Third Body Perturbation

Modeling the Space Environment

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Modeling the Space Environment

- Particle Dynamics: Basics of Orbital Mechanics:
  - Two-Body problem: Keplerian Motion — KEPLER.FOR
  - Equations for Perturbed Motion — COWELL.FOR
- Perturbations: — Models
  - Gravitational
    - Real Earth Gravitational Field: — JGM2S
    - Third body perturbation: — ECSS/DE200
  - Non-Gravitational
    - Air Drag: Static and Dynamic Models: — MSISE00
    - Radiation Pressure — ECSS-E-10-04A/DE200
    - Magnetic Field: — IGRF
    - Ionospheric Effects: — IRI
    - Debris and Meteoroid Impact: — MASTER/ORDEM
- Attitude Dynamics: Gravitational and Non-Gravitational torques

3rd Body Perturbation: Index

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- Primary formulation for N bodies
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Inertial Formulation for N Bodies

- Isolated system of N particles
- Forces: only gravitational interaction (action-reaction)
- Inertial $Oxyz$ reference system
- $\mathbf{R}_i$ position vector of particle i in $Oxyz$ axes

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- $3N$ coupled ODE's, $3N$ unknowns $\longrightarrow$ Numerical integration
- Conservation of linear momentum for the whole system: System COM moves with constant speed $\dot{\mathbf{R}}_G$

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- Take particle $i = 0$ as origin
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- Introducing relative $\mathbf{r}_i$ vectors in the inertial formulation:

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Primary Formulation for N bodies

$\ddot{\mathbf{R}}_0$ from inertial formulation for $i = 0$:

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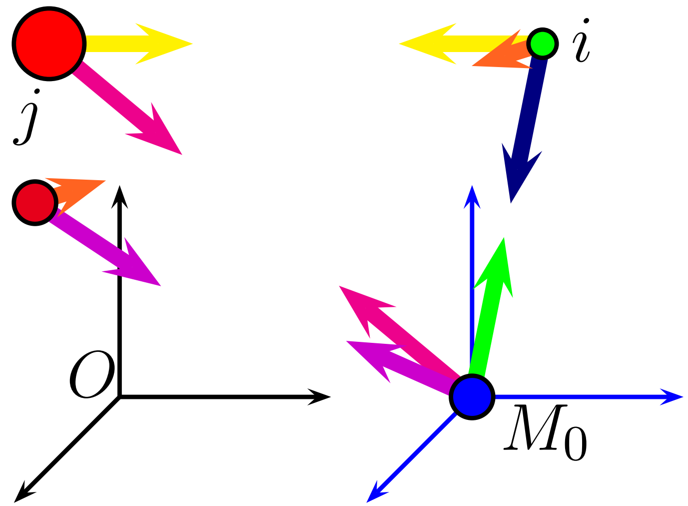

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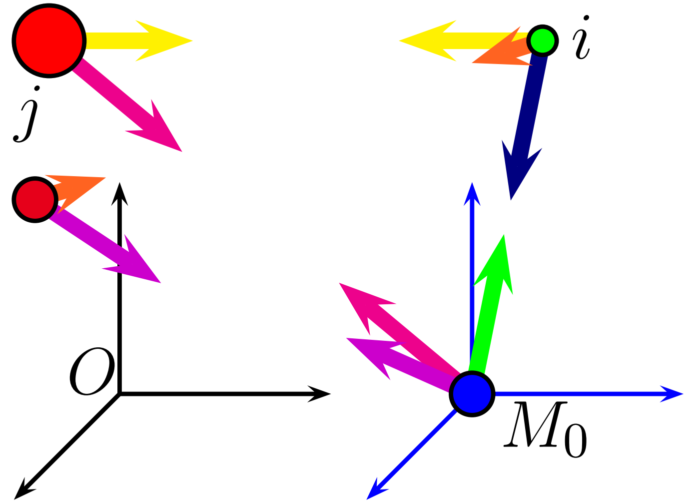

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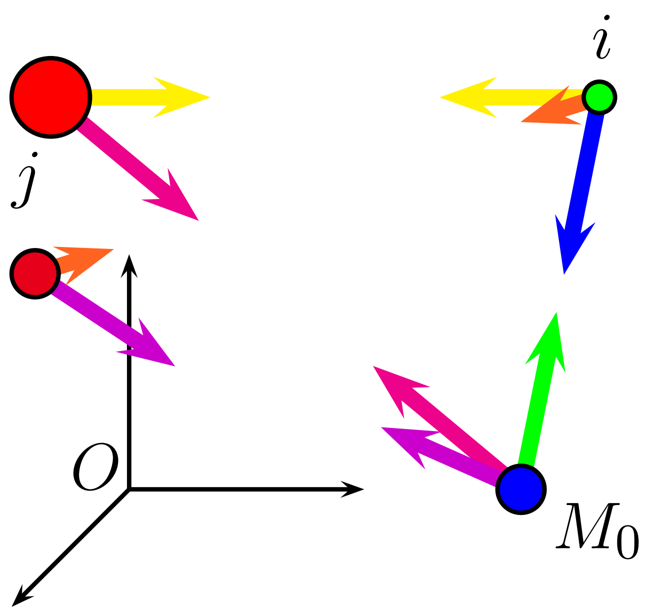
Leading to:

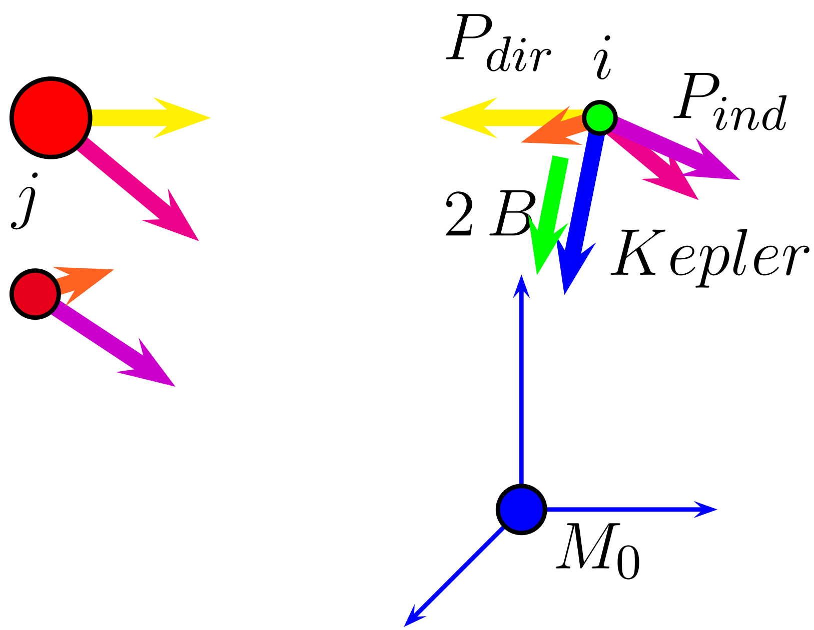
$$\ddot{\mathbf{r}}_i = -\frac{GM_0\mathbf{r}_i}{|\mathbf{r}_i|^3} + \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \frac{-Gm_j(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - \sum_{\substack{j=1 \\ i\neq j}}^{N-1} \frac{Gm_j\mathbf{r}_j}{|\mathbf{r}_j|^3} - \frac{Gm_i\mathbf{r}_i}{|\mathbf{r}_i|^3}$$

Primary Formulation for N bodies

primary inertia direct indirect

$$\underbrace{\ddot{\mathbf{r}}_i = \left\{ -\frac{G\left(M_0 + m_i\right)\mathbf{r}_i}{\left|\mathbf{r}_i\right|^3} \right\}}_{\text{Kepler 2 body}} + \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{N-1} G m_j \left(\overbrace{\frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3}} - \overbrace{\frac{\mathbf{r}_j}{\left|\mathbf{r}_j\right|^3}} \right)}_{\text{Perturbation}}$$





Application to Earth Orbits

- Primary: Earth, $M_0 = M_\oplus$
- Satellite mass always neglected in augmented mass:
  $M_\oplus = 5.973 \cdot 10^{24}$ kg, $m_{✠} < 10^4$ kg

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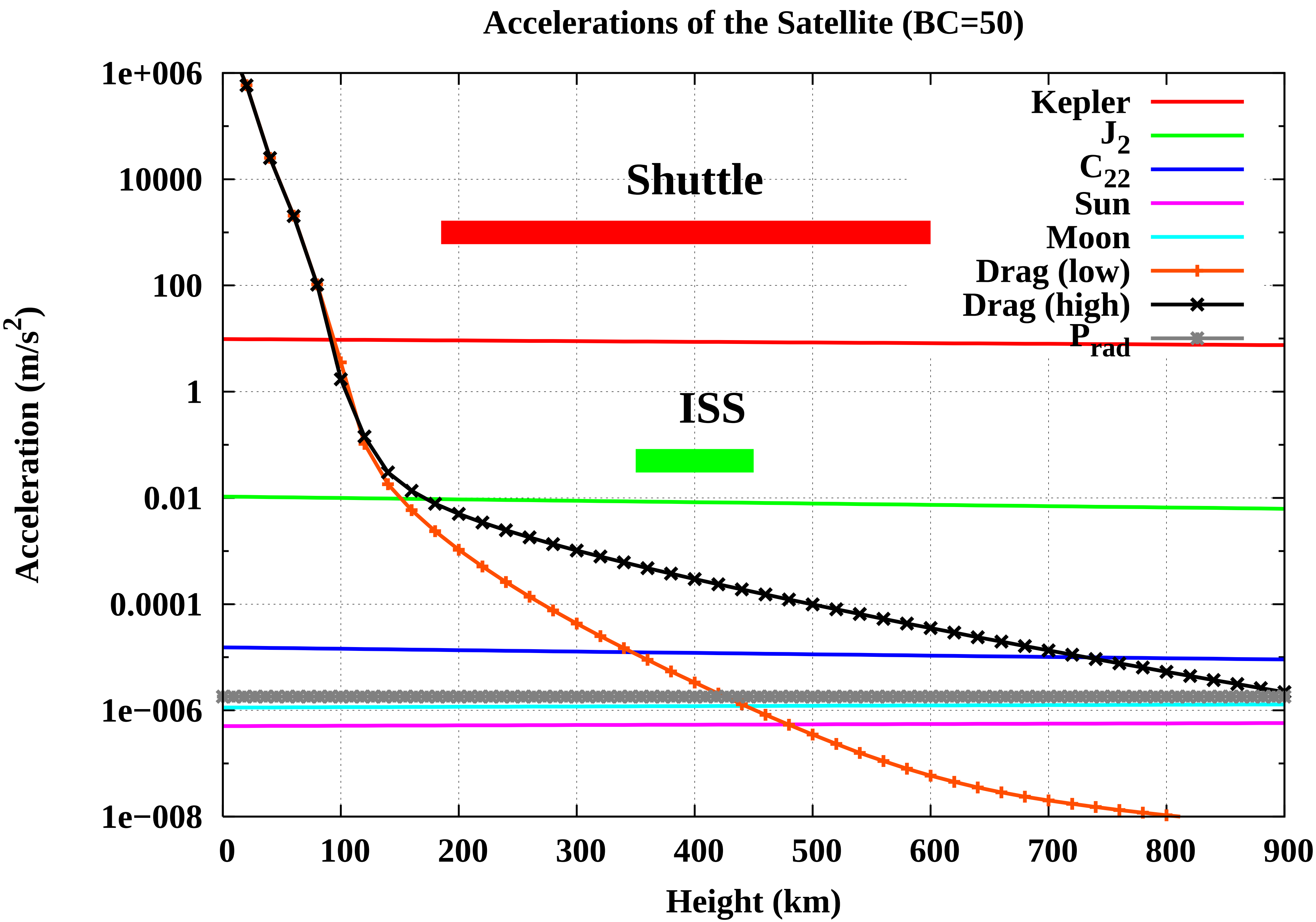
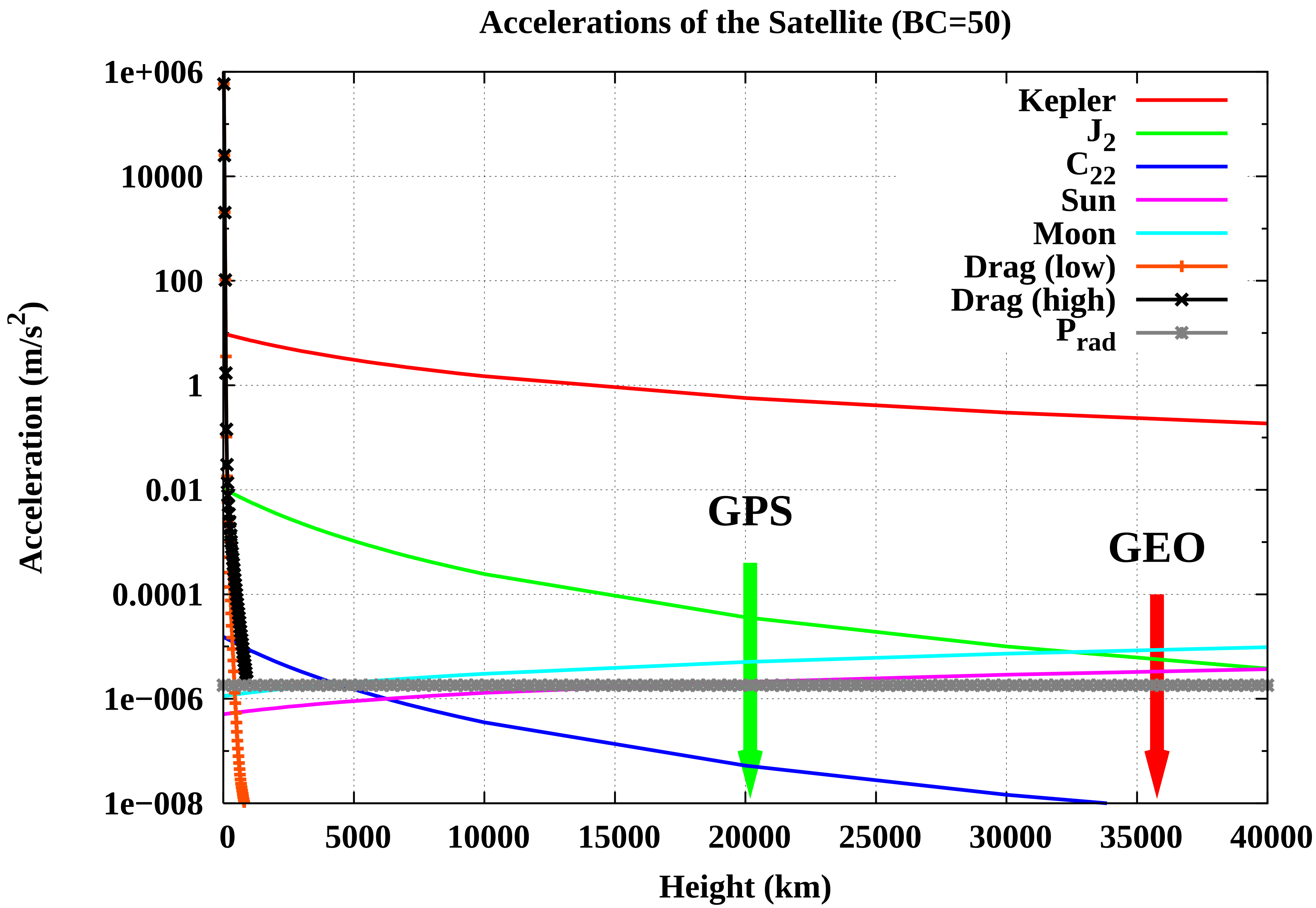
- 2 Approaches:
  - Potential: VOP-Lagrange, Analytical approximations — Battin
  - Acceleration: Numerical — Vallado, Montenbruck

Magnitude of Perturbations (LEO)



Magnitude of Perturbations (GEO)



Potential: Perturbation Function

2-Body and Perturbation terms derive from potential functions:

$$\ddot{\mathbf{r}}_i = \nabla_i \frac{G\left(M_0 + m_i\right)}{r_i} + \nabla_i R_i$$

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The perturbation acceleration

$$\sum_{\substack{j=1 \\ j \neq i}}^{N-1} G m_j \left(\frac{\mathbf{r}_j - \mathbf{r}_i}{\left|\mathbf{r}_j - \mathbf{r}_i\right|^3} - \frac{\mathbf{r}_j}{\left|\mathbf{r}_j\right|^3} \right)$$

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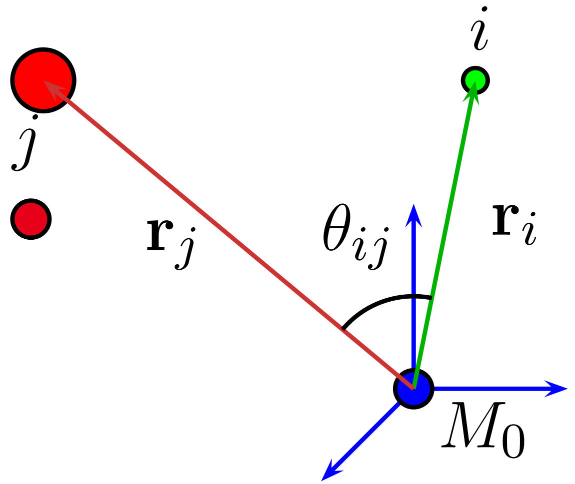
derives from Perturbation Function: $\nabla_i = \dfrac{\partial}{\partial \mathbf{r}_i}$

$$R_i = \sum_{\substack{i=1 \\ i\neq j}}^{N-1} G m_j \left(\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - \frac{\mathbf{r}_j \cdot \mathbf{r}_i}{r_j^3} \right)$$

Potential: Avoiding 1-1 Loss of Precision



- Far body over Earth/satellite system $\Rightarrow \; r_j \gg r_i$
- θ_{ij} angle between $\mathbf{r}_i$ and $\mathbf{r}_j$

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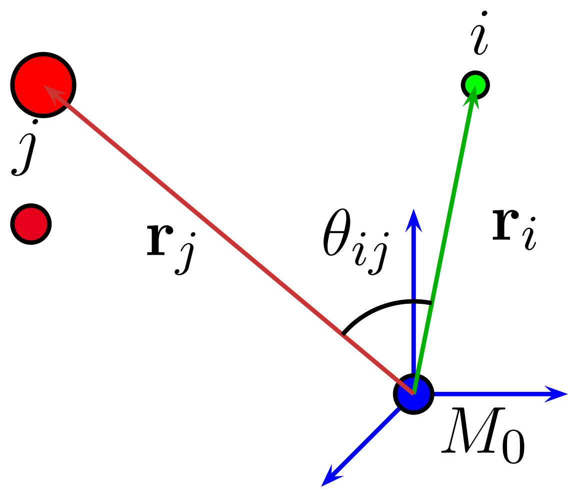

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$$\frac{\mathbf{r}_j \cdot \mathbf{r}_i}{r_j^3} = \frac{1}{r_j}\left(\frac{r_i}{r_j}\right)\cos\theta_{ij}$$

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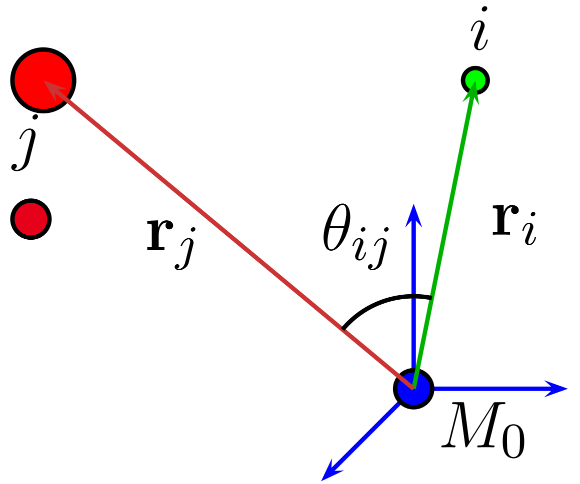

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$$\frac{\mathbf{r}_j \cdot \mathbf{r}_i}{r_j^3} = \frac{1}{r_j}\left(\frac{r_i}{r_j}\right)\cos\theta_{ij}$$

- Expansion in Legendre polynomials of argument $\cos\theta_{ij}$

$$\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} = \frac{1}{r_j}\frac{1}{\sqrt{1+\left(\frac{r_i}{r_j}\right)^2 - 2\left(\frac{r_i}{r_j}\right)\cos\theta_{ij}}} = \frac{1}{r_j}\sum_{n=0}^{\infty}\left(\frac{r_i}{r_j}\right)^n P_n\left(\cos\theta_{ij}\right)$$

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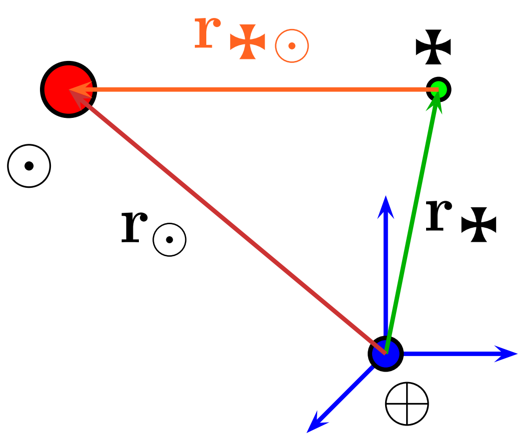
$$R_i = \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \frac{Gm_j}{r_j} \sum_{n=2}^{\infty} \left(\frac{r_i}{r_j}\right)^n P_n\left(\cos\theta_{ij}\right)$$

- No numerical problems
- Use recurrence relations
- Fast convergence for $r_i \ll r_j$: take few terms (1 for $\odot$, 2 for ☾)

Acceleration: Avoiding 1-1 Loss of Precision

- Several approaches to compute 3rd Body perturbation acceleration
- Must avoid 1-1 loss of precision
  - Gradient of truncated Perturbation Function (Maple sheet)
  - Truncated expansion of vector difference (Vallado, Montenbruck)
  - Exact $f(q)$ scalar formula for vector difference (Battin)
- Truncated methods converge fast:
  - Sun: $\frac{r_{sat}}{r_{\odot}} \ll 1$ only 1 term
  - Moon: $\frac{r_{sat}}{r_{\odot}} \ll \frac{r_{sat}}{r_{☾}} \ll 1$ at least 2 terms

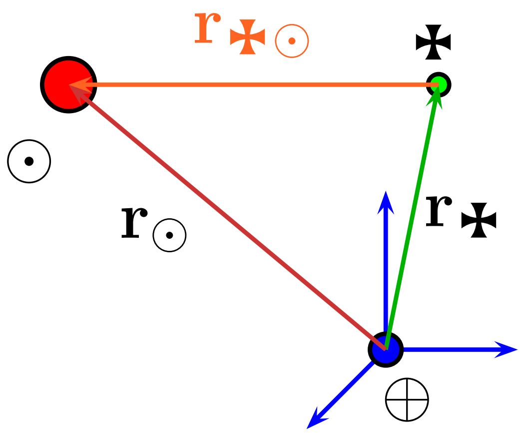
Acceleration: Truncated vector expansion



Perturbation acceleration due to Sun:

$$\mathbf{a}_{\odot} = +\mu_{\odot}\frac{\mathbf{r}_{\maltese\odot}}{r^3_{\maltese\odot}} - \mu_{\odot}\frac{\mathbf{r}_{\odot}}{r^3_{\odot}}$$

Acceleration: Truncated vector expansion



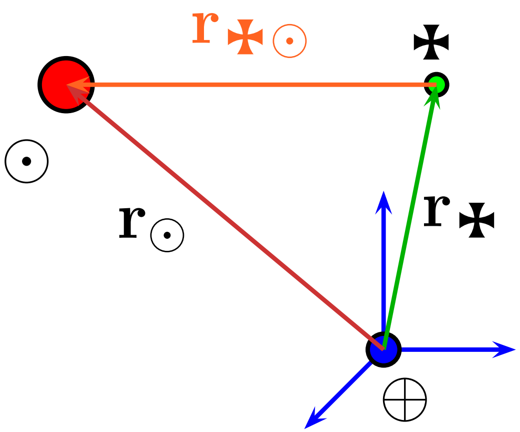
Perturbation acceleration due to Sun:

$$\mathbf{a}_{\odot} = +\mu_{\odot}\frac{\mathbf{r}_{✠\odot}}{r_{✠\odot}^{3}} - \mu_{\odot}\frac{\mathbf{r}_{\odot}}{r_{\odot}^{3}}$$

Taylor expansion of $\dfrac{1}{{r_{✠\odot}}^{3}} = \dfrac{1}{|\mathbf{r}_{\odot} - \mathbf{r}_{✠}|^{3}}$:

$$\mathbf{a}_{\odot} = -\frac{\mu_{\odot}}{{r_{\odot}}^{3}}\left[\mathbf{r}_{✠} - 3\frac{\mathbf{r}_{✠}\cdot\mathbf{r}_{\odot}}{{r_{\odot}}^{2}}\mathbf{r}_{\odot} + \ldots\right]$$

Acceleration: Truncated vector expansion



Perturbation acceleration due to Sun:

$$\mathbf{a}_{\odot} = +\mu_{\odot}\frac{\mathbf{r}_{\maltese\odot}}{r_{\maltese\odot}^3} - \mu_{\odot}\frac{\mathbf{r}_{\odot}}{r_{\odot}^3}$$

Taylor expansion of $\dfrac{1}{{r_{\maltese\odot}}^3} = \dfrac{1}{|\mathbf{r}_{\odot} - \mathbf{r}_{\maltese}|^3}$:

$$\mathbf{a}_{\odot} = -\frac{\mu_{\odot}}{{r_{\odot}}^3}\left[\mathbf{r}_{\maltese} - 3\frac{\mathbf{r}_{\maltese}\cdot\mathbf{r}_{\odot}}{{r_{\odot}}^2}\mathbf{r}_{\odot} + \ldots\right]$$

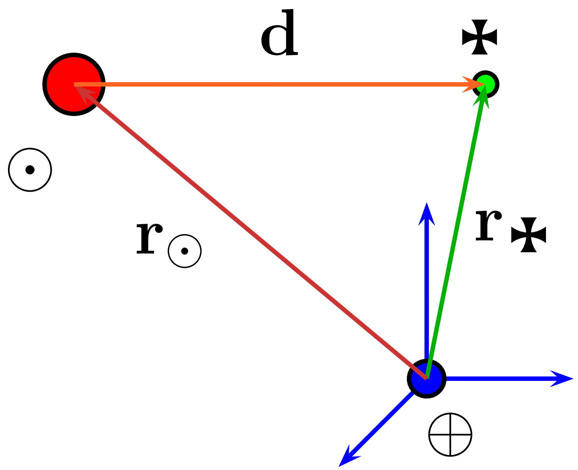
One additional term for the Moon:

$$\mathbf{a}_{\leftmoon} = -\frac{\mu_{\leftmoon}}{{r_{\leftmoon}}^3}\left[\mathbf{r}_{\maltese} - 3\frac{\mathbf{r}_{\maltese}\cdot\mathbf{r}_{\leftmoon}}{{r_{\leftmoon}}^2}\mathbf{r}_{\leftmoon} - \frac{15}{2}\left(\frac{\mathbf{r}_{\maltese}\cdot\mathbf{r}_{\leftmoon}}{{r_{\leftmoon}}^2}\right)^2 r_{\leftmoon} + \ldots\right]$$

- Vallado, 3rd Ed, p. 571 ff; Montenbruck, p. 69 ff

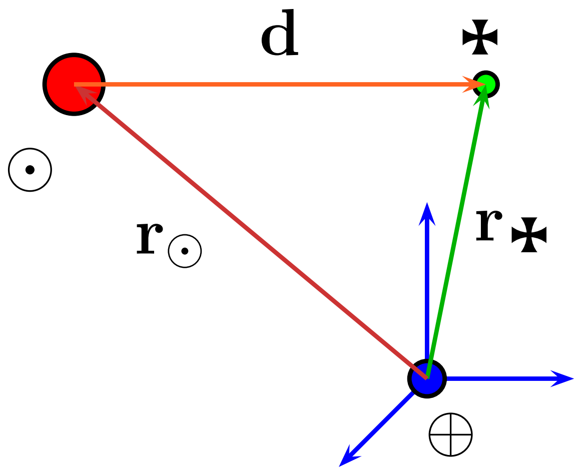
Acceleration: exact scalar function

$$\mathbf{a}_{\odot} = -\mu_{\odot}\frac{\mathbf{d}}{d^3} - \mu_{\odot}\frac{\mathbf{r}_{\odot}}{r_{\odot}^3}$$



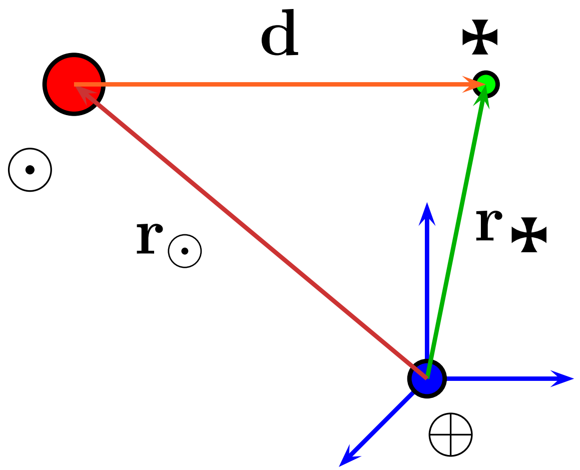
Acceleration: exact scalar function

$$\mathbf{a}_\odot = -\mu_\odot \frac{\mathbf{d}}{d^3} - \mu_\odot \frac{\mathbf{r}_\odot}{r_\odot^3} = \frac{-\mu_\odot}{d^3}\left(\overbrace{\mathbf{r}_{✠} - \mathbf{r}_\odot}^{\mathbf{d}} + \frac{d^3}{r_\odot^3}\mathbf{r}_\odot\right) =$$



$$\mathbf{a}_{\odot} = -\mu_{\odot}\frac{\mathbf{d}}{d^3} - \mu_{\odot}\frac{\mathbf{r}_{\odot}}{r_{\odot}^3} = \frac{-\mu_{\odot}}{d^3}\left(\overbrace{\mathbf{r}_{\maltese} - \mathbf{r}_{\odot}}^{\mathbf{d}} + \frac{d^3}{r_{\odot}^3}\mathbf{r}_{\odot}\right) =$$

$$= \frac{-\mu_{\odot}}{d^3}\left[\mathbf{r}_{\maltese} - \mathbf{r}_{\odot}\underbrace{\left(1 - \frac{d^3}{r_{\odot}^3}\right)}_{-f(q)}\right]$$



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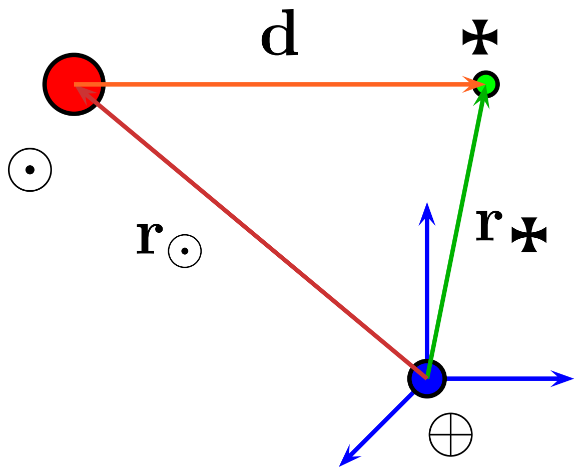

$$q = \frac{\mathbf{r}_{\maltese} \cdot (\mathbf{r}_{\maltese} - 2\mathbf{r}_\odot)}{\mathbf{r}_\odot \cdot \mathbf{r}_\odot}$$

$$f(q) = q\,\frac{3 + 3q + q^2}{1 + (1+q)^2}$$

Acceleration: exact scalar function

$$\mathbf{a}_\odot = -\mu_\odot \frac{\mathbf{d}}{d^3} - \mu_\odot \frac{\mathbf{r}_\odot}{r_\odot^3} = \frac{-\mu_\odot}{d^3}\left(\overbrace{\mathbf{r}_\maltese - \mathbf{r}_\odot}^{\mathbf{d}} + \frac{d^3}{r_\odot^3}\mathbf{r}_\odot\right) =$$

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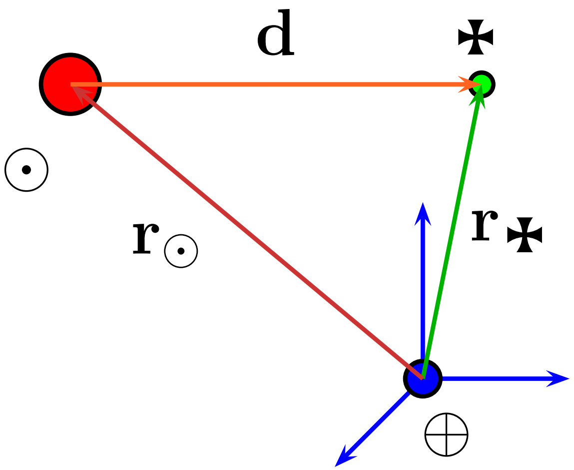

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$$f(q) = q\,\frac{3 + 3q + q^2}{1 + (1+q)^{\frac{3}{2}}}$$

- Note the different sense of $\mathbf{d}$
- The same for the Moon vector $\mathbf{r}_{☽}$, with $\mu_{☾}$.
- Cf. Battin, *Introduction to the Mathematics and Methods of Astrodynamics*, AIAA

Position Vectors $\mathbf{r}_{\odot}$ and $\mathbf{r}_{☾}$

Needed for several reasons:

- 3rd Body perturbation
- Sun's radiation pressure perturbation
- Shadow function for SRP
- Solar panel orientation, attitude through *sun sensor*, etc.

Depending on required precision:

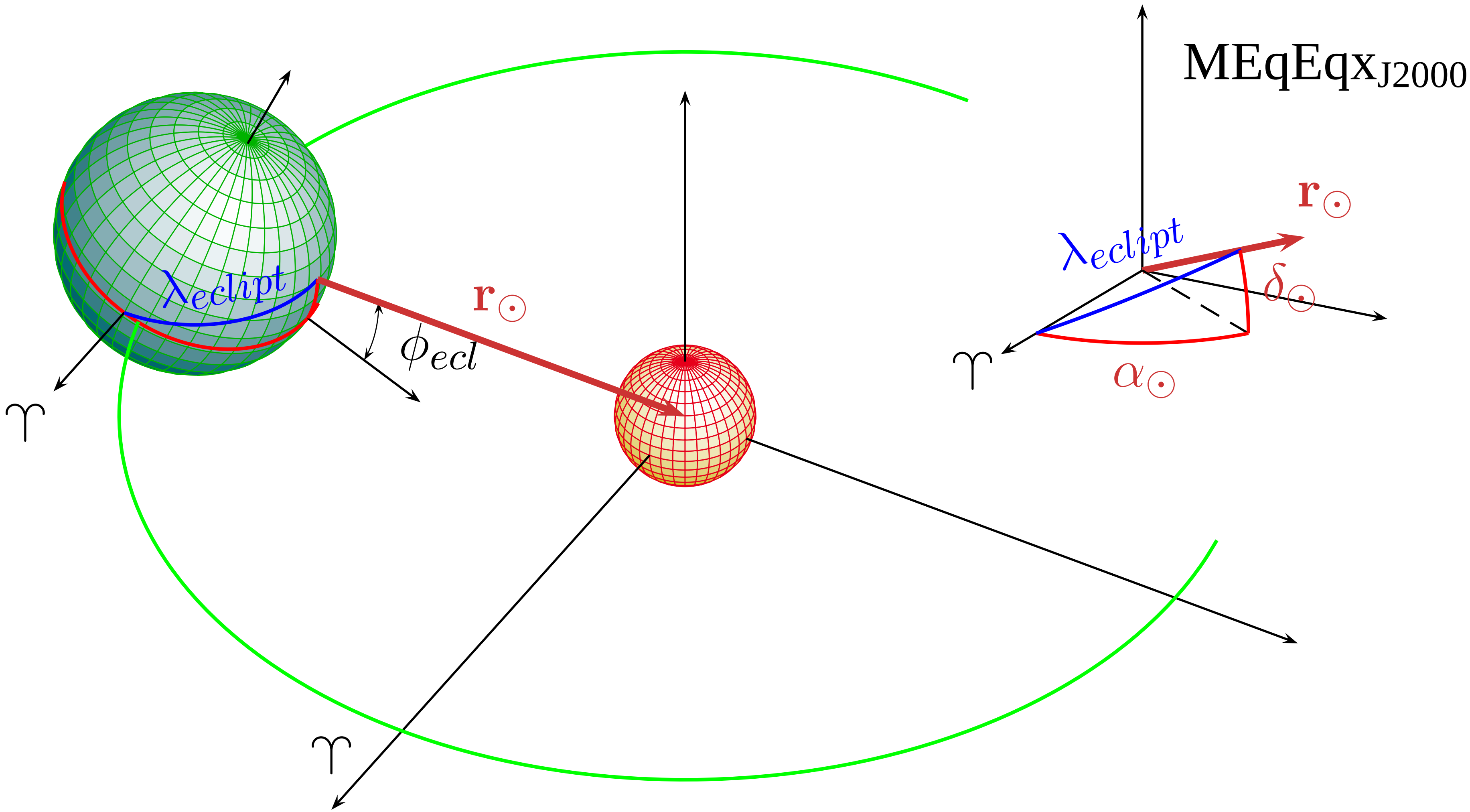
Low: Probably, no need to compute this perturbation

Middle: *Astronomical Almanach* formulas for the Sun ($\odot$) and Moon (☾) position vectors Vallado routines

High: Interpolate high-precision ephemeris files (JPL DEXXX)

- DE200 series: uses J2000
- DE400 series: uses ICRF

Geometry for Sun Position Vector



Almanach routines use Ecliptic longitude λ and latitude ϕ. Transfer to rectangular coordinates and Equatorial right ascension α and declination δ

Sun right ascension	$\alpha_\odot$
Sun declination	$\delta_\odot$
Obliquity of Ecliptic	$\epsilon \simeq \phi_{ecl}$

Sun Position Vector: Almanach

- Julian Centuries from J2000: $T_{UT1} = \dfrac{\text{JD} - 2,451,545.0}{36,525}$
- Mean Longitude $\Omega + \omega$ of the Sun (Earth+180):

$$\lambda_{M_\odot} = 280.4606184 + 36000.77005361\, T_{UT1} + \ldots$$

- Mean anomaly for the Sun

$$M_\odot = 357.5277233 + 35999.050\, T_{TDB}$$

- Reduce angles to quadrant [0,360]
- Ec. Kepler or expansion:

$$\nu_\odot = M_\odot + 2\, e_\oplus \sin M_\odot + \frac{5\, e_\oplus}{4} \sin 2M_\odot + \ldots$$

- Earth orbit eccentricity: $e_\oplus = 0.016708617 + \ldots$

Sun Position Vector: Almanach

- Sun ecliptic longitude and latitude

$$\lambda_{ec} \simeq \nu_{\odot} = \lambda_{M_{\odot}} + 1.914666471 \sin M_{\odot} + \ldots \qquad \phi_{ec} \simeq 0$$

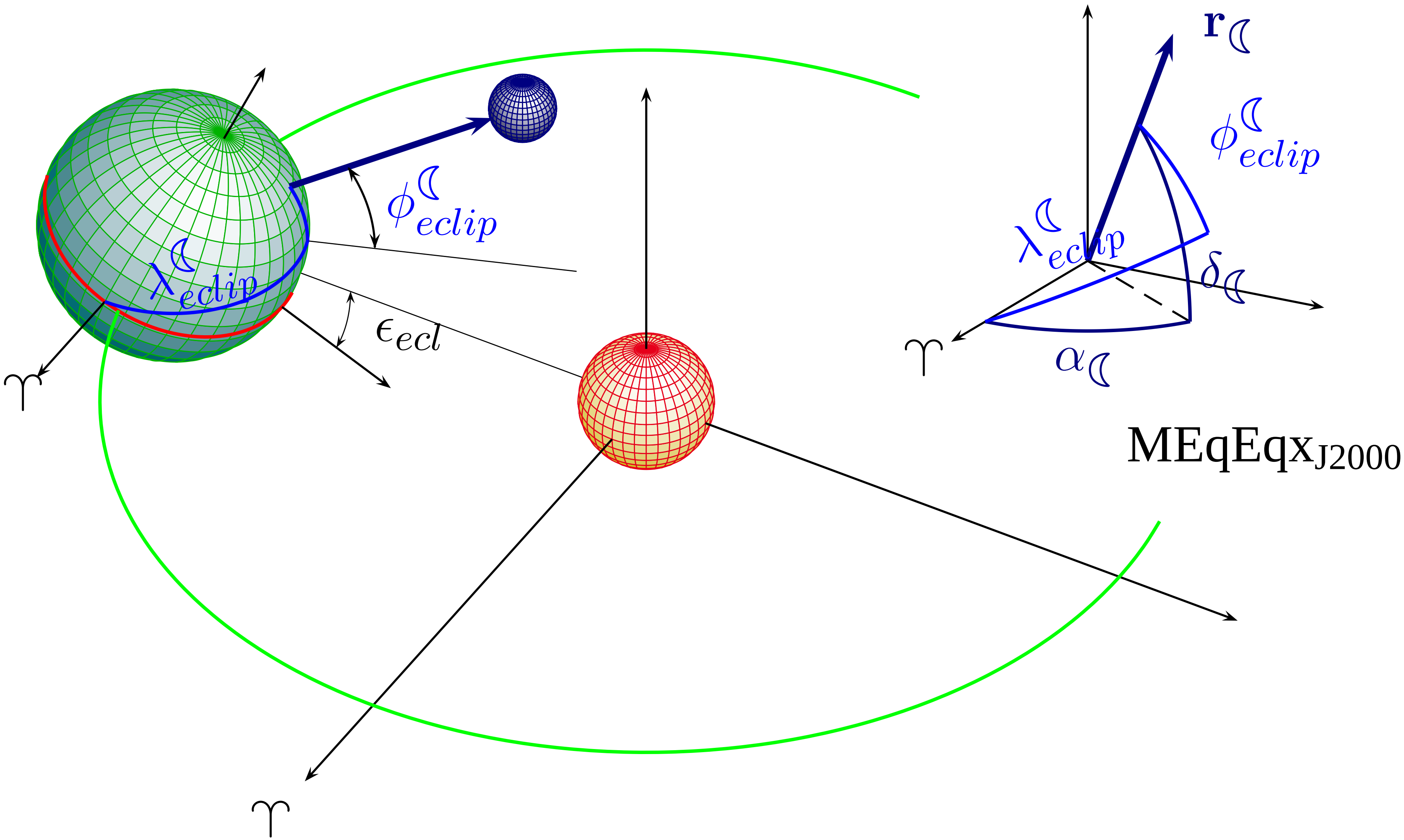
- Obliquity of the ecliptic: $\epsilon = 23.4329291 - 0.13004\,T_{TDB}$
- Sun distance (AU):

$$r_{\odot} = 1.000140612 - 0.016708617 \cos M_{\odot} - 0.000139589 \cos 3M_{\odot}$$

$$\mathbf{r}_{\odot} = r_{\odot} \begin{Bmatrix} \cos \lambda_{ec} \\ \cos \epsilon \sin \lambda_{ec} \\ \sin \epsilon \cos \lambda_{ec} \end{Bmatrix} \quad \text{Angular error} \simeq 0.01$$

Vallado:
```
SUBROUTINE SUN ( JD, RSun(4), RtAsc,Decl )
SUBROUTINE MOON ( JD, RMoon(4), RtAsc,Decl )
```

Moon Position Vector

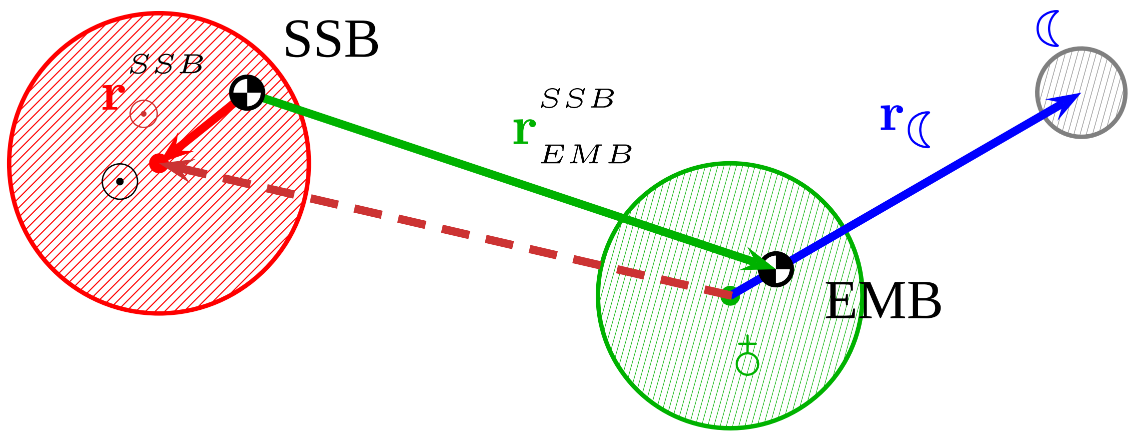


- Moon vector: More complex algorithm, because of strong Earth and Sun perturbations. Cfr. Vallado, p. 184, Montenbruck, p. 72.

JPL Ephemerides DExxx

- Complete N-body numerical integration (+nutation & libration)
  - DE200: Dynamic Equator and Equinox of J2000 (RK5)
  - DE400: ICRF (Hipparcos Cat.)
- 11 major bodies, some asteroids, No Sun-Earth vector!
  - 8 planets + Moon (relative to Earth)
  - Sun (relative to Solar System Barycenter)
  - Earth-Moon Barycenter (relative to SSB)
- Data blocks of 32 days, subdivided: 4 (☾), 16 (EMB)
- Coefficients of Chebyshev Polynomial interpolation (15-6). Recursion gives cartesian coordinates and speed of body.
- Indirect computation of Sun vector

JPL DEXXX Sun Position Vector

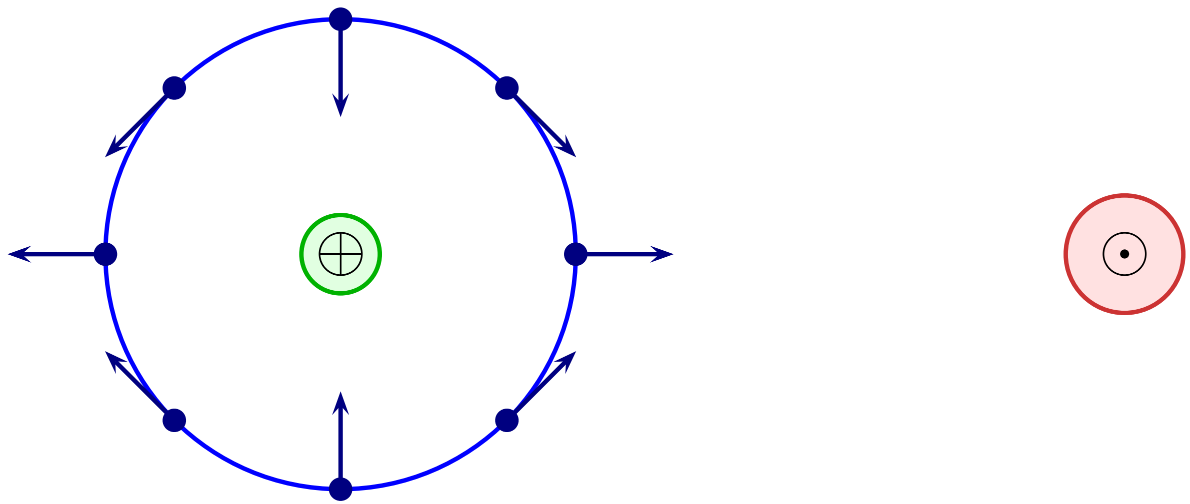


Solar System Barycenter, Eart-Moon Barycenter, Sun, Earth, Moon
Computation of Sun position vector from Ephemerides data:

$$\mathbf{r}_{♁\odot} = \frac{\mu_{☾}}{\mu_{♁} + \mu_{☾}} \mathbf{r}_{☾} + \mathbf{r}_{\odot}^{SSB} - \mathbf{r}_{EMB}^{SSB}$$

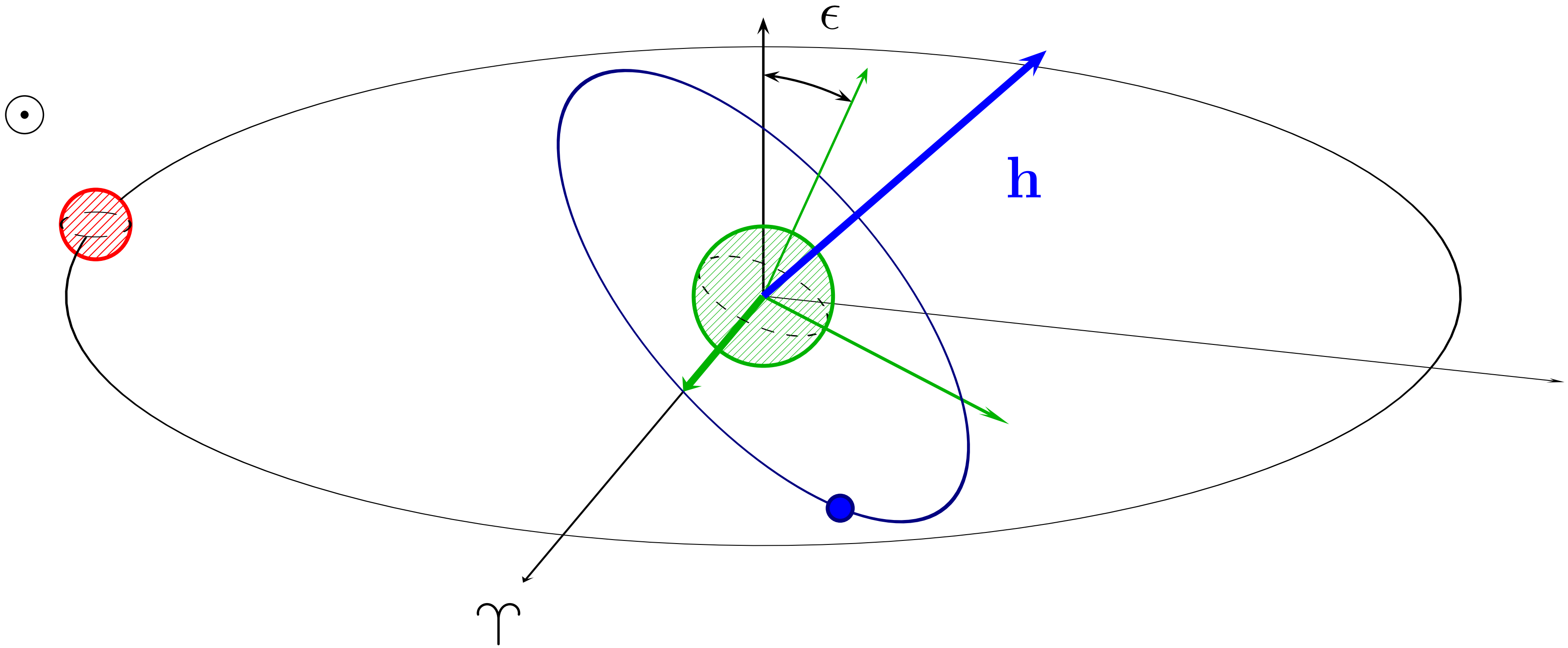
Visualization of 3rd Body Effects: Periodic

- Perturbation: difference between acceleration of satellite and that of Earth

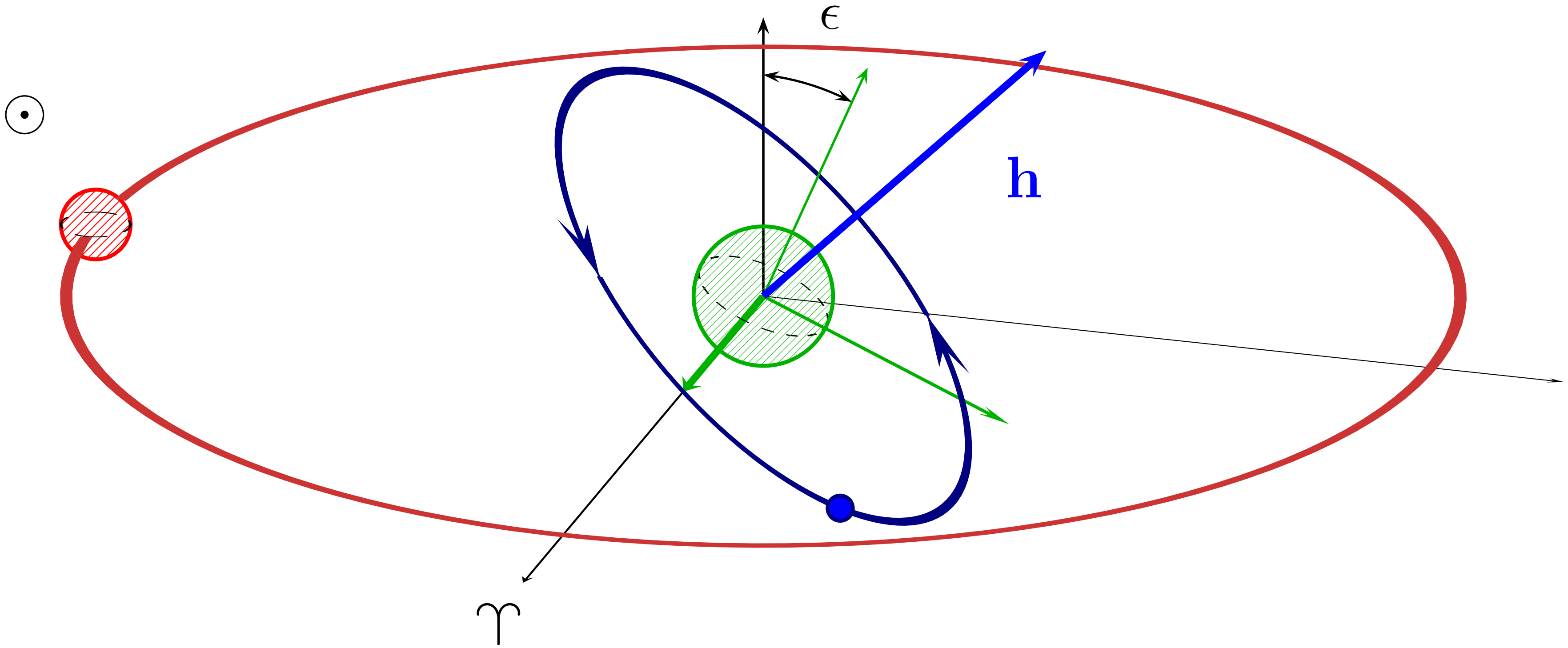


- Orbit tends to flatten in the direction of the perturbing body, which is itself rotating: $\rightarrow$ $\dot{\omega}$ (Secular), $\dot{e}$ (Periodic)

Visualization of 3rd Body Effects: Secular

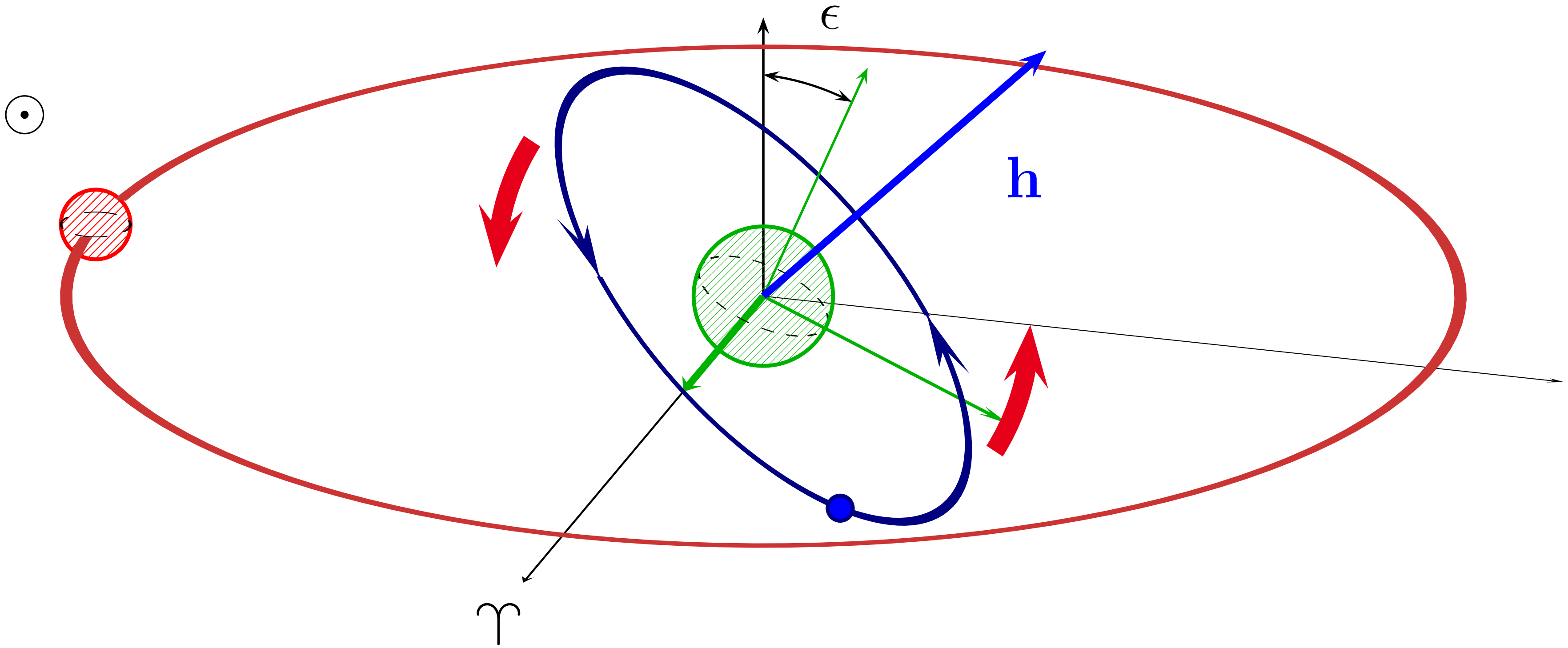


Visualization of 3rd Body Effects: Secular



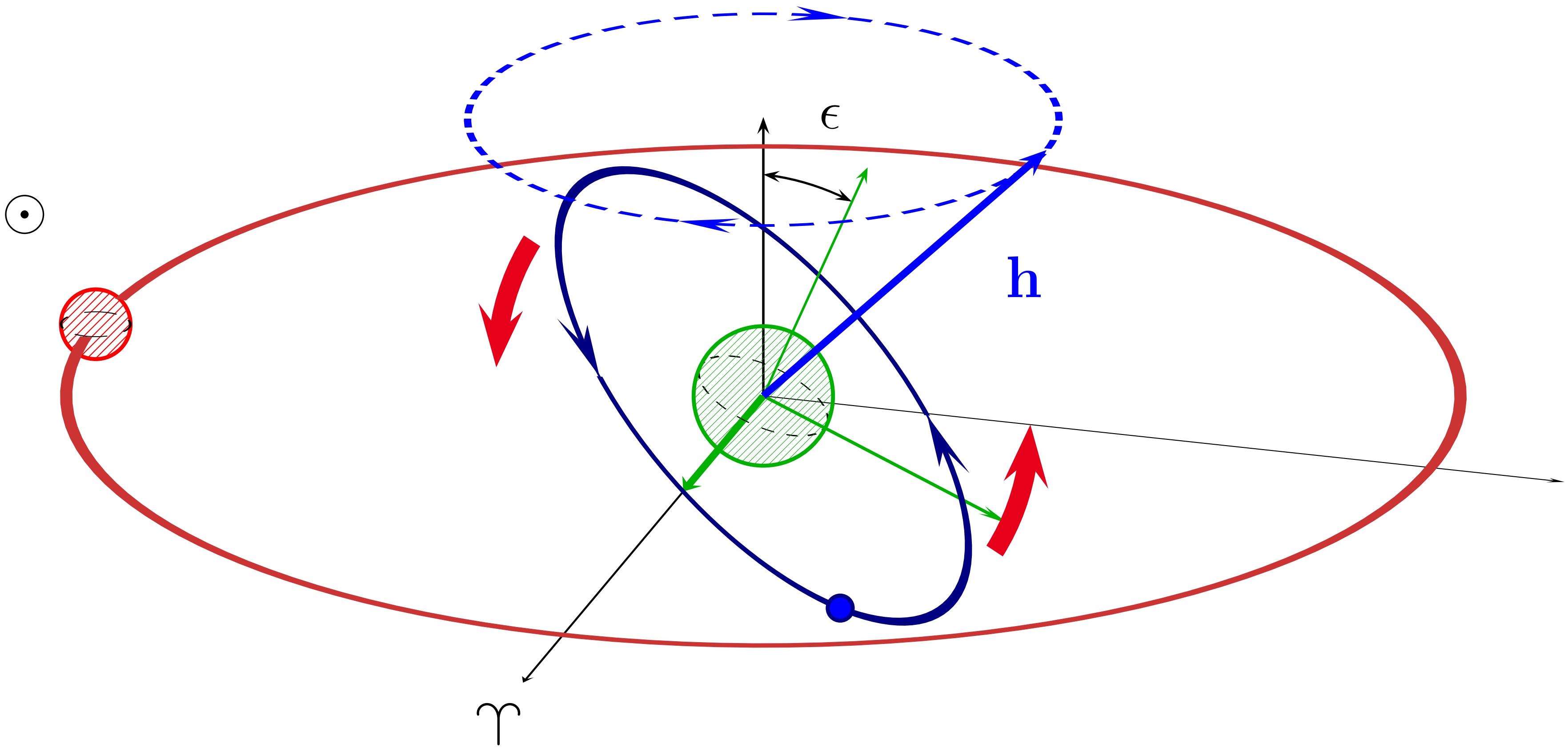
- Averaging: “smear” body over orbit: rotating rings

Visualization of 3rd Body Effects: Secular



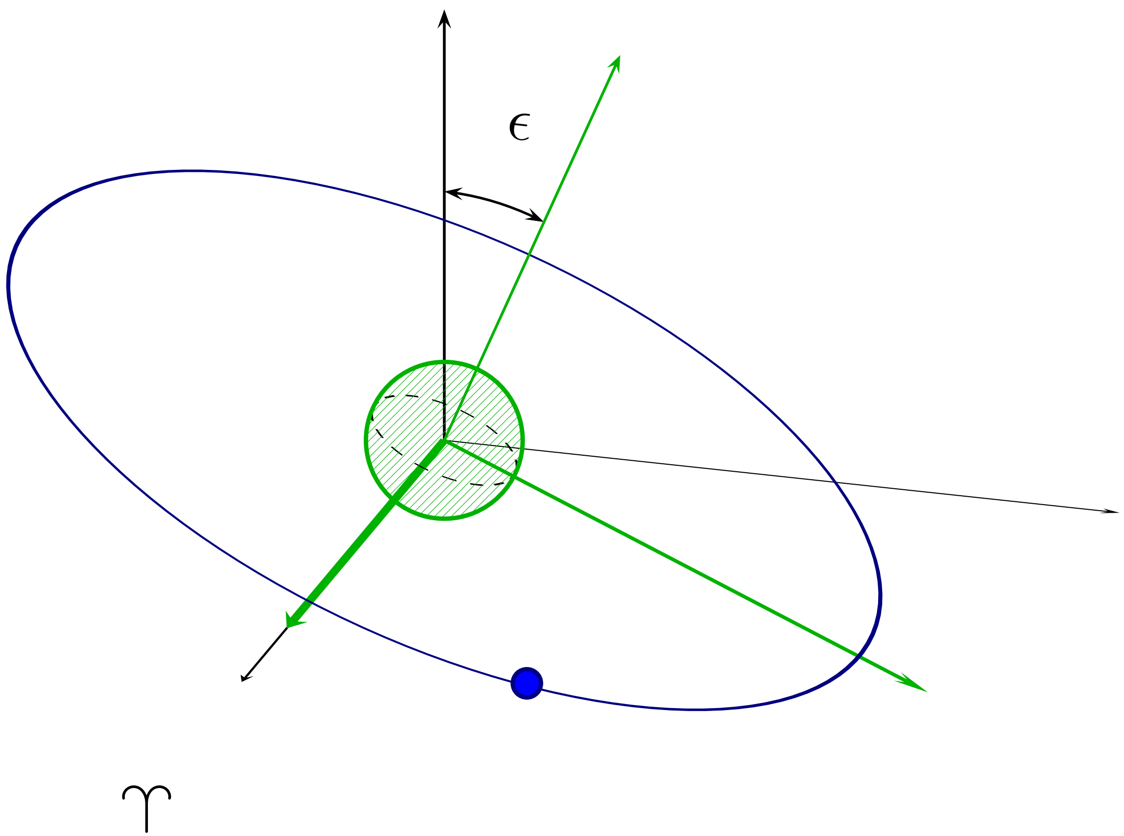
- Averaging: “smear” body over orbit: rotating rings
- Gravitational torque over satellite ring

Visualization of 3rd Body Effects: Secular



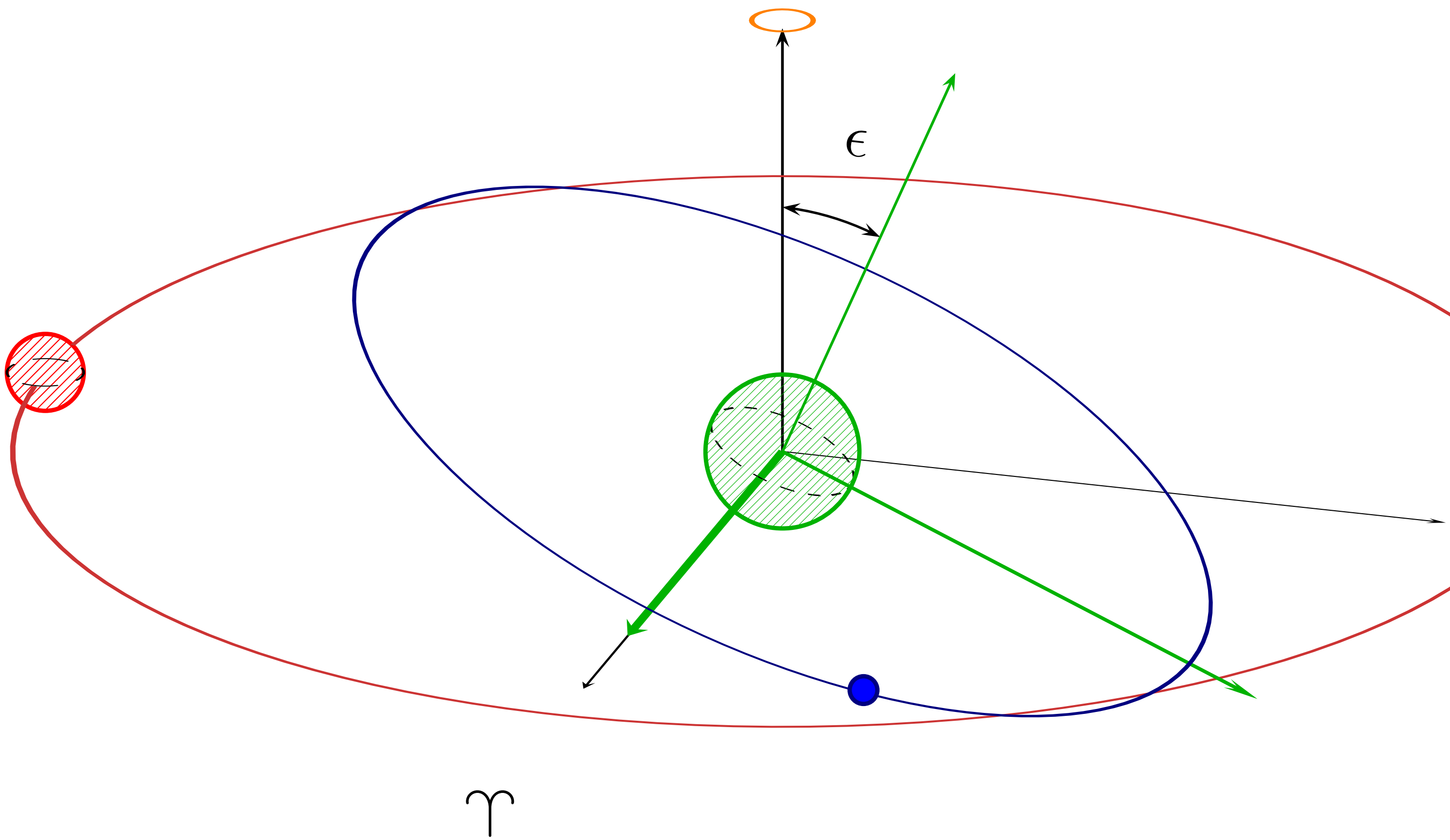
- Averaging: “smear” body over orbit: rotating rings
- Gravitational torque over satellite ring
- Precession about the Ecliptical Pole (Not Earth’s!) $\longrightarrow$ $\dot{\Omega}$, $\dot{i}$

Combined Effect on GEO Inclination



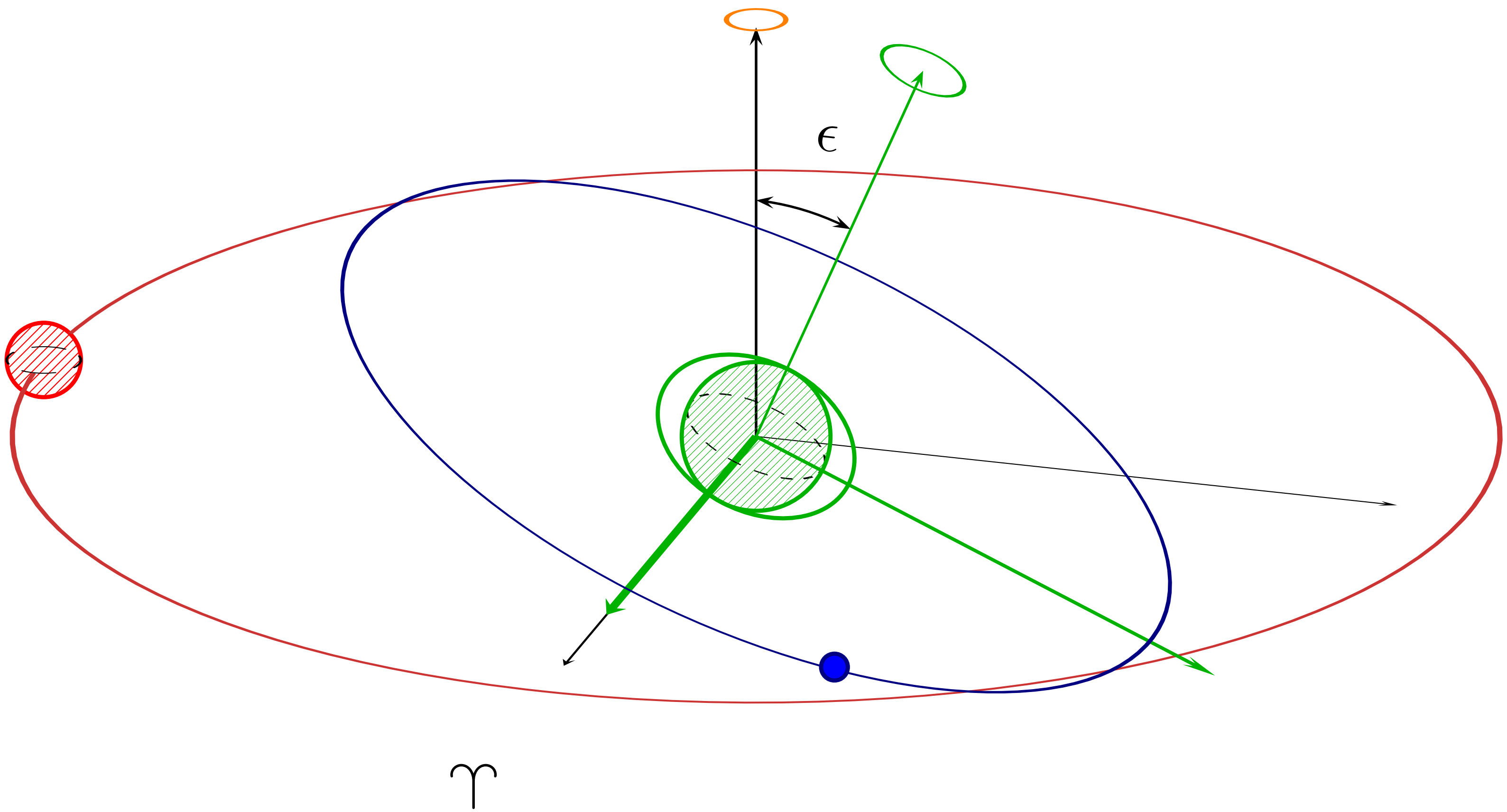
- Satellite in GEO orbit

Combined Effect on GEO Inclination



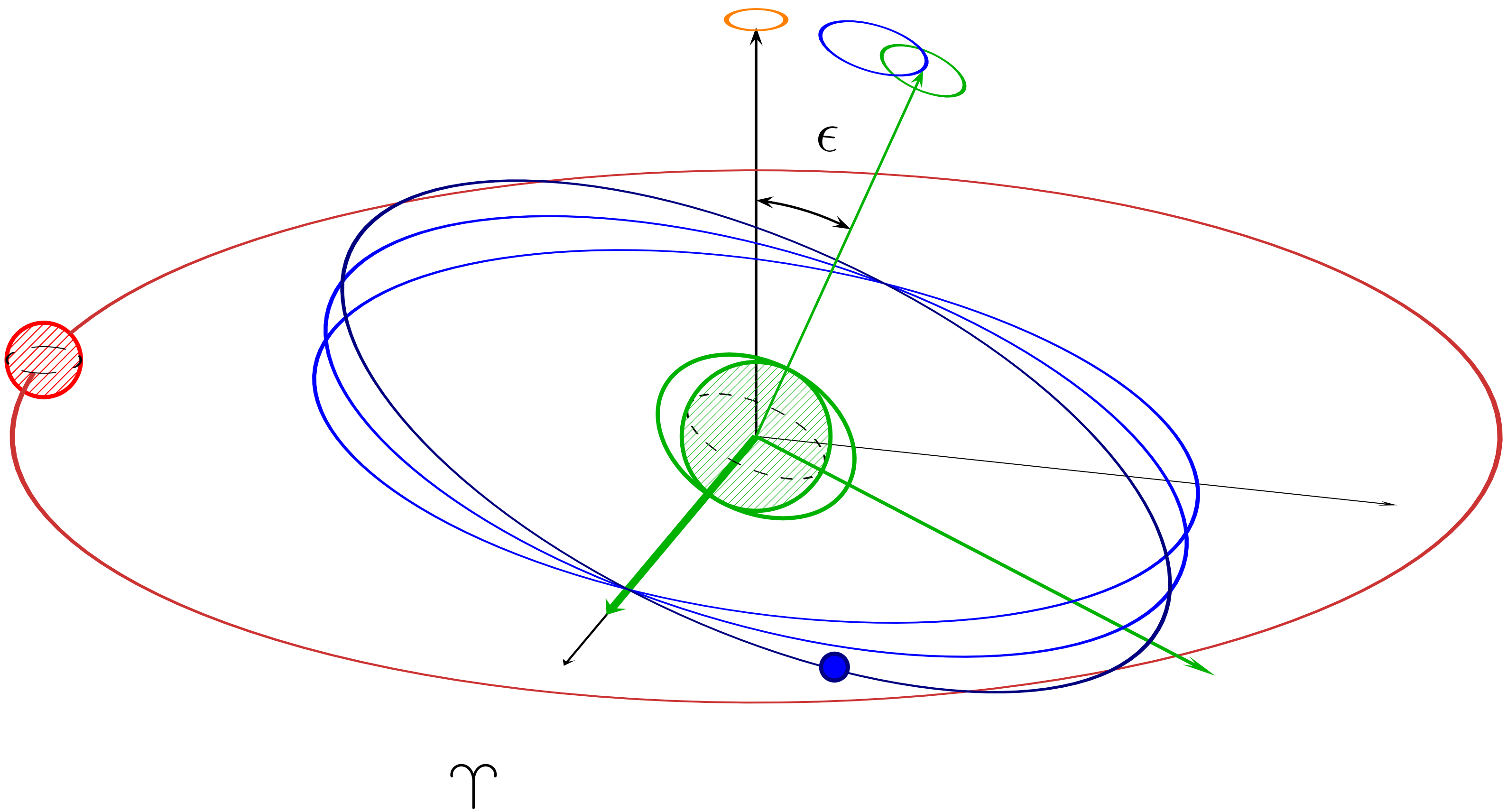
- Satellite in GEO orbit
- Sun+Moon effect: precession about the Ecliptic Pole

Combined Effect on GEO Inclination



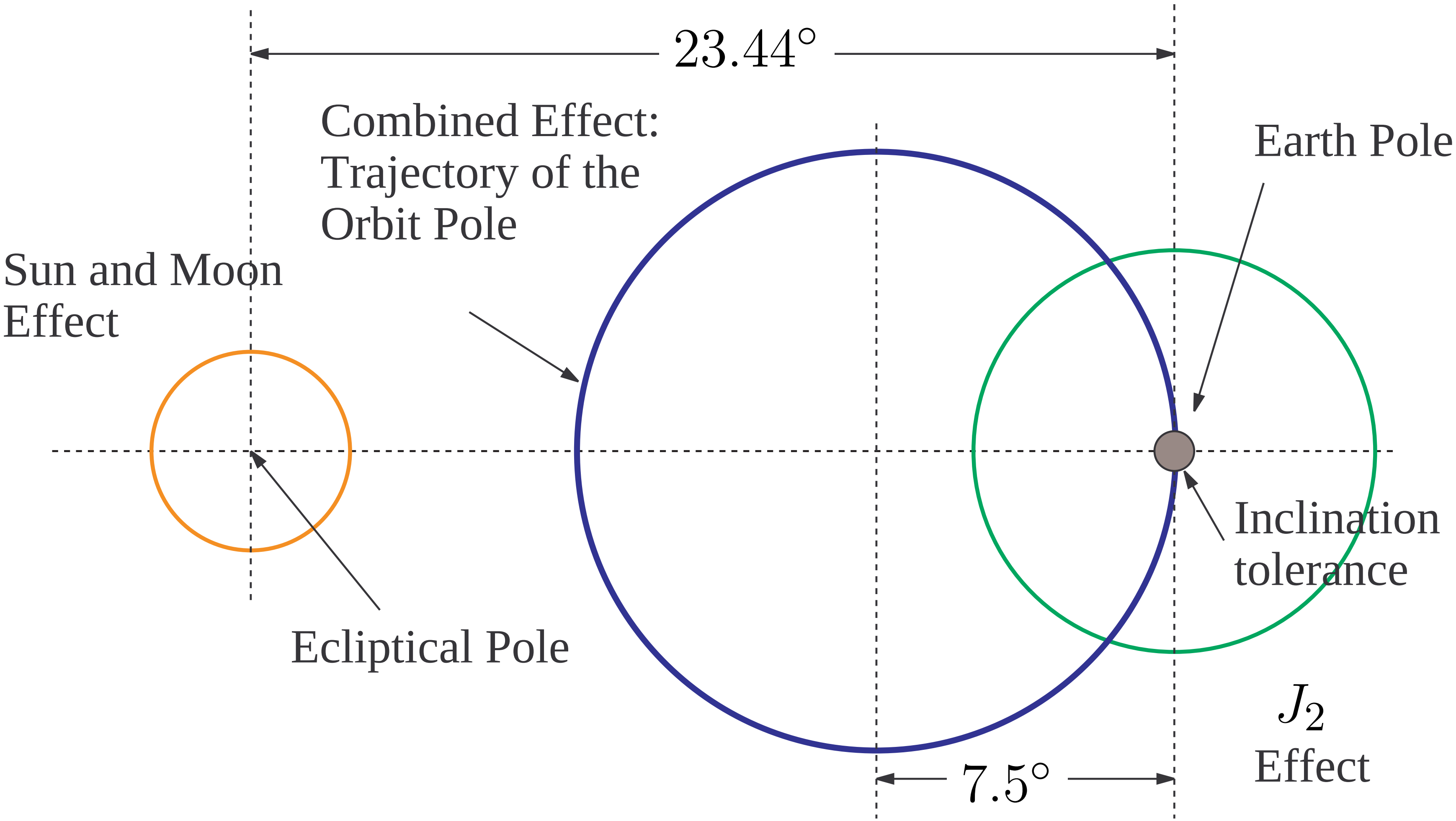
- Satellite in GEO orbit
- Sun+Moon effect: precession about the Ecliptic Pole
- Earth flattening: precession about the Earth Pole

Combined Effect on GEO Inclination



- Satellite in GEO orbit
- Sun+Moon effect: precession about the Ecliptic Pole
- Earth flattening: precession about the Earth Pole
- Combined effect: inclination change from 0-15 deg

Combined Effect on GEO Inclination



Inclination drift of a GEO satellite due to Sun and Moon 3rd body perturbation, plus J_2

Secular Effects (VOP results)

- Only Ω, ω, and M_0 suffer secular effects

$$\dot{\Omega} = -\frac{3}{8}\frac{{n_\odot}^2}{n}\frac{1+\frac{3}{2}e^2}{\sqrt{1-e^2}}\cos i\left(3\cos^2 i_\odot - 1\right)$$

$$\dot{\omega} = \frac{3}{4}\frac{{n_\odot}^2}{n}\frac{1-\frac{3}{2}\sin^2 i_\odot}{\sqrt{1-e^2}}\left(2-\frac{5}{2}\sin^2 i + e^2/2\right)$$

- Small, because of the $n_\odot^2/n$ term; $e_\odot$ is zero
- The expressions for the Moon are more complex because of $e_☾$

Other Effects

- Periodic effects in all elements, especially i and e
- Coupling of eccentricity periodic changes with drag: oscillations in e cause lowering of perigee, and periodic increase of drag in low orbits.
- Up to 15 possible resonances.
- Moon secular effect more intense than Sun's (much closer):

$$\frac{\dot{\Omega}_{☾}}{\dot{\Omega}_{\odot}} \simeq \frac{\dot{\omega}_{☾}}{\dot{\omega}_{\odot}} = \frac{r_{\odot}^3 \mu_{☾}}{r_{☾}^3 \mu_{\odot}} \frac{2 - 3\sin^2 i_{☾}}{2 - 3\sin^2 i_{\odot}} \frac{\left(1 - e_{\odot}^2\right)^{3/3}}{\left(1 - e_{☾}^2\right)^{3/3}} \simeq 2.2$$

- Just like tides.

VOP Effects

	Gravity		3rd Body	Atm Drag	Rad Press
	Zonal	Sect/Tess			
a	P	P	P	P S	P
e	P	P	P	P S	P
i	P	P	P	P S	P
Ω	P S	P	P S	P	P S
ω	P S	P	P S	P	P S
M_0	P S	P	P S	P	P S

P: Periodic
S: Secular
Also: coupling effects

Source: Vallado, Battin