

THE REGENERATED LOGIC.

THE appearance of Schroeder's *Exact Logic*¹ has afforded much gratification to all those homely thinkers who deem the common practice of designating propositions as "unquestionable," "undoubtedly true," "beyond dispute," etc., which are known to the writer who so designates them to be doubted, or perhaps even to be disputed, by persons who with good mental capacities have spent ten or more years of earnest endeavor in fitting themselves to judge of matters such as those to which the propositions in question relate, to be no less heinous an act than a trifling with veracity, and who opine that questions of logic ought *not* to be decided upon philosophical principles, but on the contrary, that questions of philosophy ought to be decided upon logical principles, these having been themselves settled upon principles derived from the only science in which there has never been a prolonged dispute relating to the proper objects of that science. Among those homely thinkers the writer of this review is content to be classed.

Why should we be so much gratified by the appearance of a single book? Do we anticipate that this work is to convince the philosophical world? By no means; because we well know that prevalent philosophical opinions are not formed upon the above principles, nor upon any approach to them. A recent little paper by an eminent psychologist concludes with the remark that the ver-

¹ *Vorlesungen über die Algebra der Logik* (Exakte Logik). Von Dr. Ernst Schröder, Ord. Professor der Mathematik an der technischen Hochschule zu Karlsruhe in Baden. Dritter Band. *Algebra und Logik der Relative*. Leipsic: B. G. Teubner. 1895. Price, 16 M.

dict of a majority of four of a jury, provided the individual members would form their judgments independently, would have greater probability of being true than the unanimous verdict now is. Certainly, this may be assented to ; for the present verdict is not so much an opinion as a resultant of psychical and physical forces. But the remark seemed to me a pretty large concession from a man imbued with the idea of the value of modern opinion about philosophical questions formed according to that scientific method which the Germans and their admirers regard as the method of modern science,—I mean, that method which puts great stress upon co-operation and solidarity of research even in the early stages of a branch of science, when independence of thought is the wholesome attitude, and gregarious thought is really sure to be wrong. For, as regards the verdict of German *university professors*, which, excepting at epochs of transition, has always presented a tolerable approach to unanimity upon the greater part of fundamental questions, it has always been made up as nearly as possible in the same way that the verdict of a jury is made up. Psychical forces, such as the spirit of the age, early inculcations, the spirit of loyal discipline in the general body, and that power by virtue of which one man bears down another in a negotiation, together with such physical forces as those of hunger and cold, are the forces which are mainly operative in bringing these philosophers into line ; and none of these forces have any direct relation to reason. Now, these men write the larger number of those books which are so thorough and solid that every serious inquirer feels that he is obliged to read them ; and his time is so engrossed by their perusal that his mind has not the leisure to digest their ideas and to reject them. Besides, he is somewhat overawed by their learning and thoroughness. This is the way in which certain opinions—or rather a certain verdict—becomes prevalent among philosophical thinkers everywhere ; and reason takes hardly the leading part in the performance. It is true, that from time to time, this prevalent verdict becomes altered, in consequence of its being in too violent opposition with the changed spirit of the age ; and the logic of history will usually cause such a change to be an advance toward truth in some respect. But this process is so slow, that it

is not to be expected that any rational opinion about logic will become prevalent among philosophers within a generation, at least.

Nevertheless, hereafter, the man who sets up to be a logician without having gone carefully through Schroeder's Logic will be tormented by the burning brand of *false pretender* in his conscience, until he has performed that task; and that task he cannot perform without acquiring habits of exact thinking which shall render the most of the absurdities which have hitherto been scattered over even the best of the German treatises upon logic impossible for him. Some amelioration of future treatises, therefore, though it will leave enough that is absurd, is to be expected; but it is not to be expected that those who form their opinions about logic or philosophy rationally, and therefore not gregariously, will ever comprise the majority even of philosophers. But opinions thus formed, and among such those formed by thoroughly informed and educated minds, are the only ones which need cause the homely thinker any misgiving concerning his own.

It is a remarkable historical fact that there is a branch of science in which there has never been a prolonged dispute concerning the proper objects of that science. It is the mathematics. Mistakes in mathematics occur not infrequently, and not being detected give rise to false doctrine, which may continue a long time. Thus, a mistake in the evaluation of a definite integral by Laplace, in his *Mécanique céleste*, led to an erroneous doctrine about the motion of the moon which remained undetected for nearly half a century. But after the question had once been raised, all dispute was brought to a close within a year. So, several demonstrations in the first book of Euclid, notably that of the 16th proposition, are vitiated by the erroneous assumption that a part is necessarily less than its whole. These remained undetected until after the theory of the non-Euclidean geometry had been completely worked out; but since that time, no mathematician has defended them; nor could any competent mathematician do so, in view of Georg Cantor's, or even of Cauchy's discoveries. Incessant disputations have, indeed, been kept up by a horde of undisciplined minds about quadratures, cyclotomy, the theory of parallels, rotation, attraction, etc. But the disputants

are one and all men who cannot discuss any mathematical problem without betraying their want of mathematical power and their gross ignorance of mathematics at every step. Again, there have been prolonged disputes among real mathematicians concerning questions which were not mathematical or which had not been put into mathematical form. Instances of the former class are the old dispute about the measure of force, and that lately active concerning the number of constants of an elastic body; and there have been sundry such disputes about mathematical physics and probabilities. Instances of the latter class are the disputes about the validity of reasonings concerning divergent series, imaginaries, and infinitesimals. But the fact remains that concerning strictly mathematical questions, and among mathematicians who could be considered at all competent, there has never been a single prolonged dispute.

It does not seem worth while to run through the history of science for the sake of the easy demonstration that there is no other extensive branch of knowledge of which the same can be said.

Nor is the reason for this immunity of mathematics far to seek. It arises from the fact that the objects which the mathematician observes and to which his conclusions relate are objects of his mind's own creation. Hence, although his proceeding is not infallible,—which is shown by the comparative frequency with which mistakes are committed and allowed,—yet it is so easy to repeat the inductions upon new instances, which can be created at pleasure, and extreme cases can so readily be found by which to test the accuracy of the processes, that when attention has once been directed to a process of reasoning suspected of being faulty, it is soon put beyond all dispute either as correct or as incorrect.

Hence, we homely thinkers believe that, considering the immense amount of disputation there has always been concerning the doctrines of logic, and especially concerning those which would otherwise be applicable to settle disputes concerning the accuracy of reasonings in metaphysics, the safest way is to appeal for our logical principles to the science of mathematics, where error can only long go unexploded on condition of its not being suspected.

This double assertion, first, that logic ought to draw upon

mathematics for control of disputed principles, and second that ontological philosophy ought in like manner to draw upon logic, is a case under a general assertion which was made by Auguste Comte, namely, that the sciences may be arranged in a series with reference to the abstractness of their objects; and that each science draws regulating principles from those superior to it in abstractness, while drawing data for its inductions from the sciences inferior to it in abstractness. So far as the sciences can be arranged in such a scale, these relationships must hold good. For if anything is true of a whole genus of objects, this truth may be adopted as a principle in studying every species of that genus. While whatever is true of a species will form a datum for the discovery of the wider truth which holds of the whole genus. Substantially the following scheme of the sciences is given in the *Century Dictionary*:

MATHEMATICS	
Philosophy	{ Logic Metaphysics.
Science of Time	Geometry
Nomological Psychics	Nomological Physics { Molar Molecular Ethereal
Classificatory Psychics	Classificatory Physics { Chemistry Biology, or the chemistry of protoplasms
Descriptive Psychics	Descriptive Physics
PRACTICAL SCIENCE.	

Perhaps each psychical branch ought to be placed above the corresponding physical branch. However, only the first three branches concern us here.

Mathematics is the most abstract of all the sciences. For it makes no external observations, nor asserts anything as a real fact. When the mathematician deals with facts, they become for him mere "hypotheses"; for with their truth he refuses to concern himself. The whole science of mathematics is a science of hypotheses; so that nothing could be more completely abstracted from concrete reality. Philosophy is not quite so abstract. For though it makes no *special* observations, as every other positive science does, yet it does deal with reality. It confines itself, however, to the universal

phenomena of experience ; and these are, generally speaking, sufficiently revealed in the ordinary observations of every-day life. I would even grant that philosophy, in the strictest sense, confines itself to such observations as *must* be open to every intelligence which can learn from experience. Here and there, however, metaphysics avails itself of one of the grander generalisations of physics, or more often of psychics, not as a governing principle, but as a mere datum for a still more sweeping generalisation. But logic is much more abstract even than metaphysics. For it does not concern itself with any facts not implied in the supposition of an unlimited applicability of language.

Mathematics is not a positive science ; for the mathematician holds himself free to say that A is B or that A is not B , the only obligation upon him being, that as long as he says A is B , he is to hold to it, consistently. But logic begins to be a positive science ; since there are some things in regard to which the logician is not free to suppose that they are or are not ; but acknowledges a compulsion upon him to assert the one and deny the other. Thus, the logician is forced by positive observation to admit that there is such a thing as doubt, that some propositions are false, etc. But with this compulsion comes a corresponding responsibility upon him not to admit anything which he is not forced to admit.

Logic may be defined as the science of the laws of the stable establishment of beliefs. Then, *exact* logic will be that doctrine of the conditions of establishment of stable belief which rests upon perfectly undoubted observations and upon mathematical, that is, upon *diagrammatical*, or, *iconic*, thought. We, who are sectaries of "exact" logic, and of "exact" philosophy, in general, maintain that those who follow such methods will, so far as they follow them, escape all error except such as will be speedily corrected after it is once suspected. For example, the opinions of Professor Schröder and of the present writer diverge as much as those of two "exact" logicians well can ; and yet, I think, either of us would acknowledge that, however serious he may hold the errors of the other to be, those errors are, in the first place, trifling in comparison with the original and definite advance which their author has, by the

"exact" method, been able to make in logic, that in the second place, they are trifling as compared with the errors, obscurities, and negative faults of any of those who do not follow that method, and in the third place, that they are chiefly, if not wholly, due to their author not having found a way to the application of diagrammatical thought to the particular department of logic in which they occur.

"Exact" logic, in its widest sense, will (as I apprehend) consist of three parts. For it will be necessary, first of all, to study those properties of beliefs which belong to them as beliefs, irrespective of their stability. This will amount to what Duns Scotus called *speculative grammar*. For it must analyse an assertion into its essential elements, independently of the structure of the language in which it may happen to be expressed. It will also divide assertions into categories according to their essential differences. The second part will consider to what conditions an assertion must conform in order that it may correspond to the "reality," that is, in order that the belief it expresses may be stable. This is what is more particularly understood by the word *logic*. It must consider, first, *necessary*, and second, *probable* reasoning. Thirdly, the general doctrine must embrace the study of those general conditions under which a problem presents itself for solution and those under which one question leads on to another. As this completes a triad of studies, or trivium, we might, not inappropriately, term the last study *Speculative rhetoric*. This division was proposed in 1867 by me, but I have often designated this third part as *objective logic*.

Dr. Schröder's Logic is not intended to cover all this ground. It is not, indeed, as yet complete; and over five hundred pages may be expected yet to appear. But of the seventeen hundred and sixty-six pages which are now before the public, only an introduction of one hundred and twenty-five pages rapidly examines the speculative grammar, while all the rest, together with all that is promised, is restricted to the deductive branch of logic proper. By the phrase "exact logic" upon his title-page, he means logic treated algebraically. Although such treatment is an aid to exact logic, as defined on the last page, it is certainly not synonymous with it. The principal utility of the algebraic treatment is stated

by him with admirable terseness: it is "to set this discipline free from the fetters in which language, by force of custom, has bound the human mind." Upon the algebra may, however, be based a calculus, by the aid of which we may in certain difficult problems facilitate the drawing of accurate conclusions. A number of such applications have already been made; and mathematics has thus been enriched with new theorems. But the applications are not so frequent as to make the elaboration of a facile calculus one of the most pressing desiderata of the study. Professor Schröder has done a great deal in this direction; and of course his results are most welcome, even if they be not precisely what we should most have preferred to gain.

The introduction, which relates to first principles, while containing many excellent observations, is somewhat fragmentary and wanting in a unifying idea; and it makes logic too much a matter of feeling. It cannot be said to belong to exact logic in any sense. Thus, under β (Vol. I., p. 2) the reader is told that the sciences have to suppose, not only that their objects really exist, but also that they are knowable and that for every question there is a true answer and but one. But, in the first place, it seems more exact to say that in the discussion of one question nothing at all concerning a wholly unrelated question can be implied. And, in the second place, as to an inquiry presupposing that there is some one truth, what can this possibly mean except it be that there is one destined upshot to inquiry with reference to the question in hand,—one result, which when reached will never be overthrown? Undoubtedly, we hope that this, *or something approximating to this*, is so, or we should not trouble ourselves to make the inquiry. But we do not necessarily have much confidence that it *is* so. Still less need we think it is so about the *majority* of the questions with which we concern ourselves. But in so exaggerating the presupposition, both in regard to its universality, its precision, and the amount of belief there need be in it, Schröder merely falls into an error common to almost all philosophers about all sorts of "presuppositions." Schröder (under ϵ , p. 5) undertakes to define a contradiction in terms without having first made an ultimate analysis of the propo-

sition. The result is a definition of the usual peripatetic type; that is, it affords no analysis of the conception whatever. It amounts to making the contradiction in terms an ultimate unanalysable relation between two propositions,—a sort of blind reaction between them. He goes on (under \mathcal{Z} , p. 9) to define, after Sigwart, logical consequentiality, as *a compulsion of thought*. Of course, he at once endeavors to avoid the dangerous consequences of this theory, by various qualifications. But all that is to no purpose. Exact logic will say that C 's following logically from A is a state of *things* which no impotence of thought can alone bring about, unless there is also an impotence of existence for A to be a fact without C being a fact. Indeed, as long as this latter impotence exists and can be ascertained, it makes little or no odds whether the former impotence exists or not. And the last anchor-hold of logic he makes (under \mathcal{Z}) to lie in the correctness of a feeling! If the reader asks *why* so subjective a view of logic is adopted, the answer seems to be (under β , p. 2), that in this way Sigwart escapes the necessity of founding logic upon the theory of cognition. By the theory of cognition is usually meant an explanation of the possibility of knowledge drawn from principles of psychology. Now, the only sound psychology being a special science, which ought itself to be based upon a well-grounded logic, it is indeed a vicious circle to make logic rest upon a theory of cognition so understood. But there is a much more general doctrine to which the name theory of cognition might be applied. Namely, it is that speculative grammar, or analysis of the nature of assertion, which rests upon observations, indeed, but upon observations of the rudest kind, open to the eye of every attentive person who is familiar with the use of language, and which, we may be sure, no rational being, able to converse at all with his fellows, and so to express a doubt of anything, will ever have any doubt. Now, proof does not consist in giving superfluous and superpossible certainty to that which nobody ever did or ever will doubt, but in removing doubts which do, or at least might at some time, arise. A man first comes to the study of logic with an immense multitude of opinions upon a vast variety of topics; and they are held with a degree of confidence, upon which, after he has

studied logic, he comes to look back with no little amusement. There remains, however, a small minority of opinions that logic never shakes ; and among these are certain observations about assertions. The student would never have had a desire to learn logic if he had not paid some little attention to assertion, so as at least to attach a definite signification to assertion. So that, if he has not thought more accurately about assertions, he must at least be conscious, in some out-of-focus fashion, of certain properties of assertion. When he comes to the study, if he has a good teacher, these already dimly recognised facts will be placed before him in accurate formulation, and will be accepted as soon as he can clearly apprehend their statements.

Let us see what some of these are. When an assertion is made, there really is some speaker, writer, or other sign-maker who delivers it ; and he supposes there is, or will be, some hearer, reader, or other interpreter who will receive it. It may be a stranger upon a different planet, an æon later ; or it may be that very same man as he will be a second after. In any case, the deliverer makes signals to the receiver. Some of these signs (or at least one of them) are supposed to excite in the mind of the receiver familiar images, pictures, or, we might almost say, *dreams*,—that is, reminiscences of sights, sounds, feelings, tastes, smells, or other sensations, now quite detached from the original circumstances of their first occurrence, so that they are free to be attached to new occasions. The deliverer is able to call up these images at will (with more or less effort) in his own mind ; and he supposes the receiver can do the same. For instance, tramps have the habit of carrying bits of chalk and making marks on the fences to indicate the habits of the people that live there for the benefit of other tramps who may come on later. If in this way a tramp leaves an assertion that the people are stingy, he supposes the reader of the signal will have met stingy people before, and will be able to call up an image of such a person attachable to a person whose acquaintance he has not yet made. Not only is the outward significant word or mark a sign, but the image which it is expected to excite in the mind of the receiver will likewise be a sign,—a sign by resemblance, or, as we

say, an *icon*,—of the similar image in the mind of the deliverer, and through that also a sign of the real quality of the thing. This icon is called the *predicate* of the assertion. But instead of a single *icon*, or sign by resemblance of a familiar image or “dream,” evocable at will, there may be a complexus of such icons, forming a composite image of which the whole is not familiar. But though the whole is not familiar, yet not only are the parts familiar images, but there will also be a familiar image of its mode of composition. In fact, two types of complication will be sufficient. For example, one may be conjunctive and the other disjunctive combination. Conjunctive combination is when two images are both to be used at once; and disjunctive when one or other is to be used. (This is not the most scientific selection of types; but it will answer the present purpose.) The sort of idea which an icon embodies, if it be such that it can convey any positive information, being applicable to some things but not to others, is called a *first intention*. The idea embodied by an icon which cannot of itself convey any information, being applicable to everything or to nothing, but which may, nevertheless, be useful in modifying other icons, is called a *second intention*.

The assertion which the deliverer seeks to convey to the mind of the receiver relates to some object or objects which have forced themselves upon his attention; and he will miss his mark altogether unless he can succeed in forcing those very same objects upon the attention of the receiver. No icon can accomplish this, because an icon does not relate to any particular thing; nor does its idea strenuously force itself upon the mind, but often requires an effort to call it up. Some such sign as the word *this*, or *that*, or *hullo*, or *hi*, which awakens and directs attention must be employed. A sign which denotes a thing by forcing it upon the attention is called an *index*. An index does not describe the qualities of its object. An object, in so far as it is denoted by an index, having *thisness*, and distinguishing itself from other things by its continuous identity and forcefulness, but not by any distinguishing characters, may be called a *hecceity*. A hecceity in its relation to the assertion is a *subject*

thereof. An assertion may have a multitude of subjects ; but to that we shall return presently.

Neither the predicate, nor the subjects, nor both together, can make an *assertion*. The assertion represents a compulsion which experience, meaning the course of life, brings upon the deliverer to attach the predicate to the subjects as a sign of them taken in a particular way. This compulsion strikes him at a certain instant ; and he remains under it forever after. It is, therefore, different from the temporary force which the hecceities exert upon his attention. This new compulsion may pass out of mind for the time being ; but it continues just the same, and will act whenever the occasion arises, that is, whenever those particular hecceities and that first intention are called to mind together. It is, therefore, a permanent conditional force, or *law*. The deliverer thus requires a kind of sign which shall signify a law that to objects of indices an icon appertains as sign of them in a given way. Such a sign has been called a *symbol*. It is the *copula* of the assertion.

Returning to the subjects, it is to be remarked that the assertion may contain the suggestion, or request, that the receiver *do* something with them. For instance, it may be that he is first to take any one, no matter what, and apply it in a certain way to the icon, that he is then to take another, perhaps this time a suitably chosen one, and apply that to the icon, etc. For example, suppose the assertion is : "Some woman is adored by all catholics." The constituent icons are, in the probable understanding of this assertion, three, that of a woman, that of a person, *A*, adoring another, *B*, and that of a non-catholic. We combine the two last disjunctively, identifying the non-catholic with *A* ; and then we combine this compound with the first icon conjunctively, identifying the woman with *B*. The result is the icon expressed by, "*B* is a woman, and moreover, either *A* adores *B* or else *A* is a non-catholic." The subjects are all the things in the real world past and present. From these the receiver of the assertion is suitably to choose one to occupy the place of *B* ; and then it matters not what one he takes for *A*. A suitably chosen object is a woman, and any object, no matter what, adores her, unless that object be a non-catholic.

This is forced upon the deliverer by experience ; and it is by no idiosyncrasy of his ; so that it will be forced equally upon the receiver.

Such is the meaning of one typical assertion. An assertion of *logical necessity* is simply one in which the subjects are the objects of any collection, no matter what. The consequence is, that the icon, which can be called up at will, need only to be called up, and the receiver need only ascertain by experiment whether he can distribute any set of indices in the assigned way so as to make the assertion false, in order to put the truth of the assertion to the test. For example, suppose the assertion of logical necessity is the assertion that from the proposition, "Some woman is adored by all catholics," it logically follows that "Every catholic adores some woman." That is as much as to say that, for every imaginable set of subjects, either it is false that some woman is adored by all catholics or it is true that every catholic adores some woman. We try the experiment. In order to avoid making it false that some woman is adored by all catholics, we must choose our set of indices so that there shall be one of them, *B*, such that, taking any one, *A*, no matter what, *B* is a woman, and moreover either *A* adores *B* or else *A* is a non-catholic. But that being the case, no matter what index, *A*, we may take, either *A* is a non-catholic or else an index can be found, namely, *B*, such that *B* is a woman, and *A* adores *B*. We see, then, by this experiment, that it is impossible so to take the set of indices that the proposition of consecution shall be false. The experiment may, it is true, have involved some blunder ; but it is so easy to repeat it indefinitely, that we readily acquire any desired degree of certitude for the result.

It will be observed that this explanation of logical certitude depends upon the fact of speculative grammar that the predicate of a proposition, being essentially of an ideal nature, can be called into the only kind of existence of which it is capable, at will.

A not unimportant dispute has raged for many years as to whether hypothetical propositions (by which, according to the traditional terminology, I mean any compound propositions, and not merely those *conditional* propositions to which, since Kant, the term

has often been restricted) and categorical propositions are one in essence. Roughly speaking, English logicians maintain the affirmative, Germans the negative. Professor Schröder is in the camp of the latter, I in that of the former.

I have maintained since 1867 that there is but one primary and fundamental logical relation, that of illation, expressed by *ergo*. A proposition, for me, is but an argumentation divested of the assertoriness of its premise and conclusion. This makes every proposition a conditional proposition at bottom. In like manner a "term," or class-name, is for me nothing but a proposition with its indices or subjects left blank, or indefinite. The common noun happens to have a very distinctive character in the Indo-European languages. In most other tongues it is not sharply discriminated from a verb or participle. "Man," if it can be said to mean anything by itself, means "what I am thinking of is a man." This doctrine, which is in harmony with the above theory of signs, gives a great unity to logic; but Professor Schröder holds it to be very erroneous.

Cicero and other ancient writers mention a great dispute between two logicians, Diodorus and Philo, in regard to the significance of conditional propositions. This dispute has continued to our own day. The Diodoran view seems to be the one which is natural to the minds of those, at least, who speak the European languages. How it may be with other languages has not been reported. The difficulty with this view is that nobody seems to have succeeded in making any clear statement of it that is not open to doubt as to its justice, and that is not pretty complicated. The Philonian view has been preferred by the greatest logicians. Its advantage is that it is perfectly intelligible and simple. Its disadvantage is that it produces results which seem offensive to common sense.

In order to explain these positions, it is best to mention that *possibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false. By varying the supposed state of information all the varieties of possibility are obtained. Thus, *essential* possibility is that which supposes nothing

to be known except logical rules. *Substantive* possibility, on the other hand, supposes a state of omniscience. Now the Philonian logicians have always insisted upon beginning the study of conditional propositions by considering what such a proposition means in a state of omniscience; and the Diodorans have, perhaps not very adroitly, commonly assented to this order of procedure. Duns Scotus terms such a conditional proposition a "*consequentia simplex de inesse*." According to the Philonians, "If it is now lightening it will thunder," understood as a consequence *de inesse*, means "It is either not now lightening or it will soon thunder." According to Diodorus, and most of his followers (who seem here to fall into a logical trap), it means it is now lightening and it will soon thunder.

Although the Philonian views lead to such inconveniences as that it is true, as a consequence *de inesse*, that if the Devil were elected president of the United States, it would prove highly conducive to the spiritual welfare of the people (because he will not be elected), yet both Professor Schröder and I prefer to build the algebra of relatives upon this conception of the conditional proposition. The inconvenience, after all, ceases to seem important, when we reflect that, no matter what the conditional proposition be understood to mean, it can always be expressed by a complexus of Philonian conditionals and denials of conditionals. It may, however, be suspected that the Diodoran view has suffered from incompetent advocacy, and that if it were modified somewhat, it might prove the preferable one.

The consequence *de inesse*, "if A is true, then B is true," is expressed by letting i denote the actual state of things, A_i mean that in the actual state of things A is true, and B_i mean that in the actual state of things B is true, and then saying "If A_i is true then B_i is true," or, what is the same thing, "Either A_i is not true or B_i is true." But an *ordinary* Philonian conditional is expressed by saying, "In *any* possible state of things, i , either A_i is not true, or B_i is true."

Now let us express the categorical proposition, "Every man is wise." Here, we let m_i mean that the individual object i is a man,

and w_i mean that the individual object i is wise. Then, we assert that, "taking any individual of the universe, i , no matter what, either that object, i , is not a man or that object, i , is wise"; that is, whatever is a man is wise. That is, "whatever i can indicate, either m_i is not true or w_i is true. The conditional and categorical propositions are expressed in precisely the same form; and there is absolutely no difference, to my mind, between them. The *form* of relationship is the same.

I find it difficult to state Professor Schröder's objection to this, because I cannot find any clear-cut, unitary conception governing his opinion. More than once in his first volume promises are held out that § 28, the opening section of the second volume, shall make the matter plain. But when the second volume was published, all we found in that section was, as far as repeated examination has enabled me to see, as follows. First, hypothetical propositions, unlike categoricals, essentially involve the idea of time. When this is eliminated from the assertion, they relate only to two possibilities, what always is and what never is. Second, a categorical is always either true or false; but a hypothetical is either true, false, or meaningless. Thus, "this proposition is false" is meaningless; and another example is, "the weather will clear as soon as there is enough sky to cut a pair of trousers." Third, the supposition of negation is forced upon us in the study of hypotheticals, never in that of categoricals. Such are Schröder's arguments, to which I proceed to reply.

As to the idea of time, it *may* be introduced; but to say that the range of possibility in hypotheticals is always a unidimensional continuum is incorrect. "If you alone trump a trick in whist, you take it." The possibilities are that each of the four players plays any one of the four suits. There are 2^{16} different possibilities. Certainly, the universe in hypotheticals is far more frequently finite than in categoricals. Besides, it is an *ignoratio elenchi* to drag in time, when no logician of the English camp has ever alleged anything about propositions involving time. That is not the question.

Every proposition is either true or false, and something not a proposition, when considered as a proposition, is, from the Philo-

nian point of view, true. To be objectionable, a proposition must assert something; if it is merely neutral, it is not positively objectionable, that is, it is not false. "This proposition is false," far from being meaningless, is self-contradictory. That is, it means two irreconcilable things. That it involves contradiction (that is, leads to contradiction if supposed true), is easily proved. For if it be true, it is true; while if it be false, it is false. Every proposition besides what it explicitly asserts, tacitly implies its own truth. The proposition is not true unless *both*, what it explicitly asserts and what it tacitly implies, are true. This proposition, being self-contradictory, is false; and hence, what it explicitly asserts is true. But what it tacitly implies (its own truth) is false. The difficulty about the proposition concerning the piece of blue sky is not a logical one, at all. It is no more senseless than any proposition about a "red odor" which might be a term of a categorical.

The fact stated about negation is only true of the sorts of propositions which are commonly put into categorical and hypothetical shapes, and has nothing to do with the essence of the propositions. In a paper "On the Validity of the Laws of Logic" in the *Journal of Speculative Philosophy*, Vol. II., I have given a sophistical argument that black is white, which shows in the domain of categoricals the phenomena to which Professor Schröder refers as peculiar to hypotheticals.

The *consequentia de inesse* is, of course, the extreme case where the conditional proposition loses all its proper signification, owing to the absence of any range of possibilities. The conditional proper is, "In any possible case, i , either A_i is not true, or B_i is true." In the consequence *de inesse* the meaning sinks to, "In the true state of things, i , either A_i is not true or B_i is true."

My general algebra of logic (which is not that algebra of dual relations, likewise mine, which Professor Schröder prefers, although in his last volume he often uses this general algebra) consists in simply attaching indices to the letters of an expression in the Boolean algebra, making what I term a Boolian, and prefixing to this a series of "quantifiers," which are the letters Π and Σ , each with an index attached to it. Such a quantifier signifies that every individual of

the universe is to be substituted for the index the Π or Σ carries, and that the non-relative product or aggregate of the results is to be taken.

Properly to express an ordinary conditional proposition the quantifier Π is required. In 1880, three years before I developed that general algebra, I published a paper containing a chapter on the algebra of the copula (a subject I have since worked out completely in manuscript). I there noticed the necessity of such quantifiers properly to express conditional propositions; but the algebra of quantifiers not being at hand, I contented myself with considering consequences *de inesse*. Some apparently paradoxical results were obtained. Now Professor Schröder seems to accept these results as holding good in the general theory of hypotheticals; and then, since such results are in strong contrast with the doctrine of categoricals, he infers, in § 45 of his Vol. II., a great difference between hypotheticals and categoricals. But the truth simply is that such hypotheticals want the characteristic feature of conditionals, that of a range of possibilities.

In connexion with this point, I must call attention to a mere algebraical difference between Schröder and me. I retain Boole's idea that there are but two *values* in the system of logical quantity. This harmonises with my use of the general algebra. Any two numbers may be selected to represent those values. I prefer 0 and a positive logarithmic ∞ . To express that something is A and something is not A , I write:

$$\infty = \Sigma_i A_i \qquad \infty = \Sigma_j A_j$$

or, what is the same thing:

$$\Sigma_i A_i > 0 \qquad \Sigma_j A_j > 0.$$

I have no objection to writing, *as a mere abbreviation*, which may, however, lead to difficulties, if not *interpreted*:

$$A > 0 \qquad \bar{A} > 0.$$

But Professor Schröder understands these formulæ literally, and accordingly *rejects* Boole's conception of two values. He does not seem to understand my mode of apprehending the matter; and

hence considers it a great limitation of my system that I restrict myself to two values. In fact, it is a mere difference of algebraical form of conception. I very much prefer the Boolean idea as more simple, and more in harmony with the general algebra of logic.

Somewhat intimately connected with the question of the relation between categoricals and hypotheticals is that of the quantification of the predicate. This is the doctrine that identity, or equality, is the fundamental relation involved in the copula. Holding as I do that the fundamental relation of logic is the illative relation, and that only in special cases does the premise follow from the conclusion, I have in a consistent and thoroughgoing manner opposed the doctrine of the quantification of the predicate. Schröder seems to admit some of my arguments; but still he has a very strong *penchant* for the equation.

Were I not opposed to the quantification of the predicate, I should agree with Venn that it was a mistake to replace Boole's operation of addition by the operation of aggregation, as most Booleans now do. I should consider the "principle of duality" rather an argument *against* than *for* our modern practice. The algebra of dual relatives would be almost identical with the theory of matrices were addition retained; and this would be a great advantage.

It is Schröder's predilection for equations which motives his preference for the algebra of dual relatives, namely, the fact that in that algebra, even a simple undetermined inequality can be expressed as an equation. I think, too, that that algebra has merits; it certainly has uses to which Schröder seldom puts it. Yet, after all, it has too much formalism to greatly delight me,—too many bushels of chaff *per* grain of wheat. I think Professor Schröder likes algebraic formalism better, or dislikes it less, than I.

He looks at the problems of logic through the spectacles of equations, and he formulates them, from that point of view, as he thinks, with great generality; but, as I think, in a narrow spirit. The great thing, with him, is to solve a proposition, and get a *value* of x , that is, an equation of which x forms one member without occurring in the other. How far such equation is *iconic*, that is, has a meaning, or exhibits the constitution of x , he hardly seems to

care. He prefers general values to particular roots. Why? I should think the particular root alone of service, for most purposes, unless the general expressions were such that particular roots could be deduced from it,—particular instances, I mean, *showing* the constitution of x . In most instances, a profitable solution of a mathematical problem must consist, in my opinion, of an exhaustive examination of special cases; and quite exceptional are those fortunate problems which mathematicians naturally prefer to study, where the enumeration of special cases, together with the pertinent truths about them, flow so naturally from the general statement as not to require separate examination.

I am very far from denying the interest and value of the problems to which Professor Schröder has applied himself; though there are others to which I turn by preference. Certainly, he has treated his problems with admirable power and clearness. I cannot in this place enter into the elementary explanations which would be necessary to illustrate this for more than a score of readers.

In respect to individuals, both non-relative and pairs, he has added some fundamental propositions to those which had been published. But he is very much mistaken in supposing that I have expressed contrary views. He simply mistakes my meaning.

In regard to algebraical signs, I cannot accept any of Professor Schröder's proposals except this one. While it would be a serious hindrance to the promulgation of the new doctrine to insist on new types being cut, and while I, therefore, think my own course in using the dagger as the sign of relative addition must be continued, yet I have always given that sign in its cursive form a scorpion-tail curve to the left; and it would be finical to insist on one form of curve rather than another. In almost all other cases, in my judgment, Professor Schröder's signs can never be generally received, because they are at war with a principle, the general character of which is such that Professor Schröder would be the last of all men to wish to violate it, a principle which the biologists have been led to adopt in regard to their systematic nomenclature. It is that priority must be respected, or all will fall into chaos. I will not enter further into this matter in this article.

Of what use does this new logical doctrine promise to be? The first service it may be expected to render is that of correcting a considerable number of hasty assumptions about logic which have been allowed to affect philosophy. In the next place, if Kant has shown that metaphysical conceptions spring from formal logic, this great generalisation upon formal logic must lead to a new apprehension of the metaphysical conceptions which shall render them more adequate to the needs of science. In short, "exact" logic will prove a stepping-stone to "exact" metaphysics. In the next place, it must immensely widen our logical notions. For example, a class consisting of a lot of things jumbled higgledy-piggledy must now be seen to be but a degenerate form of the more general idea of a *system*. Generalisation, which has hitherto meant passing to a larger class, must mean taking in the conception of the whole system of which we see but a fragment, etc., etc. In the next place, it is already evident to those who know what has already been made out, that that speculative rhetoric, or objective logic, mentioned at the beginning of this article, is destined to grow into a colossal doctrine which may be expected to lead to most important philosophical conclusions. Finally, the calculus of the new logic, which is applicable to everything, will certainly be applied to settle certain logical questions of extreme difficulty relating to the foundations of mathematics. Whether or not it can lead to any method of discovering methods in mathematics it is difficult to say. Such a thing is conceivable.

It is now more than thirty years since my first published contribution to "exact" logic. Among other serious studies, this has received a part of my attention ever since. I have contemplated it in all sorts of perspectives and have often reviewed my reasons for believing in its importance. My confidence that the key of philosophy is here, is stronger than ever after reading Schröder's last volume. One thing which helps to make me feel that we are developing a living science, and not a dead doctrine, is the healthy mental independence it fosters, as evidenced, for example, in the divergence between Professor Schröder's opinions and mine. There is no bovine nor ovine gregariousness here. But Professor Schröder and

I have a common method which we shall ultimately succeed in applying to our differences, and we shall settle them to our common satisfaction ; and when that method is pouring in upon us new and incontrovertible positively valuable results, it will be as nothing to either of us to confess that where he had not yet been able to apply that method he has fallen into error.

C. S. PEIRCE.

NEW YORK CITY.