



**Institute for  
New Economic  
Thinking**

## **Session 2, Lunch – Frank Kelly**

Presented at the Inaugural Conference @ King's College,  
April 8-11, 2010

# Networks and systemic risk

INET – Cambridge

9 April 2010

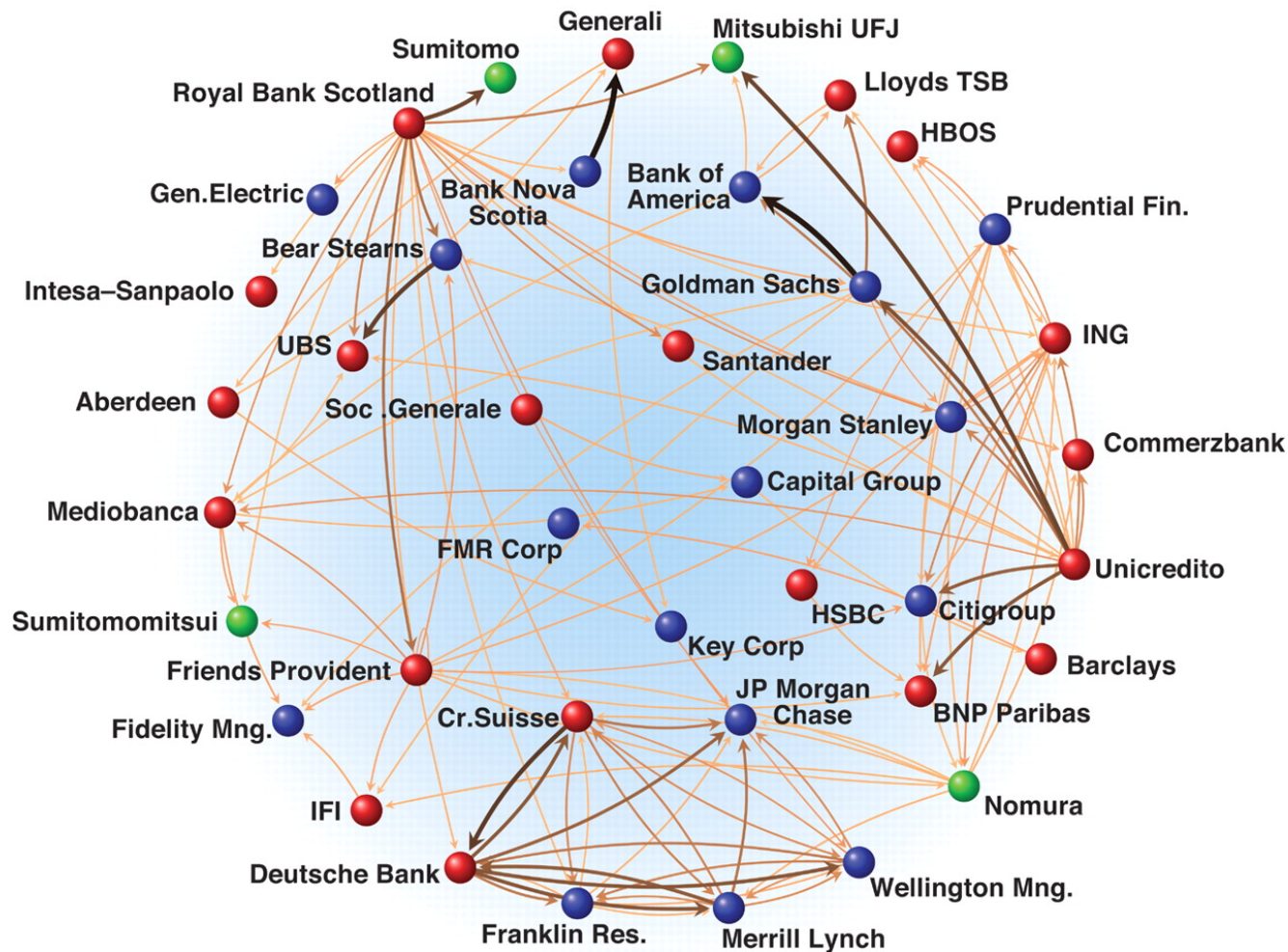
Frank Kelly

Statistical Laboratory, University of Cambridge

# Outline

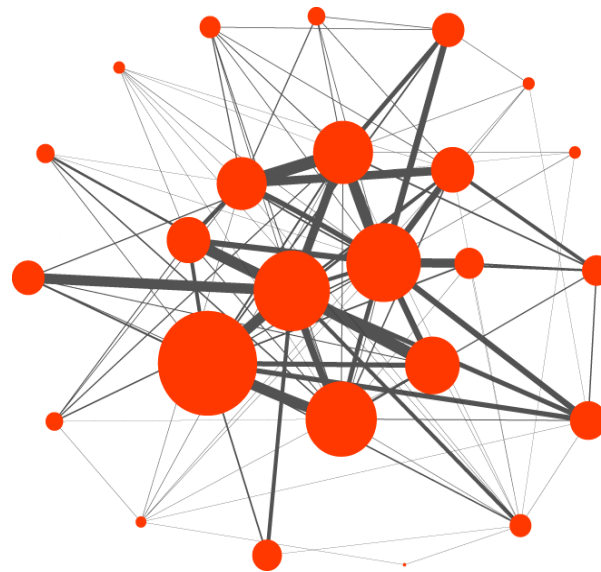
- Financial networks
- Resource pooling in communication networks
- Open questions on resource pooling

**Fig. 2 A sample of the international financial network, where the nodes represent major financial institutions and the links are both directed and weighted and represent the strongest existing relations among them**



**F. Schweitzer et al., Science 325, 422 -425 (2009)**

## Chart 3.2 Network of large exposures between UK banks(a)(b)(c)

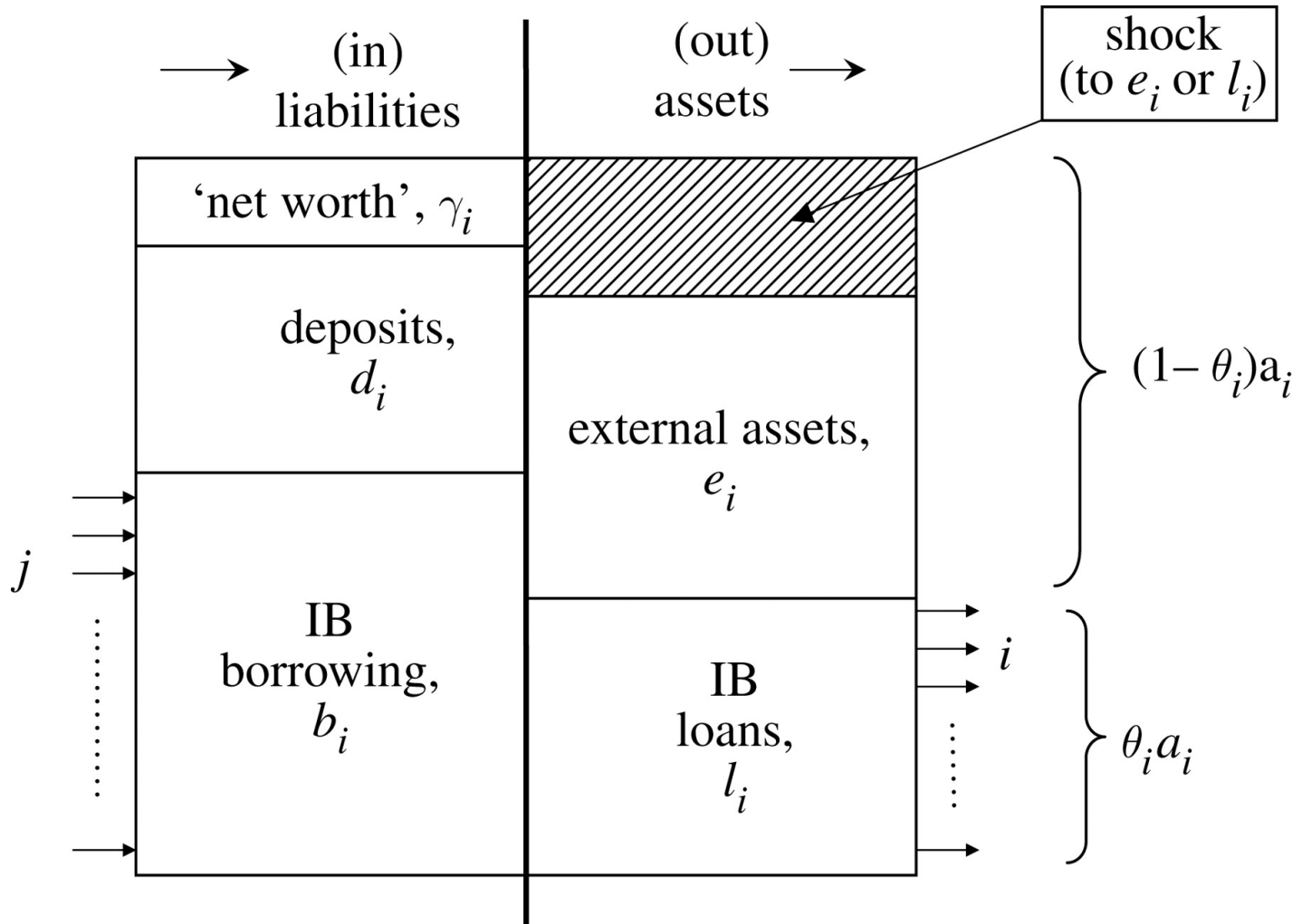


Source: FSA returns.

- (a) A large exposure is one that exceeds 10% of a lending bank's eligible capital at the end of a period. Eligible capital is defined as Tier 1 plus Tier 2 capital, minus regulatory deductions.
- (b) Each node represents a bank in the United Kingdom. The size of each node is scaled in proportion to the sum of (1) the total value of exposures to a bank, and (2) the total value of exposures of the bank to others in the network. The thickness of the line is proportional to the value of a single bilateral exposure.
- (c) Based on 2009 Q2 data.

<http://www.bankofengland.co.uk/financialstability/>

Schematic model for a 'node' in the IB network.



May R M , Arinaminpathy N J. R. Soc. Interface doi:10.1098/rsif.2009.0359

(Also: Gai and Kapadia – Contagion in Financial Markets)

# Stylised bank balance sheet

## Liabilities



## Assets



# Stylised bank balance sheet

Liabilities



Assets

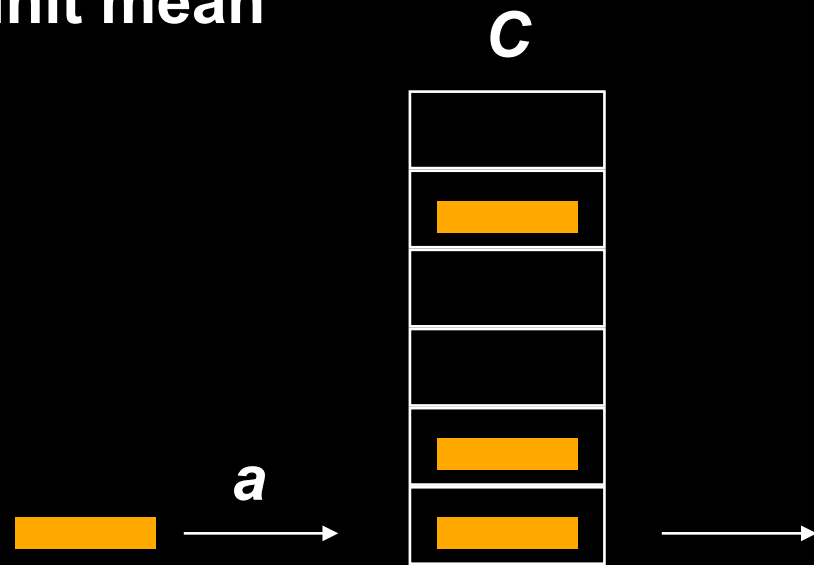




# Erlang's formula

- calls arrive randomly, at rate  $a$
- resource has  $C$  circuits
- accepted calls hold a circuit for a random holding time, with unit mean
- blocked calls are lost
- proportion of calls lost is:

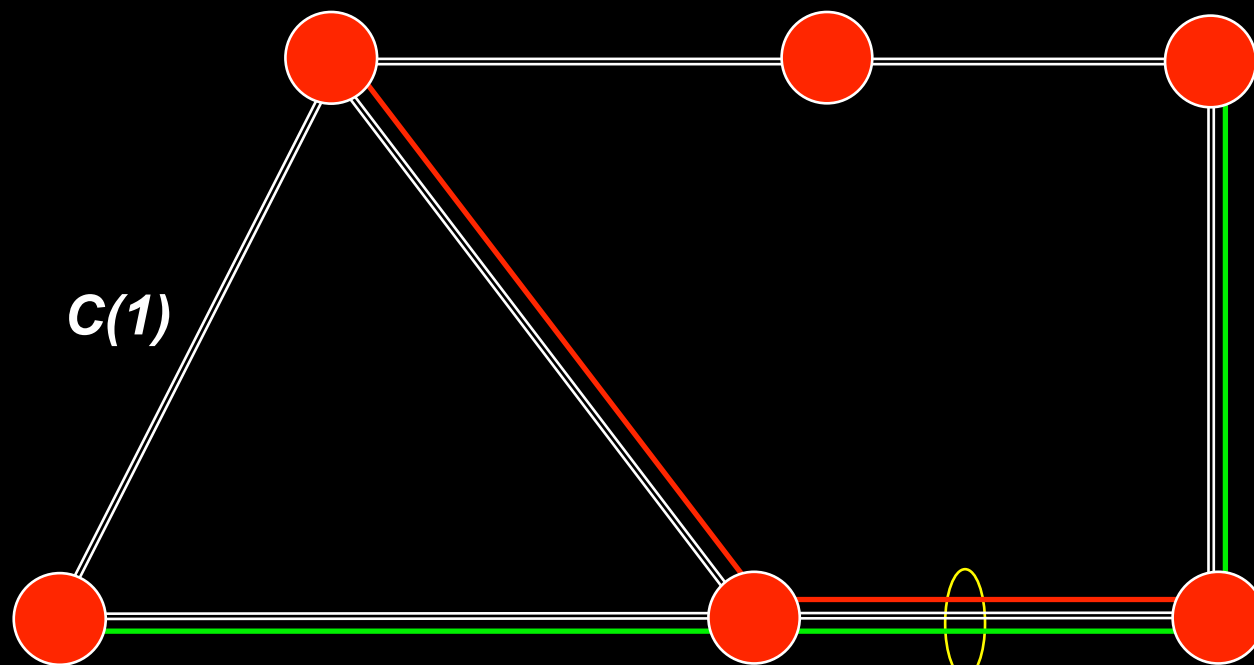
$$E(a, C) = \frac{a^C / C!}{\sum_0^C (a^n / n!)}$$



# Outline

- Financial networks
- Resource pooling in communication networks
- Open questions on resource pooling

# A loss network



Link constraint:

$$\sum_r A(j, r) n(r) \leq C(j)$$

# Resource pooling

## Aims:

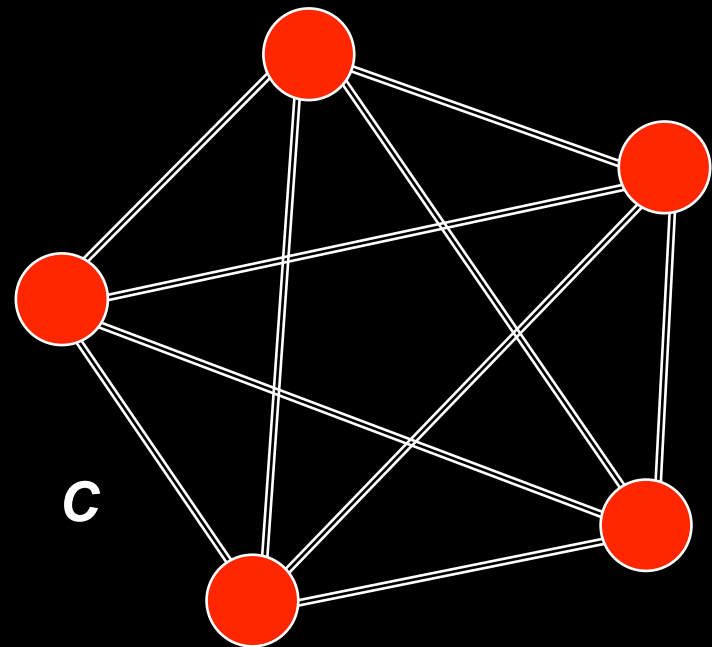
- respond robustly to failures and overloads
- lessen the impact of forecasting errors
- make use of spare capacity in the network
- permit flexible use of network resources

## Problems:

- instability
- complexity

# Example: alternative routing

- Complete graph
- All links have capacity  $C$
- Call routed directly if possible; otherwise one randomly chosen alternative route may be tried



Marbukh 1984, Gibbens, Hunt, K 1990,  
Crametz, Hunt 1991, Graham, Méléard 1993, 1994

# alternative routing

- Arrival rate per link -  $a$
- Capacity per link -  $C$
- Let  $B$  be the link blocking probability
- Then as the number of nodes grows, the blocking probability  $B$  approaches a solution of:

$$B = E(a[1 + 2B(1 - B)], C)$$

# instability, and hysteresis

link blocking  
probability,  $B$

0.45

0.4

0.35

0.3

0.25

0.2

0.15

0.1

0.05

0

0.8

0.85

0.9

0.95

1

1.05

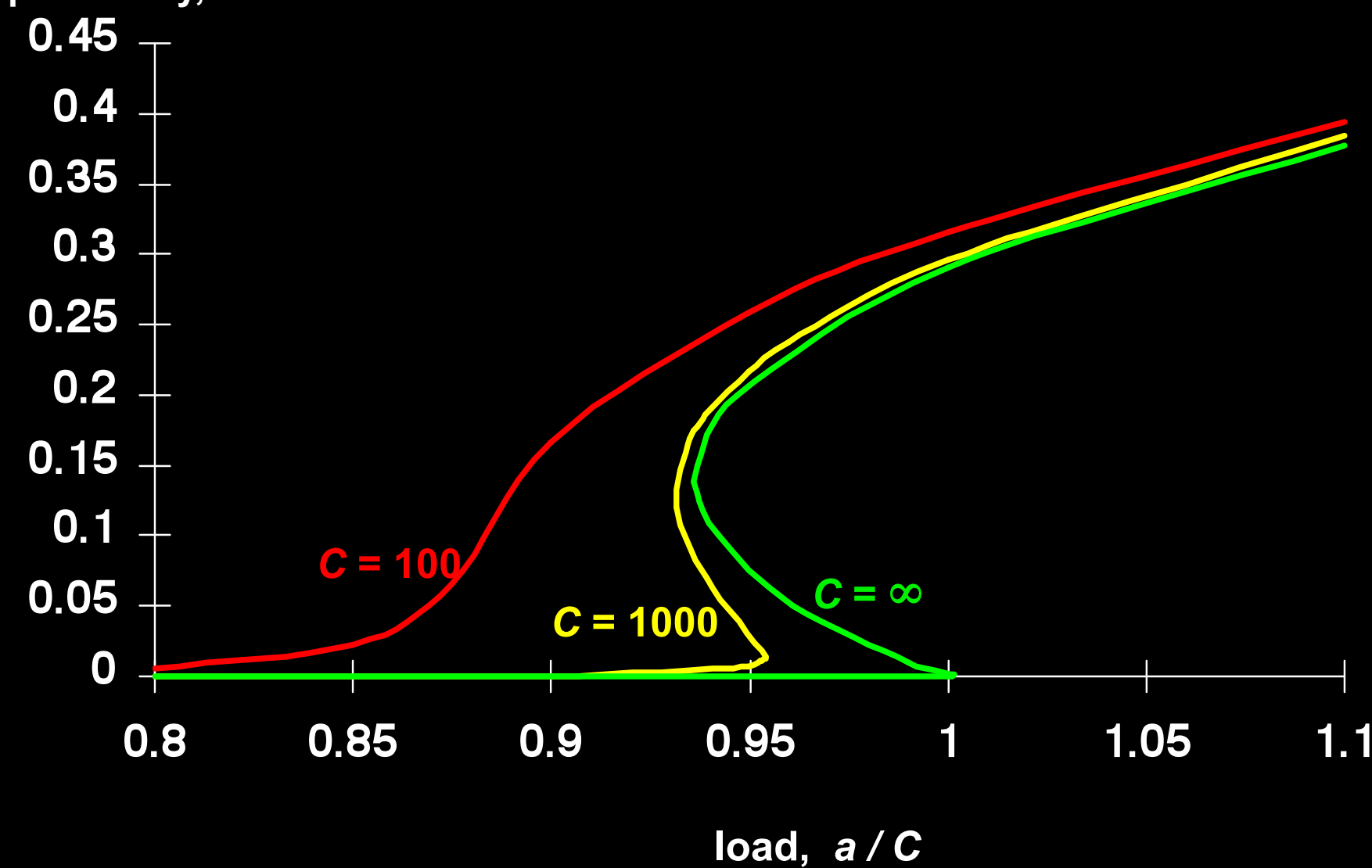
1.1

load,  $a / C$

$C = 100$

$C = 1000$

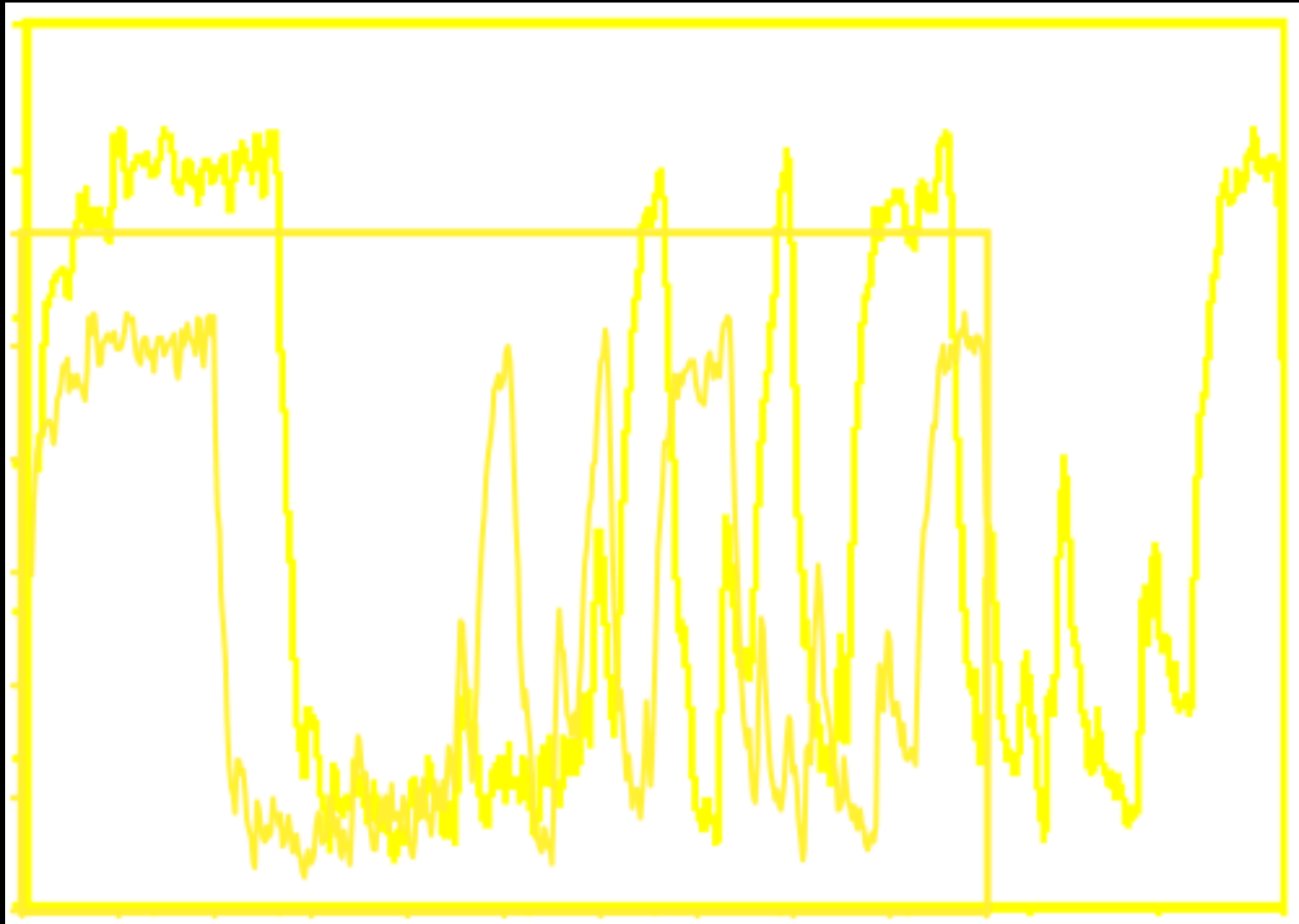
$C = \infty$



# bistability

calls in progress  
(‘000s)

50



45

0

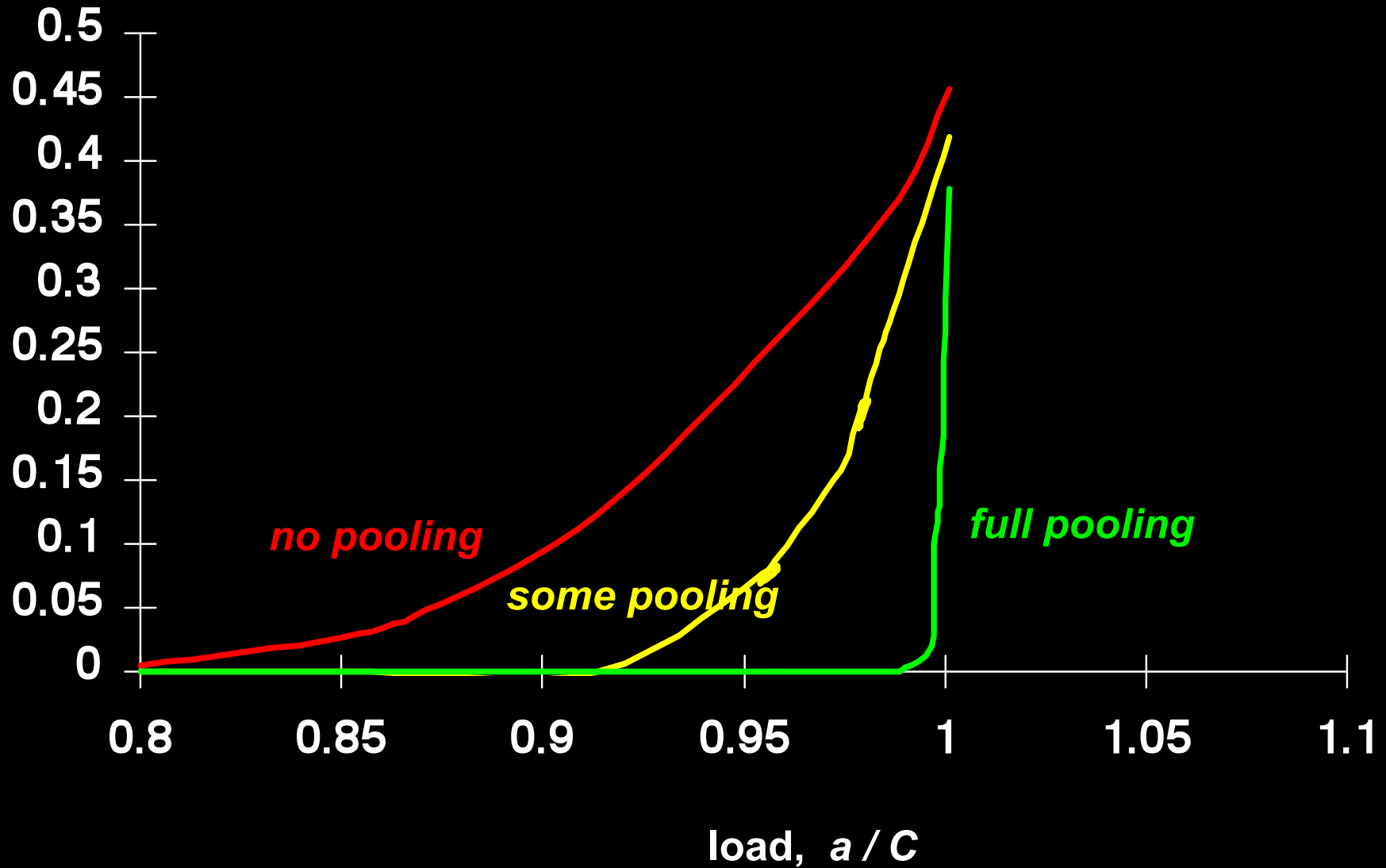
call holding times

100



# Sudden impact of capacity

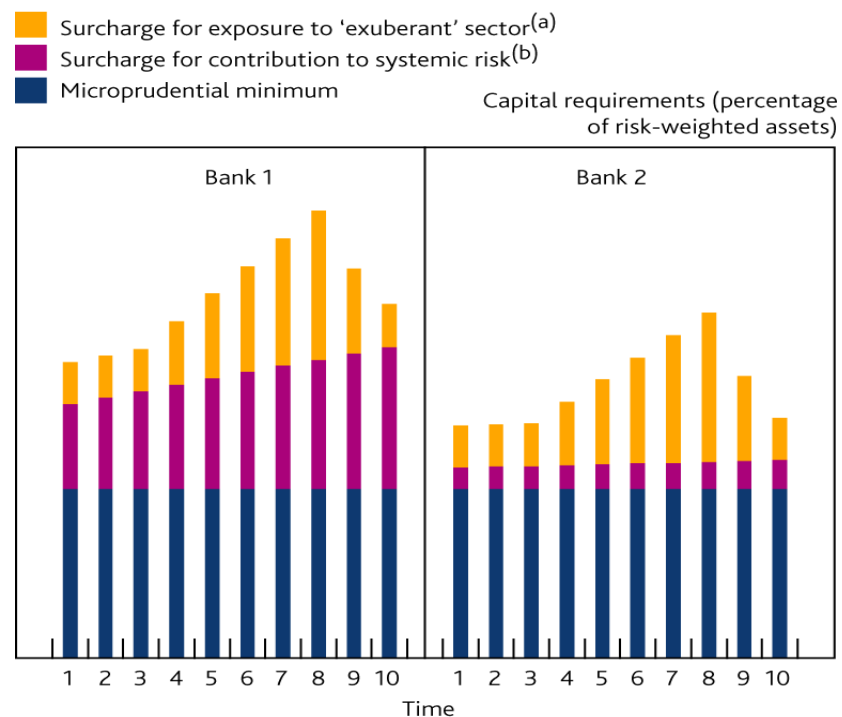
Feedback signal  
(loss, delay, price,...)



# Open questions on resource pooling

- Resource pooling does indeed
  - respond robustly to failures and overloads
  - lessen the impact of forecasting errors
  - make use of spare capacity in the network
  - permit flexible use of network resources
- But
  - can produce phase transitions if load amplified
  - obscures the approach of capacity overload
- Can decentralised control take account of system-wide risks?

## Chart 3.13 Stylised representation of a macroprudential regime based on capital surcharges



Source: Bank of England.

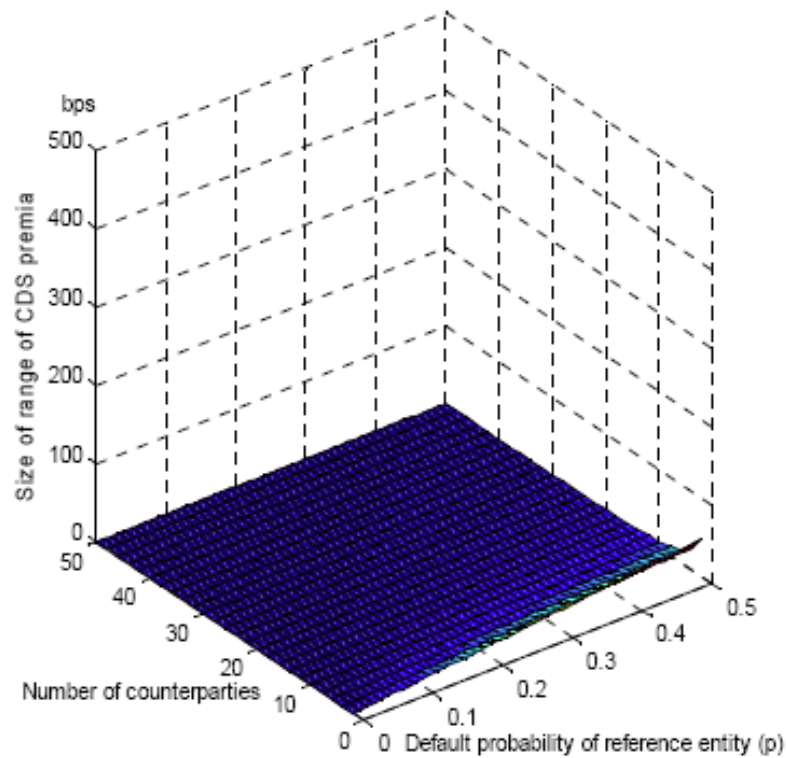
(a) Cyclical surcharge on sector that becomes increasingly exuberant through periods 4–8.

(b) Surcharge based on the contribution of each bank to systemic risk. Bank 1's contribution is assumed to be large and slowly rising

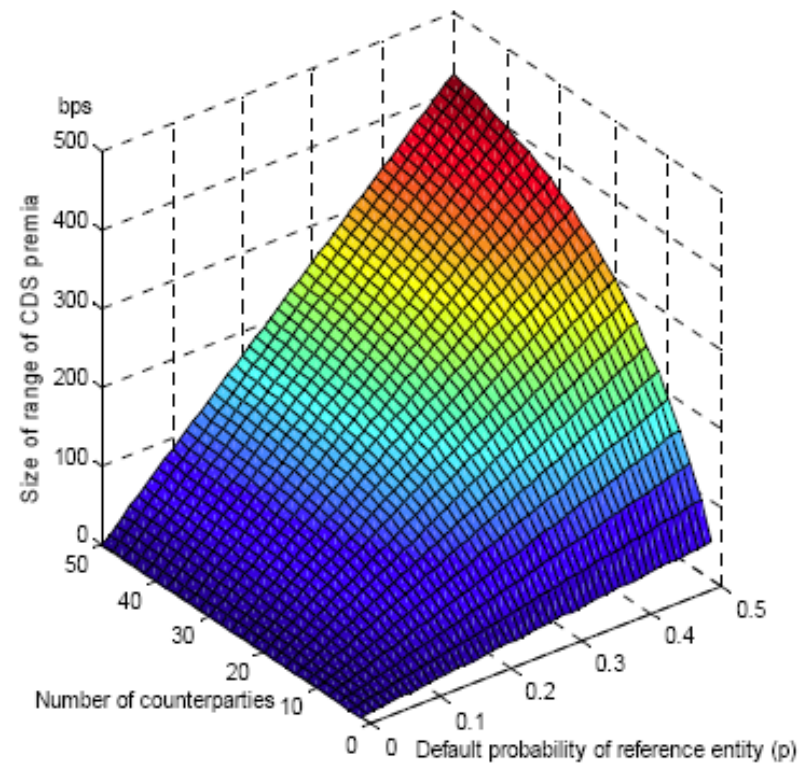
through periods 1–10. Bank 2's contribution is assumed to be smaller throughout.

<http://www.bankofengland.co.uk/financialstability/>

**Chart 4: CDS Premia and Network Uncertainty – Pre-crisis**

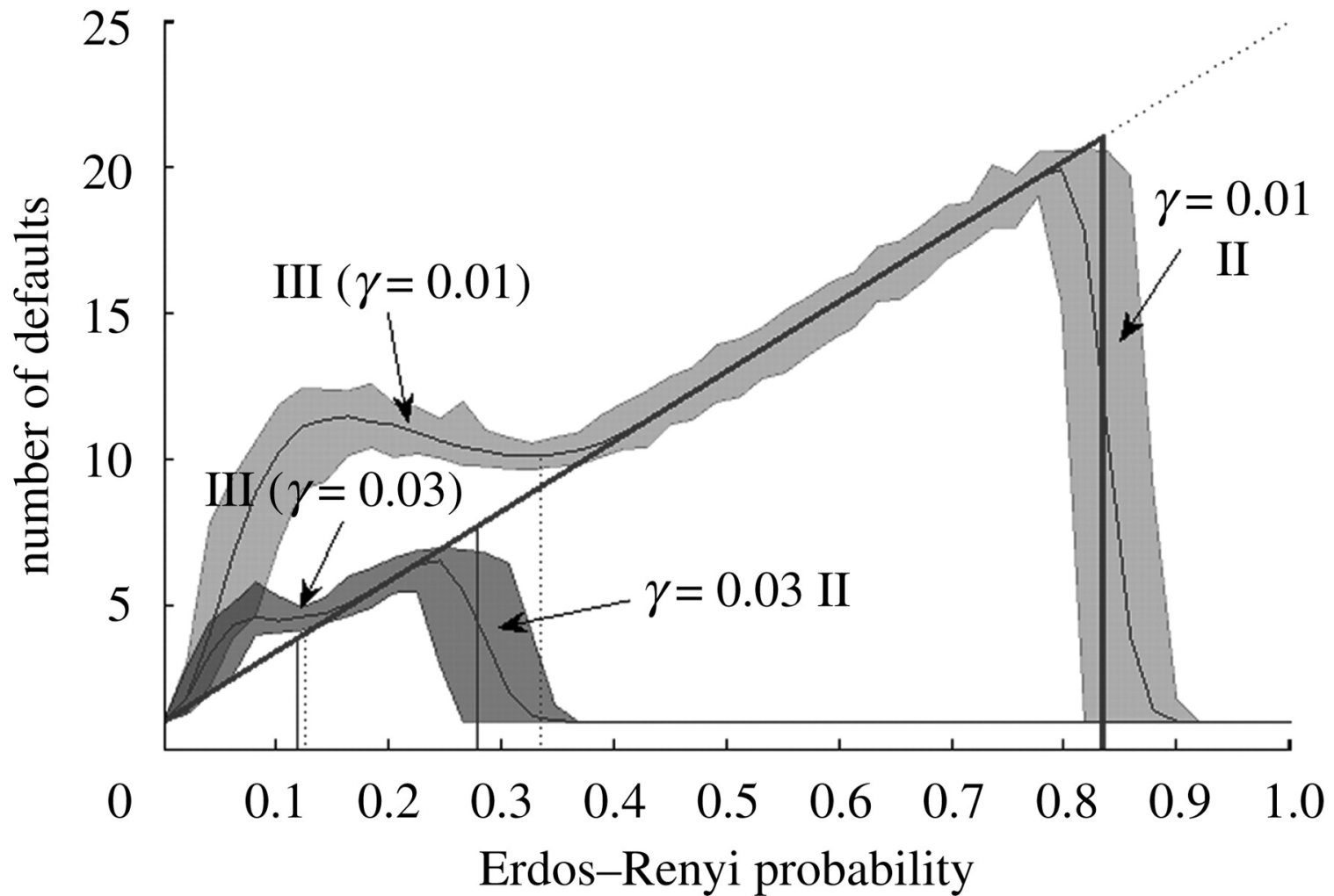


**Chart 5: CDS Premia and Network Uncertainty – Post-crisis**



<http://www.bankofengland.co.uk/financialstability/>

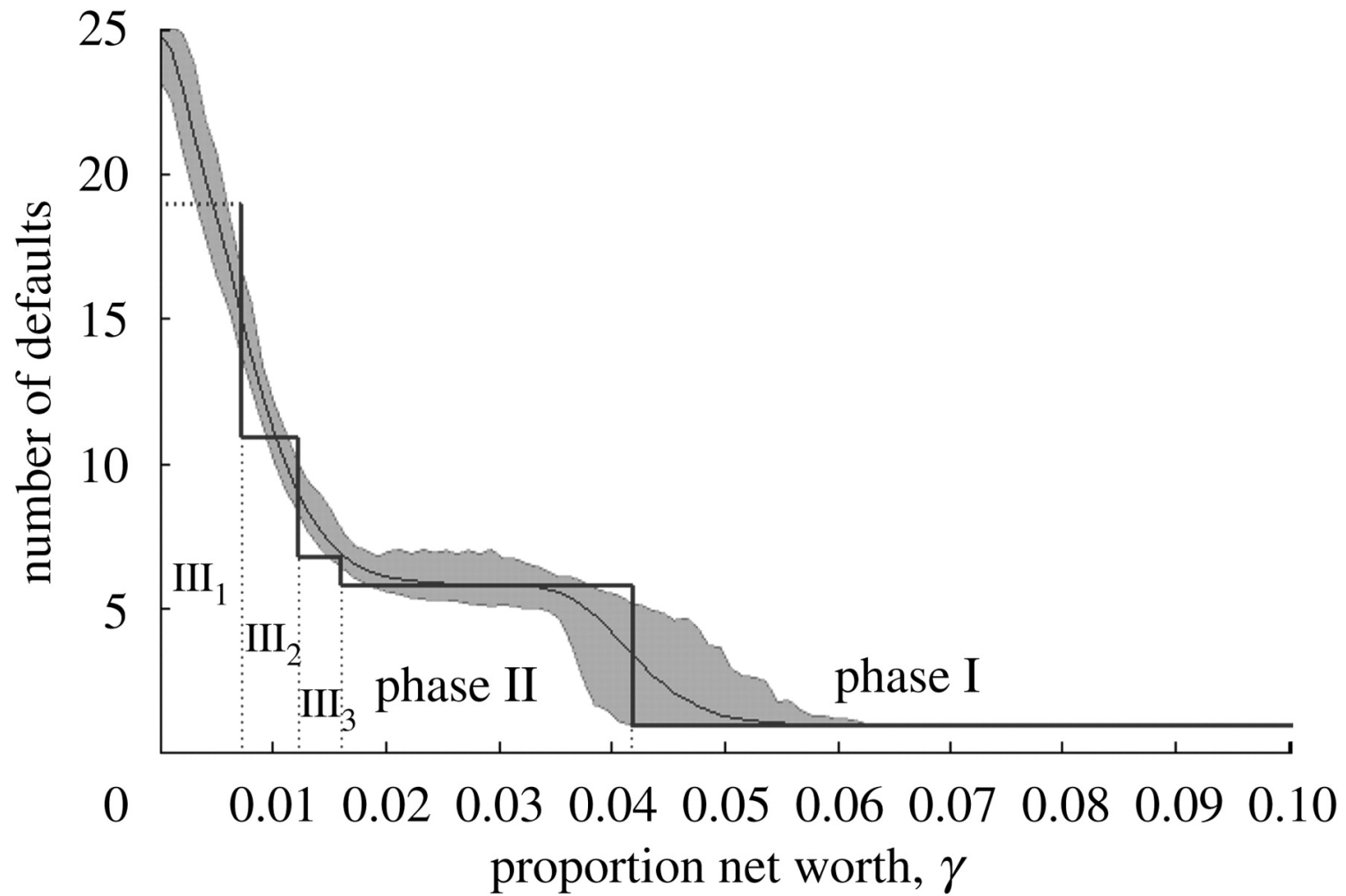
Number of banks failing as a function of the IB connectivity,  $p$ .



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Number of banks failing as a function of percentage net worth ( $\gamma$ ).



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