Current Research Interests in Production Economics

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The 3rd Workshop on Operations Management and Technology

Cartagena, Spain, May 14, 2012

Main Agenda

Main Agenda

- History and Philosophy of Production Economics
- The ***International Journal of Production Economics***
- Current Research Interests in Production Economics

Subject Area of Production Economics

- An engineering discipline
- Courses generally belong to engineering faculties
- Focus on topics treating interface between technology/engineering and economics/management
- Subject interdisciplinary in nature
- All aspects related to manufacturing and processing industries, and to production in general
- Treats allocation problems strongly linked with the optimal distribution of production resources (especially within manufacturing industries)
- Methodological approach based on the ***Theory of Production***
- Gradual integration with mathematical and statistical models and methods from *Operational Research* and *Management Science*
- Quantitative economic approach has a natural firm footing
- Strong links with industrial activities
- Globalisation of manufacturing È increasing interest in strategic issues of production
- **Production** = process of transforming one set of resources (inputs) into a second set (outputs) **having greater value to society**

Subject Area of Production Economics II:

A general theory of production must necessarily be abstract, but the cultures of various branches of technology have been so concrete, detailed and far apart that probably no need has ever been experienced for such a broad approach enabling a wide variety of interpretations: Agriculture, mechanical manufacturing, chemical engineering, etc. The economic science on the other hand - although dealing with many other issues apart from production - has had an essential need to include analyses of the opportunities to utilise resources and for that purpose to develop a framework more independent of particular areas of application.

Subject Area of Production Economics III:

Whereas the technological aspects of production concern the **opportunities** of transformation, the economic aspects concern the process of **choosing a best alternative**. From an optimisation point of view, these sets of aspects are dual, one set being the constraint when achieving the other. Both technology and economics are also normative (prescriptive) disciplines, aiming at finding, in some sense, best solutions to recommend for implementation.

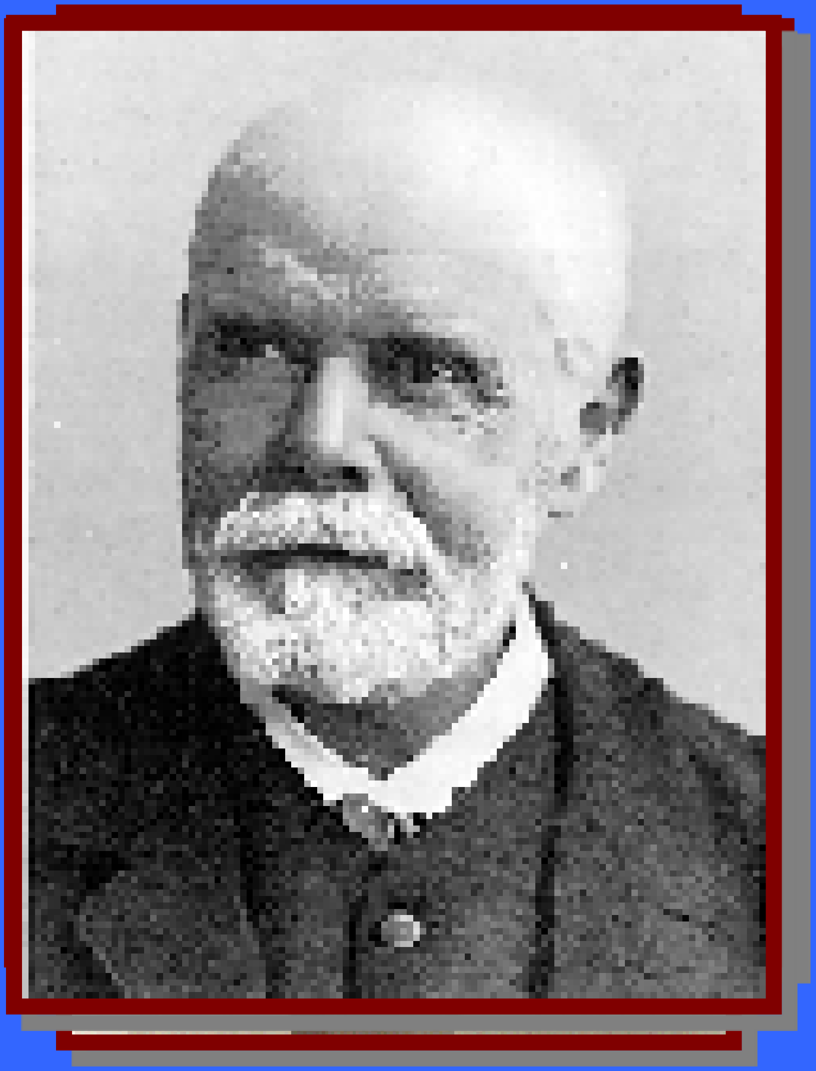
Integration of Economics and Technology

Robert W. Grubbström

Production Economics attempts to integrate technology/engineering and economics/management. The way in which this is done is, on the one hand

- to adopt methods, principles and procedures from one field and apply them in the other. In particular, the application of mathematical and statistical methodology, widely used in engineering, is applied to economic and managerial problems,

- on the other hand, technical questions and technical alternatives are analysed, applying economic principles.



Léon Walras, 1834-1910

”In any case, the establishment sooner or later of economics as an exact science is no longer in our hands and need not concern us. It is already perfectly clear that economics, like astronomy and mechanics, is both an empirical and a rational science. And no one can reproach our science with having taken an unduly long time in becoming rational as well as empirical. [illegible] fifty or two hundre[illegible] become the astronomy [illegible] chanics of Galileo to become the mechanics of d’Alembert and Lagrange. On the other hand, less than a century has elapsed between the publication of Adam Smith’s work and the contributions of Cournot, Gossen, Jevons and myself .”

Léon Walras, in Preface to *Elements of Pure Economics*, dated June 1900:

"We were, therefore, at our post, and have performed our duty. If nineteenth century France, which was the cradle of the new science, has completely ignored it, the fault lies in the idea, so bourgeois in its narowness, of dividing education into two separate compartments: one turning out calculators with no knowledge whatsoever of sociology, philosophy, history, or economics; and the other cultivating men of letters devoid of any notion of mathematics. The twentieth century, which is not far off, will feel the need, even in France, of entrusting the social sciences to men of general culture who are accustomed to thinking both inductively and deductively and who are familiar with reason as well as experience."

Now we're in the 21st Century!!

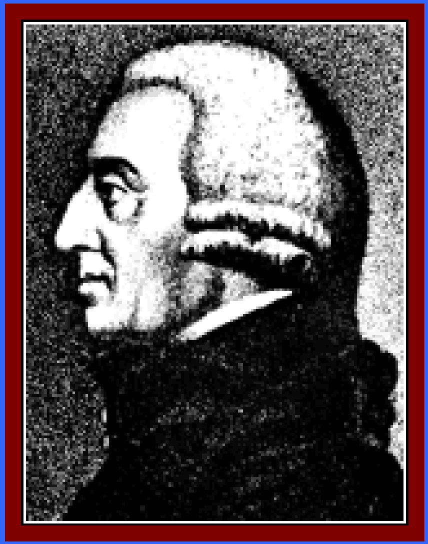
Motives for integration:

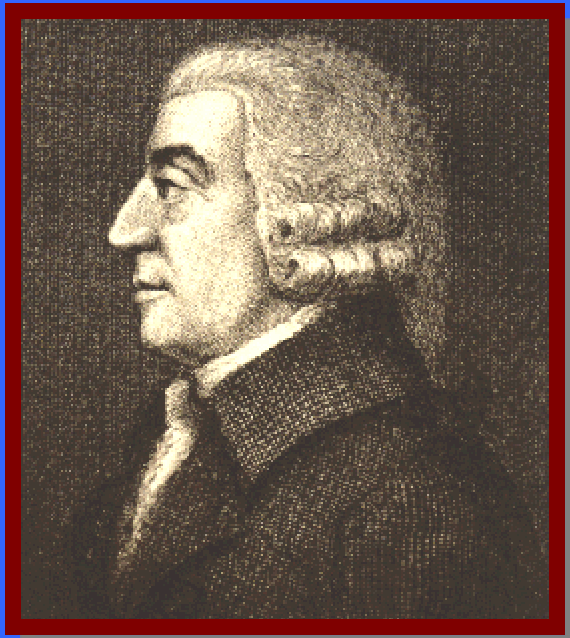
***At least three motives for striving to integrate the two disciplines*:**

- Often students of technology eventually will find themselves in leadership positions in which economic and managerial skills are essential
- Using techniques from a more advanced science in a less advanced discipline develops the latter
- Attempts to find analogies, by which results derived or developed in one discipline can be applied in the other
- Whenever this is made possible, scientific shortcuts are achieved creating efficiency

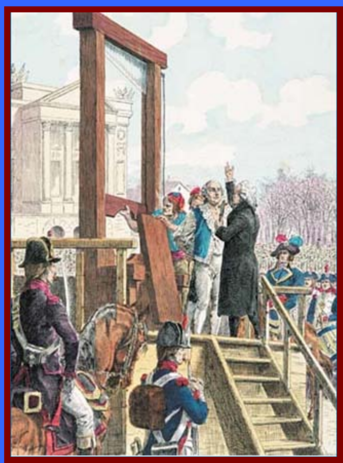
Specialisation and Efficiency

Robert W. Grubbström

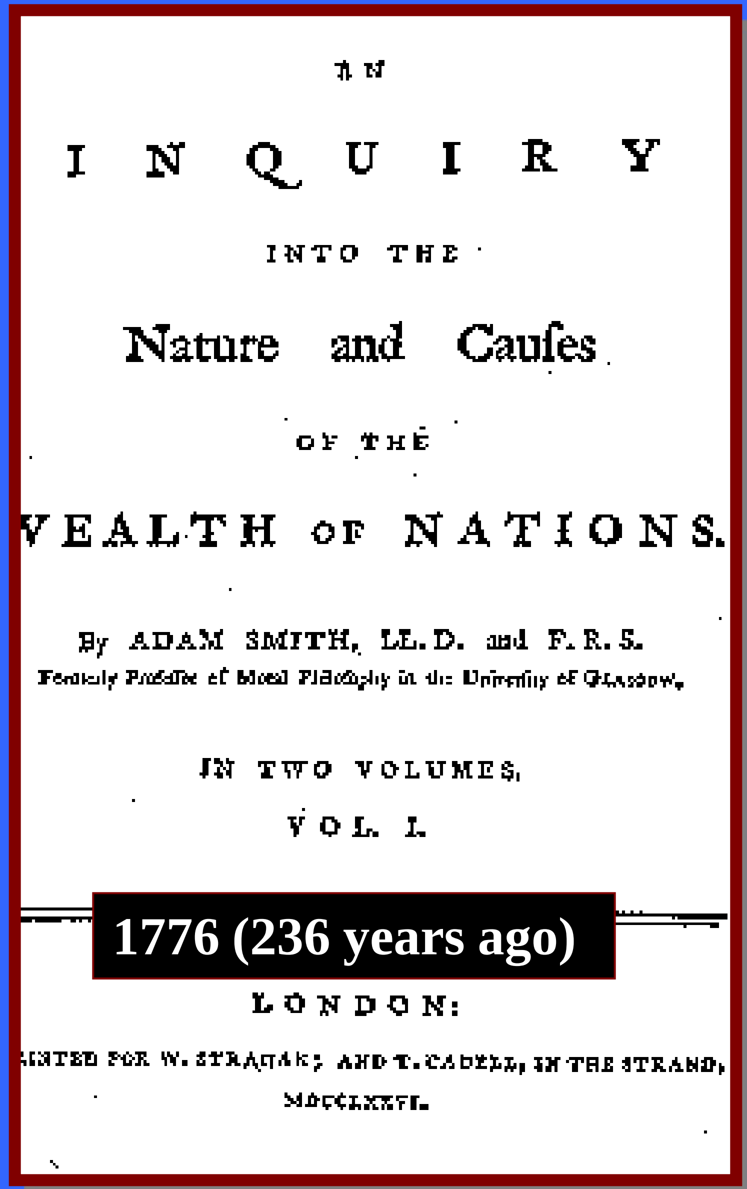




Adam Smith, 1723-1790



1787-1799



AN

INQUIRY

INTO THE

Nature and Caufes

OF THE

VEALTH OF NATIONS.

By ADAM SMITH, LL.D. and F.R.S.

Formerly Profeffor of Moral Philofophy in the Univerfity of Glasgow.

IN TWO VOLUMES.

VOL. I.

1776 (236 years ago)

LONDON:

INTED FOR W. STRAHAN; AND T. CADELL, IN THE STRAND.

MDCCLXXVI.

The First Production Economist

Robert W. Grubbström

Vilfredo Pareto, 1848-1923

The Italian Marquis and engineer **Vilfredo Pareto,** 1848-1923, was born i Paris. He wrote a thesis in solid mechanics with the title *Principes fondamentaux de l'équilibre de corps solides*, which he defended at the Technical University of Turin in 1869. He was appointed ”ordinary” Professor of *economie politique* at the University of Lausanne in 1894. He created *”Pareto's law”*, which later has become known as the ”80/20-rule”. He contributed to the foundation of *welfare economics*, by defining optimality in an economic system as a state from which no one can become better off without somebody else becoming worse off (*Pareto Optimality*).

Human Capital

Robert W. Grubbström

Frederick Winslow Taylor, 1856-1915

The Principles of Scientific Management

BY

FREDERICK WINSLOW TAYLOR, M.E., Sc.D.

PAST PRESIDENT OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

1911 (101 years ago)

HARPER & BROTHERS PUBLISHERS

NEW YORK AND LONDON

1911

John Maynard Keynes, 1883-1946

THE
GENERAL THEORY
OF
EMPLOYMENT INTEREST
AND MONEY

Despite treating a general theory of employment, not a single word about production!!!

MACMILLAN
LONDON · MELBOURNE · TORONTO
1967

(Originally published 1935-36)

Marcus Wallenberg, 1899-1982

”The institutes of technology and the natural science research institutes of the universities must continue to expand and the education must increasingly concern the teaching of methods, by which, on the one hand, one may follow the technical development, on the other, also contribute to the creation of this development. It will be increasingly important to understand

Address delivered at *Finnish Federation of Industries’ Autumn Meeting*
November 22, 1962 (half a century ago)

technical and natural sciences must more and more focus on basic sciences and possibly less time be devoted to knowledge about machines existing today, but not existing tomorrow. Over all, in the vast majority of scientific disciplines one has to give priority to the q **quantitative science** e. that one must increasingly teach **analytical methodology** and to a lesser extent feed institutional facts which very soon are obsolete.”

Technology and Finance

Robert W. Grubbström

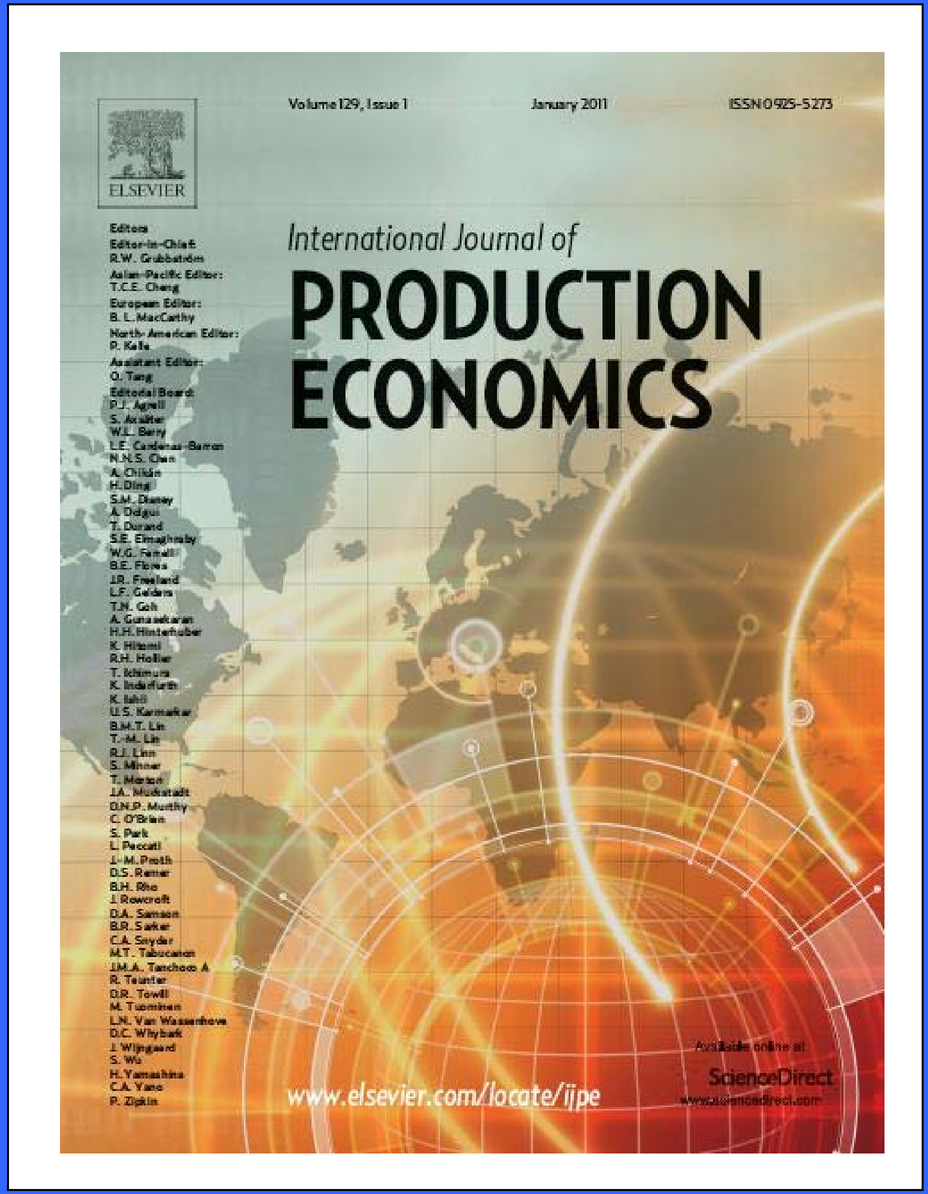
- **Ultimate economic consequences of any industrial activity are financial consequences**
- ***Trace Financial Consequences!***

Other Items related to Production Economics

- Service Production
- Risk
- Environmental Issues
- Energy-Economic Issues
- Aesthetical and Design Issues
- And more …

The International Journal of Production Economics

Robert W. Grubbström



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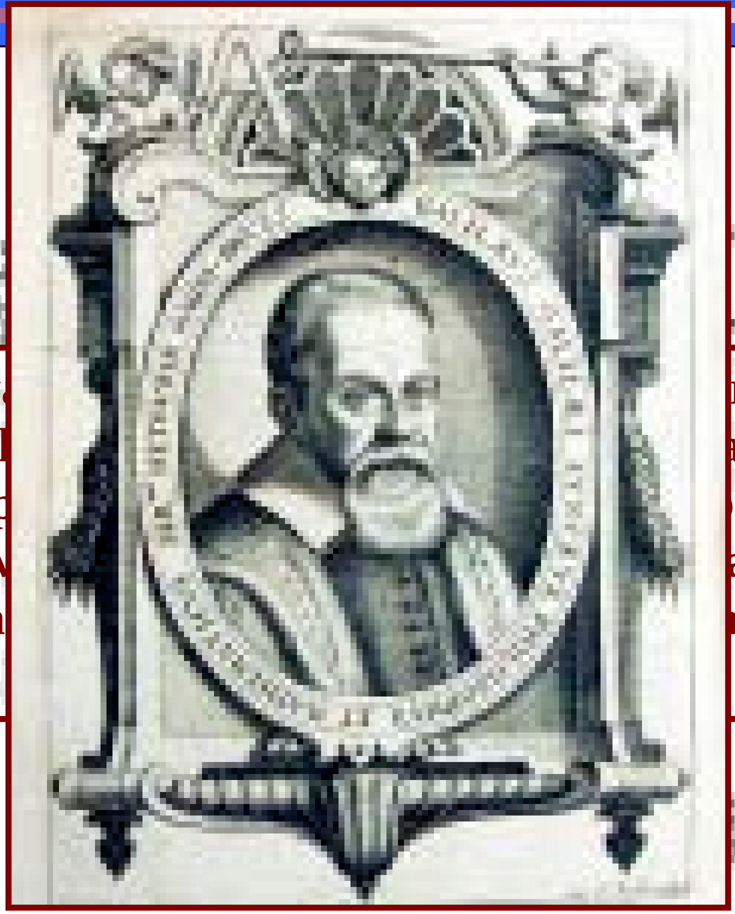
Elsevier, wa … rom a small
Dutch publ … assical
scholarship … ublishing
company w … akes its
name from … mily
publishing



ELSEVIER

The *International Journal of Production Economics* focuses on topics treating the interface between engineering and management. All aspects of the

The ultimate objective of the journal is to disseminate knowledge for improving industrial practice and to strengthen the theoretical base necessary for supporting sound decision making. It provides a forum for the exchange of ideas and the presentation of new developments in theory and application, wherever engineering and technology meet the managerial and economic

Articles accepted need to be based on rigorous sound theory and contain an essential novel scientific contribution. Tracing economic and financial consequences in the analysis of the problem and solution reported, belongs to the central theme of the journal. Submissions should strictly follow the Guide for Authors of the journal.

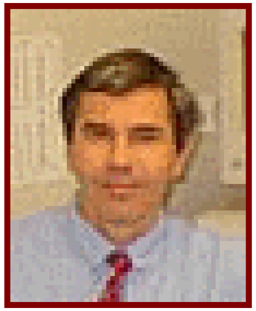
for Authors of the journal.

***International Journal of Production Economics*, Vol. 22-136… , 1991-2012 …**

***Engineering Costs and Production Economics*, Vol. 5-21, 1980-1991**

***Engineering and Process Economics*, Vol. 1-4, 1976-1979**

Editors of the IJPE



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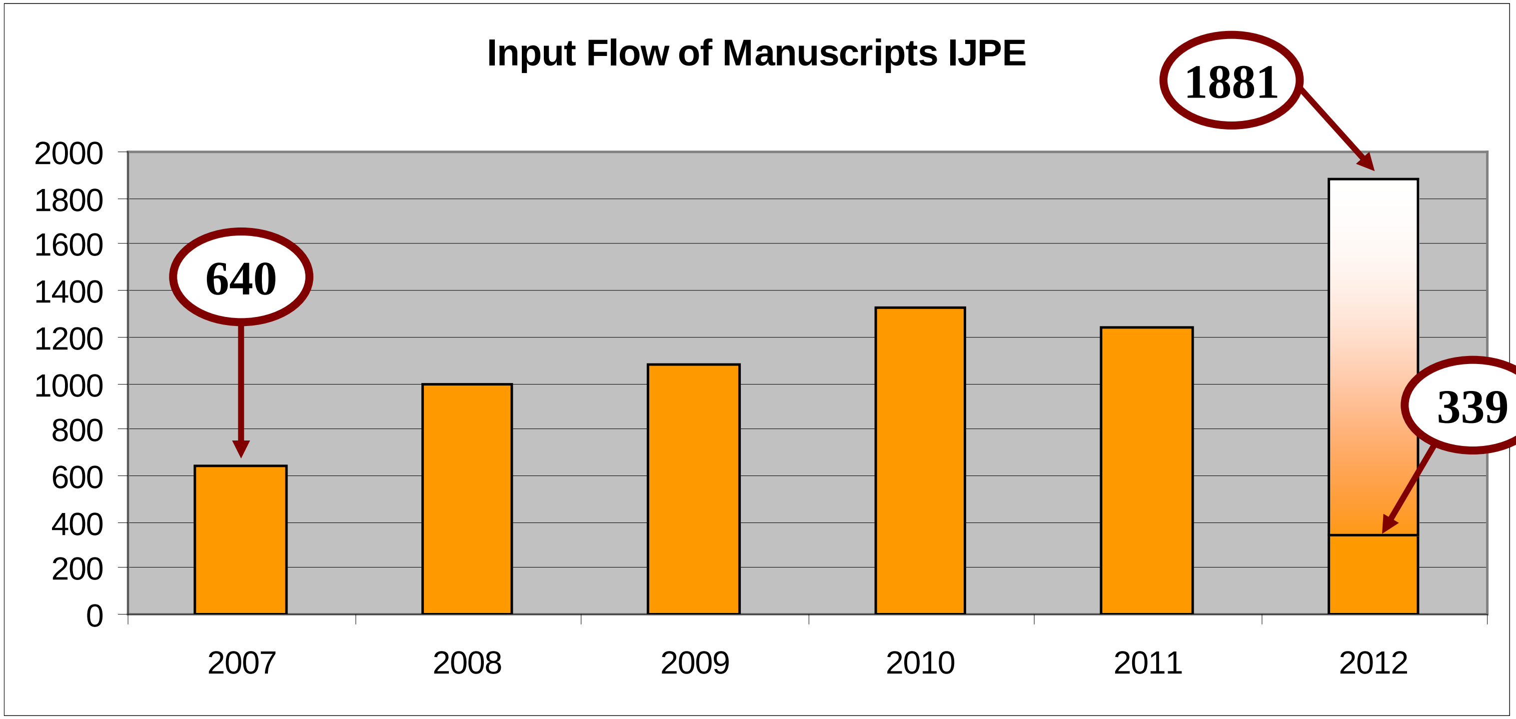

Above these figures are also minor numbers of "manually" handled manuscripts

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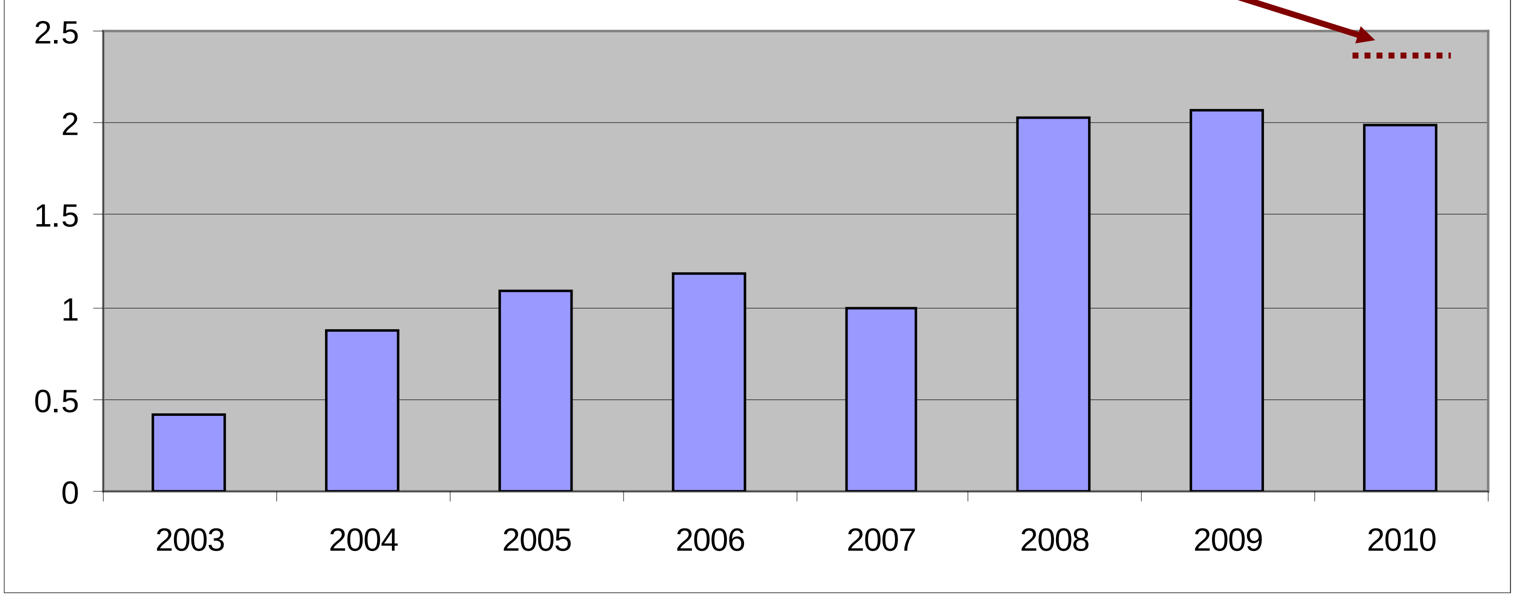
5-year Impact Factor 2010 = 2.411



Most Cited Articles 2007-

Robert W. Grubbström

Most Cited from 2007

Selection of optimum maintenance strategies based on a fuzzy analytic hierarchy process	107(1), 151-163	Wang, L. Chu, J. Wu, J.
Analyzing the benefits of lean manufacturing and value stream mapping via simulation: A process sector case study	107(1), 223-236	Abdulmalek, F.A. Rajgopal, J.
Comprehensive and configurable metrics for supplier selection	105(2), 510-523	Huang, S.H. Keskar, H.
Organizational learning culture-the missing link between business process change and organizational performance	106(2), 346-367	Škerlavaj, M. Štemberger, M.I. Škrinjar, R. Dimovski, V.
A multi-objective supplier selection model under stochastic demand conditions	105(1), 150-159	Liao, Z. Rittscher, J.
Supplier evaluation and management system for strategic sourcing based on a new multicriteria sorting procedure	106(2), 585-606	Araz, C. Ozkarahan, I.
An analytical network process-based framework for successful total quality management (TQM): An assessment of Turkish manufacturing industry readiness	105(1), 79-96	Bayazit, O. Karpak, B.
Coordination of supply chain after demand disruptions when retailers compete	109(1-2), 162-179	Xiao, T. Qi, X. Yu, G.

Most Cited from 2008(1)

RFID research: An academic literature review (1995-2005) and future research directions	112(2), 510-520	Ngai, E.W.T. Moon, K.K.L. Riggins, F.J., Yi, C.Y.
Economical assessment of the impact of RFID technology and EPC system on the fast-moving consumer goods supply chain	112(2), 548-569	Bottani, E., Rizzi, A.
Supply chain coordination: Perspectives, empirical studies and research directions	115(2), 316-335	Arshinder Kanda, A. Deshmukh, S.G.
Evaluating the business value of RFID: Evidence from five case studies	112(2), 601-613	Tzeng, S.-F., Chen, W.-H. Pai, F.-Y.
Environmental management and manufacturing performance: The role of collaboration in the supply chain	111(2), 299-315	Vachon, S., Klassen, R.D.
Selection of the strategic alliance partner in logistics value chain	113(1), 148-158	Büyüközkan, G. Feyzioğlu, O., Nebol, E.
Exploring the impact of RFID technology and the EPC network on mobile B2B eCommerce: A case study in the retail industry	112(2), 614-629	Fosso Wamba, S. Lefebvre, L.A. Bendavid, Y., Lefebvre, E.
A critical review of survey-based research in supply chain integration	111(1), 42-55	van der Vaart, T. van Donk, D.P.
The simulated impact of RFID-enabled supply chain on pull-based inventory replenishment in TFT-LCD industry	112(2), 570-586	Wang, S.-J. Liu, S.-F. Wang, W.-L.

Most Cited from 2008(2)

Assembly line balancing: Which model to use when?	111(2), 509-528	Boysen, N. Fliedner, M. Scholl, A.
Supply chain risk, simulation, and vendor selection	114(2), 646-655	Wu, D. Olson, D.L.
Analysis of the impact of the RFID technology on reducing product misplacement errors at retail stores	112(1), 264-278	Rekik, Y. Sahin, E. Dallery, Y.
Third-party logistics: A literature review	113(1), 127-147	Marasco, A.
The power of flexibility for mitigating supply chain risks	116(1), 12-27	Tang, C. Tomlin, B.
Confirmation of a measurement model for green supply chain management practices implementation	111(2), 261-273	Zhu, Q. Sarkis, J. Lai, K.-H.
Achieving supply chain agility through IT integration and flexibility	116(2), 288-297	Swafford, P.M. Ghosh, S. Murthy, N.
Economic order quantity for items with imperfect quality: Revisited	112(2), 808-815	Maddah, B. Jaber, M.Y.

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with its economic consequences.

Harris, F. W., 1913. "How Many Parts to Make at Once," Factory, The Magazine of Management, 10(2), 135-136, 152. (99 years ago)

To Celebrate the 100th Anniversary of Lotsizing of

Ford Whitham Harris 1913 original paper

the IJPE will devote a Special Issue to this Topic during 2013

Managing Guest Editor is:

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Important Issues:

- Scope
- Style, Guide for Authors
- Publishing Ethics
- Scientific Work:
  - (i) Creating knowledge (Research)
  - (ii) Disseminating knowledge (Teaching, consulting)
  - (iii) Maintaining knowledge (Libraries, archives)
- Scientific Methodology

Scientific Methodology:

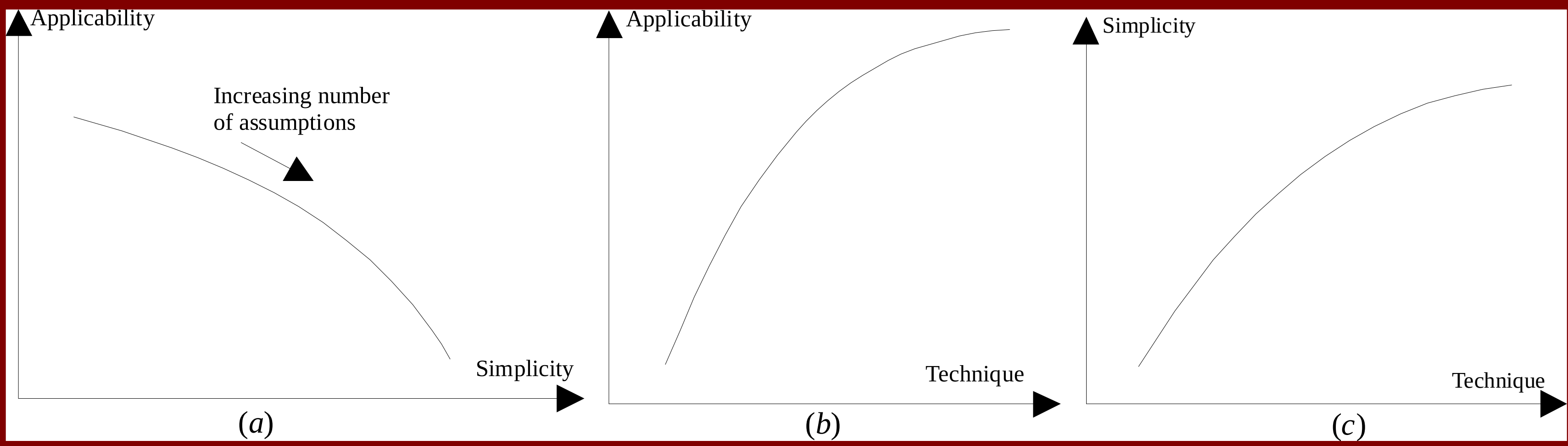
- Quality criteria:
  Rigour, novelty (contribution), completeness
- Types of scientific work:
  - (i) Description (Geography, basic Chemistry)
  - (ii) Explaining (Physics, advanced Chemistry)
  - (iii) Predicting (Metereology)
  - (iv) Retrodicting (Archeology)
  - (v) Prescripting (Engineering, Economics, Medicine, Military Science, *normative*)

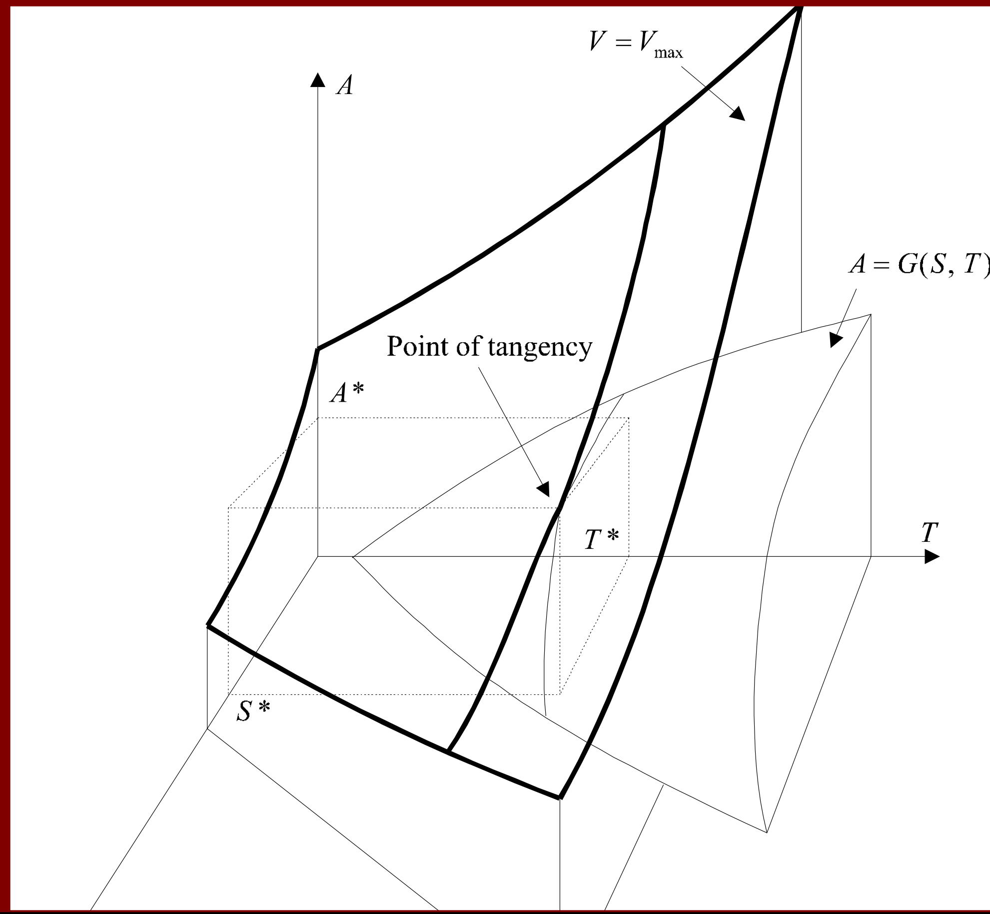
About Rigour:

- Clearly distinguish between what is reality and what is model
- Care about terminology and definitions
- Completely avoid unsupported categorical statements (that is journalism)
- Use logic:
  Learn valid rules for deductions,
  Care about what is true, what might be true, what is false,
  what might be false,
  Understand when new premises are introduced
- Care about what has been done before (references, etc)

My Criteria for Valuing a ScientificModel:

- Applicability (truthfulness, proximity to reality)
- Simplicity (explicitness)
- Level of technique





From: **Grubbström, R.W.,**
"Some Aspects on Modelling as a Base for Scientific Recommendations“,
***Kybernetes*, 30(9,10), 2001, 1126-1138.**

(Some) Current Research Interests in Production Economics

Robert W. Grubbström

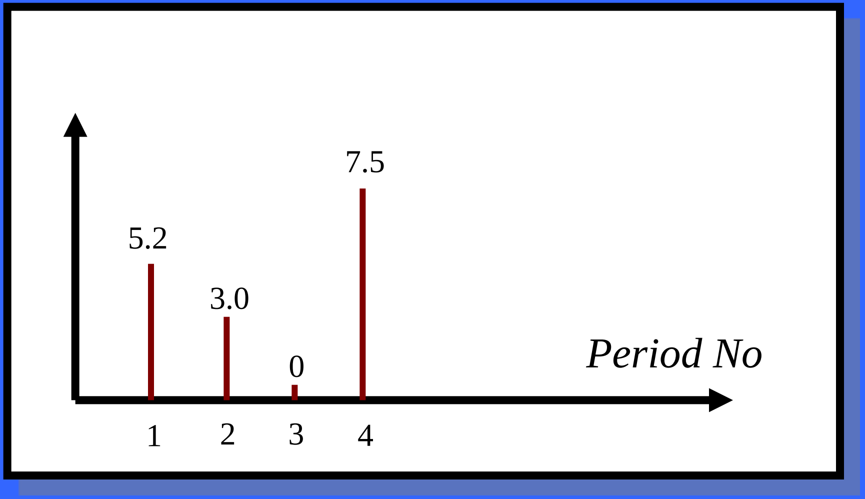
- **Economic Consequences**
- **Risk**
- **Recycling and remanufacturing**
- **Environmental Issues**
- **Energy-Economic Issues**
- **Lotsizing and MRP Theory**

Economic Consequences: In which time scale?

Discrete or Continuous Time?

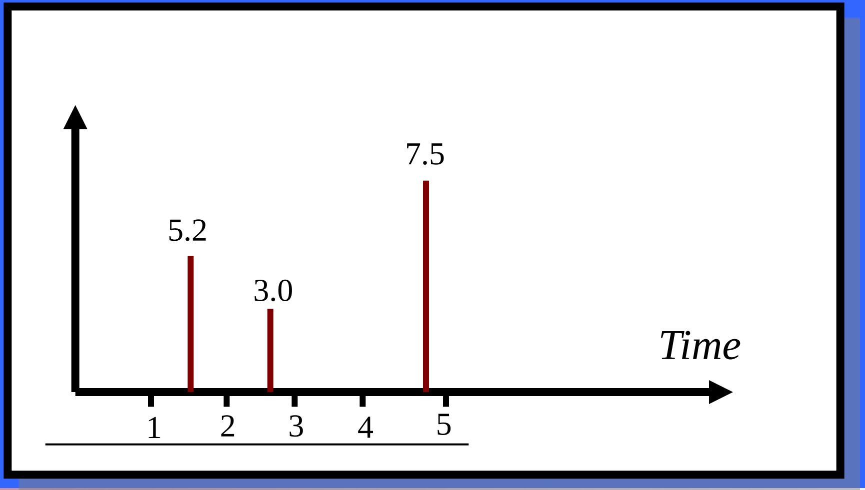
Discrete Time

Period	1	2	3	4
Requirements	5.2	3.0	0	7.5



Continuous Time

Time	1.5	2.7	4.8
Requirements	5.2	3.0	7.5



Arguments for using *continuous time* rather than *discrete time*

- A discrete time scale is embedded in a continuous time scale, so continuous time is more general.
- Globalisation makes time more continuous than before from a practical point of view.
- Often it is easier to solve problems with continuous variables.
- In discrete problems, there might be periods with no events, but empty periods simply do not exist in continuous time.

Average Cost (AC) or Net Present Value (NPV)?

The Net Present Value may be written in a completely general form,
whether or not the cash flow contains discrete payments (impulses).

Continuous interest rate

$$\text{NPV} = \int_0^\infty Cash\,Flow(t)\,e^{-\rho t}\,dt$$

The corresponding Annuity Stream is the constant flow of payments during a given time period *T* giving rise to a given NPV:

$$\textit{Annuity Stream} = \frac{\rho\,\mathrm{NPV}}{1 - e^{-\rho T}},$$

where *T* is the relevant time period

• The Net Present Value is comparable with
Total Costs/Revenues/Profits

•The Annuity Stream is comparable with
Average Costs/Revenues/Profits per time

Total Costs/Revenues/Profits are obtained as a first-order approximation of the Net Present Value with respect to the interest rate.

Similar with Average Costs/Revenues/Profits and Annuity Stream.

1964

MANAGEMENT SCIENCE
Vol. 10, No. 3, April, 1964
Printed in U.S.A.

A COMPARISON OF ORDER QUANTITIES COMPUTED USING THE AVERAGE ANNUAL COST AND THE DISCOUNTED COST*

G. HADLEY

Hadley, G., A comparison of order quantities computed using the annual cost and the discounted cost, *Management Science*, 10(3), 1964, 472-476.

the average annual cost and by minimizing the discounted cost over all future time for the simplest imaginable type of inventory model in which the demand rate is assumed to be deterministic and no stockouts are allowed. By specific numerical results it is shown that for all values of the parameters that might reasonably be encountered in an inventory problem there is only a negligible

1980

INT. J. PROD. RES., 1980, VOL. 18, NO. 2, 259–271

A principle for determining the correct capital costs of work-in-progress and inventory

ROBERT W. GRUBBSTRÖM†

In all production-inventory planning situations one of the major cost items to be

Grubbström, R.W., A principle for determining the correct capital costs of inventory and work-in-progress, *IJPR*, 18(2), 1980, 259-271.

a cost-added basis: materials, labour, share of overheads, etc.

In this paper we formulate models in which the physical production processes and their associated cash flow are analysed. By computing the present value of the cash flow as a function of characteristics of the production process, this value will also reflect a correct overall capital cost. Adjusting parameters of the physical process will then disclose what values to ascribe to products at different stages of production and assembly.

It is shown that usual accounting principles often yield too conservative

Robert W Grubbström

2011

European Journal of Operational Research 215 (2011) 115–125

Contents lists available at ScienceDirect

...search

ELSEVIER

Time reference point for discounting

Recent article suggesting an explanation to differences in Average Cost Approach and EOQ/EPQ models

Production, Man...

Holding cos... ...or Point

Patrick Beullen...

Beullens, P., Janssens, G.K., Holding Costs under Push or Pull Conditions - The Impact of the Anchor Point, *EJOR*, 215(1), 2011, 895-908.

A R T I C L E I...

Article history:
Received 1 November 2010
Accepted 6 June 2011
Available online 16 July 2011

Keywords:
Inventory
Production
Net Present Value
Average profit models
Multi-echelon inventory theory

Holding costs are traditionally determined from the investment in physical stock during a cycle. An alternative approach instead derives holding costs from Net Present Value (NPV) functions. It is known that applying both frameworks to the same system can lead to different holding cost valuations, but little explanation has been offered. By introducing the Anchor Point in a model, this paper shows, for four different systems, that traditional holding cost models (implicitly) assume pull conditions, while current NPV approaches model push conditions. This explains in part the differences between the methods. It is shown that the Anchor Point concept allows the construction of NPV models under pull conditions, giving results in better correspondence with traditional models. The traditional framework is restricted to pull conditions and important considerations could be easily overlooked, leading to wrong valuations of holding costs. NPV seems superior as such considerations are automatically incorporated. The application to multi-echelon inventory systems provides interesting insights on the roles of echelon stocks and

Robert W Grubbström

Example:
Dynamic Lotsizing

Problem

To choose the amount to produce (or not to produce) at each time satisfying given requirements and optimising an objective function (NPV or AC)

Early algorithms

The *Wagner-Whitin* (optimal, 1958) and the *Silver-Meal* (heuristic, 1973) *algorithms* for dynamic lotsizing are normally presented in terms of an *average cost* function in *discrete time*.

An additional forward optimal dynamic lotsizing algorithm has ... the *Triple Algorithm*.
...plied either time is *discrete* or ...*itional average cost* or the *Net* ...) is the objective function.

- **Single product**
- **No backlogging allowed**
- **Finite horizon**
- **Given requirements D_i at given times t_i**

1958

DYNAMIC VERSION OF THE ECONOMIC LOT SIZE MODEL*†

HARVEY M. WAGNER AND THOMSON M. WHITIN

Stanford University and Massachusetts Institute of Technology

A forward algorithm for a solution to the following dynamic ... f the
econ ... for a
sing ... riods,
we desire a minimum total cost inventory management scheme which satisfies known demand in every period. Disjoint planning horizons are shown to be possible which eliminate the necessity of having data for the full N periods.

Wagner, H.M., Whitin, T.M., Dynamic Version of the Economic Lot Size Model, *Management Science*, 5(1), 1958, 89-96.

Harvey M. Wagner
b. 1931

Thomson M. Whitin
b. 1923

Bellman Equation for Wagner-Whitin Algorithm in continuous time and NPV as objective function:

NPV

$$W_m(l) = \operatorname*{Min}_{\alpha_m(l)\in(0,1)} \alpha_m(l)\left(e^{-\rho t_m} + W_{m-1}(0)\right) +$$

$$+\left(1-\alpha_m(l)\right)\left(e^{-\rho t_m} g(t_{m-1}, t_m, t_{m+l}) + W_{m-1}(l+1)\right),$$

Cumulative requirements

where

$$g(t_i, t_j, t_k) = \frac{c}{K}\left(e^{\rho\left(t_j - t_i\right)} - 1\right)\left(\bar{D}_{k-1} - \bar{D}_{j-1}\right)$$

Robert W Grubbström

Edward A. Silver, b. 1937

1973

Silver-Meal Algorithm

According to the Silver-Meal algorithm the time average of a setup cost and holding costs is computed starting at the time of each replenishment and proceeding in steps of one time unit. As soon as there is an increase in this average cost measure, it is time for a new replenishment.

In our terminology, we interpret the average cost expression as the annuity of the out-payments. Therefore, $A(k)$ is computed for k = 2, 3, … . When $A(k+1) > A(k)$, the order at t_1 is determined as D_k.

Annuity Stream for Silver-Meal Algorithm in continuous time:

$$A(k) = \frac{\rho \ \text{NPV}}{1 - e^{-\rho t_{k+1}}} =$$

$$= \frac{\rho}{1 - e^{-\rho t_{k+1}}} \left(K + \frac{h}{\rho} \sum_{i=1}^{k-1} (D_k - D_i) \left(e^{-\rho t_i} - e^{-\rho t_{i+1}} \right) \right)$$

Obvious observations

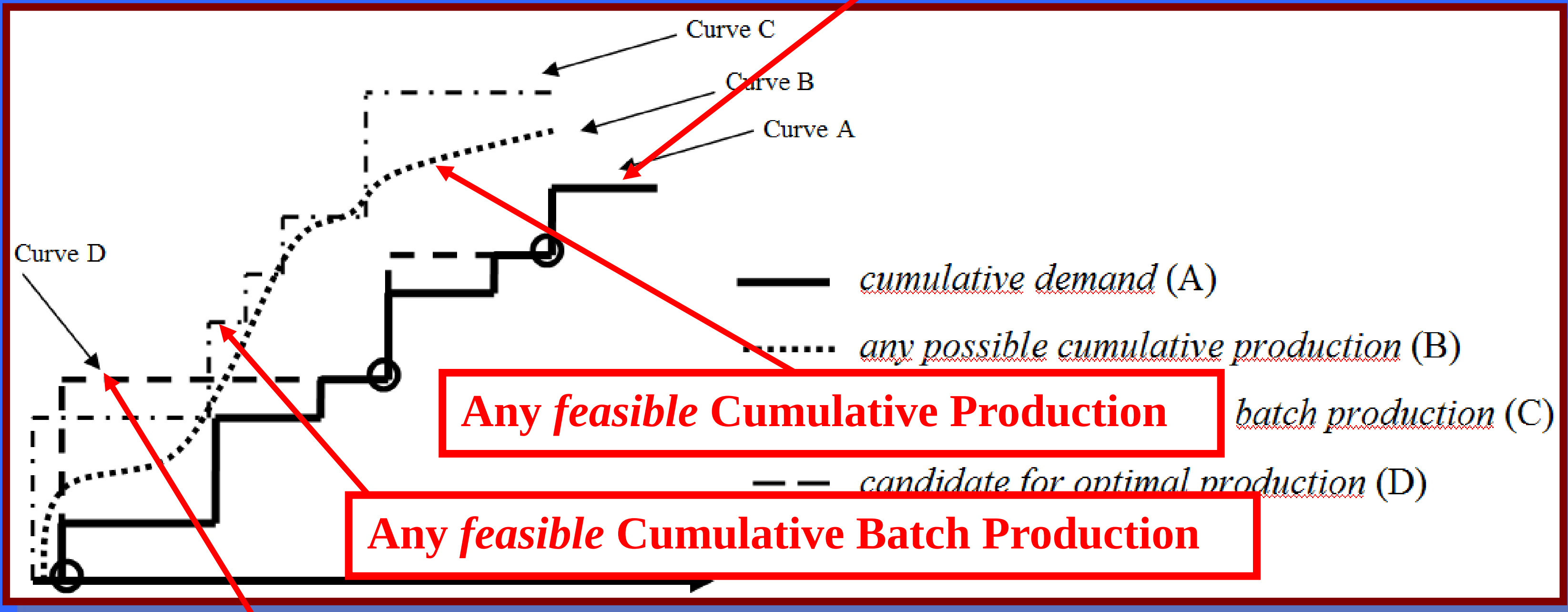
(i) When the setup cost *K* is very high compared to the holding cost *h*, there can only be one setup at the very beginning of the process (the All-At-Once, ∀@1l, solution).

(ii) With a very low setup cost *K*, there must be a setup at every time a non-zero requirement occurs
(the Lot-For-Lot, L4L, solution).

Therefore, the domain in which dynamic lotsizing exists as a non-trivial problem, is when the ratio K/h is neither high, nor low.

Inner-Corner Condition

Given Cumulative Requirements



Any *feasible* Cumulative Production

Any *feasible* Cumulative Batch Production

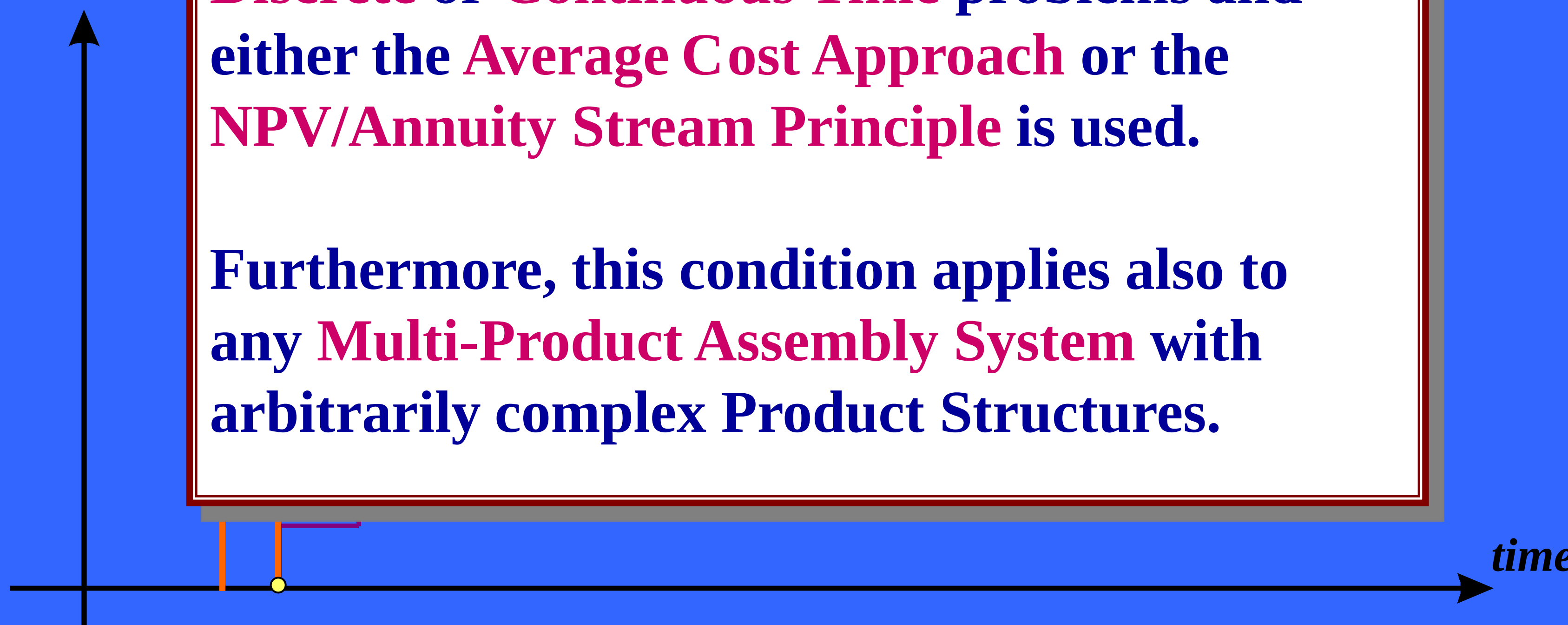
Cumulative Batch Production candidate for Optimality

Robert W Grubbström

Inner Corner Property

The **Inner-Corner Condition** is valid for either **Discrete** or **Continuous Time** problems and either the **Average Cost Approach** or the **NPV/Annuity Stream Principle** is used.

Furthermore, this condition applies also to any **Multi-Product Assembly System** with arbitrarily complex Product Structures.



1969

MINIMUM CONCAVE-COST SOLUTION OF LEONTIEF SUBSTITUTION MODELS OF MULTI-FACILITY INVENTORY SYSTEMS

Arthur F. Veinott, Jr.

Stanford University, Stanford, California

(Received October 26, 1967)

Binary properties of problem recognised already in:

Veinott, Jr, A. F., , Minimum concave-cost solution of Leontief substitution models of multifacility inventory systems. *Operations Research*, 17, 1969, 262 - 291.

inventory models with concave costs in a unified manner. Dynamic programming recursions for searching the extreme points to find an optimal point are given in a number of cases. We only give algorithms whose computational effort increases algebraically (instead of exponentially) with the size of the problem.

A MATRIX A is called *Leontief* if it has exactly one positive element in each column and there is a nonnegative (column) vector x for which Ax is

2010

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Optimal lotsizing within MRP Theory☆

Robert W. Grubbström [a,c,*], Marija Bogataj [b,c], Ludvik Bogataj [b,c,1]

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[b] Univers
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Grubbström, R. W., Bogataj, M., Bogataj, L., Optimal Lotsizing within MRP Theory, *Annual Reviews in Control*, 34(1), 2010, 89–100.

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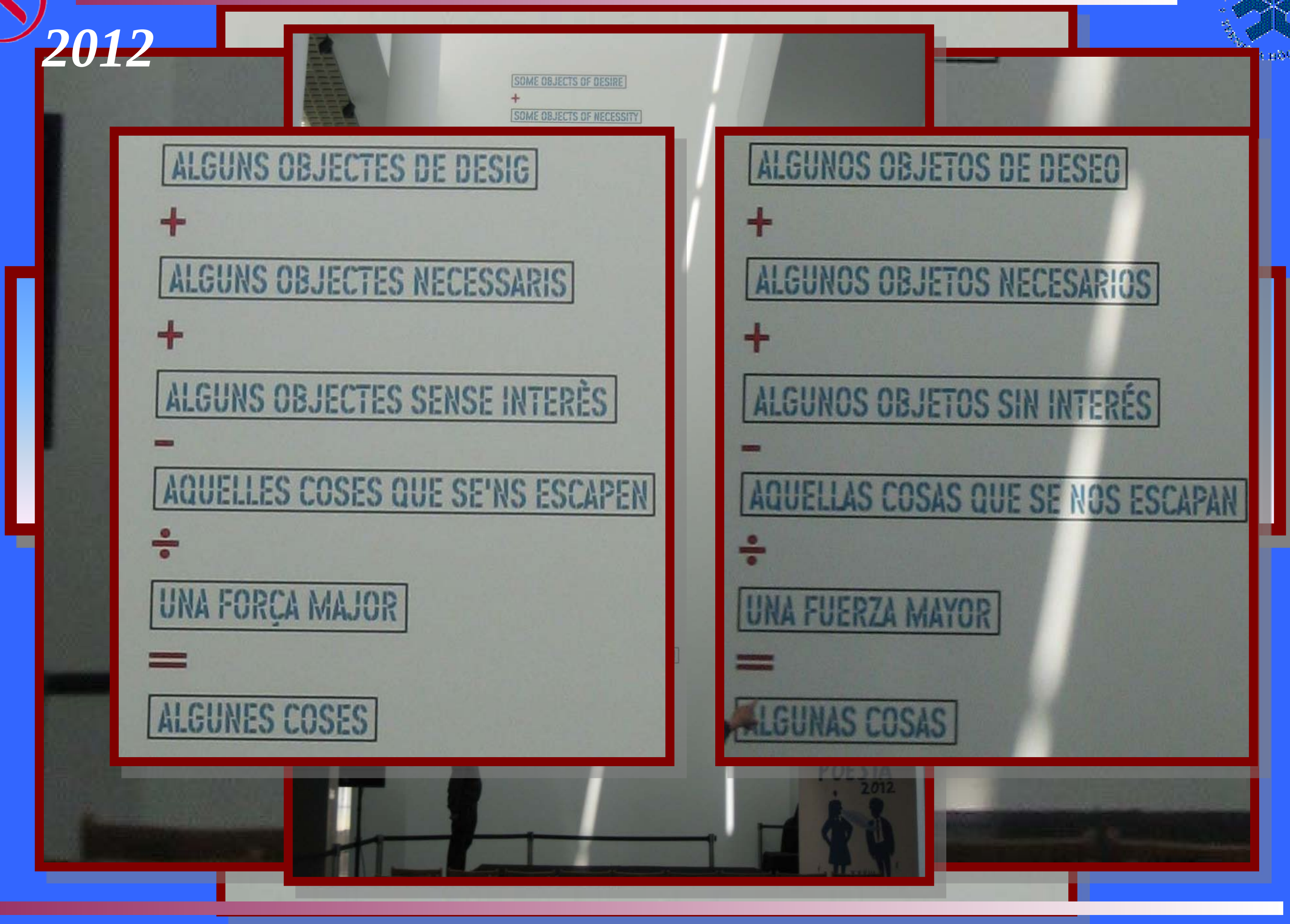
ABSTRACT

MRP Theory combines the use of Input–Output Analysis and Laplace transforms, enabling the development of a theoretical background for multi-level, multi-stage production-inventory systems together with their economic evaluation, in particular applying the Net Present Value principle (NPV).

In this paper we concentrate our attention on the question of optimal lotsizing decisions within the MRP Theory framework. MRP Theory has mainly dealt with *assembly structures* by which items produced downstream (on a higher level in the product structure) contain one or more sub-items on lower levels, but at each stage, the assembly activity produces only one type of output. This enables the *input matrix*, after enumerating all items suitably, to be organised as a triangular matrix, with non-zero elements only appearing below its main diagonal. The introduction of a diagonal *lead time matrix* capturing the advanced timing when required inputs are needed, enables compact expressions to be obtained, explaining the development of key variables such as available inventory and backlogs in the frequency

Robert W Grubbström

2012



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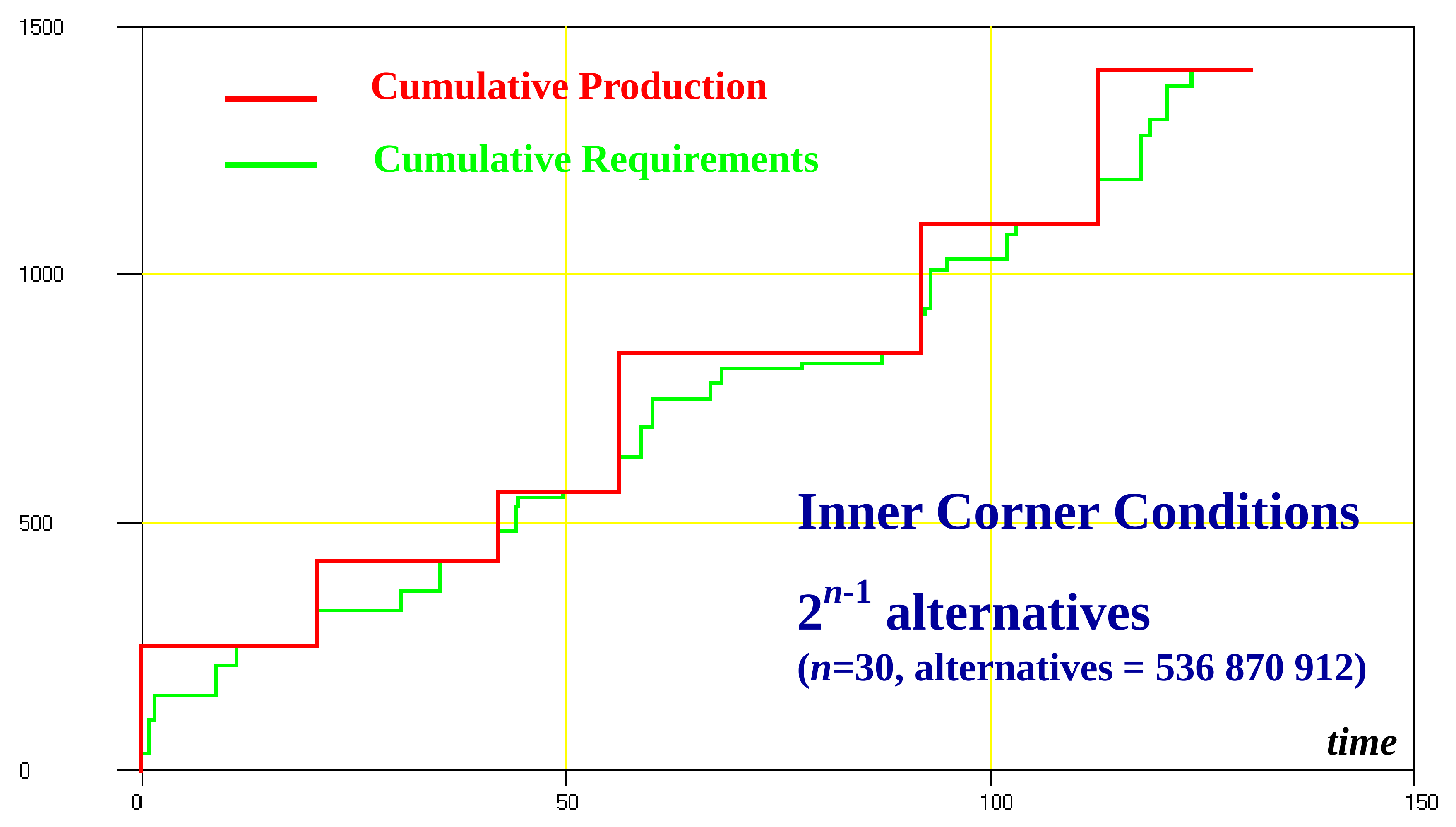
Staircase Functions



Robert W Grubbström

Complexity issues for general assembly systems are studied in:

2010

ARTICLE IN PRESS

Int. J. Production Economics ▮ (▮▮▮) ▮▮▮–▮▮▮

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Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

ELSEVIER

The space of solution alternatives in the optimal lotsizing problem for general assembly systems applying MRP theory

Robert W. Grubbström [a,b,*], Ou Tang [a]

Grubbström, R. W., Tang, O., The space of solution alternatives in the optimal lotsizing problem for general assembly systems applying MRP theory, *IJPE*, *Article in Press*, 2010, doi:10.1016/j.ijpe.2011.01.012.

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Keywords:
MRP theory
Optimal lotsizing
Assembly system
Laplace transform
Complexity

development of a theoretical background for multi-level, multi-stage production–inventory systems together with their economic evaluation, in particular applying the Net Present Value principle (NPV).

In a recent paper (Grubbström et al., 2010), a general method for solving the dynamic lotsizing problem for a general assembly system was presented. It was shown there that the optimal production (completion) times had to be chosen from the set of times generated by the Lot-For-Lot (L4L) solution. Thereby, the problem could be stated in binary form by which the values of the binary decision variables represented either to make a production batch, or not, at each such time. Based on these potential times for production, the problem of maximising the Net Present Value or minimising the average cost could be solved, applying a single-item optimal dynamic lotsizing method, such as the

Robert W Grubbström

Binary Formulation for Production Staircase

$\alpha_i = 1$, to produce at time t_i

$\alpha_i = 0$, not to produce at time t_i

Cumulative production

$$\bar{P}_j = \left(\bar{D}_j + \sum_{k=j+1}^{n} D_k \prod_{l=j+1}^{k} (1-\alpha_l) \right)$$

Lot Size (Batch)

$$Q_j = \bar{P}_j - \bar{P}_{j-1} = \alpha_j \sum_{k=j}^{n} D_k \prod_{l=j+1}^{k} (1-\alpha_l)$$

Cumulative time

$$\bar{T}_j = t_{j+1} + \sum_{k=j+1}^{n} \tau_{k+1} \prod_{l=j+1}^{k} (1-\alpha_l)$$

Length of time step

$$\tau_j = \bar{T}_j - \bar{T}_{j-1} = \left(\alpha_j \sum_{k=j}^{n} \tau_{k+1} \prod_{l=j+1}^{k} (1-\alpha_l) \right)$$

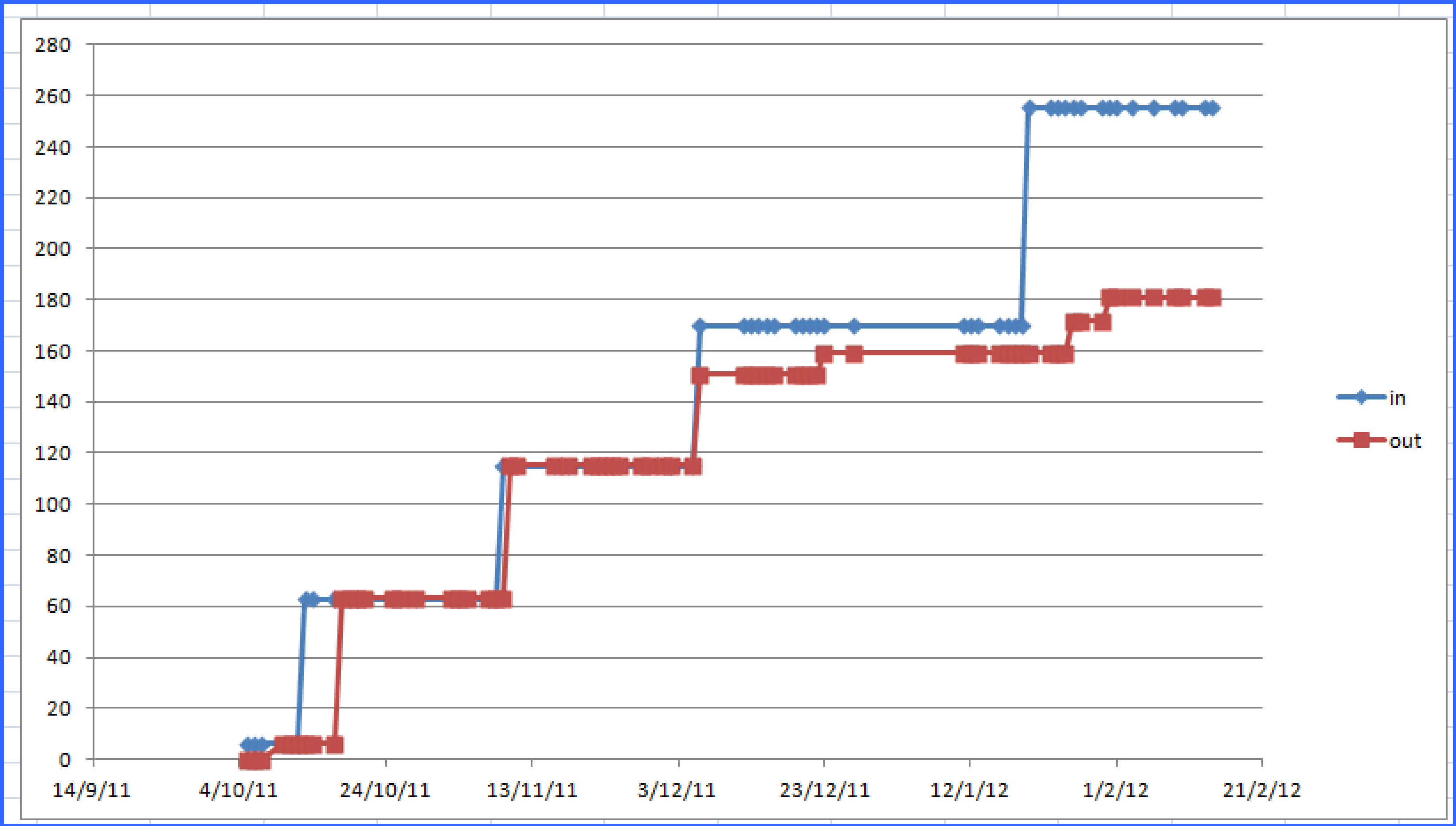
Examples of Staircase Considerations in Practice

(by Alessandro Persona and his Group at the University of Padua)

Robert W. Grubbström

INDUSTRIAL CASE 1 – SKINS

(FASHION)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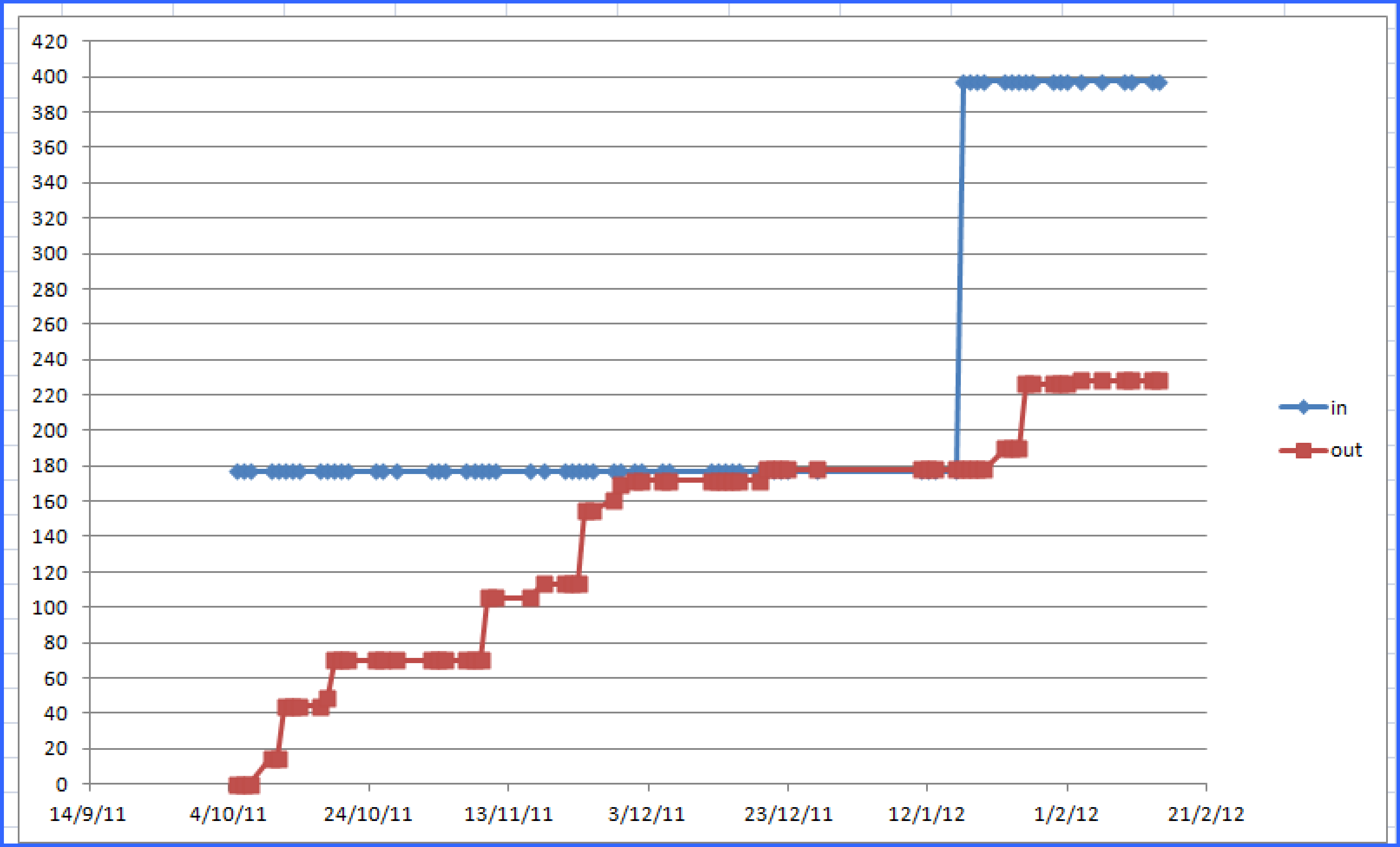

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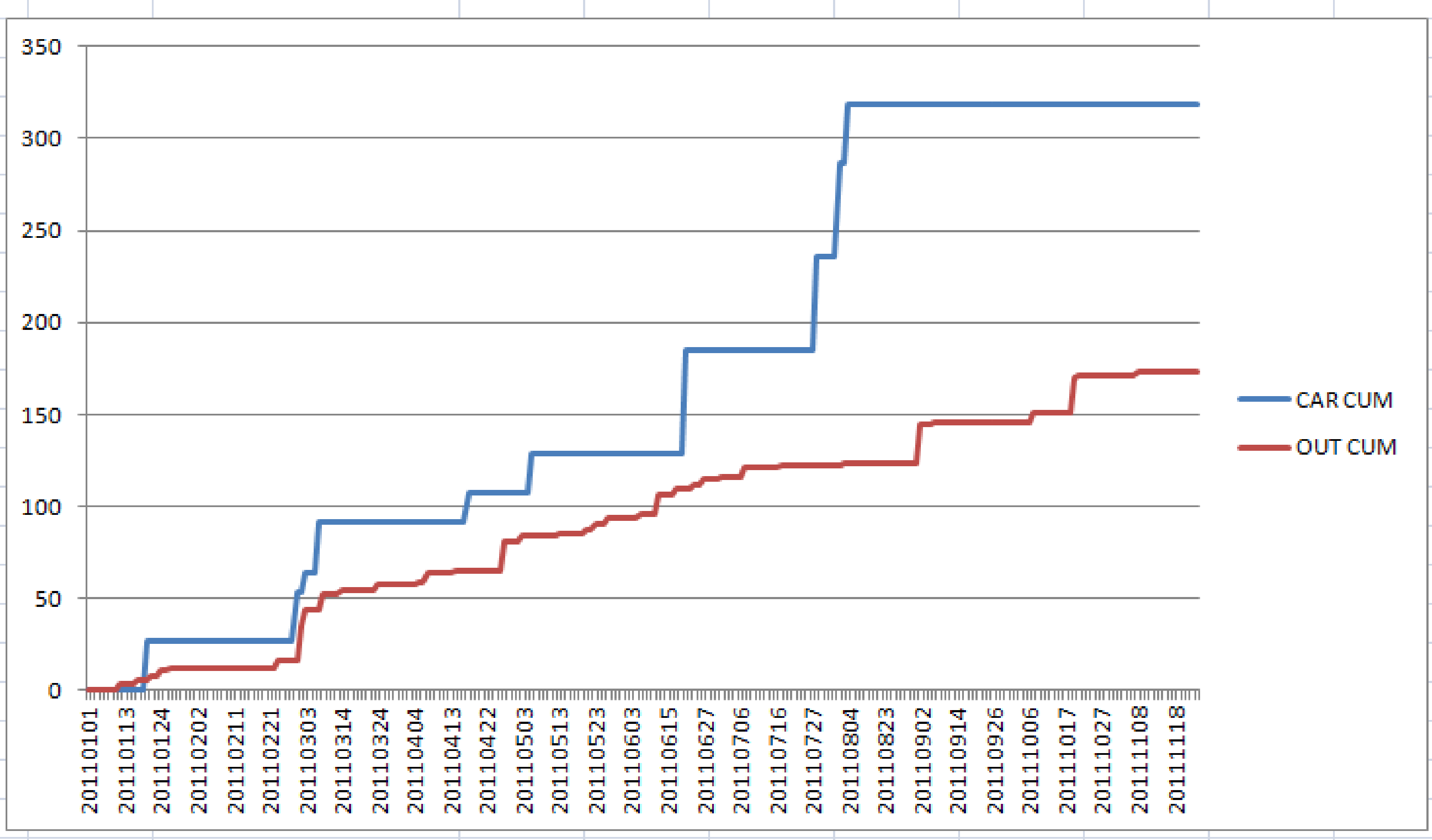
INDUSTRIAL CASE 2 – WOOD

(LONG PURCHASING LEAD TIME, HIGH CUSTOMER SERVICE LEVEL)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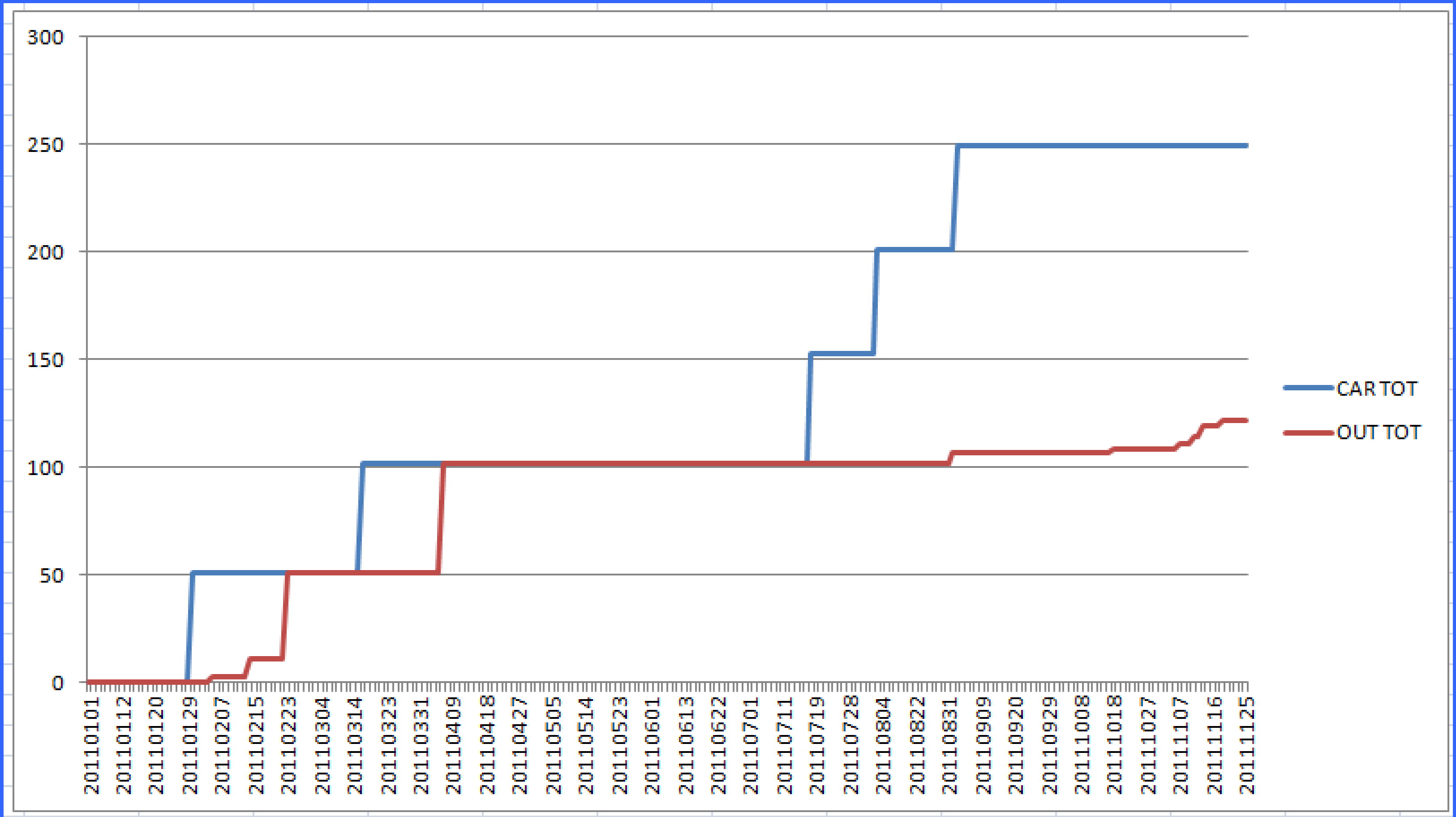
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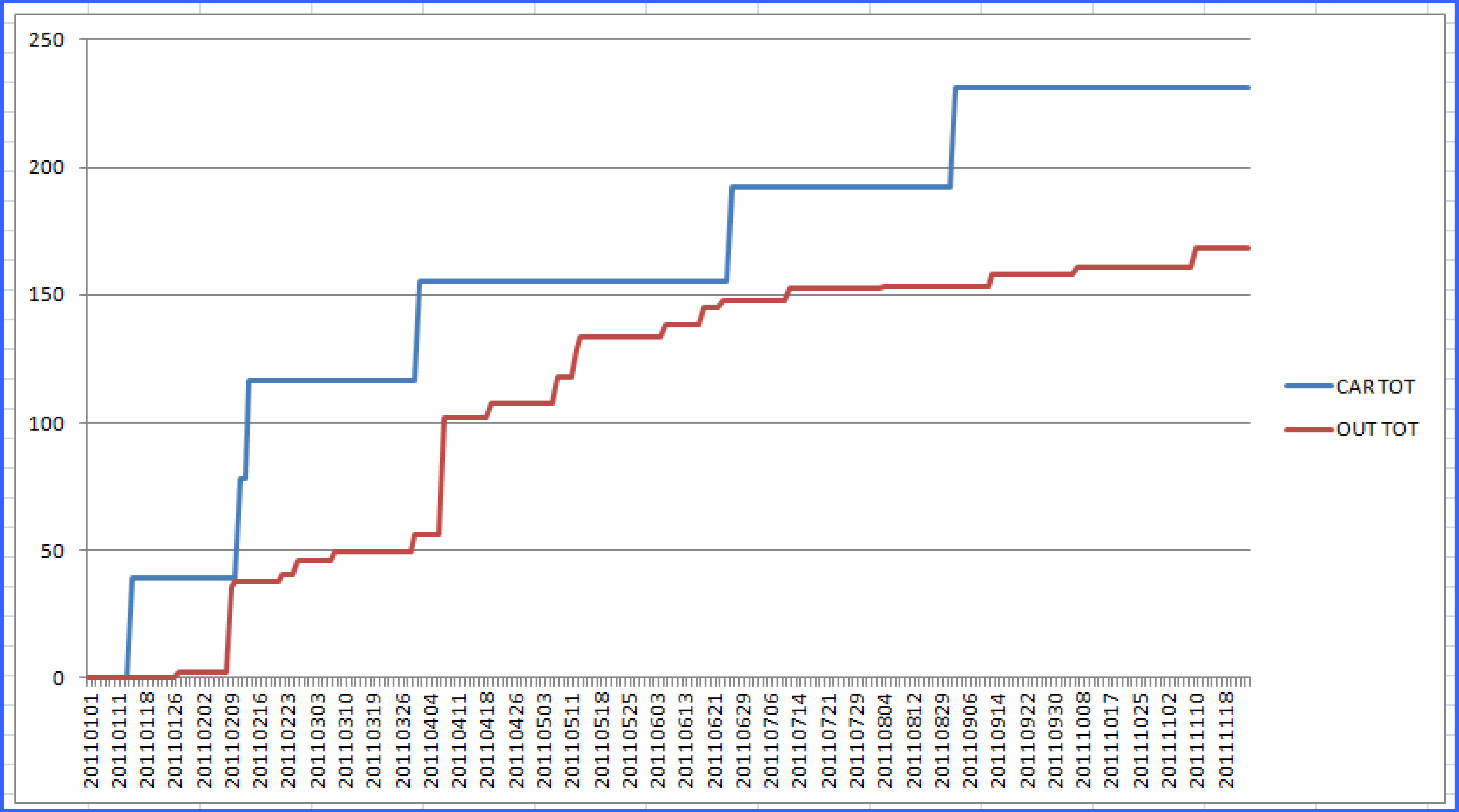
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Generalisation of Dynamic Lotsizing to Finite Production Rate *q*

Robert W. Grubbström

1997

Note: Dynamic Lot Sizing for a Finite Rate Input Process

Roger M. Hill

Department of Mathematical Statistics and Operational Research, University of Exeter, EX4 4QE, England

Received October 1995; revised August 1996; accepted 2 October 1996

Hill, Roger M., Note: Dynamic Lot Sizing for a Finite Rate Input Process, *Naval Research Quarterly*, 44(2), 1997, 221–228.

Abstract: The basic single-product dynamic lot-sizing problem involves determining the optimal batch production schedule to meet a deterministic, discrete-in-time, varying demand pattern subject to linear setup and stockholding costs. The most widely known procedure for deriving the optimal solution is the Wagner–Whitin algorithm, although many other approaches have subsequently been developed for tackling the same problem. The objective of this note is to show how these procedures can readily be adapted when the input is a finite rate production process. © 1997 John Wiley & Sons, Inc. Naval Research Logistics 44: 221–228, 1997

Roger M. Hill's sawtooth method

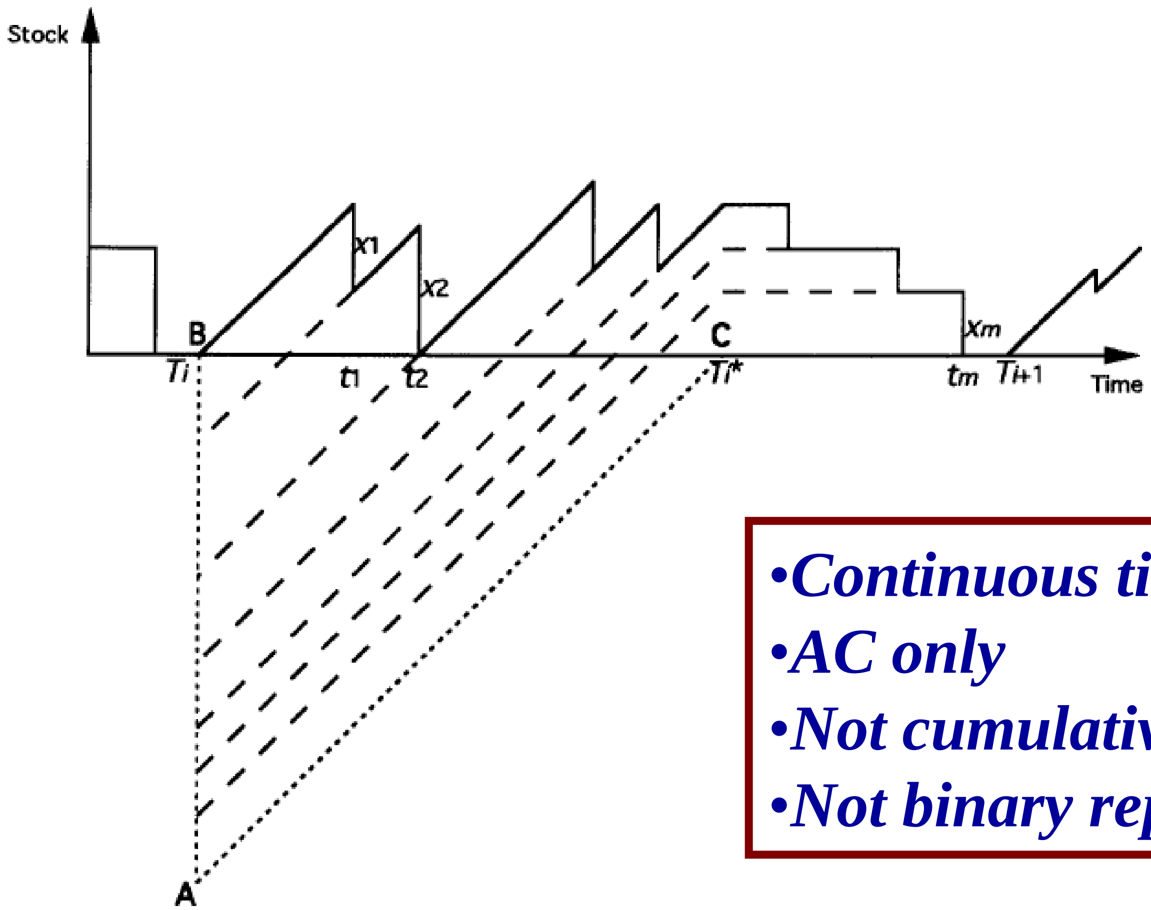


Figure 1. Plot of stock versus time, illustrating the average stock calculation.

- *Continuous time*
- *AC only*
- *Not cumulative staircases*
- *Not binary representation*

Roger M. Hill's example

Table 1. Demand values for the numerical example.

k	1	2	3	4	5	6	7	8	9	10
t_k	3	4	6	8	9	10	14	15	19	20
x_k	8	6	8	4	6	7	8	5	9	7

Table 2. Computed values for the numerical example.

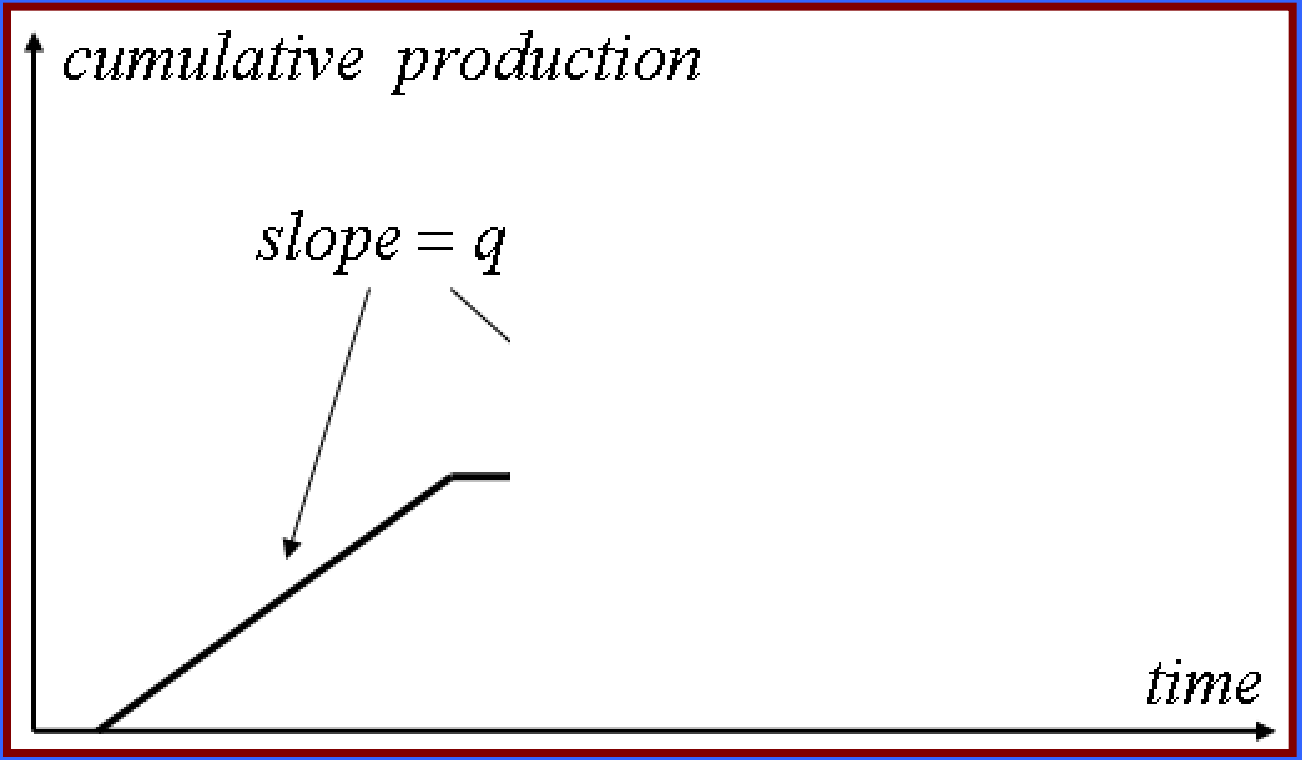
i	1	2	3	4	5
q_i	14	8	17	13	16
T_i^*	4	6	10	15	20
T_i	1.2	4.4	6.6	12.4	16.8
S_i	11.6	6.4	14.9	8.9	16.6
τ_i	–	0.4	0.6	2.4	1.6

Should be 1.8

The optimal solution is therefore to manufacture a batch of 39 ($=q_1 + q_2 + q_3$) starting at time 1.2 ($=T_1$) and a batch of 29 ($=q_4 + q_5$) starting at time 12.4 ($=T_4$), with a total cost (remembering to add back in $h(S_1 + \cdots + S_5) = 58.4$) of 176.2.

Should be 179.4

General form of cumulative production (when production rate *q* is finite)



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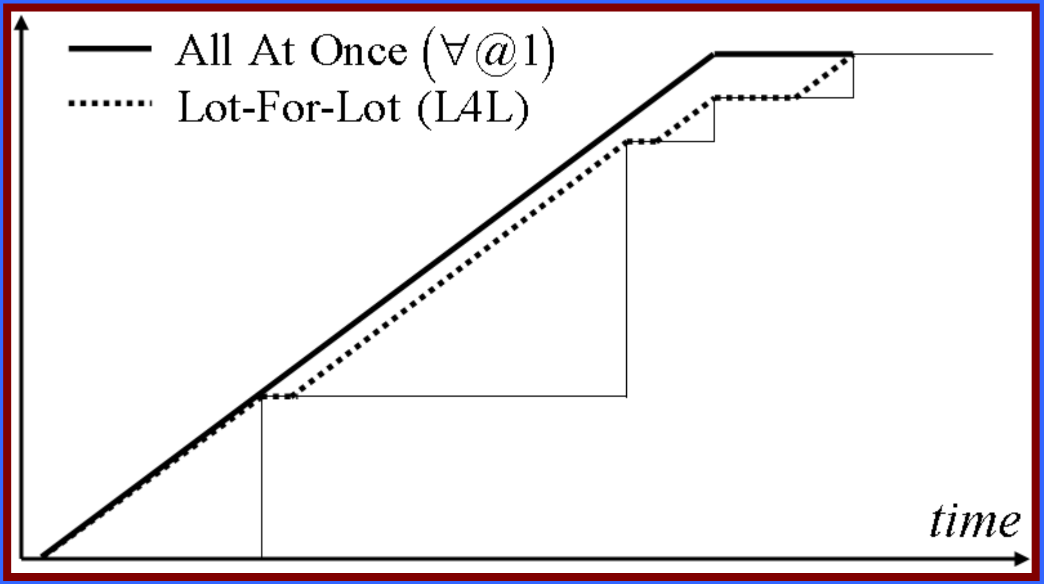
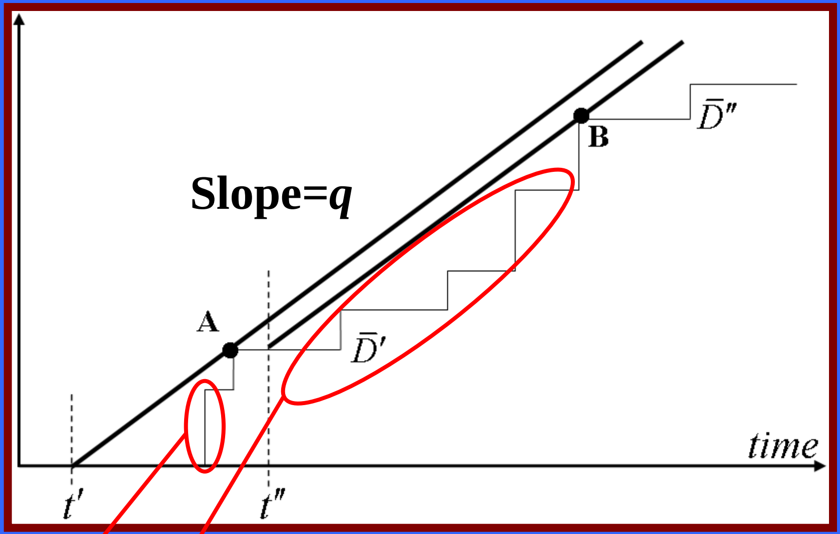
The Two Extreme Solutions



The All-At-Once (∀@1) and Lot-For-Lot (L4L) solutions to the dynamic lotsizing problem with a finite production rate.

Production Rate Restriction



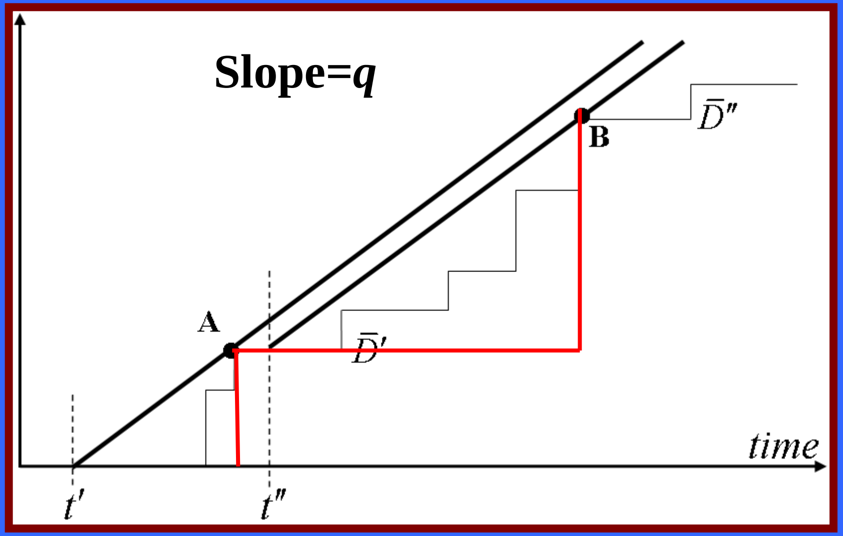
Uninteresting events

Latest time production can start

$$t' = \operatorname*{Min}_{i}\left(t_i - \bar{D}_i / q\right)$$

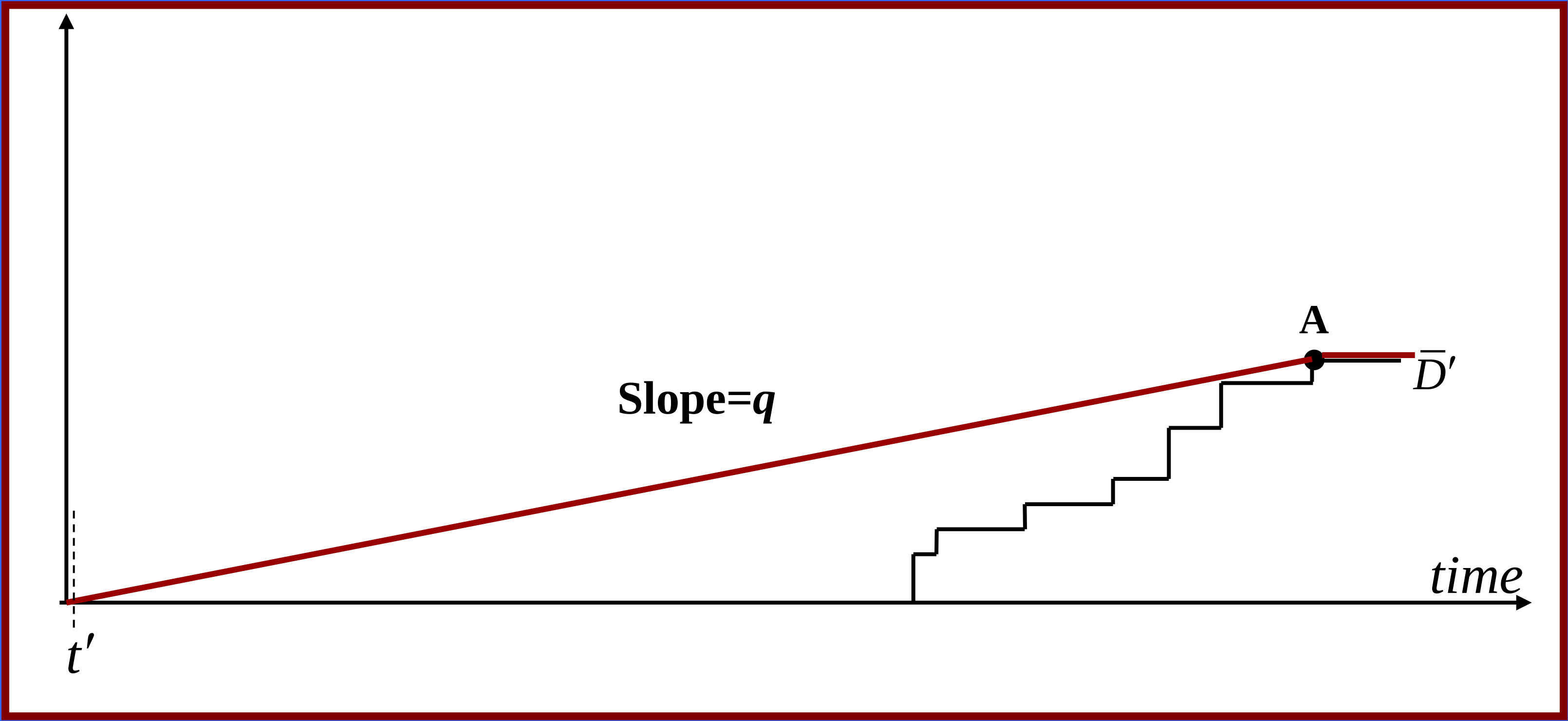
$$t'' = \operatorname*{Min}_{i,t_i > t'}\left(t_i - \left(\bar{D}_i - \bar{D}'\right) / q\right)$$

Consequence of Production Rate Restriction



Number of demand events affecting solution might decrease

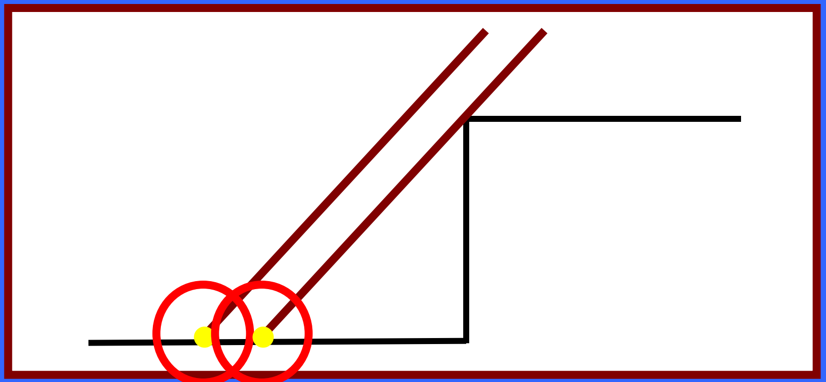
With very slow production (very small *q*)



All-at-once solution, □@1, is only feasible solution!
Similar result as if K were very high.

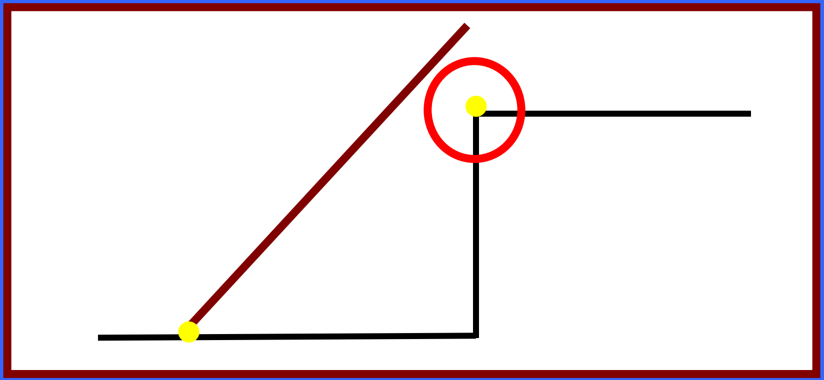
Main Theorem

Necessary for optimality, either the NPV or total cost is the objective function, is that each production segment starts from a horizontal step of cumulative requirements.



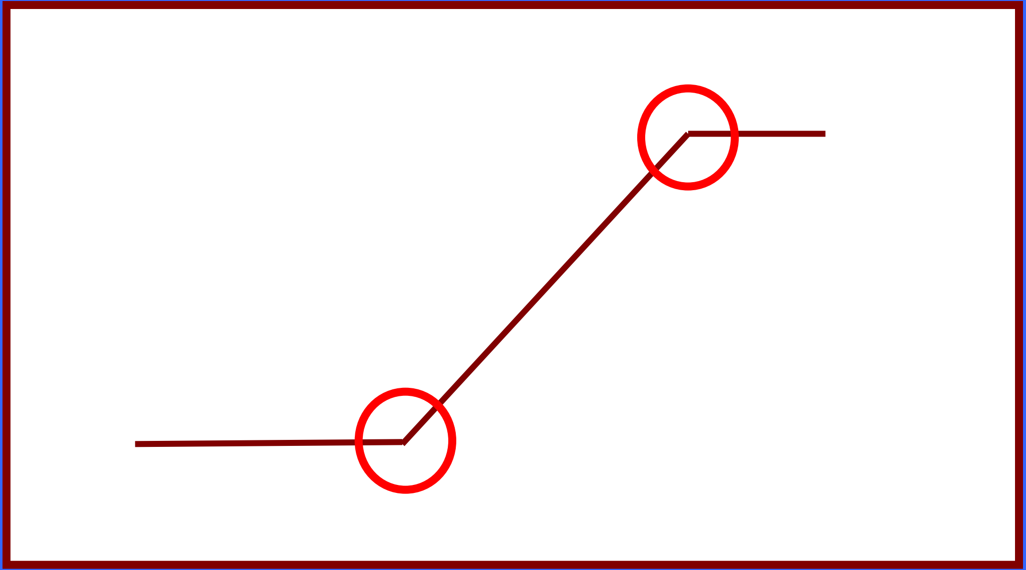
Corollary

If the production rate restriction has been applied eliminating all unnecessary requirement events,
then a segment starting on a horizontal step of cumulative requirements will necessarily touch the nearest corner of cumulative requirements.



Note

With a finite production rate, there are at least two alternative times for assigning the setup cost K.



Or, possibly, somewhere inbetween.

Another Corollary

If two adjacent segments are separated in time less than or equal to

$$\Delta t \le \frac{1}{\rho}\ln\left(1+\frac{\rho K}{cq}\right) \approx \frac{K}{cq}$$

(if setup costs are located at start of segments),
or less than or equal to

$$\Delta t \le -\frac{1}{\rho}\ln\left(1-\frac{\rho K}{cq}\right) \approx \frac{K}{cq}$$

(if setup costs are located at end of segments),

then the inclusion of both of these segments is unprofitable.

Objective Functions

$$\mathrm{NPV} = -\sum_{j=1}^{n}\left(\left(cq/\rho\right)\left(1-e^{-\rho Q_j/q}\right)e^{-\rho(t_j-D_j/q)}+\alpha_j K e^{-\rho(t_j-D_j/q+\beta Q_j/q)}\right)$$

,

$$\mathrm{TC} = -\left(cq/\rho\right)\sum_{j=1}^{n}\left[\left(1-e^{-\rho Q_j/q}\right)e^{-\rho(t_j-D_j/q)}-D_j e^{-\rho t_j}\right]_{\text{1st approximation in }\rho}$$

$$-K\sum_{j=1}^{n}\alpha_j\left[e^{-\rho(t_j-D_j/q+\beta Q_j/q)}\right]_{\text{0th approximation in }\rho} =$$

$$= h\sum_{j=1}^{n}\left(Q_j t_j - Q_j D_j/q + Q_j^{\,2}/(2q) - D_j t_j\right) - K\sum_{j=1}^{n}\alpha_j$$

The Q_j depend on the α_i .

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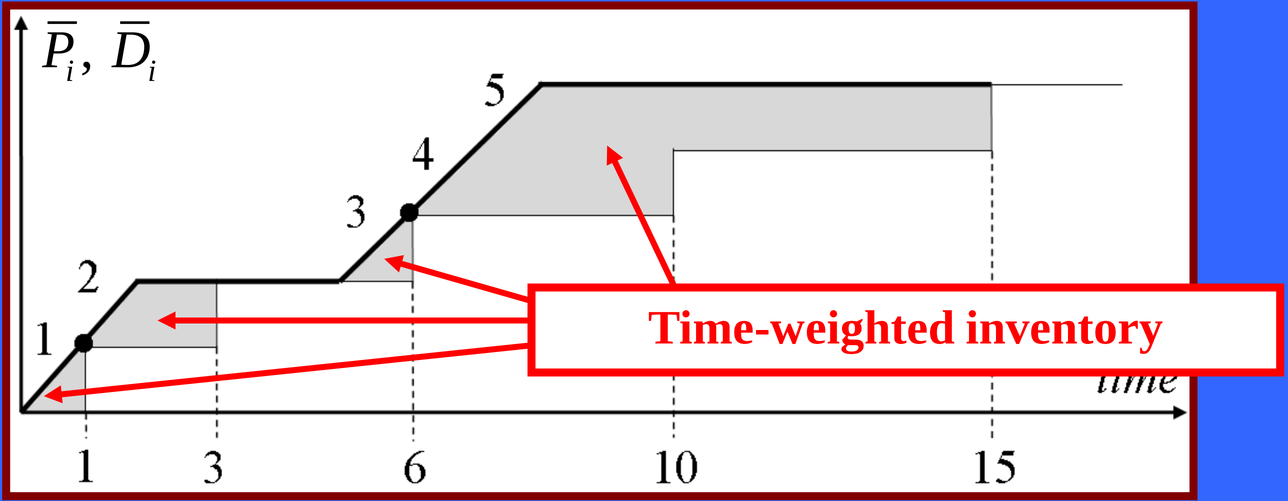
Numerical example (AC for simplicity)



,

This specific solution: $\alpha_j = 1,\ 0,\ 1,\ 0,\ 0$

$$Q_1 = 1\sum_{k=1}^{5} 1 \prod_{l=2}^{k} (1-\alpha_l) = 2$$

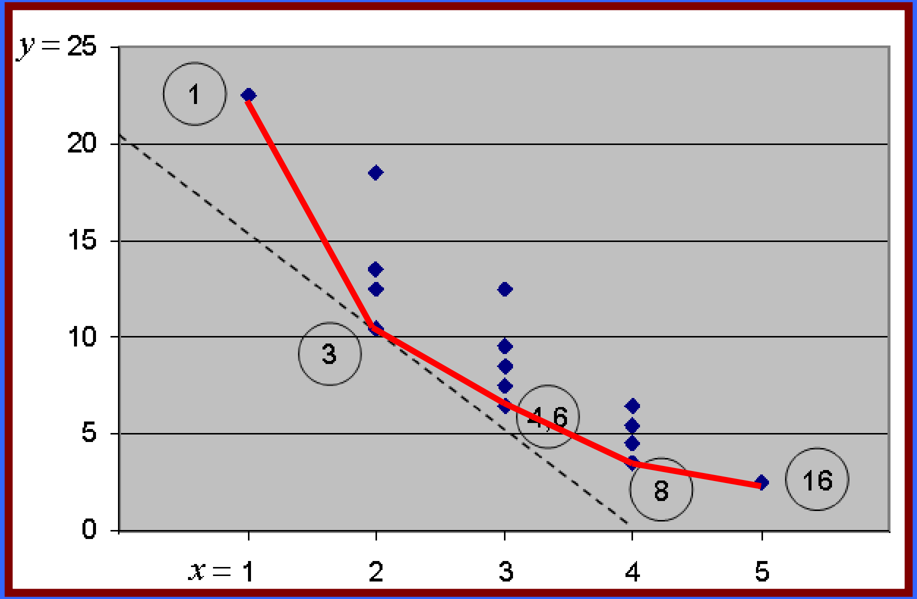
y **x**

$$\text{TC} = -h\sum_{j=1}^{5}\left(Q_j t_j - Q_j D_j / q + Q_j^2/(2q) - D_j t_j\right) - K\sum_{j=1}^{5}\alpha_j = -13.5 \cdot h - 2K$$

16 solutions

Solution No	Decisions α_j	Lot sizes Q_j	Time-weighted inventory y	Number of setups $x = \sum_{j=1}^{5} \alpha_j$
1	**1,0,0,0,0**	**5,0,0,0,0**	**22.5**	**1**
2	1,0,0,0,1	4,0,0,0,1	12.5	2
3	**1,0,0,1,0**	**3,0,0,2,0**	**10.5**	**2**
4	**1,0,0,1,1**	**3,0,0,1,1**	**6.5**	**3**
5	1,0,1,0,0	2,0,3,0,0	13.5	2
6	**1,0,1,0,1**	**2,0,2,0,1**	**6.5**	**3**
7	1,0,1,1,0	2,0,1,2,0	7.5	3
8	**1,0,1,1,1**	**2,0,1,1,1**	**3.5**	**4**
9	1,1,0,0,0	1,4,0,0,0	18.5	2
10	1,1,0,0,1	1,3,0,0,1	9.5	3
11	1,1,0,1,0	1,2,0,2,0	8.5	3
12	1,1,0,1,1	1,2,0,1,1	4.5	4
13	1,1,1,0,0	1,1,3,0,0	12.5	3
14	1,1,1,0,1	1,1,2,0,1	5.5	4
15	1,1,1,1,0	1,1,1,2,0	6.5	4
16	**1,1,1,1,1**	**1,1,1,1,1**	**2.5**	**5**

Scatter Diagram



Setup/Holding Cost Ratio	$K/h < 1$	$1 < K/h < 3$	$3 < K/h < 4$	$4 < K/h < 12$	$K/h > 12$
Optimal solution No	16	8	4, 6	3	1
Comment	L4L				$\forall$@1

Roger M. Hill's example I

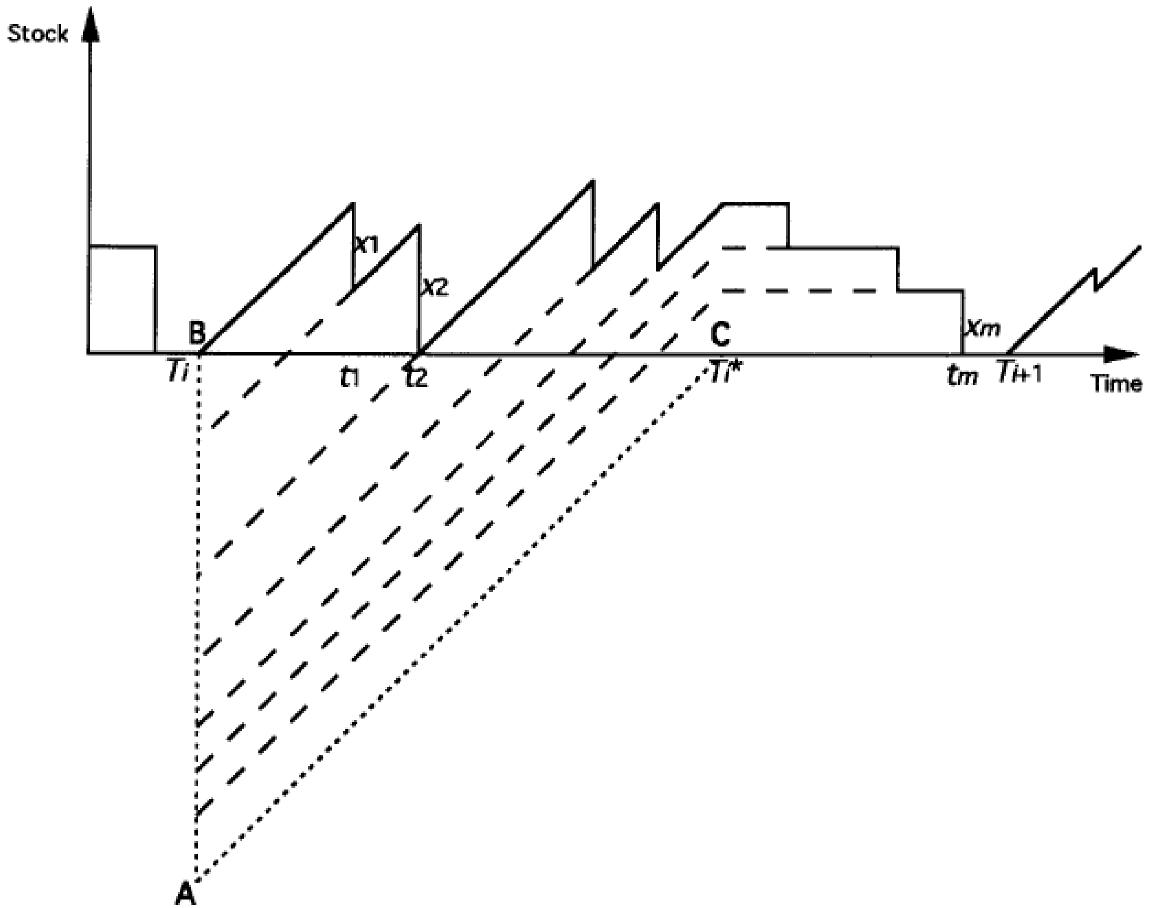
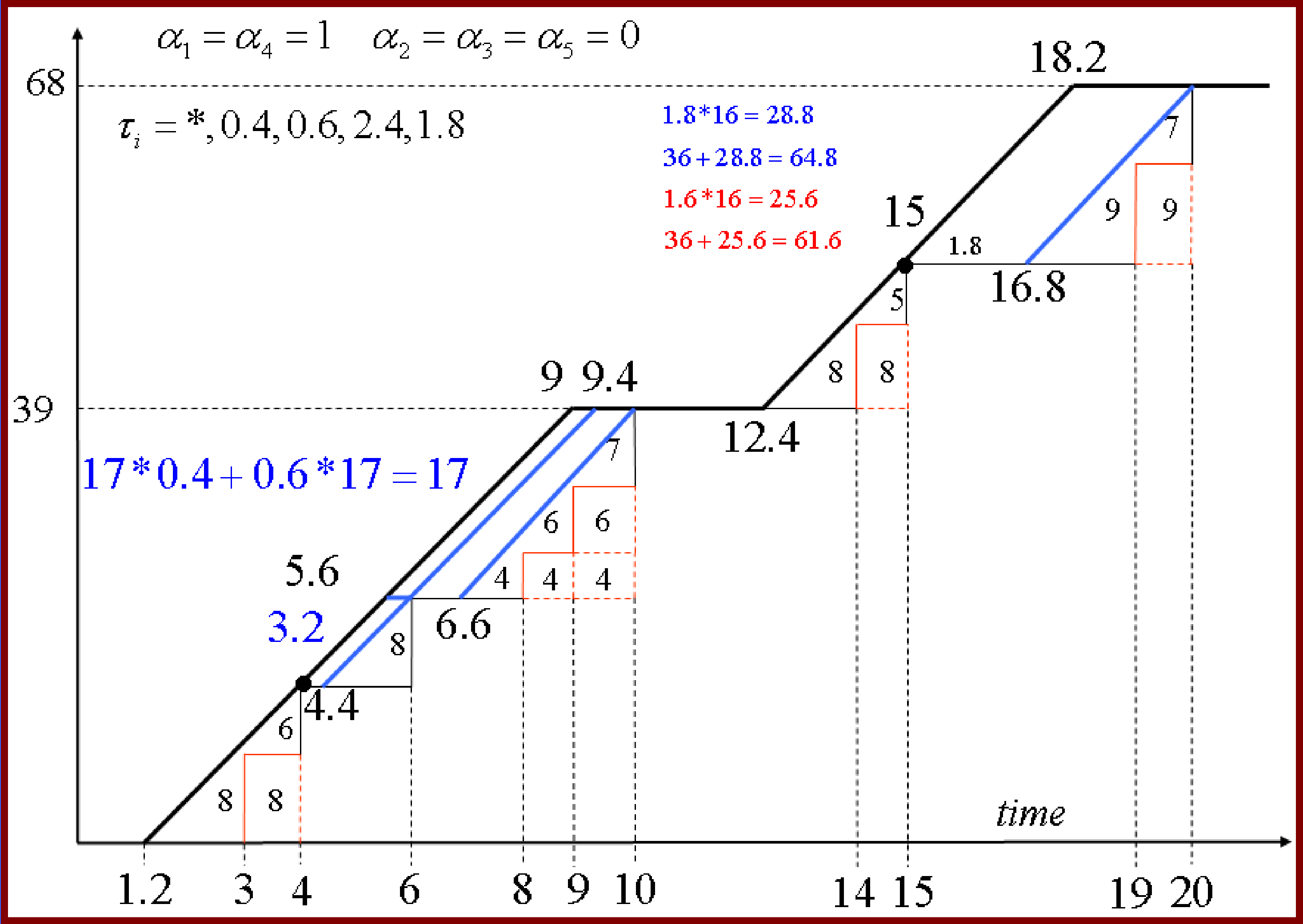


Figure 1. Plot of stock versus time, illustrating the average stock calculation.

My cumulative interpretation of Roger Hill's example



Robert W Grubbström

Summary

- ***Historical Background and Philosophy***
- ***International Journal of Production Economics***
- ***Examples of current research interest, especially Lotsizing, which is a classical example of a pure production-economic problem***

Thank you very much for your kind attention