GAUSSIAN BEAM PROPAGATION

In most laser applications it is necessary to focus, modify, or shape the laser beam by using lenses and other optical elements. In general, laser-beam propagation can be approximated by assuming that the laser beam has an ideal Gaussian intensity profile, which corresponds to the theoretical TEM$_{00}$ mode. Coherent Gaussian beams have peculiar transformation properties which require special consideration. In order to select the best optics for a particular laser application, it is important to understand the basic properties of Gaussian beams.

Unfortunately, the output from real-life lasers is not truly Gaussian (although the output of a single mode fiber is a very close approximation). To accommodate this variance, a quality factor, M^2 (called the "M-squared" factor), has been defined to describe the deviation of the laser beam from a theoretical Gaussian. For a theoretical Gaussian, $M^2 = 1$; for a real laser beam, $M^2>1$. The M^2 factor for helium neon lasers is typically less than 1.1; for ion lasers, the M^2 factor typically is between 1.1 and 1.3. Collimated TEM$_{00}$ diode laser beams usually have an M^2 ranging from 1.1 to 1.7. For high-energy multimode lasers, the M^2 factor can be as high as 25 or 30. In all cases, the M^2 factor affects the characteristics of a laser beam and cannot be neglected in optical designs.

In the following section, Gaussian Beam Propagation, we will treat the characteristics of a theoretical Gaussian beam (M^2=1); then, in the section Real Beam Propagation we will show how these characteristics change as the beam deviates from the theoretical. In all cases, a circularly symmetric wavefront is assumed, as would be the case for a helium neon laser or an argon-ion laser. Diode laser beams are asymmetric and often astigmatic, which causes their transformation to be more complex.

Although in some respects component design and tolerancing for lasers is more critical than for conventional optical components, the designs often tend to be simpler since many of the constraints associated with imaging systems are not present. For instance, laser beams are nearly always used on axis, which eliminates the need to correct asymmetric aberration. Chromatic aberrations are of no concern in single-wavelength lasers, although they are critical for some tunable and multiline laser applications. In fact, the only significant aberration

in most single-wavelength applications is primary (third-order) spherical aberration.

Scatter from surface defects, inclusions, dust, or damaged coatings is of greater concern in laser-based systems than in incoherent systems. Speckle content arising from surface texture and beam coherence can limit system performance.

Because laser light is generated coherently, it is not subject to some of the limitations normally associated with incoherent sources. All parts of the wavefront act as if they originate from the same point; consequently, the emergent wavefront can be precisely defined. Starting out with a well-defined wavefront permits more precise focusing and control of the beam than otherwise would be possible.

For virtually all laser cavities, the propagation of an electromagnetic field, $E^{(0)}$, through one round trip in an optical resonator can be described mathematically by a propagation integral, which has the general form

$$E^{(1)}(x,y) = e^{-jkp} \iint_{Input\,Plane} K(x,y,x_0,y_0) E^{(0)}(x_0,y_0) dx_0 dy_0 \quad (5.1)$$

where K is the propagation constant at the carrier frequency of the optical signal, p is the length of one period or round trip, and the integral is over the transverse coordinates at the reference or input plane. The function K is commonly called the propagation kernel since the field $E^{(1)}(x, y)$, after one propagation step, can be obtained from the initial field $E^{(0)}(x_0, y_0)$ through the operation of the linear kernel or "propagator" $K(x, y, x_0, y_0)$.

By setting the condition that the field, after one period, will have exactly the same transverse form, both in phase and profile (amplitude variation across the field), we get the equation

$$\gamma_{nm} E_{nm}(x,y) \equiv \iint_{Input\,Plane} K(x,y,x_0,y_0) E_{nm}(x_0,y_0) dx_0 dy_0 \quad (5.2)$$

where E_{nm} represents a set of mathematical eigenmodes, and γ_{nm} a corresponding set of eigenvalues. The eigenmodes are referred to as transverse cavity modes, and, for stable resonators, are closely approximated by Hermite-Gaussian functions, denoted by TEM$_{nm}$. (Anthony Siegman, Lasers)

The lowest order, or "fundamental" transverse mode, TEM$_{00}$ has a Gaussian intensity profile, shown in figure 5.1, which has the form

$$I(x,y) \propto e^{-k\left(x^2+y^2\right)}$$

(5.3)

Figure 5.1   Irradiance profile of a Gaussian TEM$_{00}$ mode

In this section we will identify the propagation characteristics of this lowest-order solution to the propagation equation. In the next section, Real Beam Propagation, we will discuss the propagation characteristics of higher-order modes, as well as beams that have been distorted by diffraction or various anisotropic phenomena.

BEAM WAIST AND DIVERGENCE

In order to gain an appreciation of the principles and limitations of Gaussian beam optics, it is necessary to understand the nature of the laser output beam. In TEM$_{00}$ mode, the beam emitted from a laser begins as a perfect plane wave with a Gaussian transverse irradiance profile as shown in figure 5.1. The Gaussian shape is truncated at some diameter either by the internal dimensions of the laser or by some limiting aperture in the optical train. To specify and discuss the propagation characteristics of a laser beam, we must define its diameter in some way. There are two commonly accepted definitions. One definition is the diameter at which the beam irradiance (intensity) has fallen to $1/e^2$ (13.5%) of its peak, or axial value and the other is the diameter at which the beam irradiance (intensity) has fallen to 50% of its peak, or axial value, as shown in figure 5.2. This second definition is also referred to as FWHM, or full width at half maximum. For the remainder of this guide, we will be using the $1/e^2$ definition.

Diffraction causes light waves to spread transversely as they propagate, and it is therefore impossible to have
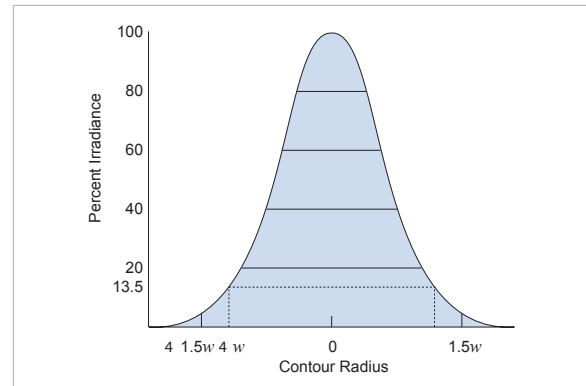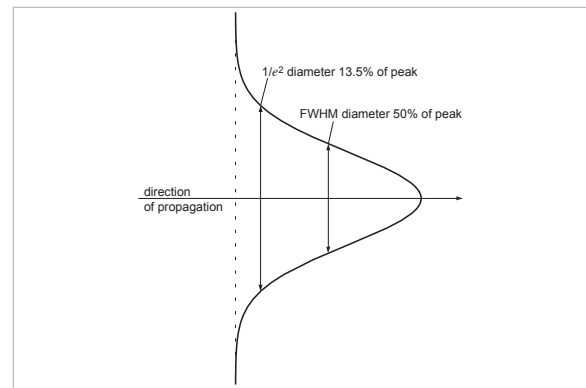

Figure 5.2   Diameter of a Gaussian beam

a perfectly collimated beam. The spreading of a laser beam is in precise accord with the predictions of pure diffraction theory; aberration is totally insignificant in the present context. Under quite ordinary circumstances, the beam spreading can be so small it can go unnoticed. The following formulas accurately describe beam spreading, making it easy to see the capabilities and limitations of laser beams.

Even if a Gaussian TEM$_{00}$ laser-beam wavefront were made perfectly flat at some plane, it would quickly acquire curvature and begin spreading in accordance with

$$R(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

(5.4)

and

$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2} \qquad (5.5)$$

where z is the distance propagated from the plane where the wavefront is flat, λ is the wavelength of light, w_0 is the radius of the $1/e^2$ irradiance contour at the plane where the wavefront is flat, $w(z)$ is the radius of the $1/e^2$ contour after the wave has propagated a distance z, and $R(z)$ is the wavefront radius of curvature after propagating a distance z. $R(z)$ is infinite at $z = 0$, passes through a minimum at some finite z, and rises again toward infinity as z is further increased, asymptotically approaching the value of z itself. The plane $z = 0$ marks the location of a Gaussian waist, or a place where the wavefront is flat, and w_0 is called the beam waist radius.

The irradiance distribution of the Gaussian TEM$_{00}$ beam, namely,

$$I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2} \ , \qquad (5.6)$$

where $w = w(z)$ and P is the total power in the beam, is the same at all cross sections of the beam.
The invariance of the form of the distribution is a special consequence of the presumed Gaussian distribution at $z = 0$. If a uniform irradiance distribution had been presumed at $z = 0$, the pattern at $z = \infty$ would have been the familiar Airy disc pattern given by a Bessel function, whereas the pattern at intermediate z values would have been enormously complicated.

Simultaneously, as $R(z)$ asymptotically approaches z for large z, $w(z)$ asymptotically approaches the value

$$w(z) = \frac{\lambda z}{\pi w_0} \qquad (5.7)$$

where z is presumed to be much larger than $\pi w_0 / \lambda$ so that the $1/e^2$ irradiance contours asymptotically approach a cone of angular radius

$$\theta = \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}. \qquad (5.8)$$

This value is the far-field angular radius (half-angle divergence) of the Gaussian TEM$_{00}$ beam. The vertex of the cone lies at the center of the waist, as shown in figure 5.3.

It is important to note that, for a given value of λ, variations of beam diameter and divergence with distance z are functions of a single parameter, w_0, the beam waist radius.

NEAR-FIELD VS FAR-FIELD DIVERGENCE

Unlike conventional light beams, Gaussian beams do not diverge linearly. Near the beam waist, which is typically close to the output of the laser, the divergence angle is extremely small; far from the waist, the divergence angle approaches the asymptotic limit described above. The Raleigh range (z_R), defined as the distance over which the beam radius spreads by a factor of $\sqrt{2}$, is given by

$$z_R = \frac{\pi w_0^2}{\lambda} \qquad (5.9)$$

At the beam waist ($z = 0$), the wavefront is planar [$R(0) = \infty$]. Likewise, at $z = \infty$, the wavefront is planar [$R(\infty) = \infty$]. As the beam propagates from the waist, the wavefront curvature, therefore, must increase to a maximum and then begin to decrease, as shown in figure 5.4. The Raleigh range, considered to be the dividing line between near-field divergence and mid-range
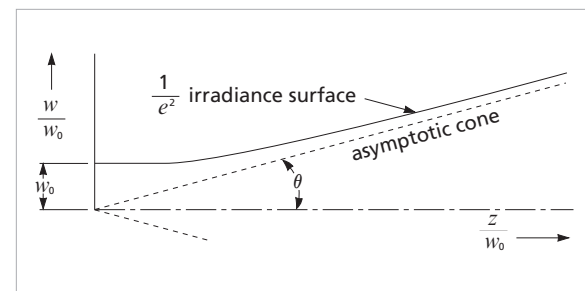

Figure 5.3   Growth in $1/e^2$ radius with distance propagated away from Gaussian waist

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divergence, is the distance from the waist at which the wavefront curvature is a maximum. Far-field divergence (the number quoted in laser specifications) must be measured at a distance much greater than z_R (usually >10 x z_R will suffice). This is a very important distinction because calculations for spot size and other parameters in an optical train will be inaccurate if near- or mid-field divergence values are used. For a tightly focused beam, the distance from the waist (the focal point) to the far field can be a few millimeters or less. For beams coming directly from the laser, the far-field distance can be measured in meters.

Typically, one has a fixed value for w_0 and uses the expression

$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}$$

to calculate $w(z)$ for an input value of z. However, one can also utilize this equation to see how final beam radius varies with starting beam radius at a fixed distance, z. Figure 4.5 shows the Gaussian beam propagation equation plotted as a function of w_0, with the particular values of λ = 632.8 nm and z = 100 m.

The beam radius at 100 m reaches a minimum value for a starting beam radius of about 4.5 mm. Therefore, if we wanted to achieve the best combination of minimum beam diameter and minimum beam spread (or best collimation) over a distance of 100 m, our optimum starting beam radius would be 4.5 mm. Any other starting value would result in a larger beam at z = 100 m.

We can find the general expression for the optimum starting beam radius for a given distance, z. Doing so yields

$$w_0 \left(\text{optimum} \right) = \left(\frac{\lambda z}{\pi} \right)^{1/2} \qquad (5.10)$$
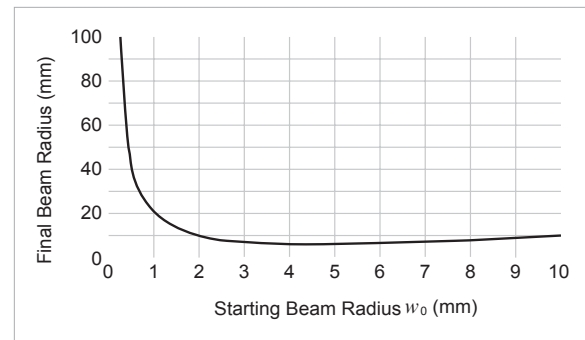

Figure 5.5   Beam radius at 100 m as a function of starting beam radius for a HeNe laser at 632.8 nm
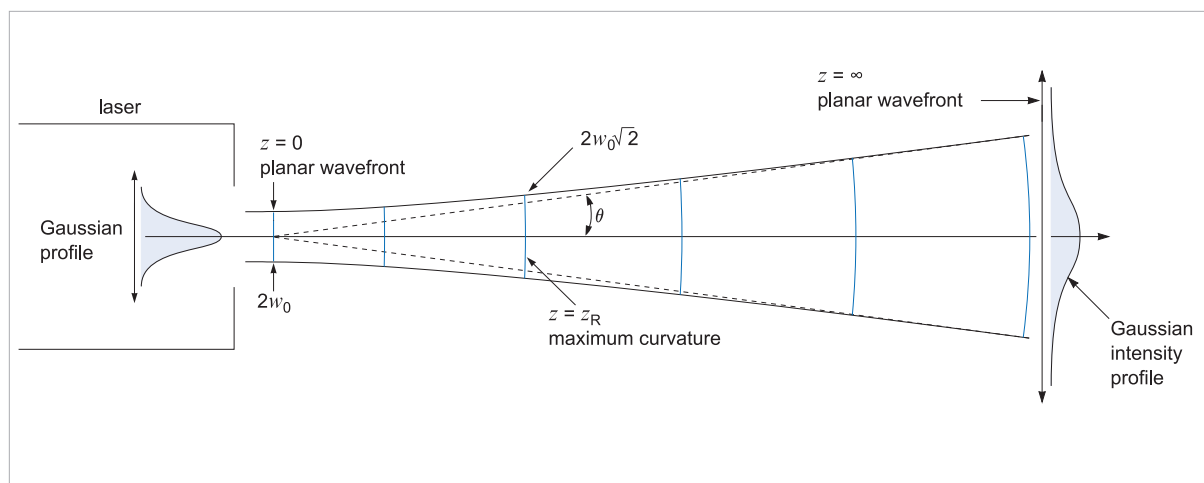

Figure 5.4   Changes in wavefront radius with propagation distance

Using this optimum value of w_0 will provide the best combination of minimum starting beam diameter and minimum beam spread [ratio of $w(z)$ to w_0] over the distance z. For $z = 100$ m and $\lambda = 632.8$ nm, w_0 (optimum) = 4.48 mm (see example above). If we put this value for w_0 (optimum) back into the expression for $w(z)$,

$$w(z) = \sqrt{2}\left(w_0\right) \tag{5.11}$$

Thus, for this example,

$$w(100) = \sqrt{2}\left(4.48\right) = 6.3 \text{ mm}$$

By turning this previous equation around, we find that we once again have the Rayleigh range (z_R), over which the beam radius spreads by a factor of $\sqrt{2}$ as

$$z_R = \frac{\pi w_0^2}{\lambda}$$

with

$$w\left(z_R\right) = \sqrt{2}w_0.$$

If we use beam-expanding optics that allow us to adjust the position of the beam waist, we can actually double the distance over which beam divergence is minimized, as illustrated in figure 5.6. By focusing the beam-expanding optics to place the beam waist at the midpoint, we can restrict beam spread to a factor of $\sqrt{2}$ over a distance of $2z_R$, as opposed to just z_R.

This result can now be used in the problem of finding the starting beam radius that yields the minimum beam diameter and beam spread over 100 m. Using $2(z_R) = 100$ m, or $z_R = 50$ m, and $\lambda = 632.8$ nm, we get a value of $w(z_R)$ = $(2\lambda/\pi)^{\frac{1}{2}}$ = 4.5 mm, and w_0 = 3.2 mm. Thus, the optimum starting beam radius is the same as previously calculated. However, by focusing the expander we achieve a final beam radius that is no larger than our starting beam radius, while still maintaining the $\sqrt{2}$ factor in overall variation.

Alternately, if we started off with a beam radius of 6.3 mm, we could focus the expander to provide a beam waist of w_0 = 4.5 mm at 100 m, and a final beam radius of 6.3 mm at 200 m.

APPLICATION NOTE

Location of the beam waist

The location of the beam waist is required for most Gaussian-beam calculations. CVI Laser Optics lasers are typically designed to place the beam waist very close to the output surface of the laser. If a more accurate location than this is required, our applications engineers can furnish the precise location and tolerance for a particular laser model.
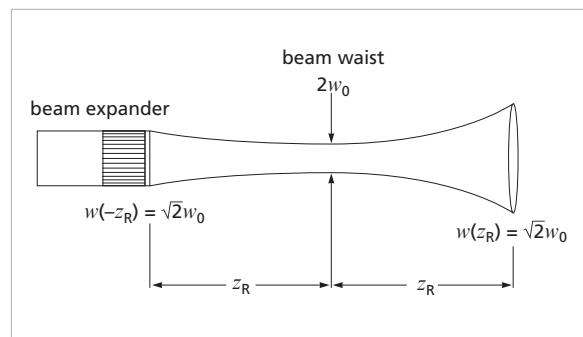

Figure 5.6   Focusing a beam expander to minimize beam radius and spread over a specified distance