Stabilizing Hybrid Data Traffics in Cyber Physical Systems with Case Study on Smart Grid

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Abstract— **In many cyber physical systems such as smart grids, communications are of key importance for controlling or optimizing the operation of physical dynamics. It is possible that the realtime measurement traffic for controlling the physical dynamics overlays delay-tolerant queuing data flows such as Internet service in the same communication infrastructure. The scheduling of the hybrid system is then important for the performance of both realtime control and data throughput. To balance both controls of the physical dynamics and queuing dynamics, they are integrated within the same framework of stochastic optimization, which is solved using approximate algorithm. The proposed algorithm is then applied in the context of smart grids and is demonstrated to achieve good performance over simple scheduling algorithms that are not aware of the physical dynamics state.**

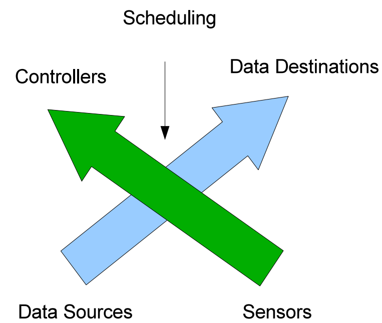

Fig. 1: An illustration of hybrid traffics.

I. INTRODUCTION

In recent years, cyber physical systems (CPS's) [3] have received substantial studies due to the wide applications in practice such as smart grids and robotic networks. In a typical CPS, sensors are used to monitor physical dynamics, whose reports are sent to controller(s) to achieve system stability and manipulate the system state to a desired one. When the sensors and controllers are not located at the same place, communications are needed for conveying the reports from the sensors. When the realtime requirement of monitoring and control is high, the communication link could be the bottleneck of the system. Hence, there have been more and more studies on the communication system design in CPS for the purpose of realtime control [1] [5] [12].

Many existing studies on the communications in CPS implicitly assume that the communication network serves only the CPS; e.g., in the advanced metering infrastructure (AMI), it is usually assumed that the AMI carries only the data of power consumptions and power price. However, in practice, such an exclusive usage of the communication network could be inefficient. Since the data rate of CPS could be time varying and less than the communication capacity, the communication network can also be used to convey other data traffics, such as Internet service, when the data rate requirement of CPS is below the network capacity. Therefore, there is a pressing need to study how to coordinate the CPS traffic and normal data traffic (or more generally, realtime data traffic for control and delay-tolerant data traffic), as illustrated in Fig. 1. To our best knowledge, there have not been studies on this topic, although there have been extensive studies on delay-tolerant traffic [11] and some studies on the CPS communication for control [7]. The problem of resource sharing between CPS traffic and delay-tolerant traffic has just emerged; however, it is important for further development of communication systems and CPS.

On the other hand, there could be both online and offline data in CPS itself. For example, in the context of communication network for phasor measurement units (PMUs) in smart grids, some data needs to be realtime such that the controllers can take quick actions to stabilize the power grids upon contingencies; meanwhile, other measurements may need to be transmitted to some storage centers in an offline manner for future analysis of system operations. Hence, it is important to balance both online and offline data traffics.

In this paper, we carry out a first-cut study by assuming simple (but widely used in theoretical study and practice) models for both CPS and delay-tolerant data traffic. Our principle is to formulate the traffic scheduling problem as a stochastic control problem, in which the physical dynamics are described using difference equation[1]. The CPS communication is modeled as the pattern of physical dynamics in the framework of hybrid systems [6]; meanwhile the delay-tolerant traffic is modeled as a constraint of the control in terms of the Lyapunov drift [11]. Note that the hybrid system framework for the CPS communication and the Lyapunov drift for delay-

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[1] In this paper, we consider discrete time system. If the system is a continuous-time one, the physical dynamics can be described using differential equations.

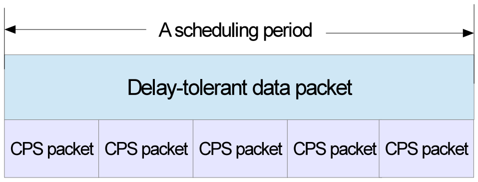

Fig. 2: An illustration of the timing structure when $M = 5$.

tolerant traffic have been studied separately. In this paper, we are the first to integrate both principles. The stochastic control problem will be solved by using Dynamic Programming (DP) and then further simplified using approximations. Then, we apply the algorithms in the scenario of smart grid.

The remainder of the paper is organized as follows. We introduce the system model in Section II. Then, we formulate the problem as a stochastic control problem, solve and simplify it in Section III. Numerical results and conclusions are provided in Sections IV and V, respectively.

II. System Model

In this section, we introduce the system model, which includes the delay-tolerant data network and the CPS. The timing structure of the system is illustrated in Fig. 2.

A. Communication Network Model

Consider a time slotted communication network in which each time slot lasts time T_s. We assume that there are N time synchronized communication nodes in the network with L communication links, which can be represented by a graph. Similarly to most existing literatures, the communication constraints of the links can also be represented by a graph, in which each node stands for a communication link and each edge means that the two incident nodes (thus the two communication links) cannot be used simultaneously due to co-channel collision. There are F delay-tolerant data flows. If a transmission succeeds, a data packet can be conveyed over a communication link in one time slot. There is no delay requirement on these data flows. There could be random transmission errors that are memoryless. We denote by q_{fn} the queue length of flow f at node n. The overall queuing situation is denoted by a vector $\mathbf{q}$.

Besides supporting the delay-tolerant data flows, the network also supports the operation of a CPS system. For simplicity, we assume that there is one centralized controller and one or more sensor in the CPS system[2]. Note that these sensors may or may not be co-located with the communication nodes for delay-tolerant data. The routing paths of the CPS system have been predetermined. When sensor n is scheduled and the corresponding communication links for CPS are activated, we denote by $\mathcal{I}_n$ the set of nodes in the delay-tolerant communication network that are interfered by the CPS communication (i.e., they are unable to transmit or receive). For simplicity of analysis, we ignore the delay from the sensors to the controllers, which is reasonable when the physical dynamics are much slower than the data transmission time; otherwise the analysis involving controlled dynamics with delay will be much more complicated. We further assume that there is no packet drop for the CPS communication; this is reasonable since the number of bits in each CPS measurement is usually small and can be protected by strong channel coding.

[2]The generic case of multiple distributed controllers is much more complicated. We will study it in the future.

B. Model of Physical Dynamics

We consider a discrete time model for the CPS system[3] in which each time interval lasts t_s. In each interval, an observation at the sensor is generated. We assume that t_s is smaller than T_s and $M = \frac{T_s}{t_s}$ is an integer; i.e., the time for delivering one data packet can be used to convey M CPS observation packets.

The dynamics of the CPS system can be written as

$$\begin{cases} \mathbf{x}(k+1) & = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{n}(k) \\ \mathbf{y}(k) & = \mathbf{C}\mathbf{x}(k) + \mathbf{w}(k) \end{cases}, \tag{1}$$

where $\mathbf{x}(k)$ is an N_x-dimensional system state at observation period k, $\mathbf{u}$ is an N_u-dimensional action taken by the controller, $\mathbf{y}$ is an N_s-dimensional observation, $\mathbf{n}$ and $\mathbf{w}$ are white random noise with covariance matrices Σ_{nn} and Σ_{ww}, respectively.

We consider the following quadratic cost function for the CPS system when the system state history is $\mathbf{x}(t)$ and the actions are $\mathbf{u}(t)$:

$$J_{CPS} = \sum_{k=1}^{T_f M} \left[\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + \mathbf{u}^T(k)\mathbf{P}\mathbf{u}(k) \right], \tag{2}$$

where T_f is the total number of time slots (recall that each time slot consists of M measurement samples for CPS), $\mathbf{Q}$ and $\mathbf{P}$ are both predetermined positive definite matrices.

III. Hybrid Traffic Scheduling

In this section, we assume that there is a centralized scheduler, which has the information of all current queue lengths $\mathbf{q}$ and the observations on the CPS dynamics $\mathcal{F}$. The scheduling is carried out for each time slot (i.e., the transmission time for one delay-tolerant data packet and M CPS observation packets). Hence, we also call the time slot a scheduling period. Although such a centralized scheduler is infeasible in practice, it provides insights for practical scheduling algorithms and an upper bound of performance, similarly to the study in [11]. We will first formulate the problem and then propose a scheduling algorithm.

A. Action and Strategy

The action of the scheduler is to schedule the activities of different communication links. For the CPS traffic, once a sensor is selected, the communication links along the path from the sensor to the controller will completely support

[3]It is easy to convert a continuous time system from a discrete time one, given the sampling rate.

the CPS observation data. If a communication link is not scheduled for the CPS data, it will be decided whether to transmit and, if yes, which packet to transmit. Hence, the action of each communication link (say link j at time slot t), denoted by $a_j(t)$, is given by

$$a_j(t) = \begin{cases} 0, & \text{if not scheduled} \\ n, & \text{if scheduled for CPS observations} \\ f, & \text{if scheduled for delay-tolerant traffic} \end{cases}, \quad (3)$$

where n is the selected sensor and f is the scheduled flow. The actions are stacked into an $L \times 1$ vector $\mathbf{a}$.

As we have assumed, the scheduler has the knowledge $\mathbf{s} = \{\mathcal{F}, \mathbf{q}\}$. We assume that the scheduler has a deterministic mapping from the system knowledge $\mathbf{s}$ to the action $\mathbf{a}$, which is the strategy of the scheduler and is denoted by π.

B. Problem Formulation

We assume that the scheduling decision is made at the beginning of each time slot. To characterize the stability of the delay-tolerant traffics, we define $\mathbf{T}$ as the set of the transient states of the queuing system and the hitting time as

$$\tau_{\mathbf{q}} = \begin{cases} \infty, & \text{if } \mathbf{q}(t) \in \mathbf{T}, \forall t > 0 \\ \min\{t > 0 : \mathbf{q}(t) \notin \mathbf{T}\} \end{cases}. \quad (4)$$

According to [11], the queuing system is stable if $P(\tau_{\mathbf{q}} < \infty) = 1$, $\forall \mathbf{q}$.

Then, we formulate the problem as minimizing the expected cost of the CPS system in (2) under the constraint of a stable queuing dynamics. Mathematically, it can be written as

$$\begin{aligned} & \min_{\pi} E\left[J_{CPS}\right] \\ s.t. \quad & P(\tau_{\mathbf{q}} < \infty) = 1, \forall \mathbf{q}, \end{aligned} \quad (5)$$

where the objective function is to minimize the expectation of the CPS cost function under the constraint of stable queuing process in the delay-tolerant traffics.

Remark 1: We have the following remarks on the problem formulation:

- Note that there could be many other policies for the co-existence of the delay-tolerant data flows and CPS observation flows. For example, some bandwidth is allocated to the CPS observation flows such that both types of flows can be transmitted simultaneously, which is more similar to frequency division duplexing (FDD). The rationale of the proposed co-existence scheme, which is more like time division duplexing (TDD), is that the CPS observation flows are triggered by only emergency situations (e.g., $\|\mathbf{x}\|$ becomes large) such that the TDD style transmission is more efficient.
- Note that the scheduling policy forms a hybrid system, in which there are two types of system states, namely the queuing states (discrete) and CPS state (continuous). The action considered in this paper is discrete. If we also design the control action $\mathbf{u}$ jointly, the action space will also become hybrid. Hence, we need to use the theory of hybrid systems to study this scheduling problem.

C. Cost Function

The key challenge of the scheduling algorithm is how to coordinate the CPS data traffics and the delay-tolerant traffics (i.e., when to dedicate the corresponding communication links to the CPS traffic and when to switch back to the delay-tolerant traffics). Although (5) provides an optimization formulation, it is difficult to solve directly. Hence, we have to resort to heuristic approaches.

As indicated in [11], the stability of the queuing system is dependent on the Laypunov function $V(\mathbf{q})$ that is a nonnegative function of the queuing lengths. As proved in Theorem 3.1 of [11], if there exists an $\epsilon > 0$ such that

$$E\left[V(\mathbf{q}(t+1)) - V(\mathbf{q}(t))\right] \leq -\epsilon, \quad (6)$$

where $V(\mathbf{q}(t+1)) - V(\mathbf{q}(t))$ is also called the Lyapunov drift [11], the queuing system is stabilized. Hence, we can replace the constraint in (5) with the inequality in (6). Since the scheduler at time 0, given the overall system state $\mathbf{s}(0)$, needs to consider both the CPS cost in the future T_f time slots and the Laypunov drift in the next time slot, we can reformulate the optimization problem in (5) as

$$\begin{aligned} & \min_{\pi} E\left[J_{CPS}\right] \\ s.t. \quad & E\left[V(\mathbf{q}(T_f)) - V(\mathbf{q}(0))\right] \leq -\epsilon. \end{aligned} \quad (7)$$

Note that the formulation of optimization problem is similar to the drift-plus-penalty strategy proposed in [9].

We can rewrite the constrained optimization in (7) into an unconstrained one, which is given by

$$\min_{\pi} E\left[J_{CPS} + \lambda V(\mathbf{q}(T_f))\right], \quad (8)$$

where $\lambda > 0$ is the Lagrange factor whose determination will be discussed later.

D. Hybrid System Formulation

The optimization of strategy in order to minimize the cost function in (8) can be fit into the framework of hybrid systems [13]. A linear switching system, as a special type of hybrid systems, has the following dynamics:

$$\begin{cases} \mathbf{x}(k+1) & = \mathbf{A}_{i_k}\mathbf{x}(k) + \mathbf{B}_{i_k}\mathbf{u}(k) + \mathbf{n}(k) \\ \mathbf{y}(k) & = \mathbf{C}_{i_k}\mathbf{x}(k) + \mathbf{w}(k) \end{cases}, \quad (9)$$

where there are K possible modes of dynamics and $i_k \in \{1, ..., K\}$ is the index of the dynamics at time slot k.

In the context of hybrid traffic scheduling in this paper, the matrices $\mathbf{A}$ and $\mathbf{B}$ do not change with time since they are determined by the physical power systems that are not changed by the scheduling. However, the matrix $\mathbf{C}$, which represents the measurement procedure, changes with the outcome of the scheduling. When sensor n is not scheduled, the rows corresponding to sensor n in matrix $\mathbf{C}$ can be considered as zeros since no observations are provided from the sensor. Hence, the index of $\mathbf{C}$ is determined by the scheduling action $\mathbf{a}(t)$, as illustrated in Fig. 3.

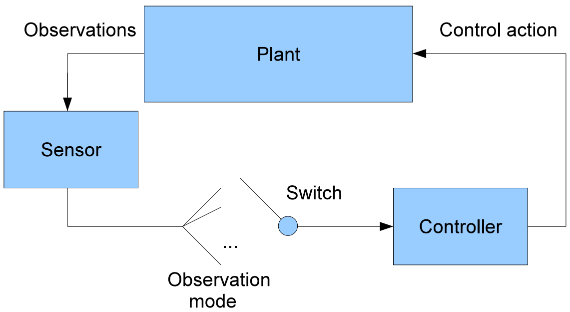

Fig. 3: An illustration of observation switching.

E. Optimal Scheduling Algorithm

Now, the scheduling problem is converted into the problem of scheduling the modes of dynamics in a hybrid system. Note that there have been many studies on the mode scheduling in hybrid systems; e.g., [10] adopted an integer programming based approach for continuous time systems. However, these studies consider only the system dynamics and do not incorporate co-existing systems such as the delay-tolerant traffic. In this paper, we will adopt the framework in [13] for a unified algorithm. In [13], the mode selection is jointly optimized with the control action in the framework of linear quadratic Gaussian (LQG) control, based on the principle of dynamic programming. The difference of our study is that what we control is the measurement system (i.e., the matrix $\mathbf{C}$) while [13] considers the control of matrices $\mathbf{A}$ and $\mathbf{B}$ (the system state is assumed to be observed directly in [13]).

1) Cost Within One Scheduling Period: Given an action $\mathbf{a}$ and previous CPS observations $\mathcal{F}_0$, we define the expected minimum cost of CPS within one scheduling period for the CPS:

$$\begin{aligned} J_c(\mathbf{a},\mathcal{F}_0) &= \min_{\mathbf{u}} E\left[\sum_{s=1}^{M}\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(s)+\mathbf{u}^T(s)\mathbf{P}\mathbf{u}(s)\Big|\mathbf{a},\mathcal{F}_0\right] \\ &= \mathbf{x}_{1|0}^T\mathbf{S}_1\mathbf{x}_{1|0}+trace\left[\mathbf{S}_1\mathbf{\Sigma}_{1|0}\right] \\ &+ \sum_{k=1}^{M} trace\left[\mathbf{S}_{k+1}\mathbf{\Sigma}_{ww}+\mathbf{S}_{k+1}^*\mathbf{\Sigma}_{k|k}\right], \end{aligned} \tag{10}$$

where the subscript c means CPS, and

$$\mathbf{S}_{k+1}^* = \mathbf{A}^T\mathbf{S}_{j+1}\mathbf{B}\left[\mathbf{P}+\mathbf{B}^T\mathbf{S}_{j+1}\mathbf{B}\right]^{-1}\mathbf{B}^T\mathbf{S}_{j+1}\mathbf{A}, \tag{11}$$

and

$$\mathbf{x}_{1|0} = E\left[\mathbf{x}(1)|\mathcal{F}_0\right], \tag{12}$$

$\mathbf{\Sigma}_{k|k}$ and $\mathbf{\Sigma}_{k|k-1}$ are the covariance matrices of the estimation of $\mathbf{x}(k)$, given observations until k and $k-1$ respectively, in the Kalman filtering (the expression can be found in Eq. (23) in [8]) and $\mathbf{S}_j$ satisfies the following Riccati equation:

$$\begin{aligned} \mathbf{S}_j &= \mathbf{A}^T\mathbf{S}_{j+1}\mathbf{A}-\mathbf{A}^T\mathbf{S}_{j+1}\mathbf{B}\left[\mathbf{P}+\mathbf{B}^T\mathbf{S}_{j+1}\mathbf{B}\right]^{-1} \\ &\times \mathbf{B}^T\mathbf{S}_{j+1}\mathbf{A}+\mathbf{Q}, \end{aligned} \tag{13}$$

and $\mathbf{S}_M=\mathbf{Q}$. Note that the second equation in (10) is cited from Eq. (24) in [8].

The different terms in (10) can be categorized into two groups:

- The terms $\mathbf{S}_k$, $\mathbf{S}_k^*$, $\mathbf{x}_{1|0}$, $\mathbf{\Sigma}_{ww}$ are independent of the observation matrix $\mathbf{C}$ and thus the action $\mathbf{a}$. They can be computed in advance.
- The matrices $\mathbf{\Sigma}_{k|k}$ and $\mathbf{\Sigma}_{k|k-1}$ are determined by the action $\mathbf{a}$.

Therefore, the cost of the CPS system can also be written as

$$J_c(\mathbf{a},\mathcal{F}_0) = trace\left[\mathbf{S}_1\mathbf{\Sigma}_{1|0}\right]+\sum_{k=1}^{M}trace\left[\mathbf{S}_{k+1}^*\mathbf{\Sigma}_{k|k}\right]+J^*, \tag{14}$$

where J^* is the remainder of the cost that is independent of the traffic scheduling.

We can also define the minimum cost of the queuing system, given the action $\mathbf{a}$ and the queuing state $\mathbf{q}(0)$; i.e.,

$$J_q(\mathbf{a},\mathbf{q}) = \min_s E\left[V(\mathbf{q}(1))-V(\mathbf{q}(0))\right], \tag{15}$$

where the subscript q stands for queuing and s is the queue scheduling policy given the queuing state $\mathbf{q}(0)$.

2) Dynamic Programming: We define the cost-to-go function for each scheduling period as

$$\begin{aligned} & J_t(\mathcal{F}_{t-1},\mathbf{q}(t-1)) \\ = & \min_{\pi} E\left[\sum_{s=(t-1)M+1}^{T_f M}\left[\mathbf{x}^T(s)\mathbf{Q}\mathbf{x}(s)+\mathbf{u}^T(s)\mathbf{P}\mathbf{u}(s)\right]\right. \\ & \left. +\lambda\sum_{r=t}^{T_f}(V(\mathbf{q}(r))-V(\mathbf{q}(r-1)))\Big|\mathcal{F}_{t-1},\mathbf{q}(t-1)\right], \end{aligned} \tag{16}$$

where $\mathcal{F}_{t-1}$ is the observations in time slot $t-1$ and $\mathbf{q}(t-1)$ is the queuing state in time slot $t-1$.

Obviously, when $t=T_f$ (i.e., the last scheduling period), we have

$$J_{T_f}(\mathcal{F}_{T_f},\mathbf{q}) = \min_{\mathbf{a}} J_c(\mathbf{a},\mathcal{F}_{T_f})+J_q(\mathbf{a},\mathbf{q}). \tag{17}$$

An efficient algorithm to obtain J_{T_f} is to carry out an exhaustive search for the scheduling of sensor (i.e., for all $\mathbf{a}$), and then compute J_c using (10) and compute J_q (given the remainder of communication links) using the optimal scheduling algorithm in pure data networks [11].

Suppose that J_{t+1} has been obtained. Then, we can compute J_t using the following Bellman's equation:

$$\begin{aligned} & J_t((\mathcal{F}_{t-1},\mathbf{q}(t-1))) \\ = & \min_{\mathbf{a}} J_c(\mathbf{a},\mathcal{F}_{t-1})+J_q(\mathbf{a},\mathbf{q}(t-1)) \\ + & E\left[J_{t+1}(\mathcal{F}_t,\mathbf{q}(t))|\mathcal{F}_{t-1},\mathbf{q}(t-1),\mathbf{a}\right]. \end{aligned} \tag{18}$$

The whole algorithm is summarized in Procedure 1.

F. Suboptimal Scheduling

The challenge of dynamic programming in (18) is how to evaluate the impact of the action on the expectation of the future cost (i.e., the term $E\left[J_{t+1}(\mathcal{F}_t,\mathbf{q}(t))|\mathcal{F}_{t-1},\mathbf{q}(t-1),\mathbf{a}\right]$).

Procedure 1 Procedure of Computing the Optimal Scheduling Law

```
for Each possible action do
    Compute the optimal cost within one scheduling period using (10).
end for
Compute J_{T_f} using (17)
for t = T_f − 1 : −1 : 1 do
    Use (18) to compute J_t based on J_{t+1}.
    Obtain the corresponding scheduling law.
end for
```

1) Representation of History: First, we need to explicitly express the history $\mathcal{F}_t$. We can represent $\mathcal{F}_t$ as a 2-tuple $(\mu_{Mt+1}, \mathbf{\Sigma}_{Mt+1})$ such that

$$p_{\mathbf{x}(Mt+1)}(\mathbf{x}) \propto \exp\left[-\left(\mathbf{x}-\mu_{Mt+1}\right)^T \mathbf{\Sigma}_{Mt+1}^{-1}\left(\mathbf{x}-\mu_{Mt+1}\right)\right]. \quad (19)$$

Then, given $\mathcal{F}_t$, it is easy to verify

$$\begin{aligned} & E\left[J_{t+1}(\mathcal{F}_t, \mathbf{q}(t))|\mathcal{F}_{t-1}, \mathbf{q}(t-1), \mathbf{a}\right] \\ = \; & E\left[J_{t+1}(\mathcal{F}_t^*(\mathcal{F}_{t-1}, \mathbf{a}), \mathbf{q}(t))|\mathbf{q}(t-1), \mathbf{a}\right], \quad (20) \end{aligned}$$

where $\mathcal{F}_t^*(\mathcal{F}_{t-1}, \mathbf{a})$ is a functional of $\mathcal{F}_{t-1}$ and $\mathbf{a}$, and the second expectation is with respect to the randomness of $\mathbf{q}(t)$. The corresponding expectation and variance matrices are given by

$$\mu_{Mt+1} = E\left[\mathbf{x}(Mt+1)|\mathcal{F}_{t-1}, \mathbf{a}\right], \quad (21)$$

and

$$\mathbf{\Sigma}_{Mt+1} = Var\left[\mathbf{x}(Mt+1)|\mathcal{F}_{t-1}, \mathbf{a}\right], \quad (22)$$

which can be computed using Kalman filtering. The details are omitted due to the limited space.

2) Monte Carlo Evaluation: In (20), we need to evaluate the expectation over all possible $\mathbf{q}(t)$. However, when M is large, it is difficult to enumerate all possible values of queue lengths after M time slots. Hence, we can use Monte Carlo simulations to numerically evaluate the expectation of $\mathbf{q}(t+1)$.

3) Quadratic Approximation: Even if we are able to reliably evaluate the expectation, it is still difficult to empirically evaluate the cost-to-go function J_{t+1}, since there are uncountably many states for $\mathcal{F}_{t+1}$ and infinitely many states for $\mathbf{q}(t+1)$. Similarly to [4], we can use quadratic approximation for J_{t+1}. In this paper, we assume

$$\begin{aligned} J_t(\mathcal{F}_t, \mathbf{q}(t))) \approx \; & \mathbf{x}_{Mt+1}^T \mathbf{\Omega}_t \mathbf{x}_{Mt+1} + trace\left[\mathbf{\Phi}_t \mathbf{\Sigma}_{Mt+1}\right] \\ + \; & \mathbf{q}(Mt+1)^T \mathbf{\Psi}_t \mathbf{q}(Mt+1), \quad (23) \end{aligned}$$

where $\mathbf{\Omega}_t$, $\mathbf{\Phi}_t$ and $\mathbf{\Psi}_t$ can be estimated from sufficiently many realizations of J_t. The details are omitted due to the limited space; the case for only $\mathbf{x}$ can be found in [4].

G. Adjustment of Lagrange Factor

Note that we fix the value of the Lagrange factor λ in the previous discussion. An improper λ may cause too much weight on either the CPS traffic or the delay-tolerant traffic. Hence, it is important to find a reasonable value for λ.

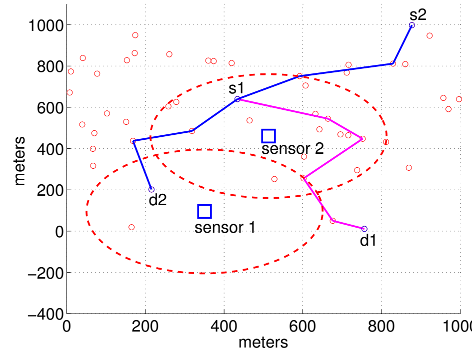

Fig. 4: An illustration of the deployment of network nodes, paths and sensors.

However, we have not found an explicit expression for λ due to the complicated expressions of the objective function and the constraints.

In this paper, we propose a simple algorithm to adaptively adjust the Lagrange factor λ. Recall that the constraint in the optimization of scheduling law is the stability of the delay-tolerant traffic. Hence, we can increase λ for the delay-tolerant traffic if the queues become unstable. The instability of the queues can be assessed by comparing the queue lengths with thresholds. Similarly to the congestion control in TCP, we can multiplicatively increase λ when the queues become unstable and linearly decrease λ when the queues are stable.

The details of the algorithm for adjusting the Lagrange factor λ are summarized in Procedure 2.

Procedure 2 Procedure of Adjusting the Lagrange Factor λ

```
Initialize λ; set thresholds γ_H and γ_L, (γ_H > γ_L), the scaling
    factor α > 1, the decrease step δλ > 0 and the minimum λ_min.
for Each scheduling period do
    if Any queue length is larger than γ_H then
        Set λ = αλ.
    end if
    if All queue lengths are smaller than γ_L then
        Set λ = max {λ_min, λ − δλ}.
    end if
end for
```

IV. Numerical Results

In this section, we use numerical simulations to demonstrate the performance of the proposed scheduling algorithms.

A. Setup

We randomly drop 50 nodes within a 1km×1km square. The maximum distance for communication is 250 meters. We assume that there are two delay-tolerant data flows (sources: s_1 and s_2; destinations: d_1 and d_2) within the network, and the shortest path routing is used to establish the flow paths. An illustration of the network is shown in Fig. 4.

We consider the voltage control in a microgrid with four distributed energy generators (DEGs). The details of the system dynamics, which is a 4-dimensional one, can be found in [4].

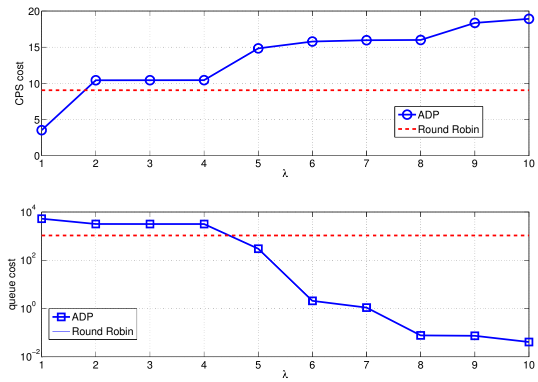

Fig. 5: Performance comparison between ADP and round robin scheduling.

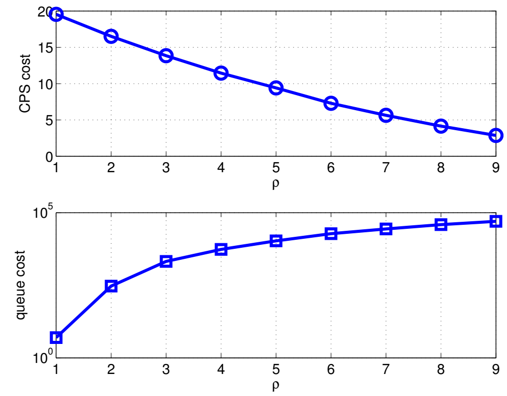

Fig. 6: Performance of probabilistic scheduling without the awareness of system state.

We assume that two wireless sensors are used to monitor the first two dimensions of the observations respectively, while the remaining two dimensions can be observed directly (e.g., they are located at the controller or there is a wired communication infrastructure for them). The locations of the two sensors are shown in Fig. 4. If one sensor is scheduled, the nodes within their interference range (marked by circles in the figure) have to stop transmitting and receiving. Hence, there are totally four possible actions for each scheduling period (scheduling or not for each of the wireless CPS sensors).

B. Performance of ADP

We used the algorithm in Procedure 1 to obtain the scheduling policy that is aware of the physical system state. Meanwhile, we tested the performances of the following two scheduling policies that are not aware of the physical dynamics state:

- Round-Robin Algorithm: The four actions are taken in turns.
- Probabilistic Algorithm: Each wireless sensor is activated with probability ρ.

The performance of the ADP algorithm in Procedure 1 is shown for various values of the weighting factor λ in Fig. 5. Only when the Lyapunov drift of queuing is close to zero, can the queue be stabilized. We observe that the queue is stabilized when λ is larger than 5. The minimum cost of the physical dynamics is around 15. The performance of the round-robin scheduling is also shown in Fig. 5. We observe that it cannot stabilize the queuing dynamics. The performance of the probabilistic scheduling is shown in Fig. 6, where the minimum cost under the constraint of queuing stability is about 20 (when $\rho = 0.1$). Hence, we demonstrated the performance gain of the proposed scheduling algorithm.

C. Adjustment of λ

We also tested the algorithm proposed in Procedure 2. Due to the limited space, we cannot show the trajectory of λ which is similar to the slow start mechanism in TCP. The numerical simulation shows that the algorithm can adjust λ such that both the CPS and queuing dynamics are stabilized.

V. Conclusions

We have studied the scheduling of hybrid data traffics serving both the control of CPS and a queuing communication network. The scheduling has been formulated as a hybrid system with the constraint of queuing stability. We have proposed an ADP based algorithm, whose performance gain over scheduling algorithms unaware of the physical dynamics state has been demonstrated by numerical simulations in the context of smart grid.

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