Tensor Model of the Rotating Universe
Exercise in Special Relativity

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Abstract

We consider a hypothetical metric of space-time, which is shown to be a model of the universe in expansion. This is an exercise in Special Relativity with transformation from an inertial frame to a rotating frame with constant rotation.

1 Introduction

Three common systems of coordinates in physics are rectangular Cartesian $Oxyz$, cylinder $O\rho\varphi z$ and spherical coordinates $O\rho\theta\varphi$. Direct and inverse transformations from the first to the second are:

$$\begin{cases} x = \rho\cos\varphi \\ y = r\sin\varphi \\ z = z, \end{cases} \qquad \begin{cases} \rho = \sqrt{x^2+y^2} \\ \varphi = \operatorname{arctg}\frac{y}{x} \\ z = z. \end{cases} \tag{1}$$

The direct and inverse transformation from the Cartesian to the spherical:

$$\begin{cases} x = \rho\sin\theta\cos\varphi \\ y = \rho\sin\theta\sin\varphi \\ z = \rho\cos\theta, \end{cases} \qquad \begin{cases} \rho = \sqrt{x^2+y^2+z^2} \\ \theta = \arccos\frac{z}{\rho} \\ \varphi = \operatorname{arctg}\frac{y}{x}. \end{cases} \tag{2}$$

The Cartesian system allows position and direction in space to be represented in a very convenient manner, but in the case of rotation the advantage goes to the cylinder and on the spherical system coordinators. So here we are working with all three.

Length of the diagonal of the cuboid with edges infinitesimal length dx, width dy and height dz is:

$$dl^2 = dx^2 + dy^2 + dz^2. \tag{3}$$

It is the spatial Pythagorean theorem for infinitesimal Cartesian coordinate system of the three dimensions $\xi_1 = x$, $\xi_2 = y$ and $\xi_3 = z$. Let us introduce the fourth, time coordinate $\xi_4 = ict$, where imaginary unit is $i = \sqrt{-1}$ and speed of light in vacuum $c = 299\,792\,458$ m/s. This is the well-known quadratic term for the interval of space-time:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2. \tag{4}$$

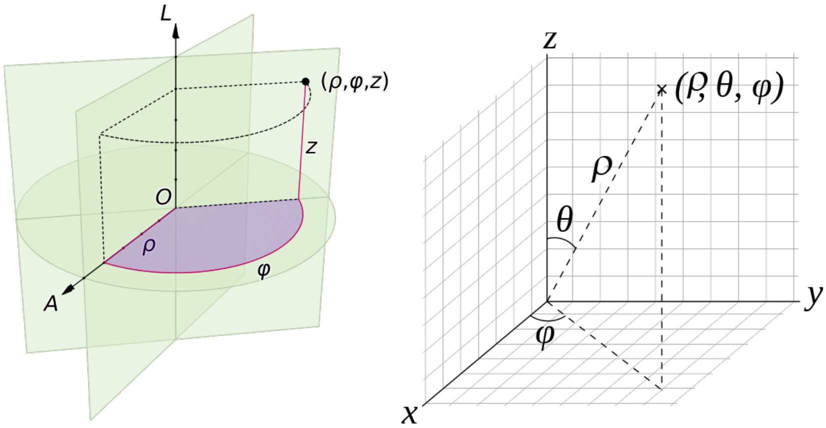

Figure 1: Cylindrical and spherical coordinates.

In Euclidean or non-Euclidean space solution of equations $ds = 0$ represents the geodesic lines, the shortest path distance between the infinitesimal points, or light paths.

By substituting the cylinder coordinates (1) and spherical coordinates (2) to (4), after simplification we have, respectively:

$$(ds)^2 = (d\rho)^2 + \rho^2 (d\rho)^2 + (dz)^2 - c^2 (dt)^2, \tag{5}$$

$$(ds)^2 = (d\rho)^2 + \rho^2 (d\theta)^2 + \rho^2 \sin^2\theta (d\varphi)^2 - c^2 (dt)^2. \tag{6}$$

Instead $(d\xi)^2$ it is common to write a short $d\xi^2$.

2 Rotation

When one cylindrical system coordinates $O\bar{\rho}\bar{\varphi}\bar{z}$ rotates in another $O\rho\varphi z$, around z axis by constant angular velocity ω, than we have transformations:

$$\rho = \bar{\rho}, \quad \varphi = \bar{\varphi} + \omega t, \quad z = \bar{z}. \tag{7}$$

Taking differentials ($\omega =$ const.) and substituting them in the expression for the space-time interval (5) we get:

$$ds^2 = d\bar{\rho}^2 + \bar{\rho}^2 (d\bar{\varphi} + \omega dt)^2 + d\bar{z}^2 - c^2 dt^2, \tag{8}$$

or

$$ds^2 = d\rho^2 + \rho^2 d\bar{\varphi}^2 + 2\frac{\omega\rho}{c}\,\rho d\bar{\varphi}^2\, cdt + dz^2 - \left(1 - \frac{\omega^2\rho^2}{c^2}\right) c^2 dt^2. \tag{9}$$

The tangential speed of a point that rotates on the distance ρ from origin is $v = \omega\rho$, and we expect it is always less than the speed of light c. So, the number $\beta = \frac{\omega\rho}{c}$ is less than one. For a short writing we also use the Lorenz coefficient:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{\omega\rho}{c}. \tag{10}$$

Gama is a real number not less than one.

Suppose that the system which does not rotate is inertial, and notice that the clock fixed to a point that rotates goes at a slower pace, according to the well-known expression for the relativistic time dilation $dt = \gamma dt_0$, or:

$$dt = \frac{dt_0}{\sqrt{1 - \frac{\omega^2\rho^2}{c^2}}}, \tag{11}$$

where dt_0 time elapsed on the clock at rest, and dt is the time measured on a clock that rotates.

To prove it we use the expression for the interval (9). The observer at the origin ($\rho = 0$, $z = 0$) also measures the elapsed time dt_0, while the clock which rotates ($d\bar{\rho} = 0$, $d\bar{\varphi} = 0$ and $d\bar{z} = 0$) measured dt. For the space-time intervals we have:

$$-c^2 dt_0^2 = -\left(1 - \frac{\omega^2\rho^2}{c^2}\right) c^2 dt^2.$$

Therefore (11).

Because of the tangential velocity $v = \omega\rho$ of the point that rotates, unlike the lack of movement in the direction perpendicular to the tangent of the circle, we also expect the length contraction, which would be consistent with the corresponding relativistic equation:

$$dl = dl_0\sqrt{1 - \frac{\omega^2\rho^2}{c^2}}. \tag{12}$$

The length dl_0 is fixed to a point that rotates and positioned tangentially to the rotation, and dl is the same length as measured by an observer at rest who viewed this rotation.

Combining the two relativistic effects (11) and (12), which arise due to the motion tangential velocity $v = \omega\rho$, we expect that it is possible to equation (9) transform into a canonical form as (5). Indeed, such a transformation is given at the end of the text [1] when done:

$$ds^2 = d\rho^2 + \frac{\rho^2 d\varphi^2}{1 - \frac{\omega^2\rho^2}{c^2}} + dz^2 - \left(1 - \frac{\omega^2\rho^2}{c^2}\right) c^2 dt^2. \tag{13}$$

where ω is some currently unknown constant.

Example 1. *Let us estimate the parameter ω.*

Solution. Let's say that the omega constant is such that the expression (13) or (8) can be applied right to the end of the visible universe.

The observable universe consists of the galaxies and other matter that can, in principle, be observed from Earth in the present day because light and other signals from these objects has had time to reach the Earth since the beginning of the cosmological expansion (quote from Wikipedia). The best estimate of the age of the universe as of 2013 is 13.798±0.037 billion years, it is stated in [2]. So, the radius of the visible universe is about $R_0 = 13.8$ billion light-years distance, centered at the point where we are. One light year is exactly 9 460 730 472 580 800 metres.

Therefore, the $1/\gamma^2 = 1 - \frac{\omega^2 r^2}{c^2}$ should be the number of 1 drops to zero, together with an increase in the distance r from zero to $R_0 = c/\omega$. Hence, $\omega = c/R_0$, or about:

$$\omega = \frac{3 \times 10^8}{(13.8 \times 10^9)(9.46 \times 10^{15})} = 2.3 \times 10^{-18}\ \mathrm{s}^{-1}. \tag{14}$$

So, it takes about 730 billion years for one rotation. □

The estimate (11), based on the size of the visible universe, has given very, very small angular velocity of rotation. However, on the edge of the universe, at a distance R_0 of us, tangential speed of a point would be very close to the speed of light.

Because of the way the universe was expanding and (11), every observer in the universe will see themselves as the center of rotation. Due to the contraction along the circle of rotation (12), in the case of extremely large distance R_0, we expect the boundary circle degenerate to (almost) a point.

3 Metric tensor

The expression (13) can be written:

$$(ds)^2 = \sum_{j,k=1}^{4} g_{jk} dx^j dx^k. \tag{15}$$

Using Einstein's summation convention, it can be written shorter:

$$(ds)^2 = g_{jk} dx^j dx^k, \quad j,k \in \{1,2,3,4\}. \tag{16}$$

This short writing is hereafter assumed. For each j,k it is $g_{jk} = g_{kj}$.

In the expression (16) the differentials of the coordinates are numbered by superscript, $dx^1 = dr$, $dx^2 = d\varphi$, $dx^3 = dz$, $dx^4 = ict$, with the multipliers in front which are the coefficients of the matrix:

$$\hat{g} = (g_{jk}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\rho^2}{1-\frac{\omega^2\rho^2}{c^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\frac{\omega^2\rho^2}{c^2} \end{pmatrix}. \tag{17}$$

Thus (17) defines two times *covariant* tensor of the second order, named *metric tensor*, which is typically a function of positions and so determines the *tensor field*. Differentials of the coordinates are *contravariant* vectors, the tensor of the first order, so they are written with indexes above.

If u^j and v^k are contravariant vectors $j,k = 1,2,3,4$, by replacing the coordinates $\bar{\xi}^m$ with ξ^n, again the upper indices $m,n = 1,2,3,4$, they are transformed according to the patterns:

$$\bar{u}^j = u^m \frac{\partial \bar{\xi}^j}{\partial \xi^m}, \quad \bar{v}^k = v^n \frac{\partial \bar{\xi}^k}{\partial \xi^n}. \tag{18}$$

Their product is then transformed according to the formula:

$$\bar{w}^{jk} = \bar{u}^j \bar{v}^k = u^m v^n \frac{\partial \bar{\xi}^j}{\partial \xi^m} \frac{\partial \bar{\xi}^k}{\partial \xi^n} = w^{mn} \frac{\partial \bar{\xi}^j}{\partial \xi^m} \frac{\partial \bar{\xi}^k}{\partial \xi^n}. \tag{19}$$

The result is two times contravariant tensor w with respect to the given affine coordinate system ξ^j.

Twice a contravariant metric tensor g is defined by

$$g^{jk} g_{kl} = \delta^j_l, \quad (\delta^j_l) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{20}$$

Where δ^j_l is Kronecker delta symbol and $(\delta^j_l) = \hat{I}$ is unit matrix of the fourth order. Thus, the matrix (g^{jk}) is inverse matrix of the (g_{jk}):

$$(g^{jk}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \left(1 - \frac{\omega^2\rho^2}{c^2}\right)/\rho^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/\left(1 - \frac{\omega^2\rho^2}{c^2}\right) \end{pmatrix}. \tag{21}$$

The matrix (g^{jk}) inverse to the matrix (g_{jk}) is called *inverse metric.*

Tensors describe linear relations between scalars, vectors, matrix, and other geometric objects. Examples of such relations are dot product, the cross product of the vectors, and linear maps like linear transformation of the coordinates. Because they express a relationship between vectors, tensors themselves must be independent of a particular choice of coordinate system. That's why they are convenient mathematical framework for formulating and solving problems in areas of physics such as elasticity, fluid mechanics, and general relativity.

Example 2. *Find transformation of equation (13) to (5).*

Solution. Note that there is no change in ρ or z coordinates. Next, suppose that:

$$\rho d\varphi = a_1 \rho d\bar{\varphi} + b_1 c d\bar{t}, \quad c dt = a_2 \rho d\bar{\varphi} + b_2 c d\bar{t}, \tag{22}$$

where a_j and b_j for $j = 1, 2$ are the unknown parameters. Substituting into (13) we find:

$$ds^2 = d\rho^2 + \gamma^2 (a_1 \rho d\bar{\varphi} + b_1 c d\bar{t})^2 + dz^2 - (a_2 \rho d\bar{\varphi} + b_2 c d\bar{t})^2/\gamma^2,$$

$$ds^2 = d\rho^2 + \left(\gamma^2 a_1^2 - a_2^2/\gamma^2\right) \rho^2 d\bar{\varphi}^2 + 2(\gamma^2 a_1 b_1 - a_2 b_2/\gamma^2)\, \rho d\bar{\varphi}\, c dt +$$
$$+ (\gamma^2 b_1^2 - b_2^2/\gamma^2) c^2 dt^2 + dz^2.$$

By equating these coefficients with (9), we obtain:

$$\gamma^2 a_1^2 - a_2^2/\gamma^2 = 1, \quad \gamma^2 a_1 b_1 - a_2 b_2/\gamma^2 = 0, \quad \gamma^2 b_1^2 - b_2^2/\gamma^2 = -1.$$

This is equivalent with:

$$a_1 = \pm\frac{1}{\gamma}\alpha, \quad b_1 = \pm\frac{1}{\gamma}\sqrt{\alpha^2-1}, \quad a_2 = \pm\gamma\sqrt{\alpha^2-1}, \quad b_2 = \pm\gamma\alpha, \tag{23}$$

where $\alpha \notin (-1,1)$ and the product of all four signs must be a plus.

For example, $\alpha = \gamma = \frac{1}{\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}$, with all signs plus. Than we have the transformations:

$$\rho d\varphi = \rho d\bar{\varphi} + \frac{\omega\rho}{c}\,cd\bar{t}, \quad cdt = \frac{\frac{\omega\rho}{c}\,\rho d\bar{\varphi} + cd\bar{t}}{1-\frac{\omega^2\rho^2}{c^2}}, \tag{24}$$

including $\rho = \bar{\rho}$ and $z = \bar{z}$. These are the inverse of the transformation cited in [1]. □

4 Christoffel symbols

Christoffel symbols [3], which are not tensors, are the numerical arrays that in coordinates describe parallel transport in curved surfaces or manifolds. The Christoffel symbols of the *first kind* are the three-index symbols that can be derived from the metric:

$$\Gamma_{jk,l} = \frac{1}{2}\left(\frac{\partial g_{kl}}{\partial x^j} + \frac{\partial g_{jl}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^l}\right). \tag{25}$$

It is always $\Gamma^l_{jk} = \Gamma^l_{kj}$.

Example 3. *Find the Christoffel symbols I for the metric (13) i.e. (17).*

Result. In our case (13), where $x^1 = \rho$, $x^2 = \varphi$, $x^3 = z$, $x^4 = ict$, we have:

$$\Gamma_{12,2} = \Gamma_{21,2} = \frac{\rho}{\left(1-\frac{\omega^2\rho^2}{c^2}\right)^2}, \quad \Gamma_{22,1} = -\frac{\rho}{\left(1-\frac{\omega^2\rho^2}{c^2}\right)^2},$$

$$\Gamma_{14,4} = \Gamma_{41,4} = -\frac{\omega^2\rho}{c^2}, \quad \Gamma_{44,1} = \frac{\omega^2\rho}{c^2}.$$

All others are zero. □

The Christoffel symbols of the *second kind* are also three-index:

$$\Gamma^m_{jk} = g^{ml}\Gamma_{jk,l}, \tag{26}$$

where is summed by index $l = 1, 2, 3, 4$.

Example 4. *Find the Christoffel symbols II for the metric (13) anent (21).*

Result. In our case (13) they which are not zero are:

$$\Gamma^2_{12} = \frac{1}{\rho}\frac{1}{1-\frac{\omega^2\rho^2}{c^2}}, \quad \Gamma^1_{22} = -\frac{\rho}{\left(1-\frac{\omega^2\rho^2}{c^2}\right)^2},$$

$$\Gamma^4_{14} = -\frac{\omega^2\rho}{c^2}\frac{1}{1-\frac{\omega^2\rho^2}{c^2}}, \quad \Gamma^1_{44} = \frac{\omega^2\rho}{c^2}.$$

It is also always $\Gamma^l_{jk} = \Gamma^l_{kj}$. □

As is known from the general theory of relativity, the differential equations of motion of *free particles* in a given field are:

$$\frac{d^2x^l}{ds^2} + \Gamma^l_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \quad j,k,l = 1,2,3,4, \tag{27}$$

The solution of these equations is called *geodesic curve.*

In the theory of relativity's space-time, called *Minkowski space*, four-dimensional velocities are often referred to $\frac{dx^j}{ds}$, $j = 1,2,3,4$. So $\frac{d^2x^l}{ds^2}$ is the gravitational acceleration in the direction of coordinate x^l. If we interpret g_{jk} as the gravitational potential, then the right-hand side of the equation of motion (27) multiplied by the mass of particles m, that is $m\Gamma^l_{jk}u^ju^k$, represents the force of gravity acting on the particle in a gravitational field. Consequently, equation (27) can be written in a shorter form:

$$\ddot{x}^l + \Gamma^l_{jk}\dot{x}^j\dot{x}^k = 0, \quad \dot{x}^j = \frac{dx^j}{ds}, \quad \ddot{x}^j = \frac{d\dot{x}^j}{ds}, \tag{28}$$

where $\dot{x}^j$ is the four-speed in direction j, and $\ddot{x}^j$ is the four-acceleration.

In our case (13), where $dx^1 = d\rho$, $dx^2 = d\varphi$, $dx^3 = dz$ and $dx^4 = icdt$, and using example 4, we get the equations:

$$\ddot{x}^2 + \Gamma^2_{12}\dot{x}^1\dot{x}^2 = 0, \quad \ddot{x}^1 + \Gamma^1_{22}(\dot{x}^2)^2 = 0,$$

$$\ddot{x}^4 + \Gamma^4_{14}\dot{x}^1\dot{x}^4 = 0, \quad \ddot{x}^1 + \Gamma^1_{44}(\dot{x}^4)^2 = 0.$$

By entering the Christoffel symbols and our coordinates, we have:

$$\ddot{\varphi} + \frac{\dot{\rho}\dot{\varphi}}{\rho\left(1-\frac{\omega^2\rho^2}{c^2}\right)} = 0, \quad \ddot{\rho} - \frac{\rho\dot{\rho}\dot{\rho}}{\left(1-\frac{\omega^2\rho^2}{c^2}\right)^2} = 0, \tag{29}$$

$$ic\ddot{t} - \frac{\omega^2\rho}{c^2}\frac{ic\dot{\rho}\dot{t}}{1-\frac{\omega^2\rho^2}{c^2}} = 0, \quad \ddot{\rho} - \omega^2\rho\dot{t}\dot{t} = 0. \tag{30}$$

The second equation in (29) has a form suitable for replacement $\dot{\rho} = u_\rho$. So:

$$\ddot{\rho} = \frac{du_\rho}{ds} = \frac{du_\rho}{d\rho}\frac{d\rho}{ds} = u_\rho\frac{du_\rho}{d\rho}.$$

Than we have the differential equation of the first order:

$$\frac{du_\rho}{u_\rho} - \frac{\rho d\rho}{\left(1 - \frac{\omega^2\rho^2}{c^2}\right)^2} = 0.$$

Hence, radial 4-velocity is:

$$u_\rho = \frac{d\rho}{ds} = \alpha \exp\left(\frac{c^2}{2\omega^2}\frac{1}{1 - \frac{\omega^2\rho^2}{c^2}}\right), \tag{31}$$

where α is a constant. No matter how small is the constant $\alpha \neq 0$, from the result (31) we see that material point in free fall at large distances $\rho \to R_0$ has very large radial velocity directed from the origin. From the second equation, in (29), we see that it has even greater the radial acceleration with the direction from the origin. The universe explodes.

Consider now a small environment of the given material point and insert result (31) in the first of the differential equations (29). We have:

$$\ddot{\varphi} + a\dot{\varphi} = 0, \quad a = \frac{u_\rho}{\rho\left(1 - \frac{\omega^2\rho^2}{c^2}\right)}, \tag{32}$$

wherein in a given environment the radial velocity u_ρ is approximately constant, that is $a \approx$ constant. Denote $u_\varphi = \frac{d\varphi}{ds}$ the angular 4-speed. Thus we have

$$\ddot{\varphi} = \frac{du_\varphi}{ds} = \frac{du_\varphi}{d\varphi}\frac{d\varphi}{ds} = u_\varphi\frac{du_\varphi}{d\varphi},$$

and the equation (32) becomes $u_\varphi \frac{u_\varphi}{d\varphi} + au_\varphi = 0$, or $du_\varphi + ad\varphi = 0$. Assuming that a is constant, we find:

$$u_\varphi + a\varphi = \text{const.} \tag{33}$$

Thus, the material point in free fall is moving with (approximately) constant angular velocity u_φ while moving away.

5 Double Rotation

Let's go back now to the spherical coordinates (2).

When one spherical system coordinates $O\bar{\rho}\bar{\theta}\bar{\varphi}$ rotates in another $O\rho\theta\varphi$, around z axis ($\theta = 0$) by constant angular velocity ω, than we have transformations:

$$\rho = \bar{\rho}, \quad \theta = \bar{\theta}, \quad \varphi = \bar{\varphi} + \omega t \tag{34}$$

Taking differentials ($\omega =$ const.) and substituting them in the expression for the space-time interval (6) we get:

$$ds^2 = d\bar{\rho}^2 + \bar{\rho}^2 d\theta^2 + \bar{\rho}^2 \sin^2\theta (d\bar{\varphi} + \omega dt)^2 - c^2 dt^2, \tag{35}$$

or

$$ds^2 = d\bar{\rho}^2 + \bar{\rho}^2 d\bar{\theta}^2 + 2\frac{\omega\bar{\rho}\sin\bar{\theta}}{c}\,\bar{\rho}\sin\bar{\theta}d\bar{\varphi}\,cdt - \left(1 - \frac{\omega^2\bar{\rho}^2\sin^2\bar{\theta}}{c^2}\right)c^2dt^2. \tag{36}$$

The tangential speed of a point that rotates on the distance ρ from origin is $\nu = \omega\rho\sin\theta$, and we expect it is always less than the speed of light c. So, the number $\beta = \frac{\omega\rho\sin\theta}{c}$ is less than one. For a shorter writing we use the coefficients:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \quad \nu = \omega\rho\sin\theta. \tag{37}$$

Measured in the overlaid coordinates, we take $\bar{\beta} \approx \beta$. In any case, gama is a real number not less than one.

Besides, suppose that z-axis ($\theta = 0$) rotates around the y-axis ($\varphi = \frac{\pi}{2}$) by the same constant angular velocity ω. Then the points on the sphere of radius ρ from the origin have about the same speed $\nu = \nu(\rho)$, which is proportional to the radius. So, we have the velocities $\nu = \omega_1\rho$, where $\omega_1 \approx \omega$ is an unknown approximately constant parameter.

From the point of view of the origin, which is in rest in an inertial system, we should notice that the clock fixed to a point that rotates goes at a slower pace, according to the expression for the relativistic time dilation $dt = \gamma dt_0$, or:

$$dt = \frac{dt_0}{\sqrt{1-\frac{\nu^2}{c^2}}}, \quad \nu = \omega_1\rho, \tag{38}$$

where dt_0 time elapsed on the clock at rest, and dt is the time measured on a clock that rotates.

It is easy to show by taking a circle rotation in the plane $\theta = \frac{\pi}{2}$ and the expression (38). Then generalize, taking such an arbitrary circle with the same radius.

Because of the tangential velocity $\nu = \omega_1\rho$ of the point that rotates, unlike the lack of movement in the orthogonal (radial) directions, we also expect the length contraction along the sphere, which would be consistent with the corresponding relativistic equation:

$$dl = dl_0\sqrt{1-\frac{\nu^2}{c^2}}. \tag{39}$$

The length dl_0 is fixed to a point that rotates and positioned tangentially to the rotation, and dl is the same length as measured by an observer at rest who viewed this rotation.

Combining the two relativistic effects (38) and (39), we expect it is possible the equation (36) be transformed into a canonical form:

$$ds^2 = d\rho^2 + \frac{\rho^2(d\theta^2 + \sin^2\theta d\varphi^2)}{1-\frac{\omega_1^2\rho^2}{c^2}} - \left(1-\frac{\omega_1^2\rho^2}{c^2}\right)c^2dt^2. \tag{40}$$

where ω_1 is some currently unknown constant approximately equal to the angular (unknown) velocity ω and $c \approx 3\times 10^8$ m/s is the speed of light. Here we can use the abbreviation

$$dl^2 = \rho^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

for comparison (39).

Lake in the example 1, assessing the angular velocity so that at the end of the visible universe $R_0 = 13.8$ billion light-years the rotation speed is almost equal to the speed of light, we find that the parameter ω_1 is approximately 2.3×10^{-18} Hz.

Example 5. *Let's find coordinate transformation that will line element (40) converted into (6).*

Solution. There are no radial changes and $d\rho = 0$, but we can expect changes in the infinitesimal dl, which lie on a sphere of radius ρ. Consider these change symmetrically, the same in all directions along the sphere. Accordingly, the expression (40) can be written in the form:

$$ds^2 = d\rho^2 + \frac{dl^2}{1 - \frac{\nu^2}{c^2}} - \left(1 - \frac{\nu^2}{c^2}\right) c^2 dt^2. \tag{41}$$

Putting

$$dl = a_1 d\bar{l} + b_1 c d\bar{t}, \quad cdt = a_2 d\bar{l} + b_2 c d\bar{t}, \tag{42}$$

where a and b are unknown parameters that depend on the ν, after a short calculation we find:

$$ds^2 = d\rho^2 + (\gamma^2 a_1^2 - a_2^2/\gamma^2)\, d\bar{l}^2 + 2(\gamma^2 a_1 b_1 - a_2 b_2/\gamma^2)\, d\bar{l}\, cd\bar{t} + (\gamma^2 b_1^2 - b_2^2/\gamma^2)\, c^2 d\bar{t}^2.$$

By equating this form with appropriate ($d\rho = d\bar{\rho}$):

$$ds^2 = d\bar{\rho}^2 + d\bar{l}^2 - c^2 d\bar{t}^2, \tag{43}$$

found the system of equations:

$$\gamma^2 a_1^2 - a_2^2/\gamma^2 = 1, \quad \gamma^2 a_1 b_1 - a_2 b_2/\gamma^2 = 0, \quad \gamma^2 b_1^2 - b_2^2/\gamma^2$$

which is equivalent to:

$$a_1 = \frac{1}{\gamma}\alpha, \quad b_1 = \frac{1}{\gamma}\sqrt{\alpha^2 - 1}, \quad a_2 = \gamma\sqrt{\alpha^2 - 1}, \quad b_2 = \gamma\alpha, \tag{44}$$

where $\alpha \in \mathbb{R} \setminus (-1, +1)$ is an arbitrary parameter. Substituting (44) into (42) we obtain the required transformation. □

Due to (44) it can be shown that expression (40) by itself means a rotation. On the other hand, the expression can be seen as a metric of a tensor field with the geodesic lines caused by effect like Coriolis and centrifugal force, as in article [1].

6 Conclusion

One seemingly not interesting, hypothetical metric (13), which due to the canonical form only at first glance does not look like a space-time which rotates, defines a model of the universe in accelerated expansion. I hope that further interpretation of the results, including further applying of the given Christoffel symbols, would be able to confirm or correct or reject the given model.

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