Creating a Third Space for Authentic Biculturalism: Examples from Math in a Cultural Context

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Tumaqcat in the Yupiaq language literally means putting the pieces together. This case demonstrates how Ms. Nancy Sharp, a Yupiaq immersion teacher, seamlessly creates a classroom space that honors and adapts her home culture while she simultaneously meets school-based mathematical standards. Ms. Sharp's Yupiaq immersion class makes patterns during the lesson described and analyzed in this case. It explores her successful implementation, according to project tests, of a culturally based math module, *Patterns and Parkas: Investigating Geometric Principles, Shapes, Patterns, and Measurement* which is part of the *Math in a Cultural Context* (MCC) series. This math module connects Yupiaq cultural activity with school-based geometry. This study is part of a much larger long-term collaboration with Yupiaq communities and math educators in the development and implementation of culturally based curriculum, MCC. The larger ongoing implementation studies have yielded consistent and repeated results favoring MCC. This article describes how Ms. Sharp develops a classroom space that not only connects her home culture and language to the culture of schooling but also shows how she resourcefully and authentically accomplished this. Through cultural and linguistic activities, she and her students engage in work that they deem important in the context of home and school. She effectively uses modeling and joint activity as a means of teaching geometric relationships, while students learn to fold and cut geometrical patterns out of paper, and learn the Yupiaq ways of putting these patterns together. She does not use typical school-based pedagogical procedures such as nominating students and evaluating their responses. Instead, students work alongside the teacher, making pattern pieces that they further fashion into a symmetrical strip pattern. Her familiarity with the cultural and linguistic context of making border patterns made it more feasible to effectively connect her cultural and linguistic knowledge with school-based math. These authentic connections resulted in better than average gain scores for her Yupiaq second language learners.

Culturally based curriculum has been touted by numerous federal reports and academics (Deyhle & Swisher, 1997; Tippeconnic, 2003) as a way to improve the academic performance of American Indian/Alaska Native (AI/AN) students. Recently, there has been a spate of articles in the popular press

as well as by national organizations such as the National Council of Teachers of Mathematics (NCTM) (*Achievement Gap Task Force,* 2004) decrying the continuing academic gap. Although common sense suggests the merits of using a culturally and linguistically based approach as a way to increase students' motivation and academic engagement, reviews of educational literature on AI/AN (Demmert & Towner, 2003) fail to make the case; and, as Demmert and Towner noted, the empirical evidence remains quite limited. Similarly, reviews of math education research for both minority groups (Tate, 1997) and mainstream groups shows a paucity of studies that reports statistically significant results (U.S. Department of Education's *What Works Clearinghouse,* 2007).

In December 2005, we edited a special issue of the *Journal of American Indian Education* (Lipka, Parker Webster, & Yanez, 2005) where we presented the results from 14 quasi-experimental studies on the implementation of *Math in a Cultural Context* (MCC), a culturally based math curriculum.[1] The 14 quasi-experimental studies showed statistically significant results and modest to strong effect sizes all in favor of rural Alaska Native treatment groups (MCC) and with one minor exception for one urban trial.

In addition, we have just completed a random experimental design study with 67 teachers and more than 750 students across 10 school districts in urban and rural Alaska. We found unequivocally that the two second-grade modules—the first on representation and measuring and the second on grouping and place value—significantly improved students' math performance, with relatively robust effect sizes (0.82 and 0.39 standard deviations; respectively, both statistically significant at the $p<0.001$ level). This occurred in both rural and urban classes (effect sizes of 1.05 standard deviations for the first module and 0.54 standard deviations for the second module in rural areas and for urban classes effect sizes of 0.83 standard deviations for the first module and 0.35 standard deviations for the second module).

This body of research, 14 quasi- and 2 experimental designs, adds significantly to the literature on culturally based and reform-oriented research for programs that work, meeting Demmert and Towner's (2003) criteria as well as the U.S. Department of Education's *What Works Clearinghouse* criteria for empirical evidence. These studies show that MCC's math curriculum makes a difference in the math performance of AI/AN students and this data shows that the MCC curriculum has the potential for closing the academic gap.

This is critically important, as a number of Alaskan school districts are feeling the pressure of No Child Left Behind (U.S. Department of Education, NCLB, 2002) and the threat of state takeover.[2] These ongoing and increasing pressures place a heavier responsibility on academic researchers to make the case to policy makers, school district decision-makers, and the local communities themselves that culturally based curriculum[3] can both improve AI/AN students' academic performance and support local knowledge, values, and ways.

Because of the increased pressure under NCLB to force school districts to respond in possibly expedient ways to meet "standards," we must not forget the importance of cultural identity, the power of ancestral language, and culture. The

following statement reinforces the importance of cultural identity, language, and cultural knowledge in schooling and also points out how resistance on the part of some members of the school community continues to erect barriers to learning. This statement was recently made by Mary Bavilla, a Yup'ik elder from Togiak, who has worked with the MCC project for many years. Her sentiment provided the impetus for this case. She stated:

> I am going to share why I willingly come to these meetings [working with the MCC project]. I have a grandson who I helped raise and taught in Yup'ik. He spoke only in Yup'ik. We wanted him to go to preschool but he chose to stay home until he was six years old. When he finally went to school, his teacher called [on him in the class] and asked who he was in English. Every time they asked who he was he answered that his name was *Aatassaaq*. One day the teacher asked me and I told the teacher that his English name was Rick [pseudonym]. I noticed that kids that were bilingual learned much faster in school. I started attending these meetings and soon realized that this group did not want to lose our Yup'ik language and our way of life. These people know that a child learns best if he has two languages (M. Bavilla, personal communication, December 1, 2006).

We have presented the results of our empirical and rigorous studies in the December 2005 special issue of the *Journal of Amercian Indian Education,* showing that MCC can improve AN students' academic performance. Therefore, this article responds to how the MCC curriculum series reinforces local knowledge, language, identity, and improves students' academic learning. We present this case study research about an experienced Yup'ik teacher who used the *Patterns and Parkas Module,* another second-grade module in the MCC series.

Background to Developing MCC

Approximately 17 years ago, we began a deliberate process to respond and alter this situation by including Yupiaq Eskimo elders' cultural and linguistic knowledge to schooling. Slowly, we learned how it relates to both academic content and pedagogical practices. Thus we began to adapt elders' knowledge to elementary school math curriculum (see Lipka with Mohatt and the Ciulistet, 1998), resulting in MCC.[4] Central to its development and implementation is a long-term collaborative educational partnership with a group of dedicated Yupiaq elders and teachers, and university educators. We developed this culturally based math curriculum and implementation project based on two complementary goals: to increase Alaska Native students' mathematical understandings and to make the knowledge of Yupiaq elders legitimate within Western schooling. In practical and theoretical terms, our works follows the work of Vygotsky (1978) and the early work of Cole, Glick, & Sharp (1971):

> We want to emphasize our major conclusion that cultural differences in cognition reside more in the situations to which particular cognitive processes are applied than in the existence of a process in one cultural group and its absence in another. Assuming that our goal is to provide an effective education for everyone…our task must be to determine the conditions under

> which various processes are manifested and to develop techniques for seeing that these conditions occur in the appropriate educational setting (p. 233).

Factors suggested by Vygotskian research (Gonzalez, Andrade, Civil, & Moll, 2001) include examining the local culture for socio-cultural conditions that may be applied to schooling. Similarly, recent research by Sternberg, Lipka, Newman, Wildfeuer, & Grigorenko (2007) with MCC points to two conditions: the inclusion of contextual and practical knowledge as a way to enhance academic knowledge. More specifically, expert-apprentice modeling (Lipka & Yanez, 1998) and joint activity (Doherty, Hilberg, Epaloose, & Tharp, 2002) appear to increase student engagement and access to learning when those features are part of the community context. In their earlier study, Lipka and Yanez (1998) describe this relationship of expert and apprentice within a Yupiaq community context. Yanez notes it is the responsibility of the novice to determine when to engage in the activity; while the experts model the culturally authentic activities, the apprentices decide when it is time to produce the activities on their own. In her classroom in a Yupiaq community, Yanez developed expert-apprentice modeling to include more expert students to model for the novice students until the entire class was involved; students assisted peers until all of the students engaged in the task. Joint activity according to Doherty et al., (2002) includes the teacher engaging in the same activity as the students beyond the initial modeling phase.

Similarly, substantial socio-cultural research (Au, 1980; Lipka, 199l; Mohatt & Erickson, 1981; Philips, 1983 emphasized the importance of reconciling differences in communicative norms between the culture of the community and the culture of the school. This research showed the dampening effect of violating the community's communicative norms in instructional practices such as "spotlighting" students, resulting in monosyllabic responses at best. Conversely, several programs embraced the communicative norms of the local community such as having multiple speakers or overlapped speech, and then incorporated these norms into a school's reading program; it appeared to increase student participation and reading outcome measures for Hawaiian (Au, 1980), and Alaska Native students (Brady, 1990).

Gee's (1996) concept of borderlands describes the space where two cultures or linguistic styles meet but co-evolve into a practice that is not strictly either and becomes a new creation. This is theoretically important. Classrooms have the potential for being these "third spaces:" not necessarily those of the dominant culture, nor in a one-to-one correspondence with the local Indigenous or ethnic minority culture. These third spaces have the potential to become productive uncharted zones between school and local cultural knowledge and norms.

From a critical pedagogical perspective, this third space has the potential for changing historically situated authority structures. Once entirely excluded from schooling, Yupiaq language and culture are woven into the culture of schooling through the enactment of MCC. This case also shows how changes in the culture of the classroom alter hierarchical relations between mainstream society and the

local cultural group as well as the authority structures accorded "the one right answer" in school-based mathematics (Alrø & Skovsmose, 2002).

This case study of the implementation of three lessons from the *Patterns and Parkas Module,* one of the second-grade modules in the MCC series, represents a milestone in our work. It highlights possible ways of effectively and authentically connecting the culture and language of the home to the culture of the school. Hermes (2007, pp. 56-73) also highlights this issue. Further, this case has additional significance because it has the explanatory power to describe an effective enactment of the MCC curriculum.

Background to the Case

Despite a hard-won Yupiaq first language of instruction program in the village of Manokotak in southwest Alaska (the case study site) in the late 1980s (Lipka, 1994), the Yupiaq language immersion program now has dwindled into a limited partial immersion program for second language learners of Yupiaq. Manokotak, a community of approximately 420 people, is almost entirely comprised of Yupiaq Eskimo. It is 370 air miles southwest of Anchorage in a roadless part of the state.

Schooling began in Manokotak in the early 1950s. There is one school building in the community and it is a K-12 school with approximately 142 students and a teacher-student ratio of 8.4 (Great Schools, 2007). As in other parts of Alaska, Manokotak is experiencing rapid language loss (Krauss, 1980, 1997); this marks the first time that the children from this region no longer come to school as fluent Yupiaq speakers. Manokotak is the last of the 26 Bristol Bay villages to experience language loss; for the other villages, the shift in language from Yupiaq to English occurred as long ago as 50 or more years and as recently as 13 years (Lipka with Mohatt and the Ciulistet, 1998).

The Manokotak students range from having passive language knowledge to having the ability to speak conversational Yupiaq. The lessons described in this case were conducted in Yupiaq to a group of students who do not display academic language proficiency in Yupiaq (Cummins, 1986). Despite this, the students in Ms. Sharp's class (a limited Yupiaq immersion program) outperformed most other classes on the project's tests as measured by gain scores according to our analysis. Further, the classroom implementation of the *Patterns and Parka* produced one of her higher gains scores while teaching MCC.

What accounts for this improvement in math performance? To examine this research question, we studied how Ms. Sharp developed productive classroom space. This case also explores how a Yupaiq teacher created a third space based on her knowledge of the cultural activity (making parka patterns) that related to the mathematical knowledge of the implemented module.

Methodology

Methods Related to the Overall Project

Specifically, we will examine the enactment of *Math in a Cultural Context* second-grade supplemental math module, *Patterns and Parkas*[5] (Pendergrast,

Lipka, Watt, Gilliland, & Sharp, 2007), in Ms. Sharp's class. Although this case does not describe the quantitative studies and the resulting analyses, it situates this case in the context of the larger research project in which it is embedded. To that end, the quantitative research and the identification of classes in which students made larger than expected gains on project pre- and post-tests was one way in which the quantitative data was used in this case. For purposes of providing additional research, the quantitative design of the larger research project is briefly described below and then the specific qualitative methods of this case.

This project, and to some extent this case, uses both quantitative and qualitative research methods. The quantitative research consists of project developed pre- and post tests within a 2 x 2 block design: urban and rural schools, and treatment and control groups. We also use a quasi-experimental approach to test the effectiveness of the treatment curriculum because teachers who originally volunteered to be in the study were randomly assigned to treatment or control conditions. Subsequently, teachers who wished to continue in the treatment group were allowed. Then, we asked for additional volunteers for both the treatment and control groups. Also, we are particularly interested in teachers who have used the treatment curriculum over multiple trials. We know from our previous research (Lipka & Adams, 2004) and from our experience working with teachers that there is a steep learning curve for many teachers learning both the pedagogical approach of connecting the culture of the community to school math and learning reform-oriented type approach to teaching school math (Kilpatrick, 2003). Thus, to test MCC against experienced users of mainstream math curricula it made sense for us to use a quasi-experimental design.

All Ms. Sharp's students took the project's pre- and post-tests; many items are modeled after standardized test items. Test scores are analyzed two ways: 1) with gain scores using an analysis of variance (ANOVA) and 2) with post-test scores using an analysis of co-variance (ANCOVA) with pre test scores used as the covariate. These two ways of determining the effectiveness of the treatment have confirmed each other to date. The resulting statistical analysis provides data concerning the overall effectiveness of the treatment curricula vs. the mainstream math curricula (curriculum in place at the different sites that we work with).

Methods Related to this Case

We determine which classrooms will be analyzed qualitatively based on one of the following criteria: (a) large class gain scores; (b) high gain scores in "low performing" school districts; and (c) classroom observations that document high student engagement or strong connections to the culture of the community. In this case, Ms. Sharp's class met all the criteria. More specifically for this case, the following 2 x 2 table summarizes the number of teachers and students in the study.

Project classrooms are videotaped, and specific lessons are transcribed. Classroom discourse and analysis of videotapes are done using a video analysis tool, *Transana* (Fassnacht, & Woods, 2005). We analyze both the discourse and

Table 1
Number of Teachers and Students

	Urban		Rural	
	# Teachers	**# Students**	**# Teachers**	**# Students**
Treatment	3	63	11	59
Control	2	35	6	45

the classroom dynamics, paying attention to how the teacher coordinates her words and her actions; how she brings in cultural ways of teaching and adjusts to the school environment; and how and what students respond to. More specifically, we paid attention to the ways in which this experienced Yupiaq language teacher created classroom space in which she uses the culture and language of the community to authentically and creatively teach math and Yupiaq language and culture in a school context. From our past work, we look for her use of expert-apprentice modeling (Lipka & Yanez, 1998) and from the work of Doherty, Hilberg, Epaloose, and Tharp (2002) we look for joint activity.

More specifically, three videotaped classroom lessons observed during the spring of 2004 constitute one data source for this case. Further, interviews with Ms. Sharp occurred periodically between March 2004 and October 2004. In addition, student artifacts were collected. These different data sources provide the basis for examining classroom interactions. Importantly, videotaped segments were shown to both groups of insiders and outsiders for varying perspectives and building a sense of the case to more fully understand the unique blend of cultures that it represents. Presenting the case at international conferences with Ferdinand and Nancy Sharp contributed to our understanding of the material.

The Case: Creation of a Third Space

Ms. Sharp created a classroom environment in which she made authentic and deeply emotional familial, cultural, and linguistic connections, while she simultaneously embedded important school math concepts through the activities of this module, *Patterns and Parkas*. Our case study describes this process.

In the sections below, we describe one example from our work with Yupiaq elders—how they make parka border pattern pieces. The example from the elder incorporates cultural and linguistic knowledge that is applied to schooling through the MCC curriculum. We describe some of the embedded mathematics that the elders employ as they make patterns. The case is a detailed description of Ms. Sharp's implementation of this curriculum taught in Yupiaq to emerging learners of Yupiaq. We provide an analysis of her classroom highlighting the emerging third space between knowing and doing in the community to effective pedagogical (school) practice, expert-apprentice modeling and joint activity. We describe and analyze how she teaches in Yupiaq connecting her cultural and linguistic knowledge of making parka patterns to school based mathematical knowledge of geometric shapes and relations. Lastly, we conclude with implications for others in the field of bilingual/bicultural education.

Curriculum Development: Based on Putting Pieces Together [*Tumaqcat*]
We have worked with elders in a variety of situations and contexts and have explored a range of subsistence-related activities from star navigating to making women's clothing (see Lipka & Yanez, 1998 for a fuller description; Lipka, Wildfeuer, Wahlberg, George, Ezran, 2001; and Lipka, Parker Webster, & Yanez, 2005). Many of the everyday Yupiaq activities relate well to Bishop's (1994) conception of math across cultures. They include measuring, designing, counting, and navigating/locating, playing, and explaining (Bishop, 1994). Based on these math topics and Yupiaq cultural activities, we developed a series of supplemental math curriculum for elementary school students in Alaska. These supplemental math curricula modules take approximately six weeks to teach. The curricula based on the elders' knowledge often includes accompanying stories that connect the cultural context and the specific cultural activity, the theme of each module such as berry picking, pattern making, and others, to specific mathematical topics such as data collection and analysis, geometric patterns, and numeration (See Parker Webster & Yanez, 2007, pp. 120-135).

Charles, a professor at the University of Alaska Fairbanks in the Alaska Native Language Center, spoke poetically about Yup'ik patterns:

> *Tumartat* is the gathering of pieces of anything and putting them together to make a whole. We look at what we sew—pieces of different types of furs, for instance—and we put them together to create one whole. We look at our patterns; different shapes are made and sewn together to create a whole. Everything around us is a pattern that creates our whole being (Pendergrast, Lipka, Watt, Gilliland, & Sharp, 2007).

Accordingly, this case highlights the authentic putting together of Yupiaq cultural and linguistic knowledge, seamlessly sewn to achieve increased academic performance in mathematics.

Patterns and Parkas Module: A specific example of everyday geometry and its potential connections to school mathematics. During the course of the past few years, we met with a small group of female Yupiaq elders. They were teaching us how they each make their border patterns (typically applied to a women's dress). In particular, we were interested in how they created their starting piece from non-uniform material. On these occasions, we observed the elder while she completed the process. The elders did not discuss each step as they worked. However when the elders completed their work we interviewed them and often had them repeat the process until we were able to make the same pattern. At these times, we also videotaped the process to ensure that it was carefully recorded and available for further reference. It is important to note that different elders choose different starting pieces and they have different ways of making their pieces. Below is an example from Winifred Beans, an elder from the village of St. Mary's, Alaska.

As we observed Ms. Beans, she took a rectangular-shaped piece of cardboard and folded the cardboard lengthwise and then she folded the cardboard in half again width-wise (see Figure 1). She held the folded rectangle, now a

quarter of its original size, and proceeded to cut across the diagonal so that the outside of the folded board fell away after the cut (see step #3 in Figure 1).

She unfolded her paper and what remained was a rhombus. This rhombus was her core piece for a parka border pattern. As Ms. Beans worked in this fashion, we noticed that each piece she made was geometrically related to other pieces. She arranged and rearranged the pieces from her set of different geometric shapes into a pleasing and symmetrically repeating pattern. By starting with the rectangle from which the rhombus was folded and cut out, and finding the various ways to combine these pieces, one can create a set of related pieces that can be used to make symmetrical patterns for squirrel parkas.

Different elders in different communities choose different pieces for their "core piece," such as square, rectangle, rhombus, and triangle. Therefore, each person's pattern set differs. Further, Ms. Beans and other elders estimated the space that they had to work with on the size of the parka border; so that when the

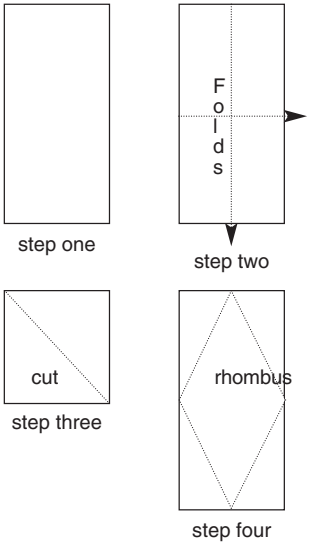

Figure 1. Creating a rhombus from a rectangle.

pieces are joined to the material, the starting and ending pieces form a continuous pattern. This process embeds both the mathematics of geometrical relationships (also estimating and filling space aesthetically) and expert-apprentice modeling as a form of teaching and communicating that we have adapted to the math MCC.

Math in a cultural context: Adapting the elders' knowledge to school math. Making parka pattern pieces has the potential to enrich the math curriculum. In our approach, students create their own patterns, often similar to those used in their community. Through this process of making pattern sets, the students gain insight into geometric relationships. This process of students making patterns differs from

Figure 2. Annie Blue in her parka.

using already made, consumer-oriented pattern pieces, which skip the important step of observing how the geometric shapes are related to each other. Further, this activity includes both spatial observations and manipulations connected directly to spatial reasoning and geometry. Although elders would not talk about their work in geometrical terms, they do show a "proof" of a rectangle (or a square) while they engage in paper folding. Elders use the Yup'ik language to describe the pieces, their symbolic meaning, and processes. They pay attention to symmetry as they cut out pieces and as they make their pattern strips and apply their rules for balance which differs from Western conceptions of symmetry. Balance is achieved through balancing black and white pieces throughout the design.

Mathematically, through spatial manipulations an elder can create parallelograms, trapezoids, triangles, and rectangles (including the square) from the rhombus and the pieces cut out from the rectangle (the triangles) and students in this lesson emulate elders and their Yup'ik teacher. Thus, this elegant example of Ms. Beans' pattern-making shows some ways in which traditional activity can be related to a Western mathematics, in particular geometry.

The classroom lesson shown below was conducted in Yupiaq. Because the pattern pieces within the Yupiaq culture have certain meanings, and, in part, the pattern pieces tell a story, Ms. Sharp takes advantage of the opportunity to convey Yupiaq cultural knowledge as well as mathematical knowledge. In part, this case tries to determine: what it is about her implementation of this module that could be associated with the students' relatively strong academic performance. Below, we show excerpts from three lessons in the *Patterns and Parkas Module* in Ms. Sharp's classroom; with some elders, the square is the starting piece.

The Case: Connecting Yupiaq cultural knowledge to teaching second-grade math. We join the classroom as students are creating parka pattern pieces. The students observe actual Yupiaq parkas and displays of patterns that Ms. Sharp has brought to class (see Figure 3 below) as a way for them to model their pieces toward a finished product.

The P*atterns and Parkas* curricula asked students to make their own pattern pieces. Ms. Sharp encourages her students to make their own unique sets. Figure 4 shows a student completing her pattern set. Each piece was made from a larger square. Some students cut their squares diagonally forming two right triangles, while some other students cut their squares into two or three rectangles or nine small squares. Other pieces are possible as well. The method for cutting pieces follows how some Yupiaq elders make their pieces.

The next day the lesson continues. Students are told that they will be making a bookmark on 1?-inch wide tag paper. Previously, they started from 3-inch square pieces. Before the students make their bookmarks Ms. Sharp spends time modeling how to do this task while simultaneously coordinating her actions such as folding paper or rearranging pieces to highlight the mathematic concepts. Below is a brief sample of this lesson. The translated discourse (from Yup'ik to English) is presented in the left-hand column, and in the right-hand column are the descriptions of physical activity and other classroom related information as well as classroom pictures. Ms. Sharp will be referred to as "T" throughout the transcription.

Figure 3. Ms. Sharp displays a Yupiaq parka.

Figure 4. Student creates her pattern set.

Classroom Discourse (Translated and transcribed from Yupiaq)	**Description of Activity**
1. T: Remember we were working on patterns yesterday? These here are patterns. Patterns. Now, say it. Students: Patterns. 2. T: These are patterns. Patterns. Yes, mother made these. Do you see these? What are they? Students: Patterns. 3. T: No, what are these? 4. Student: A square. 5. T: A square. What about this?	Time on the tape 3:45. Ms. Sharp holds up a pattern strip and shows it to the students. She points to a square shape within the pattern strip and asks the students, "What is this?" The students respond in line 4. This process continues as she points to different patterns and shapes within the strip.

Putting the Pieces Together

The discourse for this part of the lesson is reminiscent of a Western-style of teaching, direct naming of the pieces, calling on students, and the students' choral response (Cazden, 1988), but it takes place in Yup'ik. Further, Ms. Sharp coordinates each of her questions with the physical movement of her hands as she points to parts of the parka strip. More importantly, in line two she references her mother, thus personalizing and situating this school activity within this community context and the wider Yupiaq cultural context. In this opening frame Ms. Sharp makes multiple connections between the Yup'ik language and culture and its use outside of school and to the teaching of math, making patterns and geometrical patterns.

The lesson continues on in this fashion with Ms. Sharp taking different parka pieces out of a plastic bag. She holds up different parka pieces for the students to see and tells them the Yupiaq name. The students repeat the name. Once in awhile a student may say the name without prompting. In a few minutes, the lesson is resumed again; the transcription and description follow.

1. T: Only a rectangle and a triangle. And this is how she saved them, put together the two triangles. And these, I think, these are the same size, 3. If you put these together what will it become? 4. S: A rectangle? 5. S: A square, a triangle. 6. T: A square. And it can be a triangle. Then here is a square. It is like this when you do this. [a few minutes later] 7. T: Because this is like this, it is [symmetrical]. Cut it. First fold it and when it was cut, 12. T: Now, how does it go [conservation area], it is [not wasting]. It is still used. It can become a square and it is [symmetrical]. 13. T: Do you understand? Where did I get these? I got them from here. I tried to put the same ones 14. Together. [Okay], What is this? What are these? 15. S: A triangle. 16. T: A triangle. What is the name of it?	Time of the videotape: 6:26 Ms. Sharp takes two right triangle pieces and puts them together forming a square and then separates it. She coordinates her physical movements of showing the students how to arrange and re-arrange pattern pieces while she names what she is doing. She continues this process and relates the concept of symmetry (line 7). She physically shows the students that the square formed by the two right triangles has a line of symmetry. Similarly, she talks about conservation of area (line 12) and that rearranging pieces does not change its area but just its shape (two right triangles can make a square and a parallelogram). The concept of conservation in this instance is also related to theYupiaq value of not wasting resources (line 12-13).

Introducing geometry connecting to Yupiaq cultural activities. This phase of the lesson concentrates on naming the different geometric shapes in Yup'ik and how these different geometrical pieces are related to each other. For example, Ms Sharp puts two black right triangle pieces together in her hand forming a square and asks the students what they see. They respond "a square" (in Yupiaq). Then, with her finger she draws an imaginary line dividing the square along the diagonal, creating lines of symmetry, and she separates them into two right triangles. She rearranges the right triangular pieces into another shape and demonstrates that these same pieces—two right triangles and the square that was

formed—have the same area. Simultaneously, she connects the mathematical part of the lesson to the unfinished work of her mother by both showing the students unfinished strips and how they can use "spare" pieces so that they are not wasting. The cultural reasons and rules for using black and white pieces in making Yupiaq pattern pieces becomes more evident in the next scene.

17. T: [No], you know, over there in the [chart], what do they imitate? What are they called? What are these pretending to be (pointing to the triangle)? Pretend mountains. 18. T: Now say it. Pretend mountains. 19. S: Pretend mountains. 20. T: They can be pretend mountains this way, in the winter. It is dark up there [the sky]. They can pretend to be mountains in the summer. 21. T: Do you understand? During the summer. 22. T: They could be pretend mountains either way. Whoever is sewing them can decide what she wants. 26: T: I want you to see different kinds. These are also patterns, the squares. I think, they belong to this one, right? Do they look like this?	Ms. Sharp shows the students a pattern strip and they name it according to the geometrical figures and what they represent in the Yupiaq culture such as "pretend mountains." Next, she rotates the pattern so that the white triangles are on the bottom and in Yupiaq the name of this pattern changes to more of a winter scene. This possibly indicates the importance of orientation within a Yupiaq frame of reference, as well as it references math content to the students' and teacher's cultural background (lines 17-22) Mrs. Sharp's presentation of triangles in a pattern strip as a unique form called "pretend mountains" creates a second set of references for the students, one that is possibly more familiar to them. In this third space, they do not only learn of new math concepts, but are also are relating them to familiar elements available in their environment. Lines 22-26 show a distinct teaching pattern. Ms. Sharp directs the students' attention to the different pieces and looks for different ways to utilize them. Also, she acknowledges that the outcome will depend upon the person sowing. At this point, Ms. Sharp is becomes an expert or a leader. Her apprentices or students will learn alongside.

Connecting geometrical relations to Yupiaq cultural artifacts and language. In this phase of the lesson, Ms. Sharp makes a transition from the naming of geometric pieces to how they can be rotated to make other pieces to a pattern strip and what it means within a Yupiaq cultural and linguistic frame. At first, she asks the students, "What is this?" In line 17, she switches their frame of reference to what the pattern strip (black and white triangles) means in Yupiaq *[ingriruat*—pretend mountains]. Some students respond in Yupiaq by naming the Yupiaq pattern design. She further clarifies some of the cultural rules for naming the pattern pieces as she refers to darkness for summer and whiteness for winter as well as the individuality that each sewer brings to this task. Similarly, the rule for having black pieces aligned to white pieces to form a characteristically Yup'ik motif is taught through modeling and displaying examples of strip patterns.

A few seconds later she places a black right triangle next to the triangular strip pattern (pretend mountain) and forms a square. She reinforces the notion of conservation of area while a few students also state this same phrase at the same time.

Then she takes the black right triangles and forms a parallelogram, and again the students chorally name it. (See figure 5 below.) She shows them a strip and points out the parallelogram. She uses her hand to show how she would cut along a line of symmetry to cut the right triangle and then rotate these pieces to form a parallelogram. Further, she points out one side of the parallelogram and runs her finger along an edge. Then, she does the same to the opposite parallel side, providing the students with a clear visual clue to why this geometric shape is called a parallelogram. Similarly, she demonstrated the concept of congruency by placing two right triangular pieces on top of each other. One student supplied the Yupiaq word for this concept.

She holds up the pattern strip and points out the geometric shapes: triangle, square, rectangle, and parallelogram. She shows the strip and identifies a parka pattern commonly used in this type of work; in Yupiaq it is called *egaleruat* [for pretend windows]. This pattern symbolizes the window in a house.

She hangs the various pattern strips in the front of the room so that the students can see them as they begin to make their bookmarks. During this time the students remain attentive. The challenges for the students include making their

Figure 5. Ms. Sharp creates a parallelogram.

own pattern pieces that will also fit within a two-inch wide strip although their starting piece is a three-inch square.

A new lesson phase is reached when she begins more direct modeling of the activity. This phase begins at 21:25 minutes into the lesson. We rejoin the lesson as the expert-apprentice modeling phase begins.

1. T: You guys think about what you want to use. Are you going to use white or black? You are going to make a [bookmark]. Now you guys, remember I gave you this kind yesterday or this kind (holding up black and white construction paper)? Now listen first. Are we going to quickly cut them up like this?	This is the 'final' stage of the demonstration and the beginning of the expert-apprentice modeling. Ms. Sharp begins to have different students show that they know the process of working with their starting piece the square. The students also show how to fold the paper so that they can derive other pieces. She makes sure that they can figure out ways to make pieces that will fit on the two inch strip.
2. S: [Nope.]	
3. T: But how?	
4. S: Triangle.	
5. T: Triangle, after doing what?	
6. S: Fold it.	
7. S: A long square.	
8. T: How are you going to make triangles? (A kid points to something). No, first what are you going to do? First make a line? Who else can make a triangle in a different way? How are you going to do it? You are not going to quickly cut it. All one size? Look, can a big one be able to go here?	The student demonstrates with her hand how she would cut it. She draws or places an imaginary cut along the diagonal of the square as the other students in the background watch what she is doing.
9. S: [No] a small one.	When the first student completed her demonstration, Ms. Sharp asked "who else can make a triangle?" Another student volunteers and demonstrates. She begins by folding the square in half and stops. Ms. Sharp takes the piece from her and shows her that it will be too big for the 2 inch strip and returns it to her. The student folds the square along the diagonal and folds it again. Now it will fit. As this is occurring Ms. Sharp again asks for another volunteer. The rest of the class observes these students modeling.
10. S: [No, like this.]	
11. T: And then how else? Will that be good if you were going to put it here?	
12. S: [Yeah.]	
13. T: [Yeah] If you want to you can make them that size, right? Can they be this big?	
14. S: [Yeah.]	
15. T: [Yeah], and you can make pretend mountains. Do you understand? All of you are not going to make pretend mountains. You can make different kinds.	

Joint Activity

After Ms. Sharp demonstrated the process, she had the students demonstrate the process of making pattern pieces that will fit within the two-inch strip. She handed out the strips and the black and white three-inch squares. This occurs at approximately 27 minutes into this videotape. The next and final phase of the lesson takes place as students return to their desks to work. She does not directly organize the students into working groups. The students organize themselves. Some students choose to work alone, while others work in pairs or groups of three. She does not at this time tell students to "help each other" but it is clear from the videotape that students are assisting each other. Further, she does not tell them how to proceed with the task, but the students organize themselves.

As seen below in Figure 6, Ms. Sharp stays at the table and continues to do her own work. Joint activity (she has her own work) differs significantly from typical classroom teaching structures (Cazden, 1988) in which students' work is constantly monitored. At the end of the expert-modeling phase of the lesson (while the students were observing her work), she was trying to make a particular pattern piece, but through the video you can see that she had some difficulty. Once the students were involved in the task, one of the first things Ms. Sharp did was repair the pattern piece she was trying to make for her bookmark. In joint activity students have the opportunity to observe the expert make a mistake and observe how the expert problem solves.

Ms. Sharp continues to work on her bookmark while students continue to work in small groups assisting each other as needed for another 20 minutes. Students help each other and visit her to observe or ask for help. She does not micro-manage or visit each student and observe or evaluate their work. She finishes her own work.

She has substantially altered the social relations and authority structure of the classroom during this phase of the lesson. The students, not the teacher, take

Figure 6. Joint activity as Ms. Sharp works along with her students.

responsibility for their work. The students, not the teacher, decide when they need help and they either ask for assistance or they observe a peer. A peer may also assist them without invitation, or the students may observe Ms. Sharp's work. The videotaped lesson shows the students helping each other and observing Ms. Sharp's work. See Figure six shown above where Ms. Sharp works on her bookmark while simultaneously students work on their bookmarks. Although she has made classroom social relations more level, she remains the expert concerning the content of the lesson and the authority concerning classroom management. She does, however, on occasion work with students who choose to work at her table.

This is an important phase of the lesson as her pedagogical style has shifted toward the culture of the community (Lipka & Yanez, 1998). In an interview, Ms. Sharp said,

> When I was going to school they would always give us a finished product. Then they would let us make one like that. Usually I haven't seen a teacher do it along with the kids...On the other hand, the Yupiaq have a finished product and they would show that. Then they would…go along and make one together (interview May 31, 2004).

This notion of joint activity, we feel, is part of the creation of a third space—a way to put together Yupiaq and Western pedagogy and the teaching of Yupiaq culture, language, and values, and, in this case, mathematical knowledge. In fact, this cultural and linguistically-based activity of pattern making and the module's emphasis on geometrical relationships creates an integral and authentic connection between the mathematics and social-cultural norms embedded within Yupiaq culture and language and school-based mathematics.

Co-author Lipka has observed Ms. Sharp teach in classrooms, off and on, from 1981 until the present, either in person or through videotaped lessons. In all of this time, this was the first time she was observed organizing her class in a way that so deeply resonated with her local culture and language. It appears that the *Parka and Patterns Module* and its emphasis on making patterns, and possibly her increased experience with the pedagogical and mathematical approach of MCC, contributed to her students' relatively high gain scores.

Evolving an Authentic Biculturalism and Bilingualism in the Third Space

The case example shows the successful implementation of a culturally based math module taught through the heritage language using authentic products and processes from the home culture in the classroom. These pedagogical processes were not in one-to-one correspondence with learning in the community, but were creatively adapted by Ms. Sharp to fit the dual goals of teaching in a Western institution (schooling) yet transmitting Yupiaq language and culture. Although the project did not "measure" students' increasing knowledge of Yupiaq language and culture, it is clear from the transcripts and the videotaped lesson that, in fact, these were occurring. Again, we believe that her comfort with the core community activity, making parka patterns, and her deep commitment to bringing the language and the culture of the community into her classroom, made her more

comfortable and open to explore the mathematical relationships embedded within this cultural activity. It is for these reasons that we believe that her students showed relatively strong gain scores.

Simultaneously, she met the other key goal of this project, that of cultural and linguistic continuity; meeting the original goals of this project. Henry Alakayak, an elder who began this work almost twenty years ago stated:

> We shared about our way of life as Yupiaq people [to this project]. The reason we shared was for our young people, so they would not forget our way of life and our language. We gave each of the people who worked with us time to share their knowledge, then the teachers figured out what would be useful in the classrooms. By listening to them, I learned, and having the knowledge now that I learned from the group, I am able to share what I now know. Now that the group knows what they want, we add to the modules that they work on…Things that we had in the past should not be forgotten. As an elder I can share what I know… A heartfelt thanks to everyone especially the people who speak in our language and in the other language. As people who are limited in the language department, we are grateful to have people who speak in two languages. These people know who to pick as helpers for the project (H. Alakayak, personal communication, December 1, 2006).

Mrs. Bavilla added:

> I started attending these meetings and soon realized that this group did not want to lose our Yup'ik language and our way of life. These people know that a child learns best if he has two languages (M. Bavilla, personal communication, December 1, 2006).

This case has highlighted how MCC has authentically connected the culture and language of the community while simultaneously meeting the goal of improving AN students' math performance.

We feel that Ms. Sharp's classroom represents a slowly evolving pedagogical and content form that is both culturally and linguistically authentic on the Yupiaq side and mathematically challenging and appropriate on the Western side. Such an evolutionary process derived from community activity and included in school-based math curriculum may well hold promise for improving the academic performance of Yupiaq students

Conclusions

We believe that this case, and the larger body of research around MCC, provides a counter argument to the pressures that school districts now face under NCLB to meet "academic standards" at the expense of Indigenous language, culture, and identity. Schooling does not have to an either or choice. These statements have been made by many others over many years (Institute for Government Research, also know as the Meriam Report, 1928; Napoleon, 1991); however, publications such as this special issue of the *Journal of American Indian Education* showcase exemplars of how Indigenous language and culture can effectively meet both academic and cultural/linguistic goals. We hope that this case provides an impetus to resist current pressures to revert back to basics.

Jerry Lipka has 24 years of experience in the Alaskan context and has written extensively on the subject of culturally based education. He is the senior editor of *Math in a Cultural Context,* a supplemental elementary school math series based on Yup'ik elders knowledge. He has published extensively using an ethnographically oriented approach.

Ferdinand Sharp has been a consultant for *Math in a Cultural Context* and other related projects for many years. His principal role has been supporting elders and their knowledge. He has actively worked to ensure that their knowledge is included in the math modules.

Barbara Adams is an assistant professor of education at the University of Alaska Fairbanks with a background in mathematics. She is the math editor of MCC and has worked on developing culturally based curriculum, assessing its effectiveness in classrooms and conducting educational research studies for the past five years.

Nancy (Nanugaq) Sharp has been a teacher since 1985. In her school, she is the Yupiaq Teacher and speaks the language fluently. Nanugaq She has deep respect for the elders and has been working with them as a bridge to the outside world for students since she began teaching. She continues to teach with high expectation for her students to survive in two worlds.

ENDNOTES

[1]*Math in a Cultural Context* (MCC). MCC is a long-term collaborative project with Yup'ik elders, mathematicians, math educators, educators, and a number of Alaskan school districts

[2]This is based on recent personal communications with a number of Alaskan superintendents and curriculum coordinators.

[3]By culturally based curriculum we mean curriculum that is, at least in part, developed from the perspective of the indigenous group. All curriculum is culturally based. The key question is: Whose culture is it based on?

[4]*Math in a Cultural Context* is a culturally based math series based on Yup'ik Eskimo elders' knowledge. This project was funded by NSF (grant award #9618099) and more recently supported by the U.S. Department of Education (grant award #S56A03003 and grant award #R306N010012).

[5]The Patterns and Parkas is a draft module developed for the *Math in a Cultural Context* series.

[6]Yup'ik was the typical way that this indigenous group referred to their language and identity, however, more recently this becomes interchangeable with Yupiaq.

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