FUNDAMENTA
MATHEMATICAE
C (1978)

One Hundred Volumes of "Fundamenta Mathematicae"
The Editors Note

For any scientific journal the occasion of publishing its hundredth volume is a kind of jubilee and a suitable moment for assessing its contribution to world learning. In the present instance, however, the appearance of the hundredth volume is more than just a jubilee of the Fundamenta, since the founding of this journal — viewed in historical perspective — was the first sign of the rise of a new school in mathematics, which soon became known to the world as the Polish School of Mathematics.

The exact moment of the birth of a new scientific school cannot of course be established with any precision. However, when a group of mathematicians drawn together by their interest in the same range of problems, which — in outline — constitute the domain of the emerging new school in mathematics, set about to publish a journal dedicated to that very domain, one is justified in regarding the appearance of the first volume of that journal as the inauguration of the school whose organ it is going to become.

This historical date is the year 1920. Let us add that the founding of a specialized periodical exclusively devoted to *Set-Theory, its Applications and Related Problems* — as was then decided — was an integral part of a conception worked out by the creators of the new school: Janiszewski, Mazurkiewicz and Sierpiński. Answering an enquiry, instituted on the eve of the restoration of Poland's independence after many years of bondage, to estimate the needs of Polish science, Zygmunt Janiszewski published in "Nauka Polska" ("Polish Science") in 1918 an article entitled: "On the needs of mathematics in Poland", full of valuable and profound ideas, unfolding a vision of future Polish mathematics. Expounding his plan of founding a specialized journal, which was later realized in the shape of "Fundamenta Mathematicae", Janiszewski wrote:

"In my opinion, all strictly scientific periodical publications should be converted into more specialized ones: e. g. one might be dedicated to number theory and algebra, another one to projective geometry, still others to differential equations and differential geometry, trigonometric and related series, set theory, foundations of geometry, etc.

This is of course a long-term plan, which should be initiated by means of an

example. Here we see a field of activity open up before us, and the project becomes significant in yet another sense: I have in mind the acquisition of an independent status for Polish mathematics.

According to the above plan, we should found a strictly scientific journal, devoted exclusively to one of those branches of mathematics which — in our country — are pursued by eminent scholars, truly creative and numerous".

"A periodical of this kind", Janiszewski goes on to say, "would become indispensable to anybody working in the given branch of mathematics, it would find readers everywhere and would shortly gain important contributors abroad. This would ensure us our rightful place in European science, since we could make available to the world numerous works now scattered over various Polish journals, and, what is more, gain recognition not only as individuals whose nationality is not even known, but as a compact group of Poles. The existence and circulation of such a journal, published in Warsaw, would in itself provide ample evidence of our research activities. We are far from the workshops in which mathematics is being forged. If we do not want to lag behind, we must use radical means, we must get to the root of the evil. We must build such a "forge" here, in this country. And we can only do that by inducing the majority of our mathematicians to concentrate on one branch of mathematics. This tendency is forming of itself — it should only be reinforced. And, undoubtedly, the foundation of a special journal for one branch of mathematics will draw numerous scholars towards it.

Moreover, the journal might help towards establishing that "forge" in yet another way: we could become a technical centre for mathematical publications in the branch in question. We would be sent manuscript of new papers and keep in touch with other centres.

If we want to gain a suitable position in the scientific world, let us show initiative".

As can be seen, "Fundamenta Mathematicae" were meant to become an international journal. The first volume, however, contained only names of Polish mathematicians. This was not accidental. The restriction of the list of contributors to Polish authors aimed at manifesting the existence in Poland of a group of mathematicians who could and did take upon themselves the responsibility of instituting and running a publication of a character defined so exactly.

Janiszewski writes about it in one of his letters as follows: "my intention is to present in volume one practically all the Polish mathematicians working in the field of set-theory, to which the journal is dedicated". In the same letter, written one month before he died, Janiszewski says: "the printing of the first volume is almost completed, and it is my firm wish that the journal which I have conceived, implemented and edited should contain some contribution of my own starting from its very first issue".

Unfortunately, Janiszewski did not live to see the first issue leave the press. He dies on 3rd January, 1920, victim to the raging influenza epidemic which caused such terrible havoc at the time. Also, he had not had time to write a paper which

was announced in the above-mentioned letter. Volume one of the Fundamenta only contained a paper written jointly by Janiszewski and Kazimierz Kuratowski, who was then his pupil.

The list of contributors of vol. 1 of the Fundamenta contains the names of the founders of the Polish School of Mathematics and — largely — its subsequent leaders. They were: Stefan Banach, Zygmunt Janiszewski, Kazimierz Kuratowski, Stefan Mazurkiewicz, Stanisław Ruziewicz, Wacław Sierpiński, Hugo Steinhaus, Witold Wilkosz.

After Janiszewski's death the chief editorship of the journal was taken over by Stefan Mazurkiewicz and Wacław Sierpiński. In addition, the Editorial Committee included, up to the year 1928, Stefan Leśniewski and Jan Łukasiewicz, and from 1928 on also Kazimierz Kuratowski (as secretary to the Editorial Board).

Looking back over nearly sixty years, we cannot but be struck by the boldness of Janiszewski's conception, particularly on two points: firstly, publishing the papers in world languages, and secondly, as we mentioned before, making the journal strictly specialized.

The publication of the Fundamenta in languages known outside Poland made the results of our research accessible to the world at large, and at the same time it was a condition sine qua non of attracting contributions of foreign mathematicians to the columns of the Fundamenta and thus giving it an international character. It was not easy, however, to overcone the then prevalent opinion, or rather prejudice, that a Pole should publish his works only in Polish, in spite of the fact that his results might then never become known to scientists abroad. The second postulate, that of restricting the scope of the Fundamenta to only a few domains of mathematics, was even more revolutionary. In contradistinction to our present-day practice favouring the existence of a great many strictly specialized mathematical journals, in those days it was a complete novum, greeted by many mathematicians with undisguised scepticism.

There is evidence of this attitude in a letter written to Professor Sierpiński on the occasion of the appearance od vol. 1 of the Fundamenta by Lebesgue one of the leading mathematicians of the time. In addition to many favourable remarks on the papers contained in the volume, Lebesgue expressed his doubts whether a journal of such a high degree of specialization could rely on a sufficient supply of material to ensure its continued existence without any lowering of its standard. This fear soon proved to be groundless. The amount of material pouring in to the offices of the Fundamenta continuously increased, making it possible to issue the successive volumes at more frequent intervals and to raise the selection criteria.

Lebesgue himself, in an extremely interesting article "À propos d'une nouvelle revue mathématique: Fundamenta Mathematicae", published in the Bulletin des Sciences Mathématiques 46 (1922) on the occasion of the appearance of the 2nd volume of the Fundamenta, addresses to the Editorial Committee the request that the journal should include within its scope any applications of set

theory, and not only the most immediate ones, as would seem to follow from the journal's initial programme. This request resulted from Lebesgue's sincere goodwill towards the new journal and his concern for the further development of set theory.

Lebesgue wrote his article at a time when set theory was far from being generally recognized as a legitimate branch of mathematics: it was then still struggling for its rights. In Lebesgue's words "set theory was pushed beyond the pale of mathematics by the high priests of the theory of analytic functions" and if "this ostracism with respect to set theory is now on the wane", it is due to the fact that "set theory, which grew out of the theory of analytic functions, has been able to prove itself useful to its elder sister and to show to men of goodwill its merits and its wealth". It is also as a consequence of the contacts which specialists in set theory had with specialists in other fields that "they did not get involved in speculations detached from the main body of mathematics, which might have led them to the formation of a new discipline, no doubt respectable but — through its lack of topicality — destined to remain beyond the range of general interest".

This view of Lebesgue's, as well as his conviction of the necessity of laying particular stress on the applications of set theory, were shared by the editors of the Fundamenta. The same tendency is reflected in the present, modernized, formulation of the range of problems dealt with by the Fundamenta: *set theory, foundations of mathematics, real functions, abstract algebra.*

The one hundred volumes of the Fundamenta comprise 2650 papers by 1170 authors. The number of papers is not of course a conclusive measure of the journal's worth. What matters is the standard of the publications and their effect upon the development of learning. In this respect the Fundamenta can boast of a considerable number of important papers by eminent authors, papers which have played a fundamental role in the development of set theory, topology, functional analysis, theory of the functions of a real variable, and of mathematics as a whole.

By way of example, let us mention the following mathematicians, no longer living (Polish authors excluded), who have contributed to the Fundamenta: E. Borel, E. Čech, A. Denjoy, A. Fränkel, M. Fréchet, H. Hahn, G. H. Hardy, H. Hausdorff, H. Hopf, E. V. Huntington, V. Jarnik, A. Kintchine, H. Lebesgue, S. Lefschetz, B. Levi, J. E. Littlewood, N. Lusin, P. Montel, R. L. Moore, M. Morse, J. von Neumann, P. Novikov, F. Riesz, M. Suslin, P. Urysohn, G. Vitali, J. H. C. Whitehead, G. T. Whyburn, N. Wiener, E. Zermelo.

Undoubtedly the cooperation of scholars of such renown is a telling evidence of the attitude of the world of science towards our journal. From among the numerous opinions of foreign scientists about the role of the Fundamenta we shall quote those of Tamarkin and Alexandrov, voiced in connection with the appearance of vols. 25 and 50 and also of vol. 33, which was the first post-war issue.

In an article entitled "25 volumes of Fundamenta Mathematicae", published in Bulletin of the American Mathematical Society, vol. 42 (1936) p. 300, Tamarkin

wrote: "Under masterful guidance of his colleagues, S. Mazurkiewicz and W. Sierpiński, Fundamenta Mathematicae immediately developed into a unique periodical which attracted international recognition and cooperation, and whose history became the history of development of the modern theory of functions and point sets.

The richness of material furnished by Polish mathematicians and by their foreign colleagues, led to the founding (in 1929) of a new periodical, Studia Mathematica, devoted mainly to problems of functional analysis. This did not decrease in the least the interest and importance of the Fundamenta Mathematicae, which reached its silver jubilee volume in a state of youthful maturity and with hopes for coming vigorous development".

In connection with the journal's golden jubilee Professor Alexandrov wrote to the Editorial Committee that this jubilee was a feast to all mathematicians of the world.

The appearance of vol. 33, i. e. the first post-war volume of the Fundamenta, only a few months after military operations had ceased, was particularly telling. Many of the papers designed for that volume had been set up for printing in 1939. Most of them were destroyed by Hitlerite agressors and only a few escaped their vandalism thanks to the exceptional dedication and courage of the persons who concealed and preserved them. Equally exceptional dedication and energy was needed to publish that volume in the second half of the year 1945 under conditions of almost complete devastation of the material base of Polish science.

This is what Professor P. Alexandrov writes to Professor Kuratowski on receiving vol. 33 of the Fundamenta:

"The impression which that volume made upon us all was enormous and at the same time tragic. Tragic — because it would be difficult to react in any other way to the dedication of that volume to our Polish mathematical colleagues, who perished so tragically in this war, whom we all valued so much for their magnificent work and whom many of us, myself included, had had the good fortune to know personally. Their death, and the terrible circumstances in which they died will never be forgotten by those who have the cause of learning and the cause of elementary humanity deeply at heart.

The load of depression brought about by those terrible happenings makes us still more emphatic in expressing the respect — and let me be allowed to say — the admiration which we, the Moscow mathematicians, feel for our Polish colleagues and friends when we see the depth of content and the perfection of form which you have been able to give to your revived publication in spite of the difficulties you lived through during the night of occupation. It is something of a symbol of the triumph of the eternal ideals of human civilization and scientific truth over the dark and inhuman goals, in the name of which Hitlerite Germany for six years endeavoured to plunge Poland into slavery and which have now been ultimately thwarted".

The calamity of World War II was also a turning point for Fundamenta Mathematicae. The terrible years of foreign occupation from 1939 to 1944, when any form of social activity and, particularly, intelectual activity was systematically repressed by the occupying forces, brought death to about half the Polish mathematicians — usually in very tragic circumstances. These years divide the history of Fundamenta Mathematicae into two periods. The first extends from 1920 to 1939, which years saw the publication of the first 32 volumes of Fundamenta Mathematicae; the second, beginning in 1945, during which time a further 68 volumes have appeared. These two periods differ essentially in kind.

Between 1920 and 1939 there were altogether 624 papers by Polish authors, a figure significantly exceeding the number of those from abroad, which totalled only 291. Nevertheless, in that same period there were considerably more foreign contributors (145 in number) than Polish (61 in number). Papers were mainly on Topology (48 per cent), Real Analysis (about 24 per cent) and Foundations, including Set Theory, (about 16 per cent). Papers on abstract algebra were few (about half per cent) and the remaining areas made up about 11 per cent of all publications. French was the prevailing language (about 66 per cent, as against 18 per cent in German, 14 per cent in English and one per cent in Italian).

In the post-war period the papers by Polish authors (636 altogether) comprise under 37 per cent of all publications (the grand total being 1735 papers). Authors from the USA came to the fore (640 papers, i. e. about 37 per cent of the total). The break-down by subject matter is also altered. Papers on Topology still come close to 49 per cent of the total of all papers, but Real Analysis suffered a set-back (not quite seven per cent). However, the volume of work in Foundations increased (389 papers, i. e. over 22 per cent) as did, very considerably, work in Abstract Algebra (185 papers, i. e. about 11 per cent). The proportion of papers in other areas (194 in number, representing over, 11 per cent) increased only slightly. A finer analysis of the data may be found in the tables below.

There is a very marked shift in the post-war period towards the use of English. Nearly 86 per cent of all papers were in English while only 9.1 per cent were in French, about 2.6 per cent in German and about 2.5 per cent in Russian. There were no papers in Italian. English similarly prevails in other scientific journals, not only in the mathematical ones.

Whereas in the inter-war period, papers published in Fundamenta Mathematicae originated from 20 countries, those published in the post-war period now represent 43 countries. The most notable increase is to be found in the number of authors from the USA, rising to 557 (as against 207 from Poland and 405 from elsewhere). The world-wide character of Fundamenta Mathematicae has thus become much more pronounced now than in the inter-war years.

We tabulate below 15 countries that are represented by at least ten authors who published in volumes 1–100. The figures in brackets refer to the number of authors in the inter-war period.

Number of Authors:

Austria	12 (9)	German Federal Republic	29 (4)	Norway	10 (2)
Canada	43 (0)	Holland	21 (4)	Poland	212 (61)
Czechoslovakia	27 (5)	Hungary	22 (7)	Switzerland	12 (5)
England	50 (9)	India	23 (0)	USA	573 (42)
France	24 (10)	Japan	18 (2)	USSR	61 (32)

The following countries are represented by fewer than ten authors in volumes 1–100 of Fundamenta Mathematicae: Argentina, Australia, Belgium, Brazil, Bulgaria, Chile, China, Denmark, Finland, German Democratic Republic, Greece, Italy, Iraq, Iran, Ireland, Israel, Jamaica, Nigeria, New Zealand, Pakistan, Romania, Scotland, South Africa, Spain, Sweden, Venezuela, Vietnam, Yugoslavia.

The distribution of subject matter is as follows:

Volume	Abstract Algebra	Foundations of Mathematics	Real Analysis	Topology	Others	Total
1–32	4	147	221	441	102	915
33–100	185	389	120	847	194	1735
1–100	189	536	341	1288	296	2650

Distribution of publications between home and abroad:

Volume	No. of publications by Polish authors	No. of publications by foreign authors	Total
1–32	624	291	915
33–100	636	1099 (incl. 640 from USA)	1735
1–100	1260	1390 (incl. 724 from USA)	2650

Distribution between home and foreign authors:

Volume	No. of Polish authors	No. of foreign authors	Total
1–32	61	145	206
1–100	207	963 (incl. 557 from USA)	1170

Distribution by language:

Volume	English	French	German	Italian	Russian	Total
1–32	129	609	169	8	—	915
33–100	1488	158	45	—	44	1735
1–100	1617	767	214	8	44	2650

These statistics give some indication of the number of papers and authors, their classification and the changes that have taken place. There is, however, a certain arbitrariness in the calculation of the figures. Classification is, in itself, partly subjective. Equally, too, the assignation of an author's nationality is doubtful, especially as his status is liable to change due to emigration. Real difficulties were caused by papers written jointly by authors coming from different countries. Consequently the statistical data above should be treated with reservations.

As we mentioned before, Professor Mazurkiewicz (d. 1945) and Professor Sierpiński (d. 1969) were the editors of "Fundamenta Mathematicae" from the moment of the appearance of its first volume. Since 1952 the publishing of the journal is in the hands of the authors of the present note — Kazimierz Kuratowski, Editor in Chief, and Karol Borsuk, Deputy Editor.

FUNDAMENTA
MATHEMATICAE
C (1978)

Parametric inductive definitions and recursive operators over the continuum *

by

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Abstract. In this paper we consider the possible closure ordinals $|\Gamma|$ and sets $\mathrm{Cl}(\Gamma)$ of non-monotone recursive inductive operators Γ which define subsets of the continuum N^N. For an upper bound, we show that any Δ^1_1 operator has Δ^1_2 closure; it is well-known that $|\Gamma| \leqslant \aleph_1$ even for Σ^1_1 operators. On the other hand, we construct a recursive operator Γ with $|\Gamma| = \aleph_1$ and show that any Π^1_1 or Σ^1_1 set is reducible to the closure of some recursive operator. Using the notion of a *parametric* operator (essentially a class of operators, one for each real parameter, over the natural numbers N), we extend this last result in several ways.

Introduction. A great deal of research has been done in recent years on the general subject of inductive definability. The volume *Generalized Recursion Theory* cited in [3] is an excellent source for background in this area.

Briefly, an inductive operator Γ over a set X is a map from $P(X)$ to $P(X)$ such that for all A, $A \subseteq \Gamma(A)$. Γ determines a transfinite sequence $\{\Gamma^\sigma\colon \sigma \text{ an ordinal}\}$, where for all σ, $\Gamma^\sigma = \bigcup\{\Gamma(\Gamma^\tau)\colon \tau<\sigma\}$. The closure ordinal $|\Gamma|$ of Γ is the least ordinal σ such that $\Gamma^{\sigma+1} = \Gamma^\sigma$; clearly $|\Gamma|$ always has cardinality less than or equal to $\mathrm{Card}(X)$. The closure $\mathrm{Cl}(\Gamma)$ is $\Gamma^{|\Gamma|}$, the set inductively defined by Γ.

For a class C of operators, the closure ordinal $|C|$ is the supremum of the $|\Gamma|$ for Γ in C and the inductive closure $\mathrm{Cl}(C)$ is the class of subsets A of X which are reducible to $\mathrm{Cl}(\Gamma)$ for some Γ in C. (The precise notion of reducibility depending on X.)

The general problem in the field of inductive definitions is to characterize, for a given class C of operators, the ordinal $|C|$ and the class $\mathrm{Cl}(C)$.

A common restriction placed on an inductive operator Γ is that it be *monotone*, that is, for any A and B, $A \subseteq B$ implies $\Gamma(A) \subseteq \Gamma(B)$. This is a strong condition and makes monotone inductive operators easier to deal with than their non-monotone counterparts. Monotone inductive definitions over the continuum were studied in some detail in [2]. (We follow the usual convention of identifying the continuum with the Baire space ${}^\omega\omega$.)

For operators over the natural numbers, monotone and non-monotone Π^0_1 inductive definitions led to the same closure ordinal (ω_1, the first non-recursive ordinal)

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* Some of the results of this paper were announced in [1].