Loop Quantum Gravity

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String 08
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i. Loop Quantum Gravity

- Develops since the late 80s.

- About 200 people, 30 research groups.

- Several books (CR: "Quantum Gravity"):

The problem addressed:

<div style="color:red; text-align:center">

How to describe
the fundamental degrees of freedom
when there is no fixed background space

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Hypotheses

- A radical conceptual change in our concept of space and time is required.

- The problem can be addressed already in the context of *current* physical theory: general relativity, coupled with the standard model.

- In gravity, (unrenormalizable) UV divergences are consequences of a perturbation expansion around a wrong vacuum. → Confirmed *a posteriori* in LQG.

- Guiding principle: a symmetry: *Diffeomorphism invariance*.

Consistency with quantum mechanics and (in the low-energy limit) General Relativity, and full *diff-invariance* are extremely strong constraints on the theory.

Main result

→ Definition of *Diffeomorphisms invariant* quantum field theory (for gauge fields plus fermions), in canonical and in covariant form.

A comment on general relativity

- GR is: i) A *specific* field theory for the gravitational field $g_{\mu\nu}(x)$: $S[g] = \int d^4x \sqrt{-g}\,(R + \lambda)$

- ii) A *general* modification of our understanding of spacetime :

| spacetime | ⟷ | gravitational field |

Pre GR:

After GR:

spacetime   field   field   field
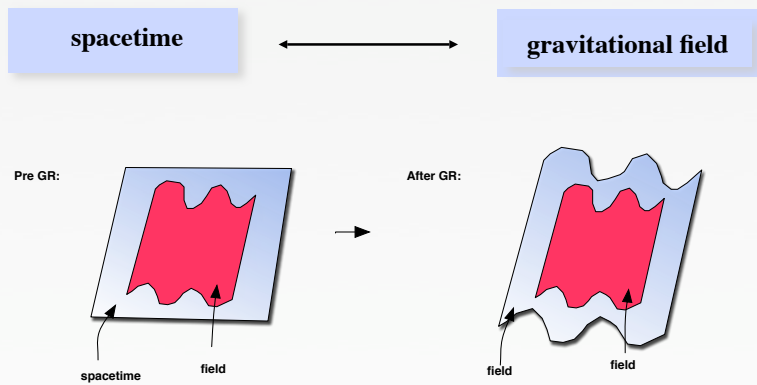
This modification is expressed by the invariance of the theory under the active action of the group of the *diffeomorphisms* on all the fields of the theory.

Why Loops ?

"Old" nonperturbative quantum gravity

Canonical (*Wheeler 64, DeWitt 63 ...*)

- *States*: $\Psi(\mathbf{q})$. $\mathbf{q}$: 3d metric of a t=0 surface

- *3d diff*: $\Psi(\mathbf{q}) = \Psi(\mathbf{q'})$ if there is a diff: $\mathbf{q} \rightarrow \mathbf{q'}$

- *Dynamics*: Wheler-DeWitt eq $\mathbf{H}\Psi(\mathbf{q}) = 0$.

difficulties**:**
- Which states $\Psi(\mathbf{q})$? Which scalar product $< \Psi, \Phi >$?
- Operator $\mathbf{H}$ badly defined and UV divergent.
- No calculation possible.

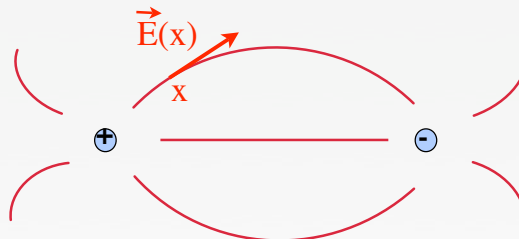### Covariant (*Misner 57, Hawking 79 ...*)

- $$Z = \int Dg \, e^{iS[g]}$$

difficulties**:**
- Integral very badly defined
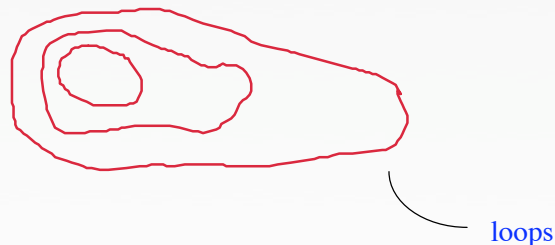- Perturbative calculations bring UV divergences back.

An old idea : Gauge fields are naturally "made up of lines"
(Polyakov, Mandelstan, Wilson, Migdal, ... Faraday)

Faraday lines:

In vacuum:

loops

The gravitational field too, can be described as a gauge field, with a connection as main variable. (Cartan, Weyl, Swinger, Utyama, ..., Ashtekar)

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Can we describe a quantum gauge-field-theory in terms of these lines?
Yes, on the lattice. (*Wilson, Kogut, Susskind, ...*)

Variable: U_e in the gauge group **G**

State: $\Psi(U_e)$ in $\mathcal{H} = L_2[G^{(\text{number of edges})}, d\mu_{\text{Haar}}]$

Operators: Magnetic field $B_{\text{Plaquette}}$: $(U_1 U_2 U_3 U_4)$
Electric field: $E_e = -i\, \hbar\, \partial/\partial\, U_e$ (Left invariant vector field)

Dynamics: $H = B^2 + E^2$

α: loop on the lattice

Loop state: $\Psi_\alpha(U_e) = \text{Tr}\,(\,U_1\,...\,U_e\,...\,U_N\,)$ $= \;\; < U_e\,|\,\alpha >$

→ $|\,\alpha >$ is an eigenstate of E_l with eigenvalue having support on α.

    $|\,\alpha >$ is a "quantum excitation of a single Faraday line".

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Generalization: spin-networks and spin-network states

Spin network $S = (\Gamma\,,\; j_l\,,\; i_n\,)$
    graph Γ
    spins j_l on links
    intertwiners i_n on nodes

spin network on the lattice: S

node

link

spin network S

Spin network state: $\Psi_S(U_e) = R^{(j_1)}(U_l)\,...\,R^{(j_L)}(U_L)\cdot i_1\,...\,i_N \;\; = \;\; < U_e\,|\,S >$

→ The spin network states $|\,S >$ form an othonormal basis in $\mathcal{H}$ (Peter-Weyl).

Can we use these loop states as a
basis of states in the continuum ?

No !

- $< \alpha \mid \alpha > = \infty$
  The loop states $\mid \alpha >$ in the continuum are *too singular*

- $< \alpha \mid \beta > = 0$ for β infinitely close to α

  States are *"too many", as a basis of the Hilbert space*.

- An *infinitesimal* displacement in space yields a *different* loop state

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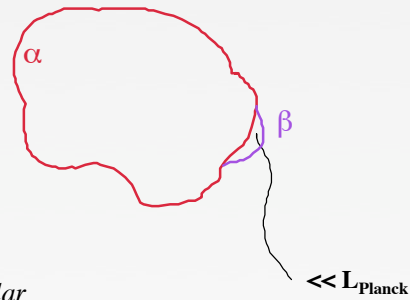
So far, just difficulties:

- "Old" nonperturbative quantum gravity does not work.

- A loop-state formalism in the continuum for Yang Mills does not work.

But the two ideas provide the solution to each other's stumbling blocks:

A *loop* formulation of *gravity*
solves both sets of difficulties

Diffeomorphism invariance :

$|S>$ and $|S'>$ are gauge equivalent if S can be transformed into S' by a diffeomorphism !

$S =$ $S' =$ $S'' =$

$< S \,|\, \Psi > \; = \; < S' \,|\, \Psi > \; = \; < S'' \,|\, \Psi >$ for any Ψ.

→ States are determined only by an *abstract* graph γ with j's and i's

s-knot states $|\, s > = \; |\, \gamma, j, i >$, where $s =$

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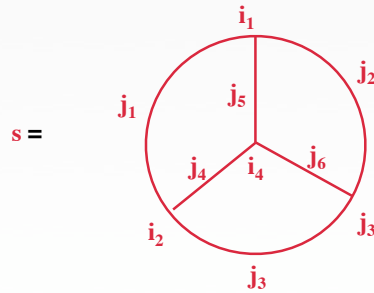
ii. The theory.

- Start from GR, or GR+standard model, or any other diff-invariant theory, in a formulation where the field is decribed by a connection A (Ashtekar).

- Do a canonical quantization of the theory, using a basis of spin network states and operators acting on these.

- Impose diffeomorphism invariance on the states.

- Study the Wheeler deWitt equation.

Result:

→ A (separable) Hilbert space $\mathcal{H}$ of states, and an operator algebra $\mathcal{A}$.

→ Basis of $\mathcal{H}$: abstract spin network states: graph labelled by spins and intertwiners.

→ A well defined UV-finite dynamics.

$\mathcal{H}$:

$\mathcal{H}_{\text{ext}}$: (norm-closure of the) space of the (cylindrical) functionals $\Psi_{\gamma_i, f}[A] = f(U_{\gamma_1}[A], ..., U_{\gamma_n}[A])$

where $U_\gamma[A] = e^{\int_\gamma A}$, equipped with the scalar product

$$(\Psi_{\gamma_i, f}, \Psi_{\gamma_i, g}) = \int_{SU(2)^n} dU \, \overline{f(U_1, ..., U_n)} \, g(U_1, ..., U_n)$$

→ This product is SU(2) and diff invariant. Hence $\mathcal{H}_{\text{ext}}$ carries a unitary representation of *Diff* and *local SU(2)*. The operation of factoring away the action of these groups is well-defined, and defines $\mathcal{H} = \mathcal{H}_{\text{ext}}/(Diff \text{ and } local SU(2))$.
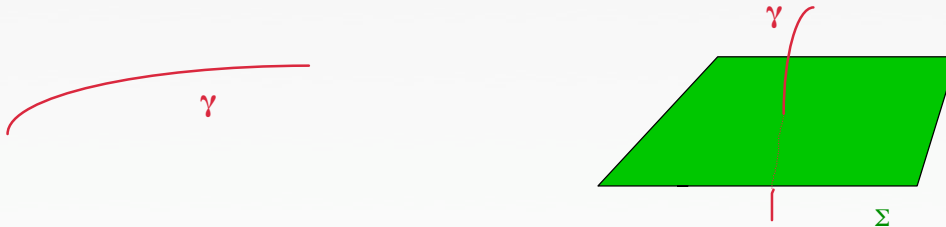
$\mathcal{A}$:

→ Operators well-defined on $\mathcal{H}_{\text{ext}}$, and self-adjoint: $U_\gamma[A] = e^{\int_\gamma A}$ and $E_\Sigma = \int_\Sigma E$

where E is the variable conjugate to A, smeared on a two-surface Σ. This acts as an SU(2) Left-invariant vector field on the SU(2) of each line that intersects Σ .

The LOST uniqueness theorem
(*Fleishhack 04; Lewandowski, Okolow, Sahlmann, Thiemann 05*)

- ($\mathcal{H}_{\text{ext}}$, $\mathcal{A}$) provides a representation of the classical poisson algebra of the observables $U_\gamma[A]$ and E_Σ , carrying a unitary representation of *Diff*,

- → this representation is *unique*.

(cfr.: von Neumann theorem in nonrelativistic QM.)

Interpretation of the spin network states $|\,S >$

S

Volume: V(R):   function of the gravitational field

$$V(R) = \int_R \sqrt{g} = \int_R \sqrt{EEE} \quad \longrightarrow \quad V(R) \text{ operator}$$

→ V(R) is a well-defined self-adjoint operator in $\mathcal{H}_{\text{ext}}$,

→ It has discrete spectrum. Eigenstates: spin networks state $|\,S >$.

- Eigenvalues receive a contribution for each <u>node</u> of $|\,S >$ inside R

R
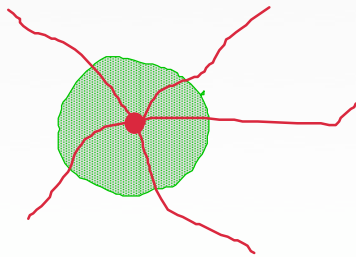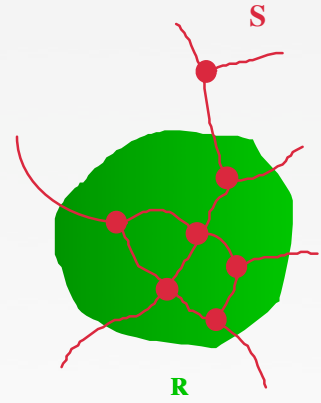
> **Node = "Chunk of space"**
> with quantized volume

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Area: A(Σ)

S

$$A(\Sigma) = \int_\Sigma \sqrt{g} = \int_\Sigma \sqrt{EE} \quad \longrightarrow \quad A(\Sigma) \text{ operator}$$

→ A(Σ) well-defined selfadjoint operator in $\mathcal{H}_{\text{ext}}$.
→ The spectrum is discrete.
→ Area gets a contribution for each <u>link</u> of $|\,S >$ that intersects Σ.

Σ
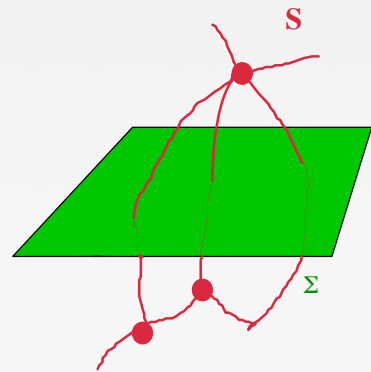
> **Area eigenvalues:** $A = 8\pi\,\hbar G\,\gamma \sum_i \sqrt{j_i(j_i+1)}$

(γ = Immirzi parameter)

j
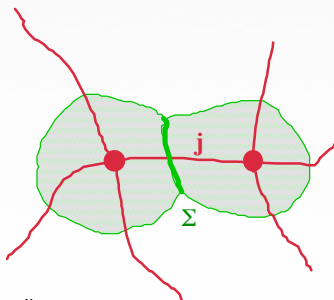
> **Link = "Quantum of surface"**
> with quantized area

Σ

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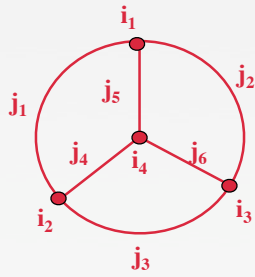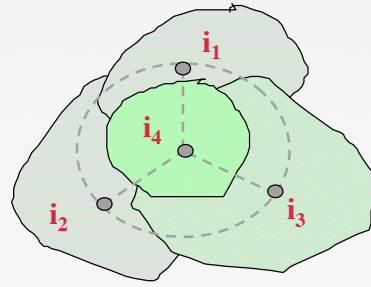
$$| \, S > \;=\; | \, \gamma \,,\, j_l \,,\, i_n >$$

s-knot state

connectivity between the elementary
quantum chunks of space

quantum numbers
of area

quantum numbers
of volume

→ Spin network states represent discrete quantum excitations of spacetime

- Spin networks are not excitations in space: they are excitations of space.
  → Background independent QFT

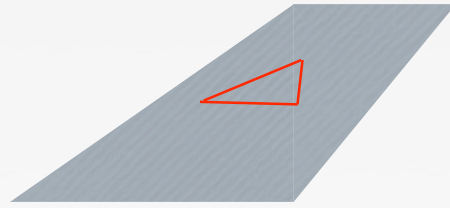- → **Discrete structure of space at the Planck scale** - in quantum sense

  Follows from:
  standard quantum theory (cfr granularity of oscillator's energy)
  + standard general relativity (because "space is a field").

Loops & strings: a cartoon comparison

If a string is:

a closed lines in space, that
forms matter and forces,

a loop is:

a closed line that forms space itself
as well as matter and forces.

III. The theory - dynamics

$$\mathbf{H(x)} \; = \; \mathbf{A}$$

Given by a Wheeler-deWitt operator $\mathbf{H}$ in $\mathcal{H}$: $\mathbf{H}\,\Psi = 0$

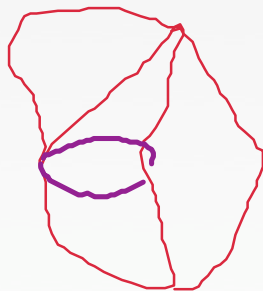• $\mathbf{H}$ is defined by a regularization of the classical Hamiltonian constraint. In the limit in which the regularization is removed.

→ $\mathbf{H}$ is a well defined self-adjoint operator, UV <u>finite</u> on diff-invariant states.

$$\mathbf{H}_\alpha \, (\mathbf{x}) \quad \sim \quad \alpha \to 0$$

$$\alpha \to 0$$

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$$\alpha \to 0$$

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$\alpha \to 0$

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$\alpha \to 0$

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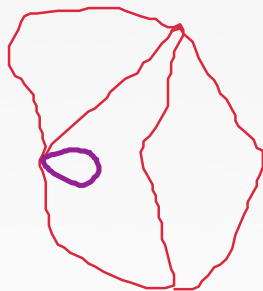
$$\alpha \rightarrow 0$$

The limit $\alpha \rightarrow 0$ is trivial because
there is no short distance structure at all in the theory !

• The theory is naturally ultraviolet finite

Matter

• YM, fermions
• Same techniques: The gravitational field is *not* special
• $\rightarrow$ UV finiteness remains
• YM and fermions on spin networks = on a Planck scale lattice !
   Notice: no lattice spacing to zero !
• Matter from braiding ?

i_n

(j_l , k_n)

γ
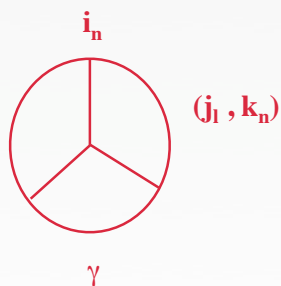
quantum numbers
of matter

$| s > = | \gamma , j_l , i_n , k_l >$

III. The theory - covariant dynamics: spinfoams
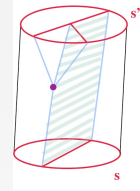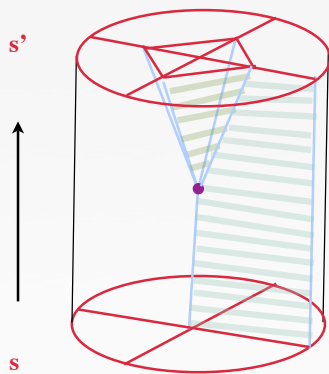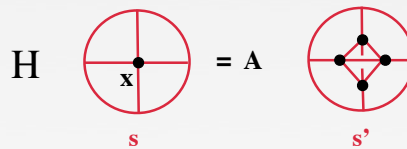

Projector $P = \delta(H)$ on the kernel of H:

$$\langle s'|P|s\rangle = \langle s'|\delta(H)|s\rangle = \langle s'|\int DN e^{iNH}|s\rangle$$

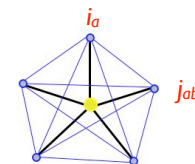$$= c_0\langle s'|s\rangle + c_1\langle s'|H|s\rangle + c_2\langle s'|H^2|s\rangle + ...$$
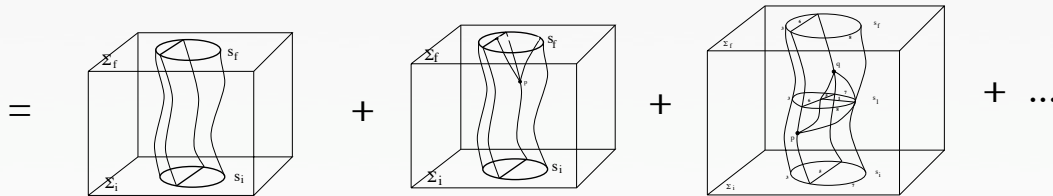
$$\langle s'|P|s\rangle = \sum_{\partial(\sigma,j_f,i_n)=(s\cup s')} \prod_f \dim(j_f) \prod_v A_v(j_f,i_n)$$

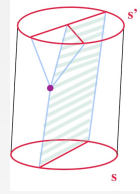Two-complex, colored with spins and intertwiners = spinfoam

Vertex amplitude: A(j,i)

$$A(j_{ab}, i_a) = \sum_{i_a^+, i_a^-} 15j\left(\frac{(1+\gamma)j_{ab}}{2}, i_a^+\right) 15j\left(\frac{|1-\gamma|)j_{ab}}{2}, i_a^-\right) \otimes_a f^{i_a}_{i_a^+, i_a^-}$$

Spinfoams

$$Z = \sum_{\sigma j_f i_v} \prod_f dim(j_f) \ \prod_v A_v(j_f, i_v)$$

$$A(j_{ab}, i_a) = \sum_{i_a^+, i_a^-} 15j\left(\frac{(1+\gamma)j_{ab}}{2}, i_a^+\right) 15j\left(\frac{|1-\gamma|)j_{ab}}{2}, i_a^-\right) \otimes_a f_{i_a^+, i_a^-}^{i_a}$$

- Can be directly derived from a discretization of the action of general relativity, on a *variable lattice*.

- Can be interpreted as a discrete version of Hawking's "integral over geometries"

$$Z = \int Dg \ e^{iS[g]}$$

- → **Z** is generated as the Feynman expansion of an auxiliary field theory defined on a group manifold. This is a 4d generalization of the matrix models (à la Boulatov, Ooguri).

- → In 3d, it gives directly the old Ponzano Regge model (A=6j). (With cosmological constant: Turaev-Viro state sum model.)

III. Physics

Loop cosmology

- Discrete cosmological time
- → Big Bang singularity removed (from Planck scale non-locality)
- → Evolution "across" the big bang (robust)
- → (Super-) inflationary behavior at small a(t)
- → scale invariant spectrum with the observed spectral index n_s.
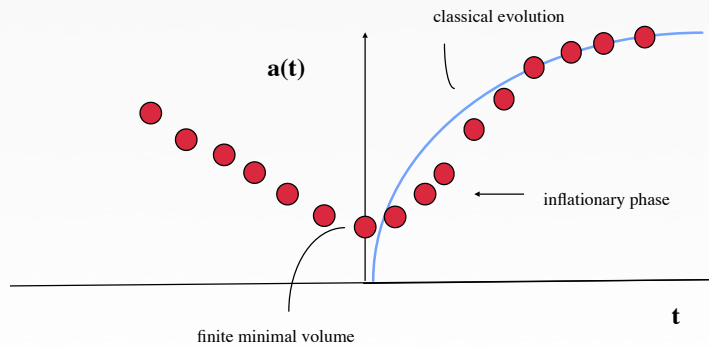
classical evolution
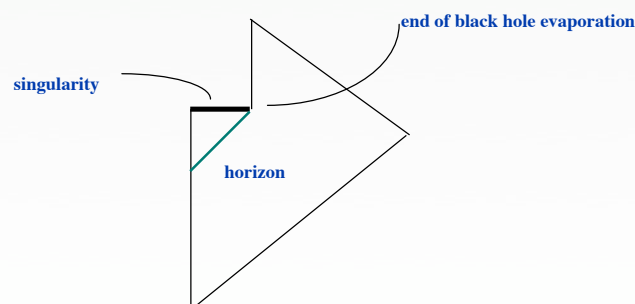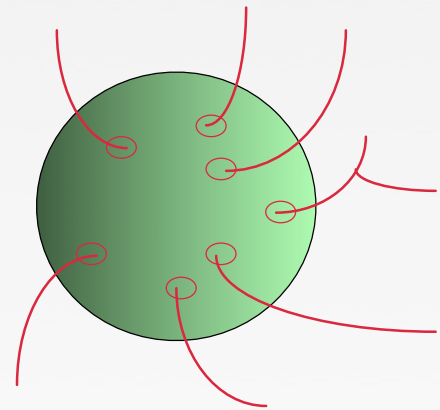
a(t)

inflationary phase

t

finite minimal volume

Black holes

- → Entropy finite, proportional to the area
- → Physical black holes
- S = A/4   if a dimensionless free parameter
  (Immirzi parameter) is fixed

end of black hole evaporation

singularity

horizon

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Black holes

- $\to$ Entropy finite, proportional to the area
- $\to$ Physical black holes
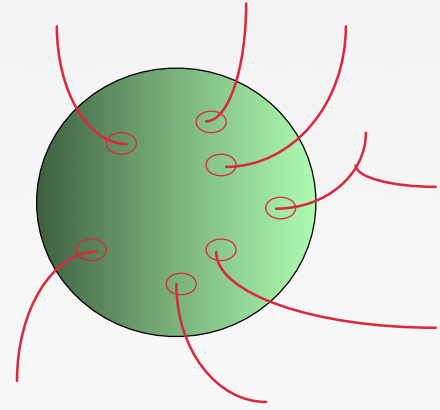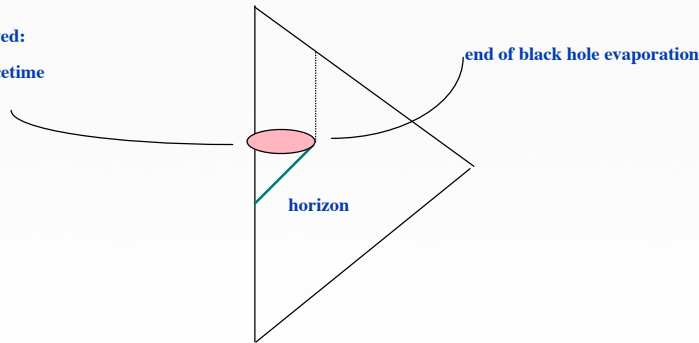- $S = A/4$ if a dimensionless free parameter (Immirzi parameter) is fixed
- $\to$ R = 0 singularity under control:
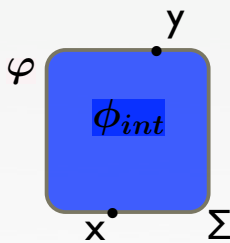

singularity removed:
non-classical spacetime

end of black hole evaporation

horizon

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background-independent *n*-point functions $\to$ low-energy limit

$$W(x, y) = \int D\phi \; \phi(x)\phi(y) e^{iS[\phi]}$$

is independent from *x* and *y* if $D\varphi$ and *S[φ]* are invariant under *Diff*.

Choose a closed 3-surface where *x* and *y* lie, and rewrite W as

$$W(x, y) = \int D\varphi \; \varphi(x)\varphi(y) \; W[\varphi] \; \Psi[\varphi]$$

where

$$W[\varphi] = \int_{\phi_{int}|_\Sigma = \varphi} D\phi_{int} \; e^{iS[\phi_{int}]}$$

Instead of being determined by boundary conditions at infinity, let the boundary state be a state picked on a given boundary geometry *q*.

$$W[x, y; q] = \int D\varphi \; \varphi(x)\varphi(y) \; W[\varphi] \; \Psi_q[\varphi]$$

This expression is meanigful in a *Diff*-invariant theory, and reduces to flat space *n*-point function for appropriate boundary state.
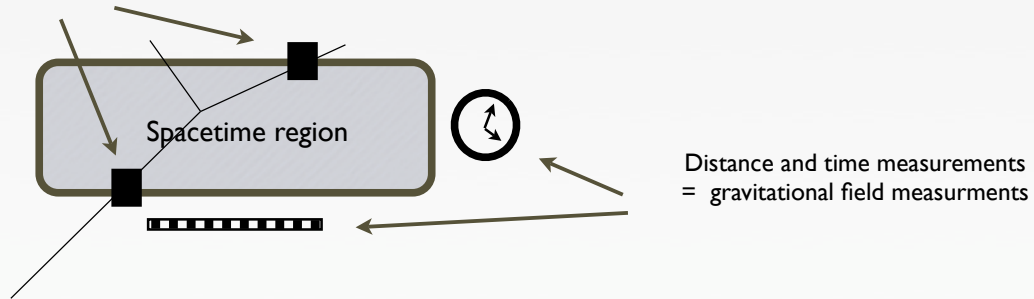
Boundary values of the gravitational field = geometry of box surface
= distance and time separation of measurements

Particle detectors = field measurements

Spacetime region

Distance and time measurements
= gravitational field measurments

In GR, distance and time measurements
are field measurements like the other ones:
they are part of the **boundary data** of the problem.

The graviton two-point function in LQC reads

$$\langle 0|g^{ab}(x)g^{cd}(y)|0\rangle = \sum_s W[s]\, g^{ab}(x)g^{cd}(y)\Psi_q[s]$$

And for large distances this is give at first order by

$$W[s] = \frac{\lambda}{5!}\left(\prod_{n<m} dim(j_{nm})\right) A_{vertex}(j_{nm})$$

→ The asymptotic expansion of the vertex gives the low-energy behavior of the theory.

→ Premiminary results yield: - free graviton propagator; - 3 point function; (hence Newton law); - first order corrections to the free graviton propagator.   Calculations in progress.

Mathematical developments

- Diffeomorphism invariant measures (Ashtekar-Lewandowski measure)
- C* algebraic techniques
- Category theory
- "Quantum geometry"
- Uniqueness of the representation

IV. Summary

- Loop quantum gravity is a technique for defining Diff-invariant QFT. It offers a radically new description of space and time by merging in depth QFT with the *diff*-invariance introduced by GR.

- It provides a quantum theory of GR plus the standard model in 4d, which is naturally UV finite and has a discrete structure of space at Planck scale.

- Has applications in cosmology, black hole physics, astrophysics; it resolves black hole and big bang singularities.

- Unrelated to a natural unification of the forces (we are *not* at the "end of physics").
- Different versions of the dynamics exist.
- Low-energy limit still in progress.

- + Fundamental degrees of freedom explicit.
- + The theory is consistent with today's physics.
- + No *need* of higher dimensions (high-d formulation possible).
- + No *need* of supersymmetry (supersymmetric theories possible).
- + Consistent with, and based on, basic QM and GR insights.