PASSAGE

Prosperity and Sustainability in the Green Economy

Tim Jackson and Peter A. Victor

Does slow growth increase inequality?

Some reflections on Piketty's 'fundamental' laws of capitalism

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Prosperity and Sustainability in the Green Economy (PASSAGE) is a Professorial Fellowship held by Prof Tim Jackson at the University of Surrey and funded by the Economic and Social Research Council (Grant no: ES/J023329/1).

The overall aim of PASSAGE is to explore the relationship between prosperity and sustainability and to promote and develop research on the green economy.

The research aims of the fellowship are directed towards three principal tasks:
1) Foundations for sustainable living: to synthesise findings from a decade of research on sustainable consumption and sustainable living;
2) Ecological Macroeconomics: to develop a new programme of work around the macroeconomics of the transition to a green economy.
3) Transforming Finance; to work with a variety of partners to develop a financial system fit for purpose to deliver sustainable investment.

During the course of the fellowship, Prof Jackson and the team will engage closely with stakeholders across government, civil society, business, the media and academia in debates about the green economy.
PASSAGE also seeks to build capacity in new economic thinking by providing a new focus of attention on ecological macroeconomics for postgraduates and young research fellows.

Publication

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Abstract
This paper explores the hypothesis that slow growth rates lead to rising inequality. This case has been made most notably by French economist Thomas Piketty. If true, this hypothesis would pose serious challenges to the project of achieving *Prosperity without Growth* or meeting the ambitions of those who call for an intentional slowing down of growth on ecological grounds.

The paper describes a simple four-sector, demand-driven model of Savings, Inequality and Growth in a MAcroeconomic framework (SIGMA) with exogenous growth and net savings rates. SIGMA is used to examine the evolution of inequality in the context of declining economic growth.

Contrary to the general hypothesis, we find that inequality does not necessarily increase as growth slows down. In fact, there are certain conditions under which inequality can be reduced significantly, or even entirely eliminated, as growth declines. The paper discusses the implications of this finding for questions of employment, government policy and the politics of de-growth.

Introduction

The French economist, Thomas Piketty, has received widespread acclaim for his book *Capital in the 21ˢᵗ Century*. The book itself (Piketty 2014a) contains 700 pages of painstaking statistical analysis. But the central thesis is relatively straightforward to describe. Piketty argues persuasively that the slowing down of growth in recent decades has been responsible for the increase in inequality witnessed in modern capitalist economies.

Under circumstances in which growth is likely to slow down further, either through global economic conditions or through deliberate policies aimed at reducing society's ecological footprint, this dynamic would tend to be exacerbated.

The aim of this paper is to unravel the extent of this challenge in more detail. To this end, we develop a simple demand-driven model of Savings, Investment and Growth in a MAcroeconomic framework (SIGMA). We then use SIGMA to test for the implications of a slowdown of growth on a) capital's share of income and b) the distribution of incomes in the economy.

By adding a government sector to the model, we are also able to explore the potential to mitigate regressive impacts through a progressive taxation system. Finally, we discuss some of the implications of these findings for the wider project of developing an 'ecological macroeconomics'.

Piketty's two 'fundamental laws' of capitalism

Piketty advances his argument through the formulation of two 'fundamental laws' of capitalism. The first of these (Piketty 2014a: 52 et seq) relates the capital stock (more precisely the capital to income ratio β) to the share of income α flowing to the owners of capital. Specifically, the first fundamental law of capitalism says that:

$$\alpha = r\beta \qquad\qquad (1)$$

where r is the rate of return on capital. Since β is defined as K/Y where K is capital and Y is income, it is easy to see that this 'law' is, as Piketty acknowledges, an *accounting identity*:

$$\alpha Y = rK \qquad\qquad (2)$$

Formally speaking, the income accruing to capital equals the total capital multiplied by the rate of return on that capital. Though this 'law' on its own does not force the economy in one direction or another, it provides the foundation from which to explore the evolution of historical relationships between capital, income and rates of return. In particular, it can be seen from this identity that for any given rate of return r the share of income accruing to the owners of capital rises as the capital to income ratio rises.[1]

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[1] We will see later that the *ceteris paribus* clause relating to constant r here is important. In fact, the rate of return will typically change as the capital to income ratio rises; and to the extent that this ratio declines with increasing β, it can potentially mitigate the accumulation of the capital share of income.

It is the second of Piketty's 'fundamental laws of capitalism' (op cit: 168 et seq; see also Piketty 2010) that generates particular concern in the context of declining growth rates. This law states that in the long run, the capital to income ratio β tends towards the ratio of the savings rate s to the growth rate *g*, ie:

$$\beta \rightarrow \frac{s}{g} \qquad (3)$$

This asymptotic law suggests that, as growth rates fall towards zero, the capital to income ratio will tend to rise dramatically over the longer term – depending of course on what happens to savings rates.[2] Taken together with the first law, this suggests that over the long term, capital's share of income is governed by the following relationship:

$$\alpha \rightarrow r.\frac{s}{g} \qquad (4)$$

In other words, as growth declines, the rising capital to income ratio could lead to an increasing share of income going to capital and a declining share of income going to labour. Unless the distribution of capital is itself entirely equal (a situation we discuss in more detail later) this relationship therefore suggests the spectre of a rapidly escalating level of income inequality. Rising wealth inequality would also flow from this.

Differential savings rates – in which higher income earners save more than lower income earners (or, equally, where there are lower propensities to consume from capital than from income) – would reinforce these inequalities further by allowing the owners of capital to accumulate even more capital and command even higher wages. The superior power of capital (op cit 22-25)) then precipitates a rising structural inequality.

It is important to stress that the relationships (3) and (4) are long-term equilibria to which the economy evolves, provided that the savings rates and the growth rates stay constant. As Piketty points out (op cit 168), 'the accumulation of wealth takes time: it will take several decades for the law β = *s/g* to become true'.

In any real economy, the growth rate *g* and the savings rate *s* are likely to be moving around continually, so that at any point in time, the economy is striving towards, but may never in fact achieve, the asymptotic result. Nonetheless, as Krusell and Smith (2014) argue, equation (4) is 'alarming because it suggests that, were the economy's growth rate to decline towards zero, as Piketty argues it will, capital's share of income could increase explosively'.

It is also interesting to point out here that the 'second fundamental law' of capitalism is somewhat familiar (although in slightly different form) to conventional economists. In fact, it is a standard textbook result (for a derivation see Krusell and Smith (2014: 4-5)) that, under certain assumptions, and along a 'balanced growth path' the capital to income ratio β' is given by:

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[2] It's perhaps notable here that savings rates tend to rise (at least in the short term) when economic growth falters, a phenomenon Keynes called 'the paradox of thrift' because of the perverse impacts it has on economic recovery.

$$\beta' = \frac{s'}{g' + \delta} \qquad\qquad (5)$$

where δ is the depreciation rate, s' is the savings rate and g' the growth rate.[3] Krusell and Smith suggest that the inclusion of the depreciation rate in the denominator of the conventional formula (which in turn follows from the slightly different definition of income, growth rate and savings rate in the conventional model) mitigates at least some of the fear about explosive increases in the share of income going to capital.

The rate at which the capital to income ratio rises depends not simply on the decline of the growth rate, but also on what happens to the depreciation rate. Since there is no particular reason to suppose that the depreciation rate declines as the capital to income ratio rises (it might well do the opposite), equation(5) suggests that any decline in growth rates is potentially 'buffered' by the presence of the depreciation rate in the denominator.

This may not be of much consolation, since the income destined to offset depreciation is essentially lost to both labour and to capital. It is, rather, a continual maintenance payment needed just to keep the capital stock intact. It is for this reason that Piketty prefers to work with the concepts of net national income, *NI,* and the net savings rate *s,* since these provide a better indication of welfare in the economy than the gross concepts.

Irrespective of these differences between Piketty's formulation and the more generally recognised form (5), the principal aim of this paper is to determine the extent to which declining rates of growth in both GDP and *NI* might lead to rising capital to income ratios and more importantly to an increasing share of income to capital. In either formulation, much depends on the parallel movements in the rate of return on capital *r* and on the savings rate *s*.

Beyond these parameters, it is clear that the eventual impacts on inequality also depend on the distribution of capital in the economy, and the redistributive role of government. In order to explore these relationships in more detail, we built a simple, closed, four-sector, demand-driven model of savings, inequality and growth (calibrated loosely against UK and Canadian data). The structure of the SIGMA model is described in the next section. The subsequent section presents our findings.

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[3] We use the variables *s'*, *g'* and *β'* here to distinguish the ratios defined in conventional economics from those defined by Piketty. In the conventional formulation, g' is the growth rate in GDP and *s'* is the gross savings rate of the economy as a proportion of income. Piketty prefers to use the concept of net national income (NI) defined as the GDP minus depreciation of capital and also defines the savings rate *s* in terms of net (rather than gross) investment as a ratio of NI.

The SIGMA Model

The SIGMA model is constructed to describe a closed economy with four financial sectors: business, government and two distinct household sectors. In a closed economy, or an economy with zero net overseas trade, the national income NI can be interpreted both as the total income in the economy:

$$NI_i \;=\; W \;+\; P \tag{6}$$

(where W represents wages, or the return to labour and P, profits, or the return to capital) and also as the sum of the total expenditure of households, firms and government on goods, services and the (net) investment in fixed capital:

$$NI_d \;=\; C \;+\; G \;+\; I_{net} \tag{7}$$

where C is consumer spending, G is government spending and I_{net} is the net investment on fixed capital goods. In this simple closed economy model, we assume for now that the demand for goods is supplied by the productive output from the total capital stock K and labour employed L, so that:

$$NI_d = NI_i \tag{8}$$

And it follows that:

$$W \;+\; P \;=\; C \;+\; G \;+\; I_{net}\,. \tag{9}$$

We suppose that the change in demand for goods and services is given by a growth rate g, which is determined exogenously in the model and may vary over time. In particular we are interested to see what will happen as this growth rate declines to zero.

Note that the growth rate in *NI* is closely correlated with the growth rate in GDP, but that for any given depreciation rate, a decline in the growth rate of NI to zero implies a decline in the growth rate of GDP to below zero. The endpoint of any scenario in which the *NI* growth declines to zero is essentially a de-growth scenario in terms of GDP. This means that we can also explore the implications for inequality of absolute de-growth (in GDP). An exogenously determined net savings rate s (which may also vary over time) determines the proportion of total *NI* that is set aside from current income (across the economy) for savings S such that:

$$S = sNI \tag{10}$$

Since we assume (for the moment) that all investment is undertaken by the private sector and that the government sector maintains a balanced budget, ie that:

$$G = T \tag{11}$$

where G is government spending and T is the tax revenue net of subsidies, this means that total (net) savings is equal to total (net) investment: [4]

$$I_{net} = sNI \qquad (12)$$

Gross investment I_{gross} is given by:

$$I_{gross} = I_{net} + D \qquad (13)$$

where D is the depreciation defined by:

$$D = \delta K_{-1} \qquad (14)$$

and K_{-1} is the lag of the capital stock K. The change in the capital stock over time is then given by:

$$K = K_{-1}.(1 - \delta) + I_{gross} \qquad (15)$$

Or equivalently, and more simply, by:

$$K = K_{-1} + I_{net} \qquad (16)$$

To close the model it remains to calculate either the return to labour (W) or the return to capital (P) or the split of income between these two factors of production. In SIGMA, we choose to determine the return to capital, r, as this is also a critical factor in Piketty's analysis, and is central to our exploration of the impacts of low growth rates on inequality.

Along with Piketty (2014a: 213-214) we assume (for now) that the return to capital is given by the marginal productivity of capital, which we denote by r_K. This assumption only works under perfect market conditions in which there are no structural features which might lead either capital or labour to extort more than their 'fair' share of the output from production. In a sense, this assumption is a conservative one for us, to the extent that conclusions about inequality are stronger in imperfect market dynamics.

Under conditions of duress, in which the owners of capital receive a rate of return r greater than the marginal productivity of capital r_K, our conclusions about any inequality which results from declining growth rates will be reinforced. Conversely, of course, we must beware of making too strong assumptions about the potential to mitigate inequality, in any situation in which the owners of capital have greater bargaining power than wage labour.

Accordingly, the next step in the model is to determine the marginal productivity of the capital stock. In this simple model, we achieve this (in a relatively conventional way) through the partial differentiation of a constant elasticity of substitution (CES) production function Y of the form:

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[4]   This follows from the well-known accounting identity: $S - I = G - T + X - M$, where X is exports and M is imports. In a closed economy $S - I = G - T$. And if the public sector is balanced, we have $S = I$.

$$Y = Y(K, L, \sigma) \tag{17}$$

where σ is the elasticity of substitution between labour and capital. Using the standard derivation developed first by Arrow et al (1961), *Y* is given by a function of the form:

$$Y(K,L,\sigma) = (aK^{\frac{(\sigma-1)}{\sigma}} + (1-a)(AL)^{\frac{(\sigma-1)}{\sigma}})^{\frac{\sigma}{(\sigma-1)}} \tag{18}$$

where *a* (as described by Arrow et al (op cit)) is a 'distribution parameter' and *A* is the coefficient of technology-augmented labour, which we will assume changes over time according to the rate of growth of labour productivity in the economy.[5]

To determine the marginal productivity of capital, we differentiate *Y* with respect to *K*, ie:

$$r_K = \frac{\partial Y}{\partial K} \tag{19}$$

To achieve this, we proceed by first factorising equation (15) as:

$$\frac{\partial Y}{\partial K} = \frac{\partial Y}{\partial Y'}.\frac{\partial Y'}{\partial K} \tag{20}$$

Where *Y'* is given by:

$$Y' \equiv aK^{\frac{(\sigma-1)}{\sigma}} + (1-a)(AL)^{\frac{(\sigma-1)}{\sigma}} \tag{21}$$

Then it follows that:

$$\frac{\partial Y'}{\partial K} = \frac{(\sigma-1)}{\sigma}aK^{(\frac{-1}{\sigma})} \tag{22}$$

And using equation (18) that:

$$\frac{\partial Y}{\partial Y'} = \frac{\sigma}{(\sigma-1)}Y'^{\frac{1}{\sigma-1}} \tag{23}$$

Using equation (18) again to substitute for *Y'* in equation (23), we find that:

$$\frac{\partial Y}{\partial Y'} \equiv \frac{\sigma}{(\sigma-1)}Y^{\frac{1}{\sigma}} \tag{24}$$

Hence we deduce that:

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[5] It can be shown that, for the special case *σ = 1*, this CES function reduces to the familiar Cobb-Douglas production function $Y = K^a.(AL)^{1-a}$. But the introduction of an explicit elasticity variable allows for a more flexible exploration of the production relationship under a variety of different assumptions about the elasticity of substitution between labour and capital.

$$\frac{\partial F}{\partial K} = \frac{\sigma}{(\sigma-1)} \cdot \frac{(\sigma-1)}{\sigma} \, a\left(\frac{K}{Y}\right)^{\frac{-1}{\sigma}} \tag{25}$$

Or equivalently that:

$$r_K = a\beta^{\frac{-1}{\sigma}} \tag{26}$$

where β (as before) is the capital to income ratio.[6] This relationship can now be used to derive profits (returns to capital) *P* through:

$$P = \alpha NI = r_K K = a\beta^{\frac{-1}{\sigma}} K \tag{27}$$

It follows that capital's share of income is given by:

$$\alpha = a\beta^{\frac{\sigma-1}{\sigma}} \tag{28}$$

Equation (28) makes it clear, as Piketty also points out (2014b: 37-39), that for *σ>1*, (and assuming that the capital to income ratio is greater than one) capital's share of income is an increasing function of the capital to income ratio. As the capital to income ratio rises, capital's share of income increases. Conversely however, when *0 < σ < 1*, then capital's share of income is a decreasing function of the capital to income ratio. As the share of capital to income rises, capital's share of income decreases. At *σ = 1*, the decline in the rate of return to capital exactly offsets the rise in the capital to income ratio, and the share of income to capital remains constant.

These considerations all underline the potential variation in economic and social structure that can arise in different kinds of societies. Actual distributions of income and wealth, even under conditions of low growth, will depend upon institutional architectures, technological possibilities and the structure of markets for capital and labour. We return to these issues in the discussion.

The production function in equation (14) can also be used in our model to derive the labour requirements in the SIGMA economy, since:

$$Y(K,L,\sigma)^{\frac{\sigma-1}{\sigma}} - aK^{\frac{\sigma-1}{\sigma}} = (1-a)(AL)^{\frac{\sigma-1}{\sigma}} \tag{29}$$

Re-arranging terms we find that:

$$\frac{1}{1-a} \cdot \left(Y^{\frac{\sigma-1}{\sigma}} - aK^{\frac{\sigma-1}{\sigma}}\right) = (AL)^{\frac{\sigma-1}{\sigma}} \tag{30}$$

And hence that:

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[6]   Note that as *σ→1*, this relationship returns to the 'first law' of capitalism (equation 1) with *a = α*. In other words, under an assumption of unit elasticity of substitution between capital and labour (as in the Cobb Douglas function, the constant *a* is given by the share of income to capital *α*.

$$L = \frac{1}{A_t}\left(\frac{1}{1-a} \cdot \left(Y^{\frac{\sigma-1}{\sigma}} - aK^{\frac{\sigma-1}{\sigma}}\right)\right)^{\frac{\sigma}{\sigma-1}} \qquad (31)$$

Since the pressure on unemployment is another of the threats from slower or zero growth, equation (31) will turn out to be a useful addition to the SIGMA model. In fact, as we shall see, it leads to some useful insights into maintaining full employment under conditions of declining growth, over and above the usual prescriptions of fewer working hours and lower labour productivity (Jackson and Victor 2011 eg).

So far, SIGMA has been articulated purely in terms of the division between capital and labour, and the relative shares that each of these achieves under different conditions. It is clear, however, that this does not in itself lead to inequality. The extent of equality or inequality in the economy depends on the actual distribution of capital and wages within the economy. If both capital and wages are equally distributed, a simple transfer of income from wages to capital has no impact at all on inequality.

SIGMA therefore divides the household sector into two groups of people, who for sentimental reasons, we will refer to (self-evidently a little tongue in cheek) as 'workers' and 'capitalists'. In the reference scenarios, we shall make no distinction between these two sets of people, who both own some capital and both earn some wages. However, as we explore the implications of changing economic structure, we will allow one set of people (the 'capitalists') to own more capital and perhaps even to command a higher share of wages than the other set of people (the 'workers').

We will also allow that 'capitalists' and 'workers' can have different propensities to consume out of their incomes. In fact, we shall see that this simple assumption (of differential savings rates) immediately introduces economic inequality, *even if the initial distribution of capital and wages is perfectly equal*. This is a fascinating insight into the structural dynamics through which capitalism has an in-built function for the divergence of incomes.

In order to reflect the levels of inequality in different scenarios, we introduce a simple index of income inequality q_y defined by:

$$q_Y = \left(\frac{Yd_{cap}}{Yd_{work}} - 1\right) * 100 \qquad (32)$$

where $Yd_{cap/work}$ represents the disposable incomes of (respectively) capitalists and workers. The index takes a value of 0 when the incomes of capitalists and workers are identical, ie there is no inequality at all, and a value of 100 when the income of capitalists is 100% higher (say) than that of workers. It can of course be considerably higher than 100 and we shall see this in some of the scenarios described in the following section.

Figure 1 illustrates the high-level structure of the SIGMA model, emphasising the dynamic relationships between income, savings, investment, capital and labour. The model itself is built using the system dynamics software STELLA. This kind of software provides a useful platform for exploring economic systems for several reasons, not the least of which is the ease of undertaking collaborative, interactive work in a visual (iconographic) environment. Further advantages are the transparency with which one can model fully dynamic

relationships and the ability to mirror stock-flow consistency within the economy (Godley and Lavoie 2007, Jackson and Victor 2014).[7]
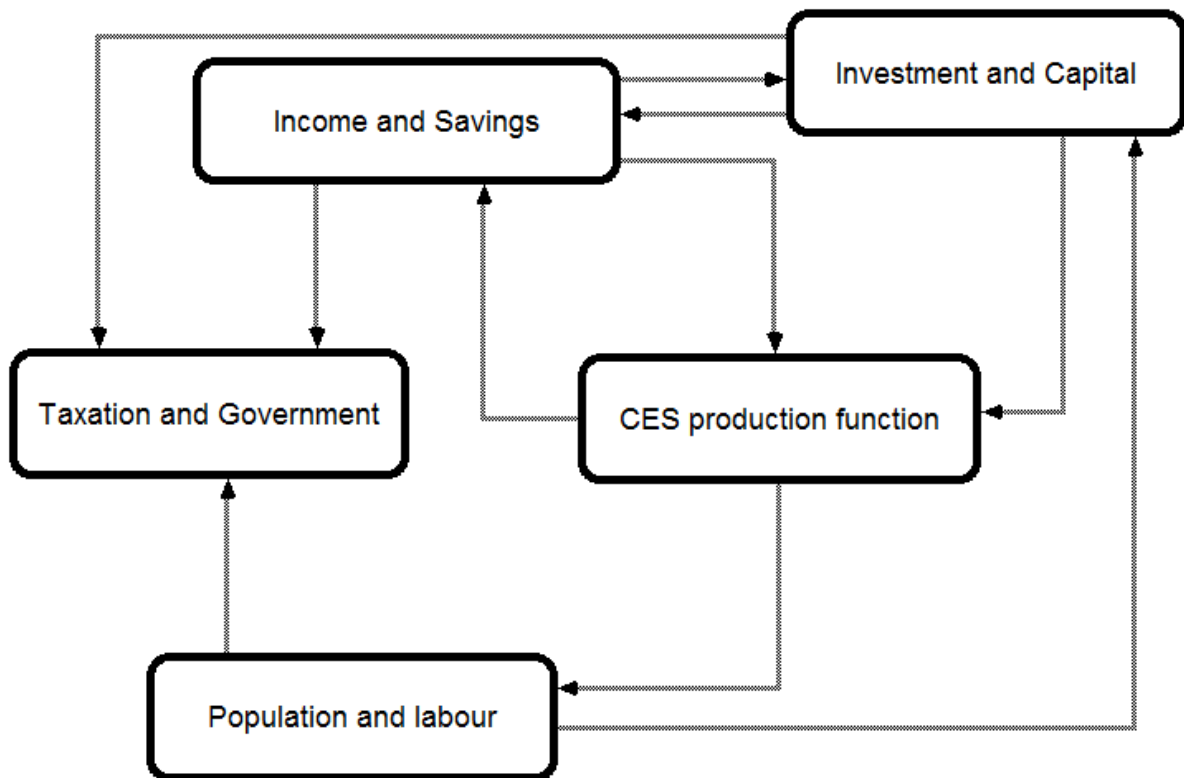
Figure 1: High-Level Structure of SIGMA model

Our principal aim in this paper is conceptual. We want to unravel the dynamics which threaten to lead to inequality under conditions of slow growth or de-growth. SIGMA is therefore not inherently data-driven. Rather it aims to model the system dynamics that connect savings, growth, investment, returns to capital and inequality. It is nonetheless useful to ground the initial values of our variables in numbers which are reasonable or typical within modern capitalist economies.

Of particular importance are reasonable choices for the initial values of the capital to income ratio, the savings rate and the share of income to capital. For the purposes of this exercise we have therefore chosen representative values (Table 1) for the SIGMA variables. Many of these are informed by our own on-going empirical work (Jackson and Victor 2011, Jackson et al 2014, Jackson and Victor 2014) with two developed economies, the UK and Canada. Some are chosen in order to ensure internal consistency within the model.


| Variable	Values	Units	Remarks
Initial GDP	1,800	$billion	UK GDP is currently around £1.6 trillion; Canada GDP is around $1.9 trillion.
Initial national Income	1,500	$billion	UK and Canadian NI are both around 17% lower than the GDP.
Initial capital stock K	6,000	$billion	Based on the estimate of capital to income ratio chosen below.
Initial capital to income ratio β	4		Capital to income ratio in Canada is a little under 3; in UK it is higher at around 5. We adopt here a value between these.
Initial income share of capital α	40%	%	The wage share of income as a proportion of NI is around 60% in both Canada and the UK and capital.
Initial net savings rate s as percentage of National Income	10%	%	The ratio of net private investment to national income in Canada was around 8% in 2012. The equivalent ratio in the UK was considerably lower, following the recession. Choosing s=8% would make the capital share of income exactly equal to rs/g for a 2% growth rate. We adopt a higher savings rate, in order better to illustrate convergence.
Elasticity of substitution σ between labour and capital	varies		In theory σ can vary between 0 and infinity. In practice values below a certain level are excluded by the existing structure of the economy – for instance when they imply a capital share of income greater than 1.
Population	50	million	The population of Canada is 34 million; that of the UK just over 60 million. This level of population gives values for per capita income consistent with advanced capitalist economies.
Workforce as % of population	50%	%	Workforces in developed nations are typically between 45% and 55% of the population.
Unemployment rate	7%	%	Typical of both Canada and the UK over the last few years.
Distribution parameter a	varies		This value is calibrated for each σ according to equation (17) at time t = 0.
Initial technology augmentation coefficient A_0	varies		This value is calibrated for each σ (and a) using the production function at time t = 0.
Initial growth rate g in reference scenario	2%	%	Growth rates (of GDP) in both the UK and Canada were slower than this in the aftermath of the financial crisis and in the UK currently a little higher. There are arguments that long-run real growth has been lower for some time. But 2% is still often used as a central policy assumption.
Initial growth in labour productivity in reference scenario	2%	%	For the reference scenario, this value is chosen to be consistent with a 2% rate of growth in the NI and the maintenance of a constant employment rate when σ = 1. The value can be varied exogenously to maintain employment with declining growth rates.
Initial tax rates	20%	%	In the reference scenario, typical economy wide net taxation rates (as a percentage of the NI) are applied to the incomes of both capitalists and workers.

Table 1: Initial Values for the SIGMA Model

Results

In the first instance, it is useful to illustrate the extent to which Piketty's 'laws of capitalism' hold true. Figures 2a and 2b illustrate respectively the convergence of β to *s/g* and the convergence of *α* to *rs/g* (when *s* and *g* are held constant) for the values chosen in our reference scenario. With these values, it is clear both that convergence occurs and also that this convergence takes some time (around a century in this case).
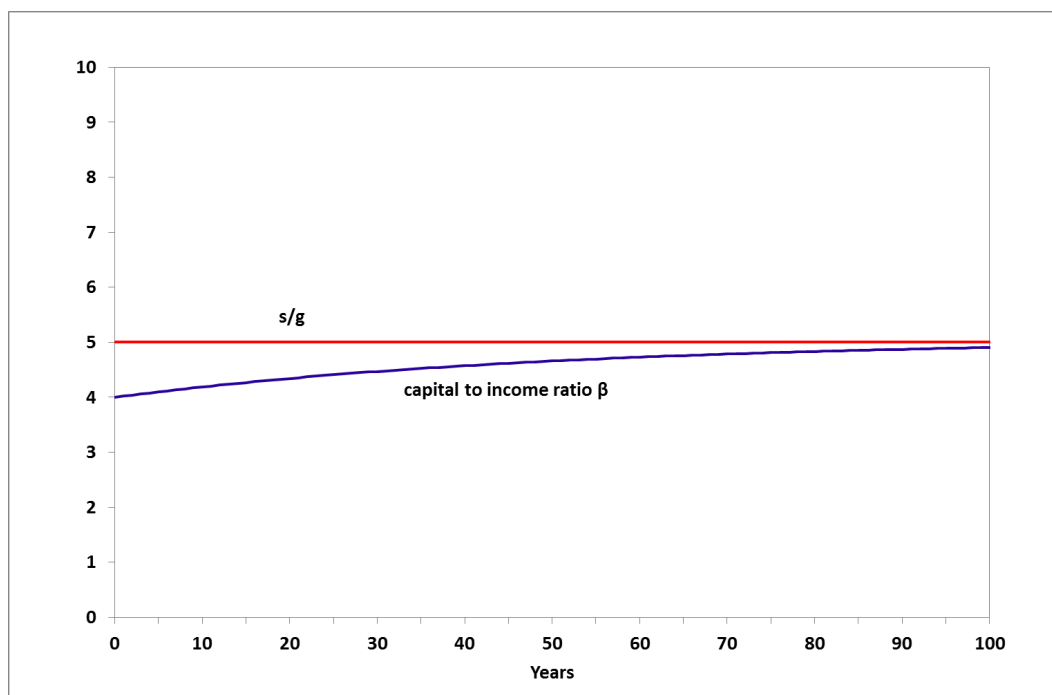
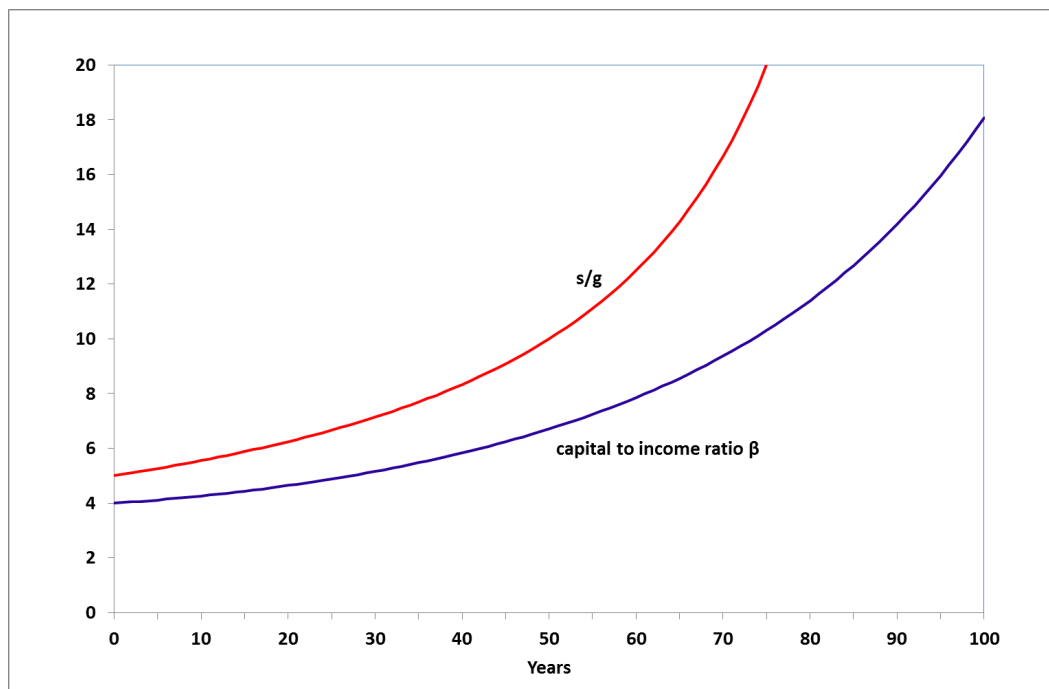*Figure 2a: Long-term convergence of the capital to income ratio with s and g held constant*
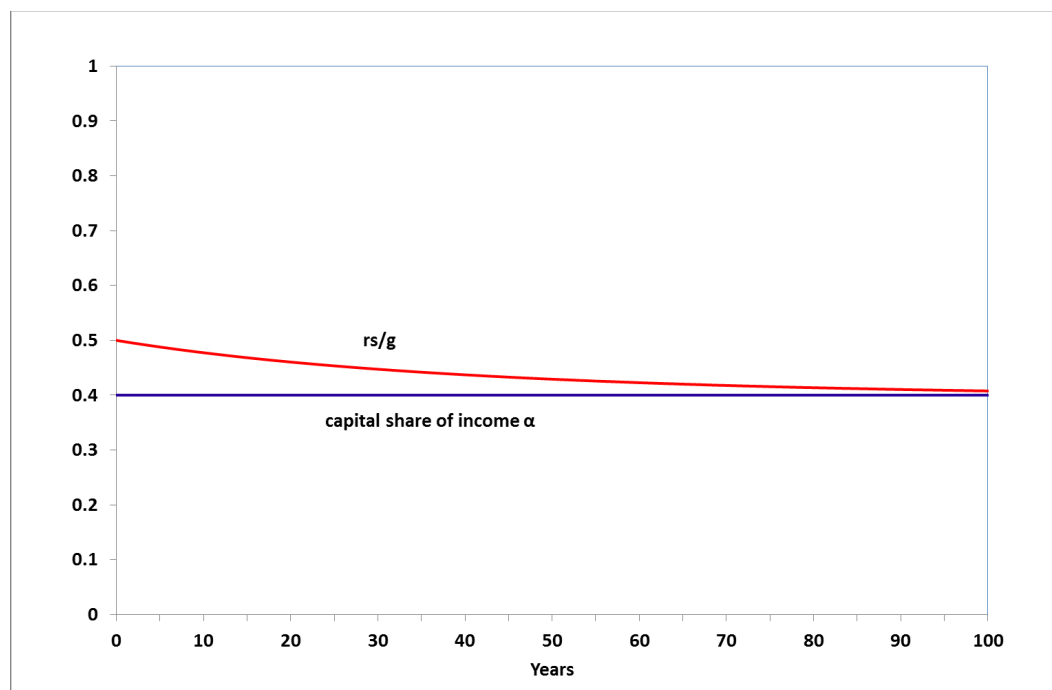
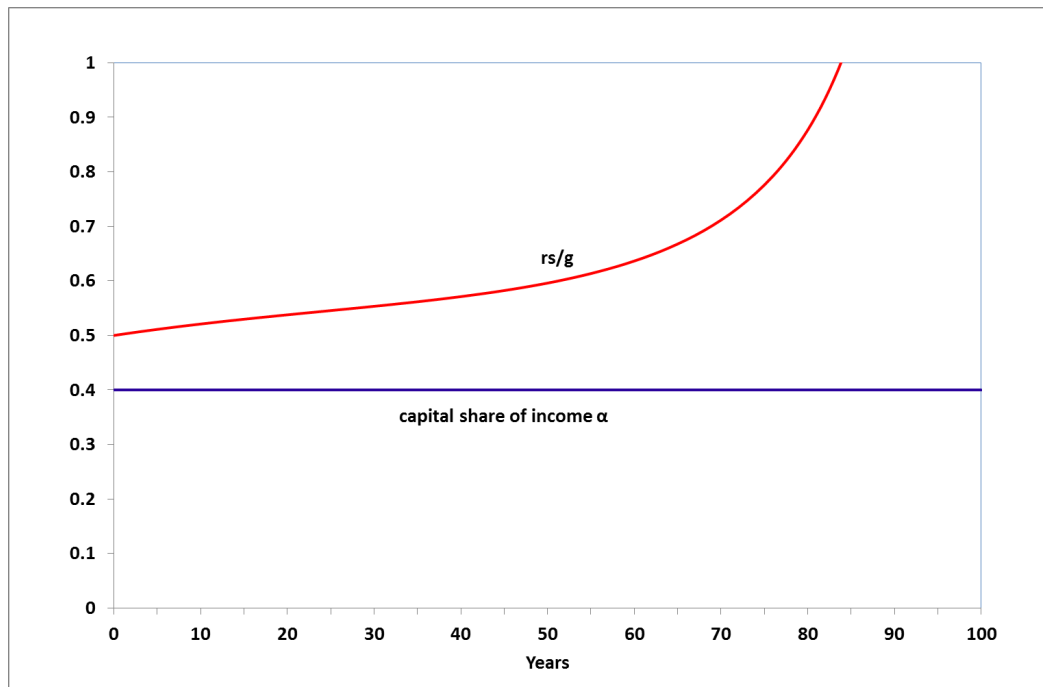*Figure 2b: Long-term convergence of incomes share of capital with s and g held constant*

It is worth remarking that if we had chosen values of s, g and α closer to the 'equilibrium' value, this convergence would have been faster. Indeed with a savings rate of 8% (close to empirical values) rather than 10%, the ratio of s to g would be 4 (identical to the capital to income ratio) and convergence would have been trivial.

It is also interesting to note here that the capital to income ratio β clearly converges *towards* the ratio s/g (Figure 2a). But Figure 2b seems to suggest that, rather than α converging towards the ratio rs/g, the ratio rs/g converges towards α. This is because of a particular feature of our initial values, the choice $\sigma = 1$. In these circumstances (see equations (26)-(28) above) the rate of return on capital (calculated as the marginal productivity of capital) moves in such a way as to exactly offset the increase in the capital to income ratio and keep capital's share of income constant. Interestingly, this remains the case whatever happens to the growth rate.

So for instance, in Figure 3, we allow the growth rate g to decline to zero. The ratio s/g therefore goes to infinity over the course of the run. As expected, the capital to income ratio β rises substantially (Figure 3a), reaching around 18 by the end of the run. It is comforting to note, however, that it does not explode uncontrollably, in spite of Piketty's second law. Even more striking is that capital's share of income α once again remains constant (Figure 3b), because the rate of return r falls exactly fast enough to offset the rise in the capital to income ratio.

Figure 3a: Long-term behaviour of the capital to income ratio as g goes to zero (σ=1)

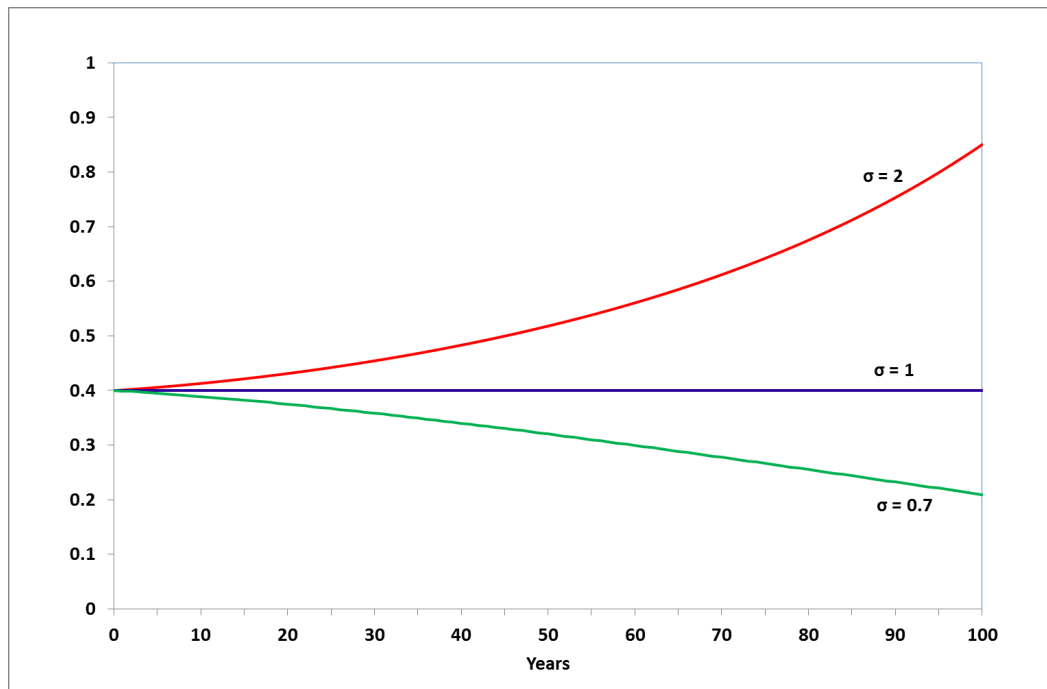*Figure 3b: Long-term behaviour of capital's share of income as g goes to zero (σ=1)*

Notice that this lack of convergence of $α$ towards rs/g is not a refutation of Piketty's law, since g is not held constant over the run. This result does go some way, however, to mitigate fears of an explosive increase in inequality as growth rates decline. Indeed, as Figure 3b makes clear, if the elasticity of substitution $σ$ is exactly one, then the decline of the growth rate to zero has no impact at all on capital's share of income.[8]

The stability of capital's share of income only holds, however, when the elasticity of substitution between labour and capital is exactly equal to one. Figure 4 illustrates the outcome of the same scenario ($g→0$) on capital's share of income for three different values of $σ$: 0.7, 1 and 2. As predicted, when the elasticity of substitution $σ$ rises above one, capital's share of income increases. Indeed, when $σ$ equals 2, capital's share reaches 85% of the total income. Piketty notes (2014b: 39) that the (less dramatic) increases in capital's share of income visible in the data over the last decades are consistent with an elasticity in the region of 1.3 to 1.6.

Conversely, however, with an elasticity of substitution less than 1, capital's share of income declines over the period of the run, in spite of the fact that both s/g and rs/g go to infinity. This is an important finding from the point of view of our aim in this paper. To re-iterate, there is no necessarily inverse relationship between the decline in growth and the share of income to capital. Rather, the impact of declining growth on capital's share of income depends crucially on technological structure and institutions. Specifically, with an elasticity of substitution between labour and capital less than one, declining growth can perfectly well be associated with an increase in the share of income going to labour.

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[8] This result (the constancy of capital's share of income) holds irrespective of the assumed behaviour of the savings rate s. Note however that there is a wide range of possible variations on the capital to income ratio, when the savings rate is allowed to vary. For instance, if the savings rate goes to zero along with the growth rate, then the ratio s/g is constant over the run. The capital to income ratio still rises (to around 11 by the end of the run) but as before capital's share of income remains constant.

*Figure 4: Long-term behaviour of capital's share of income as σ varies (g→0)*

This theoretical result is not particularly insightful without an adequate account of the relationship between capital's share of income and the distribution of capital. Under the conditions of our reference case, both income and wealth are equally distributed between workers and capitalists. For all of the scenarios so far elucidated, the inequality index therefore remains unchanged – and equal to zero. There is no inequality in such a society, *whatever happens to the share of income going to capital*.

Clearly of course, this is not very realistic as a depiction of capitalist society. One of the things we know for sure, not least from Piketty's work, is that the distribution of both capital and wages is already skewed, sometimes quite excessively. One element in that dynamic is the savings rate itself. It is well-documented that the propensity to save is higher in high income groups than in low income groups. Kalecki (1939) even proposed that the propensity to save amongst workers was zero. For the lowest income groups in the UK, the data support this view.

For illustrative purposes, we suppose next that – for whatever reason – the initial savings rate amongst workers falls to 5%, with the savings rate of capitalists rising to 15% to ensure that the overall savings rate remains at 10%. Figure 5 shows that this apparently trivial innovation has the immediate effect of introducing inequality, without any decline in the growth rate and with an entirely equal initial distribution of ownership. In Figure 5a, incomes amongst capitalists are up to 70% higher than those amongst workers by the end of the period.
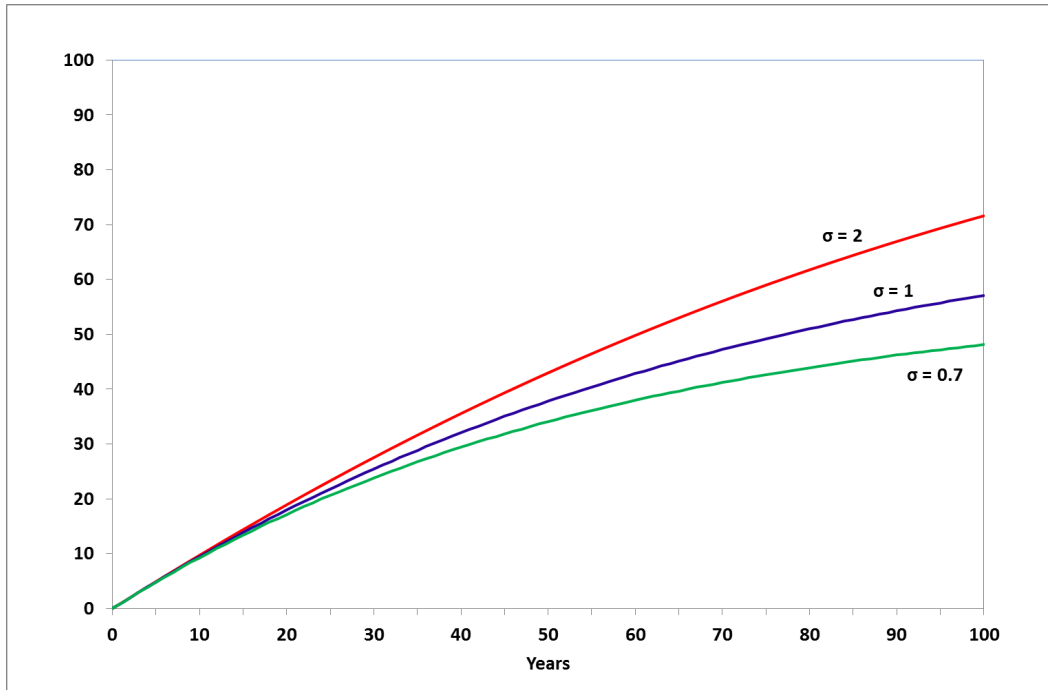
***Figure 5a: Inequality in incomes under differential savings rate (g=2%)***
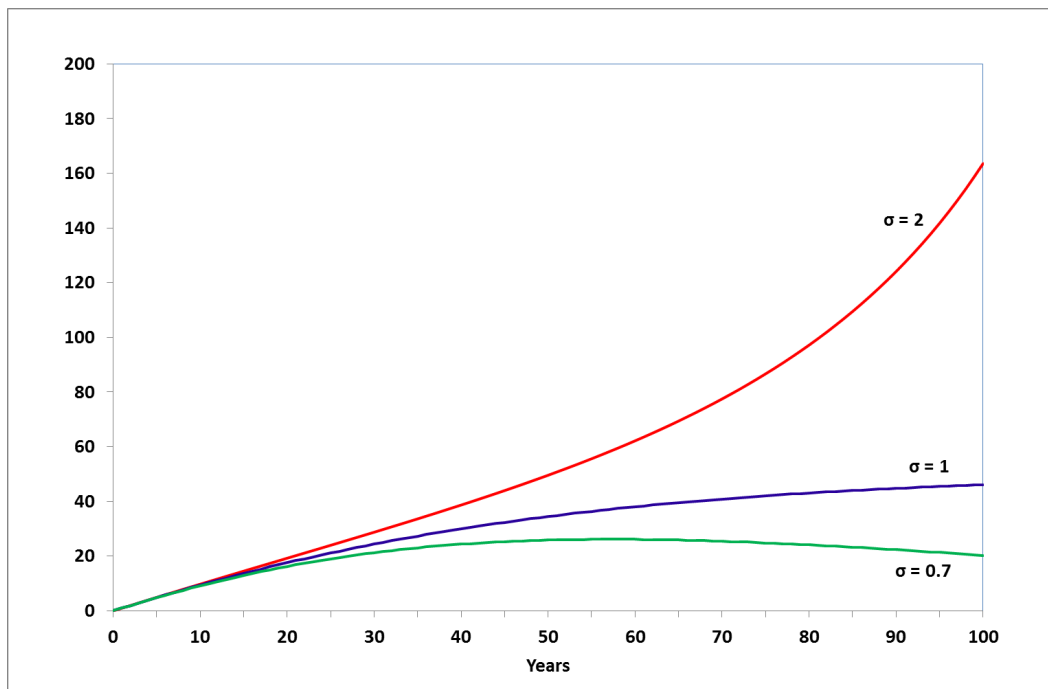
Figure 5b: Inequality in incomes under differential savings rate (g → 0)

Under conditions of slowing growth (Figure 5b), an interesting phenomenon emerges. For high σ (ie high substitutability of capital and labour), the inequality between capitalists and workers is exacerbated. When σ = 2, capitalist incomes are over 160% higher than worker incomes by the end of the scenario. By contrast, this situation is significantly ameliorated for low σ. Capitalist incomes are barely 20% above worker incomes at the end of the run when σ is equal to 0.7.

The increases in inequality shown in Figures 5a and 5b are stimulated simply by changing the savings rate, assuming a completely equal distribution of income and capital at the outset. Figure 6 illustrates the outcome, once we incorporate inequality in the initial distribution of assets. For the purposes of this illustration, we assume that capitalists comprise only 20% of the population but own 80% of the wealth – a proportion not massively unrealistic from the perspective of today's global distribution.

For the scenarios in Figure 6, we also assume (rather conservatively) that the distribution of wages remains equal between the two groups, despite the skewed distribution in asset ownership: capitalists earn 20% of the wages and workers earn 80%. Capitalist incomes are nonetheless immediately higher than workers because of their additional income from returns to capital. In fact, as Figure 5 shows, capitalist incomes start the scenario around 200% higher than worker incomes.

What happens subsequently depends crucially on the value of σ. With high σ, inequality rises even further as capitalists seek to protect returns to capital by substituting away from expensive labour. Capitalist incomes are almost 750% higher than worker incomes by the end of the run. With low values of σ, it is again possible to reverse the initial inequality, bringing the income differential down by a factor of almost ten to just over 75%.
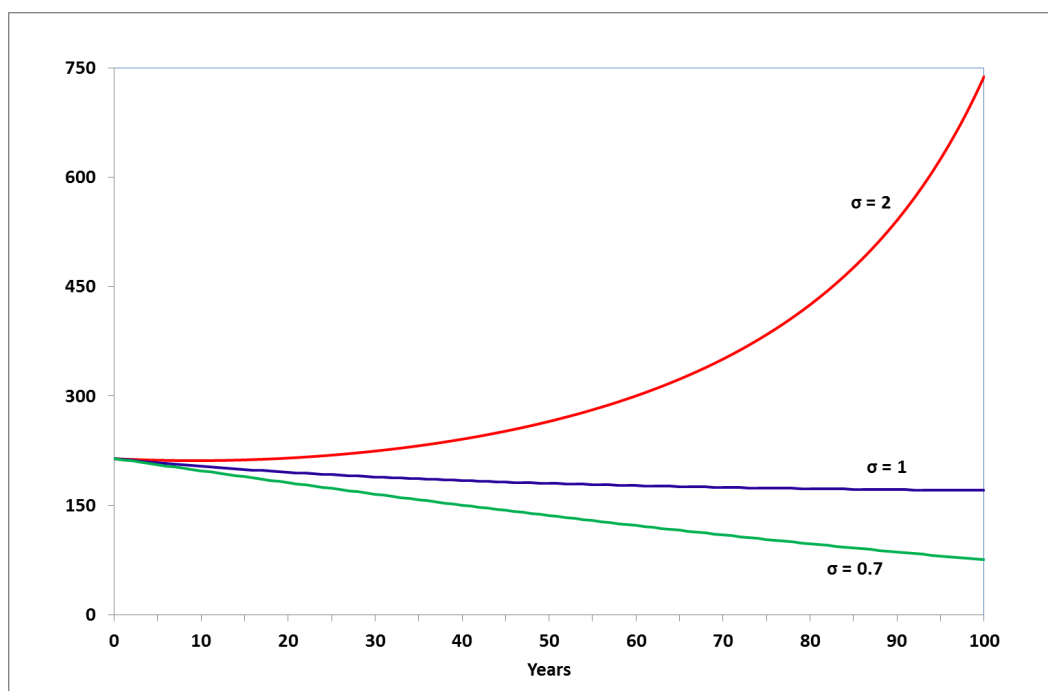

Figure 6: Inequality with skewed initial ownership of assets and differential savings

Finally, we explore the possibilities of addressing rising inequality through progressive taxation. It is clear immediately that this task will be much easier when the underlying structural inequality rises less steeply than when it escalates according to the $\sigma = 2$ scenario in Figure 6.

In fact, as Figure 7a illustrates, a modest tax differential (a tax band of 30% applied to earnings higher than the income of workers) and a minimal wealth tax (of only 1% in this

example) when taken together could equalise incomes relatively easily when $\sigma = 0.7$ but barely make a dent in the underlying inequality when $\sigma = 2$.

Figure 7b shows the per capita disposable incomes of the two segments for the low elasticity case. It is notable that towards the end of the run, this represents a de-growth scenario with a convergence of incomes, exactly counter to the fear of rampant inequality from declining growth rates which motivated this study.
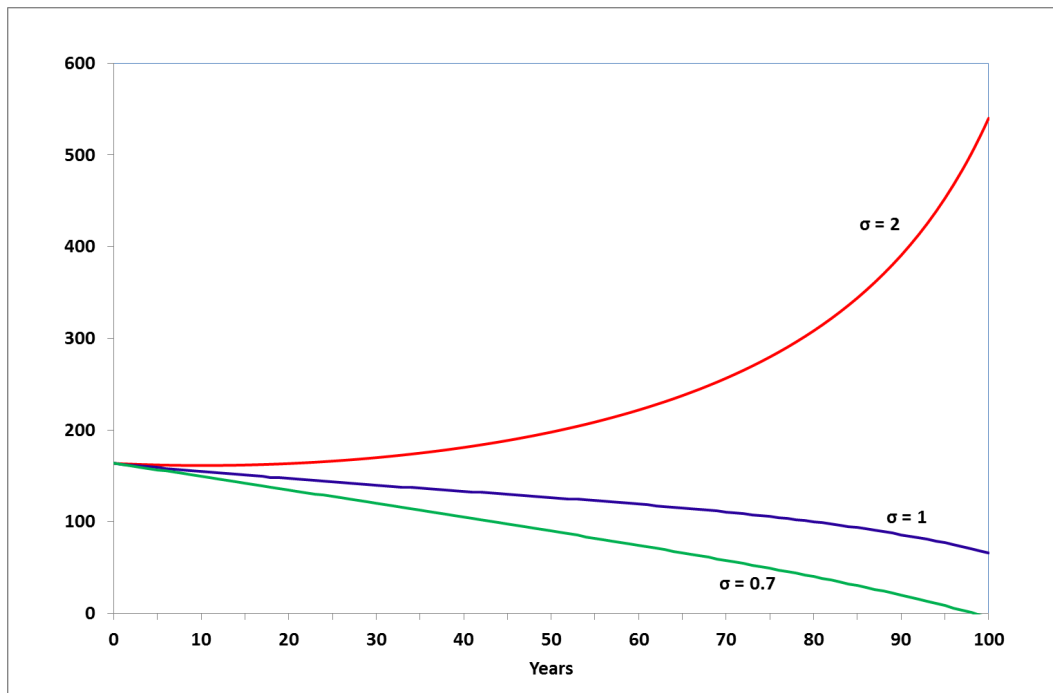
Figure 7a: Inequality reduction through differential income tax and a 1% capital tax
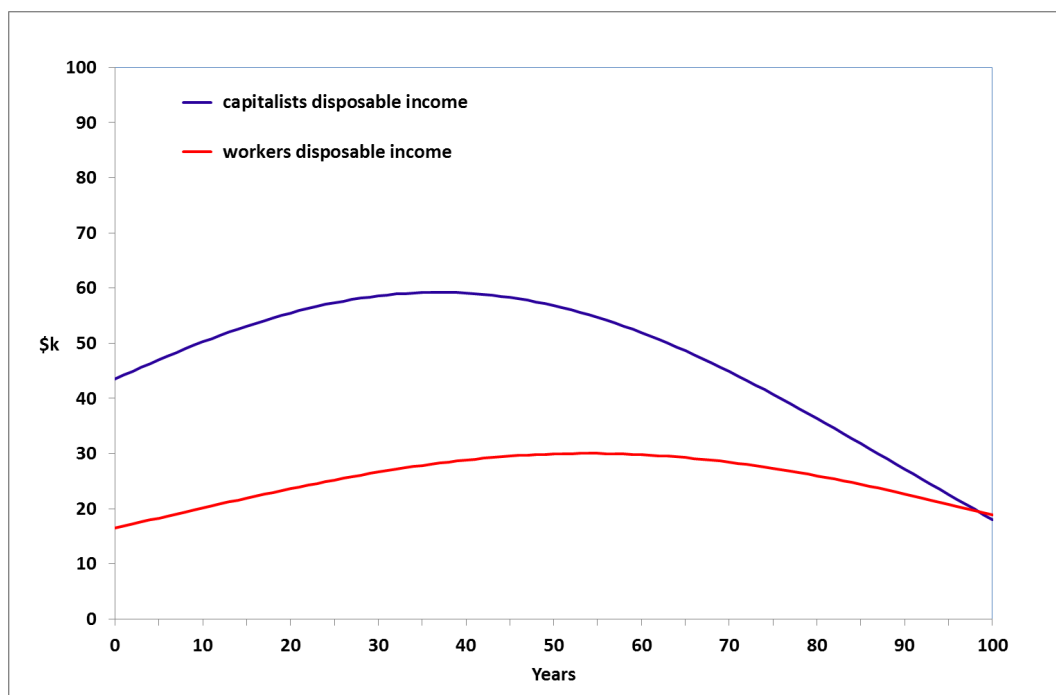
Figure 7b: Convergence of incomes under progressive tax policy (g→0; σ = 0.7)

Discussion

In his bestselling book, *Capital in the 21st Century*, French economist Thomas Piketty has proposed a simple and potentially worrying thesis. Declining growth rates, he suggests, give rise to worsening inequalities.

This thesis is particularly challenging for ecological economists. Motivated by a combination of social and ecological concerns, many ecological economists tend to be critical of growth-based economics. Some have argued forcefully for a profound shift in economic policy away from the pursuit of the GDP as an indicator of progress, and towards a different kind of macroeconomics. Our own prior work exemplifies this argument (Victor 2008, Jackson 2009). For us, the suggestion that declining growth rates precipitate inequality is a challenge that has to be taken extremely seriously.

What we have shown in this working paper is that under certain conditions it is indeed possible for income inequality to rise as growth rates decline. However, we have also established that there is *absolutely no inevitability at all* that a declining growth rate leads to explosive (or even increasing) levels of inequality. Even under a highly-skewed initial distribution of ownership of productive assets, it is entirely possible to envisage scenarios in which incomes converge over the longer-term, with relatively modest intervention from progressive taxation policies.

The most critical factor in this dynamic is the level of substitutability between labour and capital. Higher levels of substitutability ($\sigma > 1$) do indeed exhibit the kind of rapid increases in inequality predicted by Piketty, as growth rates decline. In an economy with a lower elasticity of substitution ($0 < \sigma < 1$), the dangers are much less acute. More rigid capital-labour divisions appear to reinforce our ability to reduce societal inequality.

From a conventional economic viewpoint, this might appear to be cold comfort. Lower values of σ are often equated with lower levels of development. As Piketty points out (2014a: 222), low levels of elasticity characterised traditional agricultural societies. Other authors have suggested that the direction of modern development, in general, is associated with rising elasticities between labour and capital (Karagiannis et al 2005, eg). The suggestion seems to be that progress comprises more of the same.

It is however an open question whether this is necessarily the case. The contention that progress moves inevitably in the direction of higher σ embodies numerous ideological assumptions. In particular it seems to be consistent with a particular form of capitalism that has characterised the post-war period: a form of capitalism that has come under increasing scrutiny for its potent failures, not the least of which is the extent to which it has presided over continuing inequality.

The possibility of re-examining this assumption resonates strongly with other suggestions for a more sustainable economic model. In our own work, for example, we have highlighted the importance of labour-intensive services both in reducing material burdens across society and also in creating employment in the face of declining growth (Jackson 2009; Jackson and Victor 2013). The challenge of maintaining full employment under declining growth is particularly profound. In fact, under the scenarios developed in the previous section, even if

labour productivity declines alongside growth, the demand for labour in the economy falls to less than half of its initial value (Figure 8: scenario 4).
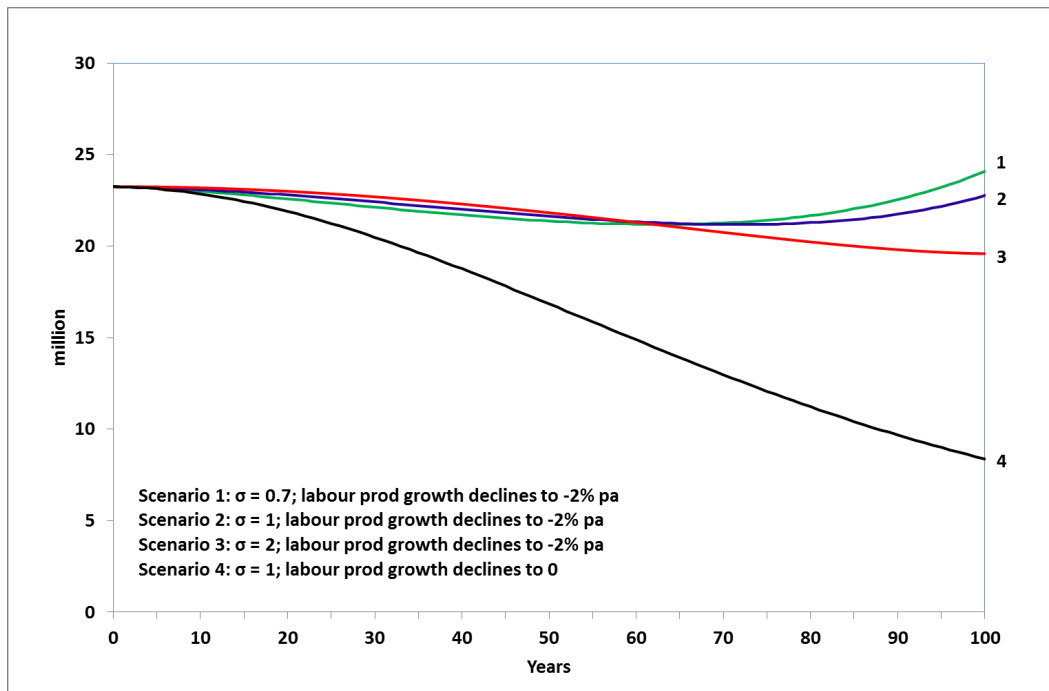
Figure 8: The demand for labour as growth declines

We have argued elsewhere (Jackson 2012, Jackson and Victor 2011) that challenging assumptions about labour productivity – or to put it another way – protecting both the quality and the quantity of labour needed in the economy against the incursions of capital, constitutes an important avenue of opportunity for structural change in pursuit of sustainability. Instead of a relentless pursuit of ever-increasing labour productivity, economic policy would aim to protect employment as a priority and recognise that the time spent in labour is a vital component of the value of the activity.

The suggestion here is that there are employment opportunities to be had by protecting the quality and intensity of people's time in the workplace. This suggestion is not a million miles from Minsky's (1986) proposal that government should act as 'employer of last resort' in stabilising an unstable economy.

The three upper lines in Figure 8 all relate to a scenario in which by the end of the run, labour productivity growth is (exogenously) determined as negative. Of particular interest in relation to these three upper scenarios is the influence of the elasticity of substitution on the achievable levels of employment. The most successful of the upper scenarios (scenario 1 on the graph) corresponds to a low-elasticity society ($\sigma = 0.7$); the least successful (scenario 3) to the high-elasticity society ($\sigma = 2$). In fact, the demand for labour actually increases in scenario 1, rising close to 100% of the workforce at the end of the run.

Up to this point, our analysis of the elasticity of substitution has been a broadly descriptive one. We have explored the influence of the elasticity of substitution between labour and capital on the evolution of inequality (and employment) in an economy in which the growth rate declines over time. It would be wrong to conclude from this that we are able to alter

this elasticity at will. Most conventional analyses assume that values of σ are given – an inherent property of a particular economy or state of development. Such analyses usually confine themselves to showing how allowing for a range of elasticity facilitates a better econometric description of a particular economy than assuming an elasticity of 1. Our own analysis here also assumes that the elasticities themselves are fixed features of the economy over time. The production function in equation (18) is predicated precisely on this assumption.

There is however a tantalising suggestion inherent in this analysis that changing the elasticity of substitution between labour and capital offers another potential avenue towards a more sustainable macro-economy, and in particular a way of mitigating the pernicious impacts of inequality and unemployment in a low growth economy. Exploring that suggestion fully is beyond the scope of this paper, but is certainly worth flagging here. It would require us first to move beyond the CES production function formulation adopted here.

The appropriate functional form for such an exercise would be a Variable Elasticity of Substitution (VES) production function. We note here that there is substantial justification and considerable precedent for such a function (Sato and Hoffman 1968, Revankar 1971). Anthony (2009) suggests that VES functions offer better descriptions of real economies than either CES or Cobb-Douglas functions. Adopting such a function would allow us to explore scenarios in which σ changes over time. This possibility is the subject of ongoing research.

We should recall here our assumption that the rate of return to capital is equal to the marginal productivity of capital. As we remarked earlier, this assumption only holds in perfect markets where capital is unable to use its power to command a higher share of income. Clearly, in some of the scenarios we have envisaged, this assumption may no longer hold. Where political power accumulates alongside the accumulation of capital, the danger of rising inequality is particularly severe and is no longer offset simply by changes in the economic structure. This question also warrants further analysis.

Finally, it is clear that a proper treatment of the distribution of incomes and wealth would need to take more account of the distribution of financial assets and liabilities than we have done here. This paper has adopted a broadly simplified view of the economy, in which wealth is accumulated through ownership of fixed capital assets, ie of non-financial assets. One of the distinctive features of modern capitalism is the rising importance of financial assets and liabilities in the distribution of wealth. In a closed economy, these assets and liabilities all sum to zero. But huge potentials for inequality arise from the asymmetry between the owners of assets and the holders of liabilities. Our own on-going work to develop a stock-flow consistent ecological macroeconomics (Jackson and Victor 2014) aims to shed light on this wider issue.

In summary, this paper has explored the relationship between growth, savings and income inequality, under a variety of assumptions about the nature and structure of the economy. Our principal finding is that rising inequality is by no means inevitable, even in the context of declining growth rates.

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