## ECONOMICAL SPACE TRANSPORTATION ROUTES BETWEEN EARTH, MOON, AND BEYOND

| TEAM MEMBERS |  |
| :---: | :---: |
| Principal Investigators |  |
| Professor Kenneth D. Mease | Department of Mechanical and Aerospace Engineering The Henry Samueli School of Engineering University of California, Irvine Irvine, CA 92697-3975 kmease@uci.edu |
| Professor Benjamin F. Villac | Department of Mechanical and Aerospace Engineering The Henry Samueli School of Engineering University of California, Irvine Irvine, CA 92697-3975 bvillac@uci.edu |
| Professor Josep M. Mondelo | Department of Mathematics Universitat Autonoma de Barcelona 08193 Bellaterra, Barcelona, Spain jmm@mat.uab.cat |
| Collaborator |  |
| Jordi Casoliva Rodon Graduate Student | Department of Mechanical and Aerospace Engineering The Henry Samueli School of Engineering University of California, Irvine Irvine, CA 92697-3975 jcasoliv@uci.edu |
| Industrial partners |  |
| Ms. Marta Escudero Catalan Industrial Partner | GTD Enginyeria de Sistemes i de Software Barcelona, Catalunya. <br> Marta.Escudero@gtd.es |
| Mr. Mike Talbot US Industrial Partner | Universal Space Lines, LLC Newport Beach, California talbot@spacelines.com |

## EXECUTIVE SUMMARY

An Earth-Moon cycler is a periodic orbit that passes close to both bodies repeatedly. If the propellant required to reach a particular cycler from the Earth and from the Moon is modest and the transit time between the two bodies along the cycler is reasonable, then having a spacecraft on the cycler can provide an important component of an economical space transportation system. Cyclers may also be useful for navigation satellites. The generation and classification of cycler trajectories for application to Earth-Moon space infrastructure is the project goal. Cyclers of two types are considered: (i) p-q resonant orbits and (ii) cyclers composed of homoclinic and heteroclinic connections between unstable periodic orbits. Though the work is focusing on the Earth-Moon system, the results and methods will be applicable to cycler trajectories in other 3 and 4-body systems.

The UCI team -- Ph.D. student Jordi Casoliva and Profs. Mease and Villac -- has had a fruitful collaboration with Prof. Mondelo at the Universitat Autonoma de Barcelona, involving frequent teleconferences and a two-week visit by Prof. Mondelo to UCI in February 2008. Numerical methods for computing cycler trajectories have been developed and many cycler trajectories have been computed and analyzed as to their practical usefulness. The collaborators are writing a conference paper that has been accepted for presentation at the AAS/AIAA Astrodynamics Specialist Conference in August 2008. There is an effort underway to obtain support from the Jet Propulsion Lab to continue the research. Dr. Martin Lo, JPL, visited UCI in May 2008 to discuss a potential collaboration, and Prof. Mondelo visited JPL in February 2008.

## WORK COMPLETED

Earth-Moon cyclers are periodic trajectories that pass close to both bodies on a regular schedule. Previous research has identified cycler trajectories that could play a role in efficiently transporting payloads between the Earth and the Moon, but to design the most effective transportation architecture, a comprehensive knowledge of efficient transportation routes in the Earth-Moon system is required. Our research is making progress in this direction. Cyclers of two types are being considered: (i) cyclers composed of homoclinic and heteroclinic connections between unstable periodic orbits and (ii) p-q resonant orbits. The metrics being used to select the most suitable cyclers for different purposes, such as cargo transport from Earth to Moon, and navigation and communication in the Earth-Moon system, are: (1) the propellant used for transferring mass to and from these orbits (2) the flight times (short flight times would be enforced for human transport, but the flight time may be relaxed for cargo delivery) (3) stability criteria associated with the trajectories (4) proximity of nearby back-up orbits, and (5) constraints on communications or mapping geometry.

In the work to date, we have used the circular, restricted three-body model for the dynamics of a spacecraft flying in the Earth-Moon system. Provided that the spacecraft trajectory does not extend too far beyond the vicinity of Earth, this model provides a close approximation to the actual trajectory that would be flown in the presence of the Sun and other gravitating bodies. The threebody problem is composed of a primary body, Earth, and secondary body, the Moon, and a spacecraft. In the circular, restricted three-body model, the gravitational forces of the primary and secondary bodies, but not the spacecraft, are accounted for, and the Earth and the Moon are assumed to be in circular orbits about the Earth-Moon barycenter. Normally, the trajectories of a spacecraft are represented in the synodic frame whose origin is at the barycenter (i.e., center of mass) of the primary and secondary bodies and is rotating with an angular velocity such that the

Earth and the Moon are at fixed points on the x-axis. The coordinates of the Earth are $(\mu, 0)$ and those of the Moon are $(\mu-1,0)$, where $\mu=0.012$ is the Moon-Earth mass ratio.

Cyclers that are p-q resonant orbits: The p-q resonant orbits are trajectories where the spacecraft makes p revolutions around the Earth between two consecutive close approaches to the Moon, while the Moon does $q$ revolutions around the Earth-Moon barycenter. An initial approximation for such an orbit, valid for very small values of $\mu$ is obtained from the analytical approach of Barrabes and Gomez. Differential correction and numerical continuation are used to compute orbits as the value of the mass ratio $\mu$ is increased to the correct value for the Earth-Moon system. Figure 3 shows 6 examples of $p-q$ resonant type cyclers.


Figure 1. Examples of $\mathbf{p - q}$ resonant type cyclers portrayed in the synodic frame. The $\mathbf{p}-\mathbf{q}$ value and Jacobi energy constant are: (a) 2-1; 0.57, (b) 2-1; 2.25, (c) 3-2; 2.10, (d) 5-2; 1.05, (e) 5-2; 2.63, (f) 5-4; 2.2.

In Fig. 2 case (c) from the previous figure is shown again to depict how the orbit looks in the non-rotating inertial frame, superimposed on the plot for the rotating synodic frame. The orbit in the inertial frame is approximately an elliptical Earth orbit. At the initial time, the spacecraft, indicated with a yellow star, is making a close flyby of the Moon. As the spacecraft follows the near-elliptical orbit, initially headed away from Earth, the Moon proceeds along its (assumed) circular orbit about the Earth. For this 3-2 resonant orbit, the spacecraft will transit the near-elliptical orbit 3 times while the Moon orbits the Earth 2 times, and then the spacecraft will again have a close flyby of the Moon and reach exactly the same position and velocity it had at the initial time.


Figure 2. Example of a p-q resonant type cycler shown by the black curve in the inertial (nonrotating) frame and by the gray curve in the synodic (rotating) frame, with the two frames superimposed.

## Cyclers composed of homoclinic and heteroclinic connections between unstable periodic

 orbits: The invariant stable manifold of an object is mathematically defined as the set of trajectories that asymptotically approach the object forward in time. They approach the base object at an exponential rate. The invariant unstable manifold of an object is defined in the same way but backward in time. It thus contains trajectories that depart from the base object at an exponential rate. If the object is a nominal orbit, invariant (stable and unstable) manifolds provide natural conduits from which we can choose suitable transfer and departing trajectories. The Genesis sample return mission has been designed in this way. When the base object is a periodic orbit, its invariant manifolds look like two-dimensional tubes. In Fig. 3(a), the invariant manifolds of a planar Lyapunov orbit around L1 in the Earth-Moon system are shown (red corresponds to unstable, and blue to stable).

Figure 3: (a) Invariant unstable (red) and stable (blue) manifold tubes of a Lyapunov orbit around L1 in the Earth-Moon system. (b) Intersection of the tubes with the hypersurface $\mathbf{x}=0$.

A homoclinic connection of an object is a trajectory that asymptotically approaches the object both forward and backward in time. An example of a homoclinic connection of a planar Lyapunov orbit around the point L1 in the Earth-Moon system is shown in Fig. 4(a). Homoclinic
connections can be computed numerically by matching the stable and unstable manifold tubes (Fig. 3(a)) in a surface of section. In Fig. 3(b), the section of the manifold tubes with the hypersurface $\mathrm{x}=$ 0 is shown. The four intersection points correspond to four homoclinic connections of the base Lyapunov orbit. One of them is shown in Fig. 4(a). This orbit may have merit as a potential EarthMoon cycler, since it has a part that can be considered as an Earth flyby. It is very high, though, due to the fact that the orbit has low energy.


Figure 4: (a) Homoclinic connection of a low-energy Lyapunov planar orbit around L1 in the EarthMoon system. (b) Homoclinic connection with a higher energy level, that flies by both Earth and Moon.

The homoclinic connections displayed are part of one parameter families that evolve across energies. Recent numerical techniques allow the continuation of these families to higher energy levels. Using these techniques, connections such as the one of Fig. 4(b) can be obtained. This trajectory can be considered as an Earth-Moon cycler, since it has a part with a Moon flyby (the Lyapunov orbit), and a part with an Earth flyby (the homoclinic connection). In the practical use of such a cycler, a spacecraft would be placed at the Lyapunov orbit flying by the Moon. When an Earth flyby is needed, the homoclinic connection can be taken, allowing for five rendezvous opportunities from Earth before it returns to the base Lyapunov orbit.

Work in progress:
(i) Classification the different families of of Earth-Moon cyclers given by homoclinic connections.
(ii) Tabulation of interesting parameters of these cyclers, such as perigee
and periselene distance and velocity.
(iii) Estimation of station-keeping costs.

## BENEFITS OF THE ACADEMIC/INDUSTRIAL PARTNERSHIP FOR CALIFORNIA AND CATALONIA

The mathematics required for this work is at an advanced level, yet it holds the key to economically viable transportation in the Earth-Moon system. Transitioning this mathematical theory into practical tools for mission designers will be an important contribution. The proposed research effort has brought together a team capable of accomplishing this transition. The project has established a collaboration between researchers at UC Irvine and at the Universitat Autonoma de Barcelona and the companies Universal Space Lines, LLC in California and GTD in Catalonia. As the work proceeds we fully expect to develop additional connections with both the U.S. and European space programs and additional U.S. and Catalan companies. A collaboration with the Jet Propulsion Lab is developing already. In addition to transitioning theory to practice, the proposed
project is leading to discoveries of desirable cycler trajectories between Earth and Moon and beyond, as well as generate software to enable others to design and analyze cycler trajectories and implement modern optimal control paradigms. The graduate student, J. Casoliva, is being trained in both the mathematical theory and the mission design and analysis aspects of the work.

## TRAVEL

February 2-16, 2008 - Prof. Mondelo in residence at UCI, collaborating with Jordi Casoliva and Profs. Mease and Villac. Prof. Mondelo gave a seminar at UCI, and also visited the Jet Propulsion Lab and gave a seminar there.

Prof. Villac will attend the AAS/AIAA Astrodynamics Conference in Honolulu, Hawaii in August 2008, partially supported by the Cal-Cat grant.

Prof. Mease plans to visit Barcelona in Sept. 2008 and collaborate with Prof. Mondelo and meet with the Industrial Partners at GTD.

## PUBLICATIONS

Jordi Casoliva, Kenneth D. Mease, Josep M. Mondelo, Benjamin F. Villac, Esther Barrabes and Merce Olle, "Families of Cycler Trajectories in the Earth-Moon System". AIAA/AAS Astrodynamics Specialist Conference, Honolulu, Hawaii, August 18-21, 2008 (accepted for publication).

