GRADE 8 MATH: 200 FREESTYLE

UNIT OVERVIEW

This unit of instruction is focused on the introduction of a precise definition of functions as students transition to the more formalized study of algebra.

TASK DETAILS

Task Name: 200 Freestyle

Grade: 8

Subject: Math

Depth of Knowledge: 2

Task Description: This task asks students to analyze graphs of functions and interpret rates of change, intersections, and points on the curves of different functions.

Standards:
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.

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TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement Common Core–aligned tasks that are embedded in a unit of instruction. We have learned through our pilot work that focusing instruction on units anchored in rigorous Common Core-aligned assessments drives significant shifts in curriculum and pedagogy. Callout boxes and Universal Design for Learning (UDL) support are included to provide ideas around how to include multiple entry points for diverse learners.

Acknowledgements: This bundle was developed by Lisa Genduso, 8th Grade Math Teacher, New Horizons Middle School; Joshua Dragoon, Senior Instructional Coach, Office of Achievement Resources; Shelley Kryger, NYC DOE Math Common Core Fellow, and Cheryl Schafer, NYC DOE Common Core Fellow.

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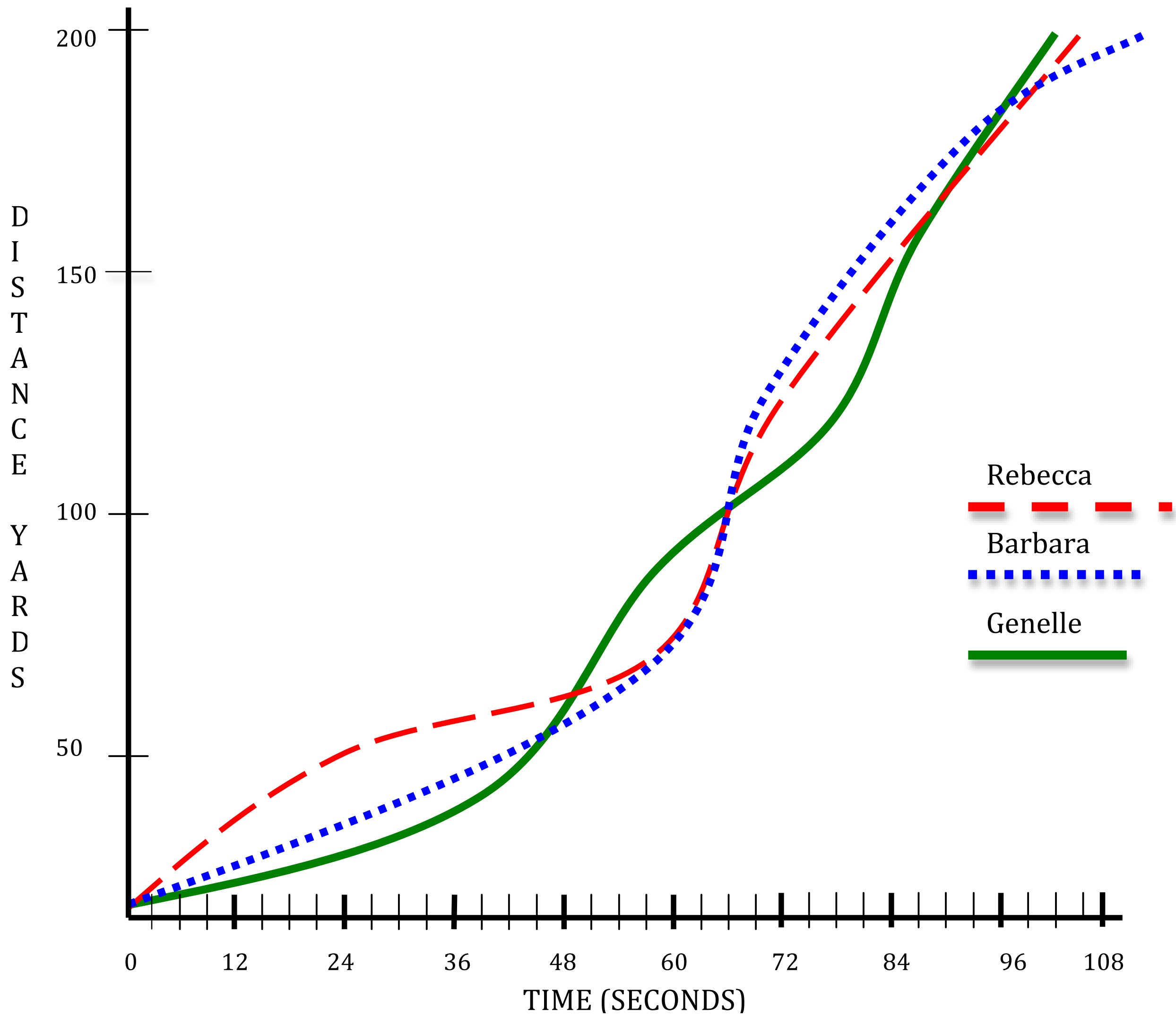
PERFORMANCE TASK

200 Freestyle

Racers Rebecca (Dashes), Barbara (Dots) and Genelle (Solid) are competing in the 200 yards freestyle race. The graph below shows the race.



1. Which swimmer is in the lead after 12 seconds? ___________________________

2. Which swimmer is in the lead at the 150 yard mark?_____________________
   Explain how you know.

3. When were all three swimmers tied? ________________________________
   Explain your answer.

4. Describe what happened during the last 50 yards of the race? ______________

____

____

____

5. What was the winning time? ______________________
   Show how you figured it out.

6. What was the average speed of the winner in miles per hour? ______________
   Show how you figured it out.

8

GRADE 8 MATHL: 200 FREESTYLE

RUBRIC

On the following pages are a scoring guide for the task and an associated rubric that provides performance level descriptions and cut scores based on the scoring guide.

SVMI Rubric

200 Freestyle		Rubric
The core elements of performance required by this task are: • interpreting a distance time graph in a given situation. • comparing functions on a Cartesian Plane • finding values of a function using the x – and y - axis • determining average speed Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **Rebecca**	1	1
2. Gives correct answer: **Barbara and** shows correct work such as: The blue line is higher vertically meaning farther in less time	1	1
3. Gives correct answer: 66 seconds (63 - 69) or 100 yards Gives an explanation such as: It is where all three curved lines intersect. That means they are at the same place at the same time	1 1	2
4. Gives correct explanation such as: With 50 yards to go Barbara slows down, Genelle first passes Rebecca and then in the last six seconds Genelle passes Barbara and wins.	1	1
5. Gives correct answer: 99 seconds (accept 96 – 102) and indicates reading off the x-axis (using a vertical line)	1	1
6. Gives a correct answer such as: About 4 miles/hour (4.13 mph) *Partial Credit:* 2 yards/seconds and shows some correct work such as 200/99	2 (1)	2
Total Points		**8**

Performance Level Descriptions and Cut Scores

Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0–2 points)

The student's response shows few of the elements of performance that the task demands. The work shows a minimal attempt on the problem and the student struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's explanation rarely uses definitions. The student struggles to recognize patterns or the structure of the problem situation.

Level 2: Performance below Standard (3 or 4 points)

The student's response shows some of the elements of performance that the task demands and some signs of a coherent attack at the core of some of the problems. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but the use of quantity is limited or not fully developed. The student's response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem, it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (5 or 6 points)

For most of the task, the student's response shows the main elements of performance that the task demands and is organized as a coherent attack at the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix when provided with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. The student often uses abstractions to represent a problem symbolically or with other mathematical representations. The student's response may use assumptions, definitions, and previously established results in constructing arguments. The student may make conjectures and build a logical progression of statements to explore the truth of these conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (7 or 8 points)

The student's response meets the demands of nearly the entire task, with few errors. With more time for checking and revision, excellent solutions would seem likely. The student's response shows understanding and use of stated assumptions, definitions, and previously established results in constructing arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of these conjectures. The student's response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The student looks closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

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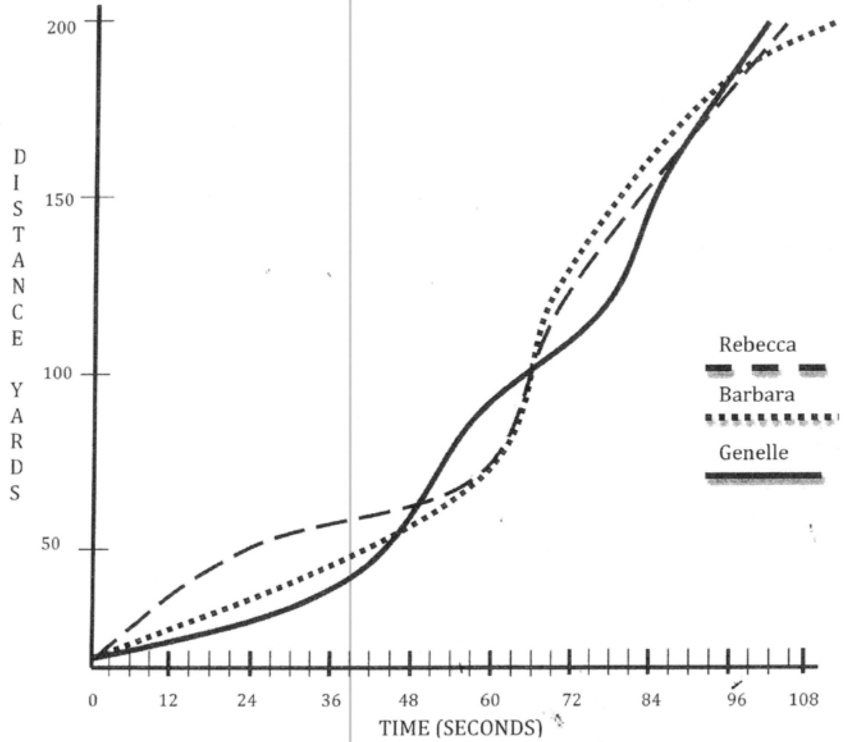
ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points. The student work shows examples of student understandings and misunderstandings of the task.

Score Point 3 (6 out of 8 points)
This student's work satisfies almost all aspects of the task. In question 8, the student correctly computes the winner's speed in yards per second and appropriately labels the answer. While the student does not compute the answer in miles per hour, this is a limited misconception and does not detract from the student's overall ability to interpret and compare graphs of functions.

200 Freestyle

Racers Rebecca (Dashes), Barbara (Dots) and Genelle (Solid) are competing in the 200 yards freestyle race. The graph below shows the race.



1. Which swimmer is in the lead after 12 seconds? Rebecca

Eighth Grade Performance Test 2012

2. Which swimmer is in the lead at the 150 yard mark? Barbara
Explain how you know.
I know this because the graph shows how she passed the others and kept that spot for a bit.

3. When were all three swimmers tied? They were tied around at 100 yards.
Explain your answer.
The were ~~a~~ tied around 100 yards because when they tied that was the closest mark.

4. Describe what happened during the last 50 yards of the race? At the last 50 yards barbara and Rebecca begin to slow down as ~~G~~ Genette toke the lead

5. What was the winning time? 46 Seconds
Show how you figured it out.
I ~~?~~ figured it out by looking at the graph and matching the end of the winners path to the time below it.

6. What was the average speed of the winner in miles per hour? ____________
Show how you figured it out.

$\frac{200 \text{ yards}}{46 \text{ seconds}}$

$200 \div 46 = 2.08$

2.08 yards per second

8

Eighth Grade Performance Test 2012

MP.1: The student accurately describes the trends in the middle of

MP.1: The student's description of how he or she determined the point at which the swimmers are tied is limited. Explaining that this is the point at which the three curves intersect would provide a more precise answer.

8.F.5: The student correctly notes that Barbara slows down. However, Rebecca's speed remains constant during this period.

MP.1: The student precisely describes a process that can be used to determine the winning swimmer's time.

MP.1: The student correctly determines the speed in yards per second (and labels the answer accordingly). However, the answer is not expressed in miles per hour, as required by the task.

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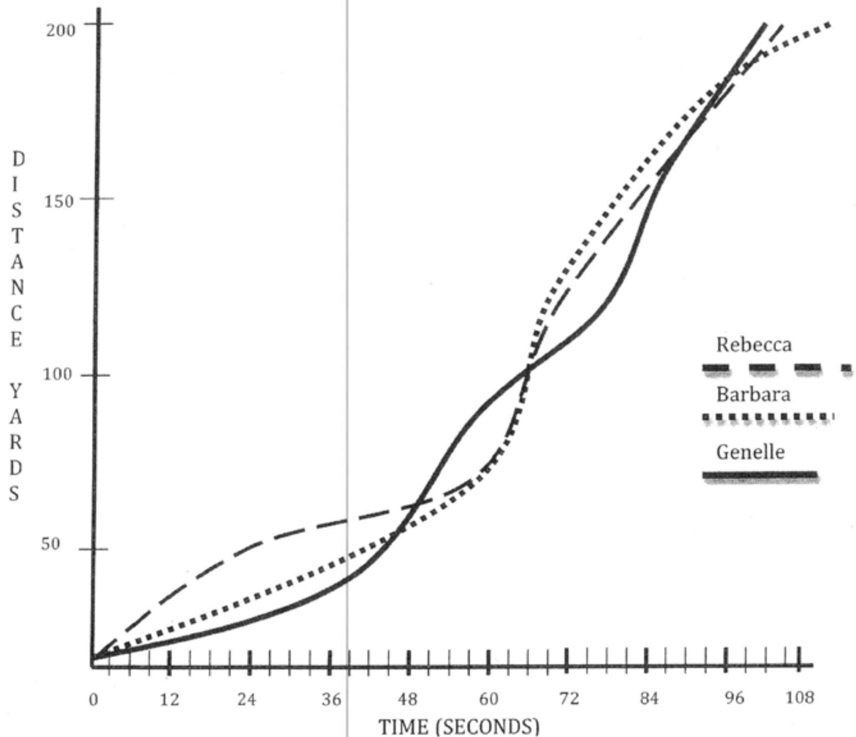
Score Point 3 (6 out of 8 points)

This student's work satisfies most aspects of the task. The student correctly answers questions 1, 2, 3, 4, and 5. The use of units in all answers reflects the student's attention to precision. The student correctly calculates Genelle's average speed; however, the speed is provided in yards per second, not miles per hour.

200 Freestyle

Racers Rebecca (Dashes), Barbara (Dots) and Genelle (Solid) are competing in the 200 yards freestyle race. The graph below shows the race.



1. Which swimmer is in the lead after 12 seconds? Rebecca

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2. Which swimmer is in the lead at the 150 yard mark? Barbara
Explain how you know. She gets there in 75 seconds

MP.3: The student provides a limited explanation as to how he or she knows that Barbara is in the lead at the 150 yard mark.

3. When were all three swimmers tied? 63 Seconds or 100 yards
Explain your answer. They all in the Same Spot

MP.1: The student correctly identifies both the time and the distance at which all three swimmers were tied and the fact that all three curves intersect.

4. Describe what happened during the last 50 yards of the race? Barbara was winning, Rebecca was in Second And Genelle was in third. Then Genelle past Barbara and Rebecca to get in First.

MP.1: The student provides a description of the order of the swimmers in the last 50 yards of the race.

5. What was the winning time? 99 Seconds
Show how you figured it out. Genelle Got to 200 Yards first and the others took longer

MP.3: The student explains why Genelle won the race but does not clearly show how he or she figured it out.

6. What was the average speed of the winner in miles per hour? 2.0 Yard Per Sec
Show how you figured it out.

$\frac{200}{99} = 2.0$

8

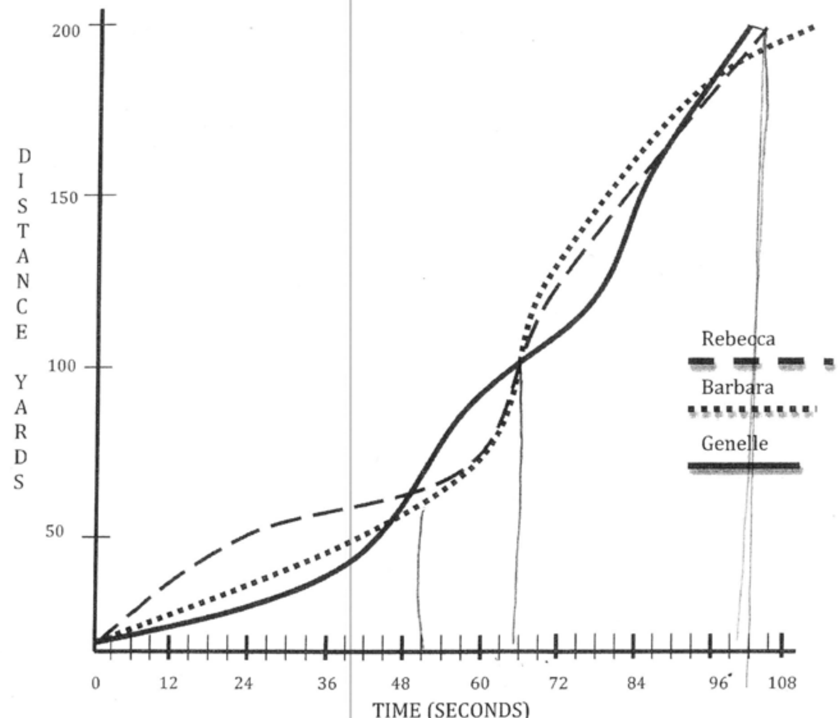
Eighth Grade Performance Test 2012

MP.1: The student correctly determines the speed in yards per second. However, the answer is not expressed in miles per hour, as required by the task.

Score Point 2 (3 out of 8 points)
This student's work satisfies some aspects of the task. The student answers question 1 correctly. Several other questions include correct numerical answers but are labeled incorrectly. These examples reflect the student's lack of attention to precision. The student demonstrates a limited understanding through the incorrect and imprecise answers and explanations.

200 Freestyle

Racers Rebecca (Dashes), Barbara (Dots) and Genelle (Solid) are competing in the 200 yards freestyle race. The graph below shows the race.



1. Which swimmer is in the lead after 12 seconds? B Rebecca

Eighth Grade Performance Test 2012

2. Which swimmer is in the lead at the 150 yard mark? Genelle's
   Explain how you know.
   Genelle's line is the Steepest, if she keeps going at this Rate she would be at the lead at 150 yards.

3. When were all three swimmers tied? 68 yards.
   Explain your answer.
   If you draw a vertical line going down it ~~touch~~ touches the x Axis At 68 Yards.

4. Describe what happened during the last 50 yards of the race? At 50 yards Rebecca and barbara were Running at the same Rate and Genelle got ~~ahead~~p and increased her speed.

5. What was the winning time? 100 minutes.
   Show how you figured it out.
   I drew a vertical line down the graph from the line that is the Steepest once it gets to 200 yards.

6. What was the average speed of the winner in miles per hour? 2 minutes
   Show how you figured it out.
   I divided the time it took her to get to 200 yards (100) and I got 2 minutes.

8

Eighth Grade Performance Test 2012

8.F.5: The student is clearly applying his or her knowledge of slope to the task. However, the answer is incorrect, and the explanation shows inadequate

MP.3: The student uses a correct procedure but does not explain why she chose this point..

8.F.5: The student incorrectly describes the rate at which the students were running.

8.F.5: The student identifies a correct numerical answer but includes the wrong unit (it should be time in seconds, not minutes). However, the explanation does not accurately show how the student could have obtained a correct answer.

MP.1: The student uses an operation that could correctly be used to determine the winner's average speed in yards per second, instead of in miles per hour as required by the task. Further, the student does not include a correct label; the student notes that it is 2 minutes, not 2 yards per second.

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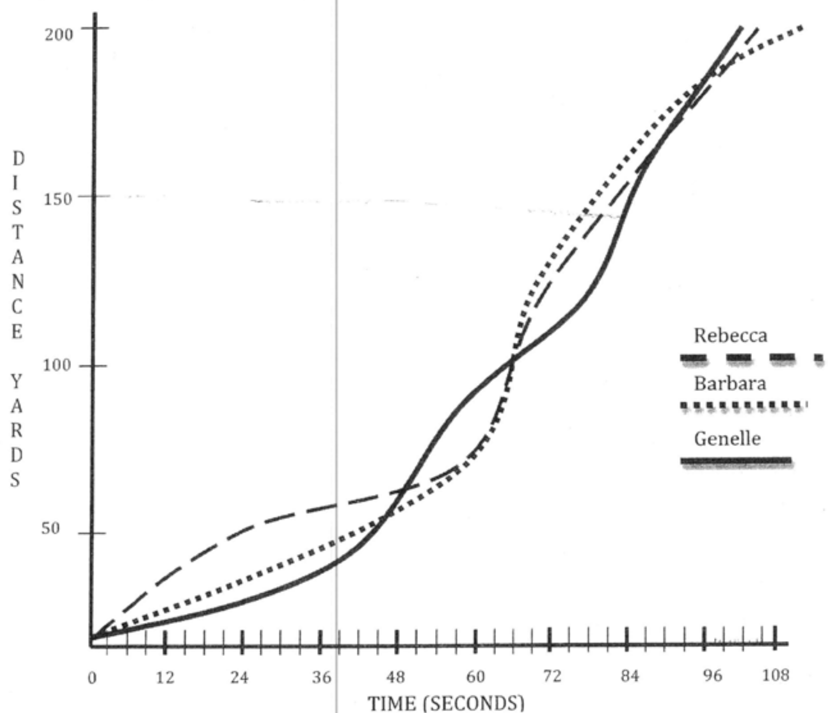
Score Point 1 (1 out of 8 points)
This student's work satisfies few of the demands of the task. The student correctly answers question 2 but provides an incorrect explanation. A number of questions are omitted or imprecise (e.g., "200 mark" instead of 200 yards).

200 Freestyle

Racers Rebecca (Dashes), Barbara (Dots) and Genelle (Solid) are competing in the 200 yards freestyle race. The graph below shows the race.



1. Which swimmer is in the lead after 12 seconds? Barbara

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2. Which swimmer is in the lead at the 150 yard mark? Barbara
Explain how you know. Because she is swimming faster

3. When were all three swimmers tied? 200 mark
Explain your answer. The all met at the same second and because they were ~~swim~~ swimming fast

4. Describe what happened during the last 50 yards of the race?

5. What was the winning time? 107
Show how you figured it out.

6. What was the average speed of the winner in miles per hour?
Show how you figured it out.

8

Eighth Grade Performance Test 2012

MP.1: While the student identifies Barbara as the swimmer in the lead at the 150 yard mark, he or she incorrectly ascribes this to the speed at which Barbara is swimming. In fact, the swimmers vary their speed in absolute terms and in relation to each other over the course of the race, and it is the distance each has traveled at that particular moment that determines who is in the lead.

GRADE 8 MATH: 200 FREESTYLE

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline

INTRODUCTION: This unit outline provides an example of how to integrate performance tasks into a unit. Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic. The length of the unit includes suggested time spent on the classroom instruction of lessons and administration of assessments. Please note that this framework does not include individual lessons.

Grade 8 Mathematics: 200 Freestyle

UNIT TOPIC AND LENGTH:

- This unit builds on prior work from Grade 7 and earlier, in Grade 8. This prior study includes interpreting and evaluating linear equations and solving systems of linear equations. Additional background on the development of these concepts can be found in the *Progression for the Common Core State Standards in Mathematics* (draft) April 22, 2011—Expressions and Equations. Prior units addressed using tables and graphs to represent the values of proportional and non-proportional relationships. This unit is intended to take 2 to 3 weeks, depending on students' prior knowledge.

COMMON CORE STANDARDS:

PRIMARY:

- **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
- **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

SECONDARY:

- **8.EE.7** Solve linear equations in one variable.
  a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

- **8.F.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Standards for Mathematical Practice

- **MP1** Make sense of problems and persevere in solving them.
- **MP2** Reason abstractly and quantitatively.
- **MP3** Construct viable arguments and critique the reasoning of others.
- **MP4** Model with mathematics.
- **MP6** Attend to precision.

Big Ideas/Enduring Understandings:

- Functions show relationships between varying quantities
- Graphs can be used to model real-world situations
- Different representations of functions provide different information
- People interpret graphs to evaluate situations

Essential Questions:

- What information can be ascertained from graphs of functions?
- Why is it useful to represent different situations on the same graph?
- How does the information provided in tables and graphs help us better interpret a function?

Content:

- Functions
- Linear functions
- Non-linear functions
- Distance-time graphs
- Scatter plot
- Exponential graphs
- Average rate of change

Skills:

- **Interpreting** linear and non-linear functions
- **Analyzing** tables and graphs
- **Understanding** the relationships between inputs and outputs
- **Explaining** and **justifying** interpretations
- **Representing** functions in multiple forms.
- **Comparing** and **contrasting** different functions
- **Identifying** functions and **distinguishing** from relationships that are not functions

Vocabulary/Key Terms:

slope, rate of change, function, equation, linear, non-linear, systems, scatter plot, rule, input, output, ordered pair, domain, range

Assessment Evidence and Activities:

Initial Assessment:

In this activity students are given the opportunity to interpret a line graph. This builds upon past knowledge of constructing line graphs but also leads them into making some connection to slope. It assesses students' ability to think about the direction and steepness of a line. In the end they are given an open-ended question to describe what could be happening in the graph. This non-routine response allows students flexibility and can be more useful in determining their true understanding.

Formative Assessment:

Interpreting Distance-Time Graphs (Formative Assessment Lesson)

This Formative Assessment Lesson, from the Mathematics Assessment Project, is a 2- or 3-period activity in which students are asked to interpret distance-time graphs and relate them to the situations they model. Students will also be expected to understand the relationship between speed and the slope of these distance-time graphs. Formative Assessment Lessons provide teachers with opportunities to both assess and develop students' understanding of math concepts. An important feature of these lessons is that they engage students in significant collaboration and discussion with peers. This lesson should be introduced two-thirds of the way through the unit. Information about Formative Assessment Lessons and additional classroom challenges are available at http://map.mathshell.org/.

Final Performance Task:

In the final performance tasks for this unit, *200 Freestyle*, students are asked to:

- Interpret the relationship between points on the coordinate plane and their relationship to functions
- Interpret points of intersections of graphical representations of functions
- Explain change over time in graphical representations of functions
- Determine average rate of change and distinguish from rates of change over different time frames

For additional detail, please see the actual *200 Freestyle* task.

Learning Plan & Activities:

- (CCLS 6.EE.6) This unit builds on students' knowledge of patterns (developed beginning in Pre-Kindergarten and explicitly identified across the grades in Standard for Mathematical Practice 7). This prior study can be connected to students' current work by beginning with patterns students can represent with tables, graphs, and equations. Consider presenting patterns in visual, verbal, tabular, and graphical formats and having students represent the same relationship in multiple ways. See below for examples:

1. Prom is coming up!

- John has been saving money for prom night. He received $23 dollars from his parents in early May. Each week he saves $7 from his salary for delivering papers. Tickets to the prom cost $80. After saving for 4 weeks, does John have enough to buy a prom ticket? How do you know? How much money would John have if he saved for 8 weeks? 12 weeks? Write an equation that would tell you how much money John had after *w* weeks.

- The table below shows the cumulative number of prom tickets sold at the end of each day. If the pattern continues, how many prom tickets will be sold on day 10? Draw a graph to represent the total number of tickets sold through day 10.

Day	**1**	**2**	**3**	**4**
Total Tickets Sold	20	25	30	35

2. Extend the pattern below and write a rule for the number of squares needed in **step** ***x***.

Step 1	**Step 2**	**Step 3**	**Step 4**
□ □□ □	□ □ □□□ □	□ □ □ □□□□ □	□ □ □ □ □□□□□ □

- ➢ Consider having student groups chart and post responses to these questions. When formally defining functions later in the unit, students can use these and compare against the criteria for functions.

- ➢ Consider using *Impact Mathematics, Course 3* lessons identified below to review content learned in prior units and introduce the content of this unit.

LESSON	**DESCRIPTION/UNIT CONTENT ADDRESSED**
1.2: “Slope”	In this lesson, students are introduced to the concept of slope through a consideration of “steepness.” Explicit connections are made to the slopes of lines.

1.3: "Write Equations"	In this lesson, students are provided with a series of investigations in which they determine whether a relationship is linear, express and interpret equations in the form $y = ax + b$, explore the relationships among various forms in which a relationship can be expressed, and write equations for graphs of linear relationships. **Investigation 1:** Students consider rates of change and identify whether a relationship is linear based on using the constant differences method. **Investigation 2:** Students represent multiple relationships simultaneously in tables and graphs and compare them. **Investigation 3:** Students consider linear equations and represent them using slope-intercept form. **Investigation 4:** Students write equations for lines, given points and slopes or a set of two points. **Investigation 5:** Students translate among verbal, tabular, and symbolic representations of relationships with explicit attention to the features revealed by aspects of equations written in standard form.
10.1: "*Functions*"	In this lesson, students are introduced to the concept of functions and asked to distinguish between examples and non-examples. **Investigation 1:** Students are introduced to a formal definition of functions. They evaluate and examine the inputs and outputs of function machines. **Investigation 2:** Students describe functions with rules and graphs. Students generate tables of values and graphs based on equations of functions and continue to distinguish examples and non-examples.

Additional Support Strategies: For students having difficulty understanding the connection between the concept of a function and the word **function**, including students who are English Language Learners, consider reminding students of words that can have multiple meanings and provide real-world examples of functions. Word walls can help students with language difficulties by reminding them of the meaning of words. When defining words, consider multiple representations of definitions, including the Frayer Model, that provide students with visual connections to meaning.

- As the unit progresses, model real-world phenomena using functions. For example, consider measuring the distance of runners as a function of time (by having students compete in a race in which some students mark the time of runners at different points along the track). You may also wish to graph temperature at different points during the day. It is important to distinguish between linear functions (e.g., increasing at a constant rate) and functions we can measure, graph, and describe that do not conform to a linear rule.

- The following resources, presented in a suggested order of instruction in this unit, can be used to develop function concepts:
  - *Early Algebra Teacher Resources* (Tufts University Department of Education)

    - In the introductory lesson "Who Shares My Function? - Linear with All Representations" students must match graphical, tabular, and verbal representations of functions. They also work to define the term *function*.
    - Additional lessons, including a follow-up to the introductory lesson mentioned above, are available on the site's middle school lessons page (https://wikis.uit.tufts.edu/confluence/display/EarlyAlgebraResources/Middle+School+Lessons).
- *Illustrative Mathematics*: 8-F Modeling with a Linear Function
    - This problem set asks students to identify whether situations could be modeled by a given linear equation. It is recommended that in addition to identifying whether the situation could be modeled by the equation, students are asked to write justifications for their answers.
- *NCTM Illuminations*: Barbie Bungee
    - In this lesson, students collect data regarding the relationship between the number of rubber bands used to form a "bungee" cord and the distance of Barbie's jump. The data this experiment yields is then analyzed, and a linear relationship is informally fit to the data. This lesson provides an opportunity to integrate technology through the use of a spreadsheet and a web-based line of best fit activity. This lesson adds standards in the 8.SP domain, notably 8.SP.1, 2, and 3.
- LearnZillion (www.learnzillion.com) tutorial videos and additional practice on the following topics:
    - Compare distance-time graphs with distance-time equations
    - Construct linear functions from a graph
    - Construct linear functions from a situation
    - Construct linear functions from tables
- *Mathematics Assessment Project* Formative Assessment Lesson
    - Interpreting Distance-Time Graphs, © 2012 MARS, Shell Center, University of Nottingham

➢ There is a strong emphasis on **accountable talk**. Students have the potential to build upon prior knowledge of interpreting linear functions. In this unit they must make connections to their understanding of systems of equations and notice how this can be applied to all functions. Their ability to communicate about these ideas and share them with others demonstrates their comprehension of this specific topic.

➢ **Presentation of Work**: In order to interpret functions, students worked in a variety of settings. Students worked individually, in pairs, and in small groups. In the "Distance and Time Graphs Activity" students worked in partnerships. Each partnership was responsible for explaining and justifying why their pairing was correct. This allowed for pairs with an incorrect response for a specific match to benefit from their understanding. This then encouraged communication and stronger understanding as to how to interpret functional relationships.

➢ **Reflections:** Many times throughout this unit students are asked to explain their understanding in a written form. This includes but is not limited to short answer responses, letters to people explaining a new idea, or full-page journals where they explain different ideas within the unit. This allows students to make comparisons and justify their understanding.

RESOURCES:

- Graph paper, calculators (scientific and graphing), markers, colored pencils, and rulers
- *Impact Mathematics, Course 3,* © 2009 Glencoe McGraw-Hill
- *Early Algebra Teacher Resources* (Tufts University Department of Education)
    - "Who Shares my Function?-Linear with All Representations"
    - middle school lessons page (https://wikis.uit.tufts.edu/confluence/display/EarlyAlgebraResources/Middle+School+Lessons).
- *Illustrative Mathematics*: 8-F Modeling with a Linear Function
- *NCTM Illuminations*: Barbie Bungee
- LearnZillion (www.learnzillion.com)
    - Compare distance-time graphs with distance-time equations
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- *Mathematics Assessment Project* Formative Assessment Lesson
    - Interpreting Distance-Time Graphs, © 2012 MARS, Shell Center, University of Nottingham

CURRICULAR RESOURCES:

***IMPACT MATHEMATICS, COURSE 3*, © 2009 GLENCOE MCGRAW-HILL**
LESSON 1.2: SLOPE
1.3: Write Equations
10.1: Functions

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Interpreting Distance-Time Graphs

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

Interpreting Distance–Time Graphs

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to interpret distance–time graphs and, in particular, to help you identify students who have the following difficulties:

- Interpreting distance–time graphs as if they are pictures of situations rather than abstract representations of them.
- Relating speeds to slopes of these graphs.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.F Construct a function to model a linear relationship between two quantities.
Describe qualitatively the functional relationship between two quantities by analyzing a graph

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work on a task designed to reveal their current understandings and difficulties. You review their work and create questions for students to answer in order to improve their solutions.
- A whole-class introduction provides students with guidance on how to work through the first task. Students then work in small groups on a collaborative discussion task, matching verbal interpretations with graphs. As they do this, they translate between words and graphical features, and begin to link the representations.
- This is followed by a whole-class discussion about applying realistic data to a graph.
- Students next work in small groups, matching tables of data to the existing matched pairs of cards. They then explain their reasoning to another group of students.
- In a final whole-class discussion, students draw their own graphs from verbal interpretations.
- Finally, students return to their original task and try to improve their individual responses.

MATERIALS REQUIRED

- Each student will need two copies of the assessment task *Journey to the Bus Stop,* a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need copies of *Card Set A: Distance–Time Graphs*, *Card Set B: Interpretations*, *Card Set C: Tables of Data*, a large sheet of paper, and a glue stick for making posters. The cards should be cut up beforehand.
- You will also need a supply of graph paper to give to students who request it. There are some projector resources to support your teaching.

TIME NEEDED

15 minutes before the lesson, a 90-minute lesson (or two 45-minute lessons), and 10 minutes in a following lesson (or homework). Timings are approximate and will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *Journey to the Bus Stop* (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Journey to the Bus Stop*.

Briefly introduce the task and help the class to understand the problem and its context.

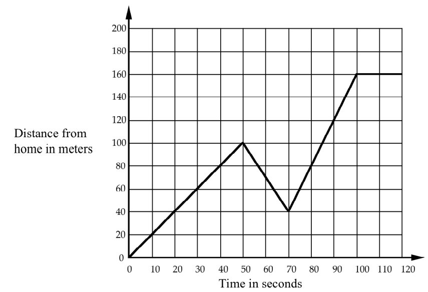
> *Read through the task and try to answer it as carefully as you can.*

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything because in the next lesson they will engage in a similar task that should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



1. Describe what may have happened.
   You should include details like how fast he walked.

   ....

2. Are all sections of the graph realistic? Fully explain your answer.

   ....

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

Common issues:	Suggested questions and prompts:
Student interprets the graph as a picture For example: The student assumes that as the graph goes up and down, Tom's path is going up and down. Or: The student assumes that a straight line on a graph means that the motion is along a straight path. Or: The student thinks the negative slope means Tom has taken a detour.	• If a person walked in a circle around their home, what would the graph look like? • If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like? • In each section of his journey, is Tom's speed steady or is it changing? How do you know? • How can you figure out Tom's speed in each section of the journey?
Student interprets graph as speed–time The student has interpreted a positive slope as speeding up and a negative slope as slowing down.	• If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like? • How does the distance change during the second section of Tom's journey? What does this mean? • How does the distance change during the last section of Tom's journey? What does this mean? • How can you tell if Tom is traveling away from or towards home?
Student fails to mention distance or time For example: The student has not mentioned how far away from home Tom has traveled at the end of each section. Or: The student has not mentioned the time for each section of the journey.	• Can you provide more information about how far Tom has traveled during different sections of his journey? • Can you provide more information about how much time Tom takes during different sections of his journey?
Student fails to calculate and represent speed For example: The student has not worked out the speed of some/all sections of the journey. Or: The student has written the speed for a section as the distance covered in the time taken, such as "20 meters in 10 seconds."	• Can you provide information about Tom's speed for all sections of his journey? • Can you write his speed as meters per second?
Student misinterprets the scale For example: When working out the distance the student has incorrectly interpreted the vertical scale as going up in 10s rather than 20s.	• What is the scale on the vertical axis?
Student adds little explanation as to why the graph is or is not realistic	• What is the total distance Tom covers? Is this realistic for the time taken? Why?/Why not? • Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not?

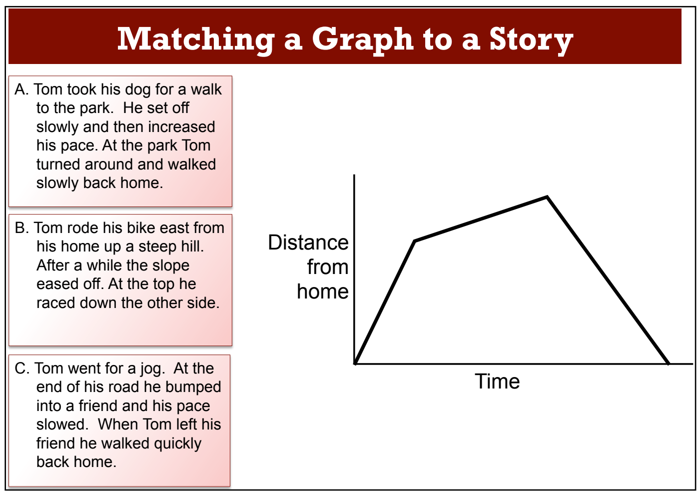
SUGGESTED LESSON OUTLINE

If you have a short lesson or you find the lesson is progressing at a slower pace than anticipated, we suggest you break the lesson after the first sharing of posters and continue it at a later time.

Whole-class introduction: interpreting and sketching graphs (10 minutes)

Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically, and question the choices put forward by others. This introduction will provide students with a model of how they should work with their partners in the first small-group activity.

Show the class the projector resource *Matching a Graph to a Story:*



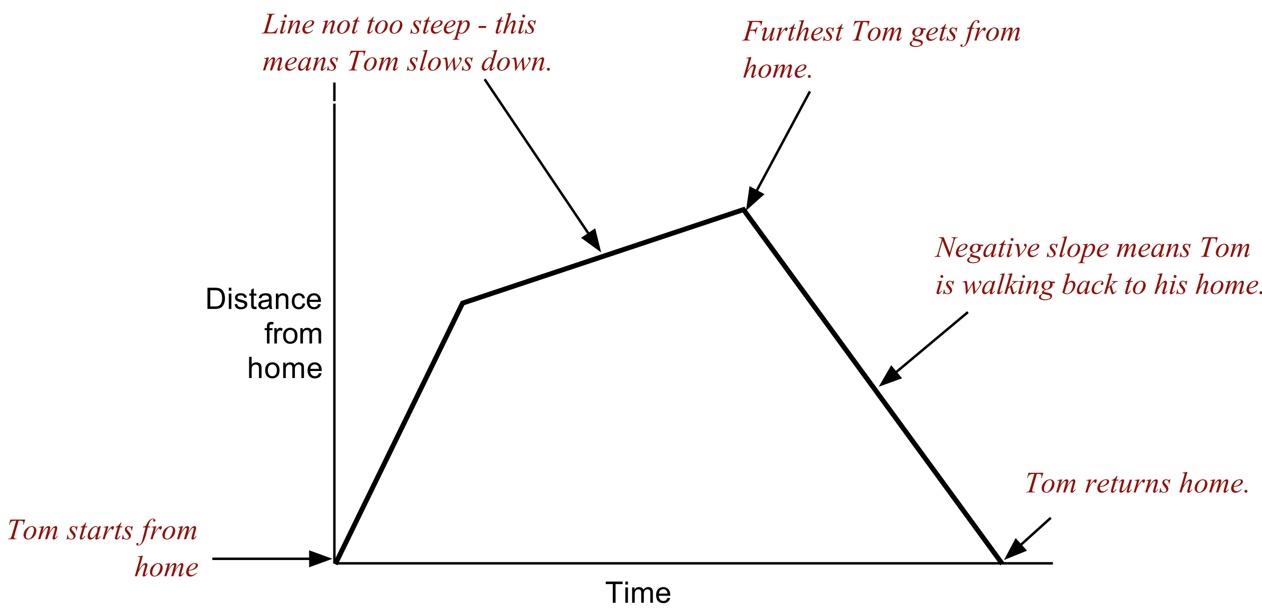
Ask students to match the correct story to the graph. They are to write down at least two reasons to support their decision.

After two or three minutes ask students who selected option A to raise their hands. Ask one or two to justify their choice. You may wish to use some of the questions on the sheet *Suggested questions and prompts* to encourage students to justify their choices and others to challenge their reasoning.

Repeat this with options B and C. Even if explanations are incorrect or only partially correct, write them next to the appropriate section of the graph. Encourage students to challenge these interpretations.

Slide P-2 of the projector resource allows you to write three different student explanations on the board at the same time.

A graph may end up looking like this:

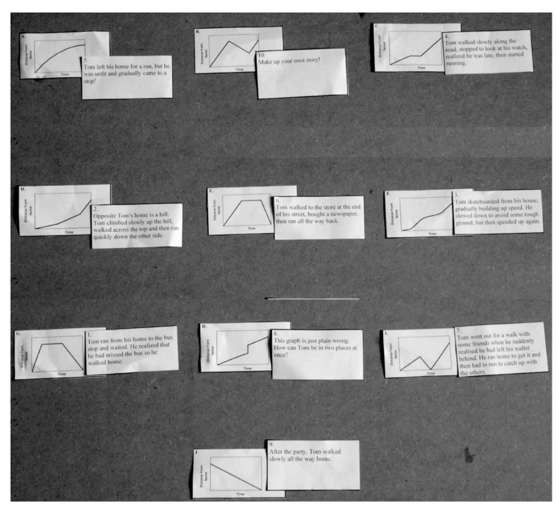


This is how students should annotate their graphs when working on the collaborative task.

Collaborative activity: matching *Card sets A* and *B* (20 minutes)

Ask students to work in small groups of two or three students.

Give each group the *Card Set A: Distance–Time Graphs*, and *Card Set B: Interpretations* together with a large sheet of paper, and a glue stick for making a poster.



You are now going to continue exploring matching graphs with a story, but as a group.

You will be given ten graph cards and ten story cards.

In your group take a graph and find a story that matches it. Alternatively, you may want to take a story and find a graph that matches it.

Take turns at matching pairs of cards. Each time you do this, explain your thinking clearly and carefully. If you think there is no suitable card that matches, write one of your own.

Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.

Write your reasons for the match on the cards or the poster just as we did with the example in class. Give explanations for each line segment.

Make sure you leave plenty of space around the cards as, eventually, you will be adding another card to each matched pair.

The purpose of this structured group work is to encourage students to engage with each other's explanations and take responsibility for each other's understanding.

Slide P-3 of the projector resource summarizes these instructions.

You have two tasks during the small-group work: to make a note of student approaches to the task, and to support student reasoning.

Make a note of student approaches to the task

Listen and watch students carefully. Note different student approaches to the task and any common mistakes. For example, students may interpret the graph as a picture or students may read the graph from right to left. Also notice the ways students check to see if their match is correct and how they explain and justify a match to each other. You can use this information to focus a whole-class discussion.

Support student reasoning

Try not to make suggestions that move students towards a particular match. Instead, ask questions to help students to reason together. If you find one student has produced a solution for a particular match, challenge another student in the group to provide an explanation.

John matched these cards. Sharon, why do you think John matched these two cards?

If you find students have difficulty articulating their decisions, then use the sheet *Suggested questions and prompts* to support your own questioning of students.

In trials of this lesson some students had difficulty stating where home is on the graph.

For this graph, where does the journey start? Is that home?

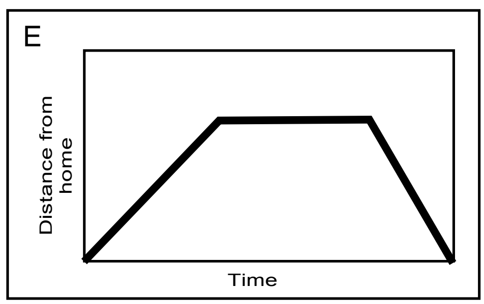
Give me a graph that shows a journey starting away from home.

For this graph, does the journey end at home? How do you know?

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.

Some of the cards are deliberate distracters. For example, a student who matches Card 2 and E indicates that they think that graphs are pictures of the situation.

2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.



Allow students time to match all the cards they can.

Sharing posters (5 minutes)

As students finish matching the cards, ask one student from each group to visit another group's poster.

You may want to use Slide P-4 of the projector resource to display the following instructions.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.

Students may now want to make changes to their poster. At this stage there is no need for students to glue the cards onto their posters as they may decide to make further changes.

If you need to extend the lesson over two days:

Once students have finished sharing posters, organize a whole-class discussion. Invite pairs of students to describe one pair of cards that they think they have matched correctly and the reasoning they employed. Encourage other students to challenge their explanations.

Finally, ask students to note their matches on the back of their poster and to use a paperclip to attach all cards to the poster.

At the start of the second lesson, spend a few minutes reminding the class about the activity:

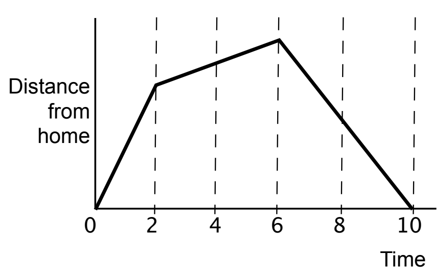
Can you remember what we were working on in the last lesson?

Return the posters to each group. The whole-class discussion on interpreting tables can serve as an introduction to the lesson.

Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide P-5 of the projector resource:

Making Up Data for a Graph



Time	Distance
0	
2	
4	
6	
8	
10	

On your whiteboard, create a table that shows possible times and distances for Tom's journey.

After a few minutes, ask students to show you their whiteboards. Ask some students to explain how they created their tables. Write their figures on the board. Ask the rest of the class to check these figures.

Is Tom's speed slower or faster in this section compared to that section?

How do you know from the graph? From the table?

Is this speed constant? How can you tell? Do the figures in the table show a constant speed for this section of the journey?

What units might these be measured in?

Are these figures realistic?

Collaborative activity: matching *Card Set C* (20 minutes)

Hand out *Card Set C: Tables of Data* and ask students to match these cards with the cards already on their poster.

You are now going to match tables with the cards already on your desk. In your group take a graph and try to find a table that matches it, or take a table and find a graph that matches it.

Again take turns at matching cards you think belong together. Each time you do this, explain your thinking clearly and carefully.

Write your reasons for the match on the poster.

Students may also wish to suggest suitable units for the distances and times on the cards.

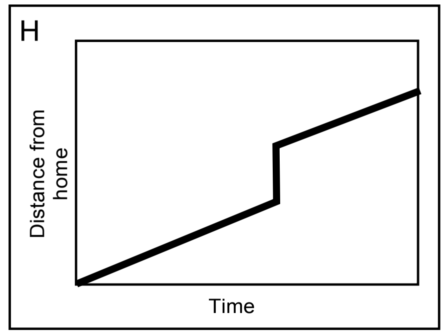
The tables should help students confirm or modify existing matches.

As they work on the matching, support the students as in the previous matching activity.

In the past, some students have had difficulty understanding the repetition in Table R. The table is intended to show the impossibility of Graph H.

R

Time	Distance
0	0
1	18
2	36
3	54
3	84
5	120



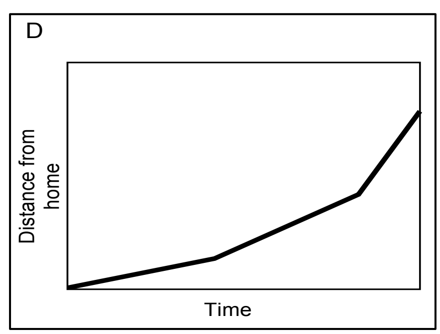
Some teachers have found that it helps students to look at the average speeds between consecutive times, by calculating differences. For example, average speeds for Table of Data Q would look like this.

Q

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

Average speed
10
10
20
20
60

This may help students to understand that the table on Card Q matches Tom's hill walk, and that the correct distance–time graph should therefore be Card D.



If some students finish quickly, encourage them to devise their own pairs of cards.

Sharing posters (5 minutes)

When students have completed the task, the student who has not already visited another pair should share their work with another pair of students. Students are to share their reasoning as they did earlier in the lesson unit.

Students may now want to make final changes to their poster. When they are completely satisfied, ask them to glue their cards onto the large sheet of paper.

Whole-class discussion (10 minutes)

Using mini-whiteboards, make up some journeys and ask the class to show you the corresponding graphs.

On your whiteboards, draw a distance–time graph to show each of the following stories:

- *Sam ran out of his front door, then slipped and fell. He got up and walked the rest of the way to school.*
- *Sara walked from home up the steep hill opposite her house. She stopped at the top to put her skates on, then skated quickly down the hill, back home again.*
- *Chris cycled rapidly down the hill that starts at his house. He then slowed down as he climbed up the other side.*

Ask students to show their whiteboards to the whole-class. Select some to explain their graph to the class. Encourage others in the class to challenge their reasoning.

Improving individual solutions to the assessment task (10 minutes)

Return to the students their original assessment task *Journey to the Bus Stop*, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

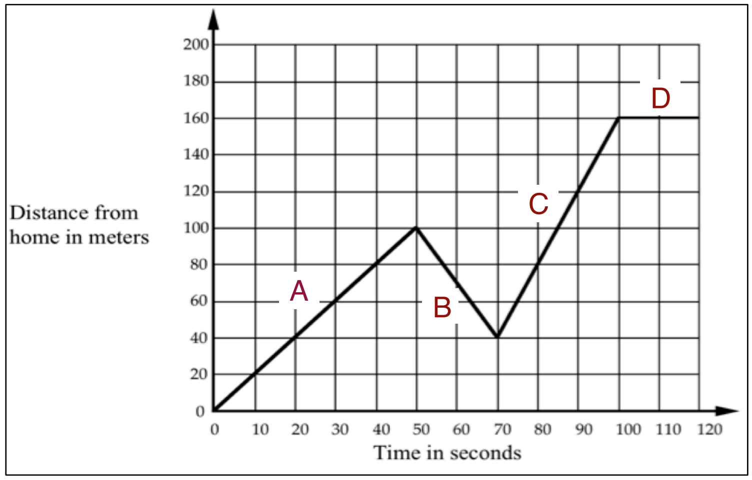
If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

If you find you are running out of time, you could set this task in the next lesson or for homework.

SOLUTIONS

Assessment task: *Journey to the Bus Stop*

1. The graph shows Tom's journey is split into four sections. The straight lines indicate that Tom moves at a constant but different speed in each section.



   A In this section of the journey Tom walks away from home at a speed of 2 meters per second (100 ÷ 50) for 50 seconds.

   B The negative slope here means a change in direction. At 100 meters from home Tom starts to walk towards home. He walks for 60 meters at a speed of 3 meters per second (60 ÷ 20).

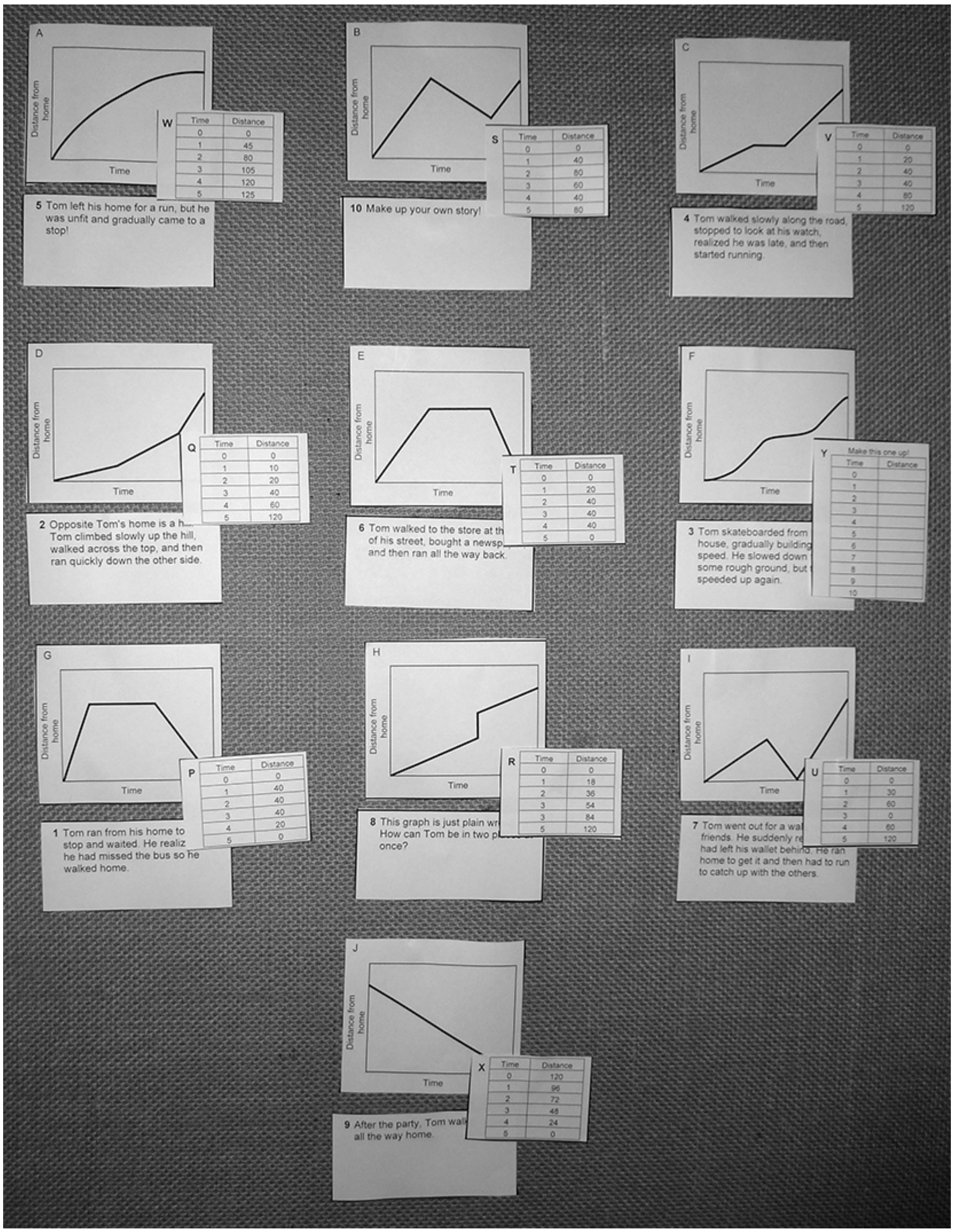
   C At the start of this section Tom changes direction. He is now walking away from home at a fast pace. His speed is 4 meters per second (120 ÷ 30). He moves at this speed for 30 seconds and covers 120 meters.

   D Here the slope is zero. This means at 160 meters from home Tom stops. It has taken him 100 seconds to get to this point.

2. The speeds provided in the answer to question 1 are realistic. A speed of 2 meters per second is a brisk walk. A speed of 4 meters per second means Tom is running.

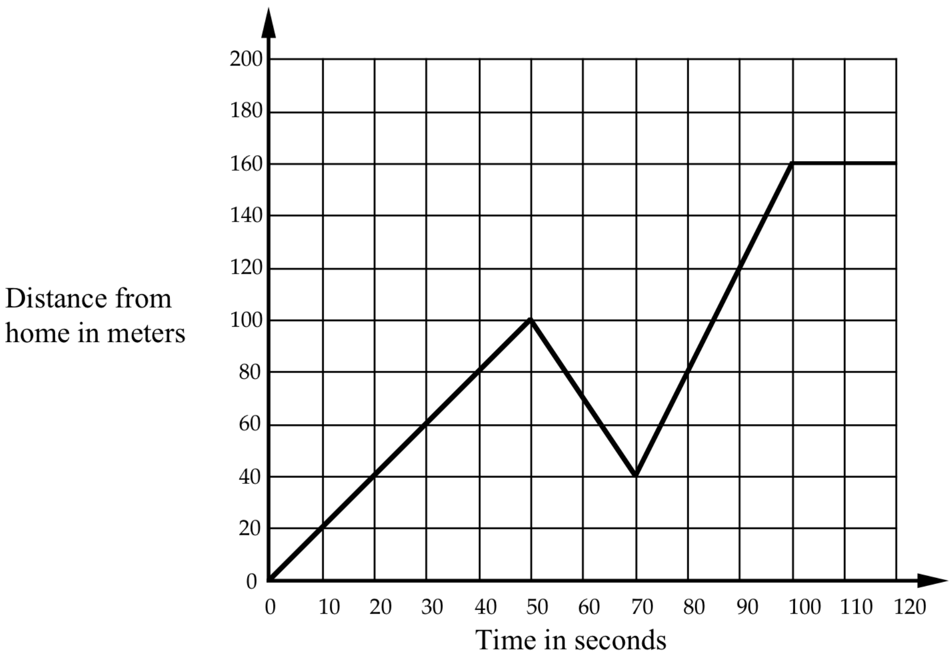
Collaborative activity

Graph	Interpretation	Table	Graph	Interpretation	Table
A	5	W	B	10	S
C	4	V	D	2	Q
E	6	T	F	3	
G	1	P	H	8	R
I	7	U	J	9	X



Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



1. Describe what may have happened.
   You should include details like how fast he walked.

....

....

....

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....

....

....

2. Are all sections of the graph realistic? Fully explain your answer.

....

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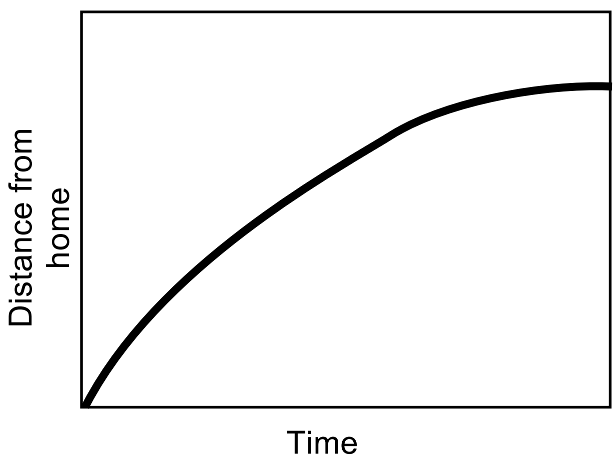
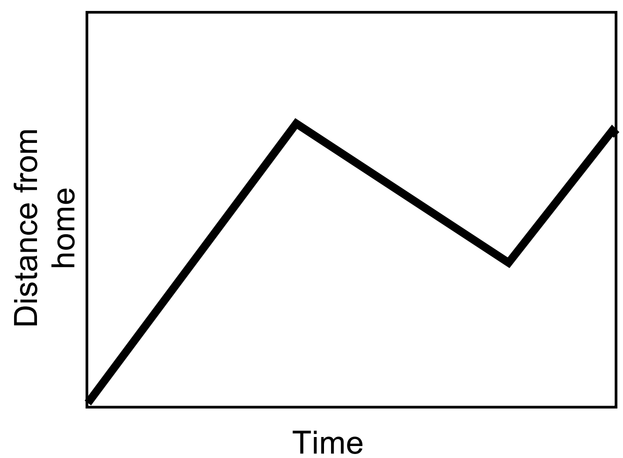
Card Set A: Distance–Time Graphs

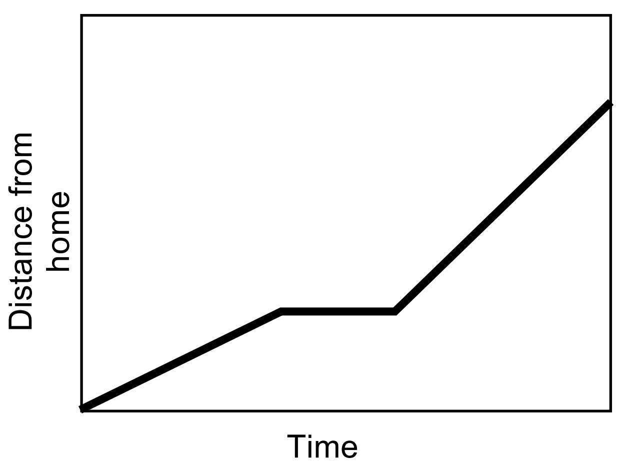
A



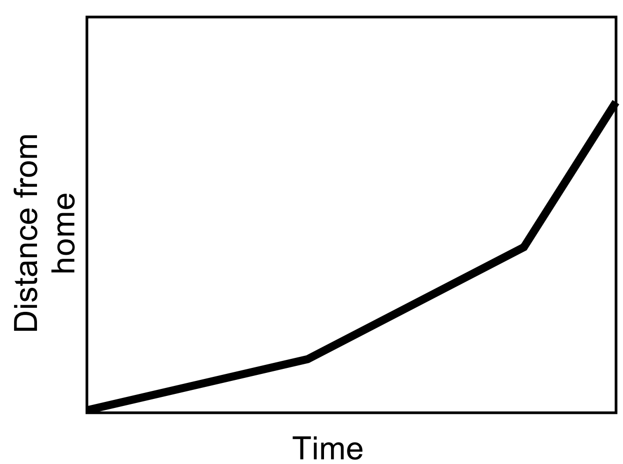
B



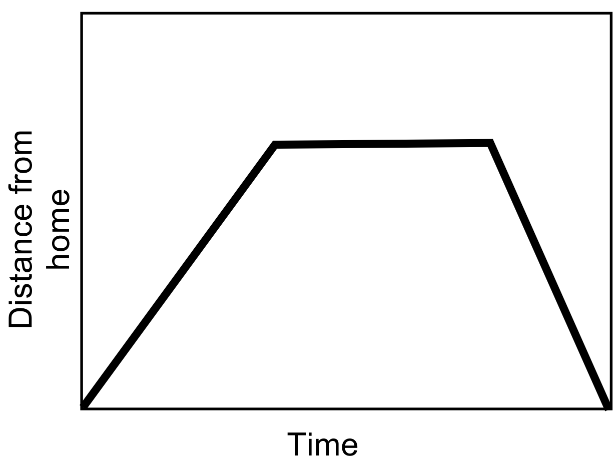
C



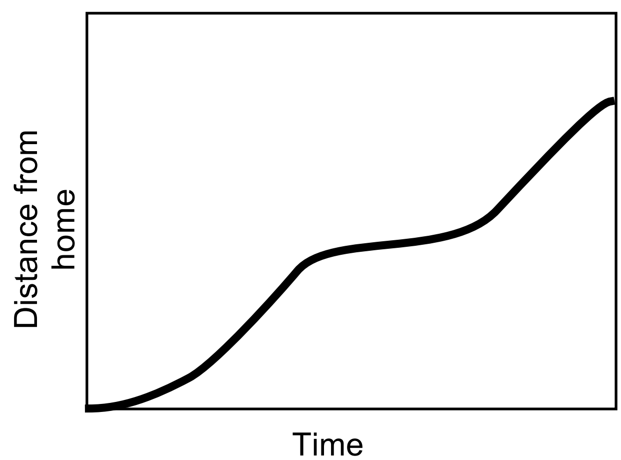
D



E

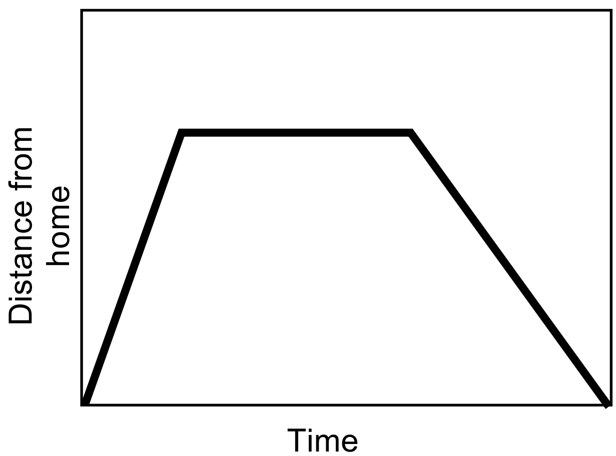


F

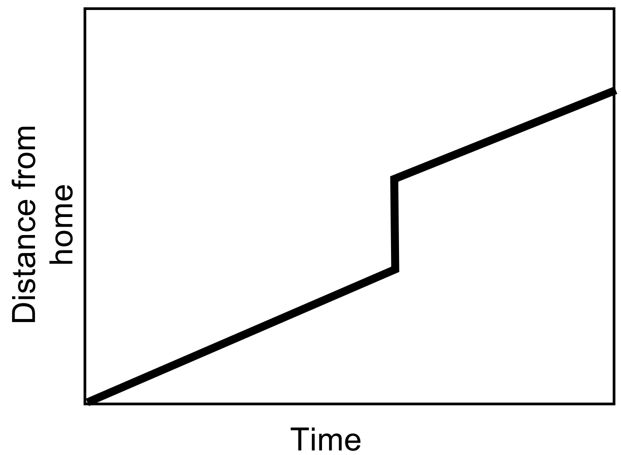


Card Set A: Distance–Time Graphs (continued)

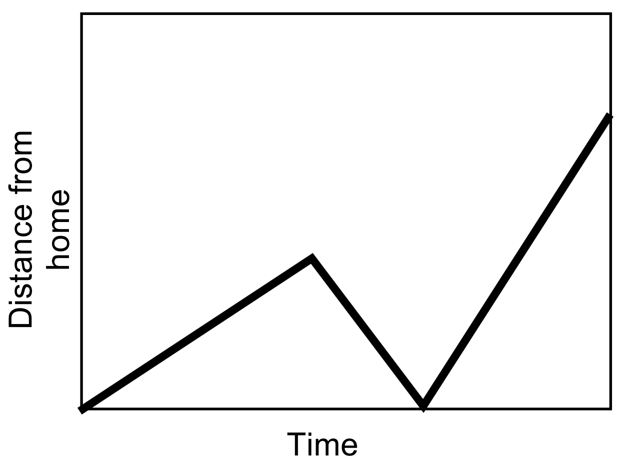
G



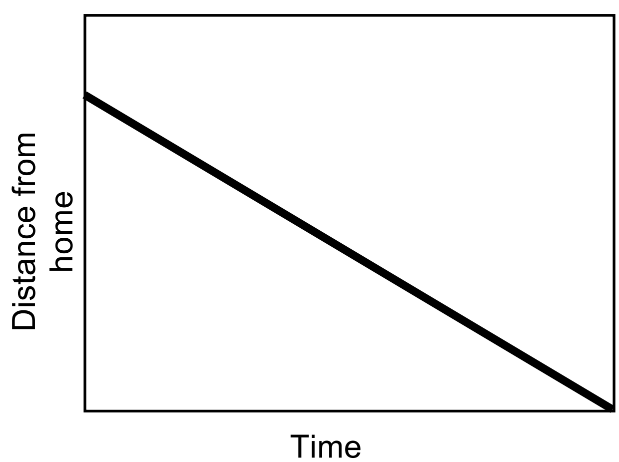
H



I



J



Card Set B: Interpretations

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	**2** Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	**4** Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	**6** Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	**8** This graph is just plain wrong. How can Tom be in two places at once?
9 After the party, Tom walked slowly all the way home.	**10** Make up your own story!

Card Set C: Tables of Data

P

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Q

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

R

Time	Distance
0	0
1	18
2	36
3	54
3	84
5	120

S

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

T

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

U

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

V

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

W

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

X

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

Y

Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Z

Make this one up!

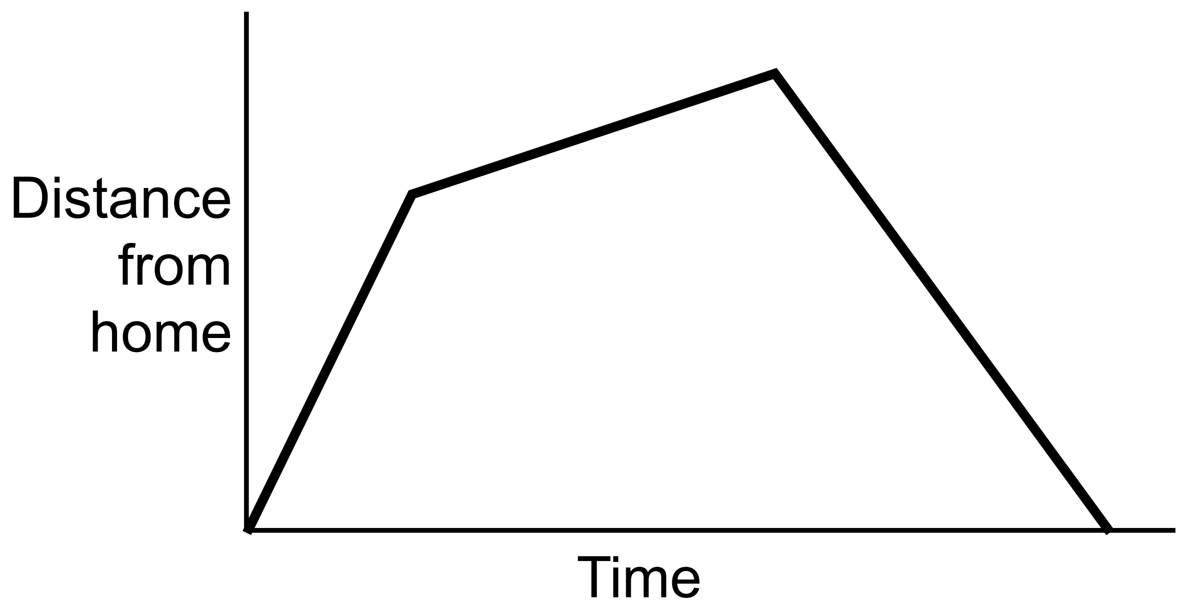
Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Matching a Graph to a Story

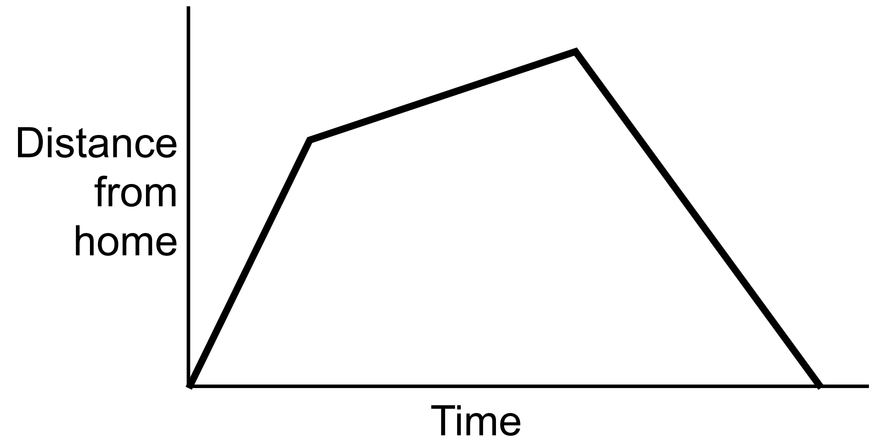
A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

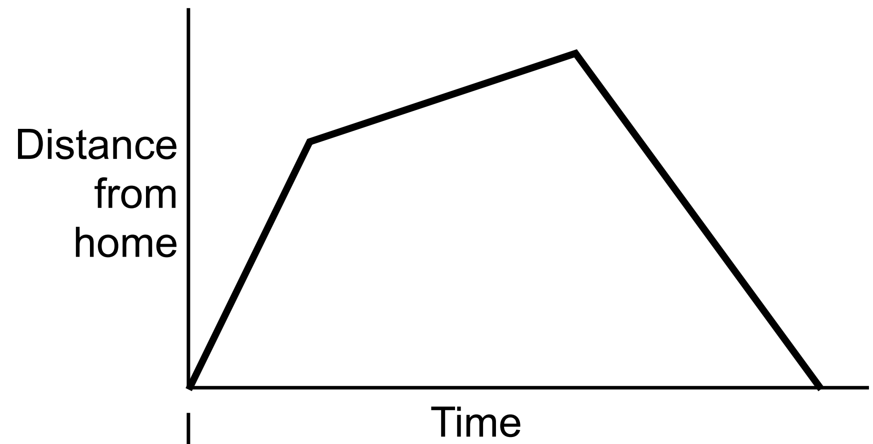
C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



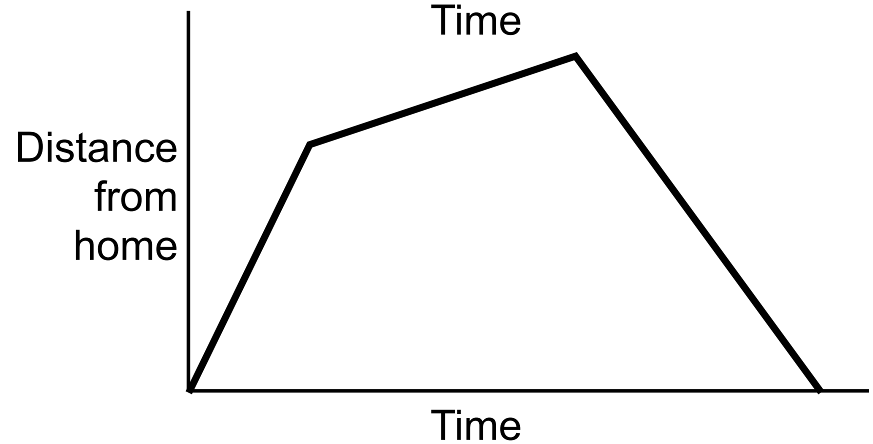
A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.



B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.



C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



Matching Cards

- Take turns at matching pairs of cards. You may want to take a graph and find a story that matches it. Alternatively, you may prefer to take a story and find a graph that matches it.

- Each time you do this, explain your thinking clearly and carefully. If you think there is no suitable card that matches, write one of your own.

- Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.

- Write your reasons for the match on the cards or the poster just as we did with the example in class. Give explanations for each line segment.

- Make sure you leave plenty of space around the cards as, eventually, you will be adding another card to each matched pair.

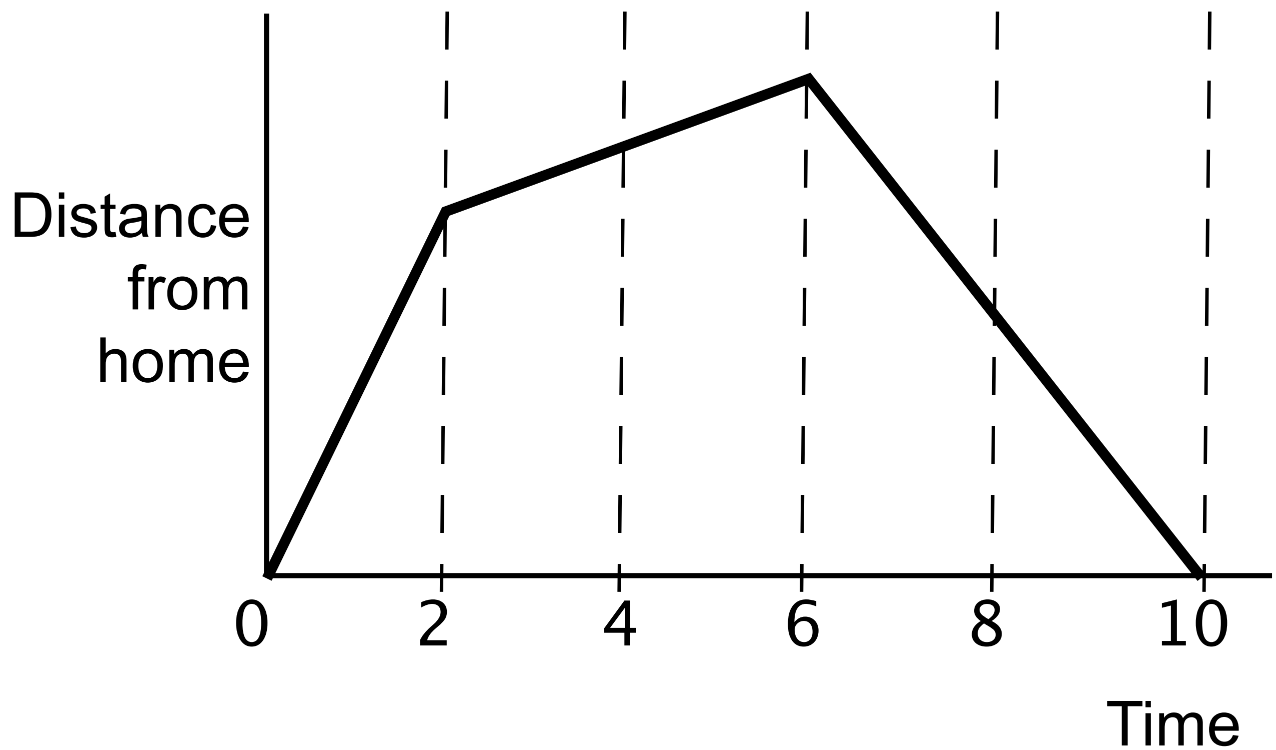
Sharing Work

- One student from each group is to visit another group’s poster.

- If you are staying at your desk, be ready to explain the reasons for your group's matches.

- If you are visiting another group:
  - Write your card placements on a piece of paper.
  - Go to another group’s desk and check to see which matches are different from your own.
  - If there are differences, ask for an explanation. If you still don’t agree, explain your own thinking.
  - When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.

Making Up Data for a Graph



Time	Distance
0	
2	
4	
6	
8	
10	

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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at the
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It was refined on the basis of reports from teams of observers led by
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based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
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