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MATHEMATICS

Unheralded Mathematician Bridges the Prime Gap

By: [Erica Klarreich](#)

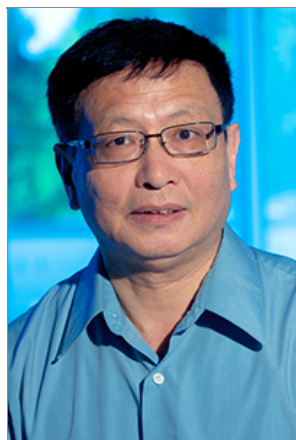
May 19, 2013

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On April 17, a paper arrived in the inbox of *Annals of Mathematics*, one of the discipline's preeminent journals. Written by a mathematician virtually unknown to the experts in his field — a 50-something lecturer at the University of New Hampshire named Yitang Zhang — the paper claimed to have taken a huge step forward in understanding one of mathematics' oldest problems, the twin primes conjecture.

Editors of prominent mathematics journals are used to fielding grandiose claims from obscure authors, but this paper was different. Written with crystalline clarity and a total command of the topic's current state of the art, it was evidently a serious piece of work, and the *Annals* editors decided to put it on the fast track.



University of New Hampshire

Yitang Zhang

Just three weeks later — a blink of an eye compared to the usual pace of mathematics journals — Zhang received the referee report on his paper.

"The main results are of the first rank," one of the referees wrote. The author had proved "a landmark theorem in the distribution of prime numbers."

Rumors swept through the mathematics community that a great advance had been made by a researcher no one seemed to know — someone whose talents had been so overlooked after he earned his doctorate in 1991 that he had found it difficult to get an academic job, working for several years as an accountant and even in a Subway sandwich shop.

"Basically, no one knows him," said Andrew Granville, a number theorist at the Université de Montréal. "Now, suddenly, he has proved one of the great results in the history of number theory."

Mathematicians at Harvard University hastily arranged for Zhang to present his work to a packed audience there on May 13. As details of his work have emerged, it has become clear that Zhang achieved his result not via a radically new approach to the problem, but by applying existing methods with great perseverance.

"The big experts in the field had already tried to make this approach work," Granville

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said. "He's not a known expert, but he succeeded where all the experts had failed."

The Problem of Pairs

Prime numbers — those that have no factors other than 1 and themselves — are the atoms of arithmetic and have fascinated mathematicians since the time of Euclid, who proved more than 2,000 years ago that there are infinitely many of them.

Because prime numbers are fundamentally connected with multiplication, understanding their additive properties can be tricky. Some of the oldest unsolved problems in mathematics concern basic questions about primes and addition, such as the twin primes conjecture, which proposes that there are infinitely many pairs of primes that differ by only 2, and the Goldbach conjecture, which proposes that every even number is the sum of two primes. (By an astonishing coincidence, a weaker version of this latter question was settled in a [paper posted online](#) by Harald Helfgott of École Normale Supérieure in Paris while Zhang was delivering his Harvard lecture.)

Prime numbers are abundant at the beginning of the number line, but they grow much sparser among large numbers. Of the first 10 numbers, for example, 40

percent are prime — 2, 3, 5 and 7 — but among 10-digit numbers, only about 4 percent are prime. For over a century, mathematicians have understood how the primes taper off on average: Among large numbers, the expected gap between prime numbers is approximately 2.3 times the number of digits; so, for example, among 100-digit numbers, the expected gap between primes is about 230.

But that's just on average. Primes are often much closer together than the average predicts, or much farther apart. In particular, "twin" primes often crop up — pairs such as 3 and 5, or 11 and 13, that differ by only 2. And while such pairs get rarer among larger numbers, twin primes never seem to disappear completely (the largest pair discovered so far is $3,756,801,695,685 \times 2^{666,669} - 1$ and $3,756,801,695,685 \times 2^{666,669} + 1$).

For hundreds of years, mathematicians have speculated that there are infinitely many twin prime pairs. In 1849, French mathematician Alphonse de Polignac extended this conjecture to the idea that there should be infinitely many prime pairs for any possible finite gap, not just 2.

Since that time, the intrinsic appeal of these conjectures has given them the status of a mathematical holy grail, even though they have no known applications. But despite many efforts at proving them, mathematicians weren't able to rule out the possibility that the gaps between primes grow and grow, eventually exceeding any particular bound.

Now Zhang has broken through this barrier. His paper shows that there is some number N smaller than 70 million such that there are infinitely many pairs of primes that differ by N . No matter how far you go into the deserts of the truly gargantuan prime numbers — no matter how sparse the primes become — you will keep finding prime pairs that differ by less than 70 million.

The result is "astounding," said Daniel Goldston, a number theorist at San Jose State University. "It's one of those problems you weren't sure people would ever be able to

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solve.”

A Prime Sieve

The seeds of Zhang’s result lie in a [paper from eight years ago](#) that number theorists refer to as GPY, after its three authors — Goldston, János Pintz of the Alfréd Rényi Institute of Mathematics in Budapest, and Cem Yıldırım of Boğaziçi University in Istanbul. That paper came tantalizingly close but was ultimately unable to prove that there are infinitely many pairs of primes with some finite gap.

Instead, it showed that there will always be pairs of primes much closer together than the average spacing predicts. More precisely, GPY showed that for any fraction you choose, no matter how tiny, there will always be a pair of primes closer together than that fraction of the average gap, if you go out far enough along the number line. But the researchers couldn’t prove that the gaps between these prime pairs are always less than some particular finite number.

GPY uses a method called “sieving” to filter out pairs of primes that are closer together than average. Sieves have long been used in the study of prime numbers, starting with the 2,000-year-old Sieve of Eratosthenes, a technique for finding prime numbers.

To use the Sieve of Eratosthenes to find, say, all the primes up to 100, start with the number two, and cross out any higher number on the list that is divisible by two. Next move on to three, and cross out all the numbers divisible by three. Four is already crossed out, so you move on to five, and cross out all the numbers divisible by five, and so on. The numbers that survive this crossing-out process are the primes.

	2	3	4	5	6	7	8	9	10	Prime numbers
11	12	13	14	15	16	17	18	19	20	2
21	22	23	24	25	26	27	28	29	30	
31	32	33	34	35	36	37	38	39	40	
41	42	43	44	45	46	47	48	49	50	
51	52	53	54	55	56	57	58	59	60	
61	62	63	64	65	66	67	68	69	70	
71	72	73	74	75	76	77	78	79	80	
81	82	83	84	85	86	87	88	89	90	
91	92	93	94	95	96	97	98	99	100	
101	102	103	104	105	106	107	108	109	110	
111	112	113	114	115	116	117	118	119	120	

Illustration: Sebastian Koppehel

The Sieve of Eratosthenes This procedure, which dates back to the ancient Greeks, identifies all the primes less than a given number, in this case 121. It starts with the first prime — two, colored bright red — and eliminates all numbers divisible by two (colored dull red). Then it moves on to three (bright green) and eliminates all multiples of three (dull green). Four has already been eliminated, so next comes five (bright blue); the sieve eliminates all multiples of five (dull blue). It moves on to the next uncolored number, seven, and eliminates its multiples (dull yellow). The sieve would go on to 11 — the square root of 121 — but it can stop here, because all the non-primes bigger than 11 have already been filtered out. All the remaining numbers (colored purple) are primes.

The Sieve of Eratosthenes works perfectly to identify primes, but it is too cumbersome and inefficient to be used to answer theoretical questions. Over the past century, number theorists have developed a collection of methods that provide useful approximate answers to such questions.

“The Sieve of Eratosthenes does too good a job,” Goldston said. “Modern sieve methods give up on trying to sieve perfectly.”

GPY developed a sieve that filters out lists of numbers that are plausible candidates for having prime pairs in them. To get from there to actual prime pairs, the researchers combined their sieving tool with a function whose effectiveness is based on a parameter called the level of distribution that measures how quickly the prime numbers start to display certain regularities.

The [level of distribution is known to be at least \$\frac{1}{2}\$](#) . This is exactly the right value to prove the GPY result, but it falls just short of proving that there are always pairs of primes with a bounded gap. The sieve in GPY could establish that result, the researchers showed, but only if the level of distribution of the primes could be shown to be more than $\frac{1}{2}$. Any amount more would be enough.

The theorem in GPY “would appear to be within a hair’s breadth of obtaining this result,” the researchers wrote.

But the more researchers tried to overcome this obstacle, the thicker the hair seemed to become. During the late 1980s, three researchers — Enrico Bombieri, a Fields medalist at the Institute for Advanced Study in Princeton, John Friedlander of the University of Toronto, and Henryk Iwaniec of Rutgers University — had developed a way to tweak the definition of the level of distribution to [bring the value of this adjusted parameter up to \$\frac{4}{7}\$](#) . After the GPY paper was circulated in 2005, researchers worked feverishly to incorporate this tweaked level of distribution into GPY’s sieving framework, but to no avail.

“The big experts in the area tried and failed,” Granville said. “I personally didn’t think anyone was going to be able to do it any time soon.”

Closing the Gap

Meanwhile, Zhang was working in solitude to try to bridge the gap between the GPY result and the bounded prime gaps conjecture. A Chinese immigrant who received his doctorate from Purdue University, he had always been interested in number theory, even though it wasn’t the subject of his dissertation. During the difficult years in which he was unable to get an academic job, he continued to follow developments in the field.

“There are a lot of chances in your career, but the important thing is to keep thinking,” he said.

Zhang read the GPY paper, and in particular the sentence referring to the hair’s breadth between GPY and bounded prime gaps. “That sentence impressed me so much,” he said.

Without communicating with the field’s experts, Zhang started thinking about the problem. After three years, however, he had made no progress. “I was so tired,” he said.

To take a break, Zhang visited a friend in Colorado last summer. There, on July 3, during a half-hour lull in his friend’s backyard before leaving for a concert, the solution suddenly came to him. “I immediately realized that it would work,” he said.

Zhang’s idea was to use not the GPY sieve but a modified version of it, in which the sieve filters not by every number, but only by numbers that have no large prime factors.

“His sieve doesn’t do as good a job because you’re not using everything you can sieve with,” Goldston said. “But it turns out that while it’s a little less effective, it gives him the flexibility that allows the argument to work.”

While the new sieve allowed Zhang to prove that there are infinitely many prime pairs closer together than 70 million, it is unlikely that his methods can be pushed as far as the twin primes conjecture, Goldston said. Even with the strongest possible assumptions about the value of the level of distribution, he said, the best result likely to emerge from the GPY method would be that there are infinitely many prime pairs that differ by 16 or less.

But Granville said that mathematicians shouldn’t prematurely rule out the possibility of reaching the twin primes conjecture by these methods.

“This work is a game changer, and sometimes after a new proof, what had previously appeared to be much harder turns out to be just a tiny extension,” he said. “For now, we need to study the paper and see what’s what.”

It took Zhang several months to work through all the details, but the resulting paper is a model of clear exposition, Granville said. “He nailed down every detail so no one will doubt him. There’s no waffling.”

Once Zhang received the referee report, events unfolded with dizzying speed. Invitations to speak on his work poured in. “I think people are pretty thrilled that someone out of nowhere did this,” Granville said.

For Zhang, who calls himself shy, the glare of the spotlight has been somewhat uncomfortable. “I said, ‘Why is this so quick?’” he said. “It was confusing, sometimes.”

Zhang was not shy, though, during his Harvard talk, which attendees praised for its clarity. “When I’m giving a talk and concentrating on the math, I forget my shyness,” he said.

Zhang said he feels no resentment about the relative obscurity of his career thus far. “My mind is very peaceful. I don’t care so much about the money, or the honor,” he said. “I like to be very quiet and keep working by myself.”

Meanwhile, Zhang has already started work on his next project, which he declined to describe. “Hopefully it will be a good result,” he said.

Correction: This article was revised on May 21, 2013, to reflect that Yitang Zhang’s doctorate was issued by Purdue University in 1991, not 1992.

This article was [reprinted on Wired.com](#) and translated into Chinese on [Wired.tw](#).


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COMMENTS FOR THIS ENTRY

Michael Currie says:



May 20, 2013 at 4:29 am

This article went into much greater depth than I expected for an article intended for the general public. And yet it was all totally understandable. Great work Erica Klarreich!



Nick Pannell says:

May 20, 2013 at 6:46 am

Such perseverance is quite remarkable.



Rick Nelson says:

May 20, 2013 at 6:55 am

When will this paper be publicly available?



BD says:

May 20, 2013 at 7:39 am

“In 1849, French mathematician Alphonse de Polignac extended this conjecture to the idea that there should be infinitely many prime pairs for any possible finite gap, not just 2.”

Of course, this is mis-stated: replace the word ‘finite’ with ‘even’ and it becomes much more sensible.



Elenore Agenbroad says:

May 20, 2013 at 10:40 am

I hate math but this intrigues me and I “get” it Lo!



Julie Dick says:

May 20, 2013 at 10:51 am

Congratulations Mr. Zhang. A huge accomplishment indeed!



James says:

May 20, 2013 at 12:43 pm

“In 1849, French mathematician Alphonse de Polignac extended this conjecture to the idea that there should be infinitely many prime pairs for any possible finite gap, not just 2.”

Am I missing something here? Wouldn't there have to be infinitely many even primes for that to work for an odd gap?



Jeremy says:

May 20, 2013 at 3:49 pm

At a time money is the pursuit of many, what an amazing story.



Kim Nilsson says:

May 20, 2013 at 7:23 pm

I have to agree with Michael Currie. This was a very well written article about something very advanced. I enjoyed it fully. I found a report on the Harvard seminar by Yi Tang Zhang, written by Emily Riehl:

http://golem.ph.utexas.edu/category/2013/05/bounded_gaps_between_primes.html



Shekar Swamy says:

May 20, 2013 at 7:58 pm

Surely, his approach and perseverance are exemplary. No matter how significant his research was,

it was done with a basic quest to find an answer. He did not do any of this for personal glory or getting published. It was a pure effort to find an answer, nothing else. What it means to me is to go through life's journey with this type of passion with no expectations. What a wonderful story!



Tommy Tang says:

May 20, 2013 at 8:14 pm

Amazing! This is huge! And an awesomely written article – it's not often we find an article about pure math that is all at once respectful, relatively in-depth, and interesting!



Yihua says:

May 20, 2013 at 8:36 pm

Congratulations, Dr. Zhang! sometimes the math is amazing!



Michael says:

May 20, 2013 at 9:09 pm

Good article.



Erica Klarreich says:

May 20, 2013 at 9:28 pm

BD and James: Good catch. Even gaps are the only ones possible, with the minor exception of the odd gaps that arise when one of the two prime numbers is 2. I decided to write "possible" rather than "even" since I thought the latter wording might throw off some readers who didn't realize that the even gaps are the only ones worth considering. But it's certainly true that odd gaps are possible, and when they do arise, it is only once, not infinitely many times.



Richard Poulin III says:

May 20, 2013 at 9:34 pm

"Simplicity is the ultimate sophistication." and the GIF animation is just that.



S b R d says:

May 20, 2013 at 9:40 pm

"There are a lot of chances in your career, but the important thing is to keep thinking," he said. I agree- thank you, author. I really enjoyed this, it leaves you wanting to know more. When you read about the depth Mr. Zhang saw in Math upon reading that original GPY paper – something many others would bypass as not much beyond a boring 2-dimensional "sieve" algorithm for determining prime numbers on a page, you it is quite an inspirational finding. I think we all have our own version of some unique thing among others that reveals such depths for us in different ways.



John Rasor says:

May 20, 2013 at 10:11 pm

What an amazing and interesting finding. I want to know where the 70 million came from. This is a very well written and enjoyable article. Bravo, Ms. Klarreich!



Tim says:

May 20, 2013 at 10:32 pm

"Study your math, kids. Key to the Universe." - Christopher Walken as Gabriel, The Prophecy 😊



Scotty McNardy says:

May 20, 2013 at 11:39 pm

Very informative article – especially for a subject with no useful application. Congratulations to Mr. Zhang. I feel some pity when such a brilliant mind uses his talent and genius on something so

useless, he needs a top job figuring out our country's budget instead! There is where this man can save lives!



Phil Kim says:

May 20, 2013 at 11:46 pm

What an inspiring story and ordeal. Sounds like a "Good Will Hunting" story. It's passion that wins in the end, not greed nor self-pride.



Kendall Helmstetter Gelner says:

May 21, 2013 at 12:10 am

Great story, and I really appreciate the level of detail this went into – I especially enjoyed seeing the animated Sieve of Eratosthenes. Really great to hear a story about people who can develop advanced results outside the traditional system.



Sam says:

May 21, 2013 at 12:27 am

I am thinking he is probably trying to get a professorship somewhere else, instead, he is working on another problem and wants to be back in obscurity. I have no doubt he will solve more important "conjecture" or whatever it means, hats off for him. I still hope he gets what he deserves, I guess that is why I am still nobody and no hope to change this pathetic situation. Good lesson and Good luck!



TronPhenix says:

May 21, 2013 at 12:29 am

I realized that he has solved a big problem but he needs a job offer.



Yizhe Z says:

May 21, 2013 at 12:40 am

Very good article. Very inspiring story.



Robyn Wang says:

May 21, 2013 at 12:56 am

Fascinating!



Brian Hastings says:

May 21, 2013 at 2:35 am

You Sir are a Gentleman and a Scholar! But you already knew that! 😊



Pavel Tomancak says:

May 21, 2013 at 2:58 am

Great story and great article. You give justice to your name Erica Klarreich 😊.



PETER CHEN says:

May 21, 2013 at 3:00 am

A truly master piece of article of introducing the work of a master!



Jonathan Verner says:

May 21, 2013 at 3:48 am

@Scotty: its funny how you think Zang is brilliant in maths while at the same you doubt his judgement in choosing the area to work in. Did it ever occur to you that doing "useless" stuff might

just be more important than the “useful” work our politicians/economists/etc do?



Zulf says:

May 21, 2013 at 6:36 am

McNardy: Newton's Laws of Motion could one day lead to us saving mankind by leaving our home or by intercepting/diverting an unwelcome wandering mountain. Newton's equations by your logic are also useless. Let us see how useless his work is after 200 years – when those who worked on the country's budget are long forgotten. A real mathematician is as addicted to his “art” as Vangogh was to his. No careers teacher will convince either person to give up their art and get a “real” job.



David says:

May 21, 2013 at 7:11 am

I wonder if this has any implications for breaking strong encryption like SHA 256 ? Anybody know?



BlueRick says:

May 21, 2013 at 7:27 am

I admire Mr. Zhang's accomplishment but I wonder if it has any practical applications and if so, what might they be?



AndreMR says:

May 21, 2013 at 7:35 am

This article is exceptionally well written. Its exceptionality rivals the exceptionality of the great work it describes.

Keep it up! Science desperately needs this high quality of journalism.



Hamid says:

May 21, 2013 at 8:09 am

Math is n't my favorite topics, most of time I learn it forced by parents .But this understanding system make me interested at math.



Enzo Scavone says:

May 21, 2013 at 9:02 am

This is very impressive. He also seems like a very congenial guy.



Bruce Lehnertz says:

May 21, 2013 at 9:13 am

As a former Iowa State University math major, and the kid that, after doing his math assignments, went back and did all the problems that weren't assigned...and always worked on all the proofs...I relate to Dr. Zhang's relentless interest in the problem, and can appreciate the difficulty of an elegant description of the problem and the proof. I was bested by Differential Equations (with an assist by my proclivity for self-indulgence during my college years. Excellent job, Dr. Zhang. Congratulations.



Erica Klarreich says:

May 21, 2013 at 10:38 am

John Rascal: Great question. The number 70 million isn't something fundamental to Zhang's analysis – it was just a convenient number for him to use (it arose out of the length of the lists of numbers he was considering, and the spacing between the numbers). Often when mathematicians are trying to prove that something has an upper bound, they use whatever numbers will push their argument through most smoothly, without worrying about whether they're getting the best

possible bound. Later on, sometimes a whole cottage industry arises around trying to push the known bound lower and lower.

One of the mathematicians I spoke to said that he thinks it's already possible to use a bound of 60 million in Zhang's arguments. I think mathematicians are fairly confident that it will be possible to push the bound much lower than that.



Mark Swanson says:

May 21, 2013 at 1:04 pm

It's already been said but I am as impressed by Erica's writing as by Yitang's work. I was going to read until I got lost (which usually doesn't take long with math) but I read all the way through to the end with understanding. Either someone slipped me a smart pill this morning or Erica is an outstanding journalist.



ESTEBAN AGOSTO REID says:

May 21, 2013 at 1:36 pm

Great story !!



Steve Bryan says:

May 21, 2013 at 1:57 pm

Mr McNardy, you are confusing arithmetic with mathematics. A number theorist's genius would be wasted on the issues involved in something like a country's budget which is more about politics and the corrosive influence of moneyed interests. Being right about economic models and their predictions seems to have little weight in public discourse – just consider the quixotic battles of Professor Krugman (Nobel laureate and NY Times columnist) against the vapid austerians.

In any case number theory has been anything but "a subject with no useful application" which you would know if you knew even a little about the subject (hint: RSA). As fields such as quantum computation continue to advance we will need ever new knowledge and understanding of fields like number theory which 'practical' people erroneously dismiss.

My congratulations to Prof Zhang and the author of this article. It is an inspiring story. The animated GIF of the Sieve of Eratosthenes is brilliant as well. I dragged the animation to my desktop and it plays very nicely entirely on its own.



Jaycal33 says:

May 21, 2013 at 2:02 pm

Great work Zhang and a well written article too!



Tian says:

May 21, 2013 at 2:10 pm

I can feel the vibe although I did not know math, it is like Linsanity all over again.



DB says:

May 21, 2013 at 4:18 pm

This was indeed a well written article; good job narrating the work for non-experts. (Though the emphasis on his relative obscurity is strong)



Mahmud Hasan says:

May 21, 2013 at 5:13 pm

Great article! A colleague shared the link with me and I thanked him for sharing this excellent article. So detail and so easily explained! Thank you very very much!!!

Andy Eppink says:



May 21, 2013 at 7:23 pm

Good work. Math is the most difficult intellectual effort there is tho I think applied math is much more appealing than theoretical.



Norman Medina says:
May 21, 2013 at 9:26 pm

Congratulations Mr. Zhang. Great story!!!



Chris says:
May 21, 2013 at 9:48 pm

Great article; very clear.



masmanz says:
May 21, 2013 at 10:04 pm

Many great contribution in science, engineering, and maths has been made by immigrants. A good immigration policy is fundamental to our success. I wish the masses knew.



Simon says:
May 22, 2013 at 12:42 am

Very well written paper. Math is fun but hard to describe. This writing is so clear! Thank you.



soomar says:
May 22, 2013 at 3:54 am

great job done I am non-expert but I like and appreciate



bigtummy says:
May 22, 2013 at 10:58 am

Thank you Erica for your wonderful story and excellent writing. It's really inspiring, and reminds me of childhood math dreams.



Tomcircle says:
May 22, 2013 at 11:36 am

Congrats to Dr. Zhang. This is another familiar story of great math theory discovered by humble unheralded man, thinking in solitude, not bothered by monetary and glory, just curious to find the truth, depends on hardwork under unfavorable social condition, and "keep thinking". His eureka by intuition is familiar with Archimedes, Poincaré and many great mathematicians in history – a result of prolonged thinking, and the 'unconscious' mind gives the hint at the most unexpected moment – while Dr. Zhang was waiting to go to concert at a lull moment.



Pushpendre says:
May 22, 2013 at 1:22 pm

Absolutely outstanding article, and incredibly inspiring dedication shown by Dr. Zhang



Sean A. says:
May 22, 2013 at 2:47 pm

This is awesome!



Richard says:
May 22, 2013 at 3:34 pm

I had the honor of knowing him at Purdue. He is a quiet person and seems to be thinking all the time. He is a friendly person, loves helping fellow students and social. His topic/interest shocked me at the time, number theory being one of them. Congratulations to his great accomplishment and perseverance. Well deserved!



Larry Siegel says:

May 22, 2013 at 4:05 pm

Great article. I'll look for more by this author.

@Steve Bryan: If you knew more about arithmetic, you wouldn't say what you did. Arithmetic is a sophisticated and important branch of mathematics. The country's budget problem is that the government needs to take in more money and spend less. That can be described as an arithmetic problem but it's really a problem in governance and human behavior.



Juan A says:

May 22, 2013 at 7:28 pm

Boiler up! It is trully inspirational to find a story of success from another person in such awe of primes! Congratz Dr. Zhang!



Sujoy Gupta says:

May 23, 2013 at 2:18 am

Let knowledge grow from more to more... Sujoy Gupta, Calcutta, India



Frank Zhang says:

May 23, 2013 at 12:18 pm

From Chinese website, it was said that he was the No.1 student in math at PKU that year. However, since knowing nothing about U.S. math program areas (in his time, no Internet and almost zero information about U.S. graduate school in China), he got into purdue's math program but not in the area he is interested. Now we all know he is interested in Number Theory.

However, he still managed to get his Ph.D. in the area he was not interested at all. That was already amazing. In addition, rumors said that his Ph.D. adviser then at Purdue apparently showed zero even negative support after his graduation, which made his life afterwards really difficult. That is one of the reason why he had such a hard time to find an academic position to support his research and he had to work at places like subway to make a living.

Even at UNH, he is not at a tenured position and has to face loads of teaching duties that any tenured professor would not have to go through.

However, he still manages to achieve this great work with such perseverance. Amazing. His fellow students at his year at PKU now mostly full professors at major institutions. However, none of them have suffered this much as he has gone through.



syhliu says:

May 23, 2013 at 3:39 pm

A wonderful story and excellent writing. "My mind is very peaceful. I don't care so much about the money, or the honor." Inspiring. I wonder if anyone is considering making Dr. Zhang's story a movie.



ronnie smith says:

May 23, 2013 at 10:46 pm

Less than 70 million? I would say "two" not more, not less- otherwise not a prime pair by definition correct?

DNS RAO says:



May 24, 2013 at 6:00 am

Absolutely remarkable story of pure dedication to find the truth without seeking any reward. Zhang must be admired for the lucidity of his writing: even a layman feels comfortable reading his article on a subject as dry as what he has tried to explore. He belongs to the same class as Einstein, who was also relatively unknown when he presented his theory of relativity to the world. Left to himself he will surely unravel greater truths in the field of mathematics.



Y Li says:

May 24, 2013 at 1:25 pm

It is amazing and inspirational that someone out there like Dr. ZHANG works on finding the fundamental truth of the universe, without any monetary rewards.



Filosofen says:

May 24, 2013 at 6:44 pm

"When I'm giving a talk and concentrating on the math, I forget my shyness," he said. "My mind is very peaceful. I don't care so much about the money, or the honor," he said. "I like to be very quiet and keep working by myself."

That's the spirit. Congratulations and greetings from Norway.

Regards, Filosofen



Eugene Seidel says:

May 25, 2013 at 2:18 am

I agree with the plaudits for Ms. Klarreich's writing. When I was in school, I really struggled with the math textbooks we had. It would have benefited me if those books had offered such clear exposition. Perhaps she should be put in charge of textbooks for secondary schools!



Gerald A. Belfer says:

May 25, 2013 at 2:46 am

Also a great lesson in humility for those of us who frequently forget we are not as smart as we think we are.



Vincent Mathew says:

May 25, 2013 at 9:42 am

Thank you very much for writing such a crystal clear article on this complex topic.



Jörgen Backelin says:

May 25, 2013 at 11:50 am

In my opinion, this is an extraordinarily good article, since it succeeds to explain a frontline research mathematics result in a way possible to understand for a more general audience. In general, this tends to be much harder as regards research results in mathematics, compared to the experimental sciences.

I have just one question. From the article I got the impression that the referees at the Annals of Mathematics were enthusiastic about the paper. Could you confirm that this means that the journal has accepted the paper for publication (with or without the usual list of minor things for the author to fix)?



GPandey says:

May 25, 2013 at 12:25 pm

Wow Congrats to Mr Zhang....



Erica Klarreich says:
May 26, 2013 at 4:07 pm

Jörgen Backelin: At the time I wrote the article, the referees had strongly recommended that the paper be accepted, and had sent a list of very minor changes. Since then the paper has been officially accepted, and is now on the Annals website (the full text is available only to subscribers): <http://annals.math.princeton.edu/articles/7954>



Wei Hu says:
May 26, 2013 at 6:54 pm

Congratulations to him! Beautiful mind and soul and truly inspiring! I even want to apply to his student if ever possible.



José Ruiz Huidobro says:
May 27, 2013 at 11:28 am

Mis felicitaciones para el Dr. Ytang Zhang y para Erika Klarreich. Ciertamente es un avance importantísimo en cuanto a la conjetura de los pares primos. En cuanto a Erika Klarreich el mérito es grande: es difícil escribir con la claridad con que ella lo hace sobre un tema incomprensible para muchos y considerado como “árido” aún por los entendidos. En cuanto a la mención sobre Harald Helfgott acerca de la conjetura débil de Goldbach (todo número impar mayor de 5 puede ser expresado por la suma de tres números primos) Helfgott ha presentado un trabajo, que pronto será sometido a una revisión por expertos (peer review), cabe anotar que Harald Helfgott, mi compatriota, nació en el Perú e hizo sus estudios escolares íntegramente en el Perú. Queda aún mucho por hacer en cuanto a los números primos pero el avance es cada vez mayor. Sólo anhelo que se pruebe mientras viva la conjetura de Riemann. NB: Volví a enviar mi comentario por cuanto por error mío lo envié antes cuando estaba incompleto.



Lorelei Leslie says:
May 27, 2013 at 6:11 pm

Thank you for writing this article in a manner so concise and clear. I think it really sweet that Mr. Zhang does not care focus on money or fame, rather he prefers peaceful solitude. How refreshing.



Chungang Shang says:
May 28, 2013 at 1:57 am

Very well written article which can help the ordinary people to understand the brilliant mind and brilliant story. Congratulations to Dr. Zhang.



Grilli says:
May 28, 2013 at 11:02 am

This is a fascinating, easy to read article and I'm appreciative. What a fairy tale for a mathematician! Congrats, to all the parties involved and much success for Dr. Zhang and I hope he reaches as far as he should go.



Sudarshan Narasimhan says:
May 28, 2013 at 8:48 pm

This is truly inspirational!!!! I really respect and admire this man's passion in Mathematics!!! Even more impressive is the fact that the solution was already capable within the existing boundaries of Mathematics today. Way to go Dr Zhang!!!! You've shown that its not always necessary to come up with a radical new idea but if you understand the existing ideas well enough, you can formulate solutions to hitherto unsolved problems still. 🙌 Salute!!!!



Byran Lucas says:
May 29, 2013 at 2:02 am

Finally, students in USA may have a scientist as a role model in addition to those movie stars and football players. Last time this happened was in the era of Richard Feynman.

**imwt** says:*May 29, 2013 at 2:22 am*

Great article. Well written! I've read the Chinese version of this article. I guess someone translated it into Chinese because little could be found about Dr. Zhang even in Chinese world. Some of his college schoolmates posted an article last week. They told more details about Dr. Zhang's work and life, which are more touching. Back to 1999, two of Zhang's schoolmates (and actually students back to college), Prof. Ge and Prof. Tang met by accident and talked about losing connection with Zhang. They decided to help him. They finally located him in a Subway restaurant somewhere in the South and were surprised that Zhang hadn't give up on math at all. Imagine you were served in Subway by a guy full of world class math problems in mind secretly. Prof. Ge works for UNH and helped to get a part-time teaching job for Dr. Zhang. Dr. Zhang told them he was working on the problem and might get something "soon". Dr. Zhang also got much help from the head of the Math Department of UNH, Prof. Kenneth Appel, who was a world class mathematician. Dr. Zhang was given a full time job as a lecturer. His daily work load is actually not too much more than a tenure tacked professor. It is believed that was because of Prof. Appel. Prof. Appel passed away in April 2013. Fortunately he learned about Dr. Zhang's break through two days before he died. That must be a very touching moment. Mr. Zhang's wife lives in California (it's not unusual for Chinese couples in US). They don't have kid. I guess Dr. Zhang has devoted all his life to math. I admire him so much and inspired so much by his story though I know little about math. Congratulations!

**Qing** says:*May 29, 2013 at 2:41 pm*

Someone should think to make a movie about him.

**optimus Prime** says:*May 30, 2013 at 6:38 am*

It's difficult to find well-informed people on this topic, however, you seem like you know what you're talking about! Thanks

**mw** says:*May 31, 2013 at 6:26 am*

Try also this sieve: <http://www.facebook.com/TableOfPrimeNumbers>

**Mathlete Coach** says:*May 31, 2013 at 10:26 am*

The valuable take-home-message here is to appreciate the many Zhang's we see around us everyday, whose "success" lies in their unrelenting and risky search for something truer and more beautiful, without -ever- "hitting jackpot"

The damaging take-home-message here is: "hard work and dedication will eventually catch people's eye." Despite hard work and dedication, the final piece of the puzzle – and whether it catches people's eye – is a chancey lottery. It usually does -not- "pay off."

But would his effort be any less inspiring if his dedication had never "paid off?" Is a rainbow any less beautiful without the pot of gold at the end?

**Hebert Perez-Roses** says:*June 1, 2013 at 8:21 am*

This is one of the best articles I have ever read, about a truly remarkable person. Chapeau !!!

**Dave Home** says:*June 2, 2013 at 7:12 am*

What a fascinating article. Thanks to my friend, Kalcidis, for the link.

**S. Le** says:*June 5, 2013 at 11:21 pm*

A historic achievement in pure thought! That we get to celebrate Dr. Zhang's achievement despite decades of his persevering in obscurity and through hardships is quite moving. For this, the editors of the Journal deserve more credit Dr. Zhang's doctoral adviser at Purdue.

**Will Hou** says:*June 7, 2013 at 7:06 am*

inspiring story and well written! a bit sad if you think about it. definitely worth to be told over and over. it reminded me of the movie A Beautiful Mind. But I am not sure the best way of reaching the producer but internet is our friend. So I tweeted Brian Grazer (my first ever tweet! had a tweeter account from several yrs ago but never used it). hopefully he can make it happen

**Berrebi** says:*June 10, 2013 at 7:25 pm*

This is an extraordinary achievement ! it seems to have no "usefulness", except in the field of number theory itself, but if i understand it correctly, it shows that the natural "distance" that we assign between numbers can be bound for infinitely many primes, so across all the natural numbers. But primes are very special by themselves, they possess a unique blend of "necessity" and "contingency" in their attributes, to talk in 17th century terms, as they multiplicatively define all numbers, filling all the gaps and being "additively" compatible with the very "natural" construction of "natural" numbers as "equally spaced" operators to number everything else. And yet they also seem to have some built-in unpredictability in their distribution that indicates a sort of higher standard that sets them apart. So the fact that subfamilies of primes can encompass the whole range of numerable infinity while keeping within bounded distance of their "next of kin" is a real discovery about the shape of the link between these sort of germs of all numbers that primes are and the texture of the numbering they generate. This opens so many speculations on possible numbers and also hard new methods to explore "our" numbers, it's fantastic beyond all notion of "usefulness"!

**Shing Lin Choy** says:*June 15, 2013 at 5:13 am*

The less useful his finding is, the greater respect I have for him. It is a rude awakening for those of us who never do anything "useless".

**Caitlin J.** says:*June 28, 2013 at 9:47 am*

An incredible achievement in its own right, this work casts an intriguing light on recent discoveries of very large primes.

**Mingwen L** says:*July 5, 2013 at 5:19 am*

"My mind is very peaceful. I don't care so much about the money, or the honor," he said. "I like to be very quiet and keep working by myself." The attitude is important!

**santanu acharjee** says:*August 20, 2013 at 1:06 am*

congrts..prof. Zhang

**L. G. Henson, Jr.** says:*September 20, 2013 at 3:44 am*

Great minds come from modest or tragic quarters. The genius mathematician Srinivasa Ramanujan of India was afflicted with a curable disease of which he died at an early age. Who knew of a nondescript Swiss patent clerk will make a monumental discovery as turning matter into

energy – Albert Einstein. The father of modern chemistry, Antoine Lavoisier was probably not from modest background since he was a taxmaster under Louis XVI, for which he was put to the guillotine by vindictive revolutionaries. These great minds as Prof. Zhang, of course, will never be forgotten, but only Einstein was allowed to grow his disheveled grey hair as a crown. Let's hope he will, too.



Dick W says:

September 24, 2013 at 5:35 pm

It is difficult for me to conceive how such a large gap could be compatible with the Goldbach conjecture. I realize that 70,000,000 may be artificially high. Still, whatever the precise maximum gap may be, it is a large number. So there are an awful lot of even numbers within this gap. I guess I have to take it on faith — I'm no Yitang Zhang — but I wish I could grasp how there are enough combinations (i.e. two primes added together) to fill up the vast number of even numbers in the region of very large numbers, when the primes have gotten so rare. (Well, Goldbach isn't proved yet, is it?)



Craig says:

November 20, 2013 at 2:26 am

Scotty McNardy, how do you propose we find a "useful" application for things not yet discovered? Perhaps you could lend me your crystal ball? I was under the impression that discoveries had to come first, before any applied use can be found...

Also, "our" (presumably you mean the U.S.) budget is already "figured out". The short answer is, you're delaying the inevitable and can soon enjoy financial bankruptcy to go with your moral bankruptcy. Have fun. God save 'Murica!



TheChairman322 says:

November 23, 2013 at 10:51 pm

Sounds typical of the elitist mindset which has become all too common amongst so-called experts (there are no 'experts') in the parallel universe of academia: i.e. Zhang didn't come from the 'proper' background with 'proper' references and 'proper' mentoring, so he was effectively black-balled from the system. How many discoveries or advances have been missed or retarded because of all the dubious (idle) 'experts' who now infest academia?



Azimuth says:

December 17, 2013 at 8:25 pm

Dick W wrote: "It is difficult for me to conceive how such a large gap could be compatible with the Goldbach conjecture."

Fortunately, the Goldbach conjecture is most at risk in the vicinity of the smaller even numbers—those which can be verified by hand. (For instance, it's easily seen that five and seven sum to $N=12$. But they are the only primes that do—Goldbach nearly fails here.)

But note that by the Prime Number Theorem, as N becomes larger there are more primes less than N , hence more candidate pairs of primes that might sum to N . (For example, $100 = 3+97, 11+89, 17+83, 29+71, 41+59, \text{ and } 47+53$ — six pairs!)

Probabilistically, Goldbach becomes more believable as N increases.



dr. Huen Yeong Kong says:

December 21, 2013 at 11:50 pm

I am an amateur number theorist who is interested in large prime gaps. From the list of large prime gaps found to date, it appears obvious that the size is increasing. I am at present working on Grimme's Conjecture which states that over every primegap, the count of distinct primes is either equal or greater than the number of consecutive composites within a primegap. To do this I define Grimme's Ratio = (count of distinct primes)/(count of consecutive composites). So far no counter-examples have been found from the list of large primegaps. But I notice the following intriguing signs that as the primegaps increase in lengths, the Grimme's Ratio starts diminishing

in values toward unity. Here are the results: Within the largest discovered primegap of 1442 starting with prime 804212830686677669; Graimme's Ratios for various lengths are: 400 1.81 600 1.72 1523.4100 sec 800 1.65 1000 1.61 1442 1.54 But at this point I am stonewalled because there is no primegap discovered beyond this todate. My conjecture is that ultimately, there will be a primegap large enough to breach below the unity value for Graimme's Ratio and thus proves that Grimme's conjecture is false. How I wish dr. Yitang Zhang's could contribute towards this goal. dr. Huen Yeong Kong

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