Introduction to Modern Cryptography

http://www.cs.tau.ac.il/~bchor/crypto07.html

Instructor: Benny Chor

http://www.cs.tau.ac.il/~bchor
School of Computer Science
Tel-Aviv University

Fall Semester, 2007-8

Prerequisites

- Linear Algebra
- Probability
- Algorithms (formerly "efficiency of computations")
- Computational Models
- "Mathematical Maturity"

Interested students lacking some prerequisites, esp. non CS students, pls talk to instructor (soon).

Administrative Details

- Intended for both 3rd year undergrads and grad students
- Grade determined by exam (70-80%) and homework (30-20%).
  In order to pass the course, you must pass the exam.
- Exam on January 21st, 2008 (Moed B on March 7th).
- Exam is closed book except for 2 double sided pages.

Administrative Details (2)

- 3-4 "dry" assignments.
- 1-2 assignments with a "wet" component (involving writing short *MAPLE* programs).
- Homework submition in groups of size one or two (but not three or more).
- Submissions after the deadlines will not be considered.
- If one member of a pair has a valid reason for late submission, the other member is still expected to meet the deadline on his/her own.
- Office hours: By e-appointment.
- E-mail: benny AT cs.tau.ac.il
- Course site: www.cs.tau.ac.il/~bchor/crypto07.html
- TA and grader: *******

Collaboration on Assignments, etc.

- Preparing homework assignments independently is a key ingredient for understanding the material (and, consequently, a successful exam :-). So it is highly recommended you and your partner make a serious effort to solve the problems on your own.
- You may collaborate with people from other groups on the problem sets, but your solutions must be written up independently, by you and your partner only.
- You are encouraged to consult online and offline sources for your solutions, but you are (a) expected to give clear cites of your references, and (b) use a write up of your own.
- Cases of plagiarism that will be detected will be dealt with severely. (For example, reducing grades for the whole course, not just the relevant assignment, and/or reporting the incident to the appropriate university authority.) If we suspect Alice had copied from Bob, both will be regarded as cheaters.

Bibliography

- Text Books:
  - J. Katz and Y. Lindell, Introduction to Modern Cryptography, Chapman & Hall/CRC Press, 2007. (The book was published in August 2007, so it will take a few weeks till it gets to TAU library. Its intro chapter is available online.)
  - D. Stinson, Cryptography Theory and Practice, CRC Press, 2005.
  - V. Shoup, A Computational Introduction to Number Theory and Algebra (Version 1), 2005. Available online at http://www.shoup.net/ntb/ntb-v1.pdf
- Other Relevant Books:
  - M, Bellare and P. Rogaway, Introduction to Modern Cryptography. Available online at http://www-cse.ucsd.edu/users/mihir/cse207/classnotes.html
  - A. Menezes, P. Van Oorschot, S. Vanstone, Handbook of Applied Cryptography, CRC Press, 2001. Available online at http://www.cacr.math.uwaterloo.ca/hac
  - B. Schneier, Applied Cryptography, John Wiley & Sons, 1996.

Course Outline

- Encryption (private and public key systems)
- Elementary algebra (groups, rings, finite fields)
- Elementary number theory
- Data integrity
- Authentication and identification
- Digital signatures
- Cryptographic hash functions
- Randomness and pseudo-randomness
- Secret sharing
- Cryptographic protocols

Class Notes and Course Site

- About 65% of lectures will be made available on the course site in the form of pdf files (generated using LaTeXBeamer package).
- The remaining 35%, mostly the number theory and algebra parts, will be given in old fashion style, whiteboard (or even blackboard) presentations. Consequently they will not be available on the course site.
- Announcements, assignments, and the like will be primarily disseminated through the course web site. Please take a look at it often. We will usually not use email for announcements.

Other Introductory Crypto Courses with Online Lectures (a very partial list)

- Doug Stinson course at Waterloo.
- Mihir Bellare course at University of California, San Diego.
- Benny Pinkas course at Haifa University.
- Eli Biham course at the Technion.

Theory vs. Practice

Following the introduction of public key cryptography, research in the area has to a large extent moved from secretive, military-like organizations to open, academic departments, and (to a lesser extent) to commercial companies.
Starting in the early 80's, theoretical foundations of cryptographic primitives, cryptographic protocols, compositions thereof, etc., were established. Provable security notions led to clearer understanding of many issues, and had far reaching (and highly unexpected) consequences in theoretical Computer Science.

These issues are mostly out of scope for the course, but there is some controversy around them. Neal Koblitz (Univ. of Wahington) and Alfred Menezes (Waterloo) published the article "Another Look at Provable Security", which criticizes several typical provable security results in modern cryptography. In his essay "On Post-Modern Cryptography", Oded Goldreich (Weizmann Inst.) responds to the Koblitz and Menezes claims.

And Now to Something Completely Different

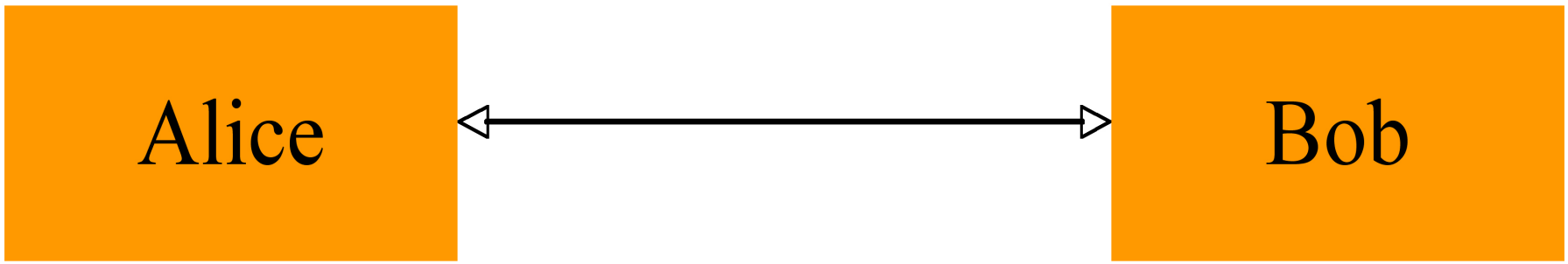
Encryption

Notations and Definitions

- Encryption function (& algorithm): E.
- Decryption function (& algorithm): D.
- Encryption key k_1.
- Decryption key k_2.
- Message space (usually binary strings, either of certain block length or unlimited stream), $\mathcal{M}$.
  Remark: Block length typically tied to key length.
- Consistency requirement: For every message $m \in \mathcal{M}$ and matching pair of keys k_1, k_2: $D_{k_2}(E_{k_1}(m)) = m$.
- So far, no requirement of secrecy.

Communication Model

Let us welcome the two major players in this field, Alice and Bob.



1. Two parties – Alice and Bob
2. Reliable communication line
3. Shared encryption scheme: E, D, k_1, k_2
4. Goal: send a message m confidentially

Threat Model

Enter the third major party

1. Two parties Alice and Bob
2. Reliable communication line
3. Shared encryption scheme: E, D, k_1, k_2
4. Goal: send a message m confidentially

Security Goals

There are some different goals we may be after

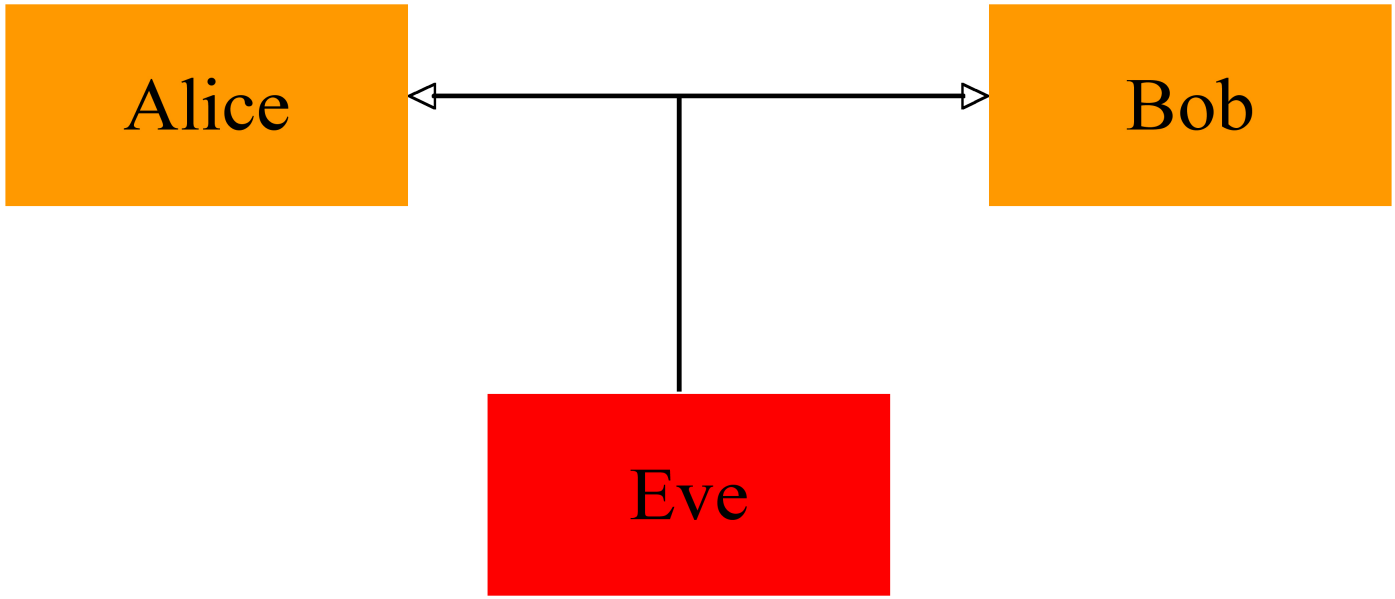
- No adversary can determine m
- No adversary can determine any information about m
- No adversary can determine any meaningful information about m.

Important questions:

- What does the adversary know or seen before?
- What are the adversary's computational resources?

Adversarial Model: Passive Eavesdropper

Enters our third major player, Eve.



- Eve attempts to discover information about m
- Eve knows the algorithms E, D
- Eve knows the message space
- Eve has intercepted $E_{k_1}(m)$
- Eve does not know k_1, k_2

Additional Definitions

- Plaintext – the message prior to encryption ("attack at dawn", "sell MSFT at 57.5")
- Ciphertext – the message after encryption ("$\Im\partial$Æ$\perp\xi\varepsilon\beta\Xi\Omega\Psi\mathring{A}$", "jhhfo hjklvhgbljhg")
- Symmetric cryptosystem – encryption scheme where $k_1 = k_2$ (classical cryptography)

Examples – (Weak) Symmetric Ciphers

- Shift cipher
- Conclusion – large key space required (can be formalized in information theoretic terms)

- Substitution cipher
- Large key space, still “easy” to break

Substitution Ciphers

Plain	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher	W	H	O	V	I	B	P	L	C	J	Q	X	D	K	R	Y	E	S	Z	A	F	T	M	G	N	U

Example:
Plaintext: `attack at dawn`
Ciphertext: `waawoq wa vwmk`

Size of key space is
$26! = 403291461126605635584000000 \approx 4 \cdot 10^{27}$.

This is large enough space to prevent exhaustive search for key (at least for old machines, and probably even today). Yet easily breakable due to known (and very non uniform) statistics of single letters, pairs of letters, triplets, etc., in all natural languages.

Perfect Cipher

- Plaintext (message) space – $\{0,1\}^n$
- Given a ciphertext, C, the probability that $D_{k_2}(C) = M$ for any plaintext M is equal to the apriori probability that M is the plaintext.
- Probability over what?

Perfect Cipher

- Plaintext (message) space – $\{0,1\}^n$
- Given a ciphertext, C, the probability that $D_{k_2}(C) = M$ for any plaintext M is equal to the apriori probability that M is the plaintext.
- Probability over what?
- Over the key space $\{k_2\}$ and the message space $\mathcal{M}$
- In a probabilistic language:

$$Pr[plaintext = P \mid C] = Pr[plaintext = P]$$

- In daily language: Knowing the ciphertext gives absolutely no information towards knowing the plaintext.

Example – One Time Pad

- Plaintext space – $\{0,1\}^n$
- Key space – $\{0,1\}^n$. The key k is chosen at random and indep. of P.
- The scheme is symmetric, $\oplus$ stands for bit-wise XOR:
  $E_k(P) = C = P \oplus k$
  $D_k(C) = C \oplus k = P$

Pros and Cons, One Time Pad

- Claim: One time pad is a perfect cipher.
- Problem: Size of key space.
- Theorem (Claud Shannon): If a cipher is perfect, then the size of its key space is at least as large as the size of its message space.
- This is bad news. Perfect ciphers are only practical for fairly small message spaces.

Computational Resources

Any serious discussion of cryptography must take into account the computational resources of all parties.
The adversary may have enough information to break a system, but if this requires resources he lacks, the threat is not real.

- Time
- Storage/Memory
- Hardware
- Theoretically: Polynomial vs. non-polynomial (probabilistic) computations
- Practically: 2^{70} steps are (barely) feasible, 2^{100} are not

Conceivable Attacks

- Eavesdropping
- Known plaintext
- Chosen plaintext
- Chosen ciphertext
- Adaptive chosen text attacks
- Physical access
- Physical modification of messages