THE SUBGROUP LATTICE OF J_2

THOMAS CONNOR AND DIMITRI LEEMANS

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
1	J_2	604800	1	2 (100), 3 (280), 4 (315), 5 (525), 7 (840), 8 (1008), 12 (1800), 14 (2016), 46 (10080)		146	1	1
2	$U_3(3)$	6048	100	17 (28), 23 (36), 35 (63), 36 (63)	1	146	2	-1
3	$3 \cdot A_6 : 2$	2160	280	6, 17 (10), 47 (36), 55 (45)	1	146	3	-1
4	$2^{1+4} : A_5$	1920	315	10 (5), 13 (6), 18 (10), 30 (16)	1	145	4	-1
5	$D_8 \times 2 : 2 : 2 : 3 : S_3$	1152	525	9, 11, 10 (3), 41 (16)	1	146	5	-1
6	$3 \cdot A_6$	1080	280	22 (12), 33 (10), 41 (30)	3	143	3	0
7	$A_5 \times 2 \times 2 : 3$	720	840	16, 22 (4), 28 (5), 31 (6), 42 (10)	1	146	7	-1
8	$A_5 \times 5 : 2$	600	1008	15, 30 (5), 31 (5), 34 (6), 47 (10)	1	146	8	-1
9	$2^4 : 3 : 2 : 2 : 3$	576	525	19, 20, 21 (2), 28 (8)	5	146	5	0
10	$2^{1+4} : A_4$	384	1575	20, 29, 37 (4)	4, 5	145	10	1
11	$2 \times 4 : 2 : 2 : 2 : S_3$	384	525	19, 29 (3), 35 (12)	5	146	5	0
12	$L_2(7) : 2$	336	1800	23, 64 (8), 93 (21), 104 (28)	1	146	12	-1

continued on next page

Date: April 16, 2014.

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
13	$2^{1+4} : (D_{10})$	320	1890	24, 43 (5), 88 (16)	4	145	13	0
14	$5^2 : 2 : S_3$	300	2016	25, 26, 27, 34 (3), 104 (25)	1	146	14	-1
15	$A_5 \times 5$	300	1008	48, 49 (5), 52 (6), 74 (10)	8	136	8	0
16	$A_5 \times 2 \times 2$	240	840	32 (3), 56 (5), 65 (6), 79 (10)	7	138	7	0
17	$3^{1+2} : 8$	216	2800	33, 80 (9)	2, 3	146	17	1
18	$2^{1+4} : S_3$	192	3150	38, 37, 36, 43 (3), 55 (4)	4	145	18	0
19	$2 \times 4 : 2 : 2 : 2 : 3$	192	525	44, 57 (12), 58 (8)	9, 11	146	5	0
20	$2 \times 4 : 2 : 2 : 2 : 3$	192	525	44, 59 (12), 60 (8)	9, 10 (3)	141	5	0
21	$2^4 : 3 : 2 : 2$	192	1050	39 (3), 44, 56 (4)	9	146	9	0
22	$A_5 \times 3$	180	3360	50, 66 (5), 75 (6), 92 (10)	6, 7	143	22	1
23	$L_2(7)$	168	1800	81 (14), 87 (8)	2 (2), 12	146	12	2
24	$2^{1+4} : 5$	160	1890	68, 115 (16)	13	145	13	0
25	$5^2 : S_3$	150	2016	40, 52 (3), 130 (25)	14	146	14	0
26	$5^2 : S_3$	150	2016	40, 53 (3), 131 (25)	14	146	14	0
27	$5^2 : 3 : 2$	150	2016	40, 54, 132 (25)	14	146	14	0
28	$A_4 \times 2 \times 2 : 3$	144	4200	58, 60, 56, 61, 67 (4), 66 (4)	7, 9	146	28	1
29	$2^{1+4} : 2^2$	128	1575	44, 43 (3), 45 (3)	10, 11	145	10	0
30	$A_5 \times 2$	120	5040	48, 82 (5), 88 (6), 105 (10)	4, 8	145	30	1
31	$2 \times 10 : 3 : 2$	120	5040	49, 65, 75 (4), 82 (5)	7, 8	146	31	1
32	$A_5 \times 2$	120	2520	50, 83 (5), 89 (6), 104 (10)	16	138	16	0
33	$3^{1+2} : 4$	108	2800	51, 106 (9)	6, 17	143	17	0
34	$5^2 : 2 : 2$	100	6048	54, 53, 52, 88 (5), 89 (5)	8, 14	146	34	1

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
35	$4^2 : S_3$	96	6300	57, 69 (3), 81 (4)	2, 11	146	35	1
36	$D_8 : 2 \cdot S_3$	96	3150	62, 69 (3), 80 (4)	2 (2), 18	140	18	2
37	$2^{1+4} : 3$	96	3150	59 (2), 62, 68, 84 (4)	10 (2), 18	145	18	0
38	$D_8 : 2 : S_3$	96	3150	62, 63 (2), 70 (3), 85 (4)	18	145	18	0
39	$2^4 : 3 : 2$	96	3150	61, 71, 83 (4)	21	146	21	0
40	$5^2 : 3$	75	2016	78, 142 (25)	25, 26, 27	146	14	0
41	$S_4 \times 3$	72	8400	66, 81, 84 (3), 92 (4)	5, 6	143	41	1
42	$A_4 \times S_3$	72	8400	67, 79, 82 (3), 92 (4)	7	146	42	0
43	$2^{1+4} : 2$	64	4725	69 (2), 70 (2), 72, 73, 68	13 (2), 18 (2), 29	145	29	0
44	$2 \times 4 : 2 : 2 : 2$	64	525	71 (6), 72 (9)	19, 20, 21 (2), 29 (3)	141	5	0
45	$Q_8 \times 2 \cdot 2 : 2$	64	4725	72, 73 (2)	29	145	29	0
46	A_5	60	10080	107 (5), 116 (6), 130 (10)	1	146	46	-1
47	$D_{10} \times S_3$	60	10080	74, 75, 76, 89 (3), 105 (5)	3, 8	146	47	1
48	A_5	60	1008	108 (5), 117 (6), 133 (10)	15, 30 (5)	118	8	0
49	$2 \times 10 : 3$	60	5040	90, 102 (4), 108	15, 31	136	31	0
50	A_5	60	840	109 (5), 118 (6), 131 (10)	22 (4), 32 (3)	108	7	0
51	$3^{1+2} : 2$	54	2800	77, 92 (12)	33	143	17	0
52	$5^2 : 2$	50	6048	78, 117, 119 (5)	15, 25, 34	136	34	0
53	$5^2 : 2$	50	6048	78, 118, 115 (5)	26, 34	135	34	0
54	$5^2 : 2$	50	2016	78, 116 (15), 120 (15)	27, 34 (3)	146	14	0
55	$D_8 : S_3$	48	12600	80, 85, 84, 93 (3)	3, 18	145	55	1
56	$A_4 \times 2 \times 2$	48	4200	83 (3), 94, 110 (4)	16, 21, 28	138	28	0
57	$4^2 : 3$	48	6300	95, 111 (4)	19, 35	146	35	0
58	$2^4 : 3$	48	4200	94, 111 (4), 107 (12), 112 (4)	19, 28	146	28	0

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
59	$Q_8 : 6$	48	6300	86, 96, 113 (4)	20, 37	141	37	0
60	$A_4 \times 2 \times 2$	48	4200	82 (3), 94, 113 (4)	20, 28	141	28	0
61	$2^4 : 3$	48	1050	94, 112 (16), 109 (4)	28 (4), 39 (3)	146	9	0
62	$Q_8 \cdot 6$	48	3150	86, 97, 106 (4)	36, 37, 38	140	18	0
63	$Q_8 : S_3$	48	6300	86, 98 (3), 105 (4)	38	145	38	0
64	$7 : 3 : 2$	42	14400	87, 103, 132 (7)	12	146	64	0
65	$D_{20} \times 2$	40	5040	89 (3), 91 (3), 90, 122 (5)	16, 31	138	31	0
66	$A_4 \times 3$	36	8400	109 (2), 111, 113, 121 (4)	22 (2), 28 (2), 41	143	41	-2
67	$A_4 \times 3$	36	8400	110, 112 (2), 108, 121 (4)	28 (2), 42	142	42	0
68	2^{1+4}	32	315	97 (10), 96 (5)	24 (6), 37 (10), 43 (15)	145	4	0
69	$D_8 : 4$	32	9450	95, 99, 97	35 (2), 36, 43	140	43	-2
70	$D_8 : 2^2$	32	9450	100, 93 (2), 99, 97, 98 (2)	38, 43	145	43	0
71	$2 \times 4 : 2 : 2$	32	3150	94, 101 (3), 100 (3)	39, 44	141	21	0
72	$2 \times 4 : 2 : 2$	32	4725	101 (4), 100, 96, 95	43, 44, 45	141	29	0
73	$Q_8 \times 2 \cdot 2$	32	4725	99 (2), 96	43, 45 (2)	145	29	0
74	$S_3 \times 5$	30	10080	102, 119 (3), 133	15, 47	136	47	0
75	$15 : 2$	30	10080	102, 118, 134 (5)	22 (2), 31 (2), 47	143	47	-2
76	D_{30}	30	10080	102, 120 (3), 133 (5)	47	146	47	0
77	3^{1+2}	27	2800	121 (4)	51	143	17	0
78	5^2	25	2016	135 (3), 136 (3)	40, 52 (3), 53 (3), 54	78	14	0
79	$S_3 \times 2^2$	24	8400	104 (3), 114 (3), 110, 122 (3)	16, 42	138	42	0
80	$4 \cdot S_3$	24	12600	106, 123 (3)	17 (2), 36, 55	140	55	-2
81	S_4	24	8400	111, 124 (3), 131 (4)	23 (3), 35 (3), 41	143	41	-3
82	$A_4 \times 2$	24	12600	108, 122, 134 (4)	30 (2), 31 (2), 42 (2), 60	141	60	-2
83	$A_4 \times 2$	24	12600	109, 125, 132 (4)	32, 39, 56	138	56	0

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
84	$D_8 \times 3$	24	12600	106, 113 (2), 124	37, 41 (2), 55	134	55	-2
85	D_{24}	24	12600	106, 105 (2), 126 (3)	38, 55	145	55	0
86	$Q_8 : 3$	24	3150	127, 134 (4)	59 (2), 62, 63 (2)	124	18	0
87	$7 : 3$	21	14400	129, 142 (7)	23, 64	146	64	0
88	D_{20}	20	30240	115, 117, 116, 137 (5)	13, 30, 34	145	88	0
89	D_{20}	20	15120	120, 119, 118, 137 (5)	32, 34 (2), 47 (2), 65	138	65	0
90	2×10	20	5040	119 (3), 138	49, 65	90	31	0
91	D_{20}	20	15120	119, 120 (2), 139 (5)	65	138	65	0
92	$S_3 \times 3$	18	33600	121, 131, 134 (3)	22, 41, 42, 51	143	92	0
93	D_{16}	16	18900	124, 123, 126	12 (2), 55 (2), 70	145	70	2
94	2^4	16	1050	122 (12), 125 (3)	56 (4), 58 (4), 60 (4), 61, 71 (3)	94	9	0
95	4^2	16	1575	128 (3)	57 (4), 69 (6), 72 (3)	95	11	0
96	$Q_8 \times 2$	16	1575	128 (3), 127 (4)	59 (4), 68, 72 (3), 73 (3)	141	10	0
97	$D_8 : 2$	16	3150	124 (3), 128 (3), 127	62, 68, 69 (3), 70 (3)	140	18	0
98	$Q_8 : 2$	16	18900	126, 123, 127	63, 70	145	70	0
99	$8 : 2$	16	9450	128, 123 (2)	69, 70, 73	140	43	0
100	$D_8 \times 2$	16	4725	125 (2), 126 (4), 128	70 (2), 71 (2), 72	141	29	0
101	$2 \times 4 : 2$	16	9450	125, 128 (2)	71, 72 (2)	141	44	0
102	15	15	10080	136, 143	49 (2), 74, 75, 76	102	47	0
103	D_{14}	14	14400	129, 144 (7)	64	146	64	0
104	D_{12}	12	25200	132, 130, 131, 137 (3)	12 (2), 14 (2), 32, 79	138	79	4
105	D_{12}	12	25200	134, 133 (2), 137 (3)	30 (2), 47 (2), 63, 85	145	85	-2
106	12	12	12600	134, 140	33 (2), 62, 80, 84, 85	106	55	0
107	A_4	12	50400	139, 142 (4)	46, 58	146	107	1
108	A_4	12	840	138, 143 (4)	48 (6), 49 (6), 67 (10), 82 (15)	50	7	0
109	A_4	12	4200	141, 142 (4)	50, 61, 66 (4), 83 (3)	108	28	0
110	2×6	12	8400	132 (3), 138	56 (2), 67, 79	110	42	0
111	A_4	12	8400	141, 142 (4)	57 (3), 58 (2), 66, 81	143	41	0

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
112	A_4	12	16800	138, 142 (4)	58, 61, 67	142	67	0
113	2×6	12	8400	134 (3), 141	59 (3), 60 (2), 66, 84 (3)	113	41	0
114	D_{12}	12	25200	132, 130 (2), 139 (3)	79	138	79	0
115	10	10	30240	135, 145	24, 53, 88	115	88	0
116	D_{10}	10	30240	135, 144 (5)	46 (2), 54, 88	145	88	2
117	D_{10}	10	6048	135, 144 (5)	48, 52, 88 (5)	118	34	0
118	D_{10}	10	1008	136, 145 (5)	50 (5), 53 (6), 75 (10), 89 (15)	48	8	0
119	10	10	15120	136, 144	52 (2), 74 (2), 89, 90, 91	90	65	0
120	D_{10}	10	15120	136, 144 (5)	54 (2), 76 (2), 89, 91 (2)	138	65	0
121	3^2	9	11200	142 (3), 143	66 (3), 67 (3), 77, 92 (3)	121	51	0
122	2^3	8	12600	137 (3), 139 (3), 138	65 (2), 79 (2), 82, 94	94	60	0
123	8	8	18900	140	80 (2), 93, 98, 99	123	70	0
124	D_8	8	3150	141 (2), 140	81 (8), 84 (4), 93 (6), 97 (3)	86	18	0
125	2^3	8	3150	137 (6), 141	83 (4), 94, 100 (3), 101 (3)	94	21	0
126	D_8	8	18900	137 (2), 140	85 (2), 93, 98, 100	141	70	0
127	Q_8	8	3150	140 (3)	86, 96 (2), 97, 98 (6)	124	18	0
128	2×4	8	4725	140 (2), 141	95, 96, 97 (2), 99 (2), 100, 101 (4)	95	29	0
129	7	7	14400	146	87, 103	129	64	0
130	S_3	6	25200	142, 144 (3)	25 (2), 46 (4), 104, 114 (2)	138	79	4
131	S_3	6	8400	142, 145 (3)	26 (6), 50, 81 (4), 92 (4), 104 (3)	108	42	-12
132	6	6	25200	142, 144	27 (2), 64 (4), 83 (2), 104, 110, 114	110	79	0
133	S_3	6	10080	143, 144 (3)	48, 74, 76 (5), 105 (5)	118	47	0

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
134	6	6	12600	143, 145	75 (4), 82 (4), 86, 92 (8), 105 (2), 106, 113 (2)	84	55	8
135	5	5	6048	146	78, 115 (5), 116 (5), 117	53	34	0
136	5	5	1008	146	78 (6), 102 (10), 118, 119 (15), 120 (15)	15	8	0
137	2^2	4	18900	144 (2), 145	88 (8), 89 (4), 104 (4), 105 (4), 122 (2), 125, 126 (2)	94	71	-16
138	2^2	4	840	144 (3)	90 (6), 108, 110 (10), 112 (20), 122 (15)	16	7	0
139	2^2	4	12600	144 (3)	91 (6), 107 (4), 114 (6), 122 (3)	94	58	0
140	4	4	3150	145	106 (4), 123 (6), 124, 126 (6), 127 (3), 128 (3)	36	18	0
141	2^2	4	525	145 (3)	109 (8), 111 (16), 113 (16), 124 (12), 125 (6), 128 (9)	20	5	0
142	3	3	8400	146	40 (6), 87 (12), 107 (24), 109 (2), 111 (4), 112 (8), 121 (4), 130 (3), 131, 132 (3)	67	42	-24
143	3	3	280	146	102 (36), 108 (12), 121 (40), 133 (36), 134 (45)	6	3	0
144	2	2	2520	146	103 (40), 116 (60), 117 (12), 119 (6), 120 (30), 130 (30), 132 (10), 133 (12), 137 (15), 138, 139 (15)	16	16	-240

continued on next page

continued from previous page

Nr.	Structure	Order	Length	Maximal Subgroups	Minimal Overgroups	Cent.	Norm.	$\mu_G(H)$
145	2	2	315	146	115 (96), 118 (16), 131 (80), 134 (40), 137 (60), 140 (10), 141 (5)	4	4	960
146	1	1	1		129 (14400), 135 (6048), 136 (1008), 142 (8400), 143 (280), 144 (2520), 145 (315)	1	1	604800

TABLE 1. Subgroup lattice of J_2

UNIVERSITÉ LIBRE DE BRUXELLES, DÉPARTEMENT DE MATHÉMATIQUES - C.P.216, BOULEVARD DU TRIOMPHE, B-1050 BRUXELLES, BOURSIER FRIA
E-mail address: tconnor@ulb.ac.be

UNIVERSITY OF AUCKLAND, DEPARTMENT OF MATHEMATICS, PRIVATE BAG 92019, AUCKLAND, NEW ZEALAND
E-mail address: d.leemans@auckland.ac.nz