

METHODOLOGICAL NOTES

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β-stability condition for the nuclei of neutral atoms

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Abstract. It is shown that a necessary and sufficient condition for β-stability of the nucleus in a neutral atom is that the mass of the atom be a minimum within the isobaric series, which is not always the same as the requirement of minimum nuclear mass often cited in the literature.

The question of formulation of the β-stability condition for nuclei arose almost at the dawn of nuclear physics [1, 2]. However, until the middle of the 20th century, insufficient accuracy and an incomplete set of experimental data on the nuclear masses of isotopes precluded the possibility of full-scale analysis of the correspondence between the theoretical predictions and experimental data. At the time, the experimental accuracy was not always sufficient to distinguish between the difference of nuclear masses and the difference of atomic masses; therefore, it seemed that the conditions of 'minimum nuclear mass', 'minimum atomic mass', and 'maximum nuclear binding energy' in the isobaric series coincided, deviations from the presumptive stability condition being regarded as exceptions [1, 2]. At present, owing to available data [3], it has become possible to formulate and to verify the exact condition for nuclear stability. Analysis of a database [3] has shown that the stability conditions that appear in the scientific literature, namely, the 'nuclear mass minimum' [4, 5] or the 'binding energy maximum' [6] in isobaric series are inaccurate, the minimum *atomic* mass of the isotope in the isobaric series being the *only absolutely accurate* β-stability condition for the nucleus of a neutral atom [7, 8].

Let us consider the stability of a nucleus with respect to processes not accompanied by a change in the number of nucleons in the nucleus, i.e., caused by weak interactions, in particular, electron (β^-) or positron (β^+) β-decay and K-capture:

$$\begin{aligned} {}^A_Z\mathrm{X} &\to {}^A_{Z+1}\mathrm{Y} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} + Q_1\,, \\ {}^A_Z\mathrm{X} &\to {}^A_{Z-1}\mathrm{Y} + \mathrm{e}^+ + \nu_\mathrm{e} + Q_2\,, \\ {}^A_Z\mathrm{X} + \mathrm{e}^- &\to {}^A_{Z-1}\mathrm{Y} + \nu_\mathrm{e} + Q_3\,, \end{aligned} \tag{1}$$

where ν_e and $\bar{\nu}_\mathrm{e}$ are the electron neutrino and antineutrino, and X, Y are nuclei with atomic number A and charge Z (in electron charge units).

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It is well known [7, 8] that the energy either released ($Q > 0$) or absorbed ($Q < 0$) in nuclear reactions (1) can be determined as the difference between the masses of the initial nuclei and the reaction products:

$$Q = M_\mathrm{N}(A_\mathrm{X}, Z_\mathrm{X}) - M_\mathrm{N}(A_\mathrm{Y}, Z_\mathrm{Y}) \mp m_\mathrm{e}\,, \tag{2}$$

where $M_\mathrm{N}(A, Z)$ is the mass of the nucleus ${}^A_Z\mathrm{X}$, m_e is the electron rest mass, the '$-$' sign corresponds to the $\beta^\pm$-decay (Q_1 and Q_2), and the '$+$' sign designates the K-capture (Q_3). Because the K-capture is always energetically more advantageous than the positron β^+-decay ($Q_3 - Q_2 = 2m_\mathrm{e}$), the possibility of positron β^+-decay does not change the stability condition of the nucleus. By definition of the nuclear binding energy W_N, we have

$$M_\mathrm{N}(A, Z) = (A - Z)m_\mathrm{n} + Zm_\mathrm{p} - W_\mathrm{N}(A, Z)\,, \tag{3}$$

where m_p and m_n are the rest masses of the proton and the neutron. The binding energy W_N is the energy that must be added to disintegrate the nucleus into the constituent nucleons.

Expression (2) is valid in the case where the nucleus has no electron shells. For decay of a nucleus incorporated in a neutral atom, one should take into account the electron binding energy. After capture of an orbital electron, the atom remains neutral, while $\beta^\pm$-decay gives rise to a singly charged $\mathrm{Y}^\pm$ ion (which is positive in the case of electron β^--decay or negative in the case of positron decay). However, since the first atomic ionization potential is not higher than 25 eV (the highest value for He is 24.58 eV), this value can always be neglected compared to the accuracy of measurement of the nuclear binding energy (~ 1 keV). Within this approximation from Eqn (2) follows, as indicated correctly in previous publications [7, 8], that the energy released upon K-capture and electron β^--decay in the decomposition of a neutral atom may be determined by

$$Q = M_\mathrm{A}(A_\mathrm{X}, Z_\mathrm{X}) - M_\mathrm{A}(A_\mathrm{Y}, Z_\mathrm{Y})\,, \tag{4}$$

where

$$M_\mathrm{A}(A, Z) = (A - Z)m_\mathrm{n} + Z(m_\mathrm{p} + m_\mathrm{e}) - W(A, Z) \tag{5}$$

is the *atomic* mass, W is the nuclear binding energy in an atom *with allowance for the energy $I(Z)$ of full ionization of the atom*:

$$W(A, Z) = W_\mathrm{N}(A, Z) + I(Z)\,, \tag{6}$$

i.e., the energy needed to disintegrate a neutral atom into constituent protons, neutrons, and electrons. To within ZI_H ($I_\mathrm{H} = 13.6$ eV is the hydrogen ionization potential), which is not below the accuracy of the measurement of the nuclear

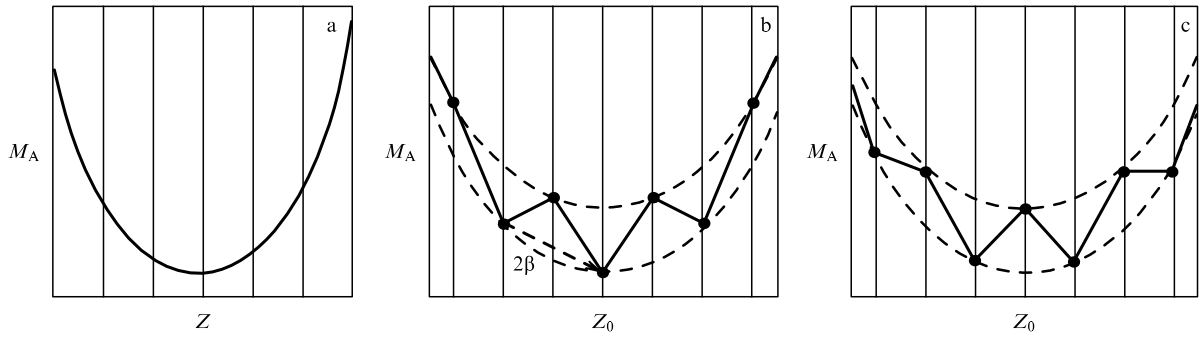

Figure 1. Dependence of the atomic mass on the charge. Z_0 is the parabola minimum. (a) For odd atomic number A; (b) for even A and even Z_0, and (c) for even A and odd Z_0.

binding energy for $Z < 100$, the energy determined in this way coincides with the energy needed to disintegrate the nucleus into neutrons and hydrogen atoms:

$$M_A(A, Z) = (A - Z)m_n + ZM_H - W(A, Z)\,, \tag{7}$$

where M_H is the hydrogen atomic mass. Since the nuclear binding energy was introduced historically to calculate the energies released in nuclear reactions involving neutral atoms, published tables [3] present precisely the atomic energies W defined by formula (6), which include the full ionization potential $I(Z)$, rather than the nucleus energies W_N. For determining the energy of the nucleus, one can also use the mass defect ΔM, which is related to M_A as follows [3]:

$$M_A(A, Z) = Am_{a.m.u.} + \Delta M(A, Z)\,, \tag{8}$$

where $m_{a.m.u.} \approx 931.5$ MeV is the atomic mass unit; for the mass defect, the normalization $\Delta M(^{12}C) = 0$ was chosen.

It is well known [7, 8] that the energetic exclusion of all possible decay channels makes up a *sufficient* condition for the β-stability of a nucleus, i.e., reactions (1) should be endothermic ($Q < 0$). The K-capture and $\beta^{\pm}$-decay processes accomplish the transformation of a nucleus with retention of the number of nucleons, i.e., migration along the isobaric series ($A = \text{const}$). Thus, on the basis of formulas (4) and (5), the sufficient condition for the β-stability of a nucleus in a neutral atom is given by the *minimum atomic mass* $M_A(A, Z)$ [which is equivalent to the minimum mass defect $\Delta M(A, Z)$], including all local minima, in the isobaric series ($A = \text{const}$).

Pay attention to the fact that we mean the minimum *atomic mass* $M_A(Z)$ rather than the minimum *nuclear mass* $M_N(Z)$ or the maximum binding energy $W(Z)$. From formulas (3)–(8), we find that the functions $M_A(Z)$, $M_N(Z)$, and $W(Z)$ are related in the following way:

$$\begin{aligned} M_N(Z) &= M_A(Z) + I(Z) - Zm_e\,, \\ -W(Z) &= M_A(Z) - Am_n + Z\tilde{m}\,, \end{aligned} \tag{9}$$

where $\tilde{m} = m_n - m_p - m_e = 782.3$ keV. Since the $M_N(Z)$ and $W(Z)$ functions differ from $M_A(Z)$ in the isobaric series ($A = \text{const}$) by the addition of terms that are monotonic with respect to Z [see formula (9)], these three functions (M_A, M_N, W) have qualitatively the same form but the $M_N(Z)$ minima can migrate towards greater Z, while maxima of the binding energy $W(Z)$ can shift towards smaller Z with respect to the minima of the $M_A(Z)$ function [these latter coincide with the $\Delta M(Z)$ minima].

The qualitative dependence of the binding energy on the nuclear charge in the isobaric series can be described using the well-known semiempirical Weizsäcker formula [7, 8]; taking into account Eqn (9), the atomic mass can be expressed as follows:

$$\begin{aligned} M_A(A, Z) = {} & Am_n - Z\tilde{m} - a_V A + a_S A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} \\ & + a_{SYM} \frac{(A/2 - Z)^2}{A} - a_P \frac{\delta}{A^P} - I(Z)\,, \end{aligned} \tag{10}$$

where $a_V = 15.75$ MeV, $a_S = 17.8$ MeV, $a_C = 0.71$ MeV, $a_{SYM} = 94.8$ MeV, and $a_P = 34$ MeV are the coefficients of the nucleus energy, namely, the volume, surface, Coulomb, symmetry and pairing energies, respectively. The δ coefficient is responsible for the pairing effect, in particular, $\delta = 0$ for nuclei with odd A, $\delta = 1$ for even-even nuclei (an even number of neutrons and an even number of protons), and $\delta = -1$ for odd-odd nuclei; the power P in the next to last (pairing) term was taken to be 1/3 to 1 by various researchers. Recall the well-known fact that follows from the Weizsäcker formula (10), namely, in the isobaric series of odd A, the $M_A(Z)$ dependence reduces to a parabola with one minimum ($\delta = 0$) (Fig. 1a), while in the isobaric series of even A, the plot of $M_A(Z)$ is a broken line confined between two parabolas corresponding to even Z ($\delta > 0$) and odd Z ($\delta < 0$) (Fig. 1b, c). In the latter case, the $M_A(Z)$ function can have (depending on A) one, two, or three minima. Figure 1b illustrates the case where the minimum of the parabola for even A corresponds to even Z, while Fig. 1 displays the parabola minimum for odd Z.

A relatively simple analysis of database [3] shows that *all stable isotopes without exception* comply with the *minimum atomic mass* $M_A(Z)$ condition in the corresponding isobaric series. Moreover, analysis has shown that all β-decay and K-capture processes *allowed from the energy standpoint* really *occur* in nature (there are no other reasons for forbidding them). In other words, the following statement is valid:

for the β-stability of the nucleus of a neutral atom (that is, stability with respect to single $\beta^{\pm}$-decay and K-capture events), it is *necessary and sufficient* that this isotope exhibits a *minimum atomic mass* within the isobaric series ($A = \text{const}$).

It is worth noting that the twelve isotopes occurring in nature and not implementing a minimum $M_A(Z)$ are unstable, although long-lived (^{40}K, ^{48}Ca, ^{50}V, ^{87}Rb, ^{96}Zr, ^{113}Cd, ^{115}In, ^{123}Te, ^{138}La, ^{176}Lu, ^{187}Re, ^{180}Ta^m); conversely,

no β-stable isotopes with atomic masses of 5 and 8 are encountered in nature, as they are unstable with respect to the decays: $^5\mathrm{He} \to {}^4\mathrm{He} + \mathrm{n}$, $^8\mathrm{Be} \to 2\,{}^4\mathrm{He}$. α-decay becomes energetically favorable for atomic masses $A > 141$, while for some isotopes with atomic masses in the $210 > A > 141$ range, this type of decay goes forbidden but all isotopes with $A > 209$ are α-active. Of special note is the naturally encountered $^{180}\mathrm{Ta}^m$ isotope, which is a long-lived (1.2×10^{15} years) isomeric excited state of the nucleus. This long half-life is due to the great difference between the spins of the isomeric (9^-) and ground (1^+) states.

To determine the relationship between the charge Z and the mass A of the nucleus of stable isotopes, we find the minimum of the atomic mass $M_\mathrm{A}(Z)$ in the isobaric series. The ionization energy $I(Z)$ is small even with respect to the small term Zm_e, which distinguishes between M_A and M_N defined by Eqn (9). (The ionization energy $I(Z)$ can be taken into account using the Thomas–Fermi model approximation [9], but this would provide excessive accuracy, as the Coulomb term in the Weizsäcker formula has a lower accuracy.) Similarly to what was done in previous publications [7, 8], we represent relation (10) in the form

$$M_\mathrm{A}(A,Z) = C_1(A) + C_2(A)(Z - Z_0)^2 - \delta(A,Z)a_\mathrm{P}A^{-\mathrm{P}}\,, \tag{11}$$

where

$$Z_0 = \frac{A}{2}\,\frac{a_\mathrm{SYM} + a_\mathrm{C}\,A^{-1/3} + \tilde{m}}{a_\mathrm{SYM} + a_\mathrm{C}\,A^{2/3}}\,,$$

$$C_2(A) = \frac{a_\mathrm{SYM}}{A} + a_\mathrm{C}\,A^{-1/3}\,, \tag{12}$$

$$C_1(A) = A(m_\mathrm{n} - a_\mathrm{V}) + a_\mathrm{S}\,A^{2/3} - Z_0^2 C_2(A) + a_\mathrm{SYM}\,\frac{A}{4}\,.$$

Since Z can acquire only integer values, the minimum $M_\mathrm{A}(Z)$ would be attained for the integer value nearest to Z_0 and determined by relation (12). This can be easily seen from the fact that parabola (11) is symmetric with respect to $Z = Z_0$. Figure 1b corresponds to the case where the Z_0 value for even A is closer to even Z, while Fig. 1c illustrates the case where the Z_0 value for even A is closer to odd Z.

The minimum nuclear mass M_N is attained under the condition similar to (12) but with the replacement

$$\tilde{m} \to \tilde{m} + m_\mathrm{e} = m_\mathrm{n} - m_\mathrm{p}\,. \tag{13}$$

It may seem that, since $m_\mathrm{e} \ll a_\mathrm{SYM} = 94.8$ MeV, the difference (13) between the conditions for minima of the functions M_A and M_N can be neglected; however, in those cases where Z_0 defined by Eqn (12) is close to half-integer values, even such a small change as $m_\mathrm{e}/a_\mathrm{SYM}$ can shift the integer value closest to Z_0 by unity.

Indeed, analysis of the published database [3] demonstrates the inaccuracy of the assumption that the minimum nuclear mass $M_\mathrm{N}(Z)$ makes up a sufficient condition of the β-stability of a neutral atom. Thus, for example, more than 30 isotopes exhibiting a minimum nuclear mass $M_\mathrm{N}(Z)$ in the isobaric series are unstable with respect to the K-capture. The following typical example can be cited: the minimum mass of the atom in the isobaric series with atomic number 55 is attained for the only stable manganese isotope ^{55}Mn, while the minimum nuclear mass is attained for the unstable ^{55}Fe isotope (decay period of 2.7 years). The ^{55}Mn nucleus is *heavier* than the ^{55}Fe nucleus:

$$M_\mathrm{N}(^{55}\mathrm{Mn}) - M_\mathrm{N}(^{55}\mathrm{Fe}) \approx 280\ \mathrm{keV}\,,$$

whereas the ^{55}Mn atom is *lighter* than the ^{55}Fe atom: $M_\mathrm{A}(^{55}\mathrm{Fe}) - M_\mathrm{A}(^{55}\mathrm{Mn}) \approx 231$ keV.

Similarly, the hypothesis of β-stability as a maximum nuclear binding energy is also rough: sixty isotopes exhibiting binding energy maxima are β-active.

It is noteworthy that the 'truly' β-stable isotopes are those exhibiting absolute minima of atomic mass M_A in the isobaric series, because the isotopes displaying local minima can decompose to an absolute minimum through a double $\beta^\pm$-decay or a double K-capture (Fig. 1b). Certainly, the probability of these processes is low but it still differs from zero. Thus, for example, a double β^--decay has been detected for ^{82}Se (10^{20} years), ^{100}Mo (10^{19} years), ^{128}Te (2.2×10^{24} years), and ^{150}Nd ($> 10^{19}$ years) isotopes. Single β^--decays are energetically forbidden for such isotopes. This situation differs from the double β^--decay of ^{96}Zr isotope, which is simultaneously unstable with respect to the single β^--decay ($^{96}\mathrm{Zr} \to {}^{96}\mathrm{Nb} \to {}^{96}\mathrm{Mo}$).

We have considered the stability condition for the nucleus of a neutral atom. It is well known that deformation of the electron shells of an atom entails a change in the β-decay period of the nucleus. The influence of the atomic electric field on the probability of β-decay of the nucleus has been considered in detail in review monographs [10, 11]. The influence of alterations in the electron shells of the atom on the β-decay of the tritium nucleus has been considered in detail in publications [12, 13], which also cite convincing experimental data.

It is worth noting that ionization of the atom may change not only the probability of β-decay for unstable nuclei, but also the conditions of nuclear stability (the stable nuclei in a neutral atom may become unstable). The theory of β^--decay into the bound electron state (in which the β-electron does not leave the atom but occupies a vacant orbit) has been expounded in a number of publications [14–17]. Attention was drawn to the fact [18] that decay into the bound state additionally expands the phase volume of the final states and, hence, increases the probability of β^--decay. Using the Thomas–Fermi model approximation for the ionization potential of an atom [9]:

$$I(Z) \cong 20.8\,Z^{7/3}\ \mathrm{eV}$$

and the expression for the ionization potential of a hydrogen-like ion (a nucleus with one last electron) [9]:

$$I^{1\mathrm{e}}(Z) = 13.6\,Z^2\ \mathrm{eV}\,,$$

we may conclude that the difference between the ionization potentials of two adjacent elements, $I(Z+1) - I(Z) \propto Z^{4/3}$, grows more slowly than the ionization potential of the hydrogen-like ion and, for almost all atoms ($Z > 7$), the following inequalities are valid:

$$I(Z+1) - I(Z) < I^{1\mathrm{e}}(Z) < I^{1\mathrm{e}}(Z+1)\,.$$

Hence, the energy of β^--decay of a completely ionized nucleus into the bound electron state is given by

$$Q = Q_0 + I(Z) - I(Z+1) + I^{1\mathrm{e}}(Z+1) > Q_0\,,$$

where Q_0 is the energy of nuclear decay in a neutral atom. In other words, when the atom is fully ionized, the β^--decay into the bound state becomes energetically more favorable than the β^--decay of the neutral atom. Analysis of the available database [3] indicates that a number of stable nuclei of neutral atoms become unstable with respect to the β^--decay to the bound state upon full ionization, namely, ^{163}Dy, ^{193}Ir, and ^{205}Tl; this point has been confirmed experimentally [19].

The theory of β^--decay into the bound state has received experimental verification [19, 20]. The neutral atom with the lowest β-decay energy is ^{187}Re (2.66 keV). The β-decay of fully ionized ^{187}Re has been investigated in work [20]. A specific feature of the ^{187}Re isotope lies in the fact of fully ionized ^{187}Re that the β-decay to the bound state provides the possibility of transition to the excited state of ^{187}Os (9.75 keV). This changes the decay scheme and substantially increases the decay probability, because the β-decay in the channel opened occurs between nuclear states whose spins differ by unity $(5/2^+ \to 3/2^-)$, whereas the β-decay to the ground state is more highly forbidden $(5/2^+ \to 1/2^-)$. The full ionization has reduced the ion decay period by a factor of 10^9 (4.3×10^{10} years for the neutral atom, and 33 years for the fully ionized atom).

Thus, when considering nuclear decay problems and even the classical stability condition, one should reasonably take into account the terms (on the order of electron mass) that are small with respect to the nuclear binding energy; in particular, the stability conditions (and the decay schemes [21]) for nuclei that reside in fully ionized atoms and in neutral atoms are different.

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