



## A Recursive Gaussian Weighted Filter For Impulse Noise Removal

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### Abstract

A recursive filter for effective suppression of impulse noise is presented in this paper. The proposed work estimates the noise corruption level and the position of impulses in the first stage. The appropriate filter parameters for a detail-preserving restoration of the corrupted pixels are determined based on these estimations. The filtering process assumes a Gaussian spatial profile in the neighborhood of corrupted pixel and interpolates accordingly. A normalized, truncated, trimmed and scaled Gaussian weighting function is proposed for the coefficients of interpolation. Extensive simulations show that the proposed filter restores fairly well even the images that are highly corrupted.

**Keywords :** Impulse noise, noise count, flag, Gaussian, scaling, Filtering.

### 1. Introduction

Impulse noise is caused by malfunctioning pixels in camera sensors, faulty memory locations in hardware or transmission in a noisy channel. Attenuation of noise and preservation of details are usually two contradictory aspects of image Enhancement. Nevertheless, both of them are important to subsequent processing stages [1]. Median filter and its variants [1-6] are the earliest proposed methods for impulse removal. However, since such filters treat all pixels of an image in the same way, they tend to modify pixels that are undisturbed by noise. The later methods [8-16] are usually of two stage, with an impulse detector where impulses are located and an impulse filter by which located impulses are filtered, thus avoiding the modification of good pixels. The noise variance is one of the key factors that determine the performance of any filter. Most of the filter parameters depend on the level of corruption. Hence, the proposed filtering technique initially estimates the percentage of corruption and the position of impulses in the observed noisy image with the help of an impulse detector [14, 15]. The filtering process

estimates the corrupted pixel from the weighted sum of its neighboring non-noisy pixels. The weights are assumed to follow a normalized, truncated, trimmed and scaled Gaussian distribution.

The remainder of this paper is organized as follows- Section (2) discusses the noise model considered. Section (3) briefs the proposed filtering technique. The results of simulations are summarized in section (4) and their significance are discussed. Some concluding remarks are presented in Section (5).

### 2. Noise in an Image

Consider an image  $I$  of size  $S1 \times S2$  and an observation image  $X$  of same size.

$$X = I + \eta \quad (1)$$

Where ' $\eta$ ' is impulse noise with random values. The noise is assumed to be uniformly distributed with a probability,

$$X_{i,j} = \begin{cases} N_{i,j} & \text{With Probability } p \\ I_{i,j} & \text{With Probability } 1-p \end{cases} \quad (2)$$

Where  $i=1, 2, \dots, S1$  and  $j=1, 2, \dots, S2$  and  $0 \leq p \leq 1$ . For 8 bit images, if a pixel is corrupted, it is replaced by positive or negative impulse values. For example, the noisy pixel takes '0' for a negative impulse and '255' for a positive impulse in the case of fixed valued impulse noise (salt and pepper noise)

### 3. Proposed Work

The proposed Recursive Modified Gaussian Filtering (RMGF) technique has two stages. In the first stage the distribution of impulses are estimated in terms of their position and quantity using an impulse detector. The second stage involves restoring the corrupted pixels as a weighted sum of their neighboring uncorrupted pixels. The parameters of the filter in the second stage are determined based on the noise estimation in the first stage. This helps in reducing the computational complexity of the proposed filtering technique.



### 3.1 Impulse Noise Detection and Noise Count

The first stage in the filtering framework is impulse detection. We make use of Adaptive Median Filter based impulse detector for finding the position of impulses [16]. Consider a  $(2m+1) \times (2m+1)$  window  $W$  around a pixel  $X_{p,q}$  given by,

$$W_{p,q}(X) = \left\{ \begin{array}{l} X_{i,j} \mid p-m \leq i \leq p+m, \\ \dots \mid q-m \leq j \leq q+m \end{array} \right\} \quad (3)$$

It could be observed that the corrupted pixels belong to the set  $\{W_{min}, W_{max}\}$ , where  $W_{min}$  is the minimal pixel value in the defined window and  $W_{max}$  is the maximum. Adaptive median filtering to the corrupted image is then applied, which yields a filtered image  $M$ . A pixel may be corrupted and assigned to a flag matrix ' $f$ ' as,

$$f(i,j) = \begin{cases} 1 & \text{if } (X_{i,j} \neq M_{i,j}) \& X_{i,j} \in \{W_{min}, W_{max}\} \\ 0 & \text{else} \end{cases} \quad (4)$$

This impulse detection scheme detects impulse noise even at higher corruption levels setting the flag matrix values as 1 wherever noise exists. This facilitates implementation of a switching strategy such that pre-selected noisy pixels alone are modified leaving other pixels intact and the estimation of noisy pixel is based on its non-noisy neighborhood rather than complete neighborhood. This technique results in providing effective filtering than uniformly applied methods. To further enhance the performance of proposed filter, we define a factor called Noise Count (NC) as

$$NC = \sum_{i=1}^{S1} \sum_{j=1}^{S2} f_{i,j} \quad (5)$$

from which the level of corruption in the image is estimated by,

$$\text{level of corruption} = \frac{NC}{S1 \times S2} \quad (6)$$

where  $S1$  and  $S2$  represent the dimensions of the image. The selection of larger or smaller neighborhood for estimation of a true pixel at a noisy location is accomplished through adaptively changing the window size of neighborhood based on percentage of corruption resulting in higher PSNR.

### 3.2 Filtering of the noisy pixel

In the second phase, the filtering of the corrupted pixels is carried out recursively. For every corrupted pixel, the estimated pixel value is a weighted sum of neighboring non-noisy pixels. A modified Gaussian weighting model is proposed which provides higher weights to the pixels near to the noisy pixel.

#### 3.2.1 Modified Gaussian Weighting Model

Conventional Gaussian filter provides output as a 'weighted average' of neighborhood of each pixel. We consider the two dimensional Normalized Gaussian weighting model (eqn 7) over the neighborhood of the corrupted pixel under consideration (Fig 1). The size

of the neighborhood ' $k$ ' is directly proportional to the noise level. For every pixel in the neighborhood of the noisy pixel  $X_{p,q}$

$$w_{i,j} = \frac{1}{2\pi} \{ e^{-|i-p|^2} \times e^{-|j-q|^2} \} \quad i,j=1,2\dots k \quad (7)$$

The area under the curve (7) is nearly one even when it is truncated to ' $k$ ' distances. We refine the weights so as to neglect all the noisy pixels in the process of estimation. The weights are reduced to null wherever there is a noisy pixel as,

$$w_{i,j} = \frac{1}{2\pi} \{ (1 - f_{i,j}) \times e^{-|i-p|^2} \times e^{-|j-q|^2} \} \quad i,j=1,2\dots k \quad f_{i,j} \in \{0,1\} \quad (8)$$

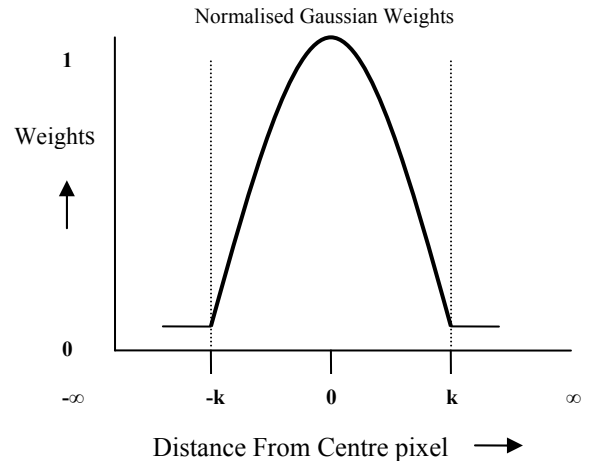


Fig 1 A Normalized Gaussian Weighting Function

The modification of weights for the corrupted pixels considerably reduces the area under the curve from unity. Hence all the weights are scaled by a factor ' $a$ ' (Fig 2) in order to maintain the unit area as,

$$w_{i,j} = a * \left[ \frac{1}{2\pi} \{ f_{i,j} \times e^{-|i-p|^2} \times e^{-|j-q|^2} \} \right] \quad i,j=1,2\dots k \quad f_{i,j} \in \{0,1\} \quad a \in R \quad (9)$$

Thus the scaling behaves as a mechanism to prevent artifacts in the output filtered image. The weights given by (eqn 9) are used to estimate  $\hat{X}_{p,q}$  as in (eqn 10),

$$\hat{X}_{p,q} = \sum_{i=1}^k \sum_{j=1}^k w_{i,j} \times X_{i,j} \quad (10)$$

Thus the noisy pixel is replaced by the weighted sum of uncorrupted pixels.



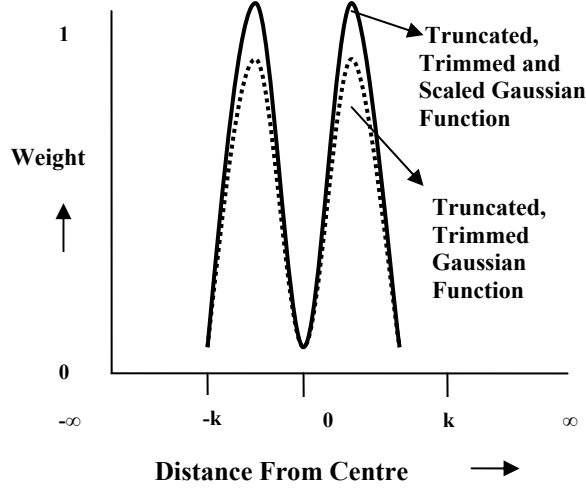


Fig 2 A truncated, trimmed and scaled Gaussian Weighting Function

## VI. Results and Discussion

Experiments were performed on two standard grayscale test images (Lena of size 512x512, and Boat of size 512x512) corrupted by salt and pepper noise with corruption levels of 1%, to 90%. These images were filtered using Standard Median (SM) filter, Progressive Switching Median (PSM) Filter, Improved Median Filter, SDROM filter; Rank order based median filter and the proposed filtering technique. Fig 3 exhibits the superior qualitative performance of RMGF in comparison with the above mentioned filters (for the images Boat). For estimating the quality of tested filters, the following image reconstruction quality measures were used

$$MAE = \frac{\sum_{i=0}^{S1} \sum_{j=0}^{S2} \left| I(i, j) - \hat{I}(i, j) \right|}{S1 \times S2} \quad (11)$$

$$PSNR = \frac{255^2}{\frac{1}{S1 \times S2} \sum_{i=1}^{S1} \sum_{j=1}^{S2} \left( I(i, j) - \hat{I}(i, j) \right)^2} \quad (12)$$

Both MAE and PSNR measure the difference in the intensity values of a pixel in original and enhanced images. Table 1 compares the performance of various filters at various noise levels in terms of MAE. The corresponding values are plotted as in Fig 4. It could be seen that the RMGF outperforms the other filters in comparison, at all noise levels. To demonstrate that the RMGF preserves image information, we take into considerations the effect of each filter on factors such as number of noisy pixels eliminated, number of pixels being replaced with the original pixel value, number of pixels that were not replaced with the original pixel value and number of uncorrupted pixels that were spoiled in the process of filtering. From the results shown in Table 2 it is observed that RMGF

exhibits higher percentage of noise removal and lower percentage of image spoiling, apart from providing high PSNR.

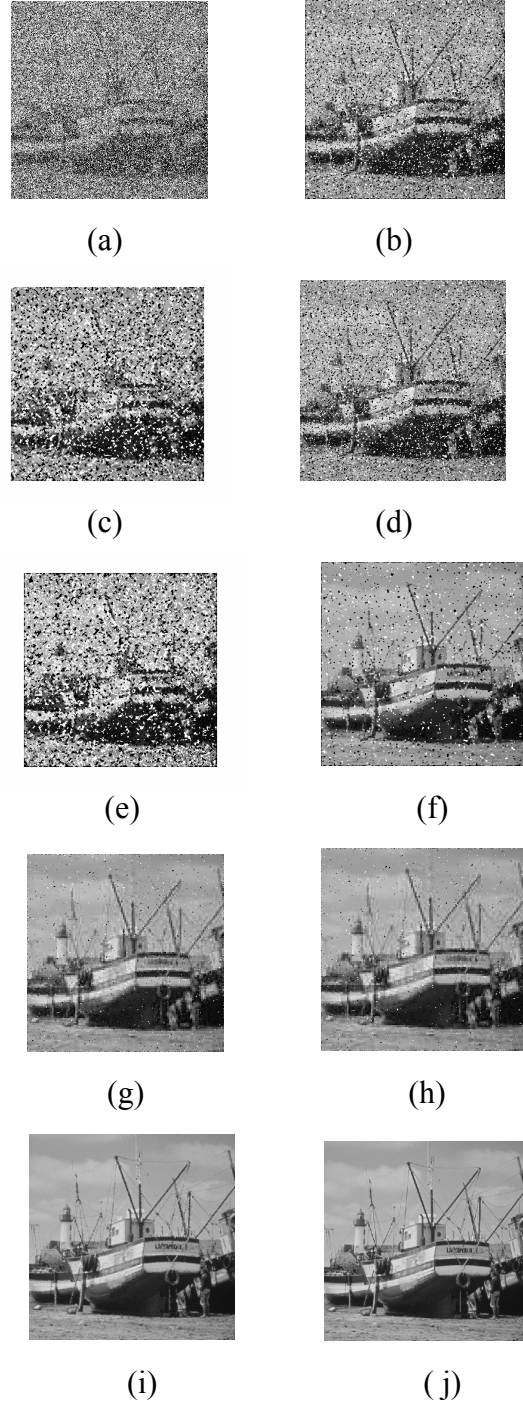


Fig 3 Restoration Results of Various Filters - a. Corrupted Boat Image with 30% impulse noise b. SM Filter c. Centre weighted Median Filter d. Improved Median Filter e. PWMAD filter f. PSM Filter g. Rank order filter h. SDROM filter. Proposed Filter i. Original Boat Image



Table 1 – Performance Analysis (MAE) of various filters

Filter	PSNR	% Spoiled	Noise Elimination	Noise Replaced With Original pixel	Noise Replaced Without Original pixel
SM	14.64	35.59	88.20	16.57	71.62
CWM	12.84	16.93	67.70	24.47	43.22
PWMAD	12.68	9.99	58.70	25.36	33.33
PSM	20.18	6.81	97.57	47.89	49.68
Improved Median Filter	14.88	11.52	84.42	36.16	48.26
SDROM	14.14	4.34	82.73	41.7	41.3
RMGF (proposed)	26.84	0.3	100	50.20	49.79

Table 2- Performance Analysis of various Filters at 70% noise

	10%	30%	40%	50%	60%	70%
SM	3.24	5.76	9.93	18.01	29.25	49.01
CWM	1.96	7.06	13.99	25.31	38.66	59.13
PWMAD	0.90	7.49	15.99	29.96	44.99	64.57
PSM	.83	2.92	4.18	6.60	11.68	25.27
Improved Medilter	1.07	4.64	8.92	17.11	28.06	47.64
SDROM	2.03	6.7	12.39	22.41	36.90	56.93
Rank Order Filetr	7.6	24.37	28.9	51.87	51.74	72.68
RMGF (proposed)	0.41	1.30	1.82	2.51	3.24	4.54

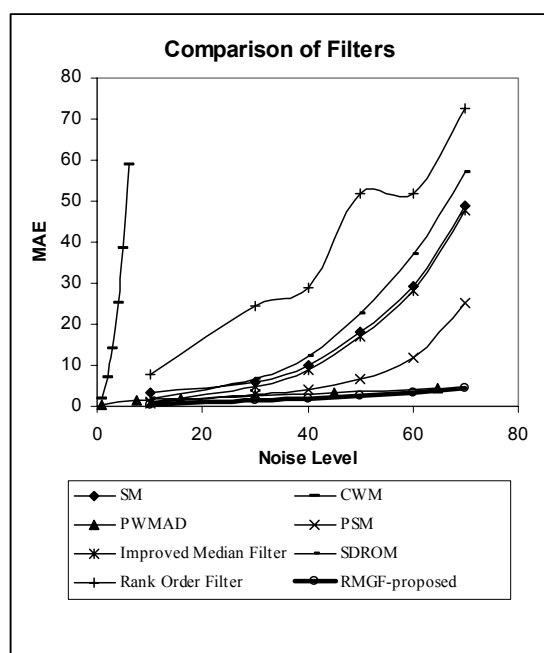


Fig 4 Comparison of Filters-Based on MAE

The proposed filter effectively suppresses the noise and filters the image without much edge degradations even at higher noise levels (>70%). It outperforms the other filters, both qualitatively and quantitatively, at all levels of corruption. The filter is capable of restoring to an acceptable quantity, a noisy image that is corrupted to a level of 90%. Figure 5 shows the Lena image corrupted with 90% noise and the corresponding output of the proposed RMGF filter.

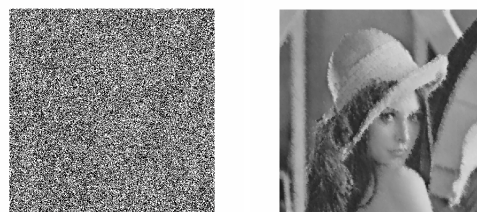


Fig 5. Corrupted and filtered images at 90% noise

Since the weights are Gaussian distributed, relatively a smaller neighborhood is sufficient to restore the corrupted pixel.

## VI. Conclusion

A new approach to impulse noise filtering is presented. The complete procedure consists of recursive Gaussian weighted filtering. It does not require any optimizing parameters or complicated pre-processing to estimate pixel values at noisy locations. The new filter outperforms most known filtering schemes both in *PSNR* and subjectively.

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