Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS SPECIALIST

ATAR COURSE

Year 12 syllabus

IMPORTANT INFORMATION

This syllabus is effective from 1 January 2016.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the School Curriculum and Standards Authority on a cyclical basis, typically every five years.

Content

Overview of mathematics courses

There are six mathematics courses, three General and three ATAR. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The Western Australian Certificate of Education (WACE) examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a General course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a General course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the WACE. It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem-solving throughout the Mathematics Specialist ATAR course. There is also a sound logical basis to this course, and in mastering the course, students will develop logical reasoning skills to a high level.

The Mathematics Specialist ATAR course provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of the Mathematics Specialist ATAR course will be encouraged to appreciate the true nature of mathematics, its beauty and its functionality.

The Mathematics Specialist ATAR course has been designed to be taken in conjunction with the Mathematics Methods ATAR course. The course contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in the Mathematics Methods ATAR course and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. The Mathematics Specialist ATAR course is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of the Mathematics Specialist ATAR course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students' learning of the course. These strands are Understanding, Fluency, Problem-solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem-solving. In the Mathematics Specialist ATAR course, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problem, for example, integration, to solve another class of problem, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this course.

The Mathematics Specialist ATAR course is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this course, there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1, vectors for two-dimensional space are introduced and then in Unit 3, vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes, and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in the Mathematics Methods ATAR course, is applied in vectors in Unit 3 and applications of calculus and statistics in Unit 4.

Aims

The Mathematics Specialist ATAR course aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 12 syllabus is divided into two units which are delivered as a pair. The notional time for the pair of units is 110 class contact hours.

In this course there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Year 11 Unit 1 and then in Year 12 Unit 3, they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn, prepares students for solving simultaneous equations in three variables.

Organisation of content

Unit 3

This unit contains the three topics:

3.1 Complex numbers

3.2 Functions and sketching graphs

3.3 Vectors in three dimensions

The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3, the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in the Mathematics Methods ATAR course is extended and utilised in the sketching of graphs and the solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students' knowledge of calculus from the Mathematics Methods ATAR course. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

Unit 4

This unit contains the three topics:

4.1 Integration and applications of integration

4.2 Rates of change and differential equations

4.3 Statistical inference

In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students' previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students will have access to an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Representation of the general capabilities

The general capabilities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities into the teaching and learning program for the Mathematics Specialist ATAR course. The general capabilities are not assessed unless they are identified within the specified unit content.

Literacy

Literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy

Students who undertake this course will continue to develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the ever-increasing demands of the information age, developing the skills of critical evaluation of numerical information. Students will enhance their numerical operation skills by working with complex numbers, vectors, the calculus of functions, and by application in statistical inference problems.

Information and communication technology capability

Students use information and communication technology (ICT) both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved, such as for statistical analysis, generation of algorithms, manipulation and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability

Students develop personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making. The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for their decision making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding

Students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results, and the social responsibility associated with teamwork and attribution of input. The areas relevant to Mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial ability and understandings are shaped by a person's environment and language.

Representation of the cross-curriculum priorities

The cross-curriculum priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate the priorities into the teaching and learning program for the Mathematics Specialist ATAR course. The cross-curriculum priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities may allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of Mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem-solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 3

Unit description

Unit 3 of the Mathematics Specialist ATAR course contains three topics: Complex numbers, Functions and sketching graphs and Vectors in three dimensions. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students' knowledge of calculus from the Mathematics Methods ATAR course. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space. The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in the Mathematics Methods ATAR course, is extended and applied in sketching graphs and solving problems involving integration.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching
- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 3.1: Complex numbers (18 hours)

Cartesian forms

3.1.1 review real and imaginary parts $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ of a complex number z

3.1.2 review Cartesian form

3.1.3 review complex arithmetic using Cartesian forms

Complex arithmetic using polar form

3.1.4 use the modulus $|z|$ of a complex number *z* and the argument Arg (z) of a non-zero complex number z and prove basic identities involving modulus and argument

3.1.5 convert between Cartesian and polar form

3.1.6 define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these

3.1.7 prove and use De Moivre's theorem for integral powers

The complex plane (The Argand plane)

3.1.8 examine and use addition of complex numbers as vector addition in the complex plane

3.1.9 examine and use multiplication as a linear transformation in the complex plane

3.1.10 identify subsets of the complex plane determined by relations such as

$|z-3i| \leq 4$, $\frac{\pi}{4} \leq Arg(z) \leq \frac{3\pi}{4}$ and $|z-1| = 2|z-i|$

Roots of complex numbers

3.1.11 determine and examine the n^{th} roots of unity and their location on the unit circle

3.1.12 determine and examine the n^{th} roots of complex numbers and their location in the complex plane

Factorisation of polynomials

3.1.13 prove and apply the factor theorem and the remainder theorem for polynomials

3.1.14 consider conjugate roots for polynomials with real coefficients

3.1.15 solve simple polynomial equations

Topic 3.2: Functions and sketching graphs (16 hours)

Functions

3.2.1 determine when the composition of two functions is defined

3.2.2 determine the composition of two functions

3.2.3 determine if a function is one-to-one

3.2.4 find the inverse function of a one-to-one function

3.2.5 examine the reflection property of the graphs of a function and its inverse

Sketching graphs

3.2.6 use and apply $|x|$ for the absolute value of the real number x and the graph of $y = |x|$

3.2.7 examine the relationship between the graph of

$y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = |f(x)|$ and $y = f(|x|)$

3.2.8 sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree

Topic 3.3: Vectors in three dimensions (21 hours)

The algebra of vectors in three dimensions

3.3.1 review the concepts of vectors from Unit 1 and extend to three dimensions, including introducing the unit vectors **i, j** and **k**

3.3.2 prove geometric results in the plane and construct simple proofs in 3 dimensions

Vector and Cartesian equations

3.3.3 introduce Cartesian coordinates for three dimensional space, including plotting points and equations of spheres

3.3.4 use vector equations of curves in two or three dimensions involving a parameter and determine a 'corresponding' Cartesian equation in the two-dimensional case

3.3.5 determine a vector equation of a straight line and straight line segment, given the position of two points or equivalent information, in both two and three dimensions

3.3.6 examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet

3.3.7 use the cross product to determine a vector normal to a given plane

3.3.8 determine vector and Cartesian equations of a plane

Systems of linear equations

3.3.9 recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations

3.3.10 examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables

Vector calculus

3.3.11 consider position vectors as a function of time

3.3.12 derive the Cartesian equation of a path given as a vector equation in two dimensions, including ellipses and hyperbolas

3.3.13 differentiate and integrate a vector function with respect to time

3.3.14 determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration

3.3.15 apply vector calculus to motion in a plane, including projectile and circular motion

Unit 4

Unit description

Unit 4 of the Mathematics Specialist ATAR course contains three topics: Integration and applications of integration, Rates of change and differential equations and Statistical inference.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout the Mathematics Specialist ATAR course.

In this unit, all of the students' previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference
- apply reasoning skills and solve problems in applications of calculus and statistical inference
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

Topic 4.1: Integration and applications of integration (20 hours)

Integration techniques

4.1.1 integrate using the trigonometric identities

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$, $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ and $1+\tan^2 x = \sec^2 x$

4.1.2 use substitution u = g(x) to integrate expressions of the form $f\left(g(x)\right)g'(x)$

4.1.3 establish and use the formula $\int \frac{1}{x}dx = \ln|x| + c$ for $x \neq 0$

4.1.4 use partial fractions where necessary for integration in simple cases

Applications of integral calculus

4.1.5 calculate areas between curves determined by functions

4.1.6 determine volumes of solids of revolution about either axis

4.1.7 use technology with numerical integration

Topic 4.2: Rates of change and differential equations (20 hours)

Applications of differentiation

4.2.1 use implicit differentiation to determine the gradient of curves whose equations are given in implicit form

4.2.2 examine related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

4.2.3 apply the incremental formula $\partial y \approx \frac{dy}{dx}\partial x$ to differential equations

4.2.4 solve simple first order differential equations of the form $\frac{dy}{dx} = f(x)$; differential equations of the form $\frac{dy}{dx} = g(y)$; and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$, using separation of variables

4.2.5 examine slope (direction or gradient) fields of a first order differential equation

4.2.6 formulate differential equations, including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved

Modelling motion

4.2.7 consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions , $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ for acceleration

Topic 4.3: Statistical inference (15 hours)

Sample means

4.3.1 examine the concept of the sample mean $\overline{X}$ as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ

4.3.2 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size *n* , including its mean μ its standard deviation $\frac{\sigma}{\sqrt{n}}$ (where μ and σ are the mean and standard deviation of *X*), and its approximate normality if *n* is large

4.3.3 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where *s* is the sample standard deviation

Confidence intervals for means

4.3.4 examine the concept of an interval estimate for a parameter associated with a random variable

4.3.5 examine the approximate confidence interval $\left(\overline{X} - \frac{zs}{\sqrt{n}}, \overline{X} + \frac{zs}{\sqrt{n}}\right)$ as an interval estimate for the population mean μ, where z is the appropriate quantile for the standard normal distribution

4.3.6 use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ

4.3.7 use $\bar{x}$ and s to estimate μ and σ to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for μ

School-based assessment

The Western Australian Certificate of Education (WACE) Manual contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Specialist ATAR Year 12 syllabus and the weighting for each assessment type.

Assessment table – Year 12

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem solving skills and algorithms. Tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to open-ended questions that require responses at the highest level of conceptual thinking. Students may be asked questions of an investigative nature for which they may need to communicate findings, generalise, or make and test conjectures. Typically conducted at the end of each semester and/or unit and reflecting the examination design brief for this syllabus.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units.

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units:

- each assessment type must be included at least twice
- the response type must include a minimum of two tests.

The set of assessment tasks must provide a representative sampling of the content for Unit 3 and Unit 4.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units. The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Specialist ATAR Year 12 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the WACE Manual for further information about the use of a ranked list in the process of assigning grades.

WACE examination

All students enrolled in the ATAR Mathematics Specialist Year 12 course are required to sit the WACE examination. The examination is based on a representative sampling of the content for Unit 3 and Unit 4. Details of the WACE examination are prescribed in the examination design brief on the following page.

Refer to the WACE Manual for further information.

Examination design brief – Year 12

This examination consists of two sections.

Section One: calculator-free

Time allowed

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Permissible items

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: nil

Additional information

Changeover period during which the candidate is not permitted to work: up to 15 minutes

Section Two: calculator-assumed

Time allowed

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Permissible items

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Provided by the supervisor

A formula sheet

Additional information

It is assumed that candidates sitting this examination have a calculator with CAS capabilities for Section Two.
The examination assesses the syllabus content areas using the following percentage ranges. These apply to the whole examination rather than individual sections.

Content area	Percentage of exam
Complex numbers	15–20%
Functions and graphs	10–15%
Vectors in three dimensions	15–20%
Integration and application of integration	15–20%
Rates of change and differential equations	15–20%
Statistical inference	10–15%

The candidate is required to demonstrate knowledge of mathematical facts, conceptual understandings, use of algorithms, use and knowledge of notation and terminology, and problem-solving skills.

Questions can require the candidate to investigate mathematical patterns, make and test conjectures, and generalise and prove mathematical relationships. Questions can require the candidate to apply concepts and relationships to unfamiliar problem-solving situations, choose and use mathematical models with adaptations, compare solutions, and present conclusions. A variety of question types that require both open and closed responses can be included.

Instructions to candidates indicate that, for any question or part question worth more than two marks, valid working or justification is required to receive full marks.

SECTION	SUPPORTING INFORMATION
Section One: calculator-free 35% of the total examination 5–10 questions Suggested working time: 50 minutes	Questions examine content and procedures that can reasonably be expected to be completed without the use of a calculator i.e. without undue emphasis on algebraic manipulations or time consuming calculations. The candidate is required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media.
Section Two: calculator-assumed 65% of the total examination 8–13 questions Suggested working time: 100 minutes	Questions examine content and procedures for which the use of a calculator is assumed. The candidate can be required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions. Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media. The candidate can be required to investigate theoretical situations involving mathematical concepts and relationships, for which they need to generalise, construct proofs and make conjectures. The candidate can be required to solve problems from unfamiliar situations, choosing and using mathematical models with adaptations where necessary, comparing their solutions with the situations concerned, and then presenting their findings in context.

Appendix 1 – Grade descriptions Year 12

<table>
<tr><td rowspan="5">A</td><td>Identifies and organises relevant information
Identifies and organises relevant information from complex and scattered sources, such as the key parameters of a simple harmonic motion function, and identifies the correct solution of a trigonometric equation from the multiple possible solutions. Describes linear motion in three-dimensional spaces. Uses answers from previous parts of a problem to carry through and solve subsequent problems.</td></tr>
<tr><td>Chooses effective models and methods and carries through the methods correctly
Solves unstructured problems by choosing the most appropriate algebraic, vector or calculus techniques. Chooses efficient methods when dealing with derivatives, integrals and statistical inference. Clearly sets out the deductive reasoning and accurately carries it through. Simplifies complicated fractions and works efficiently with algebraic expressions in fraction form. Uses well-constructed diagrams and makes appropriate geometric connections when carrying out vector proofs.</td></tr>
<tr><td>Obeys mathematical conventions and attends to accuracy
Uses the correct notation at all times with vectors, matrices, functions and calculus. Sets out reasoning in clearly defined steps that are easily followed. Draws diagrams and graphs, including the polar form, with appropriate scales and labels. Works easily with exact values such as surds, radian values or natural exponentials e^x and recognises the difference between open and closed intervals.</td></tr>
<tr><td>Links mathematical results to data and contexts to reach reasonable conclusions
Pays attention to the units, gives exact value answers, uses the correct degree of accuracy and uses radian measure when appropriate. Always takes account of the domain as defined in the problem, or by the context of the question, and excludes any results outside it. Links polar solutions of the complex equation z^n=c and the principal domain $-\pi < \theta \leq \pi$.</td></tr>
<tr><td>Communicates mathematical reasoning, results and conclusions
Sets out each step of deductive reasoning in a clear and logical sequence. Defines variables associated with diagrams and uses them consistently in the working of a problem. Carries a deductive proof through by working with only one side of the equation. Communicates the main steps of integration problems using the substitution method, including trigonometric identities. Relates the result of a problem to the context of the question by using the correct units and any related notation such as vector notation.</td></tr>
</table>

<table>
<tr><td rowspan="5">B</td><td>Identifies and organises relevant information
Identifies and organises relevant information from dense sources, for example, descriptive passages, labelled diagrams, or recognises an advanced integration method in the form $\int (f'(x) / f(x))\, dx$. Recognises that for various equations, multiple angle solutions are possible in a given domain, for example, $\sin 2\theta = \frac{1}{2}$ for $0 \leq \theta \leq 2\pi$. Identifies key information from scattered sources, for example, using answers from previous parts of a problem and bringing them together to solve subsequent problems.</td></tr>
<tr><td>Chooses effective models and methods and carries the methods through correctly
Chooses and uses the correct technique or model in unpractised situations with vectors, functions, simple harmonic motion and the calculus of trigonometric functions. Uses well-constructed diagrams and makes appropriate geometric connections when carrying out vector proofs. Translates between representations in unpractised ways, connecting diagrams to vector algebra using correct notation.</td></tr>
<tr><td>Obeys mathematical conventions and attends to accuracy
Uses the correct notation with vectors, matrices, functions and calculus on most occasions. Obeys conventions, such as using appropriate limits of integration when integrating by substitution, and setting out the reasoning in clearly defined steps. Draws clear diagrams and graphs with appropriate scales and labels. Uses exact values, such as surds and radian values using π, and recognises the difference between open and closed intervals.</td></tr>
<tr><td>Links mathematical results to data and contexts to reach reasonable conclusions
Pays attention to the units, gives answers to the correct degree of accuracy and uses radian measure on most occasions. Takes account of the domain as defined in the problem and excludes results outside it.</td></tr>
<tr><td>Communicates mathematical reasoning, results and conclusions
Carries through calculations and simplifications in a clear and logical sequence. Defines variables associated with diagrams and uses them consistently in the working of a problem. Communicates the main steps of integration problems using the substitution method. Relates the result of a problem to the context of the question by using the correct units and any related notation.</td></tr>
</table>

C	**Identifies and organises relevant information from information that is relatively narrow in scope** Identifies and organises information from relatively narrow sources, for example, identifies the amplitude and period of a trigonometric function, plots simple graphs in the complex number plane and interprets given vector diagrams. Identifies appropriate rules in calculus, such as the product, quotient and chain rules for derivatives of exponential, logarithmic and trigonometric functions.
	Chooses effective models and methods and carries through the methods correctly Applies mathematical algorithms in practised ways, for example, the product, quotient and chain rules for derivatives of exponential, logarithmic and trigonometric functions. Integrates functions with less complex algebraic expressions. Uses a familiar trigonometric identity to simplify an expression, or uses implicit differentiation that is narrow in scope. Uses given vector diagrams effectively to answer simple questions.
	Obeys mathematical conventions and attends to accuracy Obeys mathematical conventions when simplifying an algebraic expression or working with an equation. Defines the correct interval for inequalities and translates simple inequalities to graphical intervals. Obeys mathematical conventions for sketching graphs and clearly labels the main features, for example, asymptotes. Applies the conventions for vector diagrams and complex number graphs. Translates between complex number representations in practised ways.
	Links mathematical results to data and contexts to reach reasonable conclusions Gives exact values and attends to units in short responses on most occasions. Includes implied units with the solution, such as metres per second or radians per second.
	Communicates mathematical reasoning, results and conclusions Shows adequate working and justifies answers with simple or routine statements. Relates the working to any vector diagram that has been given as part of the question. Makes clear sketches of simple functions, including required features such as intercepts of graphs. Draws simple diagrams to help solve problems with vectors and applications of calculus.

D	**Identifies and organises relevant information** Identifies and organises relevant information that is narrow in scope. Identifies the amplitude of a trigonometric function. Uses the derivative to get the gradient at a point.
	Chooses effective models and methods and carries the methods through correctly Answers familiar structured questions that require short responses, for example, calculating the constant c in $y = mx + c$ given the gradient m and a point . Completes implicit differentiation with straightforward expressions. Integrates functions with simple algebraic expressions. Makes common sense connections. Recognises simple complex number inequalities and draws the regions in the complex plane. Uses a calculator appropriately for straightforward calculations, algebra and graphing. Multiplies matrices accurately.
	Obeys mathematical conventions and attends to accuracy Applies conventions for function notation. Obeys mathematical conventions by including a constant of integration. Defines linear equations and applies conventions for diagrams and graphs. Draws the complex numbers in correct position in the plane. Labels axes, shows scale and labels graphs and intersection points.
	Links mathematical results to data and contexts to reach reasonable conclusions On occasions, recognises specified conditions and attends to units in short responses. Sometimes includes units in the solution.
	Communicates mathematical reasoning, results and conclusions Shows some working. Sets calculations out in a manner that can be checked for accuracy.

E	Does not meet the requirements of a D grade.

Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

UNIT 3	
Complex numbers	
Argument (abbreviated arg)	If a complex number z is represented by a point P in the complex plane then the argument of z, denoted $\arg z$, is the angle θ that OP makes with the positive real axis O_x, with the angle measured anticlockwise from O_x. The principal value of the argument is the one in the interval $(-\pi, \pi]$.
Complex arithmetic	If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ $\Rightarrow z_1 + z_2 = (x_1 + x_2) + (y_1 + y_2)i$ $\Rightarrow z_1 - z_2 = (x_1 - x_2) + (y_1 - y_2)i$ $\Rightarrow z_1 \times z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i$ $z_1 \times (0 + 0i) = 0$ $\quad$ $Note:$ $0 + 0i$ is usually written as 0 $z_1 \times (1 + 0i) = z_1$ $\quad$ $Note:$ $1 + 0i$ is usually written as 1
Complex conjugate	For any complex number $z = x + iy$, its conjugate is $\bar{z} = x - iy$. the following properties hold $\overline{(z_1 z_2)} = \bar{z}_1 \times \bar{z}_2$ $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ $z \times \bar{z} = \lvert z^2 \rvert$ $z + \bar{z}$ is real.
De Moivre's theorem	For all integers n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.
Modulus (absolute value) of a complex number	If z is a complex number and $z = x + iy$, then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z denoted by $\lvert z \rvert = \sqrt{x^2 + y^2}$.
Polar form of a complex number	For a complex number z, let $\theta = \arg z$, then $z = r(\cos\theta + i\sin\theta)$ is the polar form of z.
Root of unity (nth root of unity)	For a complex number z such that $z^n = 1$ the n^{th} roots of unity are: $z_k = \cos\left(\frac{2k\pi}{n}\right) + i\sin\left(\frac{2k\pi}{n}\right)$ where $k = 0, 1, 2, \ldots, n-1$. The points in the complex plane representing roots of unity lie on the unit circle. The cube roots of unity are: $z_0 = \cos(0) + i\sin(0) = 1$, $z_1 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = (-1 + i)$ $z_2 = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = (-1 - i)$.

Functions and sketching graphs	
Rational function	A rational function is a function such that $f(x) = \frac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials. Usually $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of *f* is usually taken to be $R\backslash = \{x : h(x) \neq 0\}$

Vectors in three dimensions	
Addition of vectors	In component form if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j} + (a_3 + b_3)\mathbf{k}$.
Vector equation of a straight line	Let **a** be the position vector of a point on a line l, and **u** any vector with direction parallel to the line. The line consists of all points P whose position vector **p** is given by; $\mathbf{p} = \mathbf{a} + t\mathbf{u}$ for some real number t. Also given two points A and B on the plane with the position vectors **a** and **b** respectively the equation of the line passing through these lines can be written as; $\mathbf{p} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$ for some real number t.
Vector equation of a plane	Let **a** be a position vector of a point *A* in the plane, and **n** a normal vector to the plane. Then the plane consists of all points *P* whose position vector **p** satisfies $(\mathbf{p} - \mathbf{a})\cdot\mathbf{n} = 0$. This equation may also be written as $\mathbf{p}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$, a constant. (If the normal vector **n** is the vector $(\mathrm{l}, \mathrm{m}, \mathrm{n})$ in ordered triple notation and the scalar product $\mathbf{a}\cdot\mathbf{n} = k$, this gives the Cartesian equation $\mathrm{l}x + \mathrm{m}y + \mathrm{n}z = k$ for the plane.)
Vector function	In this course, a vector function is one that depends on a single real number parameter *t*, often representing time, producing a vector $\mathbf{r}(t)$ as the result. In terms of the standard unit vectors **i**, **j**, **k** of three- dimensional space, the vector-valued functions of this specific type are given by expressions such as $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ where *f*, *g* and *h* are real valued functions giving coordinates.
Scalar product	If $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ then the scalar product $\mathbf{a}\cdot\mathbf{b}$ is the real number where; $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. When expressed in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$
Vector product (cross product)	When expressed in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then; $\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$ The vector cross product has the following geometric interpretation. Let **a** and **b** be two non-parallel vectors, then $\lvert\mathbf{a} \times \mathbf{b}\rvert$ is the area of the parallelogram defined by **a** and **b** and where $\mathbf{a} \times \mathbf{b}$ is a vector normal to this parallelogram. (The cross product of two parallel vectors is the zero vector.)

UNIT 4

Integration and applications of integrations

Implicit differentiation	When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. Implicit differentiation consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example: $x^2 + xy^3 - 2x + 3y = 0$, then $\frac{d}{dx}\left(x^2 + xy^3 - 2x + 3y = 0\right)$ $\Rightarrow 2x + y^3 + x.3y^2\frac{dy}{dx} - 2 + 3\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{2 - 2x - y^3}{3xy^2 + 3}$.
Logistic equation	The logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics. One form of this differential equation is: $\frac{dy}{dt} = ay - by^2$ (where $a > 0$ and $b > 0$). The general solution of this is: $y = \frac{a}{b + ce^{-at}}$ where c is an arbitrary constant.
Separation of variables	Differential equations of the form $\frac{dy}{dx} = g(x)h(y)$ can be rearranged as long as $h(y) \neq 0$ to obtain $\frac{1}{h(y)}\frac{dy}{dx} = g(x)$

Rates of change and differenital equations

Slope field	Slope field (direction or gradient field) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment with the corresponding slope.

Statistical inference

Continuous random variable	A random variable X is called continuous if its set of possible values consists of intervals, and the chance that it takes any point value is zero. (In symbols, if $P(X = x) = 0$ for every real number x). A random variable is continuous if, and only if, its cumulative probability distribution function can be expressed as an integral of a function.

Probability density function	The probability of a continuous random variable X being located in a given interval $[a, b]$ is denoted by $Pr(a \le X \le b) = \int_b^a f(x)dx$ where $f(x)$ is defined as the probability density function (pdf). The probability density function is therefore the derivative of the cumulative probability function.
Precision	Precision is a measure of how close an estimator is expected to be to the true value of the parameter it purports to estimate.
Independent and identically distributed observations	For independent observations, the value of any one observation has no effect on the chance of values for all the other observations. For identically distributed observations, the chances of the possible values of each observation are governed by the same probability distribution.
Random sample	A random sample is a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation. The most common situation in which the term "random sample" is used refers to a set of independent and identically distributed observations. The sample mean is the arithmetic average of the sample values.