HINTS FOR THE ELUCIDATION OF MR. PEIRCE'S LOGICAL WORK.

[Charles S. Peirce has undertaken the regeneration of logic, a field that has been cultivated by only a few men, such as Boole in England and Schroeder in Germany. It is a work which entails a great deal of abstruse thinking, so that any article which would be a contribution to this line of inquiry would to the average reader be naturally difficult to understand, if not inaccessible. In order to enable our readers to see the significance of Mr. Peirce's investigation, the Editor has asked Mr. Francis C. Russell of Chicago to give Mr. Peirce's article a careful perusal and to provide our readers with a popular digest of it so as to point out the aim and course of Mr. Peirce's thought. We hope that this article on "The Amazing Mazes" of Mr. Peirce will prove helpful to the readers of *The Monist.* —ED.]

THE card curiosities described in the April number of *The Monist* demand attention only for the sake of the remark that they present in a concrete way comparatively trivial particular instances of the operation of the principles exhibited in the instalment contained in this issue. But it must not be supposed that the card curiosities have been offered to cater to "popular" interest.

It is one of the cardinal points of the method championed by Mr. Peirce that in so far as the same is possibly attainable, reasoning, indeed all serious thought, should be *iconized* (the word is mine but the idea is his); that is to say, that the idea dealt with should so far as is possible be represented by a sign or sign-complex, fit by its constitution to display in detail to the intellect all the essential features of the said idea, and especially all the various interrelations that subsist between the constituent elements thereof; in other words, that the plan, so to speak, of the said idea should be as concretely expressed as possible. To this very end Mr. Peirce has invented two schemes of logical algebra and two systems of logical graphs. Now in the present case the cards used in the card tricks fulfil an iconic office. They fulfil it in some respects in a superior way. They are not only concrete but are also corporeal.

The first third of the exposition contained in the present number is a purely mathematical account of the phenomena presented in the manipulation of the cards pursuant to the rules stated therefor, and of the reasons why the said phenomena must be so presented. If the article contained nothing different in kind than this, while it would be a paper highly interesting to mathematical experts it would nevertheless not be a paper suitable for such a magazine as *The Monist*. The sum and substance of the whole mathematical discussion stated in a summary way, is that a sufficient number of repeated dealings of a pack or packet of cards (whose order is known to begin with) into a constant number of piles according to the rules given therefor, together with repeated regatherings after each deal in the proper order of starting and following, brings round the identical known order in which the pack or packet was arranged at first.

Those who are antipathetic towards mathematics may skim over, or, if they must, may skip altogether this mathematical third of the article without serious disability for the comprehension of the rest which is really the specially important text of the article. (It begins at page 432.) The mathematical discussion involves recourse to that highly recondite region called "The Theory of Numbers" and especially to "Cyclic Arithmetic," a sub-region thereof. "Cyclic arithmetic" involves a subsidiary cyclic number system, viz., the "cyclic logarithms."

Now in these regions of mathematical exploration the explorer has one and only one master key, to wit: that operation that ought to be called "Fermatian Inference" (since it was Fermat that first brought it to light) but which is often called "mathematical induction" (a ridiculous appellation) and more often "the inference from n to $n+1$." It is plain that it must be a logical transition, and it is so analogous to the *dictum de omne*, the "chain of causation," etc., that the generalizing mind sees that there must be a single principle of some kind upon which all such mental transitions are founded and by which they are justified.

Now obedient to the generalizer's prompting, Mr. Peirce has in this article proceeded just about as though he had put to himself a certain *all important* logical problem; a problem that may be stated thus:

What is the nature and what are the leading characteristics of that relation in virtue of which whatever is true of its relate term is true also of its correlate term?

In other words, what is the pure essence of the copular relation, what immediate consequences does it entail, and *how* are the same so entailed?

Dr. Carus has somewhere called special attention to a very important and pregnant logical point that has been neglected. It is this.

Every problem, every question, implies assertion of some kind or other. Often it implies a good deal of assertion. Such implicit assertions have their logical consequences just as do all other propositions. Now the mere formulation of the problem above stated supposes and virtually asserts that there may be, perhaps, a relation of the kind described. So Mr. Peirce says (in effect): Put A as the name of that as yet unknown relation. This is altogether the same step as is taken in algebraic calculation when x is put to signify the unknown quantity. So again if there be such a relation there must be objects so related.

But as yet it is wholly unknown what these objects are and how many they are. So M is put to stand for *any* one of these objects, be they many or few or even only one single one. So again these objects being, at least, somewhat formulated by the relation A, must form some kind of a system, although it may turn out to be a system of only one member. So K is put to stand for the said system.

Now all that the formulation of the problem tells us about the relation A is that it is such a relation that whatever is true of its relate term is true also of its correlate term. So P is put to stand for *any* predicate whatsoever.

Now it must be well understood and always in mind that when any two objects, say M and *m* are in relation there is always not merely *one* relation but *two,* viz., the relation of M to *m* and the relation of *m* to M. We have a vicious habit of thinking a relation as a betweenness and as single. It is often the case that the relation of M to *m* is of the same kind as that of *m* to M, convertible as it is called, but there are two relations nevertheless, viz., the direct relation, as, say, M to *m,* and the converse relation *m* to M. Following the habits of ordinary language with respect to the voices of the verb, active and passive, and using any relation, say A, to form an example of the ways of speech used in respect to the distinction under notice, M is said to be "A to" *m* (or sometimes M "A's" *m*) in case of the direct relation, and *m* is said to be "A'd by" M in case of the converse relation.

Then (all the nomenclature and phrasings being understood) it is plain that the formulation of the problem above stated indicates and justifies certain definitional statements as follows:

K is a system of some kind or other.

K has at least one member, may be several, may be a good many, even an infinity.

The members of K are formulated by the relation A.

A is a relation such that any member M of K that may exist is "A to" one member *m* (at least) of K and in every case of the relation M is "A to" *m*, or what is the same thing in every case of the relation *m* is "A'd by" M, then if M is P, finally *in virtue of the relation* A, *m* is also P.

To say that *m* is P *because* of both these, viz., (1) that *m* is "A'd by" M and (2) that M is P, is no other than to say that *m* being in the relation A to M, to suppose that *m* was not P would be to compel the supposition that M was not P also. The object here is to bring into bold relief if possible that the relation A is such a relation that all possibilities are covered by the alternative *m* is P or else M is not P; that is to say, if *m* is "A'd by" M then either it is untrue that M is P or it is true that *m* is P. The nature of the relation supposed compels this. This is a point to be *seen*, to be intellectually intuited, for it does not admit of much if any helpful verbal explication. If A is B, then whatever is not B is not A, and every case whatever is either a case of not-A or else of B. If *m* is not P without M is P (in case M is "A to" *m*) then (since M is *any* member of K) it is plain that if P is true of any member it is true of every member, and if P is false of any member it is false of every member.

Mr. Peirce embodies these definitional statements in a complex Existential Graph. We are therefore counselled to give them a little attention. In my judgment examples are more instructive than pure description. In various cases we have no special need of making the cuts go entirely around the spots; parentheses, square brackets and braces will serve well enough for a good many cases.

In the Existential Graphs, —A, means Something is A, or A exists, or There is an A, or whatever is in substance equivalent to either of the assertions. It might just as well have been written A—, for the side on which the line is drawn is altogether immaterial. In fact, in one way of regarding the single graph-instance the line (which is also the line of identity) represents the *thing*, the letter serving only to determine that *thing* to be what the letter tells. A when inside a cut, that is to say when enclosed by a parenthesis, means denial so that (—A) means Nothing is A. The line of identity *as such* ends at the cut. When the line goes outside of the cut, and perhaps branches at a point of ter-identity, the whole line and its branches, if it has any, while still importing the idea of identity is called a *ligature*, so that a ligature may have two or three lines of identity at its ends. The cut also brackets together its contents, a feature that must be specially noticed. The single (or odd) cut also imports the universal enunciation and does *not* imply that what it contains has any existence, so that (—A) means not Something is not A, but Nothing is A, or if you please, A is *other than* something, reading the line of identity as negated and thus transformed into a line of diversity. This is one reason for making the distinction between the line of identity and the ligature. (There is no "quantification of the predicate" in the scheme of graphs.)

Since the printing offices do not contain a type to print the first horn of a parenthesis cut across by a continuous dash, the proposition Something is not-A, must be expressed here in this way, -(-A), but it must be understood and mentally supplied that the two dashes are to be taken as one long dash intersecting the parenthesis-horn. They form a ligature, (subject to the same monition). B -(-A) says Some B is not-A. Now *nota bene*. The import of a ligature in regard to the distinction between universal and particular enunciation is determined by its *least* enclosed part only *no enclosure at all* is to be taken as *twice enclosed* and so more enclosed than once enclosed. Subject to this, the rule is, oddly enclosed imports universality and evenly enclosed imports particularity.

B—A says Some B is A, but (B—A) says No B is A. Now negate (i. e. deny) the assertion. Some B is not-A, i. e. B-(-A) by enclosing it with a second cut, (indicated by square brackets) thus [B-(-A)]. Here we have a *scroll* and it should be plain that it says that it is false that there is any B that is not A, which is the same as to say that whatever B there may be is A, or in inexact enunciation, All B is A.

Whatever is doubly enclosed or enclosed in an even number of cuts we may if it suits our turn take as not enclosed at all. The outside cut of the two denies the denial of the inside cut. Two negatives make an affirmative.

The point of teridentity imports simply that whatever identity is imported by one of the lines that join there is imported by each of the two other lines, so that each line may bear three messages of identity just as a telegraph wire may bear several messages without interference.

When relative terms are introduced each will have as many lines of identity as the relation by its nature requires, usually two, occasionally three, and sometimes, though very rarely, four. In writing and in reading these relations we have to notice and obey a certain way of process from their relates to their correlates, and from their correlates back to their relates, or if the relation is triadic, from one of the correlates to the other.

There are six kinds of graphs for the expression of dual relations. Each of these kinds varies as either of the terms is positive or negative, and as the relative term is positive or negative. Thus the forms are quite numerous and I must give here only two examples the forms of which will be found in use farther on. Bearing in mind the intersection of the parenthesis-horn and taking M to stand for member and C to stand for candidate and *p* to stand for the relative term "prior to," M -(- *p* — C) says Some member is not prior to any candidate. Its negative [M -(- p —C)] says, Every member is prior to some candidate.

The definitional statements above given are diagrammed by Prof. Peirce in Fig. 11, which is a composite of three scrolls, (1) the outer scroll formed by the two outside cuts, (2) the mediate scroll, viz., that one at the right-hand side of the enclosure of the second cut, and (3) the inner and composite scroll at the left-hand side of said enclosure.

In reading the graphs it must be kept in mind that they express largely denials. This makes a style of discourse that is so full of inverted propositions and negative conceptions that it is highly unnatural. That a proposition is true is expressed by saying (diagrammatically of course) that it is false that it is false. But for exact logic such a style of expression has that high utility that it would be almost folly to neglect it. In exact logic the forms and rules must be perfectly sound for all possible cases. This makes it irksome and dangerous to try to express with perfect precision an affirmation that will serve therein. But a proposition can be expressed by way of denial with ease and precision and without any special need of providing against the "range of possibilities."

On page 444 Professor Peirce had given the precise and analytical reading of Fig. 11, and it would be useless to repeat it here. Just below it he has given the more natural reading. Now it is the outer scroll that says that K has a member (the M in the outer unshaded enclosure) and has a relation C (duly to be found out and named A-hood). It is the mediate scroll that says that every member of K is A (= C) to some member of K, and it is the inner scroll that says that whatever P is true of one member of K and at the same time untrue of another member of K, is true also of some member that is not "A to" any member of which P is true. The Roman numerals are only empty blanks. All they indicate is a relation of some kind, I indicates a relation of one blank and II a relation of two blanks, which C and the ligature say is the relation C (A-hood). I is the empty predicate which the upper part of the inner scroll says is true of some member and untrue of some (other) member. The lower part of the inner scroll says that I (that is, P) is true of some member that is not "A to" any member of which I (that is, P) is true.

Having prepared therefor by graphs and otherwise, Mr. Peirce begins at page 446 and pursues through several pages a consecutive train of perfectly flawless deduction that is so admirable that no words are available to characterize it in fit measure. As an example it teems with instruction and suggestions, and as its leading result it shows that the core, the essence, of the copular relation when it is purified of everything but itself alone, is the relation of immediate antecedence, a relation *sui generis,* and such that if A immediately antecedes B, and B immediately antecedes C, then nothing else but A can immediately antecede B, and nothing else but B can immediately antecede C, and also that nothing else but C can immediately succeed B nor can anything else but B immediately succeed A. If this seems to any one an unprofitable result for so much pains, I suppose he will keep on thinking so until he has a wider survey of the field of knowledge and of what is needed for its extension. In the train of reasoning the succession of ten immediate inferences beginning on page 450 is worthy of special notice.

The last ten pages are again so predominantly mathematical and special that few lay readers will care to bestow the study needful for their mastery and it would be officious to undertake to gloss them for mathematicians.

Up to now I have said nothing about what I regard as the most excellent text of the article, viz., that part from page 433 to page 440. My reason for this has been that my office in this article has been to explain as well as I am able some of the more difficult matters but not too difficult to make accessible to the non-mathematical reader and to the person not versed in the regenerated logic, and the part I am now speaking about needs no such explanation. In this part Prof. Peirce lays out and develops a philosophy of intellectual discovery, provides a nomenclature and illustrates his doctrine by a commentary upon the method of Euclid. In my humble opinion Prof. Peirce in this part has not only illuminated several very dark corners of the field of inquiry but has also indicated foundations and principles that sooner or later will win general acceptance.

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