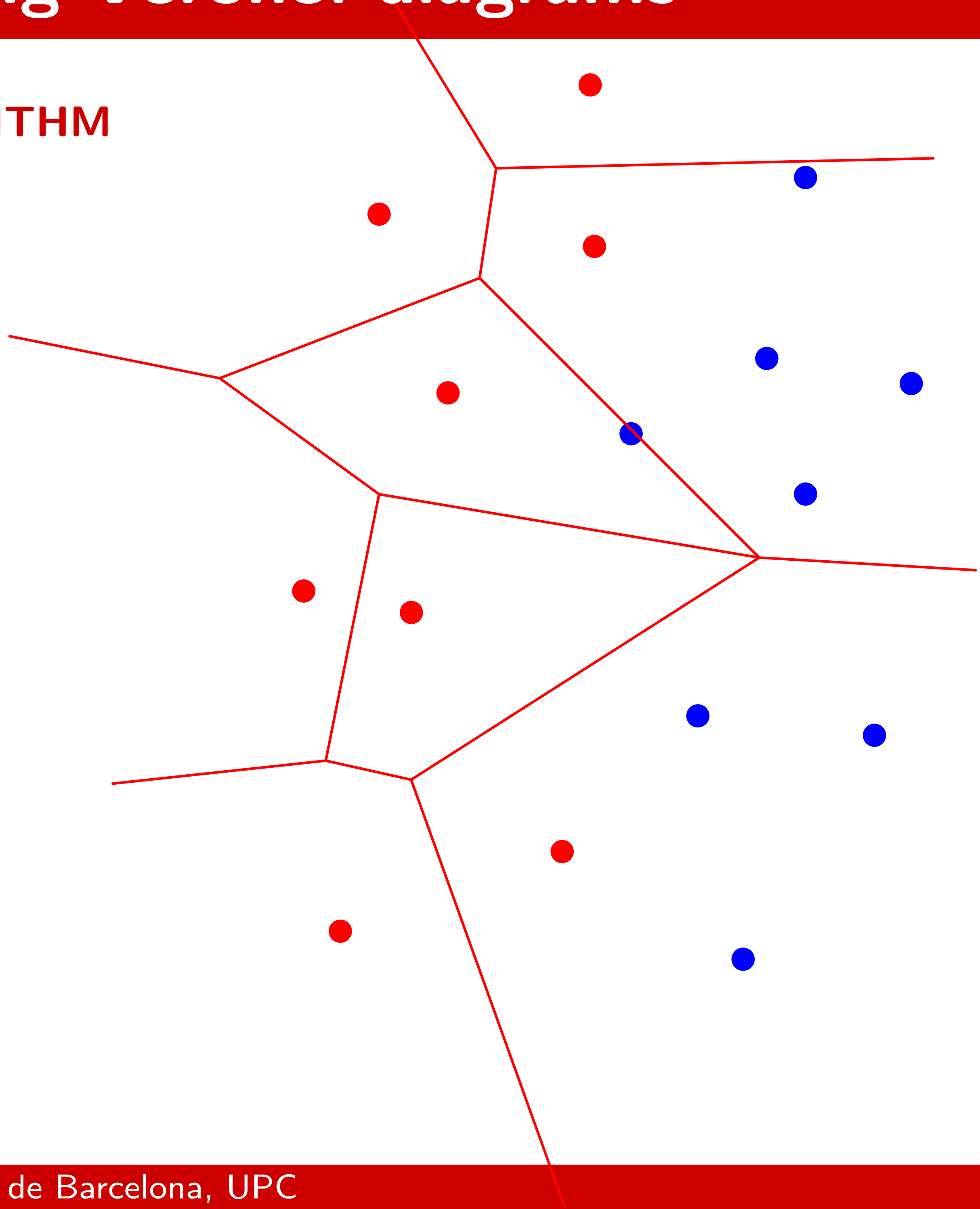


Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

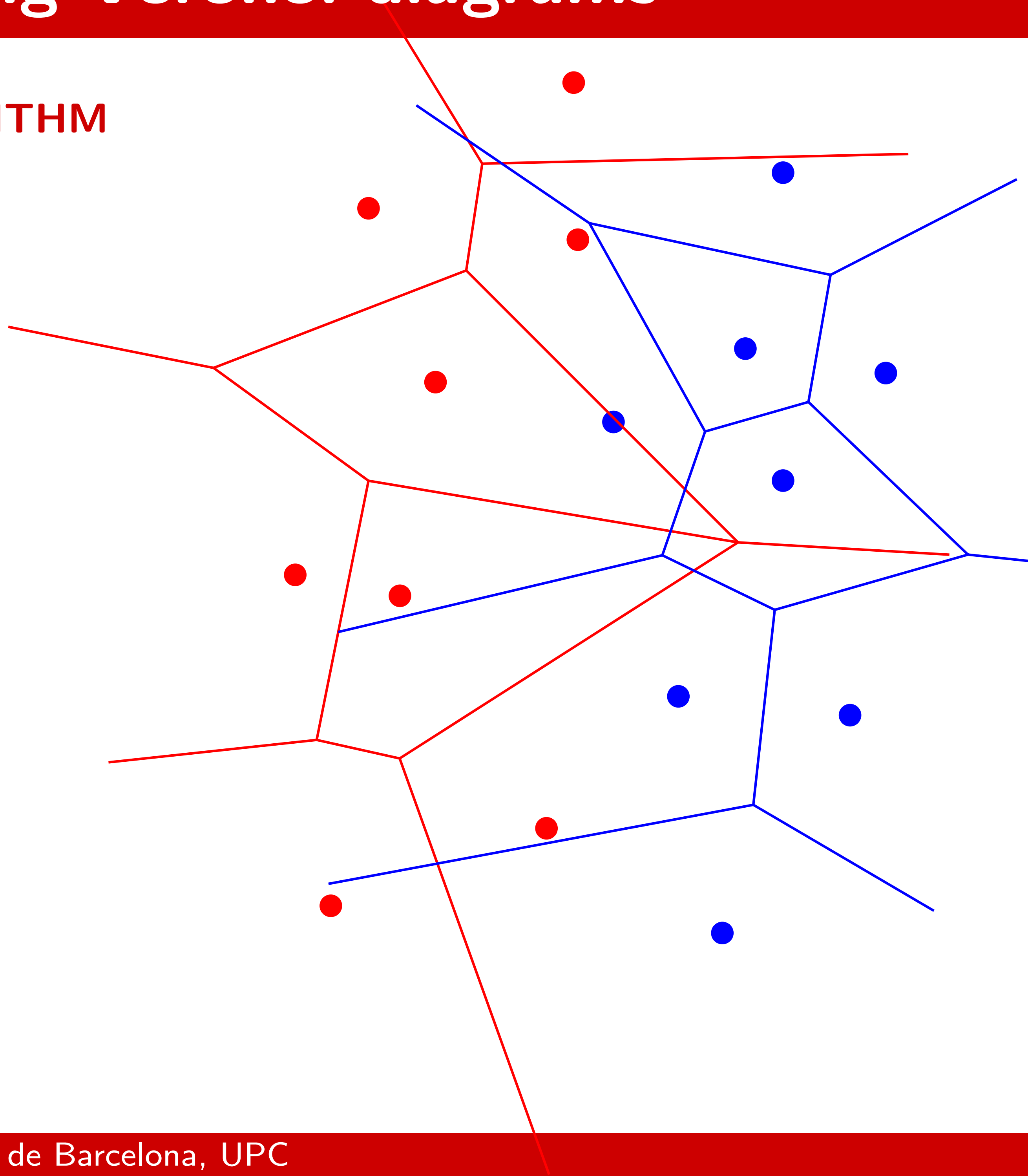


Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.



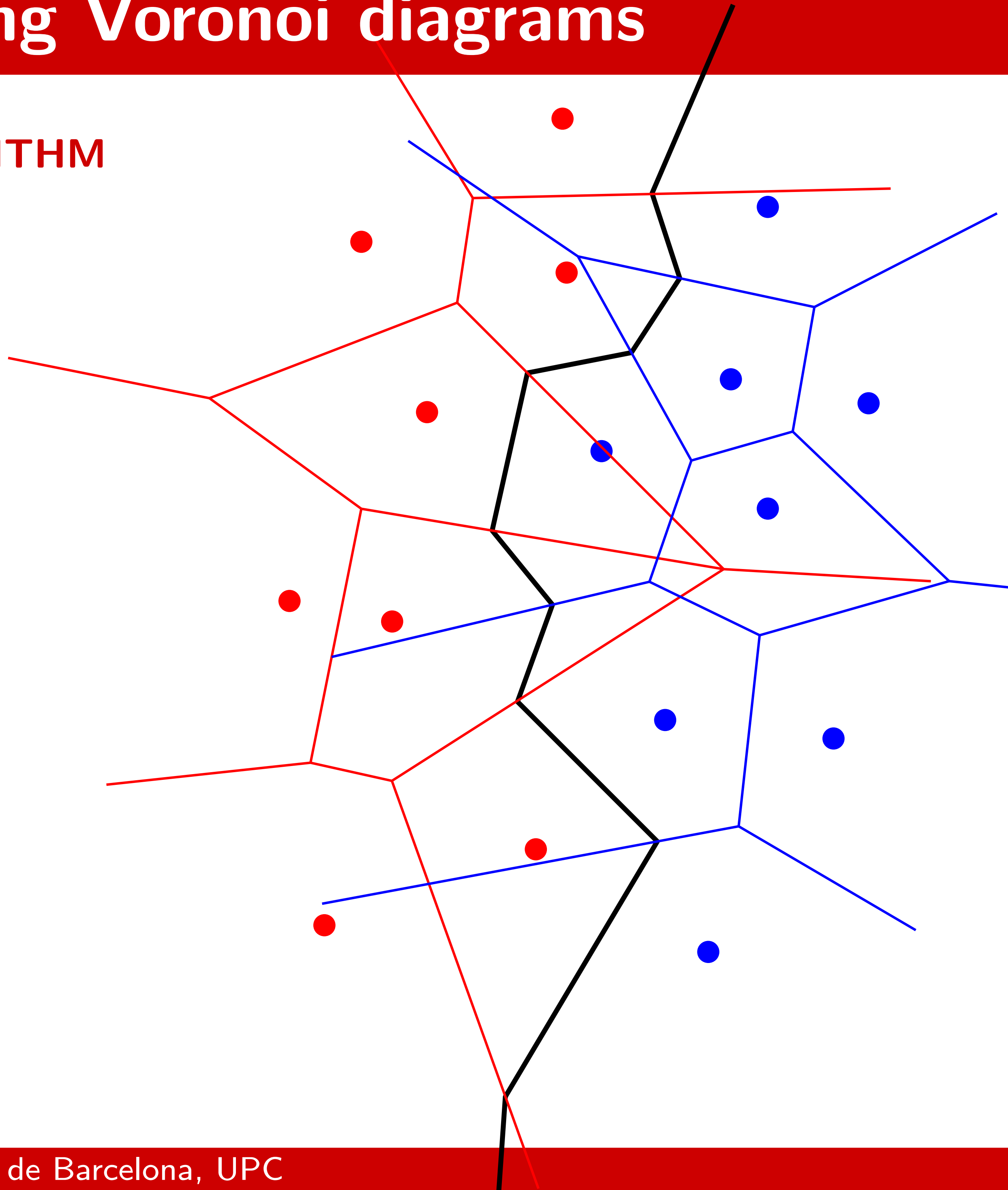
Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.



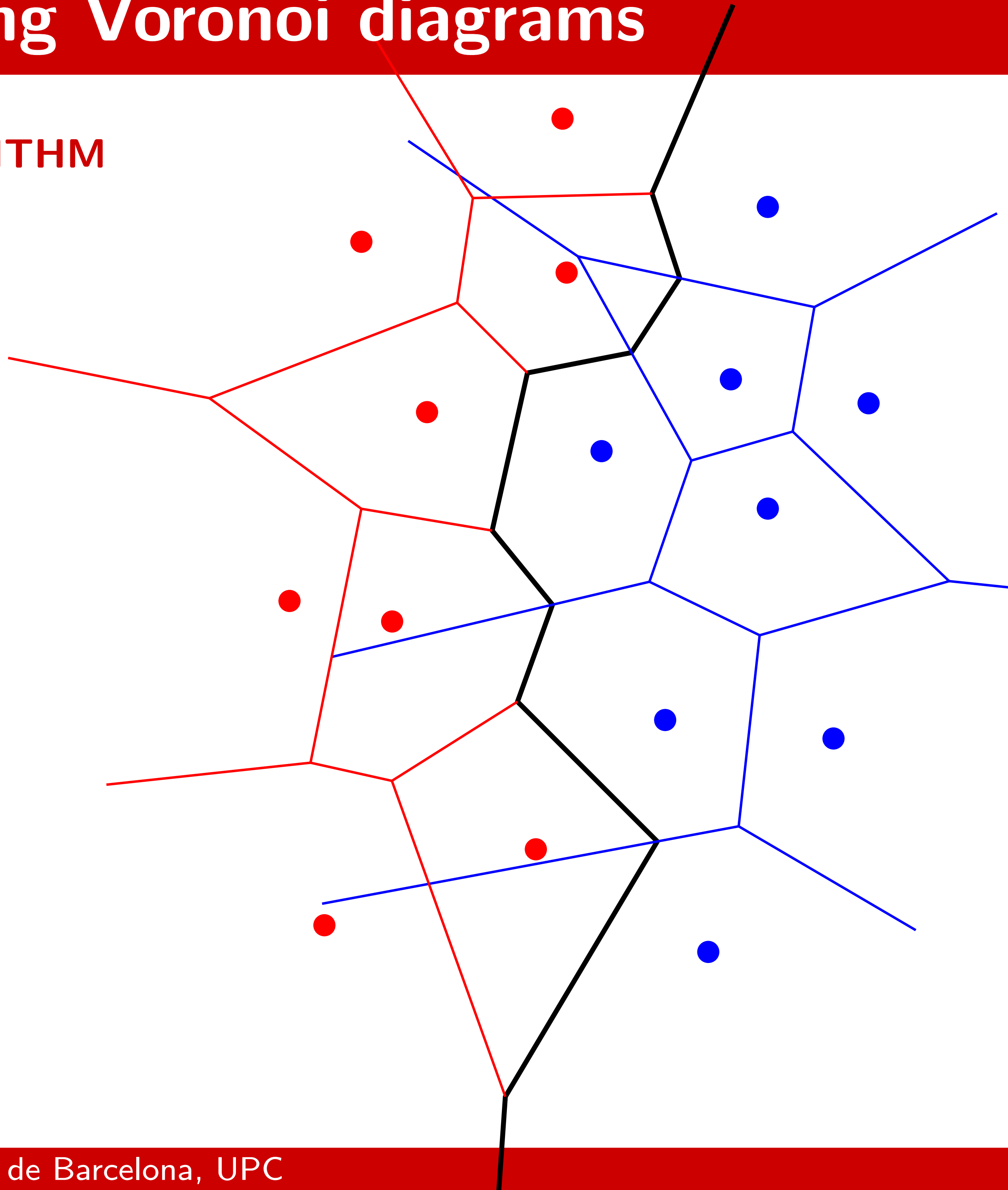
DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.



Constructing Voronoi diagrams

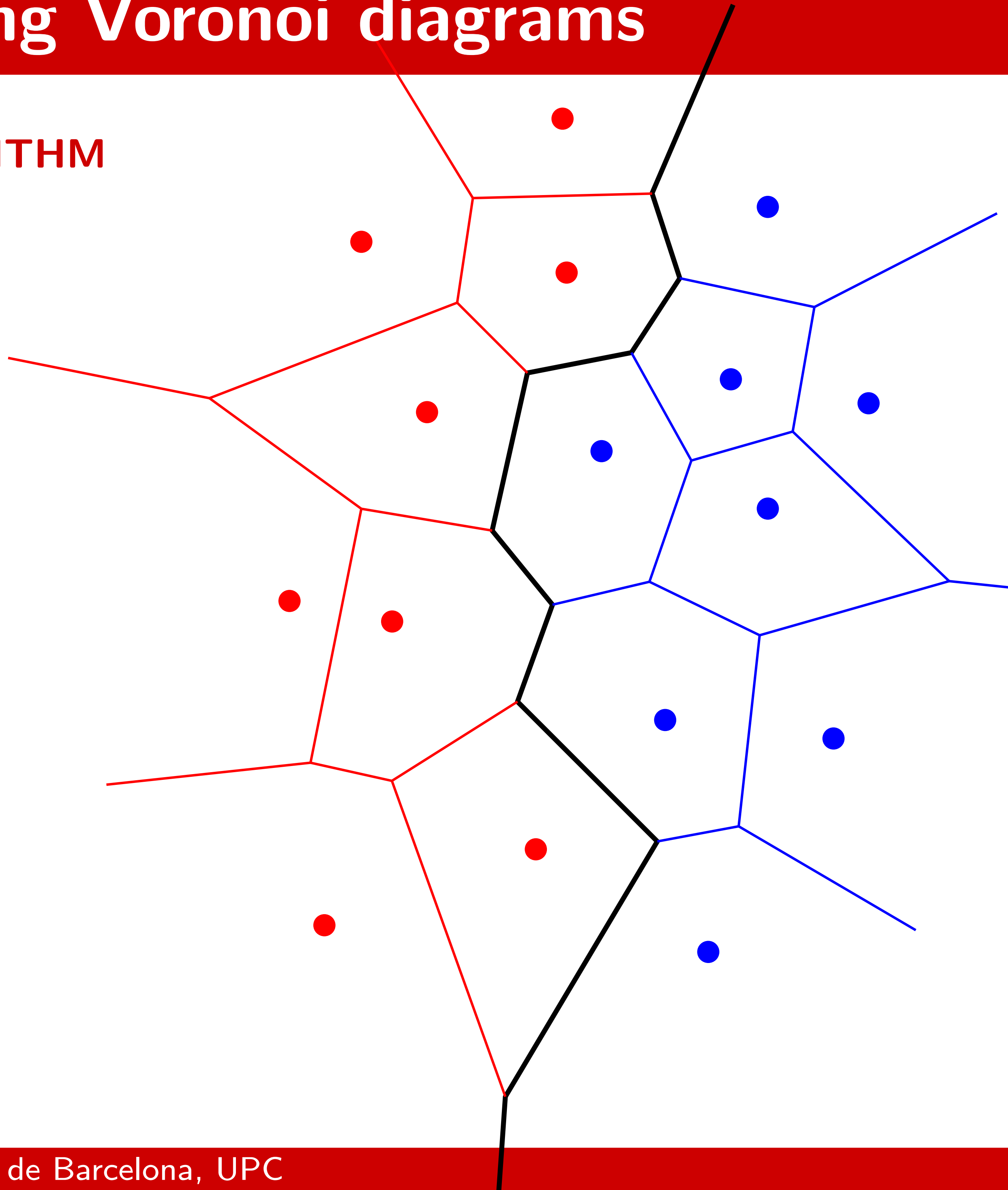
DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.



Constructing Voronoi diagrams

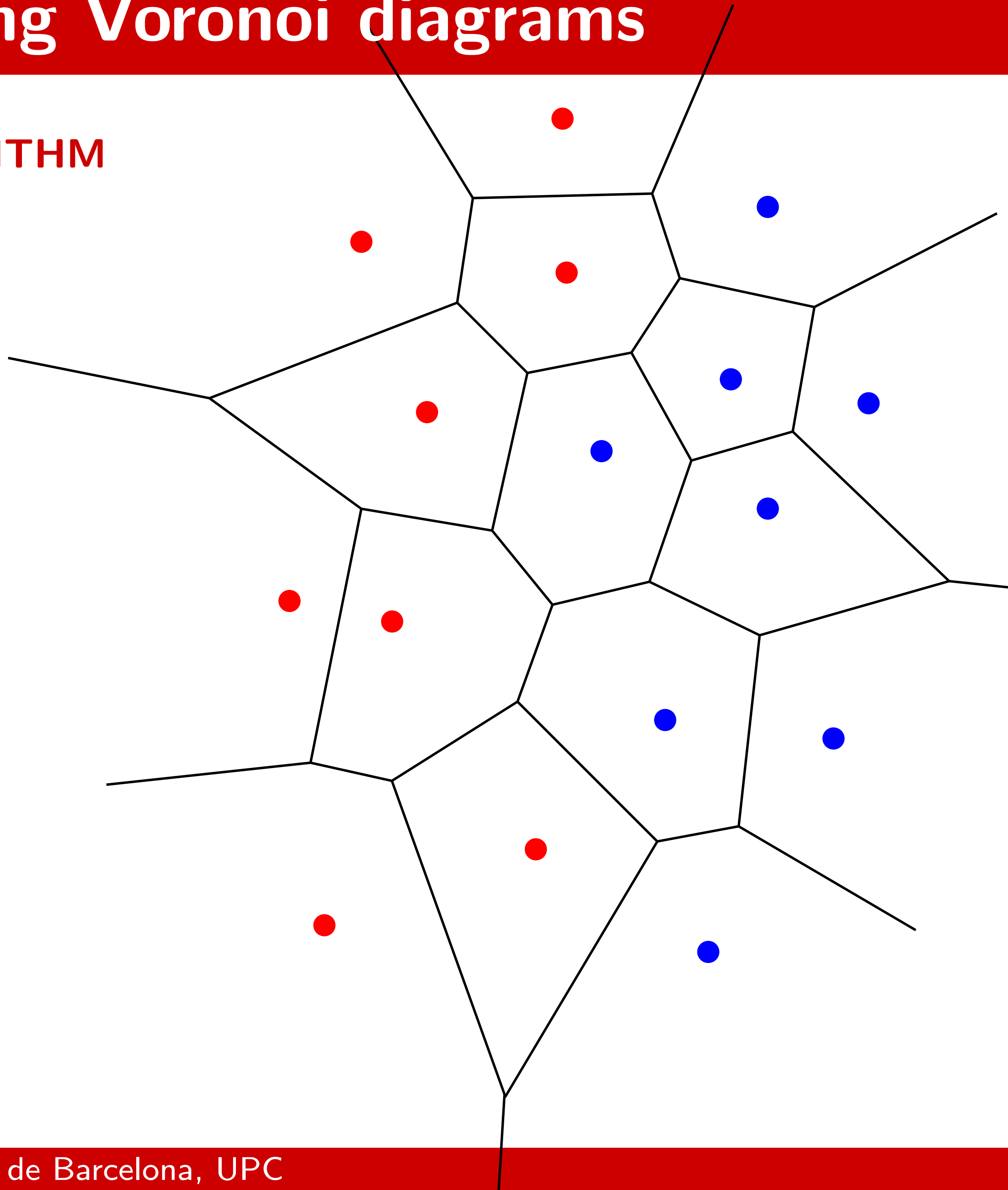
DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

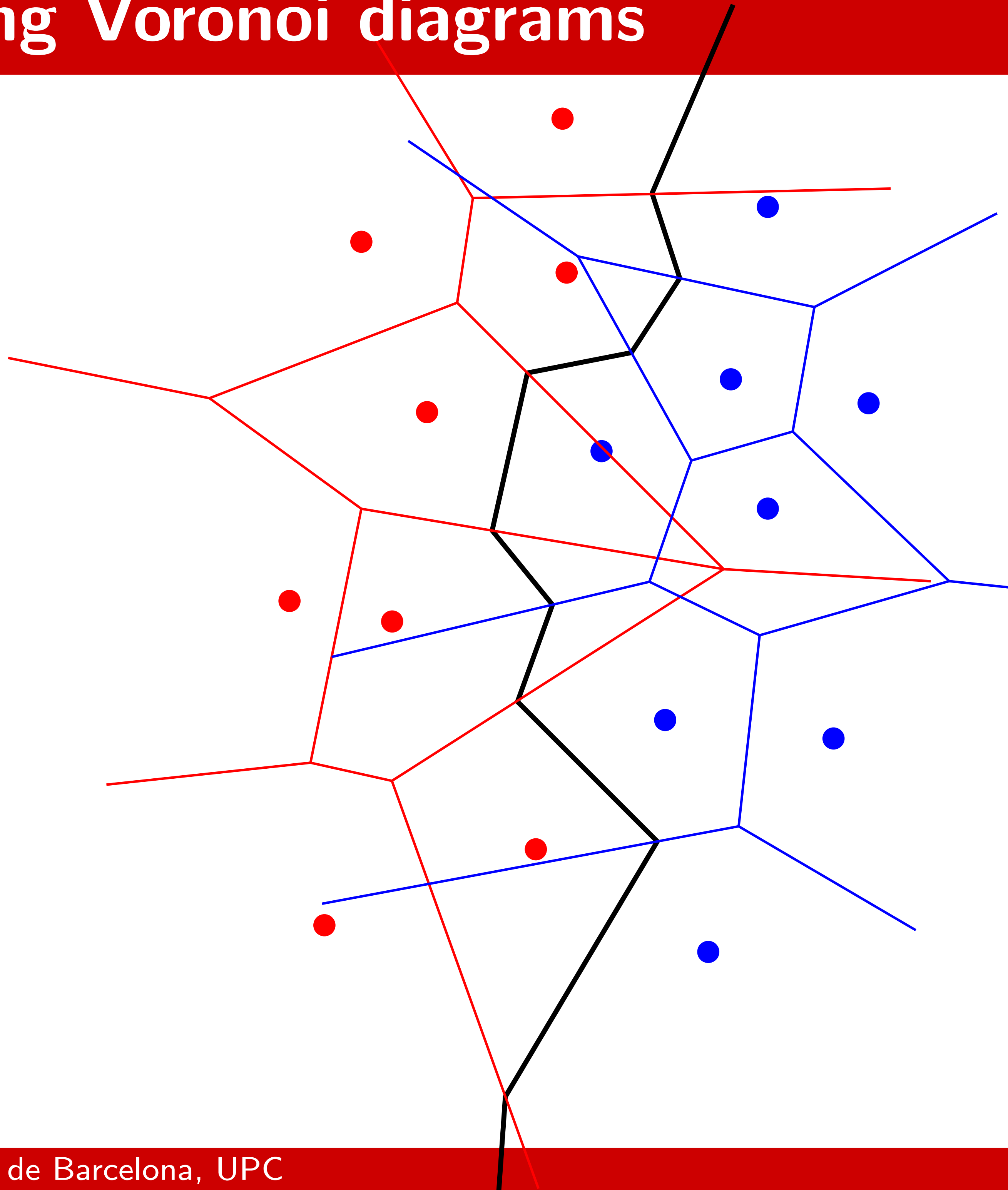
2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.



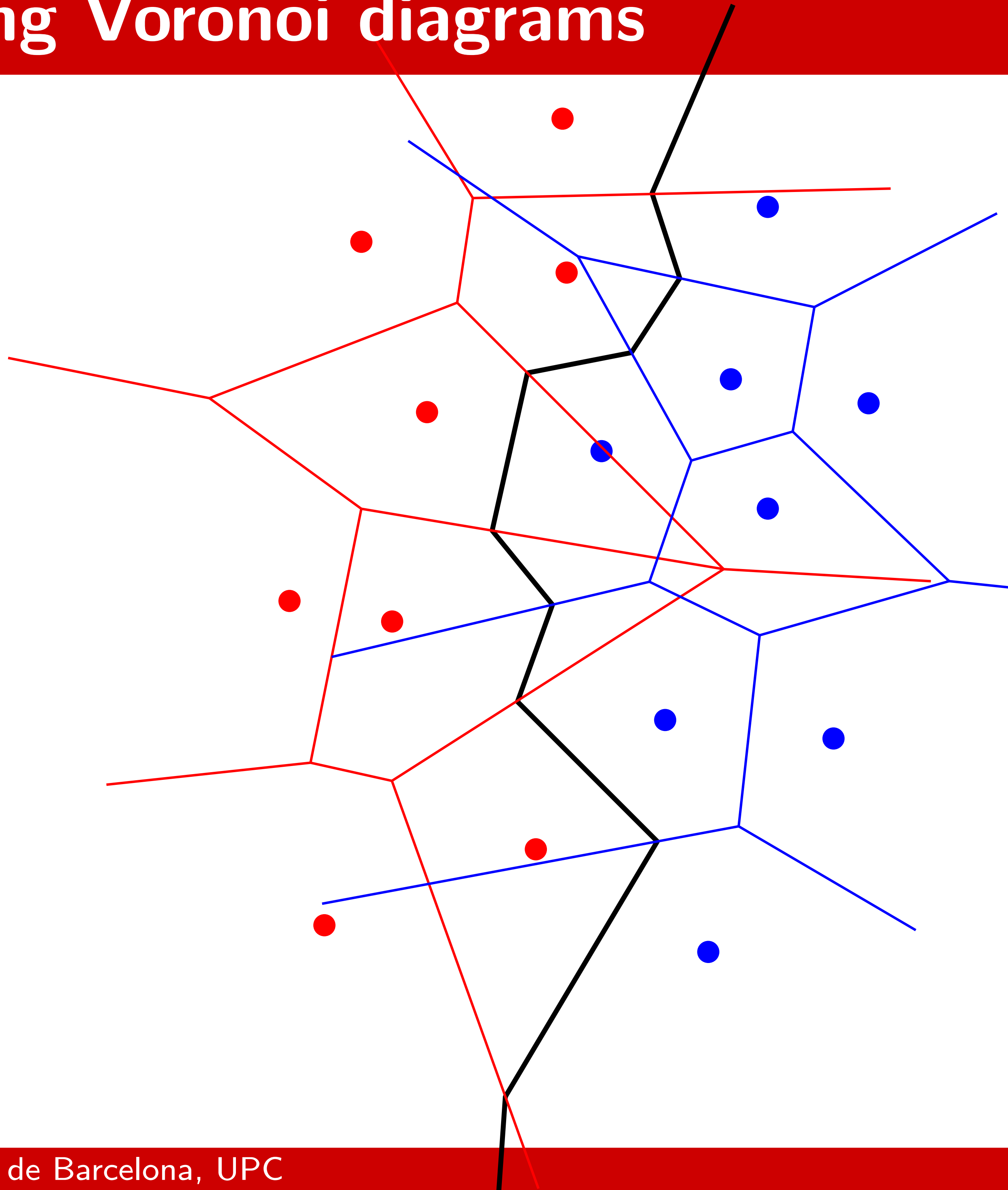
How to compute the chain?



How to compute the chain?

Initialization

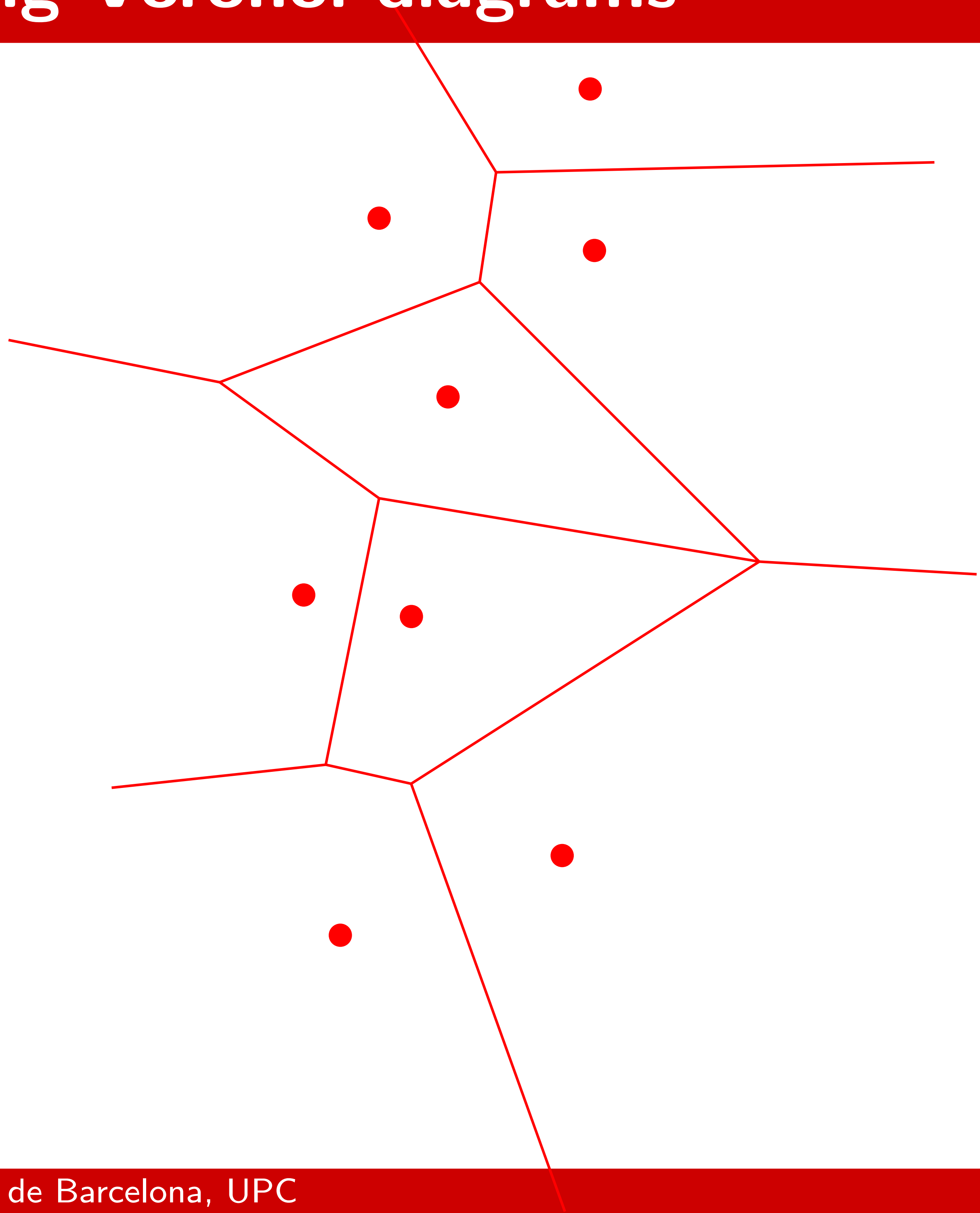
Find the two halflines



How to compute the chain?

Initialization

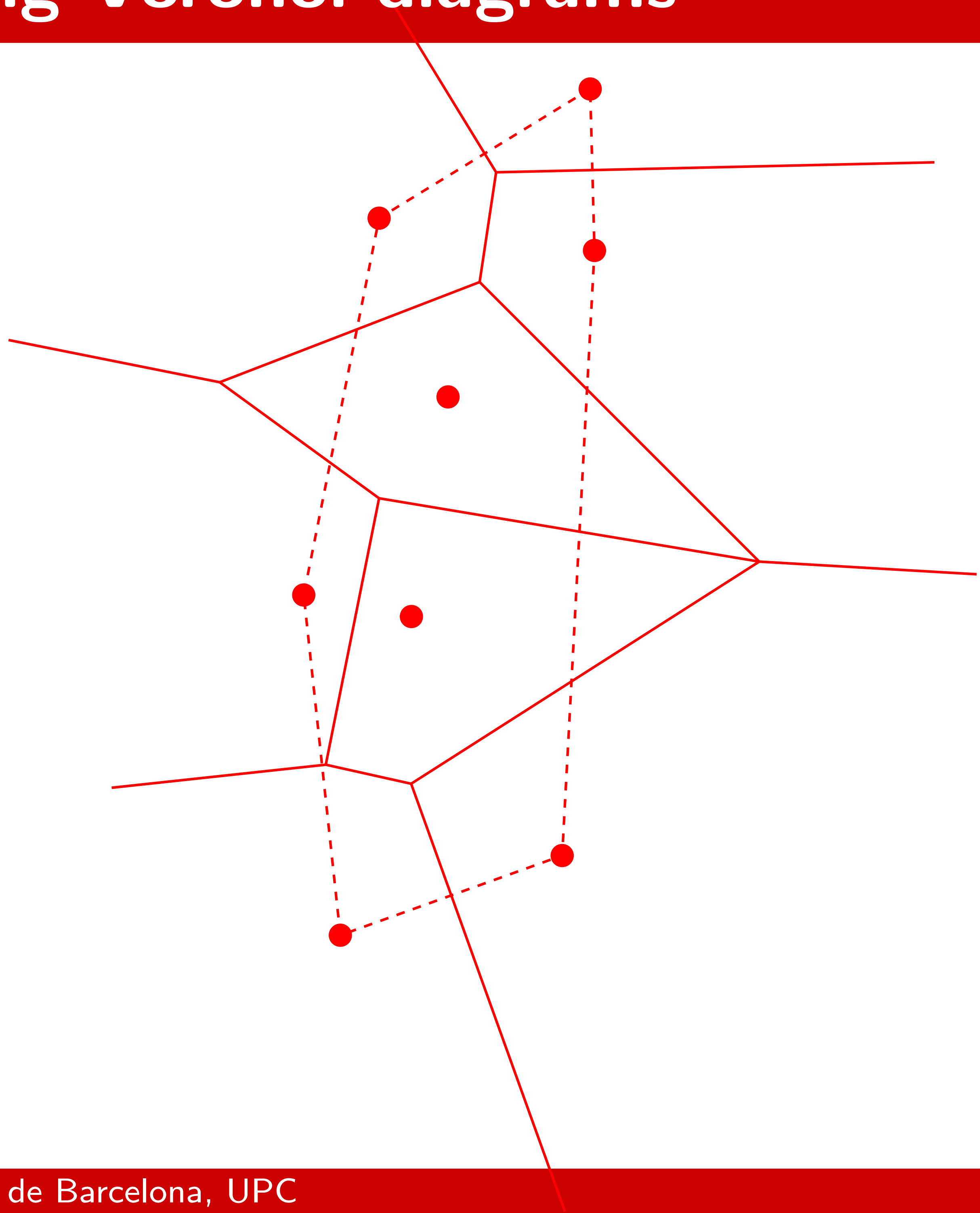
Find the two halflines



How to compute the chain?

Initialization

Find the two halflines

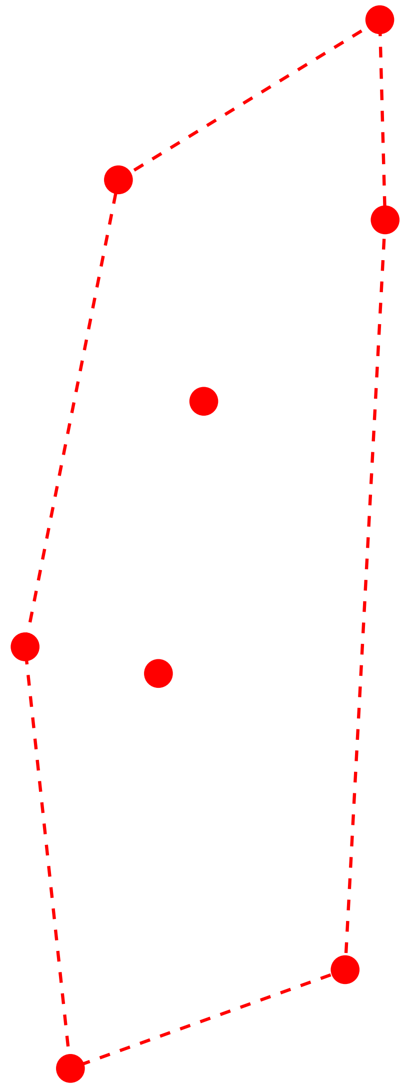


Constructing Voronoi diagrams

How to compute the chain?

Initialization

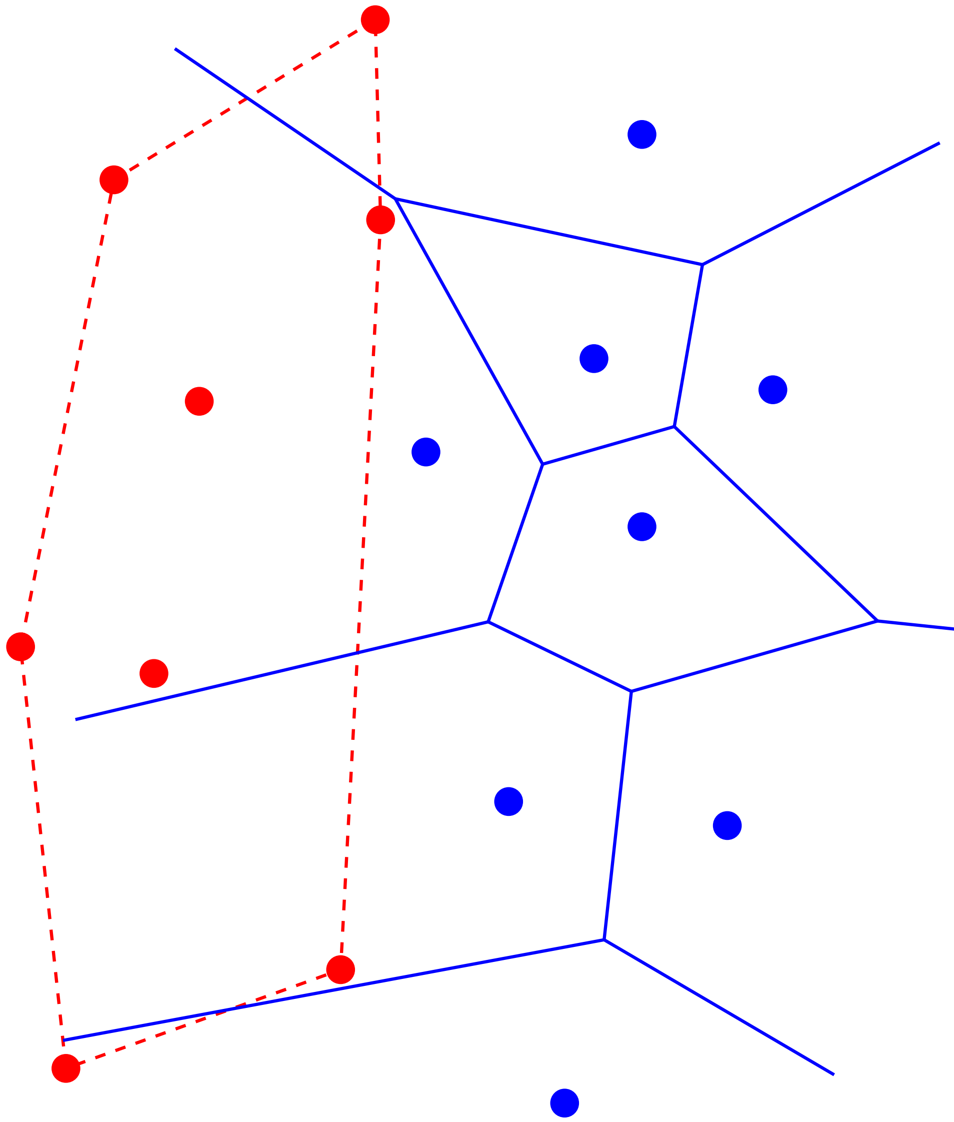
Find the two halflines



How to compute the chain?

Initialization

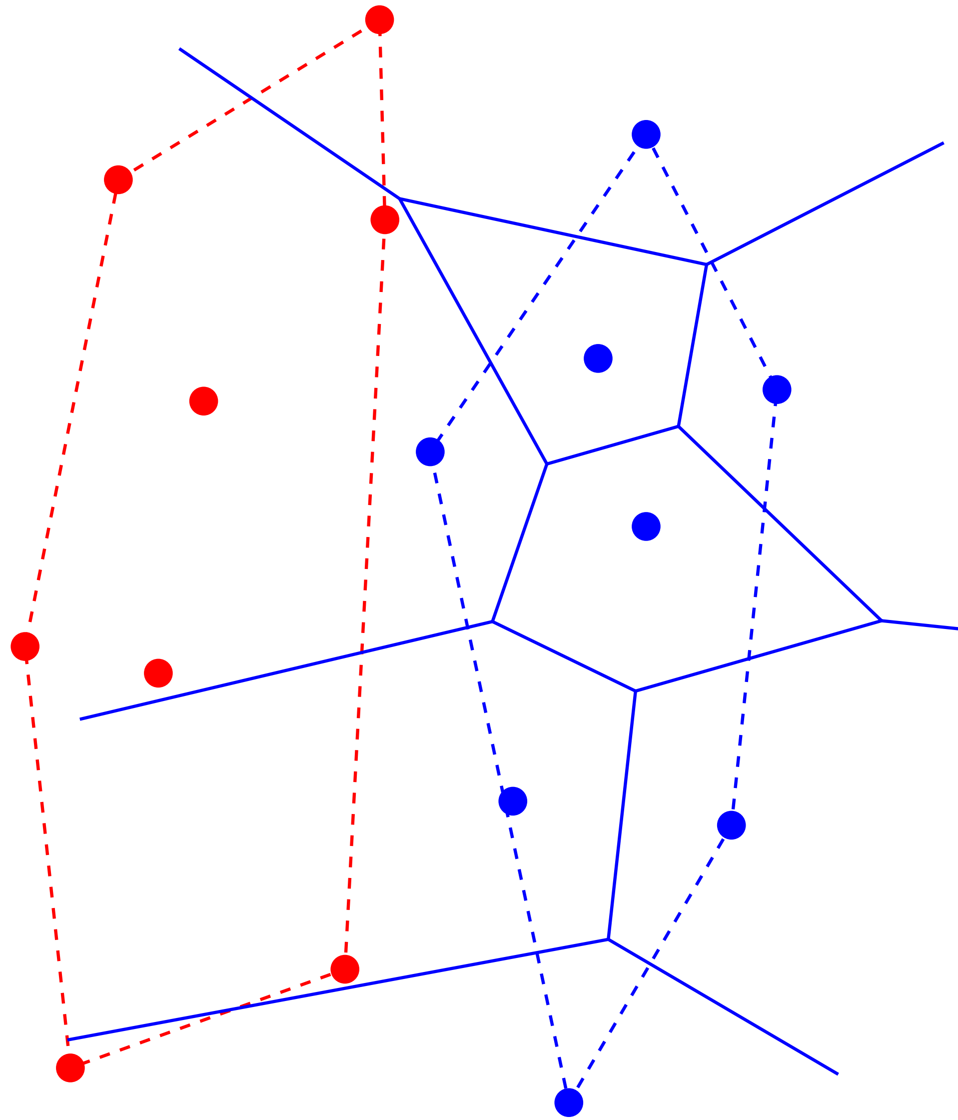
Find the two halflines



How to compute the chain?

Initialization

Find the two halflines

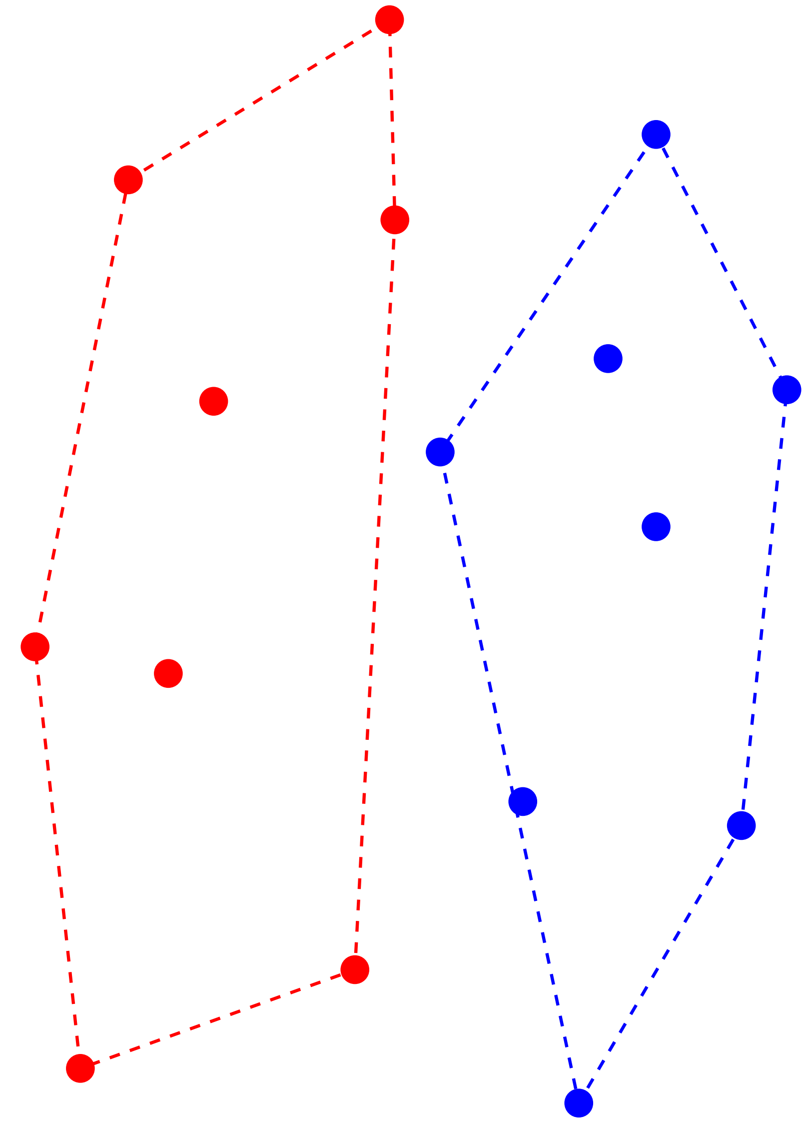


Constructing Voronoi diagrams

How to compute the chain?

Initialization

Find the two halflines

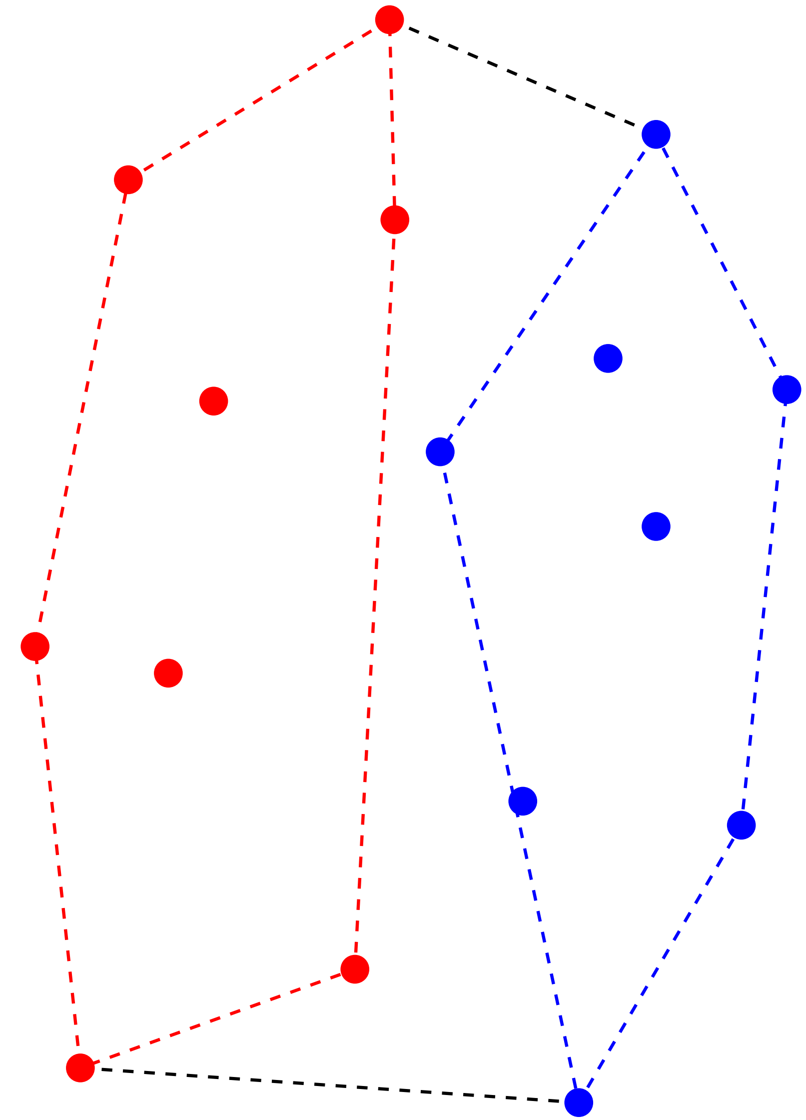


Constructing Voronoi diagrams

How to compute the chain?

Initialization

Find the two halflines

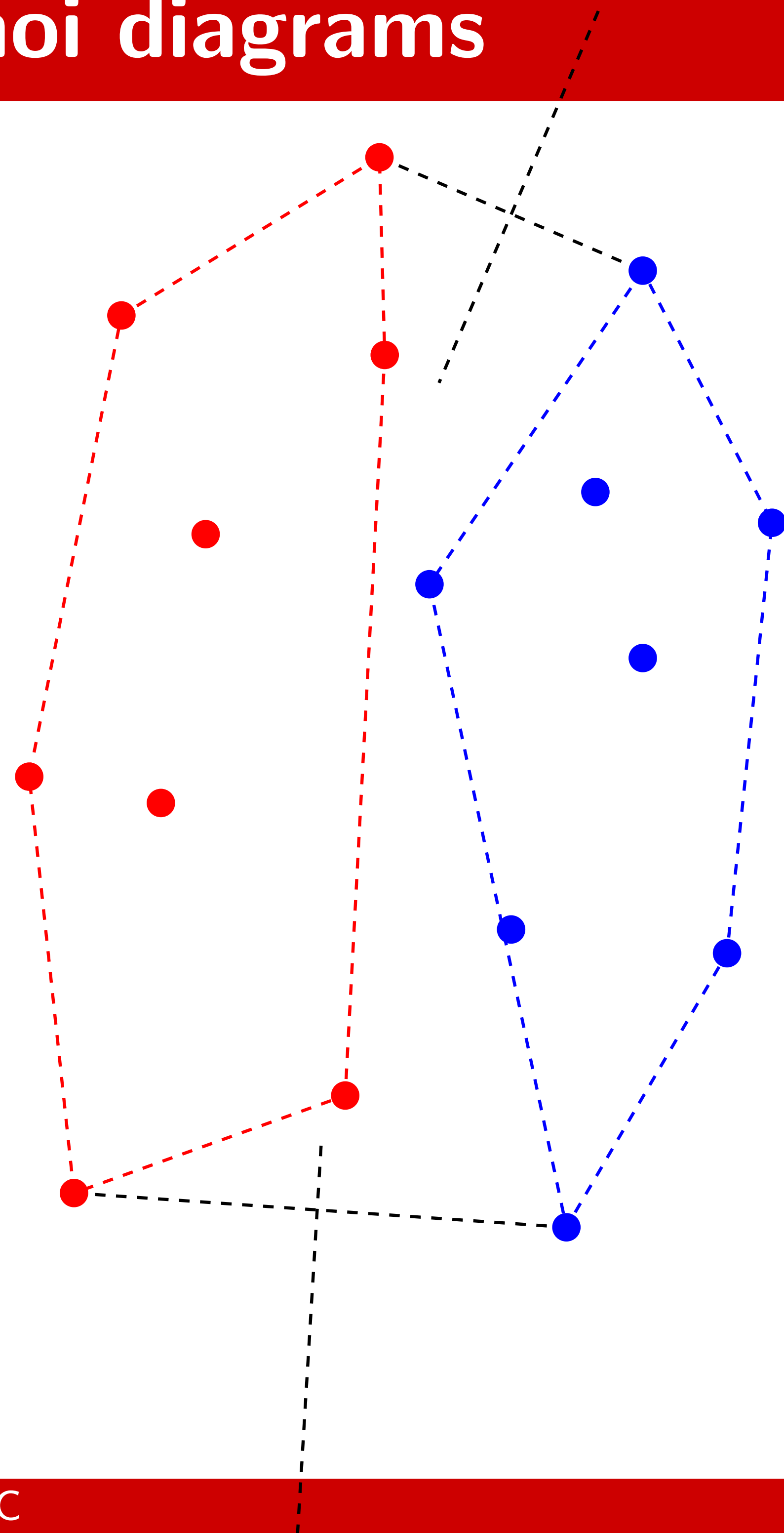


Constructing Voronoi diagrams

How to compute the chain?

Initialization

Find the two halflines



How to compute the chain?

Initialization

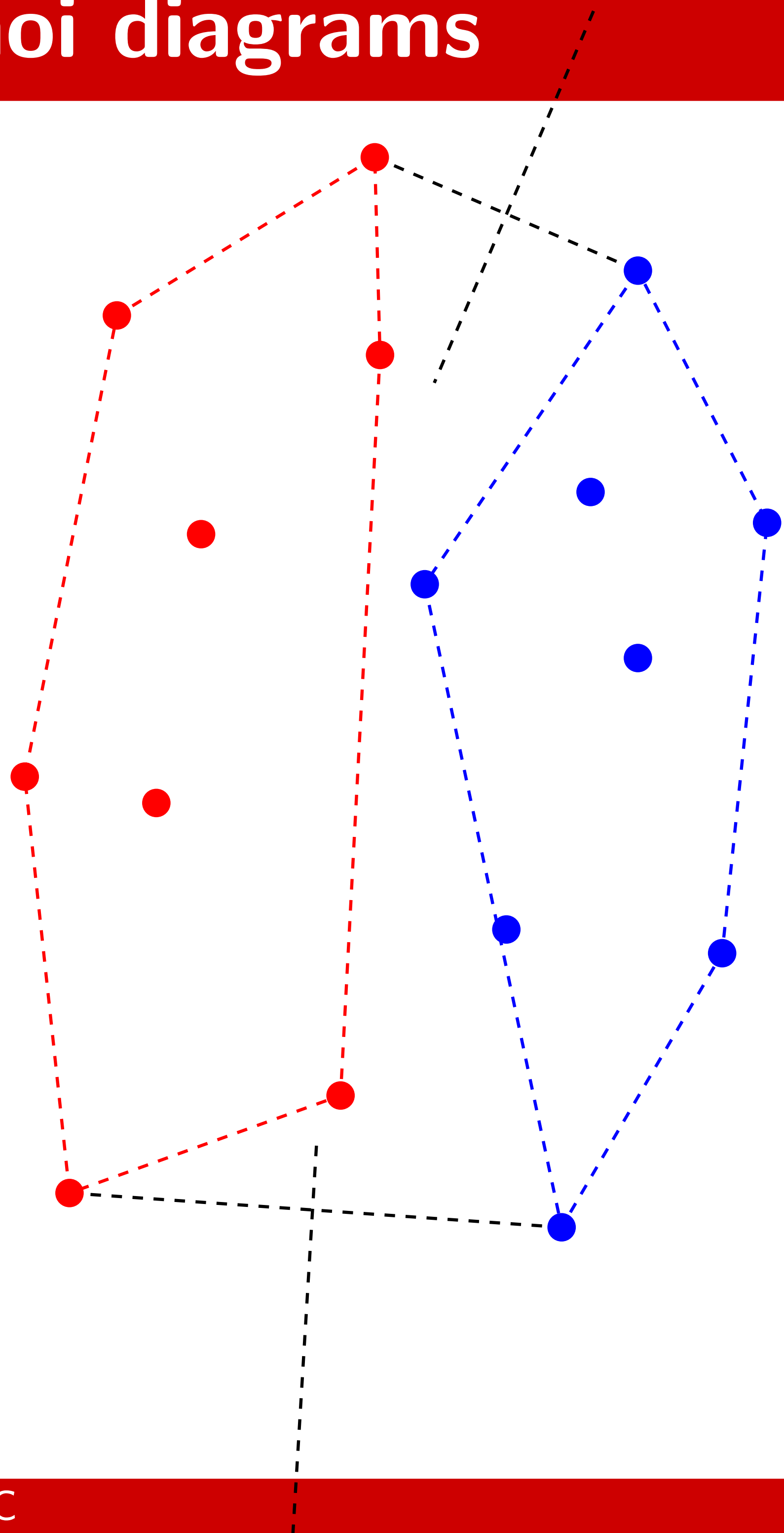
Find the two halflines

Advance

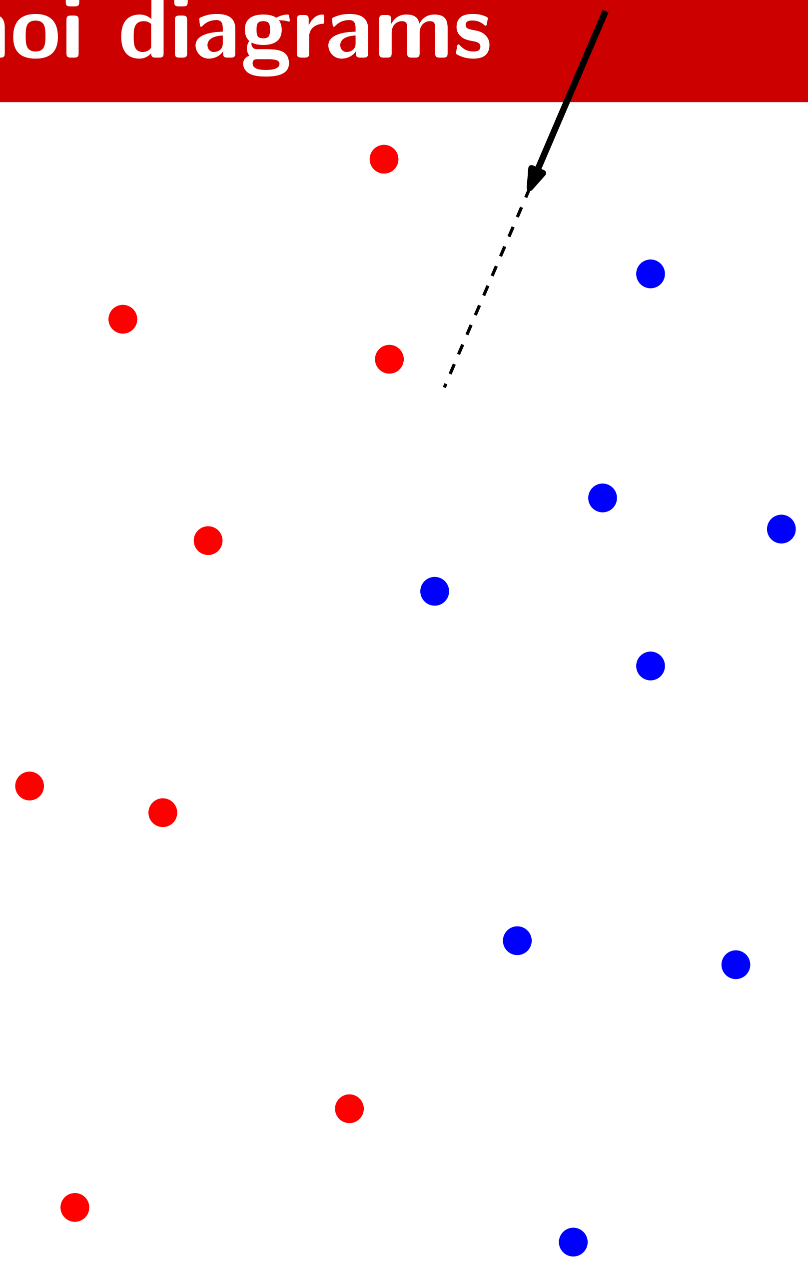
Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams



How to compute the chain?

Initialization

Find the two halflines

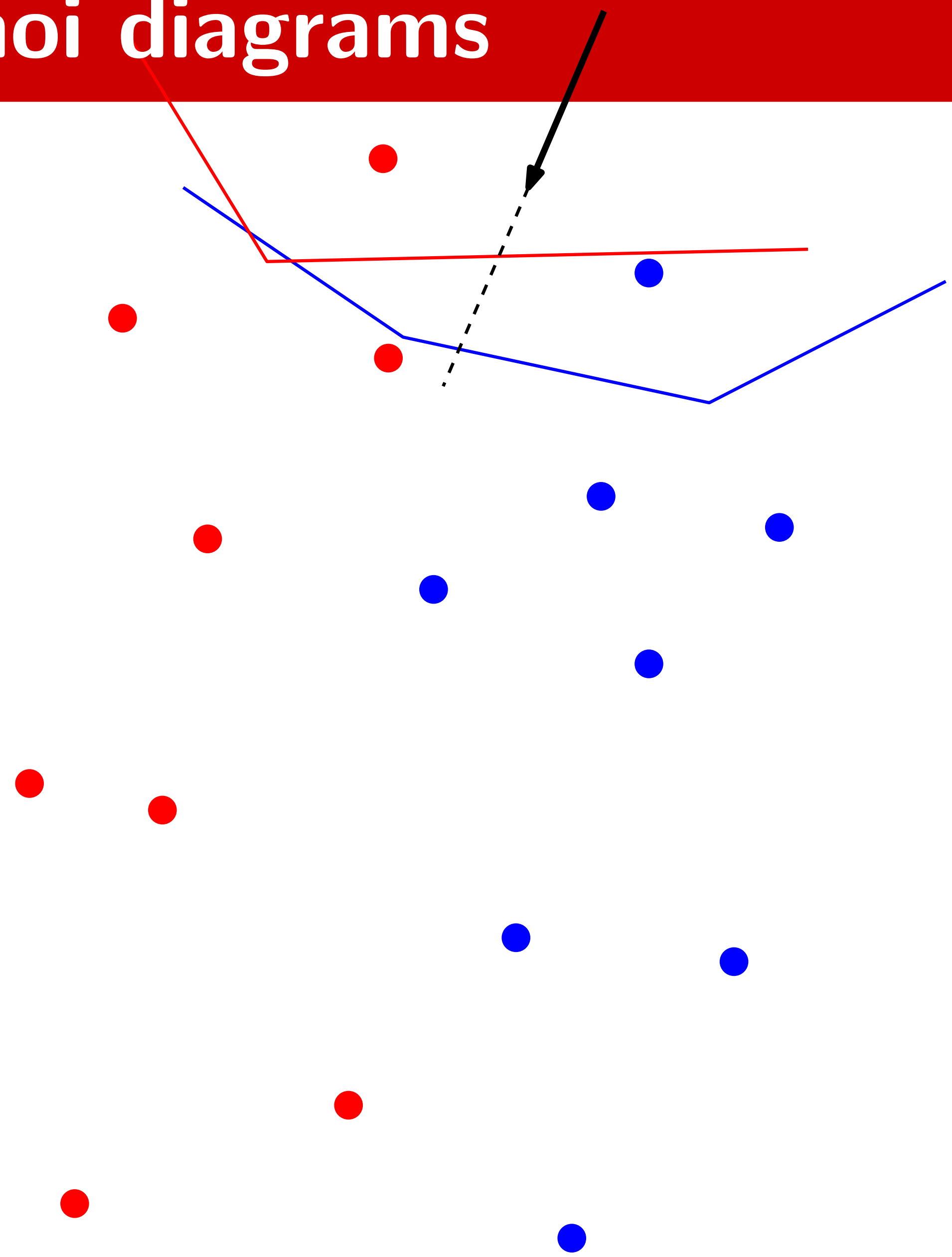
Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge

Constructing Voronoi diagrams



How to compute the chain?

Initialization

Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge

Constructing Voronoi diagrams

How to compute the chain?

Initialization

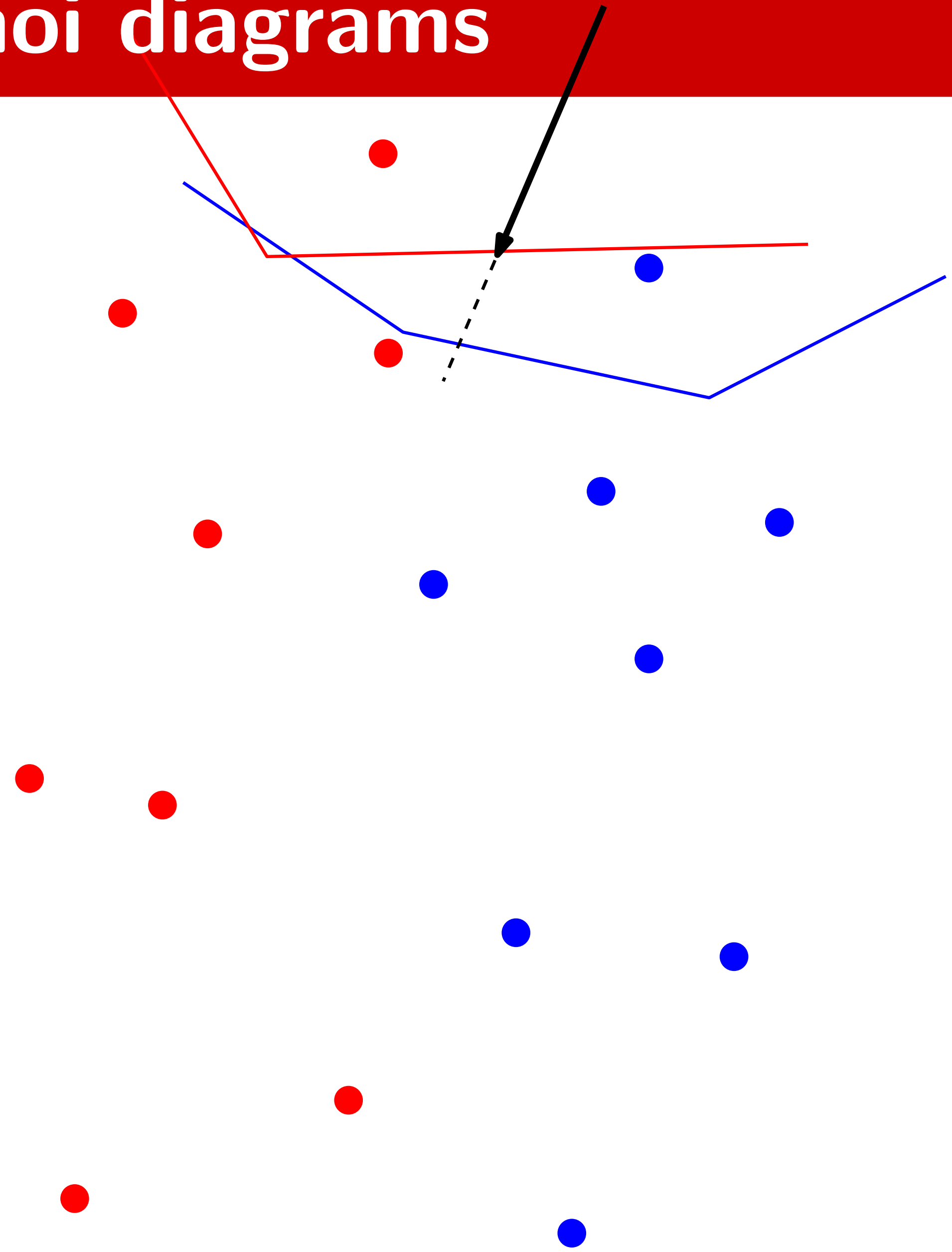
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

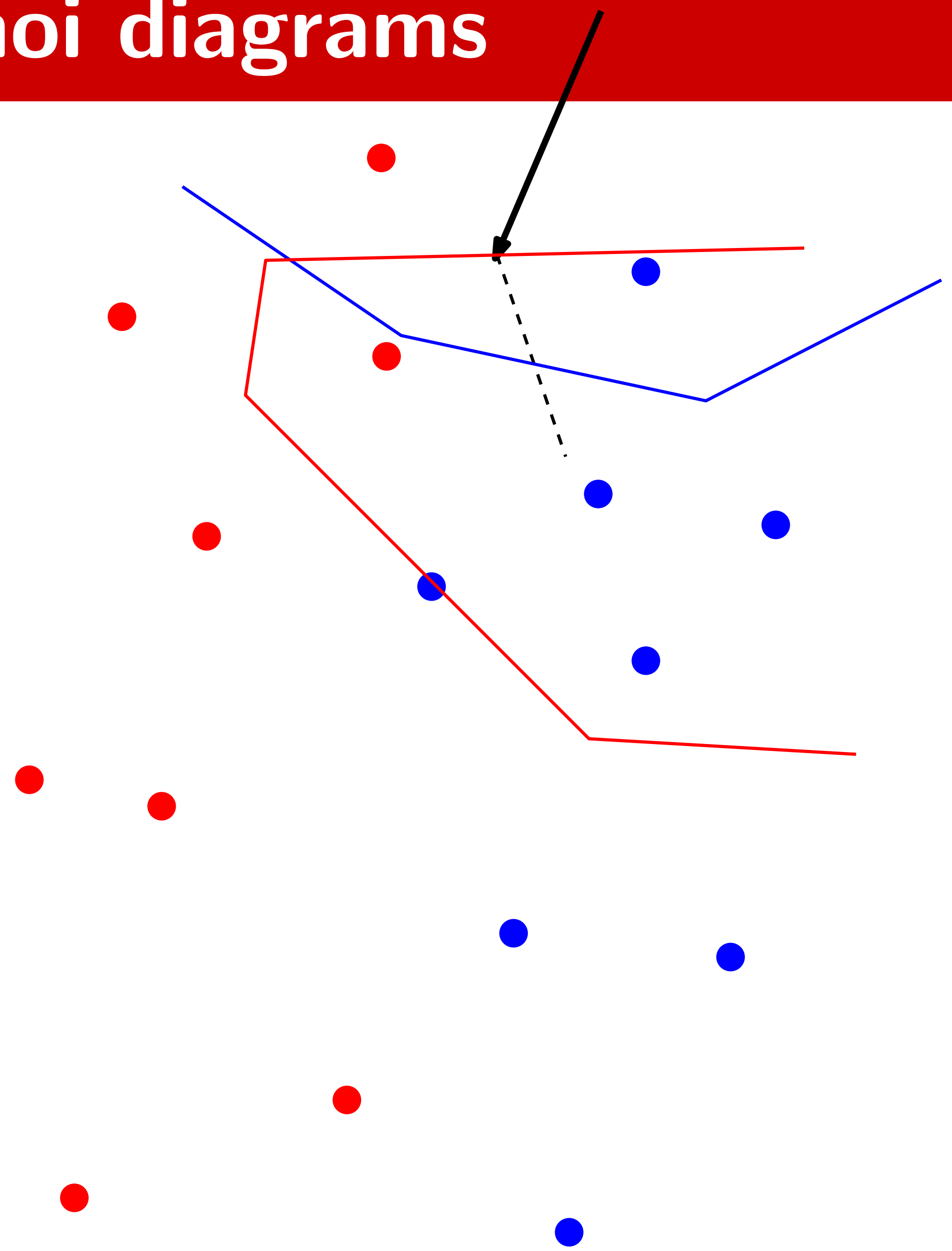
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

Initialization

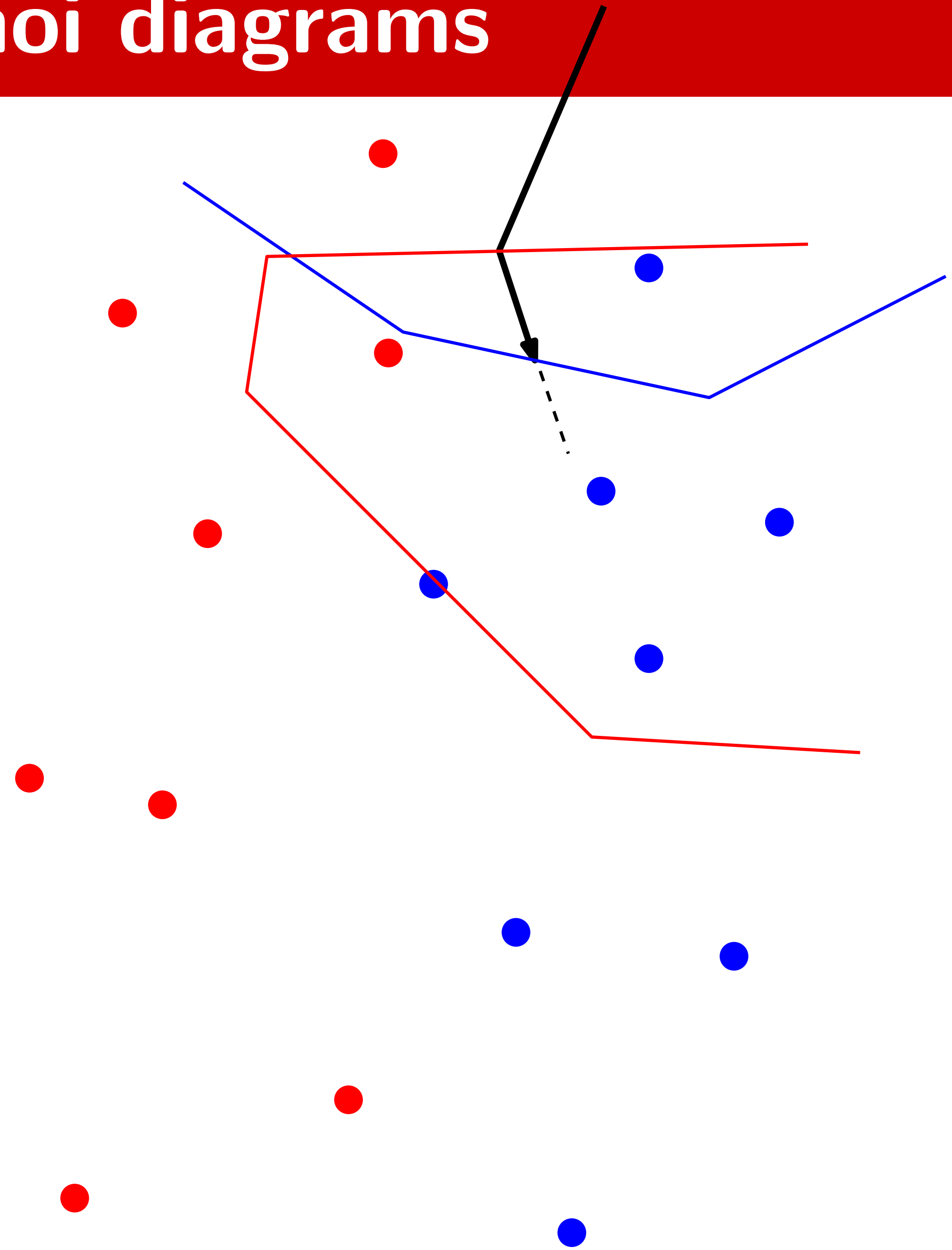
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

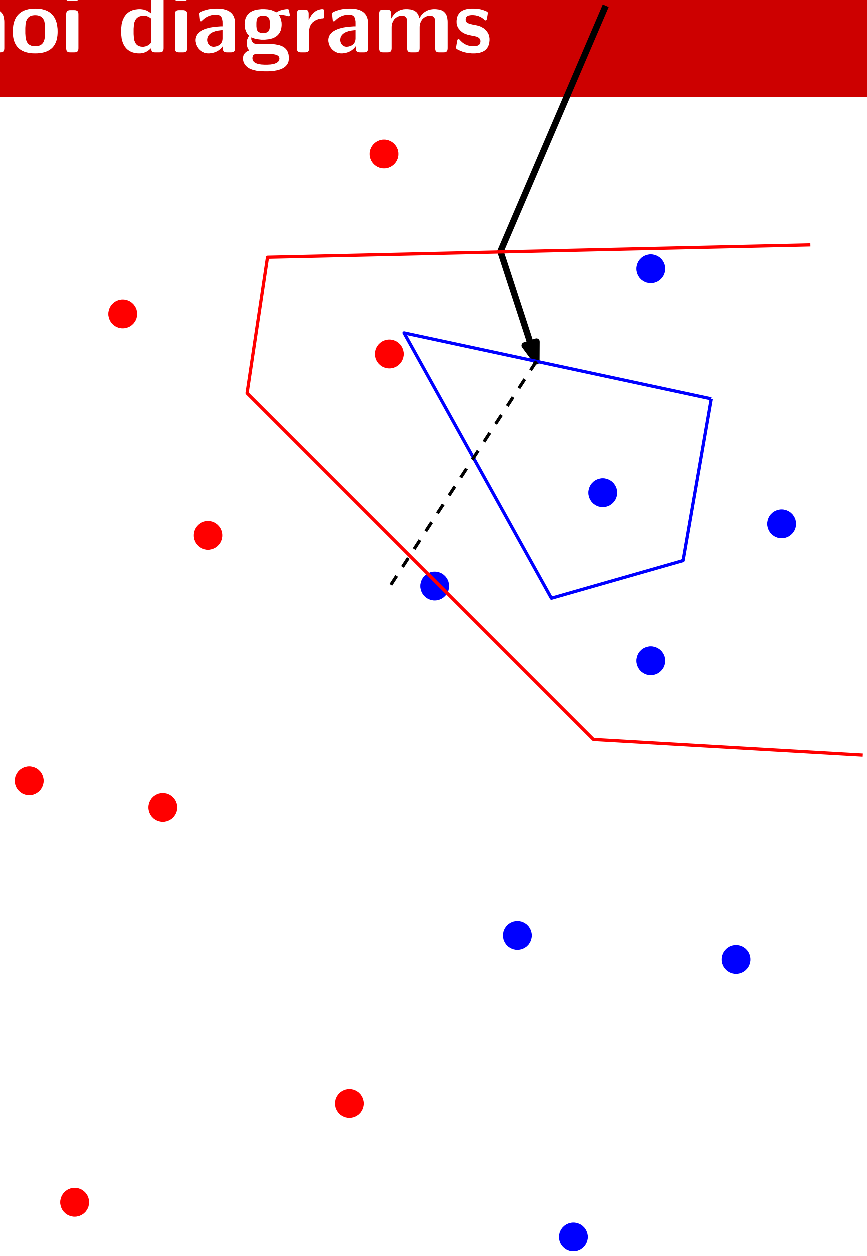
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

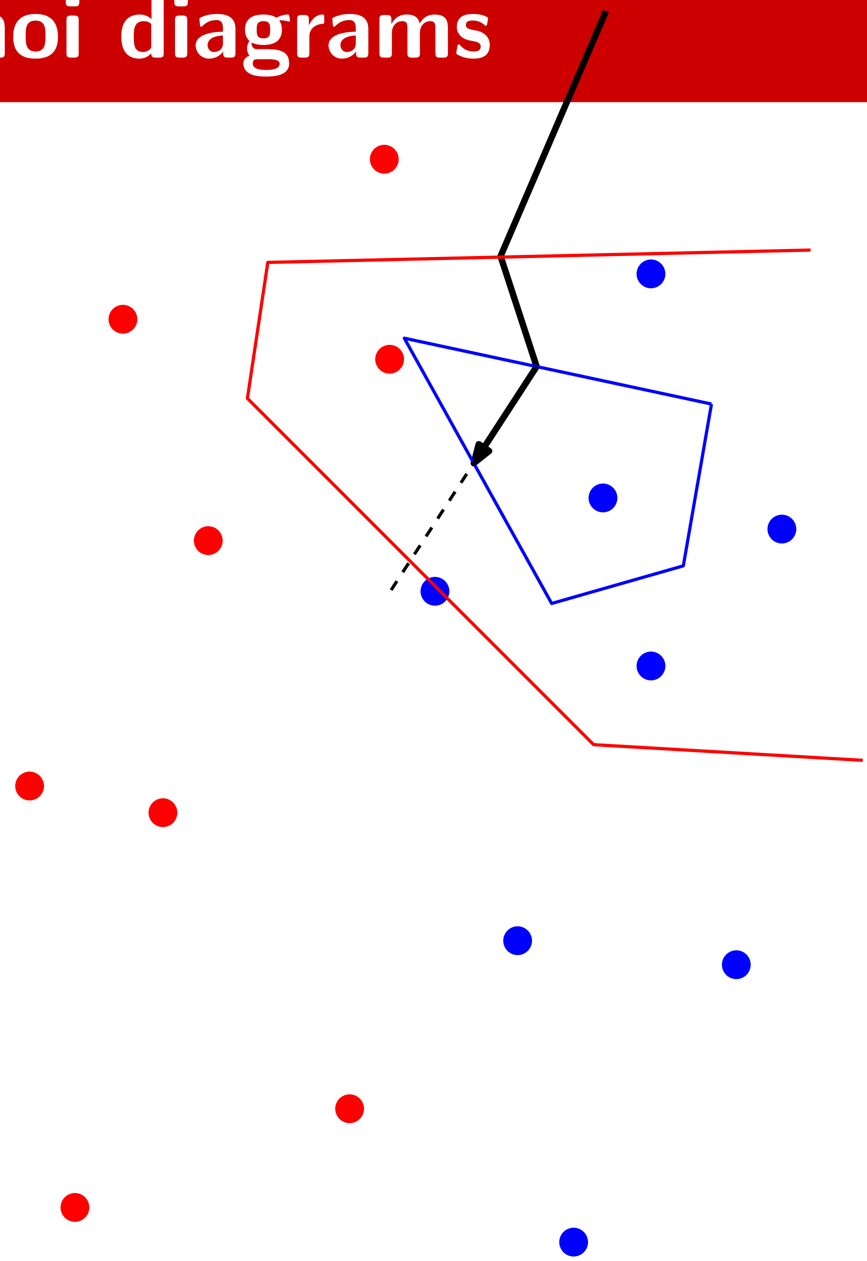
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R,B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

Initialization

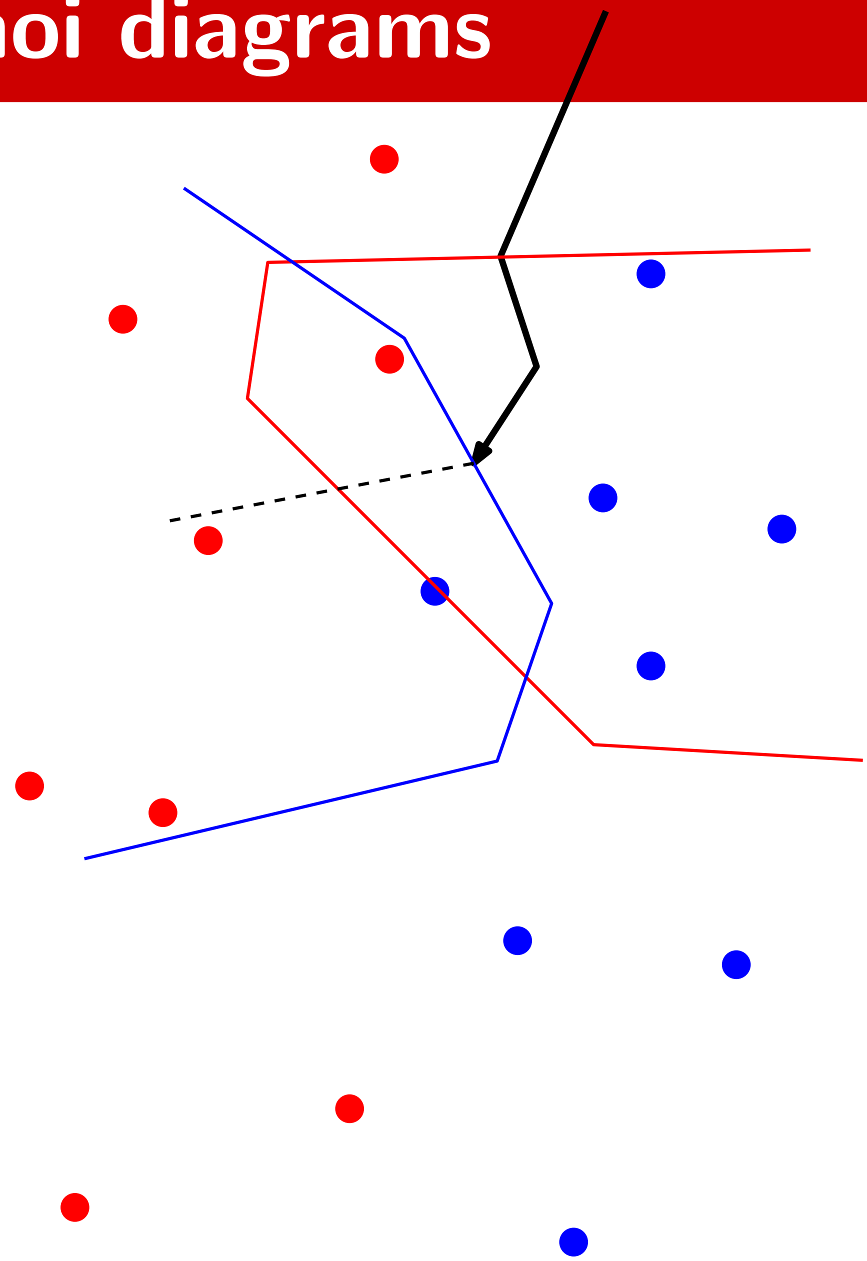
Find the two halflines

Advance

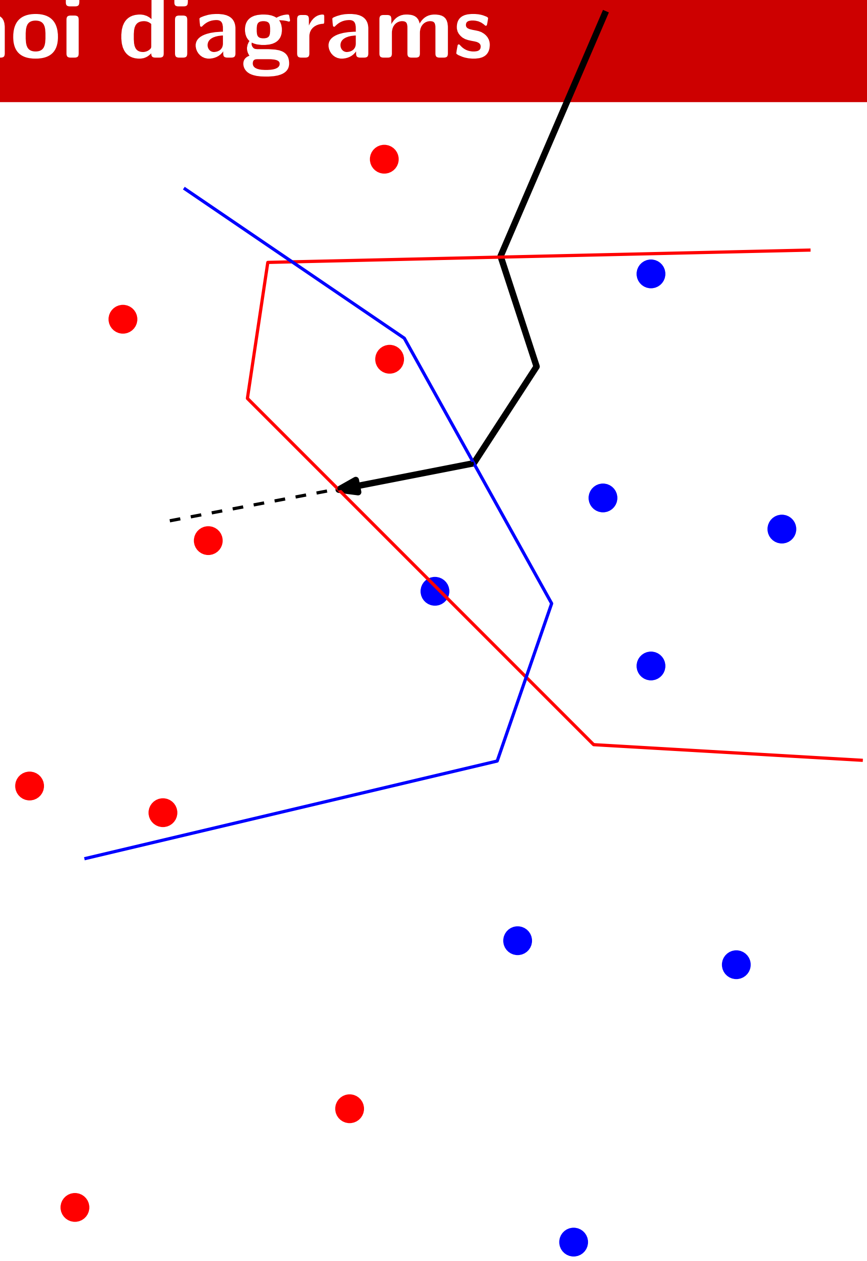
Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams



How to compute the chain?

Initialization

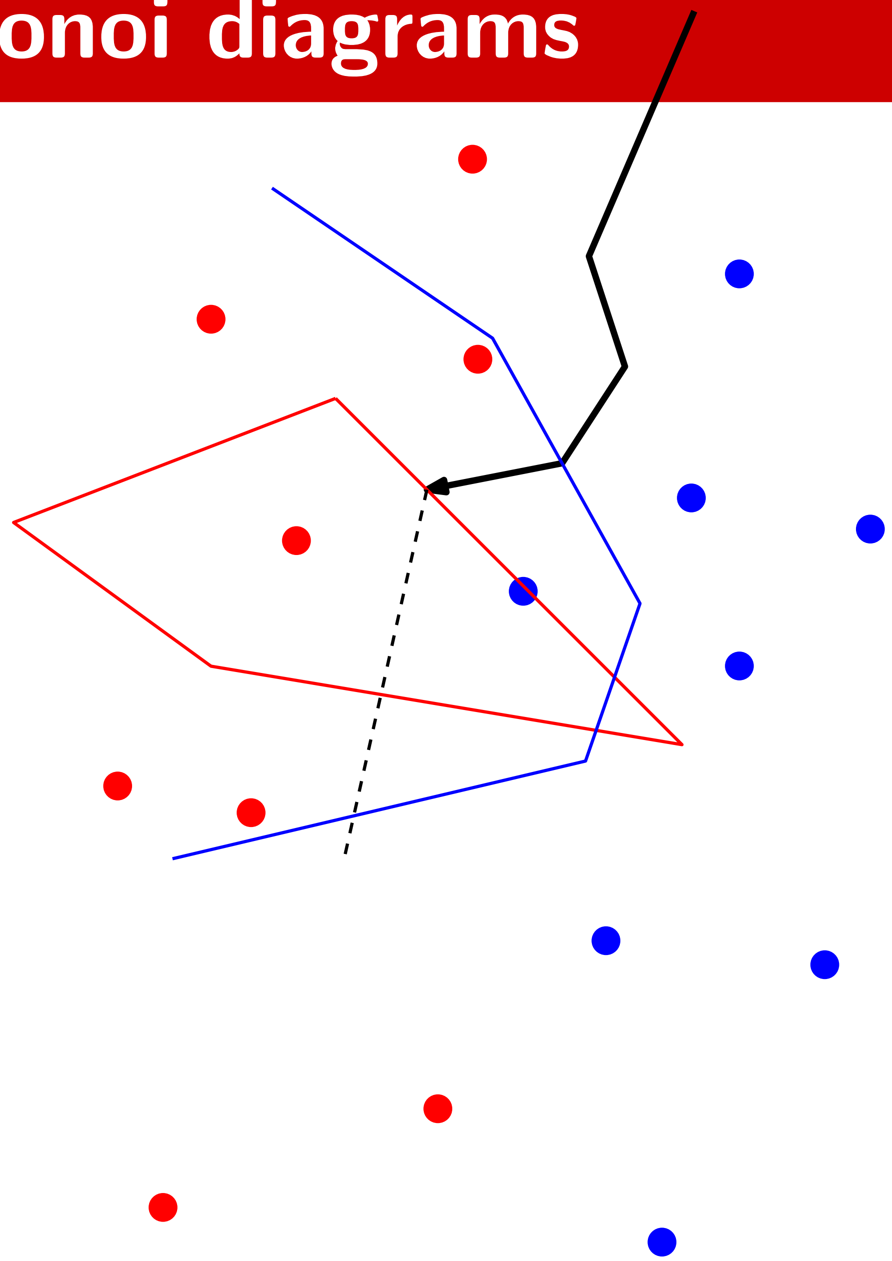
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

Initialization

Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R,B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge

Constructing Voronoi diagrams

How to compute the chain?

Initialization

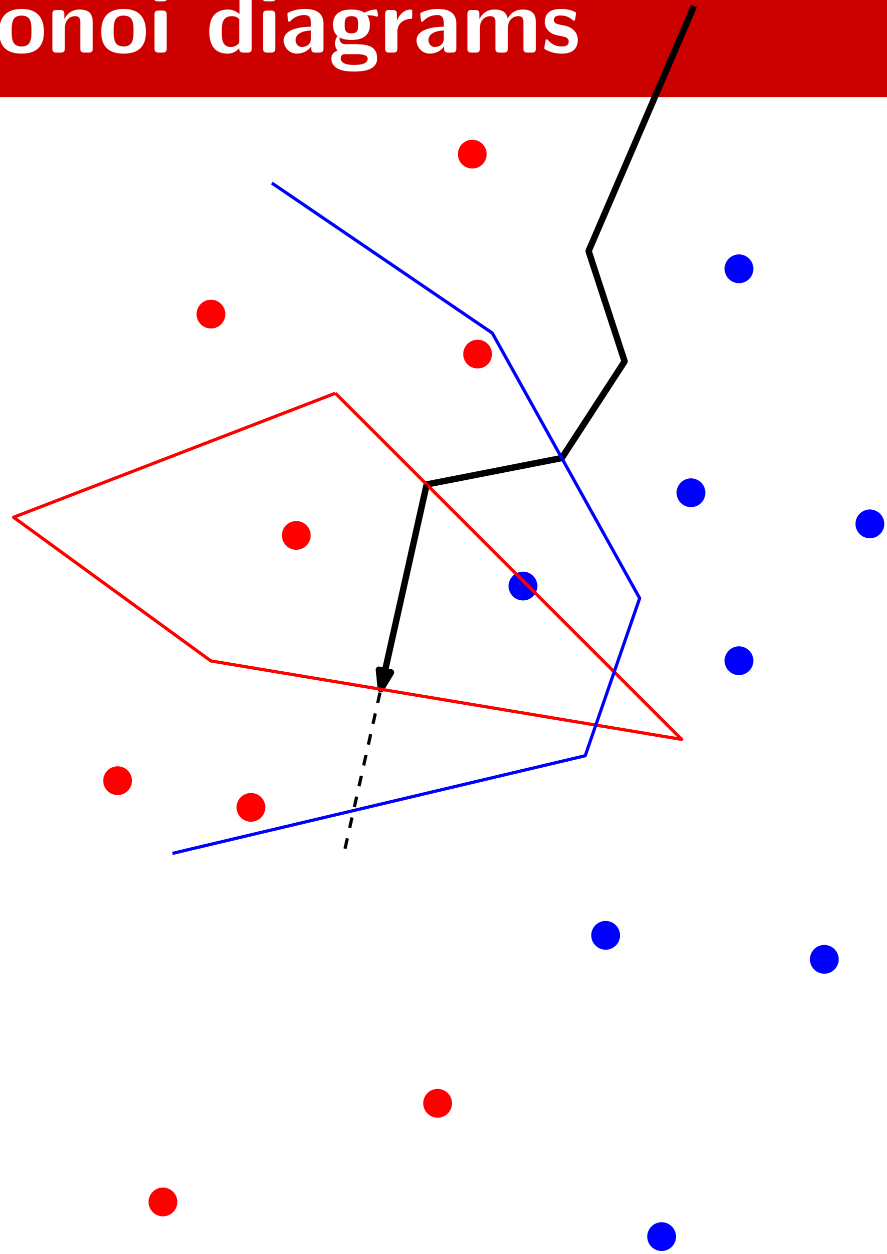
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

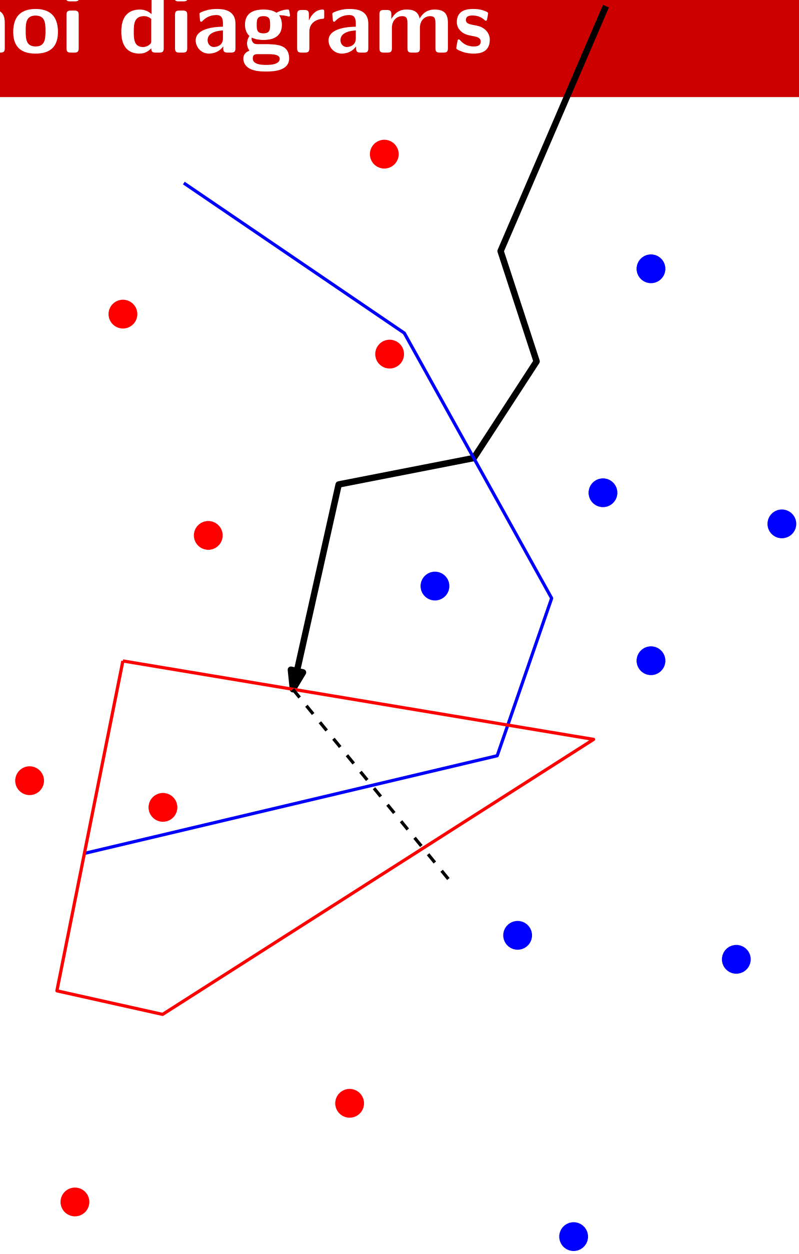
Find the two halflines

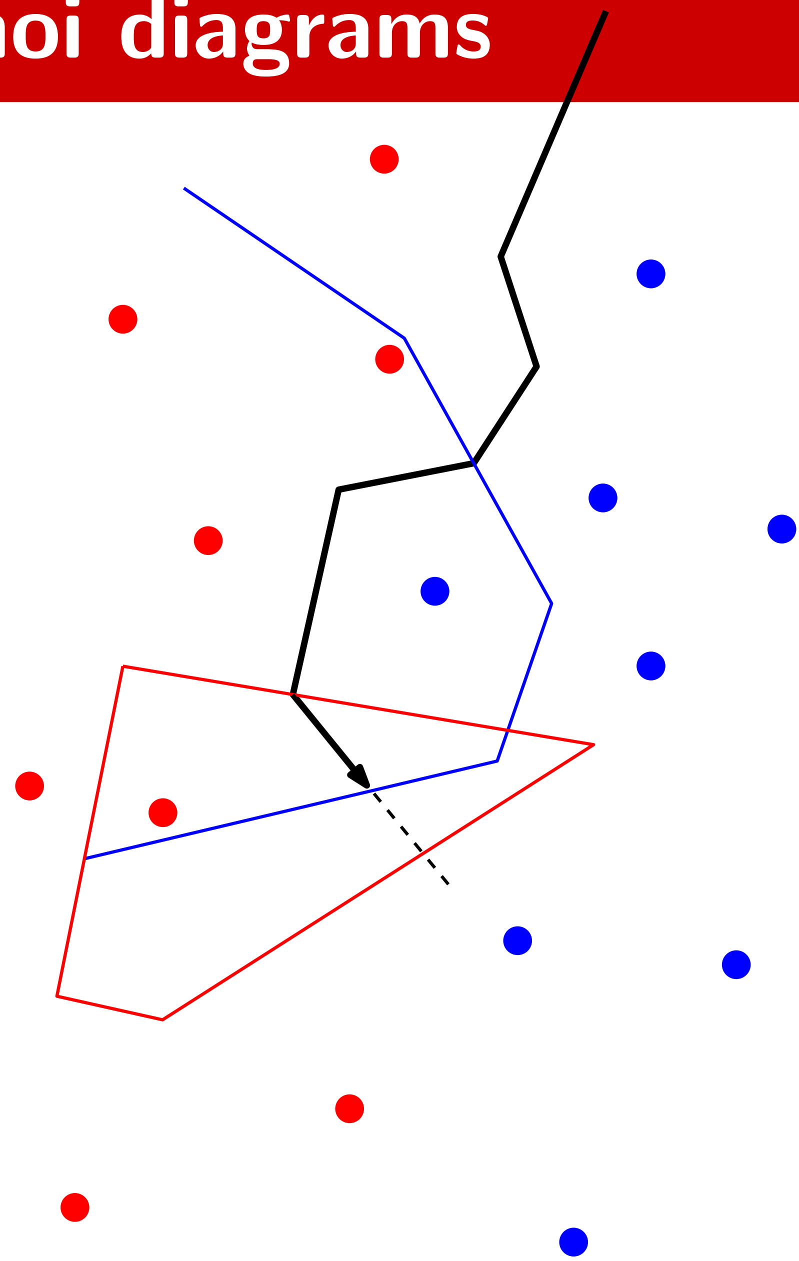
Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge





How to compute the chain?

Initialization

Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge

How to compute the chain?

Initialization

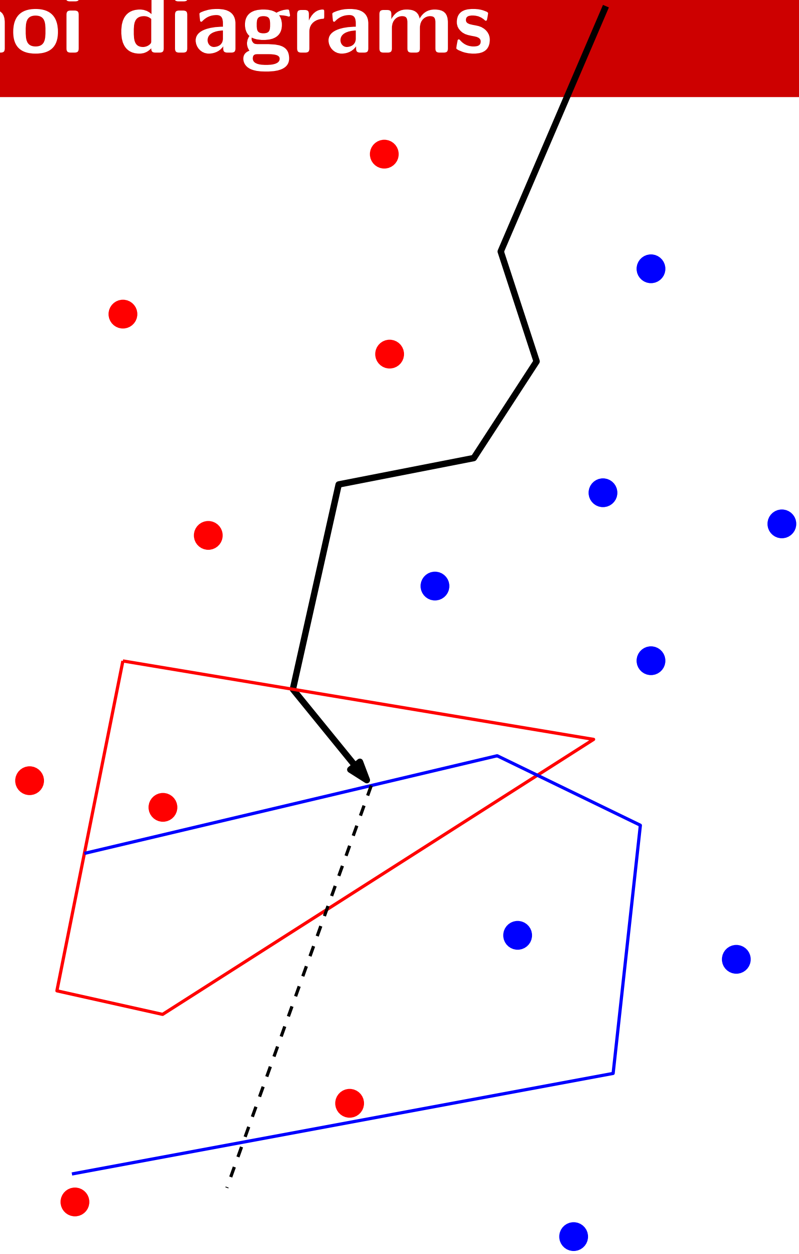
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

Initialization

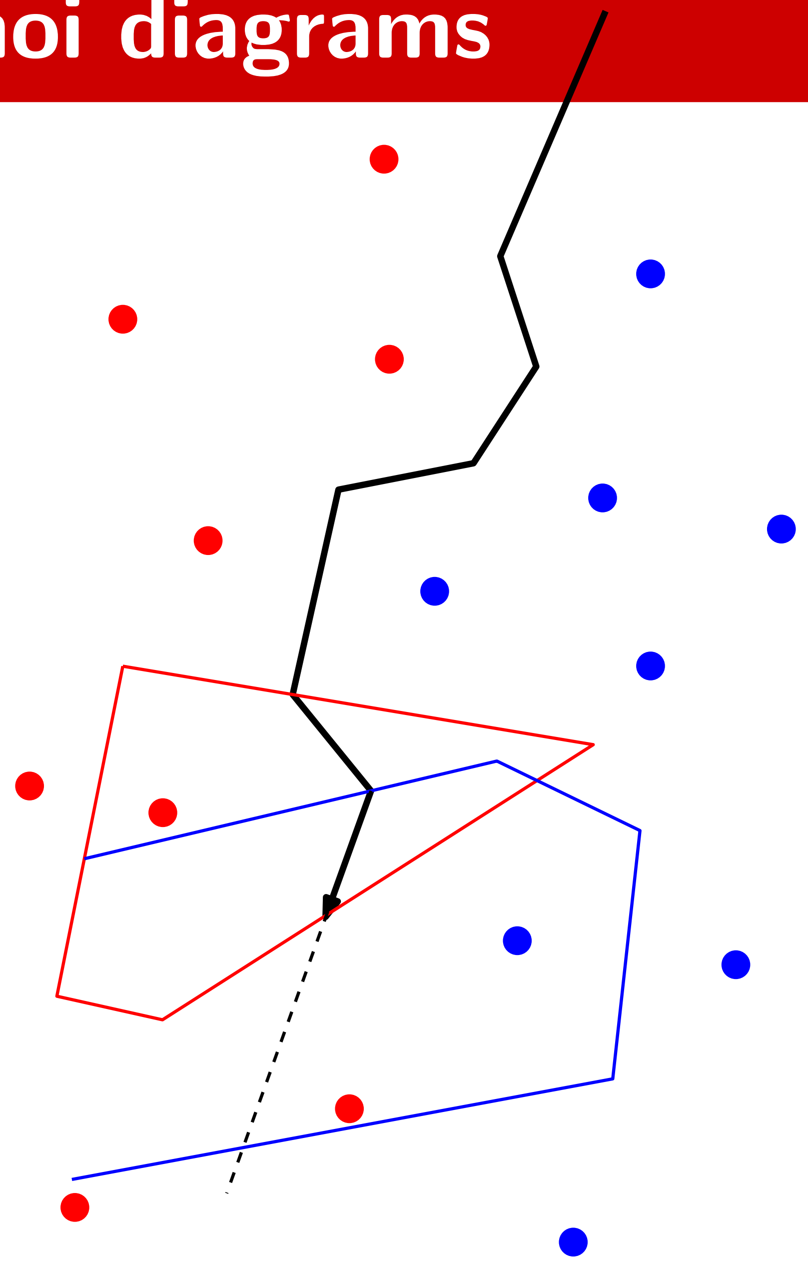
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

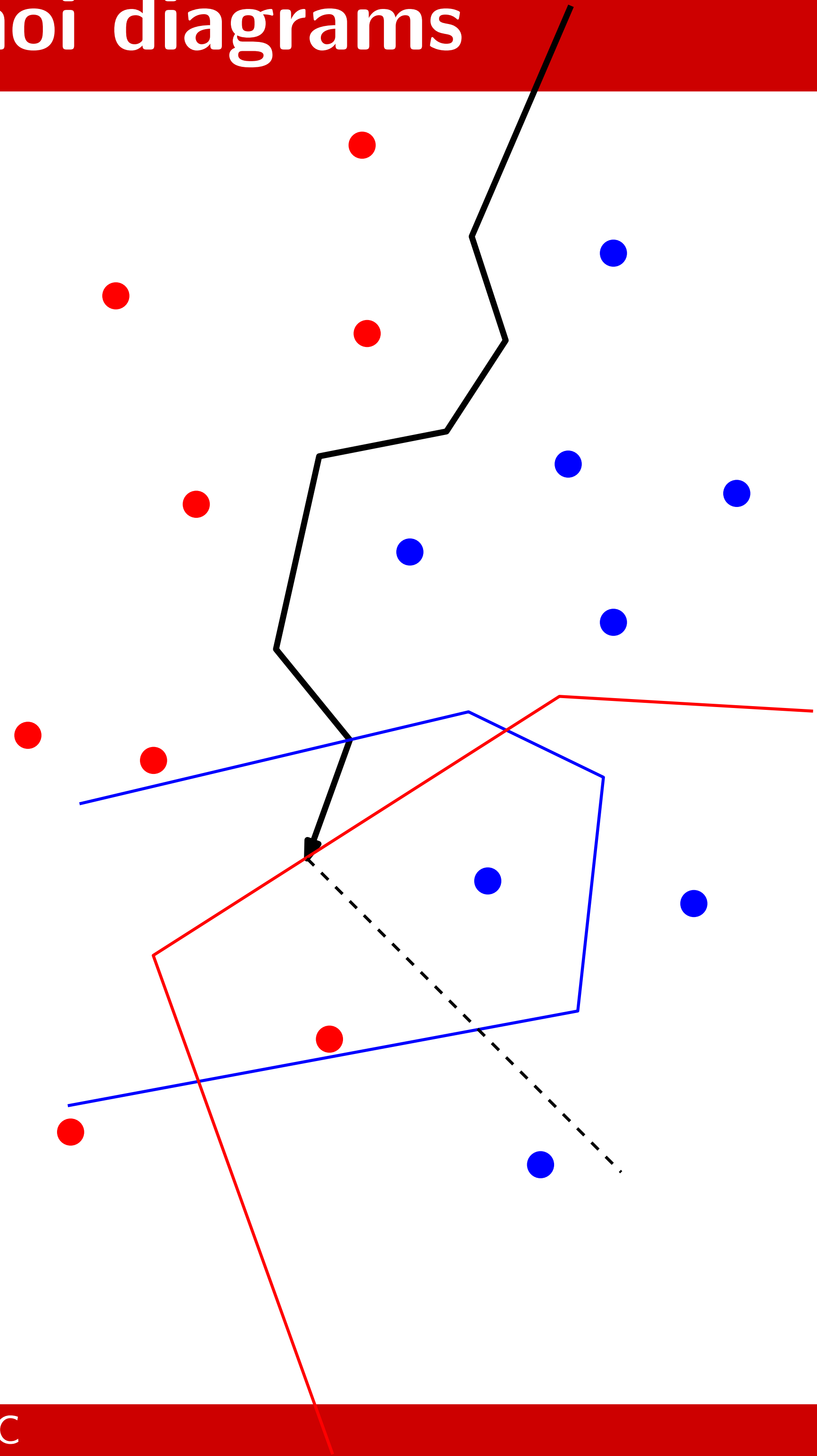
Find the two halflines

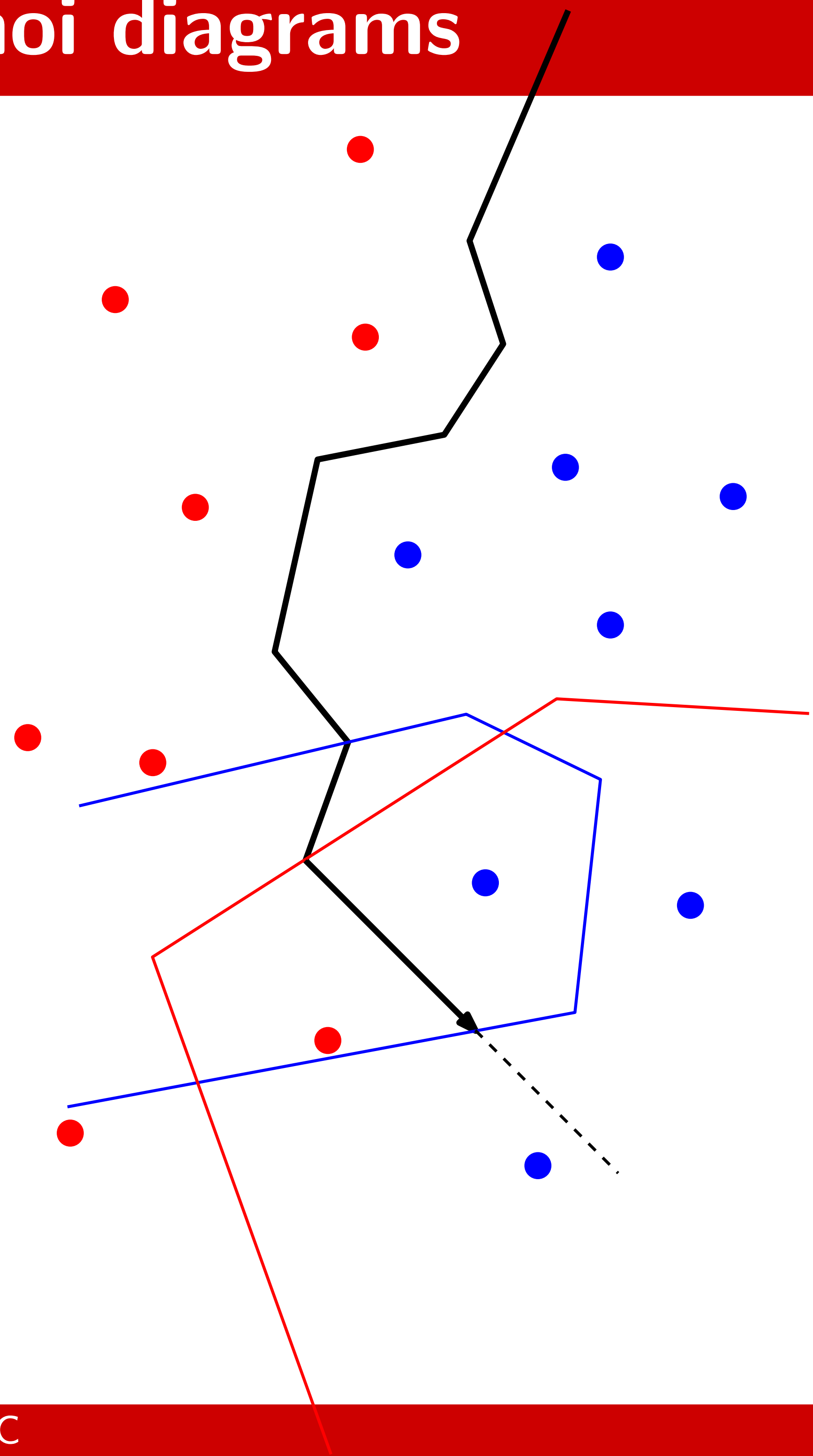
Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge





How to compute the chain?

Initialization

Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge

How to compute the chain?

Initialization

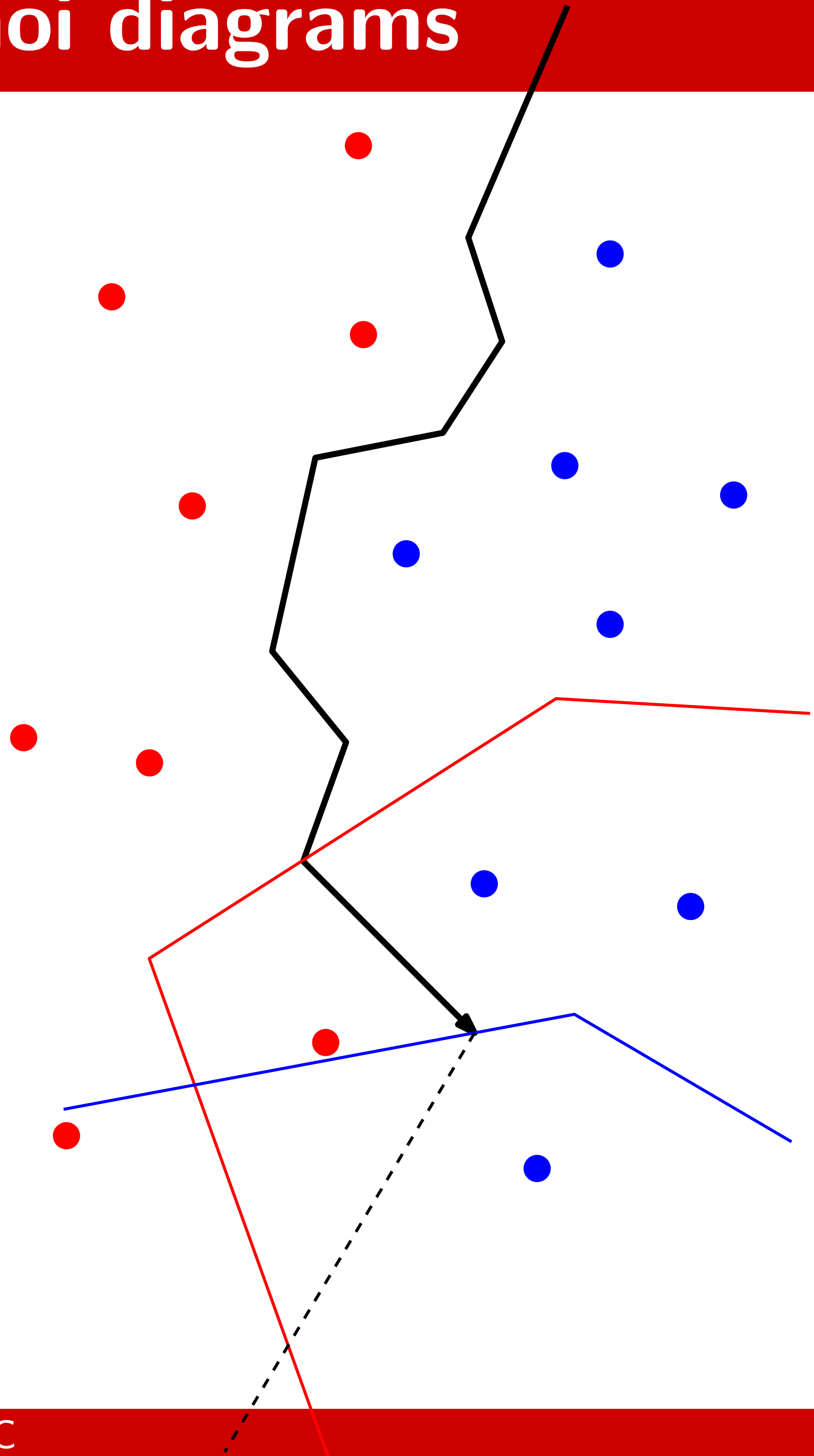
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

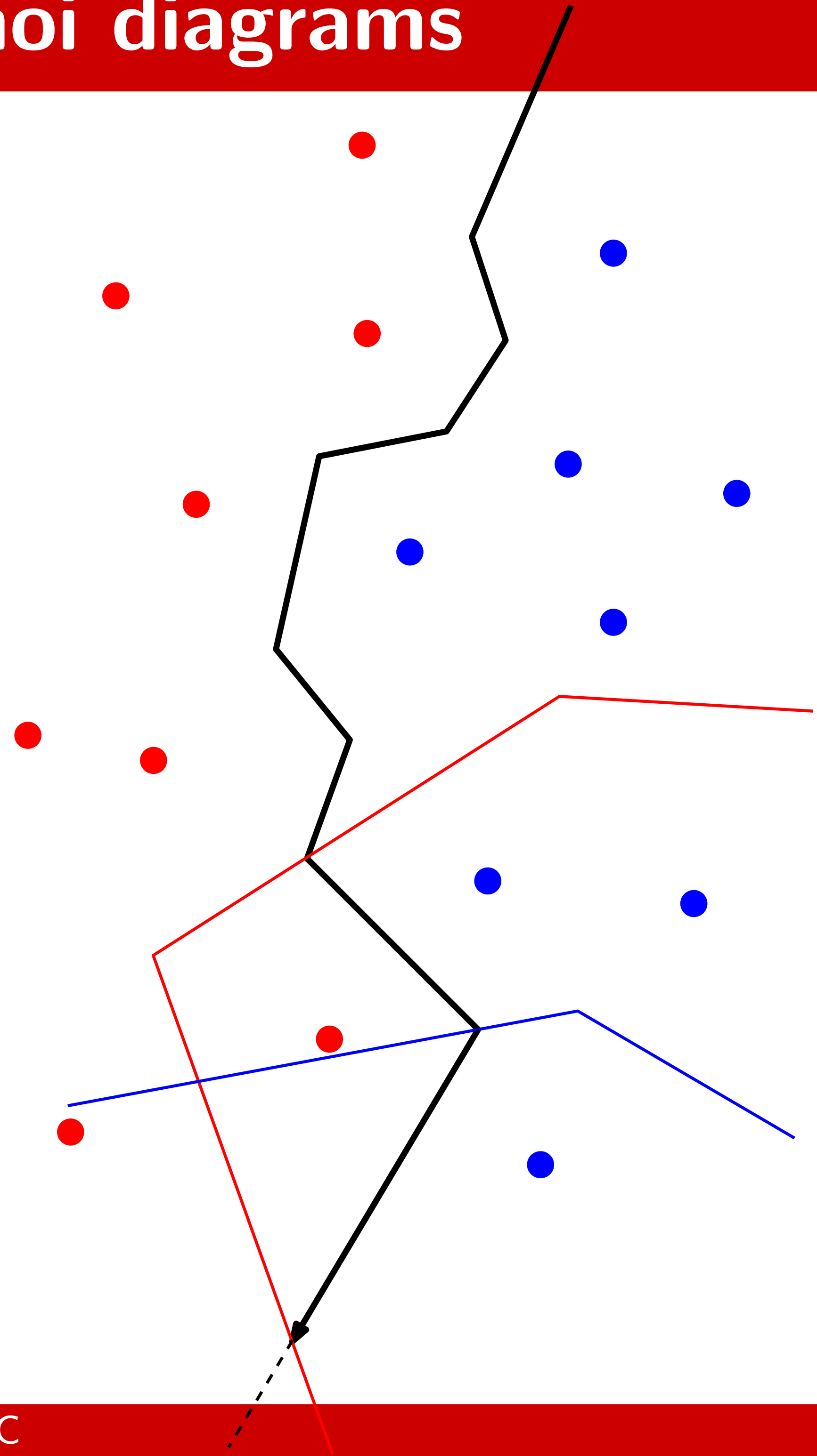
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

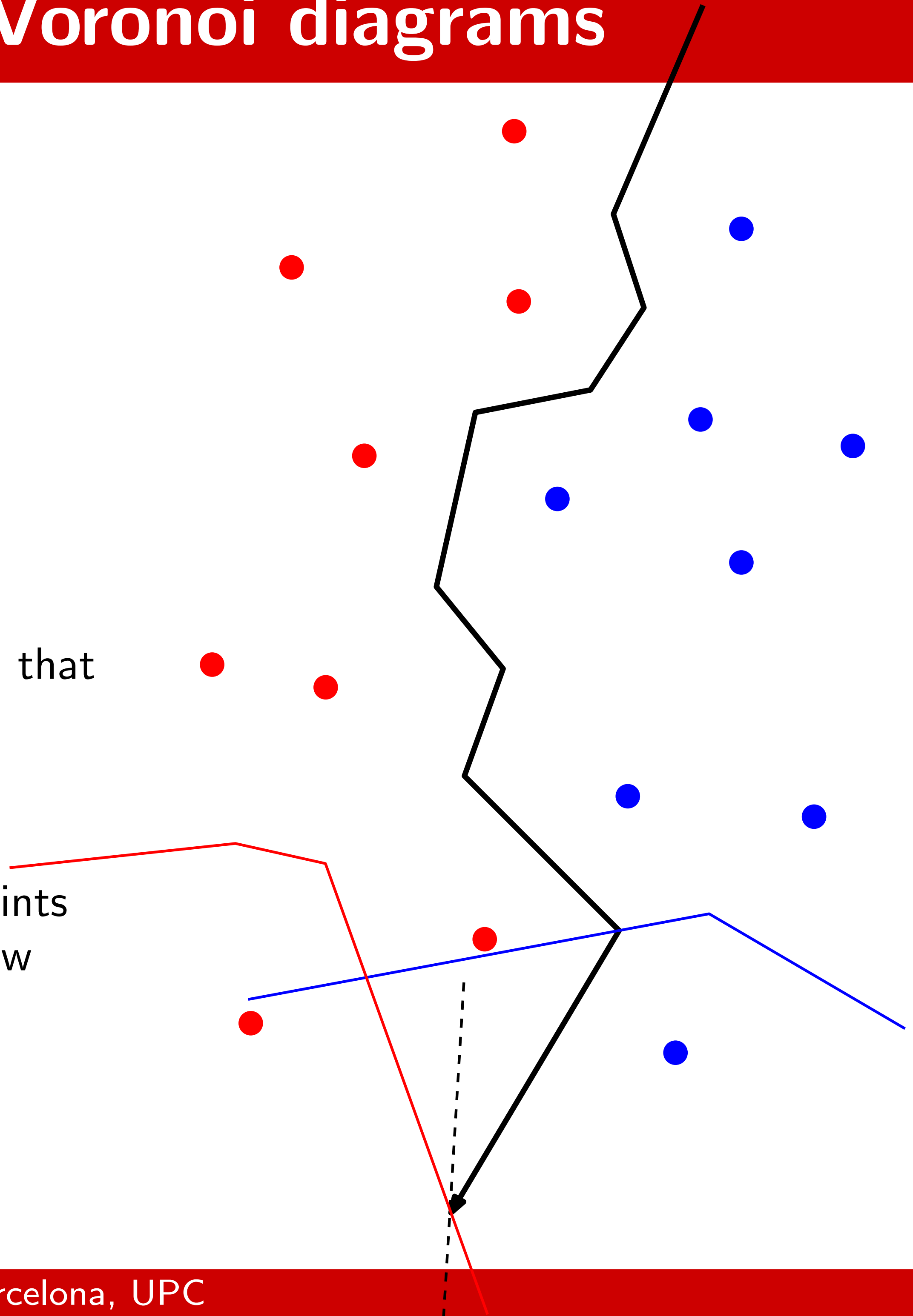
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

Initialization

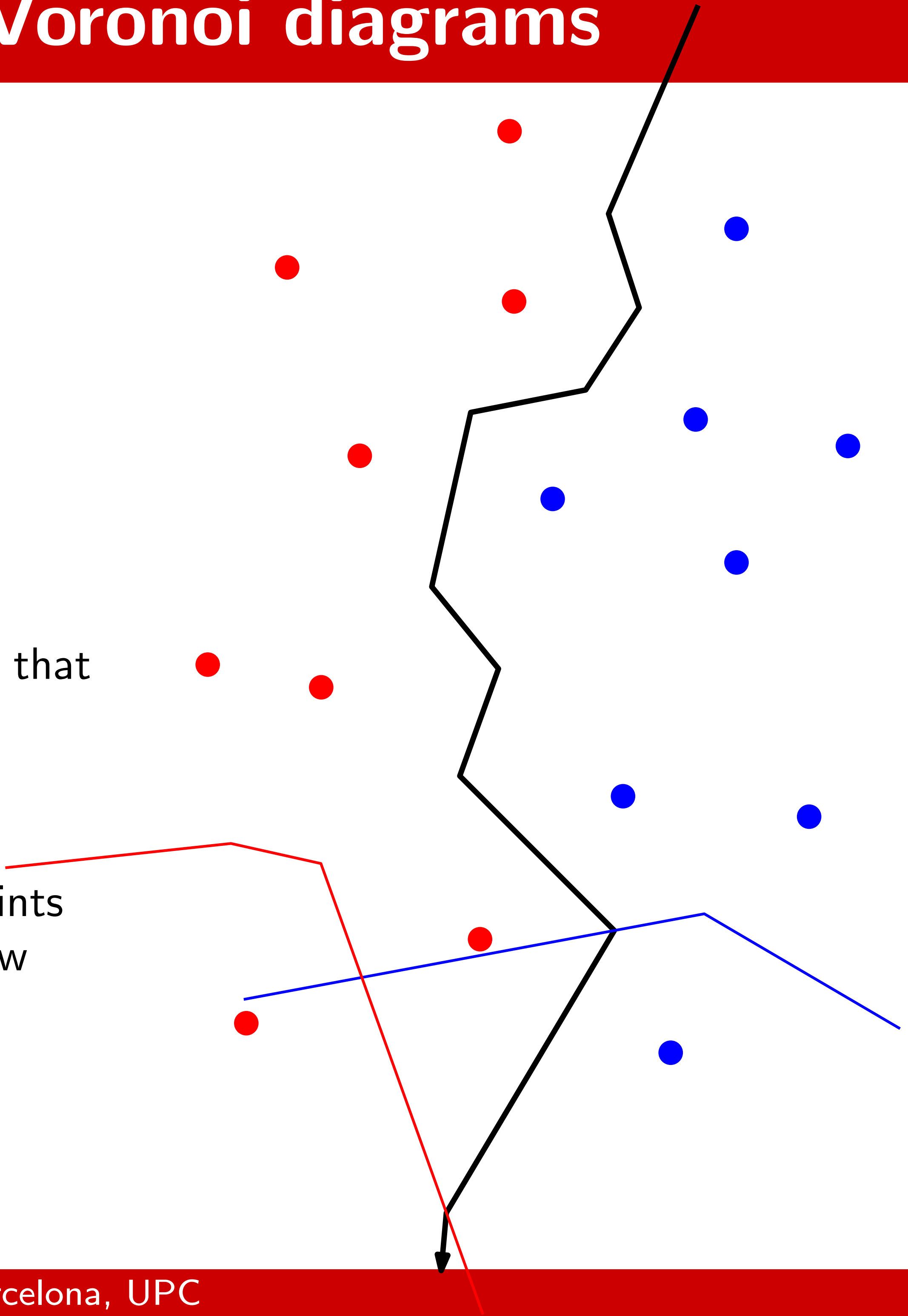
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



Constructing Voronoi diagrams

How to compute the chain?

Initialization

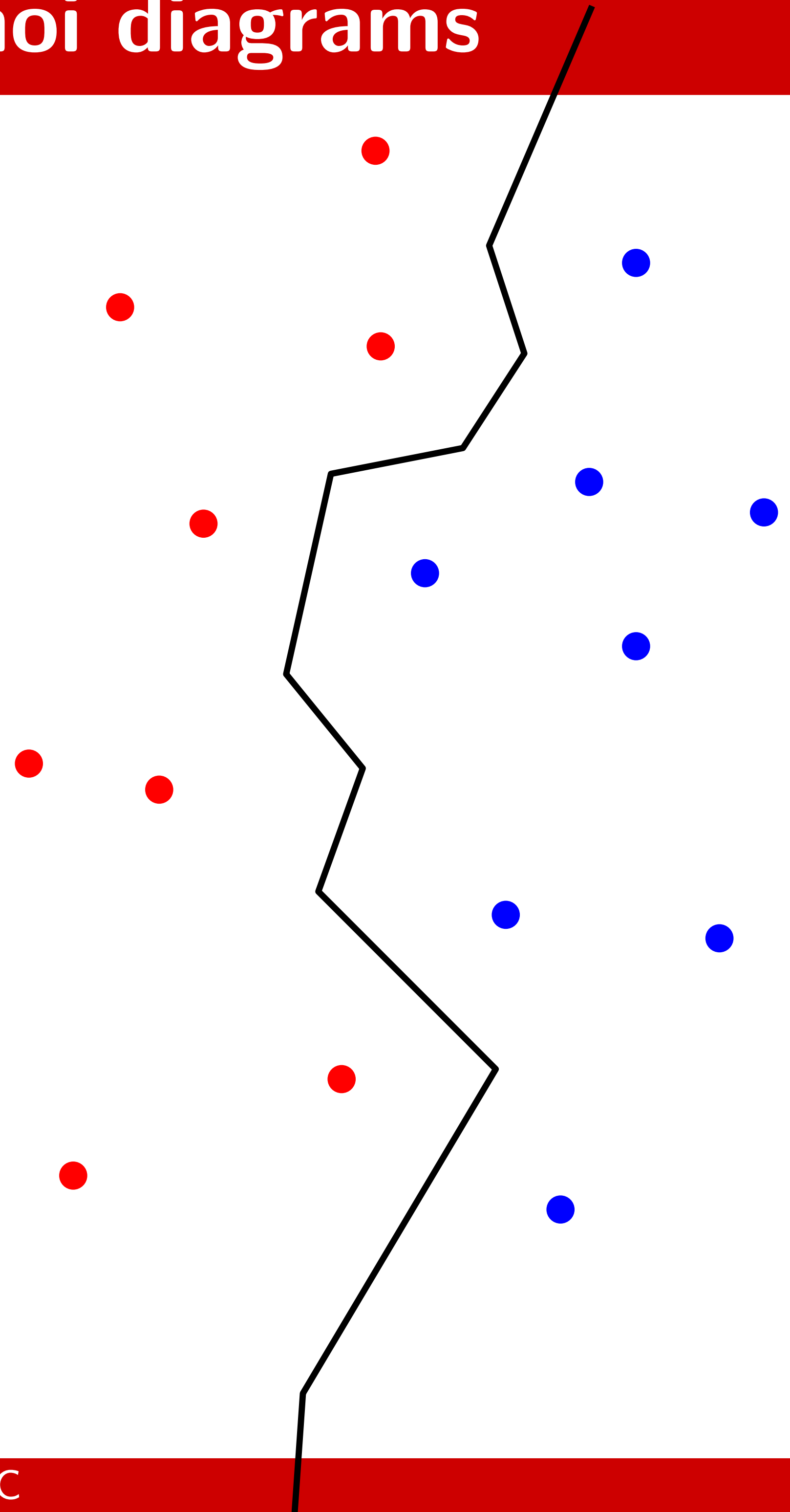
Find the two halflines

Advance

Starting with one of the halflines,
and until getting to the other one, do:

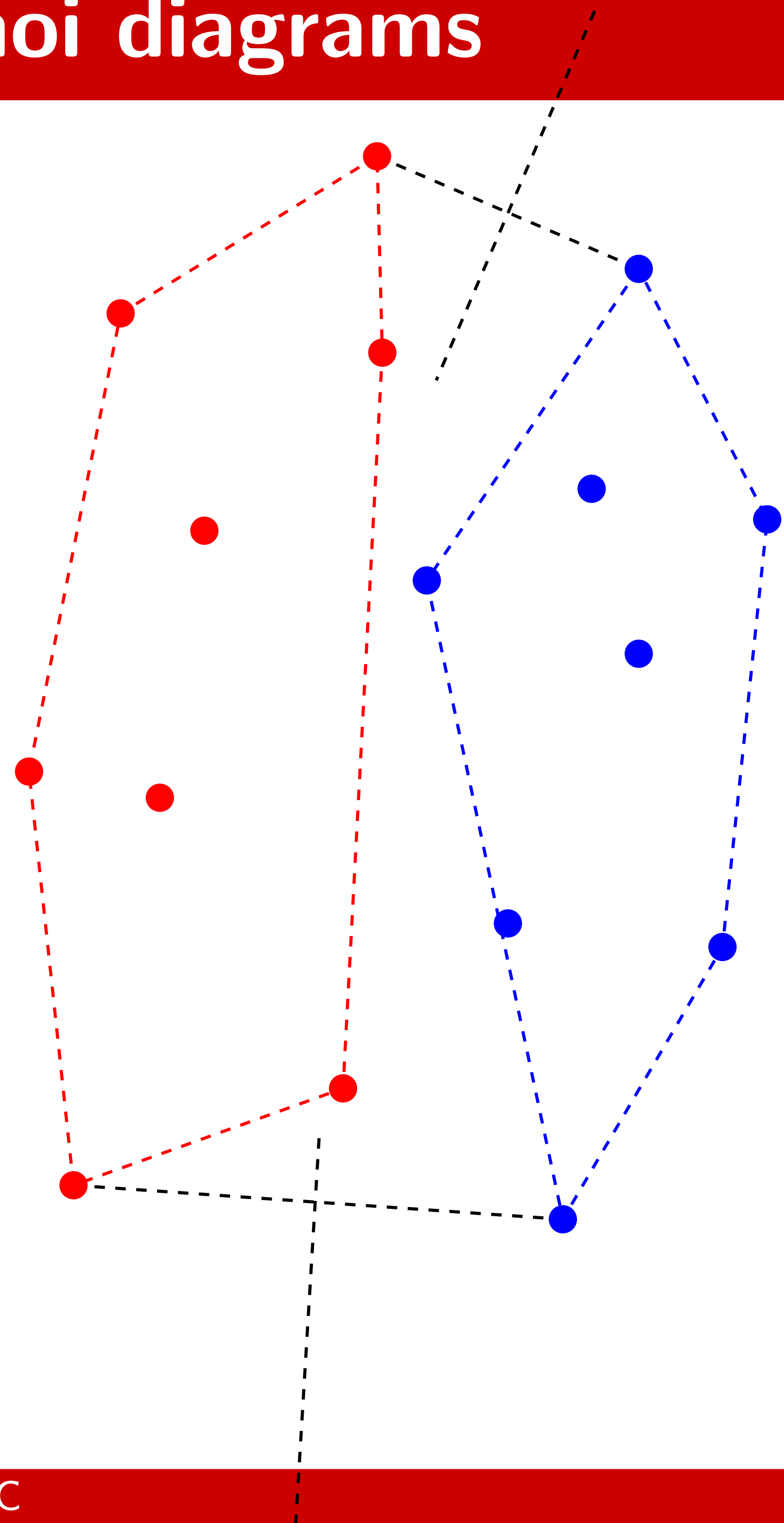
Each time an edge $e \in b(R, B)$ begins, such that $e \subset b_{ij}$, $p_i \in R$ and $p_j \in B$, do:

- Detect its intersection with $Vor_R(p_i)$
- Detect its intersection with $Vor_B(p_j)$
- Choose the first of the two intersection points
- Detect the site p_k corresponding to the new starting region
- Replace p_i or p_j (as required) by p_k
- Restart with the new edge



How to compute the chain?

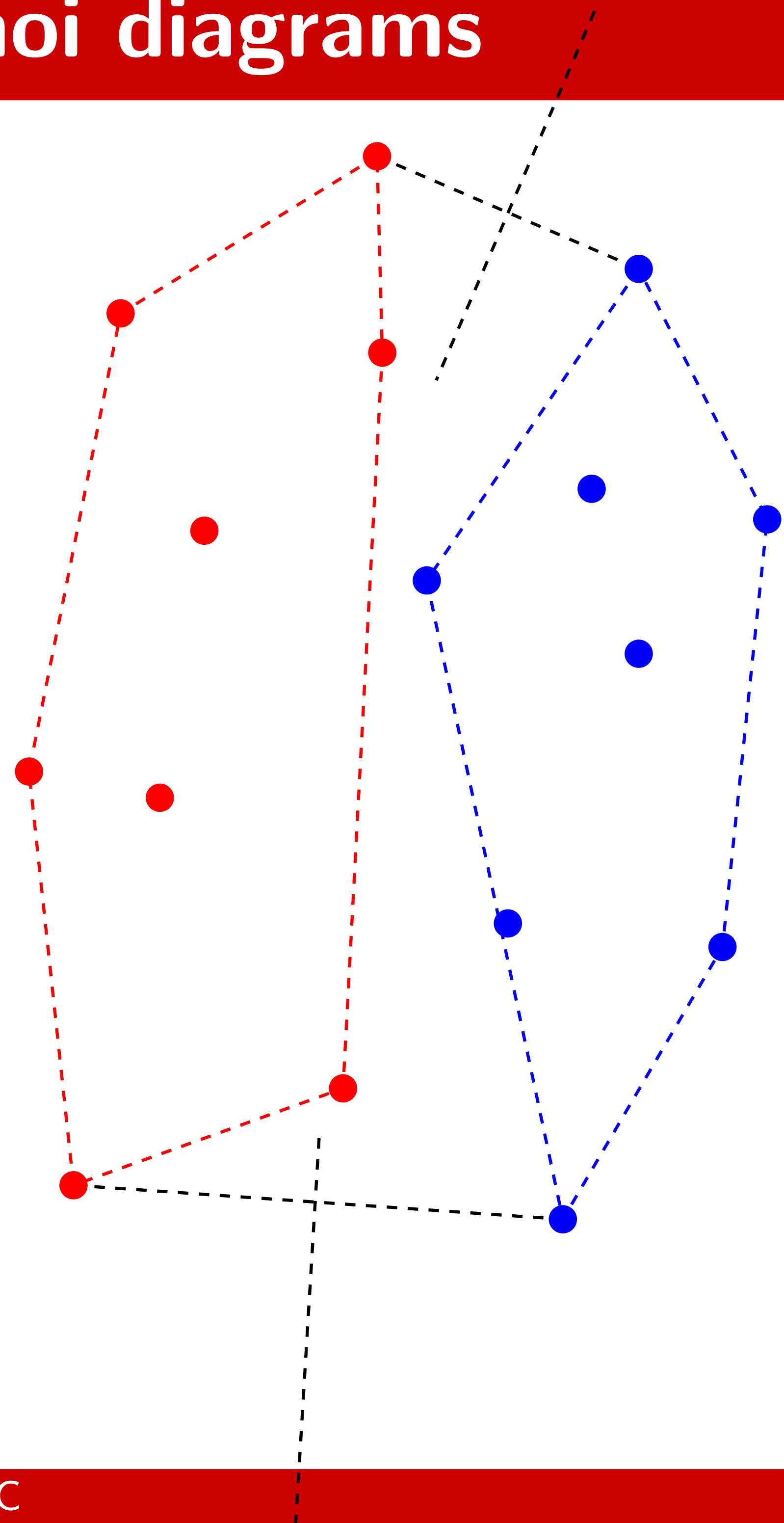
Initialization running time: $O(n)$



How to compute the chain?

Initialization running time: $O(n)$

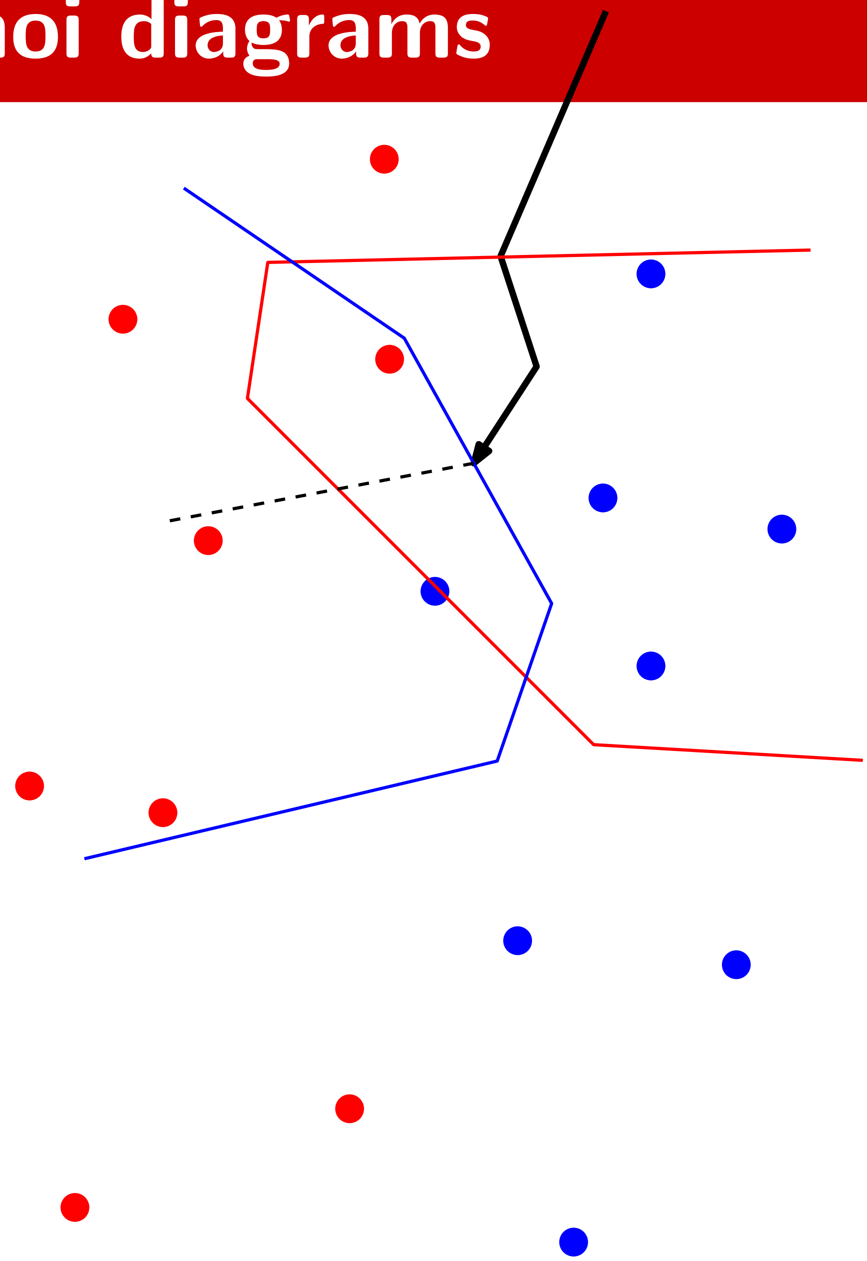
From $Vor(R)$ and $Vor(B)$.



How to compute the chain?

Initialization running time: $O(n)$

Advance running time: $O(n)$



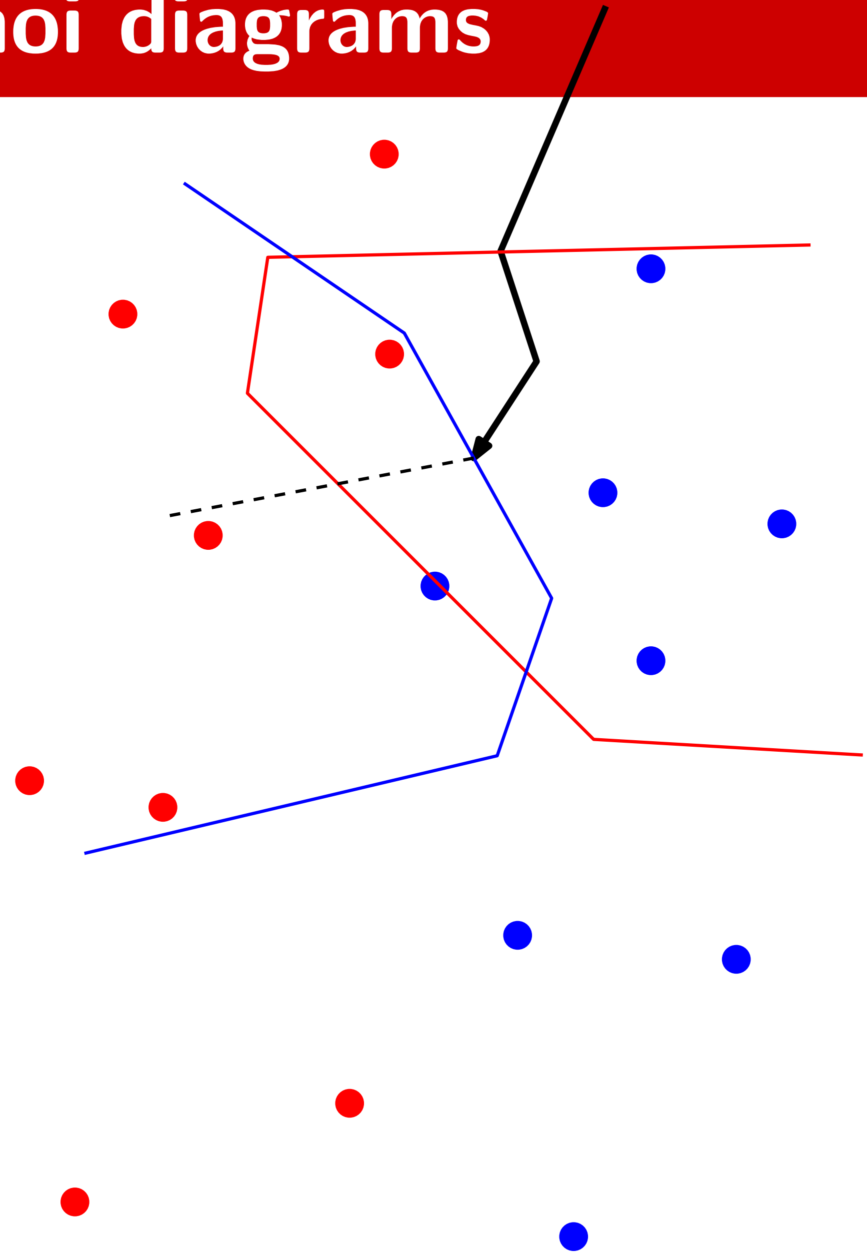
Constructing Voronoi diagrams

How to compute the chain?

Initialization running time: $O(n)$

Advance running time: $O(n)$

If e is an edge of $b(R, B)$ that entered $Vor_R(p_i)$ through some vertex $v \in Vor(P)$, then the exit point of $b(R, B)$ is found clockwise along the boundary of $Vor_R(p_i)$.



How to do the merging?

Constructing Voronoi diagrams

How to do the merging?

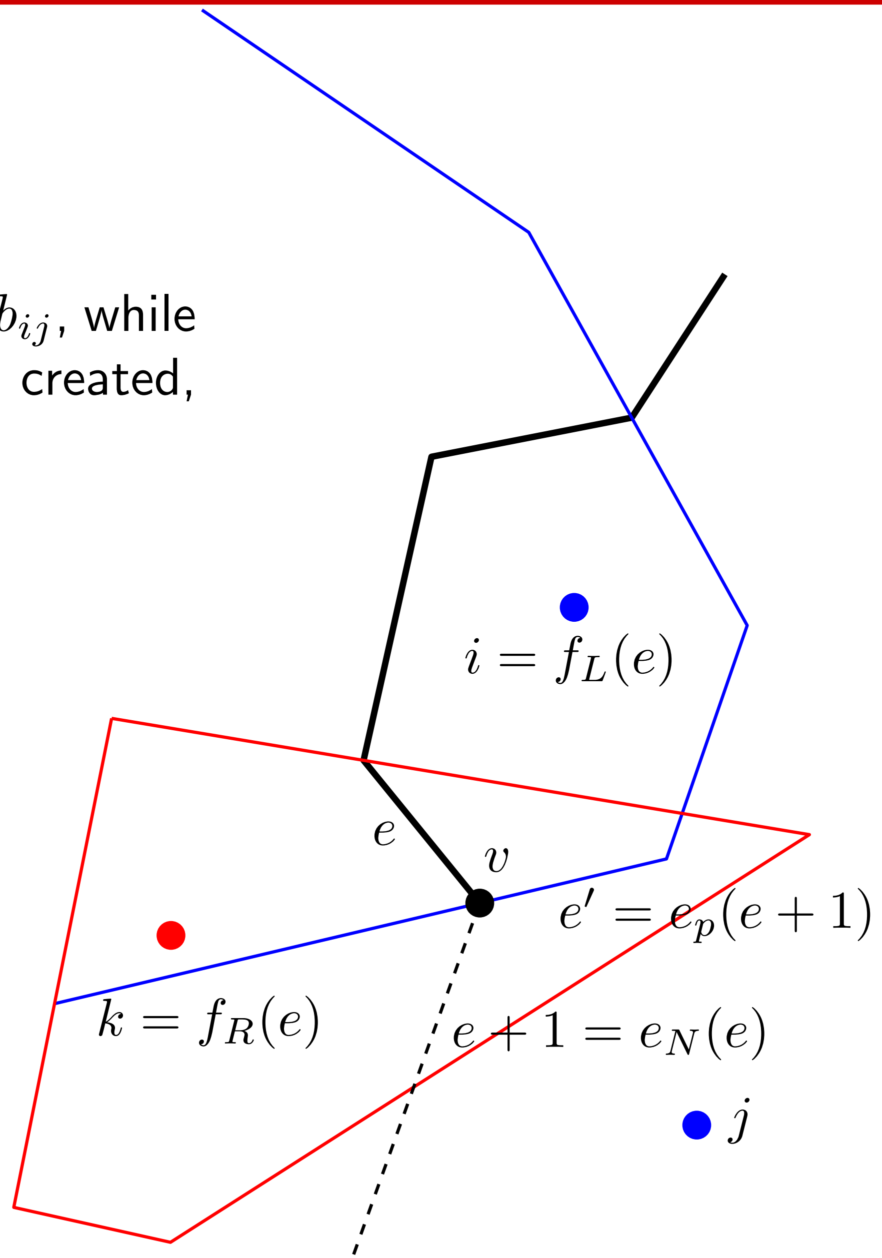
It consists in updating the DCEL:

How to do the merging?

It consists in updating the DCEL:

Each time a face $Vor_B(p_i)$ is left through an edge $e' \in b_{ij}$, while staying in the same face $Vor_R(p_k)$, a new vertex v is created, an edge e ends and another edge $e+1$ begins:

- Create $e+1$ and assign to it $v_B = v$ and $e_P = e'$
- Assign to e: $v_E = v$, $e_N = e+1$, $f_L = i$ and $f_R = k$
- Modify for e': $v_* = v$, $e_* = e+1$
- Delete all edges of $Vor_B(p_k)$ found in counter-clockwise order between the entry and exit points
- Update $e(p_i) = e$
- Create the new vertex v and assign $e(v) = e$



The procedure is analogous when exiting a face $Vor_R(p_i)$.

Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.

The total running time of the algorithm is $O(n \log n)$

Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.

The total running time of the algorithm is $O(n \log n)$

This running time is optimal, because $ch(P)$ can be computed from $Vor(P)$ in $O(n)$ time.

Constructing Voronoi diagrams

DIVIDE AND CONQUER ALGORITHM

1. Sort the points of P by abscissa (only once) and vertically partition P into two subsets R and B, of approximately the same size.

2. Recursively compute $Vor(R)$ and $Vor(B)$.

3. Compute the separating chain.

4. Prune the portion of $Vor(R)$ lying to the right of the chain and the portion of $Vor(B)$ lying to its left.

The total running time of the algorithm is $O(n \log n)$

This running time is optimal, because $ch(P)$ can be computed from $Vor(P)$ in $O(n)$ time.

OTHER ALGORITHMS

There exist other algorithms with the same running time:
- Fortune's Algorithm (sweep)
- 3D projection algorithm