On Musical Self-Similarity

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On Musical
Self-Similarity

Intersemiosis as
Synecdoche and Analogy

Abstract

Self-similarity, a concept taken from mathematics, is gradually becoming a keyword in musicology. Although a polysemic term, self-similarity often refers to the multi-scalar feature repetition in a set of relationships, and it is commonly valued as an indication for musical 'coherence' and 'consistency'.

This investigation provides a theory of musical meaning formation in the context of intersemiosis, that is, the translation of meaning from one cognitive domain to another cognitive domain (e.g. from mathematics to music, or to speech or graphic forms). From this perspective, the degree of coherence of a musical system relies on a synecdochic intersemiosis: a system of related signs within other comparable and correlated systems. This research analyzes the modalities of such correlations, exploring their general and particular traits, and their operational bounds. Looking forward in this direction, the notion of analogy is used as a rich concept through its two definitions quoted by the Classical literature: *proportion* and *paradigm*, enormously valuable in establishing measurement, likeness and affinity criteria.

Using quantitative–qualitative methods, evidence is presented to justify a parallel study of different modalities of musical self-similarity. For this purpose, original arguments by Benoît B. Mandelbrot are revised, alongside a systematic critique of the literature on the subject. Furthermore, connecting Charles S. Peirce's *synechism* with Mandelbrot's *fractality* is one of the main developments of the present study.

This study provides elements for explaining Bolognesi's (1983) conjecture, that states that the most primitive, intuitive and basic musical device is self-reference, extending its functions and operations to self-similar surfaces. In this sense, this research suggests that, with various modalities of self-similarity, synecdochic intersemiosis acts as 'system of systems' in coordination with greater or lesser development of structural consistency, and with a greater or lesser contextual dependence.

Keywords

analogy, autosimilarity, Gestalt, intersemiosis, intersemiotic continuum, invariance, musical coherence, proportion, self-dissimilarity, self-reference, self-similarity, similarity, synecdoche, translatability.

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Introduction

1.1. About this study

This research is, at least in its early development, a continuation of my master's thesis (Pareyon 2004), which addresses the fundamental symbolic relationships between music and language. That initial investigation includes examples of synecdoche's structural functions,[1] and presents simple cases of intersemiotic translation between mathematics (algebra, geometry), verbal language (syntax, semantics), and music (metre, rhythm, melody, harmony), along with proposals for graphical representation.[2]

The current research studies self-similarity in a variety of musical possibilities,[3] particularly through the processes of intersemiotic translation. From this viewpoint, this study intends to provide answers to understanding why composers, musicologists and music analysts are continuously interested on self-similarity. Accordingly, synecdoche is interpreted as a fundamental connection between different mental categories,[4] and translatability ranges are introduced, from abstract

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[1] Roughly speaking, by synecdoche is understood a substitution of a part for its whole, or of the whole for its part, involving a functional reciprocity between the general and the particular. Synecdoche is a pervasive mental operation found in a vastness of instances in language, both verbal and nonverbal. The operational definition of synecdoche, used in this study, corresponds to what is stated in 3.2.

[2] Intersemiotic translation occurs, for example, by converting a musical sound into a graphical representation, or vice versa. The subject requires development, which occupies a separate section in 3.8.1.

[3] The lack of a convention in music theory, restricting the use of the term *self-similarity*, creates a conceptual gap that this research attempt to solve from different angles. To this end unusual synonyms of self-similarity, such as 'autosimilarity' or 'self-likeness', are avoided.

[4] The concept 'mental category', used widely by authors such as Lakoff (1987), Jackendoff (1993), Child (1994), Hofstadter (1995) and Givón (2002), refers in a general way, the different groupings of ontologies in the mind/brain, which facilitate the comparison and contrast relationships of similarity and difference, considered as basic operations for deduction, memory,

to concrete kinds of analogies, in order to explore the transitions from the indeterministic paradigms, to the objective, measurable and predictable relations in musical, pre-musical and metamusical cases.

The central hypothesis of this study relies upon the assumption that musical self-similarity involves functions of coherence and consistency on several symbolic levels; in consequence, these functions can only be understood within a framework of synecdochic intersemiosis.[5] In a first development, this hypothesis stimulates the investigation of music as a self-referential phenomenon, and the cultural interpretative scrutiny of related concepts. In this context the concept of 'fractal' is analyzed.[6] Such a concept, borrowed from modern mathematics, is increasingly employed in music theory (especially after Voss and Clarke 1975, Gardner 1978, Bolognesi 1983, and Hsü 1993) but does not necessarily have a clearer or more appropriate use in the same proportion.[7] Looking for clarity in exploring this conceptualization, it is crucial to connect the—apparently new—'fractal dilemma' with problems of music theory that originate in the Pythagorean tradition. Such

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and aesthetic experience. This concept is exposed from different views on subchapters 1.2.2., 1.2.3., and 4.4.

[5] i.e. a transversal transformation of sign systems across different sign categories; for instance, transmitting and transforming a 'musical idea' with the usage of its graphic representation, its verbalized 'explanation', or its symbolic modification through physical gestures. For a definition of 'intersemiosis' see subchapter 3.8.

[6] Benoît B. Mandelbrot (1977, 1982) coined the term 'fractal' to refer a geometry with 'fractured' dimension, i.e. a geometry that cannot be described with an entire dimension, as a straight line with a dimension 1, a square with a dimension 2, or a cube with a dimension 3. Instead, it should be described with a fragmented dimension. As a metaphor of a geometry closed in itself reproducing similar figures, it is interesting the paronomasia with the ancient Greek verb φρακτὸσ, from φράσσω (φράζω), "to tighten the one against the other, spear against spear, shield against shield" (M. Anatole Bailly, *Dictionnaire Grec Français*, Librairie Hachette, Paris, 1894; p. 609).

[7] The term 'fractal', whose use in music theory is generally obscure, often replaces the concept of self-similarity. It is possible that this replacement is due to certain fashion of describing musical aspects using relatively new notions from physics, often borrowing them from the theory of dynamical systems. This fashion is evident, for instance, by noticing the absence of the term 'fractal' in the Grove's dictionary 1982 edition, compared to its inclusion in the actual electronic edition of Grove Music Online (2010). In this edition there are 12 composers associated with the term 'fractal' included as a keyword to their musical output (such composers are Marco Di Barco, Rolf Enström, Robert Sherlaw Johnson, Adolfo Núñez, Evan Parker, Elliot Sharp, Mieko Shiomi, Tomaž Svete, Jean-Claude Risset, Horacio Vaggione, Harri Vuori and Charles Wuorinen). By contrast, within the same source there is only one composer (Christopher Fox) whom, omitting the word fractal, uses instead 'self-similarity'.

dilemmas characterize the preference of a geometric abstraction over the immediate musical practice and its associated tradition. This preference can be interpreted also as an opposition between two different classes of precepts: one representing the conceptualization of the physical parts of music, and the other representing the sense of a musical practice as a social process. This sort of opposition, triggering apparent contradictions in the history of music theory, stimulates further discussion on the proper use of the concept of *musical self-similarity* in a variety of adaptations. Within this framework, this study aims to facilitate the transition from an *engineering perspective* (the constructivist-structuralist idea of musical self-similarity), to an *ecological perspective* (a post-structuralist, dynamic and holistic idea inspired by Peircean *synechism*, related after this study's subchapters 3.8. and 3.9., to Peirce's own notion of *map of the map*); in this context, the latter is invoked using the ecological figure of *house of the house*—or more generally, *worlds within worlds*.[8]

1.1.1. General background

The correspondence between the harmony of the tones and their arithmetic ratios —whose discovery in the Western cultures is attributed to Pythagoras—is a subject permanently open to debate in modern musicology. The 'Pythagorean dilemma', antecedent of the fractal dilemma, has been examined from different perspectives by such musicologists as Langer (1953), Norton (1984), McClary (1987), Smith Brindle (1987), Dahlhaus (1990), and Ockelford (2005), and is by no means a recent subject. It suffices to glance over almost any medieval treatise on music, to have an idea of how much the Pythagorean paradigm is contextually discussed in music theory. From Boëthius' *De institutione musica* (*ca.* 520) to Jacob of Liège's *Speculum Musicae* (*ca.*1340), and other subsequent treatises, different interpretations of the Pythagorean proportions are presented, with geometric and numerical analysis, and studies on the division of the monochord and the tetrachords. Nevertheless, this exhaustive work does not achieve the unification of the criteria of harmony, which

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[8] The term 'synechism' (see Peirce 1893) is adapted in this study as the –ism category for *synecdoche*, as defined in 3.2. An introduction on the concept of 'ecology' used in this study is provided in subchapter 3.7. On the notion 'worlds within worlds' see Parsegian (1968:589); this notion is connected to Jakob von Uexküll's (1940, 1957) *Umwelt*, and to its adaptation into musicology made by Mark Reybrouck (see Reybrouck 2001, 2005).

are mixed with Aristotelian *Metaphysics* ideas, crystallized in treatises by authors as diverse as Franco of Cologne (1260), Gioseffo Zarlino (1558), Pietro Cerone (1609), Robert Fludd (1618), Johannes Kepler (1619), and Athanasius Kircher (1650).

Discussion about the nature of music, either emotive or rational, can be traced to classical sources: *De musica*, attributed to Plutarch (Pseudo–Plutarch MCr 29), says that Aristotle understands two different types of musical meaning: the arithmetic type and the harmonic type, a division also related to the difference between ratio-proportion of sound, and the corporal sensation of perceived music. At the same time, Plato, in his dialogues *Timaeus* (47c7–e2) and *Republic* (331e, 412e, 461d, 531a–b, 546a–d), associates different ethical values to different types of musical practices, dividing them among those that are socially beneficial and those that are harmful. Moreover, Aristotelian dialectic separates *concrete* from *abstract*, a distinction associable to the separation between body and soul, and to the division between individual and social—an issue stressed by Cartesianism—in a wide context in which music serves as a model.[9] Sensitive to this division, McClary (1987:15) claims that

> [F]rom very early times up to and including the present, there has been a strain of Western culture that accounts for music in non-social, implicitly metaphysical terms. But parallel with that strain (and also from earliest times) is another which regards music as essentially a human, socially grounded, socially alterable construct [...] Most polemical battles in the history of music theory and criticism involve the irreconcilable confrontation of these two positions.

Likewise, criticizing Allen Forte and David Epstein, McClary indicates that the modern theoreticians "found similar correspondences between triads and properties of physical acoustics, or attempted to validate compositions on the basis of their apparently mechanical generation from pitch class sets" (*ibid.*). Successively McClary confesses her "sympathy" with what she calls the "social and human tendency" of music, but she never explains why this has an irreconcilable character opposed to the Pythagorean conception. Besides, it is curious that for McClary the mathematical aspects of music and still the metaphysical ones, are "non-social" terms, when in fact

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[9] Dialectics in constructive and philosophical systems of music is a subject that appears repeatedly throughout this study. In this regard I owe acknowledgement to Alfonso Padilla's work on *Dialectics and music* (1995).

the history and development of mathematics is inconceivable not being "essentially a human, socially grounded, socially alterable construct".[10]

Carl Dahlhaus (1990:3–11) characterizes the same dichotomy in the conflict between the harmonic models of François-Joseph Fétis (1784–1871) and Hugo Riemann (1849–1919), the first one conceived as an arrangement of relations among pitches of a scale, as "a result of mankind's historical and ethnic diversity" (Dahlhaus, *op. cit*.:3), and the second one as a result of a "physicalism" that reveals the basics of some "acoustic facts":

> Riemann took over the thesis that tonality is based on acoustical facts from a tradition of "physicalism" (Jacques Handschin) extending back to Rameau. Thus the dominant tends toward the tonic because the dominant chord is contained within the harmonic series of the tonic chord's root. But Fétis's concept of tonality represents the opposite thesis, the conviction that it is a mistake to explain musical relationships in terms of mathematics or acoustics. (Dahlhaus, 1990:7)

The idea that the concrete and the abstract notions, the corporal and the spiritual, the physical and the social, are irreconcilable oppositions, is—as it is already pointed out—a Platonic product refined by Aristotle, achieving its greater expression in the Cartesian rationalism and the concept of *res extensa* and its opposite *res cogitans*. The first one was used to denote the physical world, whereas the second one served to denote the thinking being, assuming that body and thought are made of 'substances' completely different. Yet in modern mathematical thought there is an echo of this Cartesian separation, although, as one can notice in the very voice of Benoît B. Mandelbrot (1924–2010), a doubt arises about the real separation between 'the nature' and 'the nature of mathematics': "[is] the Mandelbrot set nature or is it mathematics?" (see Peitgen, Richter and Saupe 2004:783).[11]

As it is synthesized—from very different points of view—by authors like Bateson (1972), Bohm (1980), Damásio (1994, 2000), Guerra Lisi and Stefani (1997), and

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[10] McClary's attack against Forte and Epstein, and in general against an alleged 'mathematicist' approach to music seems to have been stimulated by the controversy brought on these terms, between Forte and Richard Taruskin (see "Letter to the Editor from Richard Taruskin" & "Letter to the Editor in Reply to Richard Taruskin from Allen Forte", *Music Analysis*, vol. 5, no. 2/3, 1986, pp. 313–320, 321–337), in which Taruskin argues for a historicist conception of music, in contrast to Forte's rationalist analysis.

[11] See the complete quote on page 29.

Reygadas and Shanker (2007), the proof provided by natural sciences and experimental psychology leads to admit that the Cartesian rationalism is wrong: mind and brain, idea and body, culture and nature, are not completely different 'substances' and neither are they opposed natures. They reveal, rather, a diversity of aspects from the same nature. Upon this assumption the Cartesian individualism is also pointless, conceiving a separation between individual and society, making a parallel with the supposed separation between soul and body, or between mind and brain. Idea and action, intellect and emotion, individual and society, are, instead, coordinated aspects of the same integrity.

Musicology did not completely absorb the change of paradigm between Cartesian rationalism and dynamic holism—*holomovement* or undivided wholeness, as David Bohm started to call it around 1955, within the context of quantum physycs.[12] That is why the musicological domain still confronts positions on a social and human ideal of the musical self, in conflict with its abstract representation, supposedly 'non-human' or 'non-social', as McClary states (1987:15). This research proposes—analyzing self-similarity in music as a case study—that the Pythagorean tendencies and the abstract ideas of music are not in "irreconcilable confrontation" regarding cultural and social perspectives. On the contrary, they share fundamental features and present complementary aspects in the history of music.

It is quite obvious that music does not always operate in a manner similar to mathematics. If so, music would not have the primary need to articulate sounds. But neither does it always operate in a similar manner to verbal language. Music elaborates through itself with a greater system of cognition in which music shares some aspects with mathematics and language in general. In this way, within the thresholds of similarity and analogy, many pivotal aspects of reasoning and emotion

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[12] A step in this transition is the shift of the transformational paradigm instead the Cartesian one in the theory of David Lewin (*Generalized Musical Intervals and Transformations*, 1987). This theory initiated a fruitful debate on new rationalist strategies of musical analysis, beyond Chomskyan first generativism (1955, 1957) and its many formal revisions (e.g. Salomaa 1973). However, these new perspectives yet do not fully clarify connections with other analytical tools of musicology (e.g. Schenker 1932, Salzer 1952), musical semiotics (e.g. Nattiez 1976, Schuller 1986, Monelle 1992, Tarasti 1994), and aesthetic philosophy (e.g. Ingarden 1962, Foucault 1966, Barthes 1984). The present study proposes an initiative for this connection, via self-similarity.

unite music with the mathematical abstractions, and with numerous aspects of verbal and gestural intercourse.

The study of similarities as particular systems and relationships in music is an almost infinite field, with an endless variety of subjects and situations. Repetition in its many singularities, and that of structural correspondences, make up just two immense subsets in the universe of attributions within musical similarity. That is why this inquiry restricts its analysis to just a few examples in the redundancy of similarity: the transition from similarity to self-similarity reveals basic aspects of music as a self-referential structure and transforming system. Investigation and description of these aspects are the central tasks of this exploration.

This research is intended to contribute, on the one hand, to a critical interpretation of the concepts of self-similarity and others related to it, carrying them from their use in a mathematical context to use in musical practice and thinking. On the other hand, it is desirable to show how self-similarity is inextricably present in the perception, structuring and regulation of musical relationships. Necessarily, and in order to fulfil this purpose, this inquiry places self-similarity in a cultural context, as its fingerprint is evident in the traditions and development of the human sense of the world through music, strengthening the relationship between aesthetics, language, and ecology.[13]

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[13] In the middle of the twentieth century, and in the context of musicology, Paul Collaer (1958:66) suggested the need for a methodical measurement of similarities, in order to study how the relationships between culture and ecological environment do happen: "Comparison throws light on the existence of specific types and on the distribution of types common to several countries or peoples; it underlines the importance of melodic structures, scales, rhythms and polyphonic concepts, of musical instruments which are identical or similar found in neighboring or diverse regions; it suggests that certain kinds of music give the impression of existing in symbiotic relationship with other characteristics of culture."

1.1.2. Recent investigation on the subject

Most of available sources—including the electronic ones—oversimplify the notion of musical self-similarity, hurriedly associating it with fractals, whilst very basic issues are overlooked.[14] To this view it must be added the fact that, although there are many short articles on the subject, until now there is not any monograph which harmonizes the various lines of research, and tries to explain how different forms of musical self-similarity coexist in different operating layers.

The idea that tiny parts in music may mirror larger constructions is latent in Cooper and Meyer (1960), who suggest that rhythm and form are the same thing, and that a small formal unity extends "as an organic process", to larger sections and even to the whole shape of a piece of music.[15] This idea is also found in Roman Ingarden (1962), especially under the concept of functional relationship to parts forming a whole, as the unity of a "musical work" in the wholeness of its Gestalt.[16] However, it was not until 1980 that specialized texts started to address—explicitly— the issue of self-similarity in music, most of them as a refinement of the experiments made by Mandelbrot (1967, 1977), Mandelbrot and Van Ness (1968), and Voss and Clark (1975, 1978).

Participation in the information revolution extended music's traditional fields. Consequently, conceptual gaps were formed in relation to traditional theory and practice. As in other areas of the arts and humanities, new resources in music exceeded its capacity to assimilate technological contributions in the short term. Most specialized bibliographies, among them those of Jones (1981), Bolognesi (1983), Prusinkiewicz (1986), Yadegari (1991, 1992), Nelson (1992, 1994), Hsü

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[14] Self-similar relationships that are fundamental in language (such as multi-layered recursion and synecdoche, grammatical iteration, semantic and syntactic self-reference) exhibit structural and operational parallel with music. Intersemiotic translation, which involves the same kind of relationships in different complexity degrees, is also regarded here as a basic subject for study.

[15] The referred quote states that "As a piece of music unfolds, its rhythmic structure is perceived not as a series of discrete independent units strung together in a mechanical, additive way like beads, but as an organic process in which smaller rhythmic motifs, while possessing a shape and structure of their own, also function as integral parts of a larger rhythmic organization." (Cooper and Meyer 1960:2).

[16] According to Berlin School of experimental psychology, the concept of Gestalt refers to the relationship between a perceived form and its meaning as whole shape. Subchapter 3.5. is devoted to this issue in relation to music.

(1993), and Beran (2004), comprehend musical self-similarity as a problem of engineering.[17] In contrast, the investigations that consider its cultural significance within a certain historical tradition, are very rare (e.g. Kieran 1996, Yadegari 2004). In a brief article published in 1995, Alexander Koblyakov[18] throws a first light on this matter, making a basic statement about the study of musical self-similarity, noticing that [In principle] "there is a problem[atic] situation, which requires a revision of the research strategy. It is necessary to include the perception (*mentality*) factor in a considered phenomenon of self-similarity" (*op. cit.*:297). Unfortunately, most of the literature on the subject disregards this sharp observation.

Furthermore, warnings made by Vieira de Carvalho (1999) on technology and hegemony, and his conception of individualist avant-garde *versus* social identity, are unusual topics in the discussion of musical self-similarity. For this author, centred on Luigi Nono's dialectic of *object-subject*—inspired by Antonio Gramsci and Walter Benjamin—, the social dimension is essential to discuss musical self-similarity and self-reference. This approach comes into conflict with the conception of music as machinery or engineering.

Of course, the concept of music as engineering is not new: it is already implicit in the mechanistic conception of Pythagoreans (see Valavanis *et al*. 2007); seeing more refinement in the treatises of musical instrument construction and comes to a peak through the development of automation systems in Leonardo da Vinci's musical inventions, and with the improvement of the wheel fiddle, since the Late Middle Ages to the beginning of the Industrial Era (see Leichtentritt 1934, Bowers 1987, Haspels 1987). In the Protestant cultures of Europe and North America, the 'music as engineering' became understood as intermediation between *nature* and *machine*, and therefore as a mirroring of the human link between two worlds. Richard Kuhns (1967:264) assumes that "The engineered is *our* agency for the rejoining of art to nature", and asserts (*op. cit.*:261) that the machine, like music, architecture or art,

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[17] Madden's case (2007: xiv) is symptomatic, when he talks of an alleged 'fractal music': "I am not creating mathematics, but rather applying well developed concepts, rather like an engineer would do".

[18] Composer and mathematician, professor of composition at the Moscow State Conservatory.

comes to symbolize two aspects of the ultimate sources of all that fills our universe: the creative principle and the fact of created things. Aesthetically, they follow nature in allowing us to recognize limitless extent and infinite power, where we find sublimity, and limitation, order, balance, harmony, rhythm, decorum, where we find beauty.

By style and content, Kuhns' words reveal a cultural connection with a mythical and religious past—latent in the traditional notions of the *infinite power, order* or *decorum*. From this point of view, it is all but unclear how music and aesthetics converge with engineering as a method to solve problems. Evidently music cannot be fully defined as a list of problems to be solved. Analogously, neither language can be reduced to a vision of engineering, especially for its ties with intentionality, pragmatics, experimentation and expression. Moreover, music and language have strong ties with a poetic tradition to which the engineering can provide only a part—the measurement of a shape and the systematization of stylistic consistency—within a more complex whole (see Adorno, 1956/2002:113–122; 1970/1980:85–89).

Conversely, it is a fact that engineering, statistics, probability, and other analytical resources contribute to understanding aspects of music that would otherwise be difficult or impossible to scrutinize. From this view, according to Koblyakov's (1995:298) interpretation, the 'problems' of composition and musical analysis should not be regarded as engineering or mechanical challenges, but as "intelectual acts" in the broadest sense of the term. For Koblyakov *intellect* and *artwork* are united by the Pythagorean principle of *harmony*, under contextual relationships that give meaning to the discursive sense of music.[19]

Because of the reasons exposed here, the boundaries of the notion of musical self-similarity—systematized or not by a machine—are a recurring theme of this study. No particular preference is given to electronic music or a specific repertoire; rather, musical self-similarity is considered within a much broader spectrum of possibilities that correlate the mechanical, physiological, and pragmatic aspects of music. In this way one can say that engineering and musical language are not necessarily in conflict, but their different stages of correlation require different approaches. In any case, the main difference lies in the invertibility of the relationship *solution to a problem* or

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[19] Almost the same words, this was highlighted by José Vasconcelos (1881–1959) in his *Aesthetic philosophy* (1951:20, 52–53).

problem for a solution, whose results are completely different. This paper explores both kinds of results.

1.1.3. The ideal reader: musicologist or composer?

Many examples of self-similarity given in this study are more of an analytical or explicative character, than of a speculative nature. There are, however, several cases discussed from the (pre-)compositional viewpoint, extracted from the work of composers, and even some hypothetical examples that do not come from existent literature, but are highly probable in a compositional context. Some of these cases are essentially investigated by their association with self-similarity, rather than by their representation in scores or their actual performance. This is the case of György Ligeti's, *L'escalier du diable* (1993), studied in subchapter 6.2., or Iannis Xenakis', *Pithoprakta* (1956), studied at the end of subchapter 6.5. However, other examples include the investigation of specific scores, such as in the case of Luigi Nono's *Il canto sospeso* (1956), analyzed in subchapters 6.3. and 6.6. In short, no special preference is given to certain representational means, but to processes and relationships of musical self-similarity in general. This is why a specific example of non-Western and non-composer situation is particularly interesting, in the case of the K'miai song *Ña yohap mäshuña, ña yohap mäshuña, ña yohap mikewe*, studied on pages 383–386. This song is part of a whole repertoire that is not written, but memorized by oral tradition. Is this sort of case study interesting for composers, or is it more a matter for musicologists? Furthermore: Is the phenomenology of musical self-similarity a necessary issue of music theory, or should it rather been studied by a philosophy of music?

In many musical traditions, music is not clearly divided between compositional theory–practice, and musicological–philosophical studies. This fact includes several aspects of modern Western music. For instance, Tuukka Ilomäki (2008:iii) notes that "The relations of twelve-tone rows are of theoretical, analytical, and compositional interest". So, there should not be a conflict conceiving that musical self-similarity can be investigated from different perspectives, especially when it is evident that self-

similarity in music is a highly attractive topic for composers, analysts, musicologists and aesthetic philosophers.

Moreover, the 'reversive' mechanism of musicological analysis is—somehow—compositional synthesis. This condition seems to be clear in music, at least in part predetermined by culture and biology: musicology can e.g. analyze the features of a certain tradition, whilst the compositional work acts as a balance between continuation and change of a tradition. As a matter of fact, there is a massive amount of musical treatises that reflects a feedback between composition and musicology, theory and practice, formalization and expression. For example, Rameau's *Nouveau système de musique théorique et pratique* (1726) was originally written as a method of harmony and composition ("c'est-là le seul moyen de former promptement un bon compositeur", *op. cit.*:vii), but now is a mandatory document for those musicologists studying the structure of tonal harmony. There are, of course, many more cases that illustrate this relationship between musicology and compositional–instrumental practice in more recent times.[20] All these examples result from the output of composers with musicological interests, or musicologists with very deep knowledge on the discipline of composition.

The present study obviously requires a theoretical and empirical background in its own context. Nevertheless, in order to avoid misunderstandings or non-understanding, the reader must pay attention to the initial chapters, that introduce essential concepts. Along this study, not always is possible repeating or extending on particular definitions, which frequently are given just once, and often as footnotes when they are not capital definitions. With the aim of contributing to clarity, general definitions can also be find with the help of the index of subjects (on pages 534–552). Regarding the pertinence of mathematical jargon, it is extremely important not ignoring what is stated on sections 1.3.2.–1.3.3.

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[20] Some examples are the works of Pierre Boulez (*Penser la musique aujourd'hui*, 1963), Iannis Xenakis (*Musiques formelles*, 1963), Trevor Wishart (*On Sonic Art*, 1996), Clarence Barlow (*On Musiquantics*, 2002), Curtis Roads (*Microsound*, 2004), David Lidov (*Is Language a Music?*, 2005), Adam Ockelford (*Repetition in Music*, 2005), and Hebert Vázquez (*Theoretical Foundations of Atonal Music*, 2006).

1.1.4. Empirical evidence and concept testing

As explained in the latter section, this study focuses on specific musical materials (e.g. scores, recordings, instruments) and musical experiences (e.g. aural, emotional, gestural, and performative cases, such as those mentioned in subchapters 3.5., 4.5.– 4.8., 5.5. and 6.2.). However, the semiotic contents of pre-musical and meta-musical elaborations are also emphasized: self-similarity in pitch scales, tone rows, metrical templates, interval replication, cyclical organization, harmonic patterns and recurrence plots (mainly studied in chapters 5 and 6), are presented as central, empirical evidence of the fundamental postulates exposed in chapters 1 to 4. Such pre-musical and meta-musical elaborations are not simply judged as abstract objects lacking of symbolic and meaningful width. On the contrary, they are conceived as forms of entrainment, empathy and gestural expression, that—as solid evidence of self-similarity—can be eloquent about cultural, physiological and psychological traits of music. Thereafter, they are used for testing the basic concepts of this investigation.

A post-structuralist perspective on the empirical evidence and a meta-viewpoint on the intersemiotic model exposed in subchapter 3.8., is particularly addressed in section 1.2.4. (on intuition and experience), and in subchapters 3.5. (a Gestalt notion that considers the listener by its idiosyncratic and subsymbolic strata), 3.6. (that introduces the concept of *poiesis* in the context of human practices and experiences), 3.9. (including a debate on the *perturbation of the creative centrism*), 4.7. (that introduces the concept of *grammar-in-action*, as a necessary flexibility of the [sub]symbolic contents), 4.8. (transcending the Saussurean framework into a theory of meta-symbolic and meta-linguistic dynamics), 5.5. (proposing a clear, although still flexible notion of musical self-similarity, and claiming a pertinence for statistical determinism in music), and 6.6. (providing counterarguments against an absolute or unilateral assessment of musical self-similarity).

Subchapter 3.9., on the *perturbation of the creative centrism*, must be especially kept in mind along this study, as a key concept: musical, pre-musical and meta-musical intertwinement, and not the 'artist' and his/her 'works', are here the true case study and occupy the centre of the debate. This is essentially different to what usually occurs in a conventional methodology in structuralist musicology.

1.2. Chief lines of exploration

1.2.1. The Post-Structuralist view

Post-structuralism does, after all, perform a significant influence on the interpretation of musical self-similarities. This study particularly takes Roland Barthes' postulate into account, exposed in *Le bruissement de la langue* (1984); and Michel Foucault's in his essays *Les mots et les choses* (1966) and *Ceci n'est pas une pipe* (1973). Whereas Barthes (e.g. 1984:127) suggests analyzing languages as self-referential processes operating in cycles made of cycles, Foucault (1966:19–21) develops an epistemic study of *similarity*, *analogy*, and *repetition*, intimately involved with a variety of issues crucial to music.[21]

The work of Gilles Deleuze and Félix Guattari (1971, 1980), has been especially useful for the concept of *rhizome*, a texture made up of textures, metaphor of language and thought structures, as a self-similar elaboration that coexist in simultaneously hierarchical and non-hierarchical relationships. Studies by Tzvetan Todorov (1968) and Paul Ricoeur (2004) were also useful for their contributions to the theory of translation, in particular in the field of intersemiotic translation.[22]

Jacques Derrida's books (*L'écriture et la différence*, 1967; *Qu'est-ce qu'une traduction 'relevante'?*, 1999), spreading ideas closely related to intersemiosis and self-reference as matrices of culture, feed Yadegari's (2002) research on musical self-similarities and cultural correlation. The development of Yadegari's thought is one of the most significant contributions to literature on musical self-similarity from a post-structuralist view, bringing the discussion on musical self-similarity to the fields of linguistics, philology, and philosophy, under a deconstructionist perspective.[23]

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[21] Barthes ideas (1984) mainly feedback section 3.9.1., on 'Perturbation of the creative centrism'. Foucault's theories (1966, 1973) especially contributed to the development of chapter 3, as well as to enrich the argumentation of subchapter 5.5.

[22] See especially subchapter 3.8.

[23] Graduated in electrical engineering in 1982, Shahrokh David Yadegari completed his master degree with the dissertation "Self-Similar Synthesis: On the Border between Sound and Music" (1992). He obtained the doctorate in music philosophy with a dissertation on recursion, self-reference, and traditional Persian poetry (2004). Among the achievements of his doctoral paper

Yadegari implements his approach to musical self-similarity by developing his own theoretical (philosophy of science, philosophy of culture, music theory) and practical contributions (composition, musical analysis, computer science). He explores notions such as *tension between play and totality, multi-layered musical space, structural replication, recursive synthesis, instrumental register*, and *cultural hegemony*, producing a general theory that was of great interest to this study, and which inspired a challenging notion of consistency within these ideas. Remarkably, in his dissertation Yadegari no longer ignores language and culture as central aspects of musical self-similarity, but rather incorporates them into a correlated whole:

> Even though musical instruments are governed by physical laws, often it is the non-linear idiosyncrasy of an instrument which characterizes the unique features of an instrument, and thus, one of the jobs of the performer becomes to use these features in a musical way. As a simple example, the sound spectrum of various registers of an instrument differ from each other not only in frequency scale, but also in spectral envelope, frequency content, and form of progression over time. (Yadegari, 2004:171)

Yadegari conceives musical self-similarity as a mirroring of a range of trends, more primitive than humans, but from which and with which the human being also emerge. From this perspective, physical world and culture are no longer dissociated as happens in the Cartesian model. This notion constitutes a powerful attraction over the current research. Especially the fourth chapter, on 'The diversity of musical self-similarity', as well as the seventh, are imbued with the same spirit of conciliation between the material and the imaginary, as a consequence of a notion of harmony between finite and infinite self-similarity.[24]

The post-structuralist viewpoint in this study is completed—as it can be inferred from the previous section—with implementing pre-musical and meta-musical elaborations as case studies: empathic and entrainment processes, and gestural, emotional and mnesic loci, are treated as case studies, instead of uniquely evaluating 'music' as a mere collection of 'classical' objects (e.g. scores, melodies, chords). In this context, the theories and concepts proposed by Bateson (1972), Lakoff and Johnson

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is the Radif, "a framework for improvisation, mostly based on a collection of vocal melodies" (*op. cit.*:148).

[24] This topic is especially studied in subchapter 3.3.

(1980), Fauconnier (1985, 1997), Lakoff (1987), and Damásio (1994, 2000), and especially the Peircean semiotics introduced in subchapter 3.8., constitute a basic framework for this investigation.

1.2.2. *Mental Spaces* in Lakoff and Fauconnier

A *mental space* is a "means for conceptualizing and thinking", that participate in the representation of "any fixed or changing state of things" (Lakoff 1987:281–282). Mental spaces lack of ontological properties outside their representational operation, and are particularly useful for the elaboration of cognitive structures, including the musical ones.[25]

Fauconnier and Sweetser (1985:ix) argue that the tools of formal logic fail when confronted with the whole of phenomena of language. Consequently, Fauconnier (1997) implements a cognitive theory based on the capabilities of the human mind rather than the capabilities of mathematical systems and their historical formalization (e.g. Russell and Whitehead 1913, among many other possible examples). Fauconnier formulated a theory in which reference has a *real existence*, capable of being represented by connectors that operate as neural networks, following a few general principles. The complexity of these networks depends mainly on the interaction between these principles and contextual meta-structures that feed interpretation. Their operation resembles the application of a grammar rule, during the development of cognitive maps and general mental references.

The consequences of this perspective on music theory are remarkable: theoretical proposals made by Babbitt (1949, 1960, 1961) or Schenker (1932), for instance, become clearer as operational generalizations for imaginary representation; not anymore as musical relationships by themselves, or as forms of 'pure translation' (see Benjamin 1923). In this sense, many forms of musical similarity can be compatible, and coexist simultaneously in different strata. Each of these strata can thus mirror a

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[25] See Ockelford 2005:xv, and Ojala 2009:213–220. For a specialized description of the embedding of the musical structures in the cognitive ones, see Kaipainen 1994:15.

pattern, a path of mental spaces emerging from operations common to the species, and through dialogic processes molded by culture.[26]

Furthermore, Lakoff and Núñez (2000) find the operational foundations of mathematics in the performance of four basic mental processes, metaphorically structured and closely related: *collection, object construction, use of measuring sticks*, and *motion on a path*. Respectively, this simplified—though not simplistic—relationship between basic processes of abstraction, helps to clarify the all-encompassing operations (*collection*), geometric build-up axioms (*object construction*), generalization of analogies (*use of measuring sticks*), and mapping (*motion on a path*).

This conceptualization contributes to an understanding of the operational bases of a self-similar system. In relation to the construction of reference systems participating in the elaboration of meaning and the 'feeling of what happens', this theory is compatible with Damásio's description (2000) of cognitive processes configuration as coordination of multiple layers of perception and emotion.

Congruently, the properties of a self-similar complex in various strata of musical language can be summarized as a *collection* of similar objects, one inside the other; *object construction* based on a reference source; *use of measuring sticks* for sustaining selective processes of comparison and repetition, and for estimating aesthetic values;[27] and *motion on a path* along constructive phases in replacement and differentiation processes. Most of the examples in this study correspond to one or more of these notions.

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[26] Consider the following example: the Ptolemaic model of the universe is not in use at the current celestial mechanics. But this does not mean necessarily that the kind of mental operation required to understand the essence of the Ptolemaic model is absent in modern celestial mechanics. On the contrary, the history of science, within its theoretical deviations and findings, is also the history of a single set of operating principles in the human mind: a history of substitution of analogies (for a general introduction on the subject, see Kuhn 1962).

[27] In this area falls, for example, the 'utility' of the fractal dimension featuring a set of musical relationships. This subject is extended to the end of subchapter 5.5. (pages 300–303).

1.2.3. The concept of *cognitive domain*

For the cognitive sciences, a *domain* is a finite collection of operating principles, alongside with their operating rules and the entities to which they apply (see Gelman and Greeno 1989, and Gallistel 1990). According to Gelman and Brenneman (1994:376), the cognitive development of infants is guided by domain-specific principles. These authors highlight the importance of distinguishing between domain and simple prescription; for example, a set of prescriptions for solving a problem do not constitute a domain, which is rather a coherent set of operational principles necessary for knowledge generation. In this manner, the operational coordination between a grammar, a semantic–syntactic field, and its contextual implementation, constitutes a domain. Verbal language, mathematical language, and musical language are examples of different cognitive domains.

Throughout this study, the term *musical language* refers to the consistent set of operating principles, applications, and entities characterizing music. Verbal language is a different domain, not necessarily correlated with music. In so far as speech and music are structured by temporal sequences of sounds, embedded by metre, prosody and rhythm, it is obvious that they have a number of structural and functional similarities. But speech and music occupy different neural networks in the brain. According to Gelman and Brenneman (1994:376), there is "theoretical and empirical evidence from animal work, neuropsychology, and evolutionary theory [suggesting] that speech and music are neither subsets of each other nor parts of the same system" (see also Patel and Peretz 1997, and Peretz 2002). There is, however, a set of similar relationships between these and other domains of knowledge (such as arithmetic and geometry), comparable by their coordinative structuring, and by their dependence on the same cognitive platform and the same evolutionary context.

The concept of *Universal Grammar* developed by Noam Chomsky (1955, 1957) refers specifically to the domain of verbal language. This does not imply that there are no other innate domains, each with possible analogies and homologies concerning the innate domain of speech. The whole of these analogies and homologies in the total of the innate domains of the mind/brain, constitute the innate human *cognitive platform*. An aspect correlated with the configuration of this platform is the ubiquity

of certain patterns that throughout biological evolution characterize the functional structures of cognitive domains. The informative redundancy between context (i.e. environment of development) and cognitive platform, and the feedback between these systems, portray a complex reciprocity that exhibits self-similar traits. This is key to explaining self-similarity between context and knowledge of the context:

> [I]n the constructive view, environments conducive to knowledge acquisition must share structural relations with the model-building system. [...] For the law of frequency can be replaced by a law of redundancy. Now the argument is that learning is facilitated by the presentation of multiple exemplars of inputs that share structural description, some of which will overlap with the model-building system. (Gelman and Brenneman 1994:382)

However, given that under this scheme it is impossible to explain other basic aspects of music, subchapters 4.7. and 4.8. discuss how *rigid* aspects of musical grammar —through mechanisms of self-reference and correlation—are related. And how 'fixed' knowledge in memory and culture—with *flexible* modes of learning, practice, and recreation of music—is linked with systems that overlap and associate operational characteristics at different scales, with local and global transformations.

The concept of domain is very useful in this research as it enables the convergence of similarities between musical systems within cycles of individual expressions, collective aspects of a social context, and inter-cultural exchanges, without confusing them with analogous cycles of speech, mathematical language, and other domains of knowledge: the structural self-similarity within the whole of several domains occurs at different stages and levels, through different possibilities of analogy and intersemiotic translation.

Self-similarity in the cognitive domains

Evans (2006:1) uses a particular metaphor to explain metaphors in general. This self-definition suggests, nevertheless, a synecdochic structuring of thought rather than a metaphorical one:

> Maps are metaphors. Metaphors are building blocks in the construction of knowledge. They are the bridges we use to connect novelty or new experience to what we already know. We call this connection learning. When we learn, we successfully match (map) patterns received through our senses to patterns already stored in our memory.

It is important to consider that the process of knowing (to feel, perceive, experiment, etc.) consists of an elaboration of maps within maps in the mind/brain (see Fauconnier 1997). Assuming that to know is not merely being in contact with what is perceived, but also to emotionally associate it with what is latent in memory, it is plausible that this mapping process is achieved at various levels and categories, corresponding to different levels and categories of coordinative knowledge. The mind/brain results then from this coordination of mappings, and this coordination itself functions as a map of maps. Congruently, Schroeder (1991:11–12) characterizes the typical make-up of the human brain as a physical, self-similar structure, moulded by the evolutionary necessity of "filling a volume, whilst preserving a two-dimensional adjacency".[28] Beside this physiological level, other self-structuring levels coexist within synecdochic coordination, according to what is suggested in Chapter 4.[29]

Among the results of the functioning of the 'mind/brain'—in the usual terms of Chomskyan generativism—is, precisely, the emergence of 'independent' maps, originated through the mapping of cognition and representation. This is of utmost importance for intersemiotic mapping, within the musical representation of a spatiotemporal coordination:

> The idea of data mapping as a compositional process is generally associated with a kind of literalism of presentation. Since mapping is typically representational, it is assumed that [it] directly informs the perceived structure of composition, producing a resulting sound with similar characteristics or a similar shape. Data mapping is a process-oriented method of composition, in which the data structures that will eventually be transformed into sound exist as something independent from the composer's pre-compositional process. (Jensen 2008:243)

The 'independence' of structures to which Jensen refers here, corresponds—at least in a considerable proportion—to the nature of the intersemiosis of a *fundamental*

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[28] Schroeder thus compares thus the physical structure of the brain with the economy and structural consistency of the Hilbert curves. It is not arbitrary, then, to use the representation of these curves for the development of a musical grammar, as discussed in subchapters 6.4. and 6.5., making it part of a process of musical creation that is analogous to the brain's typical processes and forms.

[29] This is not unrelated to what Campbell (1982:262) notes: "It is a very plausible idea that the structure of a society, like a language, reflects, at least in part, the structure of the human brain."

message, based on the power laws that "exist as something independent from the composer".[30]

1.2.4. Other points of reference

Prusinkiewicz (1986, 1992) suggests for the first time that Lindenmayer systems, fractals and the Fibonacci sequences share self-organization features analogous to musical grammars.[31] Shortly after, Hsü (1993) publishes a concise, key article in which he introduces an innovative approach to systematic musicology, implementing self-similarity criteria.

Subsequently, Madden (1999/2007, 2005) carried out research into the alleged relationship between fractals and music, but unfortunately he was unable to clarify essential subjects (see criticism in Borthwick 2000 and Lanza 2006). Furthermore, Madden's handbook *Fractals in Music* (1999/2007) does not consider the observations made by Devaney (1987, 1992), about a fractal as a set of absolute and infinite self-similarity. Devaney uses the term fractal "to mean shapes that are strictly self-similar rather than statistically self-similar" (see Oliver 1992:455). Consequently, the notion of "fractal music" or "fractals in music" cannot be formulated as it is intended in Madden (1999/2007, 2005). In fact, the same objection applies to Goertzel (1997), Pérez Ortiz (2000) and Hwakyu Lee (2004), among other authors.[32] Usually, when generalizing this concept, different mental categories are overlapped: one cannot say that a segment or a piece of music is a fractal, in the same way as one cannot say that an equilateral triangle is the metaphor of a right triangle. This is about two objects whose essential differences involve also essential meanings. Commonly, when music or a certain quality of music is labelled as 'fractal', the only

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[30] See the definition of *power laws* in section 3.9.5.

[31] The Lindenmayer systems (also called L-systems) are introduced in subchapter 6.5., devoted to self-replacement strings. The topic of Fibonacci sequence is introduced in subchapter 6.3.

[32] In trying to directly apply the fractal concept in music there are too many exceptions and contradictions. This issue, directly related to the debate on musical determinism and indeterminism, occupies different sections in this study. For a critique on such direct application in musical composition, see Cooper (1991) and Vantomme (1995). Equally, Borthwick (2000) and Lanza (2006) make a criticism regarding conceptual mistakes and methodological failures in Madden's approach (1999/2007, 2005).

intention is to refer to some characteristics of self-similarity in music.[33] Another failure in Madden's work (1999/2007, 2005) is associating 'fractals' only with statistical and probabilistic analysis of music; central aspects of musical composition and interpretation are treated just occasionally, and no any issue on symbolic, cultural, biologic or environmental self-similarity are ever explored, in connection with music.

Among the composers who have published explanations and analyses of their own work with so-called 'fractal algorithms' (i.e. processes of self-similar recursion), are Charles Dodge, Gary Lee Nelson, and Tom Johnson. Dodge (1988) develops his own method for specifying relationships of pitch, rhythm, and dynamics, as well as one way to determine the different levels of structuring a composition. Furthermore, Nelson (1992, 1994) explores the notions of *random walk* and *dissimilarity*, which within this study enrich the concepts of *antiproportion* and *self-dissimilarity*.[34]

Johnson (1996, 2006) develops a much more detailed research on musical self-similarity, moving forward in four lines: *counting* ("from 1, 2, 3 to automated counting"), *transforming* (from the transformation of ones and zeros to the analytical processing), *mapping* (of numerical and geometric patterns), and *self-replicating* (of melodies, proportions, and relations). According to David Feldman (1999), the self-replication argument is particularly interesting because it clarifies the concepts of *finite* and *potential infinite* as musical practice and grammar. The central chapters of the present study consider these criteria, along with other issues addressed by Carey and Clampitt (1996), Kononov (e1998), Beran and Mazzola (1999a–b), Bigerelle and Iost (2000), and Beran (2004), and provide important material for discussion in Chapters 5 and 6.

The contributions developed by Manuel Rocha Iturbide (1999) and Curtis Roads (2004), although mainly focused on granular synthesis of sound, were of great help in better defining notions such as *acoustic particle, Fourier transform, transition matrix, noise patterns* and *chaos algorithms*, and to achieve an updated picture of the contrast between musical determinism and indeterminism. Thompson (1917), Weyl

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[33] This issue is addressed in more detail in section 1.3.4., on the 'narrow use of the term fractal'.

[34] In subchapter 6.6.

(1952), Estrada and Gil (1984), and Hofman-Jablan (e2007), were general references for research into symmetry—the last two specialized studies of musical symmetry.

In the field of musical semiotics the research developed by Kalev Tiits (2002) and Juha Ojala (2009) was useful to investigate different notions related to musical self-similarity, and to Charles S. Peirce's abductive theory.[35] Although Tiits and Ojala do not develop specific aspects of musical self-similarity, they do include discussions on geometry, logic of symmetry, space, and spatiality (Ojala); and self-organization, subsymbolic processing, and musical information (Tiits), and about other concepts related to the present study. In particular, and at different points of this research, the concept of 'abduction' is considered as a precursor to the concept of self-similarity, given the interest of Peirce in the problem of 'the map of the map' (see Peirce 1903a; from Peirce *CP*, 8.122).[36] The works of Eco (1968, 1976, 2003), Campbell (1982), Padilla (1984, 1995, 1998), Tarasti (1992, 1994, 2000, 2004), Kieran (1996) and Lidov (2005), are also related to this issue, giving further support to the exposition on grammar and style (subchapters 4.7.–4.8.).

Whereas discussions on linguistic innatism and generativism derived from Chomsky (1955, 1957, 1985) are of general interest to this work—accepting that language transformations are often comparable to their musical analogies, and generative 'trees' are also self-similar abductive models (see Peirce 1903a/1998:162; Chomsky 1956:117–118; Lerdahl and Jackendoff 1983:214), the writings of Minsky (1968), Doležel (1969, 1998), Fodor (1975, 1983), Sebeok (1977), Jackendoff (1993), Pinker (1994), Hayes (1995), Thomason (2001), and Givón (2002) were necessary for comparing their postulates with music theory.

Finally, this summary cannot ignore the seminal work of Benoît B. Mandelbrot (1924–2010).[37] Although it does not directly involve music, it formalizes many

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[35] On the Aristotle's criterion referred to as *abductio*, Peirce proposes a hypothetical inferential method, which operates in a manner different from deductive and inductive methods. "Abduction is nothing but guessing", says Peirce (*CP* 7:219); thus, for the 'map of the map' the relationship between *the map* and the *set of maps* encompassing the whole is not deductible or inducible but 'conjecturable': as suggested by Almeida (2002:13) "another word for abduction is conjecture." This concept is connected with music e.g. for the functions of *expectation* and *spatiality* studied in detail by Ojala (2009). For a detailed definition of abduction see page 459.

[36] Applying Peirce's semiotic theory to the research of musical self-similarity constitutes a method explained in 3.8.3. The problem of the *map of the map* is exposed in detail in section 3.9.2.

[37] For the associated bibliography, see Mandelbrot 1953, 1956, 1967, 1977, 1981, 1982, 1986.

aspects not organized before in general analytical approaches to musical self-similarity. Mandelbrot started a discussion that continues to be fruitful for an astonishing variety of interests in the sciences and arts. Also worthy of attention are his courses and lectures which directly stimulated musical research by composers such as Dodge, Wuorinen, Nelson, Johnson and Brothers. The refinement of Mandelbrot's seminal ideas made by Peitgen and Richter (1986), Gleick (1988), Rasband (1990), and Schroeder (1991), contributed also to Harley (1995), Steinitz (1996a–b), Slater (1998), and Roads (1999). Their contributions to the study of deterministic chaos, associated with different kinds of noise and generation of musical *order* in different parameters, are reviewed in a discussion of algorithmic composition in subchapter 5.5.

1.3. Methodology

1.3.1. Methodological summary

Literature on musical self-similarity can be considered sufficient to draw an outline of methodology, noting trends studied, including computational problems, analysis and automated generation, and—as a more recent and less abundant corpus—sources concerned with aesthetic and discursive issues. In this context, however, many fundamental ideas are adapted from Charles S. Peirce (1893, 1903a–1903d), whose notions deserve a special section in 3.8.3., with precise methodology therein detailed.

Concerning methodology related to the actual theory on self-similarity, the original texts of Mandelbrot (1953 to 1967), developed in his books of 1977 and 1982, are particularly valuable. The theories of Claude E. Shannon (1937, 1948) and Abraham Moles (1952, 1958) converge also toward this core of sources, followed by structuralists pioneering the study of self-similarity in musical codes and messages: Babbitt (1949, 1960, 1961), Pinkerton (1956), Coons and Kraehenbuehl (1958), Youngblood (1958), and Seeger (1960). However, the revision of fundamental concepts for these theories—namely *repetition, similarity, symmetry* and *recursion*—by authors such as Foucault (1966, 1973), Derrida (1967a, 1967b, 1999), Eco (1968, 1976, 2003), Todorov (1968), Deleuze and Guattari (1971, 1980), Campbell (1982),

Barthes (1984), Luhmann (1990), and Ricoeur (2004), occupies more attention. Discussion of these sources contributes to independence of postulates in this research, which is found in the concluding paragraphs of each section and each subchapter, as the study unfolds. Bridging this corpus are two major sources which have a direct impact on the current concept of musical self-similarity: Xenakis (1963/1992) who without naming it implies its quiddity,[38] particularly within an aesthetic conceptualization; and Voss and Clarke (1978), more focused on solving an engineering problem: how to use fractional noise as a source of musical information. Farther from this core of references, this scheme is gradually extended to discuss a wider variety of technical approaches.[39]

Even though the idea of self-similarity began to convert into a musicological topic at an early stage that ran from 1978 to 1992, during this period there was not any debate about a philosophy of musical self-similarity. Nor were there any aesthetics, semiotics, or sociology specifically concerned with the subject. At this stage the work is restricted to determining whether Mandelbrot's theories were indeed compatible with music theory, and to explain his measurement procedures. In 1993 Benny Shanon published the first note on "Fractal Patterns in Language", from the view of psychological linguistics. The next year Luděk Hřebíček published the first formal investigation on "Fractals in Language", from the view of statistical linguistics, and in 1995 Gal and Irvine started to talk about *sociolinguistic recursiveness*. But nobody connected their findings, then, with musicology. Hsü's (1993) and Koblyakov's (1995) short papers remained isolated. The also brief discussions published by Kieran (1996), D. Feldman (1999), and Vieira de Carvalho (1999), barely sow the seeds for a new critical theory.

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[38] The latest version of Xenakis's book, published in 1992, includes comments on the concepts of *Brownian motion, cloud of sound-points, dynamical systems, entropy, fractal, isotropy, logistic function, Markov chains, probability, randomicity, random distribution, random walk, repetition, stochastic process*, and *symmetry*, all of them linked to the notions of functional and statistical self-similarity (see Preface, xii–xiii).

[39] Among these approaches Jones (1981), Bolognesi (1983), Prusinkiewicz (1986), Fagarazzi (1988), Dodge (1988), Hsü and Hsü (1991), Yadegari (1991, 1992), Bigerelle and Iost (2000), Das and Das (2006), and Su and Wu (2006) are included for the originality of their analytic proposals.

Research studying musical self-similarity through symbolism and semiotic aspects, indicates a gradual shift away from the solution of structural and technological problems on which most literature is concentrated, conceived primarily from a pragmatic and utilitarian conception of music. Nevertheless, the complete view of these investigations may be considered more as a related whole, than as a collection of differences. To better understand this complexity, this study performs an eccentric spiral of topics, widening its cycles after simple operational definitions. Consequently, aesthetic, symbolic, and semiotic topics that have not yet been sufficiently investigated are studied herein. The conceptual framework used for their introduction includes basic notions of information and communication theories, theories of dynamical systems, and aspects of ecology intertwined with the study of musical self-similarity.

It is important to note that this study does not ignore recent developments in the pitch-class set theory, about measuring similarities (Ilomäki 2008) and self-similarity classification in Klumpenhouwer networks (Murphy e2007); but at the same time this study cannot cover them in detail, nor deepen them, since its focus is on self-similarity as an intersemiotic process, especially in its operations of analogy and synecdoche. Examples of these operations are included when considered appropriate and illustrative for a possible enrichment of pitch-class set theory.[40]

1.3.2. Special uses and codes

The reader will soon notice that this study gives preference to the term *synecdoche* over *metonymy*. The reasons for this relate to aspects of method and meaning.[41] With reference to the latter, the prefixal difference is emphasized by συν·εκ-, which implies parallelism or correlation, and μετά-, remoteness or change. The former invokes conjunction; the second, disjunction. Regarding methodology, synecdoche is also preferred for its analytical convenience in many typical relationships of music—many

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[40] This is valid for subchapters 3.4. (*Invariance*), 5.4. (*Noise in music*) and 6.6. (*Antiproportion*).

[41] Subchapter 3.2. synthesizes the etymology of 'synecdoche' based on Bailly's dictionary (1894). Some notions related are also given by Peirce (see footnote 78, page 59, of this study).

of these relationships can best be studied as synecdoche or as analogy, rather than as metaphor or metonymy.[42]

When the contrary is not suggested, the term 'frequency' is used in its broadest sense, for the repetition of a specific relationship in a system. All acoustic recurrence is a frequency or a set of frequencies identified by their repetition or their cycles of vibration, unless they are unintelligible sound clusters with no specified frequency, or *noise*.[43] The concrete case of frequencies as audible periodical vibrations, will be specified as the distinction between 'tone' (i.e. a given sound's harmonic spectrum or texture) and pitch (i.e. a specific frequency within a music scale or fixed musical space).

Italics are reserved for the introduction of abstract concepts and neologisms; within the bibliography italics are used to refer to titles of monographs. Single quotation marks ('') denote minimal definitions or condensation. Double quotation marks ("") introduce textual quotations; within the bibliography they are used to refer to dissertations, articles and minor works being part of larger ones, such as records, encyclopaedias, and other compilations. The diamond shape (◊) indicates graphs, followed by a sequential number associated with each subchapter and a sequential number for each graph.

The few expressions in alphabets other than Latin are preserved in their original typography, in order to provide a better reference source, particularly useful when trying to explain an etymology. For Greek etymologies the sources are H. G. Liddell and R. Scott (*A Greek-English Lexicon*, Oxford University Press, Oxford, 1968); P. Chantraine (*Dictionnaire étymologique de la langue grecque*, Éditions Klincksieck, Paris, 1968), and M. A. Bailly (*Dictionnaire Grec Français*, Librairie Hachette, Paris, 1894).

It is important to note that the general bibliography at the end of the text is divided into printed materials and electronic references given in a second list. In the

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[42] This is a subtlety to which Cooke (1959:1) attaches particular relevance: "Comparison between one art and another *can* help [...] when the comparison is not *metaphorical*, but *analogical*, being concerned with the artist's *intention* and *technical procedure*."

[43] In more precise terms this means $1/f^0$ noise. This concept is defined in subchapter 5.2.

main text, the latter are marked with a letter 'e' followed by the year of publication. Examples: Brothers e2009, Gerlach e2007, Snyders e2008.

1.3.3. Mathematical language

This study addresses the notion of self-similarity for its relevance to musicology, conceiving an audience with interests in aesthetic philosophy and music theory. It is therefore necessary to include equations and abstract patterns, not with the intention of replacing a comprehensive mathematical treatment, but rather provide an intuitive and informal approach to the issues discussed in the context of musicology.[44] The depth of the mathematical language involved in this approach is actually relevant to the current training for music theorists, including composers, analysts and musicologists.[45] Any reader interested in a mathematical treatment of the topic of self-similarity may consult the writings published by Peitgen and Richter (1986), Lauwerier (1987), Feder (1988), Gleick (1988), Devaney (1987, 1992), Barnsley (1988), Lindstrøm (1990), Schroeder (1991), Prusinkiewicz and Hanan (1992), Peitgen, Jürgens and Saupe (2004), and Cabrelli, Heil and Molter (2004), among many others.

In order to avoid undesirable misinterpretation of most sections in this study, the reader must be aware of the frequent usage of the term *function* in the context of *functional similarity*, as explained in subchapter 2.4. The mathematical concept of *function* (commonly denoted by f) is defined in a proper context, in subchapter 6.2. (see especially pages 295–314). Another important remark must be noted for the use of the term *fractal*, as explained below.

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[44] The term 'informal' points out the fact that in this study there is not any formalized mathematical development, but only the analysis of theoretical texts and experiments previously formalized, involving musical self-similarity. In this way, preference is given to the translation of mathematical terms into musical ones; to the empirical proofs and mathematical resources implementation for the musical synthesis and analysis.

[45] What is hereby expressed, and which concerns the rest of this research, considers Borthwick's remarks (2000:662): "very little music (if any) that currently exists can be explained entirely in terms of mathematics." In this case no any attempt is made to explain music "entirely", but only to deepen into some of its fundamental characteristics through different forms of metalanguages for musical analysis, including mathematics.

The systematic transition between intuitions of *harmony → similarity → noise* is also studied as analogy of the transition *proportion → self-similarity → chaos* (where the symbol → denotes tendency). Such a transition is also related to the Peircean *phenomenological trichotomy*, as explained in section 3.8.3. Chapters 5 and 6 pay special attention to this topic in the context of aesthetic theory in general, and musical composition and analysis in particular. The final part of subchapter 5.2. also connects this concept to a meta-theory on musical performance.

1.3.4. Narrow use of the term *fractal*

Even though this investigation provides from its beginning operational notions of the term *fractal*,[46] this section concentrates on some of them in order to facilitate its use —and prevent its misuse—during the development of this study.

After Mandelbrot's (1977, 1982) original references, and within a scientific context, the employment of the term 'fractal' is often confusing and contradictory.[47] Mandelbrot himself contributed to this situation, particularly when he looked for musical metaphors that described mathematical aspects of 'fractals'.[48] The following demonstrates such an example:

> In the Mandelbrot set, nature (or is it mathematics?) provides us with a powerful visual counterpart of the musical idea of 'theme and variation': the same shapes are repeated everywhere, yet each repetition is somewhat different. [...] Because of its constant novelty, this set is not truly fractal by most definitions: we may call it a borderline fractal, a limit fractal that contains many fractals. Compared to actual fractals, its structures are more numerous, its harmonies are richer, and its unexpectedness is more unexpected. (See Peitgen, Richter and Saupe 2004:783)

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[46] See pages 1–3. In relation to this, particular attention is due to what is stated in subchapter 3.3., in the section on 'Finite or infinite self-similarity of an image-object'.

[47] Even a few examples may lead to extreme confusion: Somasundaran (2006:2606) states that "not all fractal objects are self-similar", Foley *et al.* (1990:1020) argue that "an object that is not exactly self-similar may still seem fractal", and Lung and March (1999:113) claim that "Fractals may be considered as 'regular things which obey the law of self-similarity'".

[48] In order to confirm this assumption, see Mandelbrot (2002:193), reproduced here in subchapter 5.5. (page 305).

How does the musical idea of 'theme and variation' imply self-similarity? This statement is, actually, too vague.[49] This ambiguity leaves the difference between similarity and self-similarity unclear. It also leaves open to interpretation the question, to what extent music—in strict analogy with the Mandelbrot set—is like "a limit fractal that contains many fractals". Rather, at the end of the quotation a basic question comes to mind: what does the word 'fractal' mean?

Facing many attempts to define the term 'fractal', Oliver (1992:454–455) makes an effort at transparency, summarizing the three kinds of requisites needed for a shape to be defined 'fractal': **1.** A shape in which its *Hausdorff dimension* differs from its topological or ordinary dimension (in geometry, a dimension refers to the form in which an object or process is filling a space. The Hausdorff dimension is associated with the form in which a self-similar object or process fills a space, including irregular sets that have noninteger exponents reflecting their dimension);[50] **2.** A shape in which its parts, when amplified, reveal a similar amount of detail to the whole of which they are a part; and **3.** A strictly self-similar shape with fractal dimension (usually its Hausdorff dimension), showing exactly the same pattern for every one of its infinite scales or recursions. With this third definition—used by mathematicians such as Robert Devaney [1987, 1992]—the Mandelbrot set, one of the most popular objects in fractal geometry, "is not a fractal" (see Oliver 1992:455).[51] Accordingly, the finite self-similar curve of the *devil's staircase* is neither a fractal (see Peitgen, Jürgens and Saupe 2004:212).[52]

In practice, the first and third requirements can merge into a rigorous definition of 'fractal', without regard to any intuitive aspect of self-similarity. This study uses this combination of Hausdorff dimension and strict self-similarity as a compact definition for 'fractal set'. In contrast, this study conceives that the objects or processes that nearly match the second type defined by Oliver (1992:454–455), are

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[49] Subchapters 2.5., 3.3. and 5.2. bring elements to explain how the idea of 'theme and variations' is related to the notion of self-similarity, but not necessarily to the notion of 'fractal'.

[50] For more information on this topic, related to music, see the section *Fractal dimension: An issue of pertinence* (pages 300–303).

[51] This notion matches with the idea expressed by Mandelbrot in the previous quotation, about the Mandelbrot set.

[52] The mathematical concept of *devil's staircase* is explained on pages 354–363.

relatively self-similar objects or processes that may or may not have a fractal dimension. After this operative decision, fractal processes and objects are reduced to a generalized deductive and analytical treatment, distinct from processes and objects with relative or limited self-similarity.

A fractal exhibits a self-reference relationship between its extent and its extension rules: "The prototypical example for a fractal is the length of a coastline measured with different length rulers. The shorter the ruler, the longer the length measured, a paradox known as the *coastline paradox*" (Weisstein e2008). The *coastline paradox* occurs in music during the measurement of complex structures such as those that comprise statistical distribution of values in a whole repertoire—a massive amount of information—for instance, in a musical style in its entirety (e.g. Bigerelle and Iost 2000, Das and Das 2006, Su and Wu 2006), or in an acoustic self-similar complex such as a timbral frequency spectrum, or a turbulent air flow in a flute's beveled nozzle (see Bader 2005). Hsü (1993) also considers the self-similar shape of a melodic and contrapuntal structure of a polyphonic piece of music, compared to its own length-reduction on the proportion 1, 1:2, 1:4, 1:8, 1:16..., with an attainable fractal dimension (see also Dagdug *et al.* 2007). In this complexity, to be a fractal the musical object or process "need not exhibit exactly the same structure at all scales, but the same 'type' of structures must appear on all scales" (see Weisstein e2008). In this manner the fractal notion is associated with music in the form of two mental operations: *stereotype*, in the case of fractals whose simple scalar repetitions can be assimilated by intuition (e.g. Sierpiński's triangle), and *abduction*, in the case of the infinite potential of the map within the map (the case proposed by Peirce).[53] None of these operations signifies, however, a direct adaptation, so the deterministic rigor on the concept of "musical fractal" or "fractal music" becomes superfluous.[54] That of musical self-similarity, instead, occupies most of the development of this research.

Summarizing, virtually all musical objects and processes with self-similarity are not necessarily considered here as *fractal sets*; and even though they can be measured

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[53] Regarding the concept of *abduction*, formulated by Peirce, see pages 23 and 459. On the Sierpiński triangle, a system of equilateral triangles whose space is occupied by identical objects in its parts and its whole, see graphic representation in ◊330*a* (page 65).

[54] See discussion in subchapter 5.5.

by a fractal dimension, they are never strictly and absolutely self-similar i.e. they are never autoidentical at several points in their many structural layers. As Cooper (1991:59) states: "musical self-similarity, like that of most natural objects, is largely statistical rather than absolute, and limited to a small number of scales." Moreover, it is noteworthy that the fractal notion, and especially the concept of fractal dimension are not always associable with intuitive, aesthetic or idiosyncratic qualities that are crucial in music. For instance, Hsü (1993:27) acknowledges that "there is no resemblance to fractal geometry" in many pieces of "modern music". And even when fractal dimension can be alleged in a musical piece, this issue may be irrelevant for hearing and aesthetic appreciation. Echoing Schoenberg (1967:25), when he talks about musical symmetry, one may say that '*exact self-similarity* is not a principle of musical construction. Even if many local strata within a set repeat the general features of the same set, the structure can only be called *pseudo-fractal*'.[55]

This notion extends to the critical treatment of the revised literature. For instance, it is assumed that the "musical fractals" referred to by authors such as Dodge and Bahn (1986:185), Madden (1999/2007, 2005), Hudak (2000), and Hwakyu Lee (2004), are actually *pseudo-fractals*, *quasi-fractals*, or imaginary representations of a fractal set via mental operations such as analogy, stereotype, or abduction, commonly crossing cognitive domains as conceptualized by Zbikowski (1997).[56] From this perspective useful concepts come into play, such as self-affinity, pre-self-similarity, statistical and functional similarity, and self-dissimilarity, discussed in due course.

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[55] Arnold Schoenberg's referred text appears on page 44 (subchapter 2.3. devoted to musical symmetry).

[56] See section 1.2.3.

1.4. Content organization

This study is divided into two major parts: I. Theoretical–Methodological Framework and Basic Definitions; and II. Self-similarity in musical information and proportion: From Simple Synecdoche to Complex Intersemiosis. This division reflects the need to acquaint the reader with aspects of linguistics, semiotics, and mathematics, by linking them to specific problems of musicology, and then offer a variety of examples to prove the usefulness of the concepts introduced in the field of music theory, in the second part.

A secondary division separates this study into six chapters. The first of them corresponds to the Introduction, summarizing the essential issues that are later developed, and sketching their lines and ways of working. The second and third chapters present *basic* and *special* concepts, respectively, in order to connect general knowledge with specific definitions, which are central notions for the continuation of the study.

The fourth chapter summarizes the modes of self-similarity in the nature of musical codes and messages within their strata *corpuscular* (atomic conception of sound), *mechanical* and *acoustical*, *biological*, *phonic-phonological*, *cultural* and *ecological*, in a primary context of correlated self-similarity—a field in which intersemiotic translation occurs, necessarily involving music. The final subchapters (4.7. and 4.8.) proposes a reinterpretation of the theories that tie music with language, suggesting that grammar and idiolect are linked in a dynamical system with self-similar features, complementary to the 'migrations' between ecolect and context. In the midst of these migrations is where musical style emerges and is defined and transformed in *cycles of spirals* that never return to their initial states. In this sense, music and language share a general description as partially (pre)determined systems, simultaneously with variable features—such as emotion, intention and social transformation—highly sensitive to context and, therefore, undetermined.

The fifth chapter presents Shannon's information theory (1948) to explain the consistency between musical code and musical message; introduces the *synecdochic*

function, and reviews the most common techniques for obtaining information in data sets with self-similar features. Using basic methods of measurement such as *arc diagrams, type intervals,* and *products* and *coefficients of similarity*, to more sophisticated statistical tools such as *visual recurrence analysis*, examples are given of the different forms of self-similarity between sets and subsets in different forms of musical structuring. The final subchapter (5.5.) proposes hypotheses on how deterministm and indeterministm are coordinated forms of knowledge and intuition, that actually cooperate in different strata of intersemiosis. Particularly significant are the debates on the *fractal dimension* as "an issue of pertinence", and on *pure language* related to a (chimerical) fractal language. This discussion also gives opportunity to detail common misconceptions about fractals associated to language and music.

The sixth chapter explains the concept of proportion as an effect of specific relationships of self-reference and intersemiosis between geometry, arithmetic and aesthetic intuitions. It also investigates how the main constructive systems of the golden ratio are in fact self-similar systems—as well as its symbolic and functional conversions into musical self-references, as intersemiotic complexity. Finally, this chapter explores the concepts of 'antiproportion' and 'self-dissimilarity' as necessary contrasts to the conventions of symmetry, and as (inter)semiotic functions increasing the complexity of musical meaning.

Common notions

*There is no doubt that what is called form
is the sum total of all moments of logicality or,
more broadly, consistency of art works.*

Adorno, 1956/2002:203

2.1. Relation

"It suffices 'to perceive something', for establishing a relationship". In this sentence there is already a set of relations characterizing the same sentence as a whole: such relations may be hidden in the sentence syntax, the sequence of phonemes represented by its writing, the consonance of its vowels, etc. Just as it happens in this example, the primary ideas of *thing, object, subject*, or *fact*, are products of a relationship between percipient and perceived, or between apparent causes and corresponding actions (see Davidson 1963). After this overwhelming generality, however, one may notice that there is a tendency to grouping 'chunks' (e.g. things, objects, ontologies) of what is perceived, in operative categories (see Child 1994:68 *et ss.*; Givón 2002:40–41).

Due to the fact that the perception of spatiotemporal continuum is contrary to the individuation of operative categories, cognition divides (i.e. analyzes) the continuum in 'chunks', and relates them to 'what is around', in order to associate objects with context, by means of assimilation and appropriation. This form of association leads to *existence* of chunks, that in the logic of language corresponds to an existence of particulars and wholes. In this fashion, wholes (i.e. sets) are assemblies

of particulars that may have characteristics *equal* (i.e. identical) or *approximately equal* (i.e. similar), to enable them to be grouped within a common set (i.e. category). In turn, sets can have functions (i.e. operational, multi-relational characteristics) to enable them to establishing relationships with other sets. This game of functional assemblies contributes to mind's prospection and causality skills, to achieve systems constructing reality (see Bateson 1972, Damásio, 1994, 2000).

A category of continuous relationship between sets of pulses of perceived ontologies—resulting from the allocation of biological endorhythms upon the configuration of a *physical surface*—leads to the sensation of time.[57] Another category of continuous relationship—attributed to the arrangement of ontologies based upon the body's self-mapping—leads to the feeling of space. Another category of continuous relationship—that of sound vibrations, i.e. audible repetitions or frequencies—constitutes the basis of musical sound which, for its analysis, is segmented by discrete sets: discontinuous assemblies of accounting units.[58] Nonetheless, even analytical thinking requires an agglutination of parts by specific relationships, grouped by families or hierarchies, or by qualities related to categories that contribute to establishing systems of comparisons and constrasts.

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[57] See Augusto Fernández-Guardiola "El concepto del tiempo" ['The concept of time'] in (J. R. de la Fuente and F. J. Álvarez-Leefmans, eds.), *Biología de la mente*, FCE, Mexico, DF, 1998; p. 314. The author talks about "the idea of time". It is doubtful, however, that such an idea can be isolated from time itself, which in any way "is a product of our endogenous rhythms". Strictly speaking, more than an 'idea', time is an inherent characteristic of cognitive processes in general.

[58] This does not suggest that music, *made* in time and space, must be perceived as a segmentation. It is perceived, rather, by its tendency to continuity. Analytical segmentation is a process that follows the perception of music. As Knopoff and Hutchinson (1981:19) consider: "A scale for the musical parameters […] other than pitch or pitch differences, is hardly an array of discrete values but is instead best described in terms of a continuum of possible values. Thus we may imagine that music is, in some significant part, constituted out of a superposition of a number of continuous, expressive parameters." This idea coincides with Cooper and Meyer (1960:2) quotation on page 8 (see footnote 15).

Establishing relationships to achieve coherence is a priority of musical discourse.[59] The absence of signs leading to the interpretation of basic relationships in a musical structure by reference to a tradition, commonly leads to a feeling of 'lack of coherence', similar to the incoherence observed in linguistic incomplete relationships. The search for relationship in music is described by Stravinsky (1942:69–70) as an 'instinct of coherence': "by instinct we prefer coherence, with its silent power, instead of the restless forces of dispersion; we prefer the domain of order instead of the domain of difference". This study deals with this kind of relationships, aimed at the identification of *something*, or the preference for *something* through intuitive comparison. This implies that the traits of 'inconsistency' and 'difference' are equally important by a fundamental notion of contrast, giving meaning to the similarities of a system.

2.2. Repetition

Repetition is the frequency of an identity regarding itself. As intuitive relationship, repetition is virtually in all semiotic levels of music and most forms of continuity. In musical metre, the repetition of a series of pulses determines the identity of the measure. In addition, rhythm is characterized by a certain repetition of pulse as organized gesture; melody implies interval repetitions within a same scale, and harmony is based on pitches distributed in time, including functional relationships associated by repetition. The simplest song-form involves repeating a first section shaping most of the overall structure; this way of structural *rounding* is a trend reappearing in different vocal and instrumental forms, in different musical contexts.

In musical traditions as diverse as the vocal-instrumental festive repertoire of the Hani and Yi peoples from the Yunan province of China, and the homologous repertoire of the Uto-Aztecan family, extended in North and Central America, the

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[59] See Cruces (e2002). This author conceives four levels of musical coherence: *grammatical, textual, contextual* (pragmatic or interactional) and *sociocultural*. For the present study textual coherence is associated with symmetry, repetition, and other ways of figurative or Gestalt consistency; sociocultural coherence is associated with symbolic and representational choices. These levels are interrelated and can interact—in co-operation or in difference—within a same system, through corresponding layers of self-similarity and self-dissimilarity.

sequential repetition of one element following another has a crucial role for musical structuration. Accordingly, Hofman-Jablan (e2007:6.2) thinks of repetition as a universal aspect of music, connecting a wide variety of perceived phenomena:

> Most natural laws and occurrences, such as the coming and going of waves, the change of day and night, the changing of seasons, tides, breathing, heartbeat, pendulum movements, etc., are all different manifestations of periodicity in time. Rhythm is the repetition of occurrences or states in identical time intervals.

Based on a wide variety of sources, Ockelford (2005) concluded that repetition is the most important kind of relationship in building the sense of music, through memory. This includes repertoire composed under 'no repetition' axioms, such as integral serialism in which a tonal repetition model has been replaced by a gestural repetition.

A universal feature of music, repetition is decisive in shaping musical style. Voices pointing out the importance of repetition in different musical traditions and contexts are manifold: "repetition is a feature of all music […] all music contains repetition, but in differing amounts and of an enormous variety of types" (Middleton 1990:139); "repetition has been the decisive factor in giving shape to music […] the various devices used to integrate form are, again and again, nothing but methods of repetition" (Chávez 1961:38,41); "Intelligibility in music seems to be impossible without repetition" (Schoenberg 1967:20). Among those who acknowledge repetition as a basic element of music, Ruwet (1987:3), at the beginning of his text on musical analysis, puts the emphasis not so much on repetition itself, but on the meaningful alternation between what is repeated and what is not repeated:

> I shall start from the empirical appreciation of the enormous role played in music, at all levels, by repetition, and I shall try to develop an idea proposed by Gilbert Rouget: '...certain fragments are repeated, others are not; it is on repetition—or absence of repetition—that our segmentation is based'.

As Ruwet–Rouget understand it, *segmentation* is related to a cognitive principle: if time and space are a continuum, segmentation corresponds to how mental processes are able to establish identities. Without such segmentation the ontologies of language and music would be impossible. In addition, it can be said that repetition and non-repetition are notions analogous to similarity and difference, to be the starting point for any form of musical meaning by reference or instance.

In any musical structure, the measurement of the recurrences should make explicit the relations of continuity and discontinuity, distinguishing them from those relations which, although absent, are also relevant in music. Referring to the foundations of musical analysis, Dunsby and Whittall (1988:4) note that:

> An analytical plan will probably display certain fundamental features of musical organization that tend to occur whatever the period, or style, of a work: repetition, variation, contrast, connection, juxtaposition—bearing in mind always that 'the maximum coherence' implies a unity that embraces diversity, an emphasis on musical similarity rather than on musical contrast.

Among the few treaties dedicated exclusively to the subject of musical repetition, the one written by Ockelford (2005) asserts that the creation and cognition of musical structures come from imitation, which in turn comes from repetition. Ockelford (*op. cit.*:6–12) claims particular importance to the ideas proposed by David Lewin (1933–2003), in order to establish a music theory that assumes the question of 'what is taken into account' as central principle establishing a conceptual domain. This principle involves an epistemology of music constituted by its repetitions and similarities; and such epistemology strengthens the concept of a 'neighbourhood' between two or more elements or properties in a given set. Directly, this epistemology affects the criteria of space and relationship, and models the overall configuration of musical parameters.

Nonetheless, musical repetition is never purely quantitative. During listening, repetition demonstrates its high value by the qualitative recurrence of an element (a subject, a gesture, a pitch in a scale) or a relationship (of a specific interval, of a form of vibration in an acoustic system), strengthening or weakening the coherence of musical discourse and expression. In these terms, according to Stephen Peles (2004:58), "regularity and symmetry can contribute to musical coherence, [as] they are associated with repetition."

The repetition of musical elements can have—as analogously happens in verbal language—an expressive and rhetorical profile. In a significant way, this repetition serves to fix in mind the symbolic features of a musical identity. As used in speech, these symbolic aspects can gain psychological depth when they are omitted in a strategic way, so that the lack of repetition, as suspension or withholding of expectation, can have a greater effect than the simple repetition. Frequently, this

exchange between the mere execution of repetition and its restricted distribution, gives depth to musical planes, providing them with more interest as processes of tension and figuration.

Finally, it is essential to consider that musical repetition, like musical symmetry, never occurs in an absolute or perfect manner.[60] Unlike what may be understood in pure mathematics, the ideal accuracy of musical repetition is completely submitted to the continuous divergence of its interpretations. This notion is developed especially in subchapters 2.3. (on musical symmetry), 3.3. (basic definition of self-similarity) and 3.7. (musical recursion), to be taken as operational extensions of what is specified here.

2.3. Symmetry

In almost all musical treatises and in the music they reflect, there is a plethora of relations and functions of symmetry. Consequently, the notion of symmetry in music is too broad to be covered at once; besides, the subject has already been addressed by many authors.[61] Suffice it to summarize, then, general aspects of symmetry directly involved with musical self-similarity.

Symmetry is an intrinsic property of a set of relationships that remain invariant under such kinds of *spatial transformations* as reflection, inversion, recurrence, rotation or other abstract operations (see Thompson 1917, Weyl 1952). Under this general definition, the concept of *identity of an element*, at least as a geometric and musical entity, is closely linked to a symmetry that characterizes it and makes it unique or special in the way of relating to other elements or identities. As noted by

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[60] Walter Benjamin (1936), Theodor W. Adorno (1955), and Jacques Attali (1977) discuss at length the issue of mechanical repetition at the industrial and commercial processes of modern music. From a sociological view, they criticize forms of automated replication and reproduction as an impoverishment of music. However, from the perspectives of experimental psychology and cognitive sciences, neither this form of repetition can be absolute, as for being attested music depends on the continuous variations of perception (as suggested in graph ◊450, page 173). As Toop (1999:201) suggests, for music the absolute exactness of the repetition is of secondary interest. This matter is developed in subchapters 5.5. and 6.6.

[61] Among others: Kandinsky 1926; Chávez 1961; Howat 1977, 1983; Lerdahl and Jackendoff 1983; Estrada and Gil 1984; Mazzola 1990; Hahn 1998; Lerdahl 2001; Madden 2005; Vázquez 2006; and Hofman-Jablan e2007.

Weyl (*op. cit.*:3), symmetry gathers in a compact way the notions of unity and relation: "symmetry is [...] something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole." In music, symmetry covers most of the operations and functions of a constructive element in the mixture with which it interacts. This notion of symmetry has a simultaneous realization in musical time and space.

For the ideal representation of music there is a correspondence between intuition and rational justification of symmetry, as in the following example, which suggests a repetition of spatial values with a complete feasibility of comparison between them. These values also reflect a musical intentionality for the ordering of temporal reciprocity, with equal durations, and prosody, with an accentuation equally distributed:
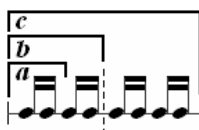
◊230. Schematic representation of symmetry as musical repetition and emergence of distributive hierarchies (adapted and simplified from: G. Weber 1821/1851, I:92–97). For each item repeated there is an axis of symmetry. At the same time, for every level of grouping there is an axis defining *a*) the repetition of the original item, *b*) the repetition of the original system, and *c*) the repetition of *a* and *b*, or generalized set.

The reason for this idealized representation is to propose a *rigid order*, similar to that proposed by a grammar rule, in order to implement a general framework of usage, and—on the other hand—facilitate a consensus on its interpretation, comparable to a *flexible order*, decisive for the pragmatic orientation and the intentional nuances of music, by analogy with what happens in verbal language. From this very simple case (◊230) may derive other relations of imperfect or incomplete symmetry, and other arrangements of symmetry / asymmetry in varying degrees, as suggested in ◊231.
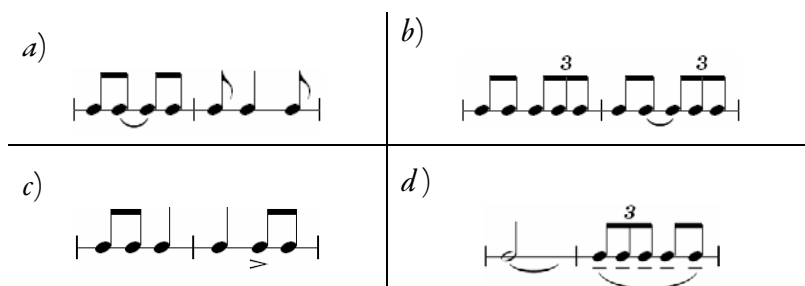
◊231. Examples of an identical time-span system with symmetric alterations:
   a) bilateral symmetry with prosodic asymmetry (the orthographical difference suggests a prosodic shade differentiation);
   b) bilateral symmetry with articulatory asymmetry (the tie represents a change of articulation);
   c) radial symmetry with prosodic asymmetry;
   d) bilateral symmetry with prosodic and articulatory asymmetry (the length of both segments is the same, but their prosody and articulation differ).

Carlos Chávez (1961:47–50) notes that "rhythm and symmetry are the essentials for musical construction. Rhythmical and symmetrical repetition operate as the basic and universal principle for rudimentary as well as larger structures." Chávez conceives four fundamental types of symmetry in music, from a basic relationship of identity: (*a*) bilateral symmetry, (*b*) reflection or radial symmetry or mirror, (*c*) 'recurrence' or inverted symmetry or cancrizans, and (*d*) double inversion symmetry or reversed cancrizans:
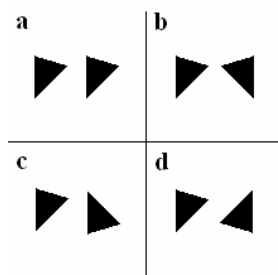
◊232.

To facilitate their formalization, Estrada and Gil (1984:31) labeled these typologies with the letters dbqp, corresponding to their symmetrical accommodation into a musical structure; for example:
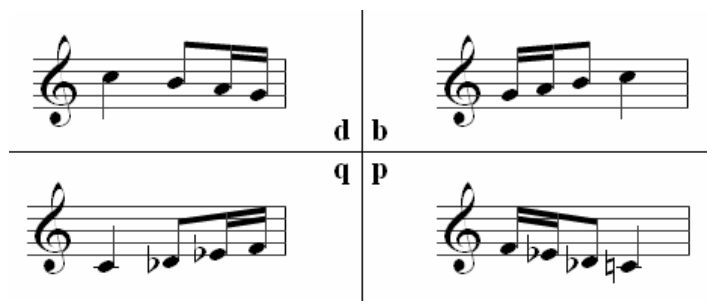
These four typologies are the foundations of a system of similarities established in the music of the Western culture since the Middle Ages to the present. The canon, one of the most exploited resources after the eighth century, implements "four forms of movement [...] the initial movement, its inversion, its retrograde, and the inverted retrograde" (Hahn 1998:405). Equally, Estrada and Gil (1984:31) note that "The recurrence, *cancrizans*, like the reflection, *mirror*, are formal procedures of counterpoint. In the twelve-tone technique they are the basic operations for the series, and in integral serialism, besides pitch structuration, the series orientation affect length, timbre, and loudness." These authors undertake an analysis of the orbits of the cube,[62] assuming that it consists of a symmetry that "can be useful" in a comparative analysis of tonal harmony.[63]

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[62] The orbit of a geometric figure is the set of the characteristic relationships between the points of the geometric figure that allows us to define it as such. For example, the set of relations among the points *forming* a cube fulfill the orbit of the cube. Correspondingly, the orbit of a point x in X is the set of elements of X to which x can be moved by the elements of G, where G is a group acting on a set X (in this example, a geometric figure).

[63] To this end, Estrada and Gil (*op. cit.*:14–15) suggest distributing the intervals of tonal harmony within the orbit of the cube, to make them coincide with the cube's eight vertices (analogous to the elements of the octave), twelve edges (intervals of second, third and fifth), twelve lines diagonal to the faces (intervals of second, third, fourth, sixth and seventh), and six principal planes (intervals of second, third, fourth and sixth).

Witnessing a vastness of possibilities for adapting symmetry into music, this study focuses on a limited selection of relations. Preference is given, thus, to the scalar symmetry in which some typical relations featuring a particular process or object are repeated or transformed within a more general structure. In consequence, special emphasis is paid on the synecdochic function that operate through the symmetric identification of the whole in its parts, or of the part in its whole.

Regarding the acoustic frequencies integrating a tone, the typical relationship between fundamental and first harmonic complete a symmetry that extends to smaller intervals into a spectrum which can be analyzed as a Fourier series.[64] Concordant to graph ◊230, in this case the harmonic intervals are also 'easier' to be perceived and described when they are doubles or halves. As a matter of fact, the best known examples in Western tonal music are the intervals of *octave* (the double of the base or *fundamental* frequency, equivalent to the ratio $^2/_1$), and *fifth* (the half of the base frequency, equivalent to ratio $^3/_2$, whose metrical interpretation, called *sesquialtera*, is a ubiquitous proportion in many different musical traditions).[65]

It seems obvious that any set of frequencies represented as integer (pitch, tone, note) can be represented equally as a sum of two halves. In successive layers, the total of parts building up a set of metric pulses, complete a symmetry by the relationship of halves made of halves (see Zuckerkandl 1956:179). This model is useful because —perhaps due to an empathy with the human symmetry—it is much easier to identify halves or doubles, and pair sets in general, than odd, asymmetric sets. It is clear, however, that this is a stereotype and an overall summary of relationships that *do not really exist*, neither in the body symmetry nor in music, but as an idealization. This is why Arnold Schoenberg (1967:25) conceives that: "Real symmetry is not a principle of musical construction. Even if the consequent in a period repeats the antecedent strictly, the structure can only be called *quasi-symmetrical*".

According to Chávez (1961:47), musical symmetry "is an adjective quality implying the existence in time or space of two or more equal or identical elements, placed at equal intervals or distances". For this description, the word 'adjective' has a special significance, as it suggests that symmetry does not mean exactly the same in

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[64] See subchapters 4.1. and 4.2.
[65] On this particular see also figure ◊310, within the context of analogy.

music and in mathematics: for the latter symmetry is 'substantive', it is a primary category of the mathematical thought. Rather, musical symmetry is a category always subordinate to sonority. This is thoroughly confirmed by Hofman-Jablan (e2007:5.1.) noting that the mathematical symmetry is not completely equivalent to the musical one, but analogous:

> [W]e have translated the terminology of the theory of symmetry to the language of music and have established the relationship between analogous rules. Because of the specific nature of the laws of symmetry in music, the theory of symmetry laws do not permit their direct (mechanical) transposition and application. This is why it is necessary to first define all specific properties, in the first place those dealing with local and global symmetry, and to create a special system of symmetry laws [...]

In parallel to what is discussed on the Pythagorean dilemma at the beginning of the Introduction, this study suggests that the principles of symmetry stimulate a confrontation between what is perceived as feasibility of comparison, and what is proven mathematically by statistics and geometry. This discussion extends throughout the following chapters, going deeper into the concept of musical self-similarity and its intuitiveness.

Isometry

In mathematical parlance, the term *isometry* refers to a relationship of equivalence and correspondence between each of the elements of a set, preserving the distances along a set transformation (see Mason 1969:381). By this definition, any musical invariance is isometric; therefore, musical (self-)similarity—an invariant phenomenon—implies a kind of isometry (from Greek ισός, same, and μέτρον, measure), including identity, equivalence and proportional sameness.

The replication of the unity of any musical symmetry operates on a basic relation of isometry, which Hodges (2003:98) represents by the equality of two adjacent symbols: ●●. Musical isometries can have a variety of analogies, so that any simple repetition can be the starting point for a self-referential system.[66] For example, the simplest musical isometry, which ratio is represented as 1:1, has the analogies of *unison* in a harmonic interval, *whole measure* in a metric interval, or *whole period* in a

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[66] For an operative definition of 'self-reference' see subchapter 3.6.

phraseological interval. Hodges (*ibid.*) explains that, in their spatial representation, the four basic kinds of musical symmetry (see ◊232) operate under the principle of not altering the distance between two points in the plane: "Transformations with this property are called *isometries* because they don't alter the scale of distances". For the same reason, isometries are often a defining feature of self-similarity in music, holding relations of identity and equivalence at different scales.[67]

Setting the self-referential foundations of music, the isometries have a key role in the construction of musical meaning, as repetitions consolidating the periodic form of a recurrent pulse or a harmonic system. In a practical context, a tuning fork or a metronome beating at equal intervals, supply the ear with referential isometries for the articulation of music. Then, the isometries act as a *rigid frame* with which repetition subordinate recursion, of *flexible nature*.[68]

By repeating patterns uniformly, the instrumental forms and the typical metre of a stylistic repertoire make use of isometries as structures for general referentiality (see Katzarova 1970:31–32). Resources like the repetition of a measure in a uniform metrical scheme, or a uniform figure, or a section completing a larger uniformity, work as functional references after an isometry. A similar relationship does happen in the functional use of the unison and the doubling of harmonics, as acoustic technique in classical instrumentation. Furthermore, conceiving that such isometries operate in a very wide variety of layers and parameters, it can be stated that a whole repertoire plays a structural role by its repetitions, involving (pre)self-similar relationships.[69] This outlook becomes more complex when one estimates that musical proportions, and not only musical repetitions, operate as analogies of a primary isometry, replicating itself at a smaller scale. Accordingly, the musical styles conform self-similar structures within which there are varying degrees of identification with an original reference, transferred by isometries and isomorphisms

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[67] For example, in chapter 6, when comparing ◊620*Ca* with its origin ◊620*Aa*, the distances between specific subsets of points are equivalent. In this example, by duplication of the original design, it appears that some distances are divided or multiplied by 2; isometries prevail as basic structural relationships.

[68] On recursion and recursiveness, see subchapter 3.7.

[69] See subchapter 3.8.

of key importance to the listener's memory, and to the collective memory of a musical tradition.[70]

Isomorphism and self-similarity in music

A musical system that preserves sets and relations among elements, is called *isomorphism* (ισός, same, and μορφή, form). Echoing John D. Cuciurean (1997:1), it can be stated that 'the unfolding of a compact set into a larger structure exhibiting an isomorphic relationship with the smaller set is the essence of self-similarity'.[71] Such a set can be constituted either by algebraic or geometric relations, and set in time and space, so the basic musical relationships may behave in a tendency to isomorphism —or *isomorphisms of isomorphisms*. If such an isomorphism appears in several scales or subsets of a universe, then it is said that there is a self-similar musical relationship.

Cuciurean (*cit.*), and later Amiot (2003), Vázquez (2006:273–276), Murphy (e2007), and Ilomäki (2008:35–53), interpret the consistency between isomorphisms and functional similarity as the basis for musical structuration, since such consistency produces "maximal diversity on the surface, whilst maintaining a simple underlying structure" (Cuciurean 1997:1). Murphy (e2007) finds also that there are degrees of musical consistency ranging from generalized self-similarity to the progression of inconsistency in a self-similar model; this relationship is called *self-dissimilarity*—a concept developed here in subchapter 6.6.

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[70] See subchapters 4.5. and 4.6.

[71] Cuciurean (*op. cit.*) emphasizes this property in group algebra, not in set theory. However, the principle of self-similarity still the same. The original text says: "The unfolding of a compact algebraic group into a larger structure exhibiting an isomorphic relationship with the smaller group is the essence of self-similarity".

Musical automorphisms

A transformation is a 'map' of an object or process, into another, corresponding and symmetrically similar object or process. A vast array of musical objects and processes are transformed in different operations such as *dilation, expansion, reflection, rotation or stretch*.[72] Some of these transformations are characterized by an invariant line and a scale factor, and many preserve sets and relations among their elements; therefore it is said that they are isomorphic. An isomorphism mapping a group onto itself in a one-to-one account of elements and relations, is an *automorphism* (see Morris 1987:167, Lewin 1990:86–87). Automorphisms are 'engines' of self-similarity, because they preserve distances, intervals and functions of identities at different rates of spatial and temporal coordination.

Lewin (1990) suggests that pitch-class transformations commonly correspond to isographic networks (*Klumpenhouwer networks*) related to specific automorphisms. Klumpenhouwer (1998:82–83) also hypothesizes that—for a generality of harmonic systems—there are commutation, combination, inversion and partition 'protocols', grammatically enabled as algebraic properties characterizing musical constructions. Accordingly, Carey and Clampitt (1989:196) associate scalar automorphism as a pre-requisite for well-formedness in musical (grammatical) consistency. Nonetheless, automorphisms do not only occur in harmonic structures, but also—and at least—in a variety of metric, rhythmic, melodic, counterpunctual and textural relationships (see Amiot 2008:164–165,171). Because of this condition, Chapter 5 pays special attention to a variety of musical self-similarity related to isomorphisms and automorphisms (in particular, see 5.4. and 5.5.).

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[72] In this context, the concept of 'affine transformation' is introduced in following subchapters 2.4.–2.5. It is further developed in 6.2.–6.5.

2.4. Functional similarity

The concept of similarity is extended to virtually all uses of language, and permeates all forms of human communication based on the invocation of signs of some things in others, as relationships of identity, equality, proximity and association. Typically, the notion of similarity is latent in the construction of categories and processes of perception and comparison, so it is difficult to grasp the same notion for its analysis as it involves its own elements for description and definition. It is therefore essential to narrow a definition of similarity, at the same time distinct from and connected to other concepts to which it is often associated.

The term *functional similarity* means the order established between two systems of comparable relationships.[73] "Order, then, can be identified with similarity, and disorder with difference" (Feibleman 1968:3). According to Rips (1989), the decision to attach an object to a category depends on the degree of similarity assigned to that object, regarding the known members of that category. A category is each class in which, by their similarity, elements or relations are grouped in 'families'.

> To put this in more cognitive terms, if you want to know whether an object is a category member, start with a representation of the object and a representation of the potential category. Then determine the similarity of the object representation to the category representation. If this similarity value is high enough, then the object belongs to the category; otherwise, it does not. [...] This simple picture of categorizing seems intuitively right, especially in the context of pattern recognition. (Rips 1989:21)

Despite the transparency of this definition, there are criticisms against it. These criticisms come from philosophical discussions on similarity, particularly in

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[73] A special case of functional similarity is the *mathematical functional similarity* (e.g. in a logistic map). In order to avoid confusion along this study, employing this term, the definition of *mathematical function* (f) is given in a proper context, in subchapter 6.2. (see especially pages 346–348).

Goodman (1970) and Quine (1969), which have gradually gained ground in psychology. Accordingly, Murphy and Medin (1985) point out that similarity is a notion too weak to explain categorizations in an appropriate manner, and empathizing with Goodman (1970), they concludes that the similarity is highly relative and context dependent. Especially because judgments of similarity depend on objects, properties, relations and categories learned through experience and stereotyped by culture (see Rips 1989:21–22; Lakoff 1987:281–282).

The functional operation of a particular group of temporal, spatial and causal relations in a musical structure, facilitates the investigation of similarity degrees to explain—at least partially—such a structure.[74] But the extension of the criteria of similarity in music as a language has several obstacles. This issue is related to the difficulty in distinguishing the so-called *universals* of musical language (see Padilla 1998, Mâche 1998). In short, whether such universals do exist, there are also relationships between them, knowable as *fundamental similarities*. Kaipainen (1994:50) proposes the same argumentation in logical terms, saying that if music is permissible as a special case of cognitive processes in general, then "anything said about general cognition can also be said about music".

The objections of Goodman (1970) and Quine (1969) address the 'uniqueness' of the similarities outside their own context. Similar objections can be argued against the concept of a 'universal music' with fixed values. In any case, the identification of similarities in music is limited to the analogy and comparison of knowable practices and measurements, typified within a specific tradition and encoding, and subject to a cultural orientation, as happens with language. The similarities and differences in music "cannot be explained adequately as part of a closed system without reference to the structures of the sociocultural system of which the musical system is part, and to the biological system to which all music makers belong" (Blacking 1973:30–31).

In an intuitive manner, Stravinsky (1947:32) observes that "Similarity is hidden; it must be sought out, and it is found only after the most exhaustive efforts". Consistently, Rips (1989) and Smith (1989) consider that similarities can be found in

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[74] Foucault (1966:321) acknowledges that such a similarity is established by affinity and sympathy between two things, "the sign of affinity is analogy; the cypher of sympathy lies in proportion." The close relationship between analogy and proportion is studied later in subchapter 3.1.

a variety of ranges between cognitive surface and cognitive depth. A superficial similarity would correspond to the attribution of common properties between two sound frequencies emitted by two different physical sources. A deep similarity would be, by contrast, that which can be explained as a psychologic link in the use of both sound frequencies with different origin, within the context of a convention (a treatise of harmony, a specific grammar orientation, a contextualized use, and so on). On the other hand, there should be a *global similarity* associated with the relationships between cognitive system networks, for example in the length and pitch classification within a chord; and a *dimensional similarity* between systems of compatible parameters, within the relationships linking the abstract systems of a musical content, such as *modalities, extensions*, and *intensions*.

Linda B. Smith (1989:173) distinguishes between *perceptual similarity* and *similarity in complex concepts*. With the first one she refers to a relationship between comparable groups, as just described above; with the second one she refers to a degree of sympathy in the identification of one thing respect to another, by common, measurable or immeasurable values. By following this approach, some examples of perceptual similarity would be basic ideas groupable under the types *strong, sharp,* or *consonant*. By contrast, some examples of similarity in complex concepts would be ideas groupable as *proportional* or *stochastic similarity*. This very basic separation already appears on the intuition with which Mandelbrot (1982:4–5) distinguishes two basic types of self-similarity:

> The combination *fractal set* will be defined rigorously, but the combination *natural fractal* will serve loosely to designate a natural pattern that is usefully representable by a fractal set. For example, Brownian curves are fractal sets, and physical Brownian motion is a natural fractal.[75]

Speech and music share many relations of perceptual similarity. They are also related as complex systems.[76] But they are also associated in a deferred way that can be separated as *the musical similar to speech, the musical similar to non-verbal language*, and *the musical similar to itself*. This concurs with Charles Seeger's idea (1960:226), depicting music and speech as symbolic functional systems with cases of *identity*

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[75] This notion is discussed in subchapter 5.5.
[76] See the beginning of chapter 5, within the section devoted to Zipf's law.

(homology), *similarity* (analogy), and *difference* (heterology). This picture is enriched when the analysis of the similarities reveal that there are distinct degrees within the same category, in which a thing or a relationship 'is similar' to another one.

2.5. Statistical similarity

For its usefulness in achieving concrete results in the exploration, analysis and interpretation of data concerning musical relationships, the concept of statistical similarity is much more relevant in this study than the general concept of similarity. *Statistical similarity* is the considerable degree of comparison between values which are not identical or equivalent. This notion is commonly used in computer science, to identify similar patterns in heterogeneous data bases, which is usually associated with grammatical consistency criteria (see Wang, Xu and Zeng 2006).

A starting point in the exploration of musical similarity statistics can be the finding of symmetrical/asymmetrical relations, noting common figures between two sets of relationships. Even when it is not explicitly used, or not used with its many resources, statistical similarity is an intuitive and common tool for musical analysis: Lerdahl and Jackendoff (1983) use it to give support to their theory on *grouping preference rules*. According to these rules, at a basic level, the spanning of time and space determines a greater or lesser tendency toward the perceptual grouping. In the case of music, the 'spaces' are also frequencies—in the broadest sense of the term— with grouping trends dictated by a perception and a near-term memory (Wertheimer 1923); "groups are perceived in terms of the *proximity* and the *similarity* of the elements available to be grouped. In each case, greater disparity in the field produces stronger grouping intuitions and greater uniformity throughout the field produces weaker intuitions." (Lerdahl and Jackendoff 1983:41).

The notion of similarity is also affected by the rate of repetition of the perceived values. As a whole, the measurement and possible prediction of musical values by repetition, proximity, or functional affinity, constitute the basis of statistical similarity in music, and are useful to identify aspects of relation, symmetry, and proximity.
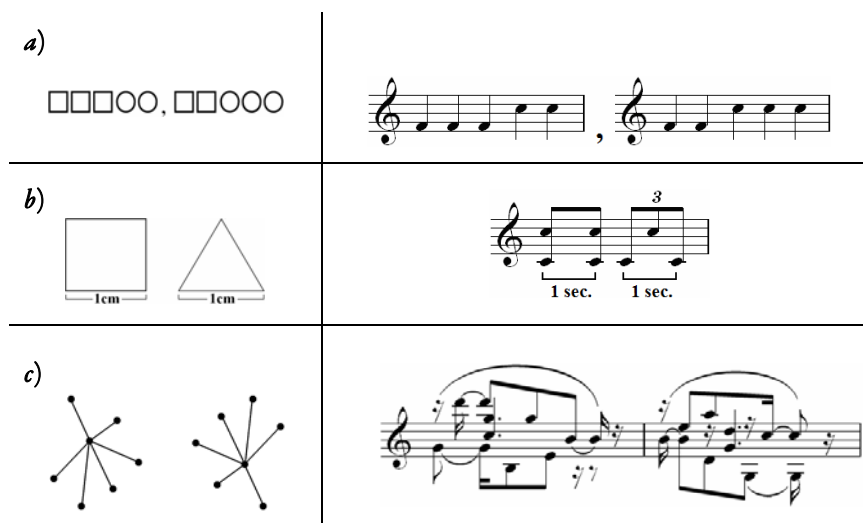
◊250. Simple symmetries with statistical similarity. In the boxes to the left, examples of contiguous visual empathy in two dimensions (width and height). To the right, examples of *musical* empathy in typical dimensions {*x*} = *time*, and {*y*} = *pitch*. It is worth noting another level of similarity shared by all these examples as representational models: the employment of dots and lines for suggesting structural relations (an aspect studied in subchapter 6.1.) The sources for this examples are adapted from (*a*) Lerdahl and Jackendoff 1983:41; and (*b*)–(*c*) from Pareyon 2004:15–16.

The observation and classification of the frequencies (repetition) and their relative positions (distribution) is very important in the definition of statistical similarity. In ◊250, for example, specific frequencies and positions are evident for each case. In (*a*)-left there are two sets with two sorts of elements. Clearly, there is an immediate relationship between the subsets containing squares, both for their spatial proximity and for the affinity between the figures. The same type of relationship can be attributed to the subsets containing circles. Both sets grouped in (*a*)-left are 'similar between them' because for each case their elements are not the same and they are not distributed in the same way—otherwise they would be identical sets—although there is a partial identity when considering the relationship element by element. Something similar may be pointed regarding (*a*)-right, where there are not two different geometry classes, but two different pitch classes. Here again there is a spatial tendency for grouping equal pitches (read distributions) {F-F-F | C-C} and {F-F | C-C-C}. The intuited affinity in their distribution occurs by a mechanism comparable to that

seen between the elements in (*a*)-left. Nevertheless, regarding (*a*)-left it is worth to mention that the unit taken for comparison changes in (*a*)-right from a spatial form, to a temporal form (represented by coordinate *x*), whilst the pitch happens to be represented as verticality (musical frequencies in coordinate *y*).

The following case in ◊250, (*b*)-left, shows two elements related by the length of their parts (1cm) but not equal in their distribution. It is not therefore the case of identical but statistically similar elements in a relationship that can be represented as $4l \leftrightarrow 3l$. Between (*b*)-left and (*b*)-right there is an analogous spatial distribution which, as for the previous example, has come to a coordination of pitch and duration. It should be noted that in this case there is a measurement which is not presented in the previous example. In this case there is a 'scalar affinity', because the measuring scales can be compared with the horizontal intervals in the analogy 1cm ~ 1sec.

The example in (*c*) shows an analogy of proportional relationship representing a finer spatial distribution, with a hierarchical systematization in which a 'dominant' element coordinates other elements, 'subordinates': in (*c*)-left the dominant element is tied with the others through converging lines, whereas in (c)-right the dominant element appears as the reference for a group of intervals. In these two cases the property of *transitivity* is remarkable, whereby two unconnected elements show a relationship via a third element (in logical terms: if $x \in A$, and $y \in x$, then $y \in A$). Theorizing examples like those shown here, it is possible to discern the transitivity in transitive classes for the interpretation of functional sets in a musical structure. This notion is demonstrated by the examples of transformation in subchapter 2.3., and is developed throughout chapters 5 and 6, together with the concept of mapping coordinates of length, loudness, pitch, and timbre.

Obviously, the complexity between statistically similar sets can be much greater than that seen in ◊ 250 (see e.g. Lewin 1990, Murphy 2007, Ilomäki 2008). The forms of similarity can range from the level of the sets with one or few modes of relationship, to systems of sets with functions of gradual similarity and diversified group behaviours. Consequently, the variables submitted to the analysis of statistical similarity can be very abundant and distinct among them. The following chapter gives an introduction to this diversity.

Special notions

3.1. The two modes of analogy

In his treatise on the beginnings of Greek mathematics, Árpád Szabó (1978:144–169) devotes many pages to discussing the classical meaning of the word *analogy*. In this discussion the concepts of musical harmony, geometric proportion, and harmonic ratio, are closely related.

A first etymological approach may produce a very broad definition for this concept: the preposition ἀνά means *backwards, following the same path*, whilst the complement λόγος, *ratio*, derived from λέγειν, means to reason or discern. An analogy would be, then, not a reversal of things, going backwards, but rather a recovery of the sense of one thing into another that is comparable. The approach made by Szabó (*cit*.:145–146), however, reveals a very specific definition:

> The relation of *proportion* (or sameness of ratio) was called ἀναλογία in Greek geometry. [...] Modern variants of the word ἀναλογία are to be found in all European languages. Moreover, they all have roughly the same meaning. 'Analogy' means similarity, conformity, relationship, or the extension of a rule to similar cases. [...] It is less well known that this same word was not originally a grammatical or linguistic term, but a *mathematical* one. The root of the word ἀναλογία is obviously λόγος, which in mathematics meant the 'ratio' between two numbers or quantities (*a* : *b*). Ἀναλογία itself described a 'pair of ratios'. Following Cicero, it was translated into Latin by *proportio* (*a* : *b* = *c* : *d*). The Greek grammarians of Helenistic times undoubtedly borrowed their term ἀναλογία from the language of mathematics.

Thus, the concept of analogy can be interpreted as a precursor of self-similarity, which typically involves notions of relation, proportion, repetition, and convergence, and an "extension of a rule to similar cases" (compare this with the notion of *scale* in

Mandelbrot 1981). For cases of proportional systems in music, this definition of analogy also involves the self-reference of a harmonic system, as detailed in Chapter 6.

Szabó's conceptualization relating analogy to its meaning of proportion, whilst musical and mathematical, is not opposed to the notion of analogy as a paradigm or insight through comparison. Rather, it implies some functional coordination. Foucault (1966:19–21) examines the relationship of similarity under the episteme of *pre-Cartesian resemblance* in four modalities that are "fundamental for the construction of knowledge in the Western cultures": convenience (*convenientia*), emulation (*aemulatio*), analogy (*proportio*), and sympathy (*sympathies*). For the meaning of their etymologies, but also for their modes of operation, convenience and emulation are associable with *paradigm*, whereas analogy and sympathy are associable with *proportion*. Both forms of relationship, such as proportion and paradigm, are coordinated, and eventually subordinated one to each other (see Foucault, *op. cit.*:27–32) so that it is possible to devise a *paradigmatic analogy*, or a *proportional analogy*, according to the balance of their functions (i.e. convenience, emulation, analogy, and sympathy).

In explaining the possible functional similarity between music and language, Seeger (1960:226) prefers analogy to metaphor (μετά, *change*; φέρειν, *to bring* or *to lead to*), whose operation involves changing original meanings. Whereas there are few musical relationships that can be explained as a metaphor or as a mere imitation or simile, analogy covers almost all structural aspects of music; even for those fundamental aspects in which music operates in intersemiosis with other forms of expression. As Schoenberg (1967:25) suggests, "The term symmetry has probably been applied to music by analogy with the forms of the graphic arts and architecture". This paradigmatic analogy occurs whenever their aspects can be explained by a system of comparisons, according to a logical tradition.

The paradigmatic analogy consists of pairing two systems of comparable relations. For instance, Hindemith (1937/1941:57) conceives that "If we think of the series of tones grouped around the parent tone C as a planetary system, then C is in the sun, surrounded by its descendant tones as the sun is surrounded by its planets". This analogy is firmly entrenched in the theory of tonal music, as can be seen in a

more recent description by Ashton (2001:10): "A natural [acoustic] pattern quickly evolves, producing seven discrete *nodes* (or *notes*) from the starting tone (or *tonic*), separated by two *semitones* and five wholetones, like the sun, moon and five planets of the ancient world". According to Vosniadou (1989:416–417), as symbolic pairing, the paradigmatic analogy works as follows:

> 1. The [cognitive] system retrieves a familiar source example together with an explanation of how this source example satisfies some goal.
> 2. The system maps the explanation derived from the source onto the target and attempts to find out if this explanation is justified by the target example.
> 3. If the target used justifies the explanation, the conclusion is that it satisfies the goal.

This explanation is based on the causality that characterizes the description of a set of relations *B*, by parallelisms with another set of relations *A*. Whether the causes and effects featuring *A* are comparable to those featuring *B*, then a paradigmatic analogy can be made between the two sets. It must be noted, at this point, that incongruence between conceptual domains "is *not* a defining characteristic of analogical reasoning" (Vosniadou *op. cit.*:417). The defining characteristic of analogical reasoning is, as Vosniadou asserts (*ibid.*), "similarity in underlying structure"; i.e. similarity as a system of manifold deep references (psychological, cultural, contextual, etc.).

◊310. Manifold analogy in a simple musical system. In this example at least two parallel systems of proportional analogy can be acknowledged: *a*) *additive analogy* in comparing the number of the grouped elements, equal to the multiplication of the measure 3×2. *b*) *subtractive analogy* in pitch comparison, equal to the division of the measure 3÷2, interpreting the result 1.5 as an 'octave' (*integer* or first harmonic interval) plus a 'fifth' (*first half* or second harmonic interval). In this same example, the multiplication operates as an analogy of the sum, whilst the division operates as an analogy of the subtraction. From the point of view of intersemiosis, a paradigmatic analogy can also be noted: the conceptual similarity between the graphic representation (the example's image) and its conversion into sonority, by a symbolic convention. This kind of analogy can be summarized as the link between a pipe and its drawing, as suggested by René Magritte's painting *La trahison des images* (1929), with the inscription "Ceci n'est pas une pipe".

Analogy, as a system of comparisons, is conditioned by a tendency to establish such comparisons in a given context. The cases in which analogy may be associated with a *universal* comparison are due to an intuitive ability to relate sets of relationships, as well as to the identification of their context. Some examples associated with the latter are symmetry, proportion, and functional reciprocity between systems that can be judged as analogous.

The relationship between musical metre and harmony can be strictly analogous in a broad comparative framework, and in the mathematical framework of ἀναλογία, since the comparison between their symmetries can be exact. The interval of fifth can be represented by the ratio 3:2, whilst the same ratio may represent an analogous metre by the ratio of sesquialtera (see ◊310). This example contains the germ of a profound coordination on self-similarity, based on the two classical modes of analogy, summarized as proportion and paradigm.[77]

From this subchapter it can be concluded that, in musical thought, analogy is meaningful to the extent that it contributes to the identification of an entity and its modalities, such as frequencies (repetitions) and characteristic combinations. According to Bent and Drabkin (1987:5), "By comparison [analysis] determines the structural elements and discovers the functions of those elements [...] comparison of unit with unit, whether within a single work, or between two works, or between the work and an abstract 'model' [...] The central analytical act is thus the test for identity". Thus, the basic tools of analysis are, by opposition or complement, forms of analogical comparison.

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[77] In order to support the operational use of these two concepts in this study, the following distinction is proposed: all analogy that is not proportional is paradigmatic. Additionally, analogies exist that are proportional and paradigmatic, at the same time (see for example ◊310).

3.2. Synecdoche

Based on classical texts by Plutarch and Quintilian, among other ancient writers, Bailly's dictionary (1894:1850) defines συνεκδοχή, synecdoche, as "Figure of words which consists in using a term in a more comprehensive sense, for example, taking a singular collectively for a plural".[78] Bailly presents other related terms, like συνέκκειμαι, which means "to be exposed [together] with", and συνεκκρούω, "to invert with, together, or at the same time". The same author entered other examples using the prefix συν·εκ-, implying simultaneity (temporality) and continuity (spatiality); for all these cases, the notions of parallelism and togetherness are generalized.[79]

In broad terms, within the study of language, synecdoche (pronounced sɪˈnɛkdəki) is a figure in which the part is taken by the whole (*pars pro toto*) or the whole is taken by the part (*totum pro parte*), for example, taking the genre by the species or the species by the genre. When Boris Cyrulnik (2004:4) asserts: "To speak is to create a piece of world",[80] he uses a grammatical metaphor in which speech is part of language, i.e. the *world*. The same example also operates as an *abstract synecdoche*, taking the particular speech as a global language, or a 'piece of the world' [morceau de monde] by reference to the world. In contrast, the lexical metaphor commonly corresponds to a *concrete synecdoche*. An example of this is the sign that acts as an index, making a correlation and involving an appearance of *A* in *B* (see Peirce 1903a/1998:163).[81] This also happens in a measurable physical relation, asserting that the relation *A* : *B* is proportional to the relation *B* : *C*.

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[78] The original text says: "συνεκδοχή. Figure de mots qui consiste à employer un terme en un sens plus compréhensif, par example, en prenant un singulier collectivement pour un pluriel". The alternative writing συνεχδοχή, not recognized by Bailly (*op. cit.*), makes more explicit the sense of continuity according to Charles S. Peirce (1893/1998:1): "The word *synechism* is the English form of the Greek συνεχισμός, from συνεχής, continuous."

[79] For the operative difference between *synecdoche* and *metonymy*, see section 1.3.2.

[80] Verbatim: "Parler, c'est créer un morceau de monde".

[81] A simple case of *lexical metaphor–concrete synecdoche* is the interpretation of the concept *clouds* as *storm* in the utterance "the clouds from the West will bring the first storm of the year." Demonstrative pronouns and in general the deictics explained in subchapter 3.6., directly and specifically indicating real objects, also fall into this account. A concrete case in music, in this

Both forms of synecdoche, concrete and abstract, are common in music, often involved in forming and structuring systems of coherence through self-reference. By using the notions of *point*, *group*, and *mass*, Karlheinz Stockhausen (1989:33–37) employs symbolic systems in which the part is correlated with the whole. Also, the synecdochic concepts of timbral, harmonic, and durational configuration are evident in theories on musical self-similarity put forward by Fagarazzi (1988), Hsü and Hsü (1991), Xenakis (1992:292–293), Hsü (1993), and Yadegari (2004).

The use of synecdoche in music is common even for the very definition of music e.g. under the general criteria of *musurgia* (Kircher 1650) or *natural music* (Tiessen 1953). These criteria in principle fix the notion that a partial definition of music —e.g. the basic features of melodic displacement—can extend to a general definition. As explained below, according to the theoretical approach of Carey and Clampitt (1996), synecdoche also serves as a basic constructive principle in tonal music, associating the diatonic and chromatic scales, as explained below.

Synecdoche, proportion, and well-formedness

Both sorts of sound distribution, *stochastic indeterministic* and *absolute deterministic*, may share a general notion of self-similarity, based on the criterion of proportion, which is closely related to that of harmony.

It is said that the Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, 13...) is self-referential because each adjacent number of the sequence depends on the sum of the two preceding elements, which in turn (from 2) depends on a constant ratio, approximate to 1.618033988749894... usually represented by the symbol φ. It also is said that the sequence is self-similar because each of its 'growing' segments are distributed in the same proportion (*golden ratio*, ~ φ), regardless of the size of the segment or its geometric projection.[82]

For the tonal music of the Western tradition, the overall relationship between the diatonic and the chromatic sets can be considered within a relationship

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sense, is the interpretation of a pitch as (part of) a chord in the context of functional harmony (e.g. a C taken as index of C major).

[82] According to Madden (2005:xi) the Fibonacci sequence and the golden mean "are self-similar members of a discipline called fractal geometry".

analogous to the Fibonacci sequences and the golden mean, since their initial values are based on a self-reflection principle that can be described as $x_n = x_{n-1} + x_{n-2}$, a formula that synthesizes the Fibonacci sequences, whose initial intervals can be associated as 0 = *perfect unison*, 1 = *minor second*, 1 = *Major second*, 2 = *Major third*, 3 = *perfect fifth*, 5 = *octave*, 8 = *minor sixth*, 13 = *Major sixth*; with their inversions 1 = *Major seventh*, 1 = *minor seventh*, 2 = *minor third*, 3 = *perfect fourth* (0, 5, 8 and 13 generate equivalences), counting the semitones of the chromatic row:
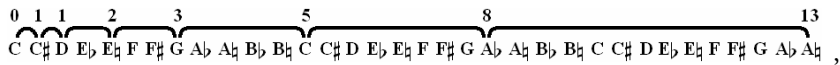
or counting the whole tones of the diatonic scale, generating the series 0 = *unison*, 1 = *Major second*, 1 = *Major third*, 2 = *perfect fifth*, 3 = *octave*, 5 = *Major sixth*, 8 = *Major seventh*; con las inversiones 1 = *minor second*, 1 = *minor third*, 2 = *perfect fourth*, 5 = *minor sixth* and 8 = *minor seventh* (0 and 3 generate equivalences):
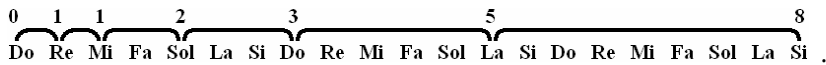
Accordingly, Carey and Clampitt (1996:65) assert that "The diatonic scale is self-similar in the following respect: the distribution of semitones within any diatonic interval is approximately equal to the overall distribution of semitones within the octave, namely two in seven" (i.e. $^2/_7$ as the distribution of half steps in diatonic intervals compared with half steps per octave). These authors associate this self-similarity to the quality of *well-formedness*.[83] It is said that a scale is 'well-formed' "if its generator always spans the same number of step intervals" (Carey and Clampitt 1996:63). In addition, Lerdahl and Jackendoff (1983:308) consider that "The closest analog to linguistic grammaticality in music theory is adherence to well-formedness rules." This notion is developed in Chapter 5, which through Zipf's law explains the relationship between the structural self-similarity in 'fractional noise' $1/f$—ubiquitous

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[83] The term *well-formedness* is borrowed from linguistics originally developed by David Rynin (1949:386: "Every meaningful expression is well-formed.") after a conjecture of Reichenbach (1947:25: "The formation rules [...] delimit the domain of meaning; i.e., they determine what expressions we wish to consider as having sentence meaning."). The notion of well-formedness does not refer to a grammatical correctness in itself, but to the acceptance of each specific case in a given constructive context, according to what the same context allows.

distribution of music—and the common grammaticality in language in general, and in music in particular. By its typical function, the self-similarity described by Carey and Clampitt, and suggested by Lerdahl and Jackendoff (*loc. cit.*) is comparable to a rhetorical or grammatical formula, in which the whole is recognized in the part: "Diatonic scale segments thus possess a synechdochic property: the part reflects the organization of the whole with a minimal, but inevitable degree of distortion." (Carey and Clampitt 1996:66).

In contrast, stochastic self-similarity is present in processes of preference and vulnerability of grammars, within the dynamics between ecolects and idiolects as a creative intercourse.[84] This is evident, for instance, at the variable agogic within the same musical piece, and in the stylistic variations within the creative processes in general. Many other examples from self-organization systems in music, match with this description. As Blackwell (e2006) acknowledges, "A self-organising system might produce appealing music, not so much by breaking rules, but by allowing new rules to spontaneously emerge." Therefore, vulnerability of grammars is particularly meaningful if it corresponds to an elaborating recursiveness.[85]

Under this conception not only the parallelism between absolute deterministic and stochastic indeterministic self-similarity are distinguishable, but also, a parallelism between synecdochic grammatical distribution and undetermined stylistic tendency. Prior to the development of the concept of musical self-similarity, Schenker's theory of tonality (1932) approaches this description through its intuitive notions of *Schichten* (strata) and *verborgene Wiederholung* (hidden repetitions), which in this study help to support the notion of 'sound nesting', developed in subchapter 5.4. Summarizing, musical self-similarity can be found as a coordinated process, simultaneously in general *stable* form, and in local *unstable* behaviour.[86] The possible alternation of these properties and the embedding of one into the other determine the form and function of their nestings.

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[84] This is discussed in detail in subchapter 4.8.

[85] See the definition of recursion and recursiveness in subchapter 3.7.

[86] Ultimately, the potential effectiveness of Schenkerian analysis and its post-tonal refinements (e.g. Dubiel, 1990) lie at the basis of an analytic reductionism processing selections between similarities and differences as points of coordination within a musical system.

3.3. Self-similarity

After a preamble containing essential details, an outline of the central concept involving this research finally becomes possible: self-similarity indicates the repetition at different rates, of the relations featuring an analogous set of symmetries (see Mandelbrot 1967, 1977; Peitgen and Richter 1986).[87]

Being similar to itself, a broccoli is an example of a self-similar structure assuming that a component of its own whole is similar to the whole. Another example of a self-similar object is the graphical representation of a set of relationships that involve, in two or more scales, the reproduction of the same set of relationships—something that actually happens in design and the visual arts, with the so-called *Droste effect*.[88]

There are, however, 'more complex' structures with endless self-similarity, in which each part, however small, implies its whole—something that happens in the so-called fractals; and there are others, as seen in large segments of language, in which a relative self-similarity does not appear as a kind of 'scale-bound object within a similar object', but as a statistical feature: distributions, and not shapes, are found in a self-similar relationship.

Although self-similarity is a phenomenon present in a wide variety of musical aspects, it was very little studied before Voss and Clarke (1975, 1978) demonstrated its cognitive and structural relevance. It was only until recently that it was explored by its cultural significance and contextualized interpretation, aspects in which stand the pioneering developments of Koblyakov (1995), Kieran (1996), and Yadegari (2004). Moreover, its application in recent fields of musicology, including stylometry (see e.g.

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[87] An example of self-similarity already mentioned in the Introduction is the Peircean 'map of the map', which has inspired many literary fictions. The drawings by M.C. Escher (1898–1972) are also mentioned, with their motifs assembled in various directions pointing to a seeming infinity, and which Yadegari (1992:62–66) uses to formulate his own definition of musical self-similarity.

[88] The name *Droste effect* is due to writer Nico Scheepmaker (1930–1990), who by 1979 named such a relationship of self-similarity shown on the front cover of a box with a commercial product (Droste cocoa powder), in which the figure of a nun appears holding a tray with the same box, in a way that the same image is repeated several times in smaller scales. As a philosophical matter, this question had already been exposed earlier by René Magritte (1898–1967) in his painting *Les deux mystères* (1966).

Beran and Mazzola 1999a, or Bigerelle and Iost 2000), and pitch-class set theory (Murphy e2007) is currently producing its first results.

Under intuitive, functional, and statistical criteria, the manifold relations of self-similarity in music are of theoretical, analytical, and compositional interest. Such relations are fundamentally connected with the principles of musical self-reference and coherence, and they reflect the way music elaborates by following operative rules, within successive formulations and transformations. This means that the measurement and comparison of self-similar relations can eloquently reveal how the internal structure of a certain music is configured.

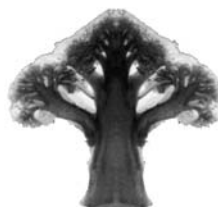*Finite or infinite self-similarity of an image-object*

If an 'object' is defined as a whole that can be identified by given set properties, then an object may well be any of the cases of symmetrical self-similarity appearing in ◊330. Nonetheless, caution is in order, taking into account the words expressed about symmetry, in subchapter 2.3, where it is stated that self-similarity and symmetry are not equivalent concepts. Too freely, Foote and Cooper (2001:1) say that "Music is generally self-similar. [...] Structure and repetition is a general feature of nearly all music. That is, the coda often resembles the introduction and the second chorus sounds like the first".[89] This description contemplates symmetry, not self-similarity. Whether symmetry is based on relationships of equality, repetition, correspondence, or simple similarity, self-similarity demands that these relationships occur as a convergence of themselves. Symmetry does not depend on self-similarity —the equilateral triangle is not made of equilateral triangles. Rather, self-similarity depends on a symmetry of reflexive symmetries—the Sierpiński triangle is made of symmetries of identical symmetries. What is interesting in examples of absolute or relative self-similarity, is not a main axis of symmetry, but the relationship of that axis with subsequent axes, throughout several self-structuring levels.

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[89] Apparently, "introduction" here means the *exposition* of a musical theme. The same mistake reappears in Cooper and Foote (2003:19).

a)
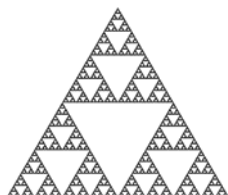
b)

c)

◊330. Three different cases of self-similarity representation:

(*a*) Finite stereotype of an infinite fractal: the Sierpiński gasket.
(*b*) Finite stereotype of a finite self-similar object: a broccoli with radial symmetry.
(*c*) Finite stereotype of a self-similar musical object, potentially infinite.

The three examples involve intersemiotic translation: (*a*) requests for a mathematical, abstract interpretation, beyond its limited graphical representation; (*b*) requires comparison with completely different objects, the broccoli pieces used commonly in a kitchen; (*c*) requires, also, an interpretation in the world of musical sound, different from its graphical representation. Obviously, such a world can be contained within another world (e.g. the world of a musical genre or style); successive embeddings of musical worlds are further referred to by using the figure 'worlds within worlds'.

Returning to the initial example of this subchapter, it may be said that a broccoli is self-similar, because the overall shape seems to keep the same relation to its parts, at least at five or six steps or scales. The same kind of relationship can be found in other vegetables, and generally in many plants and biological structures, including the nervous and vascular systems of animals (see Mandelbrot 1977:150). In all these cases the relationship of self-similarity is limited and is becoming less evident throughout the step or scale of insight. By contrast, certain geometric objects, like the Sierpiński gasket (pictured in ◊330-*a*), are absolute and infinitely self-similar. It is irrelevant

whether their printed representation on paper do not perfectly fulfil their endless self-similarity; what is relevant is that their structural relations can continue into smaller scales, in an unlimited way. This implies that at every level the object has the same geometric relationships with respect to itself and its parts. Due to the impossibility of representing them materially, these objects are represented by the continuous iteration of a function, i.e. as an infinite mapping of themselves. They exist, however, only as an abstract mental image, as their infinite self-similarity means that charecterizing them would take an infinite time, using the most sophisticated automated means (see Lauwerier 1987:33). This imaginary condition is crucial in understanding a point of convergence between the aesthetic qualities and the logico-mathematical propositions extracted from self-similarity: the fact that the infinitude of an absolutely self-similar object is understood using a finite model is related to the processes of language in general, and with the role of synecdoche in particular. For instance, the word 'tree' does not refer specifically to a tree, but to its generality, potentially infinite (see Wittgenstein 1953, 1968; Rhees 1968).

The logico-mathematical conceptualization of an object or a whole system of absolute self-similarity is based on the context of a set of axioms that allow us to assume, without empirical testing, that self-similarity continues into the infinite. Analogously, colloquial language does not need to prove that a word or sentence can be used in an infinite variety of circumstances. By having a conversation, no one explains in detail the implied infinity that constitutes a meta-context in the mind of the listener—e.g. a take-for-granted endless polysemy of a single verbal expression. Such a meta-context operates like the implication of musical similarities in which an object (or a process, e.g. a gesture) can be repeated in different contexts and in a potentially infinite plurality of relationships. Roughly, this is how Peircean abduction operates within the musical relationships implicating synecdoche and analogy.[90]

The understanding and representation of absolutely self-similar objects depend on a finite imaginary context and a finite range of experiences and concepts. A *generalized* mode of conceiving textures featured with absolute self-similarity suffices to assume that they are infinitely self-similar objects. This issue emerges from a basic

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[90] The concept of *abduction* is introduced on pages 23, 31–32.

level of geometric representation, for instance, conceiving that two lines converge in a corner, assuming that a line is an infinite sequence of segments. Thus the convergence of these two lines is infinite, and therefore it is impossible to draw a triangle.[91] This contradiction illustrates, nevertheless, how in mathematics, as in music and language in general, the pragmatic context affects the determination of a concept, even—and especially—when it implies infinity.[92]

Consequently, it is not impossible to implement infinite self-similar sets as grammars for music. The opposition between actual and potential infinity is not necessarily absolute, but instead offers chances for negotiation to define analytical and constructive strategies in music.[93] In this context, David Feldman (1999:80) criticizes the opposition between actual and potential infinity, supported by John Cage:

> Cage's [musical] Nature suffers a profound limitation. By dint of his interest in meditation and contemplation, Cage privileges the actual and so neglects the potential. The laws of mathematics constrain Nature by logically delimiting the possible; confronting those laws brings us to Nature unbridled by the chains of history. Beyond the challenge of exploring the physical space about us lies the difficulty of charting the vast conceptual space that comprises Nature's mathematical objects which proliferate exponentially.

Coherently, Thomas H. Lee (2003:344–345), when talking about the infinite self-similarity of $1/f$ noise,[94] suggests that the apparent paradox comprising an infinite phenomenon is solved simply on the coexistence of the actual and the potential:

> A question that often arises in connection with $1/f$ noise concerns the infinity at DC implied by a $1/f$ functional dependency. [...] The resolution of the apparent paradox thus

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[91] See Aristotle, *De Lineis Insecabilibus*, book II.

[92] In Peirce's words (1904/1998:323), "every endless series must logically have a limit". Operationally this notion is closely related to the theory of Georg Cantor (1845–1918) on transfinite sets.

[93] A good example of this kind of negotiation between finite and infinite is found in a Milton Babbitt (1964:92–93) exposition about the written representation of the musical sound: "The electronic medium, perforce, provides regulable and measurable control of frequency, length, loudness—and therefore, of envelope, spectrum, and mode of succession. This, in an adequate notation, would require means of signifying a continuous infinity of values in each of these dimensions, but realistic musical needs apparently are satisfied by a discreet, finite collection of values. Thus the creation of an adequate and efficient symbolic notation depends upon the acquisition of knowledge of aural perception."

[94] This concept is explained in subchapter 5.3.

> lies in recognizing that true DC implies an infinitely long observation interval, and that humans and the electronic age have been around for only a finite time. For any finite observation interval, the infinities simply don't materialize

This same kind of resolution can be observed in other physical aspects of music, like its harmony, length, loudness, pitch, rhythm, voice leading, and timbral organization. It is not absurd then to make analogies between potential and actual terms within these parameters, for example, in the tonal system. As a matter of fact, Dubiel (1990:243) finds a functional similarity of the actual and the potential matching the tonal organization proposed by Schenker (1932), in which the foreground (*Vordergrund*) corresponds to the realization surface, the actual ground, moving itself not onto a fundamental structure (*Ursatz*), but onto the broadest width of the "free composition" (*freie Satz*), the potential ground.[95] The musical structure coordinates, in this fashion, the contingencies and dependencies inherent in the generating system and the acting conditions. The same can be extended to a variety of cases in which the combination between contingencies and dependencies provides the sense of music from the determination of the operating system and its operational flexibility. Correspondingly, the relevant issues of analysis revolve around interpreting the meaning of the actual towards the potential (see Dubiel, *loc. cit.*).

It is worth observing in this context that not all self-similar finite sets are translatable into sets of relations for musical practice, nor are all self-similar infinite sets are useless for this practice, due to fact that they are infinite. The important point to grasp here is that for each case of the various types of musical self-similarity an adequate conceptual treatment is required. Obviously, the sense of musical self-similarity is not the same as the one characterizing a mathematical operation, e.g. the iterative continuum in a function of infinite self-similarity; but at the same time there is a strict analogy between both kinds of self-similarity. Whether or not this analogy is associated with the concept of *deterministic self-similarity*, the concept of *stochastic self-similarity* is related to the irregularities of practice, e.g. the variable distribution of frequencies in the performance of the 'same' musical piece, perceived as causal or casual subtleties; or the probabilistic trend for coherence within a given style.

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[95] On the polysemy of the term *Satz* and its many possibilities for translation, see page 190 (footnote 257).

Conceptually, the approach is also different: it can be said that a musical composition contains itself in infinite turns, at infinite scales, without the necessity of being listened to infinitely. Its audible version reaches some stages of self-similarity, but its holistic version is 'unknowable' as it cannot be experienced.[96] Still, whether the general relationships of the knowable version are always repeated in an equal manner, such infinite composition is in fact knowable because its essential relationships and characteristics are knowable.[97]

To synthesize, the only two possible operations involving infinite fractals and music are the *stereotype*, in the case of fractal objects of intuited continuity,[98] and the *abduction*, in the case of the infiniteness of the 'map of the map'. Other uses of the term fractal in music, like those introduced by Madden (1999/2007), R. S. Johnson (2003), Hwakyu Lee (2004), or Pegg *et al.* (e2008), are vaguely metaphorical and need to be revised.[99]

For the stereotype, the intuitive ease with which we can memorize the general configuration of the object is related to the feasibility of its musical use. Whether it is possible to sufficiently represent the general relationships of a fractal set in order to intuitively recognize it, the same intuition can be used to make an auditory representation of the same set, using intersemiotic translation. In the case of an auditory representation of the Sierpiński triangle, equal intervals in groups of three segments should be obvious (see Pickover 1990).

For abduction the conception of a 'simple' geometry at different scales is not necessary. It is necessary instead to intuit that a relationship is repeated in a similar way at different scales. As regards the problem of the 'map of the map' (Peirce *CP*, 8.122), the very relevant aspect for intuition is the observer perspective, guessing the existence of a 'tunnel' that binds and penetrates the myriad of self-mapping maps.

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[96] This subject is treated in more detail in subchapter 5.5.

[97] The possibility of discovering the infinite through the finite depends, according to Samuel Taylor Coleridge, on the power of the symbol, which operates "above all by the translucence of the Eternal through and in the Temporal" (*The Statesman's Manual*, 1816:437–438).

[98] This applies, for instance, to the cases of the Sierpiński triangle, the Cantor set, the Pythagoras tree, and other typical fractals, including the proportional successions with exact-infinite self-reference, mentioned in chapter 6.

[99] On the mistaken use of the term in Madden (1999/2007), Hudak (2000), and Hwakyu Lee (2004), see pages 21–22, 32. This theme is developed throughout subchapter 5.5. Concerning its metaphorical misuse in Pegg *et al.* (e2008), see page 202.

Self-similarity in a musical process

A process is a system of joint relations in time and space, which—as often happens in the physical processes—may have a tendency towards stability or instability (see Thomson 1851; Shannon 1937, 1948; Wiener 1948).

Carey and Clampitt (1996:62) suggest that a musical process "is one that exhibits parallel construction at different levels of scale". This criterion applies to the classic analysis of 'motifs' within musical segments, and in general to periods forming larger periods. Nonetheless, the same criterion is valid for the psychological processes involving the appreciation of musical structures, and for the physical processes that can often only be observed using automated synthesis or analysis.[100]

A typical process in music is the sequentiation of pitches and lengths in a melodic system. If this sequentiation meets the basic description of self-similarity, it can be said that there is a 'self-similar melody'. For instance, Hudak (2000:301) defines a self-similar melody as follows:

> Start with a very simple melody of *n* notes. Now duplicate this melody *n* times, playing each in succession, but first performing the following transformation: the *i*th melody is transposed by an amount proportional to the pitch of the *i*th note in the original melody, and is shifted in tempo by a factor proportional to the duration of the *i*th note.

Johnson and Amiot (2006:3) remark that a self-similar melody must coincide with itself when its original form is articulated in a slower time.[101] This sort of musical structure (examples in ◊331) actually has a precedent in the Alberti bass,[102] in typical binary forms in which the harmonic accompaniment mimics long durations of

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[100] Chapter 5 specifically deals with this issue.

[101] The original text says: "Une mélodie qui coïncide avec elle même quand on la joue à un tempo plus lent".

[102] The Alberti bass is a kind of instrumental accompaniment, very common in the keyboard repertoire—particularly in Mozart. It articulates with triads arpeggiated in four notes (with equal durations) succeeding in the order lowest, highest, middle, highest, forming the pattern for the harmonic bass (left hand in the keyboard). See G. Burdette: "A Thorough Harping on Alberti", *Music Research Forum*, vol. 4, no. 1 (1989); pp. 1–10.

◊331. Two examples of an 'autosimilar melody': notes are distributed so that a melodic line is proportionally identical to another, in a way that the pitches coincide for each vertical relation. (Examples based on Johnson 1996, Hudak 2000, Johnson and Amiot 2006, and Amiot 2008).

melodic structure with short durations. More precisely, though, the Alberti bass corresponds to a pre-self-similar structure,[103] and according to Johnson and Amiot (2006:23) it produces a symmetric and invariant melody that can serve as a self-similar relationship with a parallel melodic line. This structural form, in which melody and harmony are correlated through durations, is also the basis for various compositional procedures developed by Milton Babbitt, Morton Feldman, and Tom Johnson (see Johnson and Amiot, *op. cit.*).

Both examples in ◊331 are elaborated using simple rules: ($r1$) keep the proportion of 1 to 3 (or 1 to 6) along the duration of notes in both melodies; ($r2$) keep the same intervals in parallel motion; ($r3$) make the pitches coincide for each vertical relation; ($r4$) collect pitches only in sets of one or several paired notes. Nevertheless, as Amiot (2003, 2006) and Johnson (1996, 2006) consider, there are mathematical conditions for elaborating more complex self-similar melodies and rhythmic canons.

Johnson and Amiot (2006:7) obtain conclusions that can be summarized in three points: [104] [1] for a primary relation of self-similarity (relation a) a melody (m_k) where m_k designates the kth periodical note in a period n, is self-similar if for all k, $m_{a \times k} = m_k$; [2] the maximal number of notes in a self-similar melody with period n is $3n/4$; and [3] a melody with relation a and period n is invariant over a relation b, if and only if b is a power of a.

This concept of melodic self-similarity differs from that of scalar self-similarity with direct symmetric transformations, either for the type of intervals, discrete in one case, dense in the other; or for the metrical relationship, present in one case and absent in the other one.[105] Such a metrical relationship, fundamental in the concept

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[103] See a definition of pre-self-similarity on pages 75–78.

[104] It is understood that such a melody is elaborated after a and n, arbitrarily preselected.

[105] Mandelbrot (1981:45) generalizes these differences for all the scalar systems, suggesting a distinction between *scalebound* and *scaling object*: "I propose the term *scalebound* to denote any object, whether in nature or one made by an engineer or an artist, for which characteristic elements of scale, such as length and width, are few in number and each with a clearly distinct size. […] A scaling object, by contrast, includes as its defining characteristic the presence of very many different elements whose scales are of any imaginable size. There are so many different scales, and their harmonics are so interlaced and interact so confusingly that they are not really distinct from each other, but merge into a continuum. For practical purposes, a scaling object

of 'autosimilar melody' used by Johnson and Amiot (2006), is the basis of their own idea of 'autosimilar rhythmic canon', as that slightly emerging within the rhythmical skeleton of the examples in ◊331 (more developed in Amiot 2003). The differences between these two modes of 'melodic' self-similarity are mediated by a third form e.g. in the self-similarity in Conlon Nancarrow's melodic canons for pianola, which although repeating the same melodic pattern in different rates of tempi, do not correlate horizontal development with vertical affinity; neither do they work within a model of dense intervals.

At this point, something worth noting is that there is not one, but several forms of self-similarity in melodic processes. In addition, and in a way comparable to the variety of musical parameters associable to integral serialism, it must be noted that there is a wide variety of parameters—not only those which can be related to a melody or an interval or intonation period—associated with the different forms of self-similarity described in the following chapters.[106]

A musical process as 'time series'

According to Barenblatt (1996:14):

> A time-developing phenomenon is called *self-similar* if the spatial distributions of its properties at various different moments of time can be obtained from another by a similarity transformation.

This 'transformation' implies the symmetric displacement of a first group of relations. According to this notion, Oliver (1992:322–323) explains the 'affine transformations' (*shrink, squish, stretch, spin,* and *skew*), comparable to the transformative operations mentioned in subchapter 3.4. within the context of pitch set invariance. For now, suffice it to take a simple case with the proportional segmentation of a straight line.

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does not have a scale that characterizes it. Its scales vary also depending upon the viewing points of beholders."

[106] According to Kieran (1996:44): "there may be patterns of varyingly complex shapes or sounds that develop out of and are related back to a fundamental structuring element. That is, there is a deep structure of self-similarity: at the simplest level there is symmetry of a kind across scale, patterns evolving into patterns, recursions, and so on. Of course, in music, the relations get ever more complex and intricate. Still, the adequately sensitive listener may understand a piece of music by grasping the deep structuring role of the inter-relations among sounds."

Let a straight segment k_1 be analogous to the length of a musical note. Then let k_2 be a parallel straight segment whose length measures a third of the first segment k_1 :


Thereafter, let the relation $k_2 \to k_1$ be symmetrical and adjacent to a third segment k_3:
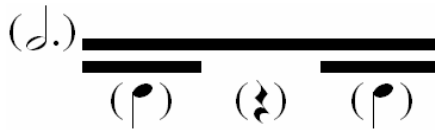
By repeating the same relationship $(k_1 \to k_2 \to k_3)$ in a subsequent scale $(k_2 \to k_5 \to k_6)$ a following level of durations is symmetrically obtained with the same proportion:
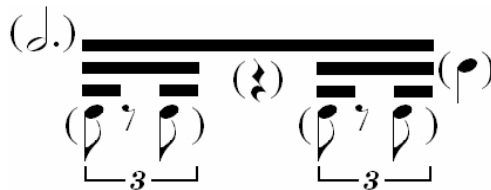
If this is repeated endlessly, in subsequent steps, then this procedure gives what in mathematics is known as 'Cantor set',[107] a set of intervals characterized by infinite self-similarity:
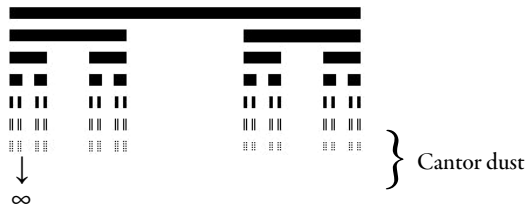
Cantor dust

◊332.

Based on this example, it can be stated that the consecutive relationships between the durations represented as horizontal spaces, work altogether as 'time series' (i.e. an infinite summatory function). Beran and Mazzola (1999a), Bigerelle and Iost (2000), Das and Das (2006), Su and Wu (2006), and Dagdug *et al.* (2007), among others, use this analogy to analyze the stylistic variations in affine sets of musical information. It is quite obvious that, for music, the meaningful events nested in time series do not occur with the regularity and monotony characterizing the interval grid in the Cantor set. The investigation of the deviations in the series is precisely the object of analysis to determine how differences occur in music, between the fixed relationships (*grammar*) and the flexible relationships (*style*).[108]
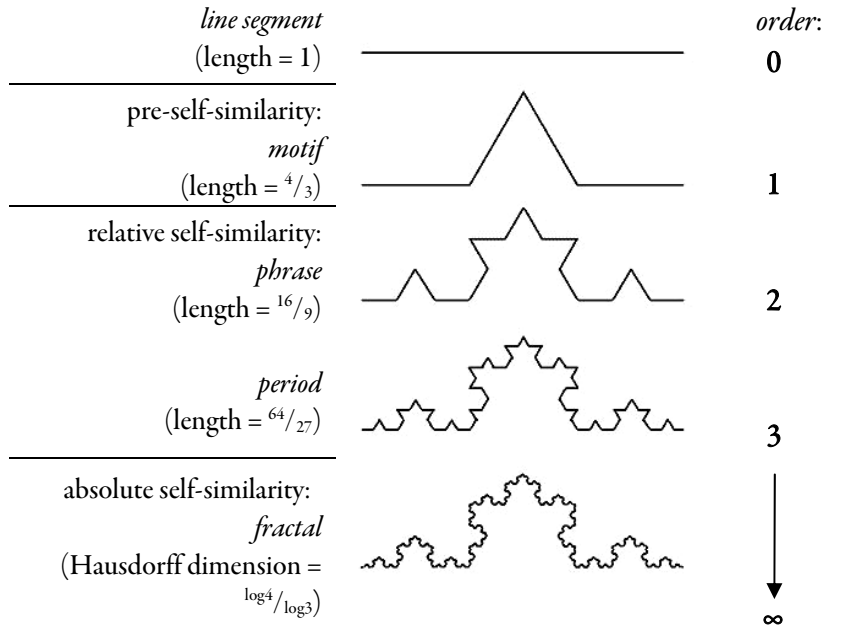
Pre-self-similarity

Barenblatt (1996:xiii–xiv, 200) uses the concept 'pre-self-similar' to indicate the first stage of construction in an asymptotic self-organizing process. The stages may vary depending on the process involved, but the constructive principle is basically the same as the primary transition from a simple relation to a self-similar, potentially infinite process. The example shown in ◊333 corresponds to the generation of the Koch curve from a straight line segment to the absolute self-similarity. In this scale, the first 'order' of the self-similar construction represents the *motif* at the stage of pre-self-similarity. The term *motif* was introduced in this context by Lauwerier (1987). The terms *phrase* and *period* are used here as a deliberate analogy with a musical structure, an idea put into practice by György Ligeti in his *Étude I: Désordre* (1986), for piano, as a method of intuitive consistency through statistical self-similarity.[109]

The notion of pre-self-similarity is useful in the study of musical self-similarity, and in the metaphor of language as a *natural fractal*, i.e. like a quasi-self-similarity or approximated self-similarity (Pareyon 2007c), including aspects of "self-affinity", as

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[108] Subchapters 4.7. and 4.8. delves deeper into these notions. For more detail on the role of the 'time series' as method for music analysis, see pages 143–145, 238–241, 288–290, 300–303; the latter corresponds to a section devoted to the pertinence of the fractal dimension in music.

[109] In *Désordre* (1986), Ligeti makes an intersemiotic translation of the first structuring levels of the Koch curve (see ◊333). Richard Toop (1999:201) correctly asserts that, for this case, "The exactness of the analogy is of secondary interest: what the scientific model offers here is inspiration, not legitimation."

defined by Mandelbrot (1982:395; 2002:50, 85).[110] Consistently, the present study reserves the term 'affinity' for the compatibility between set distributions, and to express the overall relationships in an *affine transformation*.[111]

		order:
line segment (length = 1)		**0**
pre-self-similarity: *motif* (length = $^4/_3$)		**1**
relative self-similarity: *phrase* (length = $^{16}/_9$)		**2**
period (length = $^{64}/_{27}$)		**3**
absolute self-similarity: *fractal* (Hausdorff dimension = $^{\log 4}/_{\log 3}$)		**∞**

◊333. Generation of the *Koch curve*. The 'order' 1 represents the stage of *pre-self-similarity* or *motif*, as a first constructive step, typical in fractal algorithms. Lower orders are 'nested' within higher orders e.g. the figure corresponding to order 2 can be exactly found in the figure corresponding to order 3. The proportion of the nesting is characterized by the ratio written in the leftmost column (the length between brackets). The absolute self-similar case (i.e. fractal) at the infinite order is approached by proportional analogy (a *synecdoche* in logico-rhetorical terms, and an *abduction* in terms of Peircean semiotics).

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[110] Madden (1999:4, 21) introduces the term 'self-affinity' to refer to this approximate self-similarity, within a statistical context of musical analysis.

[111] An affine transformation is any geometric transformation preserving collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). Source: Weisstein (e2008).

The table in ◊333 presents a progression of an intervallic system as an analogy of a musical system, gradually more complex and less intuitive. The stages in this example can also be compared with audible frequencies or tones, from the pure to the complex tone and the timbral pattern; and with metrical distribution, from the ordinary measure to the complex period and the metrical groups in polytempi. This scheme signified a proportional and paradigmatic analogy of geometrical self-similarity in its early stages (from the minimal *quantum* to the development of phrases, periods, or larger systems), regarding the self-similar relationships in the typical proportions of musical harmony and rhythm.

◊334. Pre-self-similarity in a harmonic texture for four voices. The bass pattern is similar to the beginning of the highest voice (C, D, B, E). Strictly there is no self-similarity in this example; however, assuming the continuity of the relationship between the two extreme voices (i.e. putting a ½ length F below the bass line), a continuous 'imitation' arises with characteristics of relative self-similarity. The two middle voices would then design an intermediary self-similarity between originality and imitation. If consecutive layers repeat at scale the middle voices and the same relationship between low and upper voices, a self-similar global texture can be surmised. This is suggested by the sign ∞, as a potentially infinite extension in the parameters of pitch and duration.

Nelson (1994:3) uses the notion of pre-self-similarity—although without explicit mention—to explain the transition between the starting point of a self-similar system, and the primary 'two dimensional outline' of the same system:

> One of the simplest fractal models is the two dimensional outline of mountain ranges. Initially, we draw lines to represent the major peaks and valleys. These lines are subdivided

by a recursive process to produce the next level of detail. When we continue subdivision through generations of ever shortening lines, an image emerges that reminds us of the patterns found in natural landscapes.

In the asymptotic deterministic processes—i.e. with a known overall behaviour, as happens with iterative geometric transformations in many patterns of musical invariance,[112] the stage of pre-self-similarity is crucial for determining the successive patterns. This is why a *motif* can be very relevant in a schematic prediction for self-similarity. Nonetheless, it is important to note that in asymptotic stochastic processes a constructive stage can be determined by the immediately preceding stage, regardless of a stage of pre-self-similarity—something that occurs, for instance, in Brownian noise, Markovian structuration, and random walks.[113]

3.4. Invariance

In the present study, the term 'invariance' is restricted to the analytical context in two modalities: (1) absolute persistency of typical relationships within transformations of mechanical/acoustical systems, and their graphic representation; or (2) as specific reference to the forms of invariance identified and studied by pitch-class set theory. In general literature, as explained below, the concept of invariance may have a parallel definition to that of self-similarity.[114]

Mandelbrot and Van Ness (1968:423) postulate that "self-similarity' [is] a form of invariance with respect to changes of time scale"; whilst Schroeder (1991:xiii) assimilates the whole notion of self-similarity to that of invariance:

> Self-similarity, or invariance against changes in scale or size, is an attribute of many laws of nature and innumerable phenomena in the world around us. Self-similarity is, in fact, one of the decisive symmetries that shape our universe and our efforts to comprehend it.

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[112] See following subchapter 3.4., on invariance.

[113] For a general description of Brownian noise, see page 241. For the concepts Markovian structuration and Markov chains, see 226–235, 241–242. On random walk, see pages 241, 248–249, 252–253.

[114] Seeger's idea of musical invariance (1960), which is exposed at the end of this subchapter, can be associated with the latter perspective.

Besides the fact that invariance can be found in a diversity of functions in pitch sets, pitches themselves can be invariant sets. Fourier's analysis provides evidence for supporting this conceptualization,[115] revealing a trend, from sound generation itself, to the self-structuring and self-referentiality that characterize music.[116] Gerlach (e2007) investigates the cause of the pervasive power of Fourier analysis in physics from a more general perspective, when he observes that this capacity is due to generalized translation invariance:

> Suppose a linear system is invariant under time or space translations. Then that system's behaviour becomes particularly perspicuous, physically and mathematically, when it is described in terms of translation eigenfunctions, i.e., in terms of exponentials which oscillate under time or space translations. [...] In other words, it is the translation invariance in nature which makes Fourier analysis possible and profitable.[117]

It is obvious that this argumentation cannot be used to assert that musical relationships are absolutely predetermined by physical laws.[118] But it is true, after all, that a substantial part of these relationships depends on the fundamentals of mechanics and acoustics. Music emerges within the complexity of several layers that are correlated and simultaneous; and these layers are not uniquely or necessarily hierarchical and evolutive, as positivism presupposes.[119] The need to identify these levels—at least the most relevant—determines the organization of Chapter 4 in several thematic sections.

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[115] For a general definition of 'Fourier analysis' see pages 143–145.

[116] For an introduction to this notion, see Voss and Clarke (1975, 1978), Hsü and Hsü (1991), Bigerelle and Iost (2000), and Su and Wu (2006). This concept is developed in chapter 5.

[117] Gerlach (*loc. cit.*) adds a remark to this text: "*Nota bene*: real exponentials are also translation eigenfunctions, but they won't do because they blow up at $+\infty$ or $-\infty$".

[118] In other words, it is impossible to assert that the physical laws can explain the general and the particular concerns of the social sciences (considering that music, remarkably, is also a social phenomenon).

[119] The positivist doctrine persists in certain approaches to musical self-similarity. The sentence "Progress depends on organized skepticism" is not precisely a motto coined by Auguste Compte (1798–1857), but the inaugural epigraph in Madden's book (2007:1) about "fractals in music" (*sic*). The debate over the idea of progress in music is extensive, linked to the engineering approach on music as a collection of problems to be solved. This study's Introduction presents some of the more evident topics of this issue, related to the conceptualization of musical self-similarity. For a philosophical discussion on the subject see e.g. Dent (1928), Benjamin (1936), Adorno (1970/1984:300–303), Ballantine (1984), Kieran (1996) and Vieira de Carvalho (1999).

Invariance in pitch-class sets

A simple example of invariance in the context of arithmetic is the displacement between two integers—represented by an arrow—which does not change length when the same quantity is added to both ends of the interval; the displacement is invariant under translation by addition (this does not occur equally in the multiplication of distances; then the interval is not invariant in multiplication). Symmetry has other cases of invariance: in the examples shown in ◊331–◊334 the fundamental relationships remain the same, regardless of position changes or symmetric transformations.[120] This kind of invariance is also found in pitch set elaborations.

The implementation of a principle of 'no repetition' in musical scales, as found in different theories of Chávez (1961), Schoenberg (1922, 1967, 1975) and in many cases of musical scores written under the methods of integral serialism,[121] involves the recurrence of basic relations in a gamut of parameters (i.e. length, loudness, pitch, tempi, etc.). Such recurrence commonly leads to self-similar patterns, rather than repetitive ones, because it involves the reproduction—in different ways—of the relations already comprised in the original scale, or in the set of rules considered as the starting point of the constructive process. As a form of statistical self-similarity, this sort of recursive patterning is usually studied by classical methods of mathematics, like the measurement of the Euclidean distance, or the probabilistic classification of neighbours (using the *k*-nearest neighbours algorithm), depending on the scaling rules implemented in the system under investigation.[122]

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[120] This principle is the basis for transformation theory. For an introduction to its arithmetical aspects, see E. T. Bell (1932). "A New Type of Arithmetical Invariance", *American Journal of Mathematics*, vol. 54, no. 1; pp. 35–38. For an introduction to its implementation in pitch-class set theory, see Vázquez (2006) and Ilomäki (2008).

[121] Under this principle, for a pitch-class set {0,1,2,6} sequences like {1,2,6}→{0,2,1} are allowed, but {1,2,6}→{2,1,6} is not allowed. In short, this principle of non-repetition operates as an algorithm for the development of partial differences, in a universe of inevitable similarities, given the finiteness of the set and the symmetry between its segments.

[122] The *k-NN classifiers* method is used to estimate the probabilistic density, or the direct probability, to know that an element x belongs to class C_j given the information contained in a set of prototypes (Fix and Hodges 1951). The exactness of this algorithm is inversely proportional to the amount of noise present in the form of irrelevant neighbourhoods. For this reason the mode of implementation of correlated algorithms is particularly significant when

If a *finite set of pitches* [123] is implemented to generate many sequences of pitches at various levels of organization (e.g. intervals, trichords, hexachords, melodies, or harmonic textures) the features of the set will be present in a variety of proportions and combinations—depending on their different levels of order—in the sequences themselves.

In *pitch-class set theory*, Babbitt (1949, 1960, 1961), Perle (1977), Lewin (1982), Morris (1987), Vázquez (2006), and Ilomäki (2008), among others, employ the notion of invariance in the pitch-class space in reference to how a segment of a pitch set remains similar, or the same, under inversion, transposition, or retrogradation. In a first attempt to adopt the term 'invariance', Babbitt (1949:383) notes that Bartók uses inversion, retrogression, and free permutation, "essentially" as a traditional method "concerned with varying linear characteristics while preserving their relative contours. Never does he use inversion, for instance, in its abstract structural role of maintaining the harmonic invariance of successive dyads, as is done in twelve-tone music."

Pitch-class set theory interprets a number of concepts involved with intervallic invariance, namely: *set classes, pitch rows, aggregates, arrays, partitions, collections* and *networks of collections*, among others.[124] The term *set* designates a specific pitch order; *set class* designates the classes of all sets, i.e. all forms of order of pitch classes that can be transformed according to classical symmetry operations. A *pitch row* is any finite ordering of *n* pitch-classes (Babbitt 1960:248); for instance, the twelve-tone row used as a constructive paradigm. The rows, as well as the *aggregates*, can be 'partitioned', i.e. presented in segments that keep the original intervallic and combinatorial relationships. The aggregates are 'recognizable compounds' of set segments, with some or almost no any resemblance to the set; they can articulate in different time scales in which the segments plot generates self-similar relationships. According to Dubiel (1990:220):

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optimising the characteristics of scalability, as distinguished from *scaling shapes* and *scalebound* (see Mandelbrot 1981:45).

[123] Sets of pitches are, for example, the chromatic and the diatonic rows, a chromatic or diatonic hexachord, as well as any finite set of fixed pitches with special intonation (i.e. an intonation different to the chromatic-diatonic one).

[124] These concepts are widely treated by Babbitt (1960), Mead (1983, 1984), and Dubiel (1990).

> The unfolding of similar structures at different rates, with members of the faster ones figuring in the slower ones and, especially, with the slower ones more narrowly constrained by the system, is, moreover, a strikingly 'Schenkerian' thing to have achieved within the twelve-tone system [of Babbitt].

In the music of Babbitt—and in the theoretical corpus he himself contributes to building—aggregates operate on a plane of 'horizontal' durations and 'vertical' pitches, sometimes with structural interdependence; some other with relative independence. For example, in *Three compositions for piano* (Babbitt 1948), horizontal aggregates are fully ordered, whilst the vertical, or *arrays*, "in general are not" fully ordered (Dubiel 1990:220). According to Mead's formalization (1984:312) an *array* "consists of two or more strings of sets, from one or more set classes, unfolding simultaneously." A *collection* is an "unordered bunch of pitch classes", and a *collection class* is the "class of collections which may be transformed into each other under transposition and/ or inversion".[125]

Typically, during the aggregate-formation processes, a proportional relationship between the chromatic collection (12 elements) regarding a hexachord (6), and a hexachord regarding a trichord (3), is evident not only for the symmetry of its partitions, but particularly for the way they interact with the assembled segments, and the different layers which, in depth, result in an audible plot. Mead (1984:324) believes that the consistent use of aggregates on the audible surface, involving various forms of invariance, produces a strong sense of self-reference in the musical structure. Therefore the aggregate inversions, transpositions, and retrogradations, powerfully contribute to segments (trichords, tetrachords, etc.) mapping themselves, combining information redundancy and structural variety—a notorious behaviour in many self-similar sets.[126]

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[125] This implies that there may be two classes of compositions with the same set class, whilst having no sets in common. For instance, {0,1,2} is not equivalent to (0,1,2); the first example refers to a pitch-class set, the latter to a set of intervals.

[126] This notion of invariant set as 'self-mapping' is evident in the analysis by Mead (*loc. cit.*) of Babbitt's music: "A trichord, by content alone, is sufficient to identify a single set and its retrograde, or at most two sets and their retrogrades, in the case of trichords which can be mapped into themselves under inversion. With such set classes it is possible to saturate sections of a composition with referential details which, while containing considerable internal variety, may point to a remarkably circumscribed number of sets." In this source, the notion of

As noted by Dubiel (1990:246) and Mead (1983:108)—though not exclusively in Babbitt's music—what occurs rapidly on the 'surface' does not necessarily owe its significance to a reflection of what occurs slowly in the 'deep layer', or conversely; it may be, rather, that the 'superficial' details reflect the underlying array structure of the set class. In this sense, Dubiel (1990:247) conceives that the array "is merely one of the levels at which the set is influential—not the level of its true manifestation". Moreover, generalized consistency between pitches, regarding a set class, is not sufficient to lay out and establish a whole set of significant musical relationships —Babbitt (1950:57) says that the mere identification of relations "is trivial"; a correlate between different types of functionality is required, and systematic coordination between functional similarities and differences. According to Dubiel (*op. cit.*:248):

> The 'contingencies and dependencies' of particular pitches are where the set's 'influence' is to be observed; it is the development of various kinds of functionality, like these just illustrated, and not just the assurance of interval-class correspondence between various pitch successions, that makes the set part of a piece.

Most of the reseach on pitch-class set theory does not devote much attention to the shape of the possible combinations between the rows, the aggregates and their partitions, and especially to the permutative properties that characterize a system of compositional or analytic operations. The numerous possible combinations—that can be simplified as successive factors—is nothing more than a collection of potential features; what is relevant to musical criteria is rather how categories are established in order to reduce set properties in a few apprehensible expressions. Invariance traits resulting from these operations are due to a need for self-similarity categorizing a *whole* into a group relatively easy to grasp and manipulate.[127]

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synecdoche as a functional relationship between 'part' (*trichord*) and 'whole' (*two sets and their retrogrades*…) should be noted.

[127] Ilomäki (2008:6) clearly identifies the reduction of the combinatorial factor in the pitch rows, as a cognitive process: "These are daunting numbers for a human; there is no way we can examine each pair of rows or even each row separately. In coping with this multitude, a typically human approach is to place rows and their relations into categories".

Richard Swift (1988:29) conceives the transformation of rows and aggregates as expansion of the potential self-reference of a generating set.[128] Additionally, he contemplates invariance in these processes, not only as exclusive property of pitch-class sets, but also by its extension to *isometric series* representing the temporal flow of music (*time-point rows*). This is already suggested by Babbitt (1955, 1962) in the context of integral serialism, adapting similar constructive criteria to functional relationships between pitches, lengths, and intensities, but also to abstract functional relationships such as aggregates and rows. In this way Swift (1988:29) connects pitch intervals criteria with tempi and polyphony: "The resultant polyphony of smaller pulses (or tempi) serves to clarify the larger pulse and its relation to a series of similar pulses, whilst ensuring the saturation of as many aspects of the music as possible by the row and its operations."

Invariance in communication processes according to Seeger

Charles Seeger (1960), for whom the musical processes are processes of communication, provides the basis for a generalized theory of musical self-similarity, considering the relationships of invariance as highly significant:

> In presenting the pattern of design qua moods of a logic in a unified, partially closed system, I shall not describe them in terms of universal and particular, for they are both; nor in terms of affirmation and negation, for they are neither; but in terms of the variance and invariance of progression of the four simple functions or resources of the compositional process, without whose full participation a sound-signal is not a music-signal and therefore not a music-message, i.e., music.

The 'four functions' to which Seeger refers are four 'simple' communication functions (*pitch, loudness, tempo* and *proportion*), in contrast to other 'compound' functions (*timbre* and *accent*, combinations of pitch and loudness, and tempo and proportion, respectively). Congruently, Seeger (*op. cit.*:235–237) conceives a *variance*

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[128] Among the methods used by Swift (1988) for control and development of this self-reference as a compositional process, are inversion and transposition of pitch rows with or without modular bounding. For example, converting the row {1,5,3,2,4} into {5,1,3,4,2} (modulo 6), or transposing the first row as {2,10,6,4,8} by the multiplication of values (× 2, in this case). In the present study this subject extends to a section on *modular bounding* (on pages 393–395) which in turn is extended to the topic of *scales generation* (pages 395–398).

of direction and a *variance of extent*, associated with the four simple functions. In this fashion he concludes that music operates through four generative branchings: *single melody, successive combinations of melodies, simultaneous combinations of melodies,* and *successive combinations of simultaneous combinations of melodies* (*op. cit.*:235–237).[129] Noticeably, this picture with systems of relations within systems of relations, subjected to common operational rules and principles, anticipates a correlate of self-similarity in all strata of music.[130] This notion motivates Seeger to support a theoretical position in which it is not the universal versus the particular that may substantiate an analysis of sound codes as musical messages, and neither the affirmative or negative, but rather the coordination between similarities and differences at different levels of musical communication.[131]

3.5. Gestalt

Gestalt theory provides a psychological approach to explain how the mind-brain processes self-similar relations, attributing partial qualities to the whole embracing the parts.

The Gestalt concept, associated with *figural theory*—mental configuration of similarities in the patterns and grouping related categories, was introduced to modern philosophy and experimental psychology by Christian Christian von Ehrenfels (*Über Gestaltqualitäten*, 1890), after ideas first put forward by Goethe and Kant. Later, Max Wertheimer helped to develop the notion that the operating principle of the mind is of a holistic and analogue kind—operating in both senses of proportion and paradigm, with a tendency of self-organization. The work of Edmund Husserl and Ernst Mach also prepared the foundations for a theory of form

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[129] Melody consists of rhythmic formulas with a third dimension (pitch). A rhythmic formula enters this scheme as two-dimensional layout (length and loudness); the 0-dimensional entity, analogous to points in geometry, is the pulse.

[130] Consider the relevance of this formalization regarding ◊331, and in general, for the context of the 'autosimilar melodies' as defined by Johnson (1996, 2006), Amiot (2003, 2006), and Johnson and Amiot (2006).

[131] The structural role of self-dissimilarity in such coordination, is a topic explored in subchapter 6.5.

and figurality, according to which objects are perceived depending on their distribution and accommodation in an intuited whole. This arrangement would correspond to an innate perception that determines the order in which objects are perceived in relation to the physical context. The reproduction of this order, through language, is due in parallel to laws of an innate grammar: what Chomsky (1957) calls *Universal Grammar*, whose empathy with the principles of order in music highlight Lerdahl and Jackendoff (1983), and Jackendoff (1993:165–183); and to a process of learning and interpretation of the 'figures' in a cultural context: what Stephen Peles (2004:58) summarizes as "typically complex and idiosyncratic" in a variety of subsymbolic aspects crucial to music.[132]

The Gestalt, as intuition of coherence and belonging, also has a seminal, poietic faculty in the multiplication of structural relations;[133] so it works as a basic mechanism of musical elaboration, as Schoenberg conceptualizes (1995:297–299):

> [Radial] symmetry is one of the simplest principles: to the right of the axis is the same thing as to the left of the axis. In principle the mirror, the retrograde, and the inversion are also basically similar. The advantage, in addition to easy comprehensibility, is that [radial symmetry] nonetheless offers a new gestalt that in reality contains the same inner relationships.[134]

In the analysis of tonal music, it is obvious that an adaptation of the Gestalt in the notion of *Urlinie*, conceived by Schenker (1932) as *Urgestalt*, incorporates it into a theory of patterns in which the whole and its parts are correlated. This is an important issue in Babbitt's musical thought (1950:57): "What of the significance of

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[132] Peles (*loc. cit.*) refers to ideas on musical organization in Arnold Schoenberg: "the things Schoenberg had to say on the subject of symmetry, since his relation to the idea was typically complex and idiosyncratic." For an approach to the theory of subsymbolic strata in music see Tiits 2002.

[133] Roughly speaking *poiesis* means 'creativity' or 'productivity', usually in a biologic context. From the ancient Greek verb ποιέω, 'to create'.

[134] The original text says: "Symetrie [*sic*] ist eines der einfachsten Prinzipen: rechts von der Achse befindet sich dasselbe (in gleichen Abständen, in gleichen Massen etc.) wie links von der Achse. Das Prinzip des Spiegels, des Krebs, und die Umkehrung sind im Grund auch dasselbe. / Ihr Vorteil ist, nebst leichter Fasslichkeit, dass sie doch eine neue Gestalt darbieten die in Wirklichkeit die gleichen inneren Verhältnisse hat […]". The concept of *Gestalt* in Schoenberg operates *sui generis*, and does not always coincide with the standard definition of the term (for instance in Husserl or Mach); for this reason, in this specific case, readers are referred to the comments of Patricia Carpenter and Severine Neff, in their edition of the texts of Schoenberg (edited 1995).

the event at precisely its own moment of occurrence, at its own tonal level, and in its relation to other such events and to the work as a whole?" [135]

The concept of Gestalt is also of paramount importance in Roman Ingarden (1962:55, 107–108), both for the perspective of musical structure, as for the theory of perception:

> A particular tone formation might form a whole which is so unified that one does not regard it as composed of the individual tones: this is especially true when the whole in question is a Gestalt. [...] What the listener will *hear* with special distinctness as the chief element is precisely the Gestalt. [...] What creates the unity can be nothing else than Gestalt, and indeed a Gestalt that unfolds itself in time during the execution of a performance.[136]

This notion has implications in a variety of analytical approaches, mainly developed in the study of melody (Viret 1982), and musical temporality (Tenney and Polansky 1980, Grisey 1987). According to Ames (1982:46), the efficiency of this approach is based on two fundamental criteria: the relative proximity of events or objects in one or more dimensions; and the relative similarity in one or more aspects of the figure. Both criteria are explored here, especially in Chapters 3 and 4, as aspects of *functional self-similarity*.

Grisey (1987:269) anticipates an ontology of musical self-similarity, suggesting the investigation of musical processes as relationships between *objects*: "the object allows us to understand the process in its Gestalt, and to effect a system of combinations". For Rowell (1983:161–162, 173) these combinations form 'patterns' with a variety of structural features, which in coordination with other 'values', generate different levels of gestalt complexity.

The adaptation of this notion to music theory, rekindling the concept of *gatom* (Gestalt atom) first formulated by Gestalt theorists, facilitates the identification of relationships that generate a self-similar system.[137] According to Cope (1987:36) a

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[135] This quotation comes from a context in which Babbitt criticizes the theories of René Leibowitz (1913–1972).

[136] Verbatim: "So sehr das betreffende Tongebilde, ein einheitliches Ganzes—insbesondere eine „Gestalt"—bildet, das sich aus den einzelnen Tönen gar nicht zusammensetzt […] obwohl das, was sich dem Zuhörer als Hauptelement mit besonderer Ausgeprägtheit aufzwingt, eben die Gestalt ist" (55). "Dieses Einheitschaffende kann nichts anders sein als eine „Gestalt", und zwar eine Gestalt, die sich im Vollzug einer Ausführung in der Zeit entfaltet." (107–108).

[137] This concept of 'atom' is developed in subchapter 4.1.

gatom "represents a class of object instances that has as its variables of values [such as] interval, duration, dynamic, and definition". Based on Tenney and Polansky (1980), Ames (1982:47) uses the concept of Gestalt to develop a procedure for post-serial composition, in which

> No entity will have structural precedence over any other entity at the same level, since such precedence would contradict the hierarchical organization. The process terminates along each of its multitudinous paths when it has refined the total aggregate into a description of an individual tone.

This procedure brings together relevant aspects of algorithmic composition with self-similar objects, for which recursion ensures consistency between gatom and Gestalt, between individuality and system (see Leman 1995, 1997), achieving structural coherence through synecdoche and analogy in various operational levels.

Mazzola and Zahorka (1996) introduce into musical Gestalt a useful *topology of motifs*, a set of compositional archetypes formalized in terms of architectural bodies (*architektonischen Körper*), using the analytic theory of Réti and Kopfermann (1982). Interestingly, this approach connects a general notion of functional self-similarity and recursion with the notions of *paradigm* and *proportion* (included within the labels *Motivische Paradigmatik* and *Gestaltete Motive*), which the present study recognizes as 'classical forms of analogy', based on the research developed by Szabó (1978).[138] Additionally, Mazzola and Zahorka (1996) support their theorization by the *intra-* and *inter-Gestalt* coordinative relationships under operating conditions than can be *rigid* or *flexible* (*Starre Gestaltabbildung* and *Elastische Gestaltabbildung*), interpreted in the present study as negotiations between grammar and style (on foundations established by Meyer 1956).[139]

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[138] See subchapter 3.1.
[139] See subchapter 4.7.

3.6. Self-reference

A *self-reference* is an expression 'saying' something about itself, by itself.[140] This also holds in a form or a structure that contains itself, like those self-similar processes in which a segment implies all parts of the system to which they belong. In fact, Yadegari (1992:69) assumes that "self-similarity should be thought of as a portrait of a self-referential entity".

In his 'Introduction to self-reference', Smullyan (1994) makes this concept understandable thanks to the simplicity of the first example he gives: "John is reading". It is an example which comes true *if and only if* the action is true (if John is reading that he reads), because then there is no reference to a second subject, but to the same reader who validates the sentence as an ontology (John), as an action (to read), and as a knowledge (to read to read)—all at the same time. This example reminds us of the self-referential exercise made by writer Salvador Elizondo (1932–2006) in his poem *The Graphographer*, in which ontology, action, and epistemology (a *trichotomy* characterized in section 3.8.3.) are intertwined:

> I write. I write that I write. Mentally I see myself write that I write and I can also see myself seeing that I write. I remember writing already, and also see myself that I wrote. I see myself remembering that I see myself writing and I remember seeing myself remember that I wrote and I write seeing myself writing, to begin by rehearsing that I remember having seen myself the sighs, the gazes, write that I was seeing myself write the gestures, the undulations, that I was remembering having seen that I write.[141]

In this kind of reflexive plot, Smullyan (1994) recognizes the weight of the *indices* in the construction of self-references. The indices are symbols or words whose denotation depends on the context, as when the pronoun 'myself' depends on the individual articulating that word; or when the length of a musical tone depends on the place where it appears in a composition. Clearly, the example becomes more complex in the case "John is reading this sentence", because the index 'is' works as

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[140] In these terms, a good example of self-reference is the first paragraph of subchapter 2.1. Accordingly, this footnote is itself also a system of self-reference and redundancy within this study's framework.

[141] Salvador Elizondo, *El Grafógrafo*, Joaquín Mortiz, Mexico, DF, 1972.

self-referential 'calling' or 'quotation' (Smullyan *op. cit.*:2). A paradigmatic analogous relation can happen in music with self-referential indices, as explained below.

Although it is obvious that music never means the same as verbal language, it is possible to build systems of musical quotations by means of a symbolization of relations that are presupposed in a larger symbolic structure.[142] An example of this is the music that encompasses, by the use of quotations, its own *history*—its course as temporal listening—simultaneously within the history of references that makes itself possible. Whether considered a quotating-articulation that is both audible reference and historical referentiality, Luciano Berio's *Sinfonia* (1968) has those characteristics as a whole made of self-referential segments. Something similar happens in many other cases of intertextuality which base their operational and logical identity in a system of prior relationships, understood or presupposed within the same identity.

Berio's *Sinfonia* articulates by a finite set of references comprising its own history through other histories which in turn contain the piece, or at least contain, strategically and operationally, significant segments of it—like seeds containing the genetic code of a biological structure.[143] The quotations Berio uses operate as windows to other historical compositions 'already containing' the germ of his *Sinfonia*. In this manner, subsequent compositions following Berio's Sinfonia, making use of its segments, will be, by abduction, an extension of those windows, as Charles S. Peirce suggests for the case of the map of the map.

For Kuhns (1978), but also for Price (1988), Fiske (1990), and Davies (1994), music is self-referential in a generalized sense. "Many musical signs or 'gestures' refer 'congenerically', that is, to other parts of the musical work or musical style. One theme recalls another or 'refers' to its own recurrences or treatment." (Davies 1994:10). Accordingly, all music inscribed in a tradition and style, operates with some degree of functional self-similarity, as surface of its own self-referencing.

This form of musical self-reference aside, self-reference also exists through the complexity of the signs of culture, in the relative instability of ecolects and styles of

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[142] See footnote 97, page 69, on Coleridge's idea of the symbol by its synecdochic function.

[143] This, however, cannot be a fully automated relationship. This question stimulates the discussion presented in subchapter 5.5., on the negotiations between determinism and indeterminism.

music; in the musical idiolect, and even in the *musical idea* immersed in solipsism: what Kaipainen (1994:23–24) calls 'autocommunication', using the example of an individual who listens to a recording in spatial isolation from other individuals. In this case—as Kaipainen conceives—music does not seem to be communication, at least in the ordinary sense of the term; which does not imply, at all, that music is not a language. It is a language that in some respects is comparable to speech, including traits of solipsism and isolation, such as in silent reading, in monologue, or in soliloquy.[144]

Self-reference plays an important role in the relationships between message and context, and is commonly associated with the pragmatic use of the indices. In rhetoric, a basic typology of self-referential relations includes *anaphora, cataphora*, and *endophora*, parallel to their musical analogies. Anaphora is the continuous repetition of symbols, gestures, or structures, by way of emphasis, parody, or contrast. Cataphora is an anticipation of a predictable structure in which the symbolic content of an element reappears. Endophora is an intralinguistic reference, a textual vehicle within the text itself, such as in the case of the self-referential index in the given example "John is reading this sentence."

In music many deictic functions—in general those indices structuring possible or necessary relationships—are self-referential, and according to Reybrouck (2009), they affect several mechanisms for delimitation, comparison and operativeness. Equally, the recursion of a rule in a musical system operates as an endophora, preventing the results of applying the rule to the ontological principles of the rule itself.[145] In contrast, exophoric references may be identified in musical styles with a narrative development in which, for instance, a chord or a motif is playing a role comparable to that of a character in a story—as happens with a musical quotation.

Deixis can also be a type of exophora. When one says, "not that, but the other", the object referred to is not verbally specified. It is specific rather on the contextual use of the expression. Hence the exophoras, such as 'callings' or 'quotations', act as referential instruments of relationships *in absentia*, which are typical traits of the

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[144] For shades of analogy that compare music with speech, see pages 51, 56, within the context of Charles Seeger conceptualization on the subject (Seeger 1960:226).

[145] This conceptualization is expanded in subchapter 6.5.

deictics and remain in the memory of the speakers, but not in the explicit structures of language. A musical structure can also make use of relationships between a present and an absent element, serving as a reference and creating discursive tension, assuming that the element missing in the articulation of speech, may be present in the memory of the listener (Tarasti 1994:277).

Self-reference and autopoiesis

Based on the research of Maturana and Varela (1973, 1980), several authors relate self-reference to *autopoiesis*. The latter refers to the self-organizing processes of living organisms, as represented in the growth of Lindenmayer systems, explained in subchapter 6.5. Specifically, according to Varela, Maturana and Uribe (1974:188),

> The autopoietic organization is defined as a unity by a network of production of components which (i) participate recursively in the same network of productions of components which produced these components, and (ii) realize the network of productions as a unity in the space in which the components exist.

The widespread extension of the notion of self-organization into studies on culture and societies has been criticized due to its reductionism (see e.g. Roth 1981). Among thinkers implicated in this controversy are philosophers such as Ludwig von Bertalanffy (1901–1972), author of a theory of 'social physics' based on the general systems theory; and Heinz von Foerster (1911–2002), author of a hypothesis about the "causal circularity and feedback mechanisms in biological and social systems" (see Foerster 1949); as well as sociologists such as Niklas Luhmann (1927–1998). However, Luhmann acknowledges an overgeneralization of the causal systems of self-reference (see Luhmann 1990:1), observing that:

> At first sight it seems safe to say that psychic systems and even social ones are living systems too. [...] However, we immediately get into trouble in precisely defining what the 'components' of psychic and social systems are whose reproduction by the same components of the same system recursively defines the autopoietic unity of the system.

Luhmann (*ibid.*) believes that organisms are "special types of systems", and suggests that culture and social systems are also *sui generis* systems, distinct and only in certain respects parallel to living systems. In order to clarify this idea, he elaborates a scheme of analogies (see upper section in scheme ◊360).
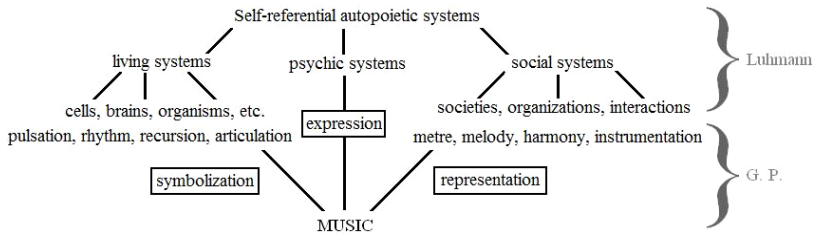
◊360.

This scheme, says Luhmann, "must not be understood as a description of an internal system of differentiation; it is not a scheme for the operation of systems, but for observation". Consistently, Luhmann (*op. cit.*:2) explains that his intention is to differentiate types of systems or modes of autopoiesis, also according to different modes of self-reference. In ◊360 Luhmann's original box is marked by a key (right, top) with his name, whilst the complementary scheme in the bottom (corresponding to the key marked G.P.) indicates its adaptation for the present subchapter, suggesting a parallel of self-referential systems in music.

Interestingly, along with this scheme for the conceptualization of self-reference, comes a useful interpretation of Tarasti's inquiry (2004:2), wondering if "music does represent, signify, or express […] Where is here the level of content or meaning?". The schematization in ◊360 suggests that music is a complex system within a flexible, and usually open convergence of structures of representation, meaning, and expression, each of which is characterized by different forms of poiesis, somehow compatible among them. The "level of content or meaning" cannot be situated, thus, at a unique or specific layer, but at several relatively flexible layers of cognition and knowledge intercommunication.

3.7. Recursion

In different disciplines, the term *recursion* may have different meanings, but usually it implies the notions of *iteration* (in contrast to *repetition*), and *self-generation* as a potential quality of a function iteration.

In linguistics *recursion* is the property of some systems of rules by which the result of applying a rule is used again to undergo the same rule or another related rule, generating syntax structures. Pinker (1994:481) defines this recursion as "A procedure that invokes an instance of itself, and thus can be applied repeatedly to create or analyze entities of any size: [for example] A *verb phrase* can consist of a verb followed by a noun phrase followed by a *verb phrase*."

In logic and mathematics there is a similar use of this notion to define systems or processes that involve themselves in their own definition, usually formed by the *iteration* (i.e. loop executing) of a function. An example from arithmetic, which can also be interpreted in set algebra, is the definition of a natural number: 1 is a natural number; each natural number has an integer successor, also being a natural number. Another typical example is the Fibonacci sequence defined at the beginning of subchapter 6.3.

In music and in speech, the recursions allow a lack of pre-established limits for the combination of the elements of a motif, a phrase, or a larger structure. A convention is plausible that allows the articulation of a tone followed by another different tone and another, and so on, forming a *phrase* or a *theme*. The same system can allow a series of tones to complete differently each time, making possible a *poiesis* (creativity) in the symbolic and structural relationships of music. There are abundant examples of this kind of operation. In the case of verbal language, names can be associated with many adjectives and complements, without rules that may sanction a specific number of combinations. In the case of music a relatively small set of tones can be articulated through a systematic recursion, a very long—potentially infinite—string made of analogous strings with their own functional attributions.

In a philosophical framework involving aspects of sociolinguistics, Marrades Millet (1998:54–55) explains 'creativity' (i.e. self-generation), as a typical feature in many recursive systems, distinct from mechanic repetition. This definition also approaches the notion of musical recursion as creative separation of identity and difference:

> Contrary to common belief, following a rule is not to repeat the same in each turn, but in a sense it involves doing something different from what was originally shown, in such a way that it equals to act in each case under the same rule. [...] Following a rule is to make something identical in different ways.

For Bolognesi (1983:26) the notion of musical recursion is straightforwardly linked to the notions of *hierarchy* and *self-similarity*: "A self-similar structure is one whose parts recursively repeat the whole structure, and this immediately implies a hierarchy with an infinite number of levels." Accordingly, Lerdahl and Jackendoff (1983:14–16) observe that, as 'at all levels' the rules that in a musical pattern relate in a similar way some dominant groups to other subordinated groups, "this uniformity from level to level [reveals] that grouping structure is *recursive*; that is, it can be elaborated indefinitely by the same rules." These authors also facilitate the connection of "the principle of recursion" with the concept of self-similarity:

> The principle of recursion says that the elements of metrical structure are essentially the same whether at the level of the smallest note value or at a hypermeasure level (a level larger than the notated measure). [Lerdahl and Jackendoff 1983:20]

Whether the fundamental relations—not so much the elements between them, which can present significant differences—are 'essentially the same' at the different metrical levels of a music structure, then it can be said that there is a relationship of self-similarity intertwining these levels. As seen in Chapter 5, an analogous kind of relationship also appears at different levels of harmony, loudness, melody, timbre, and durational distributions, linking the macro- with the micro-structures through self-similar systems.

As an effect of some functional recursion, self-similarity is also directly linked to the appropriation or imitation of discourse 'in style' as considered by Haugen (1950), Stephan (1979), and Escal (1981). This recursion usually has the appearance of

borrowing, variation, pastiche, or collage, or of synthesis of a musical theme on another theme. In this sense, recursion is also an aspect of self-reference.

It is worth clarifying the employment of two terms, *recursiveness* and *recurrence*, which do not mean exactly the same as recursion. Recursiveness is the contingency of recursion in a relationship. It is sometimes seen as synonymous with self-similarity, which is incorrect: self-similarity is a relationship defined by itself, repeated in its own image (auditory, visual, etc..), whilst recursiveness is the potentiality of a set of rules that are organized by themselves throughout their own iteration. Self-similarity is also an intuitive aspect of the form, whilst recursiveness is only an operational quality within a system that may or may not be self-similar. The cyclic use of a same rule does not guarantee similarity across different scales.

On the other hand, recurrence (or *recurrence relation*), as described in subchapter 2.3. on symmetry, does imply the symmetrical inversion of an interval or a set of intervals. It could be argued that recurrence results from applying a grammatical precept, as indeed happens in classical counterpoint, and even that it is found in an unlimited number of cases. However, recurrence is an effect of recursion, not the recursion itself. Consequently, both terms cannot be regarded as synonyms.

Recursion and self-organization

The development of collective websites is a palpable example of the relationship between recursion and self-organization. There may be, in this case, a basic set of rules to make descriptions and frames for actions, which, without a fully explicit agreement, contribute to a functional whole, well organized even in the variety of its contents. In this example, explicit rules for elaboration determine the basic models for the recursion and its operational symbols; the use of these models, meanwhile, ordered by a common need for information, generates a self-organized whole, with a flexibility of approaches and interpretations not encompassed by the initial rules.

Musical traditions are also examples of recursion and self-organization, insofar as a set of conventions can serve for a continuous reworking of the musical discourse in which a massive input of stylistic variants converges into a consistent repertoire, without a necessarily explicit agreement for classification, action, or interpretation. It

is said, then, that there is a *coordination* between the fixed or rigid parts of music, with other flexible parts, typical of negotiations between *idiolect* (an individual or particular form of language) and *ecolect* (modality of an immediate, collective environment).

The term 'self-organization' was introduced to philosophy by Immanuel Kant (1724–1804),[146] and it was adapted to systems theory, and especially developed by cybernetics following the work of Heinz von Foerster (1949), Claude E. Shannon (1948), and Norbert Wiener (1948). Because of its common association with the concepts of recursion, self-similarity and self-reference, its use was easily extended to the investigation of dynamical systems. Similar to what occurs with the concept of self-reference, its adaptation and overgeneralization into the social sciences led to sharp criticism. The idea of self-organization, however, is still useful in describing a variety of relationships in emerging symbolic systems, such as language in general and music in particular.[147]

According to Josephson and Carpenter (1994,1996) music is a phenomenon based on the self-organization of codes, from which messages are constructed in the strata of musical traditions and culture. From this radical perspective, "perception of music is not 'mere perception' but perception allied to the presence of a different, more fundamental system" (Josephson and Carpenter 1994:3). In this study, Chapter 4 does not particularly favour this fundamentalism, rather it suggests that music emerges as a complex environment in which various stages and cycles participate simultaneously in synecdochic intersemiosis (as suggested e.g. by Bateson 1972).[148]

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[146] *Kritik der Urteilskraft* (1790:B292): "In einem solchen Produkte der Natur wird ein jeder Teil, so wie er nur durch alle übrigen da ist, auch als um der anderen und des Ganzen willen existierend, d. i. als Werkzeug (Organ) gedacht: [...] sondern als ein die anderen Teile (folglich jeder den anderen wechselseitig) hervorbringendes Organ, dergleichen kein Werkzeug der Kunst, sondern nur der allen Stoff zu Werkzeugen (selbst denen der Kunst) liefernden Natur sein kann; und nur dann und darum wird ein solches Produkt als organisiertes und sich selbst organisierendes Wesen ein Naturzweck genannt werden können."

[147] Kalev Tiits (2002:89–108) offers a historical account on the concept of self-organization in music, and analyzes its implications for musical information systems.

[148] This is especially related to the section *The concept of abduction in Bateson* (see page 158).

Recursion and ecology

The notion of recursion is similar to that of recycling, since it embodies an idea of cycles among objects, images and concepts through (re)generation of qualities. The notion of recycling is commonly used in the urban societies, and postmodern society takes this notion in order to reuse melodies and songs, in an operation partially controlled by commercial and political interests. However, the essence of this kind of operation is already visible in more general processes of language, as borrowing, variation, transformation, etc., which are also found in different forms of expression and communication in animal communities (see Bateson 1972).[149]

One of the main needs in recycling is the 'knowing what' is going to be recycled and 'knowing how'. So, the classification of objects, images and concepts, is followed by a selection of operations that make recycling possible. For Kaipainen (1994) the meaning of the processes of recursion in music is concentrated in the 'dynamics of musical knowledge', and therefore, its performance in ecomusicology has an explicit interest for cognition: *knowing what* and *knowing how* in the sonic world, would be connected to the way humans interact with their environment, with all cultural and biological aspects implicit. Kaipainen, Toiviainen and Louhivuori (1995) deal with this kind of knowledge in order to elaborate a map (a model of routines) which manages to mimic human behaviour in the recursion 'in style' of melodic patterns.

For Kaipainen (1994) the notion of 'world' corresponds with that of οἶκος (origin of the prefix *eco–*, 'house'), and his idea of ecomusicology is constricted to the study of actions in the 'world' through sounds as cognitive processes. Instead, the present study focuses, not on the investigation of routines or neural maps reproducing a behaviour in the human 'world' of sounds, but on the investigation of the relationship 'house within a house', which is compatible with Parsegian's (1968:589) ecological view on "worlds within worlds", also related to the concept of *Umwelt–* niche, a coupling of ideas first put forward by Jakob von Uexküll (1940, 1957) and Thomas A. Sebeok (1977), and Donald L. Hardesty (1972), respectively.[150] This is

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[149] This issue is discussed in a general way in 2.7., and more specifically in subchapters 4.7. and 4.8.

[150] See subchapter 4.3. and 4.8.

precisely the notion of ecology used in this research, particularly developed in chapters 2 to 4.

The ecology of music, studied by a new discipline called *ecomusicology* (see Hambræus 1974, Kaipainen 1994, Trochimczyk 1995, Reybrouck 2001, 2005), is concerned with the meaning of music as 'made' in context. Previously, Merriam (1964:161–162) connected this notion with the concepts of *environment* and *habitat*:

> The questions which surround the learning of music are very important ones, for they provide us with a knowledge of how music is produced, as well as an understanding of techniques, agents, and content of music education in a given society. [First of all,] at birth, the human infant enters a man-made environment which acts as a buffer between him and the raw habitat.

This same idea is explicitly defended by Blacking (1973:58) "The origins of music that concern me are those which are to be found in the psychology and in the cultural and social environment of its creators"; as well as by Reybrouck (2001:600, 626–627) "Music deals with man-made environments, that are cultural constructions. ... [A] central issue [here] is the way how listeners as subjects experience their own phenomenal world, and how they can make sense out of their sonic environment." From this viewpoint, the 'sense' of music does not preexist, but is created as part of a relatively subjective environment—e.g. a Gestaltic sound complex.[151]

Habitat, context, environment, Umwelt, niche, all these ideas are captured by the same notion of *house* or οἶκος, which always is nested within another οἶκος (see Meystel 1998). Obviously, the concept 'house of the house', is directly related to both, ecology and biological self-similarity. Chapter 4 is entirely devoted to investigate this relationship from a musicological conceptualization. The Peircean concept of 'the map of the map' (Peirce *CP*, 8.122), closely related to this issue, is introduced in sections 3.8.2.–3.8.3., and developed in section 3.9.2., with significant applications in chapters 4, 5 and 6.

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[151] The notion of Gestalt is explained in subchapter 3.5.

3.8. Intersemiosis

Charles S. Peirce (*CP* 5.484) writes that "Σημειωσις in Greek of the Roman period, as early as Cicero's time, meant the action of almost any kind of sign; and my definition [of semiosis] confers on anything that merits the title of a 'sign'". Thus, *semiosis* is the process or processes of signs 'in action', assuming that "a sign mediates between the interpretant sign and its object" (*CP* 8.332). As Martinez notes (1997:66), it is necessary to observe that Peirce does not limit this approach to speech, but his ideas of *sign, language,* and *interpretation* are operationally extended, for example, to the elaboration of a melodic phrase, or to abstract reasoning in mathematics.

Dorányi (2004:256) notes that "geometry, as a vehicle of communication, can carry different kinds of semantic, aesthetic, affective or functional content". Similarly, different types of musical content with semantic, aesthetic, emotional or functional attributes may also include various communicative or expressive aspects of geometry. This kind of networking of content in different systems of signs is called *intersemiosis*.

Intersemiosis is the operation or—using Peirce's concepts—the 'action' of a sign or sign system corresponding to a category, under influence, transformation or transduction onto other categories of signs. Jakobson (1959:232) uses the notion of *intersemiotic fact* when referring to the transmission of a system of signs, such as a visual system, into another system of signs, for example, a musical sign.

The concept of semiosis has been consistently used in a variety of applications in music theory,[152] whereas that of intersemiosis is less developed in this field, despite a relationship as basic as that between a sound and its written representation—that might be investigated as an intersemiotic fact.

Indeed, Cooke (1959:1) conceives of music as a language within an intersemiotic context, immersed in intertwining systems of analogies:

> Although all the arts are essentially autonomous, owing to the different materials and techniques which they employ, there is clearly a kind of bond between them. We speak of the 'architecture' of a symphony, and call architecture, in its turn, 'frozen music'. Again,

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[152] Some examples of this application are the investigations by Nattiez (1976, 1987), Monelle (1992), Tarasti (1994), Martinez (1997), Tagg (e1999), Lidov (2005), Almén (2008), and Ojala (2009).

> we say that certain writing has a 'sculptural' quality, and sometimes describe a piece of sculpture as a 'poem in stone'.

The 'kind of bond' which Cooke identifies between different systems of aesthetic signs, is nothing else but intersemiosis.

The concept of *multimodal grammaticality* is also intimately linked with that of intersemiosis. O'Halloran (2008:233) notes that "Semiotic resources are viewed as interlocking semantic phenomena in the systemic functional multimodal discourse analysis approach, giving rise to the concept of *multimodal grammaticality* where linguistic, visual and symbolic choices function together to construct meaning in mathematics discourse". Something similar can be said about a convergence of applications and elections in several semiotic levels, forming meaning in musical discourse.[153] O'Halloran (*op. cit.*:234–235) conceives that the mathematical discourse has a 'registeral' mix of field or *content*; a mix of idiolect-ecolect negotiations or interpersonal relationships or *tenor*; and a generic mix of *genres* and structural functions of *hierarchy* "which function to construct mathematical reality." The same relationship applies to multimodal resources in the elaboration of musical discourse (see Fauconnier and Turner 1996, Zbikowski 1997).

O'Halloran (2008) postulates that the intersemiotic systems for the construction of experiential meaning include *discursive systems* (e.g. intersemiotic ideation) and *grammatical systems* (e.g. relationships of transitivity, lexicalization, symbolization, and visualization). Simultaneously, she proposes categories of 'mechanisms' to explain how the trends and elections of visual and symbolic language—hearing should be added—are integrated to produce the semantic expansion of mathematical discourse in a way analogous to the semantic expansion of musical discourse. Such mechanisms formulated by O'Halloran are: *semiotic cohesion* (multimodal reference), *semiotic adoption* (incorporation of semiotic elections between grammars), *semiotic mix* (structural combination), *juxtaposition* and *spatiality* (spatial arrangements, concrete analogy, proportion), *semiotic transition* (explicit change to another semiotic resource), and *semiotic metaphor* (metaphorical expressions, abstract analogy, paradigm). All of these 'mechanisms' are explored throughout the present study, given its relationship to processes of musical self-similarity. O'Halloran devotes

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[153] See subchapters 4.7.–4.8.

special attention to the role of intersemiotic metaphor in a similar way to which this research focuses on the functions of intersemiotic synecdoche.

Intersemiosis and multimodal grammaticality have manifold theoretical and practical aspects of interest for music. This study does not intended to thoroughly cover both issues, but instead concentrates on the derived concepts of *intersemiotic translation* and *synecdochic intersemiosis* because of their relevance for the investigation of musical self-similarity as a complex phenomenon—i.e. a coordinated wholeness of semiotic layers.

3.8.1. Intersemiotic translation

Music's relationship with other arts is dialogical. This means that music can persuade other arts, or can be persuaded by other arts to modify its symbolic content and meaning. Moreover, the comparison of components of music, with the components of other kind of aesthetic forms, strengthen the sense of music and—in general—the sense of aesthetics (see Sickles & Hartmann 1942, Ingarden 1962, Monelle 1992, Lidov 1999, Tarasti 1993, 1994).

The ordinary language a musician uses to refer to music operates as a continuous intersemiosis: when Walter Piston (1947:13) asserts that "The outline of a melody may be perceived by simply looking at the music", he is searching for a synaesthetic method of intersemiotic translation. The functional correspondences at this crossroad of semiotic systems are not arbitrary, but reflect the dialogical relationships between cognitive domains, essential for inferential reasoning based on comparison and analogy, of vital importance for music (see Zbikowski 1997).

Étienne Souriau's (1892–1979) treatise on comparative aesthetics, entitled *The correspondence of the arts* (1947), essays to formulate a general theory of intersemiosis in the arts. It is based on the assumption, also defended by Ghyka (1938:78), that it is perfectly possible to transpose a spatial notion of rhythm, as it is found in design or architecture, into a temporal notion of rhythm, as it is found in music. Both authors also assume that aspects such as proportion or enumeration, are fully empathic in this kind of transposition or *intersemiotic translation*—as defined below, including aspects of harmony, timbre, and texture in their broadest sense. After all, the transposition of a spatial notion of rhythm into a temporal notion of rhythm, occurs whenever the

written representation of music is interpreted as actual sound: intersemiotic translation is a phenomenon clearly and continuously present in a variety of musical practices.

Though there is abundant research on the 'translation' of paintings into music,[154] the literature pays less attention to the general processes of intersemiotic translation, often overlooked during the musical conversion of self-similar sign systems. With a name that may seem far-fetched and not quite conceivable on first contact, Roman Jakobson's definition of *intersemiotic translation*, is nothing else but describing a forest, as in Sibelius's music, or articulating a narrative through sound, as happens in distinct instrumental traditions (see e.g. Rowell 1983, Almén 2008). The terms of *ekphrasis, transposition, transmutation*, and *intersemiotic translation* are often considered as homologous and used interchangeably for similar cases. There are, however, convincing reasons to adopt the last of them in the context of the relationships of self-similarity and the theory of maps within maps presented by Charles S. Peirce.[155]

Umberto Eco (2003:110) suggests that "when a verbal text describes a visual artwork, the classical tradition speaks of ekphrasis".[156] A good example is the painting of *Las Meninas* (or *The Maids of Honour*) by Diego Velázquez, described by Foucault in the introduction to his literary essay *Les mots et les choses* (1966). Eco (2003:111) also comments the case of 'hidden ekphrasis' when the references from the source to be translated are not explicit; for instance, when describing a picture—one may use the same example of *Las Meninas*—without using any information about its title and author, and omitting details from the painting. The recreation of the scene painted

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[154] The pioneering essay by Eero Tarasti (1993) figures among the few works published on musical intersemiosis. It should also be noted that issue 105 of the magazine *Pauta* (January-March 2008), directed by Mario Lavista, is entirely devoted to aspects of 'interdisciplinarity' between music and other arts. This issue includes the writings of Sotelo and Sánchez Escuer itemized in the bibliography of this study. However, none of these precedents analyze musical intersemiosis in function of self-similarity, neither do they explore the role of synecdoche in intersemiotic translation, or in pre-musical and meta-musical environments. The present study seeks to address, precisely, this absence, developing a theoretical framework.

[155] This concept, investigated under the name *abduction*, is introduced on pages 23, 31–32, and then developed in subchapter 3.3., on self-similarity (pages 66–69, 76), as well as in following sections and further subchapters (see also 'abduction' in the Index of subjects).

[156] From the Greek ἐκφράζω, to expose in detail; from ἐκ, out, and φράζω, to explain with signs and words. A related term, φράσις means "statement of facts in the prologue to a tragedy" (M. A. Bailly, 1894).

by Velázquez is achieved, then, by reproducing relationships that allow a reconstruction, although partial, of the objects or figures in question. Thus the spectator can guess the source of references named by Peirce (*CP* 7.219) as *abduction*: the mental process which elaborates the assumptions that allow the identification of the source. Consistently, in his treatise on music, painting, architecture, and film, Ingarden (1962) does not speak of 'intersemiotic translation', but of 'kinship' (*Verwandtschaft*) [*op. cit.*:229] and 'means of reconstruction' (*Rekonstruktionsmittel*) [*ibid.*] of specific aspects across different disciplines or aesthetic fields.

In particular, the concept of 'transposition' refers to passing information from one symbolic medium to another. Jakobson (1980:90) describes it as follows:

> There are not only translations, but also transpositions into another art. [A] poem could be transposed into a painting [...] However, something quite different will come out because the semiotic structure is different. It will be an intersemiotic fact. Transposition is permissible. [...] I consider Blake's illustration of Dante very beautiful; however, it is not Dante, but something quite different. On the other hand, [a] poem could also be put into music or could be filmed. All these transpositions show that there is a common element in all these art forms. Something remains. Most of it is gone, though; I would not say it's lost, but it is altogether transformed.

Jakobson uses the word 'transposition' as a synonym for 'intersemiotic fact' and with it encompasses the concept of ekphrasis. But with its use a problem arises: the word transposition is already employed for very specific cases in linguistics and in music theory, having no direct relationship with the idea exposed by Jakobson. In music 'transposition' means to move a system of notes from an instrumental register to another; this process is close to the concept of *intralinguistic translation*, also coined by Jakobson (1959). Similarly, in translation theory, transposition "involves replacing one word class with another without changing the meaning" (Vinay and Darbelnet 2004:132). This definition of transposition implies, simply, to say 'the same' in other words.[157]

Jakobson (1959:232–233) defines three types of translation: *intralinguistic*, which corresponds to the interpretation of verbal signs by means of other verbal signs of the same language or the same system of symbols; *interlinguistic*, which corresponds to the interpretation of verbal signs by means of other verbal signs of a language other

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[157] However it is interesting to compare this notion with the concept of recursion ("following a rule") a defined by Marrades Millet (1998:54–55) [see quotation on page 95].

than the language of origin (i.e. standard concept of translation); and *intersemiotic*, which is the transmutation of a system of signs, verbal or nonverbal, into another different system of signs. In his approach to these definitions, Eco (2003:110) repeats the equivalence originally made by Jakobson: "intersemiotic translation, that is, [the] transposition from a given semiotic system to another, as happens when a novel is transformed into a movie or a painting is described by a poem." In this way Eco assumes the equivalence of the concepts of ekphrasis, transposition, and intersemiotic translation.

For this study, and in order to achieve an operative use of the term *intersemiotic translation*, its use is restricted to the definition that Jakobson suggests (*cit.*), as a system of signs transmuted into another different system of signs. This definition does not only fit Eco's examples between novel and film, or between painting and poem. It can also fit exchanges between many other kind of complex semiotic systems, such as the translation of a topography into a map, or a musical score (a form of acoustic map) into musical sound.

The operativeness of exact self-similar systems, such as fractals,[158] does not imply any internal translation—as they are consistent in themselves. In contrast, the operativeness of relative self-similar systems may involve a (quasi-)intralinguistic translation process as a system of references displaced within the same system of signs: a tree structure can be described by the generic translation of a main structure, into main branches similar to the overall system, and a successive self-translation into its subsets. This form of translation implies a poietic recomposition of the original sign system, that is, the creation of 'something new' ("something quite different") nevertheless reflecting consistency between source and recreation (see Jakobson 1992:102–103).

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[158] See notes on the strict use of the term 'fractal', in section 1.3.4.
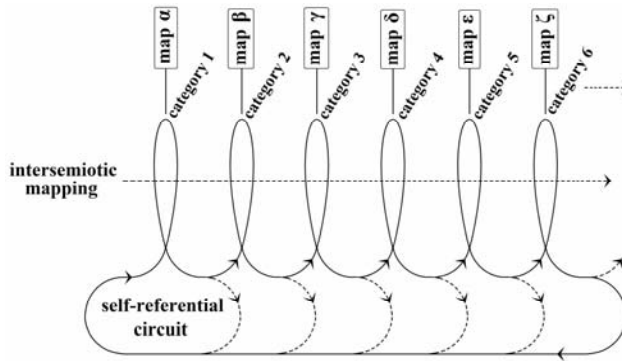
◊381. Self-referential circuit representing the intersemiotic flow of adjacent categories forming systems of analogies. The lower part of the chart invokes the Peircean category of *firstness* (explained in section 3.8.3.) as cyclical phenomenon of 'being' or self-reflection resulting from self-reference. The medium part of the chart points to *secondness* as cyclical referentiality of a knowing process; and the upper part points to *thirdness* as an elaboration of emergent sub-categories. The upper part also resembles the structure of Chapter 4, in adjacent categories of self-similarity (e.g. corpuscular, mechanical, biological, structural-linguistic, stylistic, and transcultural self-similarity).

Going deeper into this system of recreations, the intersemiotic translation works as a link between empathic categories of references in different sign systems. As intersemiotic flow, this form of translation plays an essential role in the interpretation based on the correlation of categories, in which are established analogies, chiasmi, comparisons, metaphors, metonymies, parallels, similes, synaesthesias, synecdoches, and systems of polysemy—including body-instrumental gestures and sounds, silences, and other symbolic resources common in musical performance.

In this scheme (see ◊381) the concept 'map' indicates a symbolic space for the representation of ontologies with specific qualities and relations, related to other corresponding ontologies. These ontologies are grouped in categories or domains, which in turn may be associable as contiguities. An example of this association is the following string of relations: a score for violin is a map of actions and relations in the musical space and time; interpretation of that score is realized by a mental map in a massive coordination of neural networks; recording of this *music* on a tape is a map of electromagnetic signals, read by an automated system; the conversion of the analogue

recording into a digital file involves a binary map which, when expressed by a visual editor, generates a new (bit)map. This string can stop in a circuit, or continue indefinitely, supposing that such a (bit)map stimulates a reinterpretation of the original source (in this case a score), producing a continuous information feedback. For each cycle of this circuit there are different operative categories, connected by a continuous flow of intersemiotic translation. The sequence of maps composing this translation is here described under the label 'intersemiotic mapping', and its theorization serves to explain the correlation between neighbouring layers of self-similarity within musical processes in general.[159] Such a correlation is illustrated throughout Chapter 4, describing the intertwining among six typical categories of self-similarity: *corpuscular, mechanical, biological, structural* (*linguistic*), *stylistic endomorphisms*, and *transcultural self-similarity*. At the same time the notion of intersemiotic translation provides a theoretical framework for the description of Markov chains, Lindenmayer systems, recursion trees, and fractional noises as musical sources, as explained in Chapter 5, and for the study of the conversions of spatial and visual patterns in general, into music, as suggested in Chapter 6.

Intersemiotic translation may weave semiotic strings between different sign systems.[160] For instance, Felguérez' *Steel Equation* has characteristics abstracted from mathematical thought, materialized into the language of modern sculpture.[161] In its turn, *Steel Equation* can be translated into a set of pictures, being recomposed through a new set of equations and geometric relations (the mathematical representation of the same sculpture), or into a musical complex.

Another similar case is the collection of sculptures by Eduardo Chillida, described by Mauricio Sotelo (2008:15) as "silent Sea of sounds". Chillida carves stone blocks and moulds metal pieces giving them *rhythms* and *motifs* borrowed from

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[159] Useful in the context of semiotics and aesthetic theory, the concept of 'intersemiotic mapping' is equivalent to *cross-domain mapping*, employed in cognitive sciences and experimental psychology (for its adaptation to music theory see Zbikowski, 1997:200).

[160] However, an intersemiotic translation is not always feasible or does not always consists of an explicit negotiation or migration of signs. This notion is suggested above under the concept of *hidden ekphrasis*.

[161] Manuel Felguérez (born 1928) is a sculptor and painter, author of an innovative output displayed in international collections.

the music of J.S. Bach, or from the sound of the sea.[162] Something comparable occurs with the architectural spaces created by Luis Barragán,[163] transformed into piano sonorities in Gabriela Ortiz' *Patios serenos* (see Sánchez Escuer, 2008). The specific case of Francisco Guerrero also belongs to this tradition in which form, space and movement are conceived, not from a mathematical conceptualization—as it is commonly asserted for the music of this composer, but rather from a highly *cultural intuition* of the world as an organized system of systems. This difference escapes to most of authors suggesting that Guerrero 'used fractals' to compose music, as in Woolf's (e2000) review:

> *Oleada* (1993), for a string orchestra of 50 real parts, is based on 'the fractal movement of a wave' in micro-polyphony too complex to perceive, superficially like Ligeti's experiments, the parts 'sliding from one voice to another as in a whirlwind', [with] every detail deduced mathematically and notated with precision. Guerrero saw the form almost as a live organism.

The relevant fact in this idea, is neither "the fractal movement of a wave"—as if such a thing could exist beyond engineering schemata—nor the "every detail deduced mathematically and notated with precision"—how precisely could a wave be humanly notated?! The keyword here is 'almost'. Then, the relevant question is how something musical can be 'almost' a living organism, or 'almost' mathematics. The answer seems to be at the perceptual and cultural mechanisms of intersemiosis, as the central chapters of this study suggest.

All these results can be applied over and over again to other intersemiotic translation processes, forming infinite semiotic strings in an *intersemiotic continuum*. For instance, Guerrero's music can also be described by its fractal dimension,[164] and, conversely, certain (pseudo)fractal object or process can be described by Guerrero's music. In a bias similar to analogy—expressible as proportion or as paradigm—the intersemiotic translation can also have *deterministic* or *indeterministic* orientation. Accordingly, an analogy may combine aspects of both proportion and paradigm, and

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[162] Eduardo Chillida (1924–2002) is a sculptor known for his monumental abstract works, many of them associated with landscapes and open environments.

[163] Architect Luis Barragán (1902–1988) became the second winner of the Pritzker Prize (1980).

[164] The concept *fractal dimension* is explained in a special section in subchapter 5.5. (see pages 300–303).

intersemiotic translation may combine aspects of determinism and indeterminism. Zbikowski (1997:207–209) identifies this dichotomy as a constructivistic difference between *atomistic buildings* and *chains-of-being*, with their own systems of hierarchy and recursion. Specifically alluding to musical constructivism, Zbikowski (*op. cit.*:210–212) associates *atomistic buildings* with 'rationalist organization' (i.e. musical determinism), and *chains-of-being* with causalities based on tautological or metaphysical attributions (i.e. musical indeterminism).[165]

An example of determinism in intersemiotic translation is the direct conversion of a geometric object into a musical system, e.g. a regular hexagon into a hexatonic whole-tone scale, or an abstract two-dimensional map into a system of pitch-lengths with absolute correspondence of intervals. So, under explicit rules, Felguérez' *Steel Equation* can be translated into sonority. Now, examples of indeterminism in intersemiotic translation are the recreation of a 'forest' in Sibelius music, or the 'sea' in Debussy's *La mer*. In these cases, the resulting musical systems operate by paradigmatic analogy, dependant on interpretative variations in their cultural context.

Something to be emphasized is that, as happens in the two classical modes of analogy, intersemiotic translation is coordinative: it can at the same time be of deterministic or indeterministic character, depending on an interpretative approach. The fact that the music of Sibelius or Debussy is based on a metaphor does not mean that the same metaphor is not also working as an analogy, with an intuitive usage of ratios and proportions, in consistency with what is evident in certain physical characteristics in *woods* or the *sea*. In any case, the difference between determinism and indeterminism in intersemiotic translation lies in the balance of the ordering of signs, guided by the rigor of analytic processing and its deterministic grammaticality, or by the expressiveness of an indeterministic idiolect.

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[165] In the present study, musical ontologies in both forms of 'constructivism' are mirrored by the *atomist* or *gestaltic* notion, as well as by a *continuist* or *integralist* counterpart.

3.8.2. Synecdochic intersemiosis

In both cases, the notions 'forest' and 'sea' are transmitted by synecdoche: in music —including its figurative notion as musical performance and written formation—as in other kinds of language, is not mandatory to explicitly present all the physical qualities of the referred systems. It suffices transmitting a single aspect, enough intuitively and adequate to cultural context, to reproduce, by paradigmatic analogy, the images 'forest' or 'sea'.

In music, however, the synecdochic functions deeply affect the recreation of objects and spaces. For example, Fourier analysis (whose theoretical bases are explained in subchapter 4.2.) permits interpreting the periodic systems of frequencies of sound as complexities resembling the partial layers forming each system. This is reflected in a very elementary fashion, by the ordinary nomenclature in a tonal scale: when playing a tone of a specific gamut with a violin, one may say that 'a pitch x from the scale X is sounding' (e.g. a c_4 in a C major chord). This clearly constitutes a synecdoche. At the same time, there is another, hidden synecdoche: one may say that the pitch x has certain qualities, but 'x' implicitly means the set of its subharmonic components e.g. those which are typical of the violin strings. Summarizing, x signifies, in its singularity, the polysemy of its manifold particularities.

A synecdoche is a generalized relationship structuring meaning and sense. From the point of view of psychiatry, experimental psychology and cognitive science, "Sense is not in things. It is in the living being who uses things giving them a sense" (Cyrulnik 2004:13). Then, a synecdoche is not a property characterizing the universe, but a cognitive entrainment that extends—by an abduction of aesthetic character— the intelligible over the unintelligible; the known over the unknown; and the dominated over the indomitable, opening the possibilities for conjecture and prediction.[166]

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[166] This idea is foreseen by Locke, although from an interpretation restricted to rationalism: "General and universal belong not to the real existence of things; but are the inventions and creatures of the understanding" (Locke 1690:II, 21).

Being considerable part of the human capabilities and biases, music is one of the most blatant manifestation of intersemiosis.[167] Under this premise, Chapters 2, 3 and 4 present a selection with examples of synecdochic intersemiosis concerning musical self-similarity. These examples provide, within their different contexts, an overview of the 'map of the map',[168] according to the concept of abduction founded by Peirce: the whole spectrum of the partial relationships having implicit the totality to which they belong, produces a comprehensive picture of functional similarities within a subset of similarities with respect to a bigger set of similarities, or, using terms of Guerra Lisi and Stefani (2004, 2008), within a *globality of languages* that achieve coherence and consistency through a self-similar intersemiosis. In harmony with this ideas, Cruces (e2002) brings into musicology the figure of 'worlds within worlds', under the connotation 'worlds of sense', in self-similar cycles that—according to Cruces—are expressed in four levels: *grammar of the sound system, musical text, interactive situation*, and *practical, isomorphic, and synaesthetic schemata of sociocultural congruence*.[169]

3.8.3. Applying Peircean semiotics to the IC theory

The *intersemiotic continuum theory* (IC theory), introduced in section 3.8.1. (see especially chart ◊381), is inspired by two Charles S. Peirce papers (1893, 1903a, with some ideas from the latter work developed in Peirce 1903b and 1903d).[170]

Peircean semiotics is a widely researched metalinguistic model (for a general introduction to the subject see Eco 1976, Lidov 1999), that equally has been widely applied to musicology (see Monelle 1992, Tarasti 1994, Martinez 1997, Lidov 2005, Ojala 2009, among many others). Since Peirce's theories are explained elsewhere, this

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[167] This is why, assuming cultural variation, the concept of music is so difficult to be enclosed in a simple, unified definition.

[168] i.e. the ecologic concept of the 'house of the house' proposed in this study.

[169] In particular, the association of "practical, isomorphic, and synaesthetic schemes of sociocultural congruence" invokes the relationship established in the present study, between functional self-similarity and intersemiotic translation. In music these two concepts are intertwined. On *isomorphism* and *isometry* see subchapters 2.3. and 6.2. On the concept *sociocultural congruence* see subchapters 4.6.–4.8.

[170] Peirce 1893, 1903a–1903d partially correspond to Peirce's *Collected Papers* 5.66–81, 88–92, 5.180–212, 7.565–78, 8.122, and *MS* 51. See bibliography.

section is limited to summarizing what the Peircean concept of *firstness, secondness, thirdness* means, and how it is adapted to the present study.

Among the classes of signs that Peirce states, his phenomenological trichotomy (*firstness, secondness, thirdness*)—elaborated as a revision of Hegel's triadic categories of *being-nothing-becoming*, and *being-essence-concept*—is suitable for systematically embracing other Peircean semiotic trichotomies (such as *Qualisign, Sinsign* and *Legisign*; or *icon, index* and *symbol*; or *rheme, proposition* and *argument*). Given that this model (i.e. the phenomenological trichotomy) is accurately designed for the study of signs in general, it is adopted here as a general framework for the investigation of musical signs and musical-sign processes. This decision also involves the study of self-similarity in pre-musical and meta-musical objects and processes (e.g. geometric objects or dynamical processes expressed as music, and vice versa).

As it can be inferred from Peirce's writings (*cit.*), *firstness* is typically characterized as self-reflection, perceptual immediacy, first contact or feeling; *secondness* as reaction or relation; and *thirdness* as mediation, representation or conceptualization, following from the previous relationships. These phenomenological categories also have a deictic association, as vagueness or 'some' (firstness), specificity or 'this' (secondness), and generality or 'all' (thirdness). These notions are closely related to basic concepts explained in Chapter 2; but more importantly, they are used to investigate the generalized relationship between source and analogy (usually forming a synecdoche), in most of the pre-musical cases studied in Chapter 4, and the meta-musical cases studied in chapters 5 and 6.

The main advantage of adopting this method for investigating analogy and synecdoche in music, is that it allows us to explain them in terms familiar to music theory, identifying specific, symbolic roles in relationships such as *theme* to *variation*, or *subject* to *countersubject*, or *antecedent* to *consequent*.

Peirce's phenomenological trichotomy is not limited to establishing causal relationships, but remarkably contributes to understanding the association between *feelings, reactions* and *representations*, even in the more abstract models of music, linguistics and mathematics. A very clear example of this is given here in the section

Musicological interpretation of the Cantor function, in subchapter 6.2.,[171] exploring the form in which a 'simple' ontology—an abstract firstness, in this case a straight line segment—is synecdochically associated with itself, generating a secondness (i.e. a geometrical first-order analogy), and then forming a second-order analogy, till elaborating successive segmentation or thirdness (a process suggested by chart ◊381). Moreover, as subchapter 6.1. explains it, this logic is essential for proportion in general. The same schemata can serve to investigate many other series and geometric objects associated, by synecdoche and analogy, to music (see e.g. Fripertinger 1999). This is why, for instance, charts ◊623 and ◊624 show analogous images of transitions *harmony → similarity → noise* (equivalent to *proportion → self-similarity → chaos*, arrow denoting tendency), suggesting a parallel relationship with firstness, secondness, and thirdness, in the Peircean sense of *abduction*.[172]

Peirce establishes an ontological dependence between categories, based on the epistemic-empirical association between *non-relative quality* and *first analogy of the quality*. Therefore, he establishes a consequential context for 'first-categories' or 'first-ontologies' from which other subcategories can be 'abducted' (see ◊381). Peirce (1903d:270) himself explains this as follows:

> It is possible to prescind Firstness from Secondness. We can suppose a being whose whole life consists in one unvarying feeling of redness. But it is impossible to prescind Secondness from Firstness. For to suppose two things is to suppose two units; and therein has Firstness, even if it has nothing recognizable as a quality. Everything must have some non-relative element; and this is its Firstness. So likewise it is possible to prescind Secondness from Thirdness. But Thirdness without Secondness would be absurd.

This model is actually useful for any string of semiotic categories in which firstness, secondness and thirdness are relative or non-relative states of semiosis, rather than 'pure' ontologies or absolute modalities of understanding. Thus, ennumeration (1, 2, 3...) is trivial; what really matters here is the synecdochic action of *a* on *b*, of *b* on *c*, and so on, elaborating a semiotic continuum through (partial) transitivity. This can serve to study how strings of signs—even infinite strings based on the same principle

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[171] See pages 360–362. It may require, however, reading previous pages on the Cantor function (pages 354–360).
[172] For a definition of *abduction*, see pages 23 and 459.

of extension into subsequent analogies and synecdoches—can have epistemological continuation: for example, how numerical series or rhythmic patterns in music can proceed from 'something', to a specific pattern, and then to conjecturable sub-patterns.[173] This is especially valid in the context of subchapters 5.1. to 5.4., whose topics depend on the recursive self-reference of an initial state (i.e. firstness); as well as in the context of subchapters 6.3. to 6.6., on the recursive relationship of an initial self-reference (*idem*).

Connecting Peirce's *synechism* with Mandelbrot's *fractality* is one of the main objectives of the present study.[174] Using a precise language, Peirce formulates three concepts familiar to self-similarity and fractal theorizing. Namely: (1) the *self-sufficiency*, (2) the *economy of the Universe*, and (3) the *map of the map* (see Peirce 1893, 1903a). Respectively, these notions are directly connected to the modern physico-mathematical concepts of (1) *self-reference*, (2) *power laws*, and (3) *self-similarity*, closely related among them, and all of them explained along this Chapter (see 3.3., 3.6. and 3.9.).

Finally, the cyclical model for the intersemiotic continuum (IC)—synthesized in graphic ◊381—suggests that Peirce's trichotomy is also compatible with an infinite cyclical model of recursiveness, powering semiotic variety based on self-reference and recursion, as explained in subchapters 3.6. and 3.7.

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[173] Pareyon (2010b:35–36) suggests that this principle actually governs all self-referential sequences: "This is not to say—at all—that Peirce was looking for representing Fibonacci series; neither this would be relevant for this case. The true relevance in this elaboration is that one can find here the same intuition for order and hierarchy through self-reference, starting from very few elements and very simple conditions: what Peirce identifies as *firstness, secondness,* and *thirdness,* for a mental process. The mind of Peirce works here like the mind of a composer or a mathematician, in search for the most simple—but still operational—model for the synthesis, its extension, and the methodification of self-referentialities."

[174] Synechism (see Peirce 1893) is the Peircean doctrine of synecdoche, as the source for an existential philosophy. For a definition of synecdoche, see subchapter 3.2.

3.9. Philosophical implications

3.9.1. Perturbation of the creative centrism

Among the consequences of exchanging the author and the spectator, in music, in a comparable way to literature, perplexity arises from self-similarity when confusing source with interpreter, or referential origin with destination, as J. L. Borges suggests (*Other Inquisitions*, 1952:68–69):

> Why does it make us uneasy to know that the map is within the map and the thousand and one nights are within the book of *A Thousand and One Nights?* Why does it disquiet us to know that Don Quixote is a reader of the *Quixote*, and Hamlet is a spectator of *Hamlet*? I believe I have found the answer: those inversions suggest that if the characters in a story can be readers or spectators, then we, their readers or spectators, can be fictitious.

The same idea is fostered by Lotman (1988/1994:383) when he meditates on the problem of the 'story within the story', in a text within another:[175] "Specularity exists between the two texts, but whatever appears to be a real object turns out to be only the deformed reflection of something that was itself a reflection." Lotman conceives, however, "more complicated" cases in the intertwining of self-references, where the reflection is completely confused with the 'real object', "where the text and its frame are interwoven, so that both frame and text are framed." (*ibid.*).

Without entering into conceptual labyrinths of self-reference as Lotman does (*cit.*), Roland Barthes (*La mort de l'auteur*, 1968) intersperses views on the subject with meditations on the author vanishing through their own creative references. The same issue concerns Michel Foucault (*Qu'est-ce qu'un auteur?*, 1969) with certain uneasiness: "Who speaks truly, when we conceive the voice of an author?"[176]

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[175] An example of this is the structure of the first volume of *Don Quixote* (1616), in which a narrative structure encompasses others.

[176] In "The Backward State of Metaphysics" (1898), Peirce wonders if there is a distinction between the world's 'external' and 'internal'. Later, in "The Categories Defended" (1903) he postulates a self-similar relationship between the categories of thought (*quality, reaction, representation*), in turn with their own branchings of subsequent categories (see Pareyon 2010b). In this postulate, Peirce does not use the term 'self-similarity', but formulates the equivalent concept of 'map of the map' or 'map of itself': "the map of itself with the map of the map within its boundary. Thus there will be within the map a map of the map, and within that a map

Analogously, for music makes evident a 'perturbation' of the creative centrism, assuming that—from the perspective of synecdochic intersemiosis—*music* has its centre everywhere and its limits nowhere.[177] This occurs, indeed, when assuming that the physical characteristics of certain timbral spectrum or fractional noise are 'nestings' of structural characteristics of music, and of language in its most fundamental aspects.[178] Following this plot of self-reflections, the notions of *author* or *composer* are also confused with those of *work, interpreter*, and *spectator*, because whether the cycles of a musical style are part of a larger complex of trends, equally cyclical, and the creative periods of a musician are reflected in periods of their own music made of comparable cycles, the overall picture constitutes a correlated whole from which neither an ultimate representation can be extracted, nor an initial cause.[179]

Necessarily, the investigation of these relationships must employ criteria to describe holistic patterns instead of producing descriptions that, whereas looking for extreme accuracy, lose sight of the most meaningful general relationships. This implementation is inspired by the theory of dynamical systems initially developed by Henri Poincaré (1854–1912): "To Poincaré, a global understanding of the gross behaviour of all solutions of the system was more important than the local behaviour of particular, analytically-precise solutions" (Devaney 1987:viii; see also Poincaré 1886, 1907).[180]

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of the map of the map and so on *ad infinitum*." (Peirce 1903a/1998:161–162). This issue is discussed further in the next section.

[177] This figure points out simultaneously the *music of the spheres* and the *infinite sphere* in the history of Western thought, according to the postulates of Alain de Lille, Nicholas of Cusa, Giordano Bruno, and Blaise Pascal, on a sphere with "its centre everywhere and its limits nowhere". The notions of harmony and harmonic expansion accompanying these ideas and their relationships with music have roots in Plato (*Timaeus, Republic*) and, above all, in Aristotle (*De Caelo* B9, 290b12).

[178] On the concept of 'nesting', see subchapter 5.4.

[179] i.e. the 'universal cause' (ἄιτον) in Aristotelian teleology.

[180] This idea is absorbed in musicology by the thought of Jan P. LaRue (1969:450–451): "once we comprehend the wholeness, the parts fall into a proper perspective. The opposite process yields less insight, for a study of the parts does not usually help us to sense the whole; in fact, it tends to fragment any broader view, obscuring it with a multiplicity of detail. Hence, it becomes essential to begin with large overviews." This notion enriches the discussion on musical style in subchapter 4.8.

Assuming these considerations, this study focuses more on the investigation of global processes, than on the specific work of a composer or a musical style. The peril of attempting to cover a too broad field is avoided by limiting this investigation to basic concepts such as *analogy, intersemiosis, intersemiotic translation, map, musical ecolect, musical idiolect, proportion* and *antiproportion, recursion, self-similarity* and *self-dissimilarity, style,* and *translatability*, which give operational consistency to this research.

3.9.2. A self-comprehending map

The notion that something can contain itself, repeating its own form in infinite scales, has left evident traces in modern thought. Philosophy, especially in its pragmatist trends—permeating poetry, fiction literature, and the graphic arts—communicates this notion through the varied output of authors like Charles S. Peirce, Thomas de Quincey, Octavio Paz, René Magritte, and M.C. Escher. The drawings of the latter, especially his tessellations, anticipate aesthetic aspects of fractals. Paz' poetry reaches an analogous effect by the concept that an author can be reading and writing of himself: "I too am writing and at this very moment someone spells me out" (*Brotherhood* in *Collected Poems*, 1987). The case of De Quincey is particularly remarkable by his astonishing lucidity revealing the nature of sound as a self-similar complex (De Quincey, *Autobiographic sketches*, 1790–1803; 1862:122):

> Even the articulate or brutal sounds of the globe must be all so many languages and ciphers that all have their corresponding keys—have their own grammar and syntax; and thus the least things in the universe must be secret mirrors to the greatest.

The perplexity induced by this idea adopts the image of a 'map of a map' in Peirce's thought (Peirce *CP*, 8.122):

> Imagine that upon the soil of England, there lies somewhere a perfect map of England, showing every detail, however small. Upon this map, then, will be shown that very ground where the map lies, with the map itself in all its minutest details. There will be a part fully representing its whole, just as the idea is supposed to represent the entire life. On that map will be shown the map itself, and the map of the map will again show a map of itself, and so on endlessly. But each of these successive maps lies well inside the one which it immediately represents. Unless, therefore, there is a hole in the map within which no

point represents a point otherwise unrepresented, this series of maps must all converge to a single point which represents itself throughout all the maps of the series. In the case of the idea, that point would be the self-consciousness of the idea.

In the field of mathematics this idea later bears fruit in the work of Mandelbrot (1967, 1977, 1982) who coined the term fractal to express—using the same metaphor as Peirce—that the edges of Britain could not be represented by a simple set of straight line segments with an Euclidean dimension, but with a 'fractured' dimension.

The concept of 'idea' mentioned by Peirce also has relevance to this respect: beyond the notion of structural self-similarity in music, a musical *idea* can also be self-similar as it germinates within itself, being part of the idea of the person who imagines it and carries it out. From this viewpoint the exemplary work of composers such as Stockhausen, Xenakis, Ligeti or Nancarrow should be considered, who, shortly after 1950 began to suggest that the relationships characterizing a musical piece could be constituted by similar relationships, one inside another. The continuity of this scheme produces, with a variety of means, a musical repertoire with properties of approximate self-similarity, reinforcing the idea of potentially unlimited musical grammar operating by functional cycles within larger cycles—a latent conception in the analytical research of Bolognesi (1983) and Fagarazzi (1988). This evolving conceptualization, intertwining musical repertoire and analysis, also implies the "self-consciousness of the idea" proposed by Peirce.

3.9.3. Stochastic distribution

Several concepts of stochastic distribution, such as Markov chains and the fractional $1/f$ noises (where 1 refers to a spectral energy unit correlated with f, which means the frequency of a varying process),[181] are of paramount importance in this research, for a simple reason: in a dynamical system—be it a piece or fragment of a musical work—its components can be distributed under the same stochastic pattern at different levels. This means that whereas the relationship among these components is not intuitively self-similar in the form 'object inside a similar object', there may be a distributional self-similarity characterizing the system.

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[181] The definition of $1/f$ noise, including examples of its typical relationships and applications in music, is given in subchapter 5.3.

The term *stochastic*, from the ancient Greek στόχος (something tossed or thrown at random) indicates a process whose behaviour cannot be expressed as a simple cause–effect relationship, but as the intrinsic probabilities of the process. Shannon and Weaver (1949:10, 102–103) suggest that a stochastic process corresponds to the probabilities of an initial relationship (called 'source') to produce a sequence of symbols in a spatial or temporal development.[182]

> A system which produces a sequence of symbols (which may, of course, be letters or musical notes, say, rather than words) according to certain probabilities is called *stochastic process*, and the special case of a stochastic process in which the probabilities depend on the previous events, is called a *Markoff process* or a Markoff chain. (*op. cit.*:102).

Often, the initial state of a stochastic process is determined by known conditions, as happens with the first iterations of functions invoked to produce some Lindenmayer systems, fractals, or Markovian chains. These processes, however, can lead to undetermined changes in the short term with growing chaotic behaviour due to the initial high sensitivity of the system.

Observing the behaviour of $1/f$ noise, Voss and Clarke (1978), and later Hsü and Hsü (1991), found that the changes in a sample frequency of tonal repertoire (J.S. Bach and W.A. Mozart are among their examples) are statistically self-similar and reflect 'scalar independence',[183] a property related to the so-called *fractal dimension*.[184] Jones (1981, 2000) also explores the relationship between Markov chains, the Fibonacci sequence, and Lindenmayer systems, highlighting 'common symmetries' between poetic metre and musical rhythm: "They possess self-similar qualities which are related to fractal models [...] and can provide a keen insight into some quite profound inter-relationships between the arts and sciences" (Jones 2000:1). In relation to statistics and information theory, the Chapter 3 provides more detail on these aspects, which are discussed further in musical examples in Chapters 4 and 5.

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[182] This is schematically represented by ◊510.

[183] This 'scalar independence' means that the basic relations in a system reappear regardless of the scale with which the system is observed. Consistently, Mandelbrot and Van Ness (1968:423), state that "self-similarity [is] a form of invariance with respect to changes in time scales."

[184] On this subject, see the separate section at the end of subchapter 5.5. (pages 300–303).

3.9.4. On the concept of chaos

Feinberg (1985:264) defines chaos as "a type of behavior of physical systems in which the evolution of the systems cannot be predicted because of its sensitive dependence on minor changes in the properties of the system". This implies that—like chaotic systems found in *nature* rather than one emulated by computers—even if the system's general conditions may be predicted for an initial period, the diversity of factors influencing in the system makes it impossible to predict in the mid to long term.

Quite commonly, the word 'chaos' is used to describe a lack of order, regularity, or symmetrical agreement. This employment, however, is vague and inadequate for a careful study of changing phenomena, such as music, as 'lawlessness' may be due to an effect of immediate intuition, but not a deep state of things. A more precise use of the word chaos is thus associated with deterministic evolution, predictable at least as an initial process, that can result in chaotic dynamics. For instance, as part of a process of musical creativity, Kagel (*vid*. Padilla 1984:121) highlights the relevance of 'planning chaos'. This notion of chaos is opposed to that of 'randomness' in which it is impossible to establish the predictability bases for any state of the observed phenomena. It is important, therefore, to bear in mind the close relationship of chaos with determinism and predictability, by contrast to the relationship between randomness and indeterminism.[185]

For a specialized musician, albeit with modest proficiency in mathematics, it is easy to perceive the difference between deterministic chaos and indeterministic randomness in the following terms: in an orchestral tutti written in a full score with all details and possible explanations for maximum control on the predictable effect, the musicians play at one time, with full dynamics ($fffff$) in the most powerful register of the instruments and with accurate rhythmic figures in fast tempo under control. The ultimate effect is similar to a complex deterministic chaos. Conversely, assuming that another example of orchestral tutti is suggested in a score with vague indications of intensity and speed, in a variation of pitches and lengths without further explanations, the result will be similar to random chaos—perhaps with some

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[185] This is the central topic in Eco's essay (1962) on openness and indeterminism in the aesthetic processes.

musicians playing sounds with the greatest discrepancy, and some even playing small fragments of the last work studied. This resulting disparity is not trivial in musicology because both sorts of processes are present, to some extent, in all forms of music, on the grounds that in music it is always possible to identify a notion of control (*determinism*), by opposition to a lack of control (*indeterminism*), for any of the musical parameters.[186]

Consistently, the notion of chaos is associated with musical processes that are stable at first, becoming unstable as they move apart from the initial relationships of their original sources. The emulation of such phenomena is equally of great interest in the study of acoustic self-similarity, for example in the forming turbulences whilst blowing on a wind instrument embouchure (see Bader, 2005); than for compositional synthesis (see Roads, 2004:886–889), and stylistic analysis (Bigerelle and Iost 2000, Su and Wu 2006, Dagdug *et al*. 2007). In this context the concept of chaos is closely involved with the generation of stochastic processes, described above, as well as with the predictability in correlation of global and local music systems

Regarding compositional synthesis, the generation of chaos is common from the iteration of a function that is clearly determined in its first applications, and that in the medium term results in subtle variations leading to chaos. Eventually, these results can be juxtaposed with random processes, as happens in some examples of Western musical repertoire from the second half of the twentieth century (see Bennett 1995, Harley 1995, Little 1993, Steinitz 1996a–b, Maurer e1999, Lochhead 2001, Salter 2009).

In principle, due to the recursion of a generative process and to the self-reference of its corresponding system, self-similarity is related to a variety of chaotic processes. Roads (*op. cit.*:887) points out a kind of order 'nesting' in chaos; a continuous

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[186] Ironically for a conventional musical approach, chaos may be associated with the notion of noise as a system of frequencies in which its inherent relationships are very unstable and even highly non-correlated, as in the case of $1/f^0$ noise or *white noise*, which, however, can be produced by a deterministic algorithm. Here, the term 'deterministic algorithm' implies a controlled set of instructions for the generation of noise. On this subject see subchapters 5.2.–5.3. and 5.5.

bifurcation of ordered behaviour that, as with massive process, does not intuitively correspond to an image of order or harmony, but to one of chaos or noise.[187]

3.9.5. Power laws

With a reasoning that illuminates the generalized relations of self-similarity and its significance for music, Carlos Chávez (1961:38) conceives that there are universal patterns of symmetry and repetition independent of the scale with which they are observed: "We humans are part of the universe, ruled by the same over-all laws governing light spectra, acoustical resonance, principles of life, capillarity, osmosis, cyclical phenomena, and the like. There is a primeval kinship between them and us".[188] This is particularly verifiable in the case of power laws.

When the probability of measuring a specific value of some quantity varies inversely as the power of that value, the quantity is said to follow a *power law* (Newman 2005:323). Many examples of this are observed in the proportional relationships between two physical frequencies: a random process acts under a power law if a frequency of the origin (of the process) is increased by a higher proportion compared to the rest of the frequencies (Schroeder 1991:4–16). In a very rough sense, frequencies soon decay insofar as a frequency is established with a rate considerably higher than the rest of its group. This applies to physical phenomena, including all systems that operate under the same law, for example, in language: according to the Zipf's law described in subchapter 5.1., words that are about twice longer than 'big words', are four times more rare than those in the verbal repertoire. Assuming that this repertoire is a random, endless and *countless* set of phonetic arrangements from a finite (i.e. alphabetical) series, the total of its arrangements is comparable to a Cantor set (see schematic representation in ◊332). Therefore, the *language* in question is necessarily self-similar, self-structured—by hierarchies of words, and has a fractal dimension. (See Schroeder *op. cit.*: 37).

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[187] On the concept of 'nesting', see subchapter 5.4.

[188] Chávez (*loc. cit.*) makes this claim explicitly in the context of musical symmetry and repetition.

Power laws are observable in a wide variety of physical, chemical, biological and social phenomena. Burioni, Cassi and Vezzani (2004:36) consider that "All real physical structures (embedded in three-dimensional space) have been found up to now to exhibit power law behaviour in the low-frequency density of vibrational states". This principle also holds true in various aspects of psychology. For example, Wickelgren (1974) found a power law specific to human memory processes in the short and medium term, governing the regular rhythms of forgetfulness under statistically normal conditions.

Much of the current theoretical work linking physics and mathematics is beset with the investigation of power laws (see Newman 2005). They are a central issue in the study of the connection between structural self-similarity with generalized physical laws like the *Second law of thermodynamics*, which states that the entropy of any thermodynamically isolated system tends to increase over time (see Thomson 1851). This implies that, when a part of a closed system interacts with another part, energy tends to divide its power equally, until the system reaches thermal equilibrium. Consistently, for a power law, the frequency of an event in a system tends to a progressive subdivision, so that the 'secondary events' of the system are many and the 'critical events' are few.[189] This conceptualization serves to understanding an idea synthesized by Hsü (1993:24): "Musical notes have been compared to elementary particles, and their relationships seem to bear a parallelism to the structure of the universe and seem to follow the same laws of thermodynamics as crystals, gases or any other measurable substance." Such an idea stimulates the development of subchapters 4.1.–4.2. and 6.1.–6.4., in their turn connected with other sections in this study, dealing with concepts on culture, language and environment.

Schroeder (1991:34) conceives that the power laws are, by definition, of a self-similar character. They are valid at all scales or, rather, are independent of the scale at which they manifest themselves. In addition, Weisstein (e2008) states that all self-

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[189] Using the metaphor of the Great Flood for a unique or very rare event, Mandelbrot and Wallis (1968:909) call such a phenomenon *Noah effect*. Later physics, including acoustics, adopted the term *soliton* to describe a single event of unusually large magnitude with respect to the periodic behaviour of a system (Aubry, 1979). For an association of this concept to fractional noise, see subchapter 5.3.

similar objects "are defined by a power law". Accordingly, power laws involve fundamental aspects of music as sound propagation and resonance, intensity, timbre, harmonic hierarchy, metrical distribution, figural recursion, agogic and prosody, as well as the functional orientation and probabilistic trend in a prescriptive musical grammar, as suggested in Chapter 5.

3.9.6. Intersemiosis and power laws

One of the aspects of the deterministic intersemiotic translation (henceforth DIT) is the intentionality of *exact* analogy between different phenomena of information, subject to a specific power law. The maintenance of quantitative relations, for example, in the case of alphabetic transliteration into codal units with dots and dashes in Morse code, is insufficient to preserve the *fundamental message* in translation, because in this case, the code is just a vehicle for transliteration, and not translation itself, with context and meaning (see Eco 2003:9–11). DIT requires transmission of a compatible system of analogies.[190] Intuitively, Walter Piston (1947:13) draws attention to such a compatibility: "We permit ourselves the use of the term 'melodic curve' to describe this outline, although an accurate graphic representation of a melody would show a series of angular movements". This intuition is also valid for the musical DIT of a fractal curve, such as the Hilbert and Peano curves (described in subchapter 6.3.), or the devil's staircase (described in subchapter 6.2.).

A DIT involving abstract relations, for instance, between music and geometry, or between painting and music, cannot be supported by a lexical reference or any equivalent, indeterministic means. This is an essential difference with intralinguistic or interlinguistic forms of verbal translation (as theorized by Jakobson 1959:232–233). On the other hand, a DIT may use physical references perceived as space, time, texture, intensity, or colour, reflecting power laws, which determine basic relations such as uniqueness/repetition, symmetry/asymmetry, information/entropy, and self-

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[190] In the given example, using Morse code, dots and dashes are not in analogy with the transmitted message, unless the message is self-referent i.e. a Morse code message whose semantic content is "dash dot dot / dash dash dash / dash" = DOT, "dash dot dot / dot dash / dot dot dot / dot dot dot dot" = DASH, and so on, following sequences of the same kind.

similarity/self-dissimilarity, that are crucial to preserving and transmitting a message between different semiotic systems in a deterministic environment.

Whereas the *message* in intralinguistic or interlinguistic translation may consist of a lexical meaning, with a pragmatic or figurative use, the *fundamental message* in the DIT between two systems of analogous self-similarity, may consist of very elementary relations between message and code. For example, whilst in any linguistic translation (a)symmetry is eventually dispensable as a basic reference of meaning —captured by lexicon or intentionality—in a DIT translation any non-lexical reference is extremely significant. Hence, it may be that the *fundamental message* within a DIT, is the common symmetry between two elements or two systems, or the form in which points are scattered within a swarm. A DIT converts swarms of symbols into 'other' swarms of symbols, and so, into a 'new' symbol embedded in a system of invariance, provided that, as Meystel (1998:348) acknowledges, "swarms of symbols are also symbols". Whether this condition is disregarded, then expectations in translation may be too high, for example, if the *fundamental message* is intended to be transmitted entirely by a rather limited intersemiotic mapping:

> The problem with much fractal music is that the self-similar deep structure of the sequence often does not translate readily to a musically interesting deep structure. The structures that are so obvious when looking at a plot of a fractal sequence often go unnoticed when the sequence is heard. (Biles e1998).

However, a point-by-point rendering in an itersemiotic mapping, cannot guarantee the transfer of a symbolic reference in its entirety. In any case, there must be considered which are the relevant symmetries and repetitions to be transferred, as well as their justification under specific power laws. This is clear in the case of Peter Gena (e2006), when associating similar systems of probabilities—including symbolic complexes as diverse as the *I Ching*, the genetic code, and music—and implementing some generalized structural similarities to justify the intersemiotic mapping:

> By coincidence, DNA and the *I Ching* are structured in remarkably similar ways. The *I Ching* contains 64 possible hexagrams, and genetic code is likewise made up of 64 codons. 64 is a power of 2 (2^6), and both systems make use of it because powers of 2 can be readily represented with binary units. Because of the ways in which they are represented, each hexagram [in the *I Ching*] and each codon [in DNA] contains virtually identical information content, with a hexagram derived from a space containing $2^3 \times 2^3$

possibilities, and a codon derived from a space containing 4^3 possibilities. [...] This similarity has nothing to do with the function of either system. The point of comparing the two is to illustrate the aptness of comparing the approaches taken in DNA mapping with Cage's systematic composition with chance [i.e. with the *I Ching*]. (Jensen 2008:252).

In Gena (e1999 e2006) and in the revision of his work by Jensen (2008), priorities, constraints and scope of the intersemiotic mapping become clear. Quite obviously, this mapping is not limited to the symbolic and structural correspondences between DNA and *I Ching*: it can be expanded to a general theory of intersemiotic translation, based on the self-similarity/self-dissimilarity relations determined by power laws.

Intersemiotic variety in musical self-similarity

This chapter shows that the relationships of self-similarity in the musical language appear in 'reciprocally related worlds' and—paraphrasing Parsegian (1968:589)—in 'worlds within worlds', an intuition already explicit in the philosophical doctrine of Emanuel Swedenborg (1688–1772).[191] Different boundaries of correlation are found, thus, for the self-similarity in a sound grain regarding a broader physical system, in simple vibratory systems within a more general mechanics, in the tissues within organic complexities, in the phonic within the biological, and in the phonological within the cultural, forming a vast array of synecdochic intersemiosis.

The patterns of self-similarity in structures within other structures reveal the redundancy that enables a consistent long-term self-organization within the emerging systems (see Lewes 1875; Moles 1958, 1963; Maturana and Varela 1973; Schelling 1978; Maturana 1980; Reygadas and Shanker 2007). The collective behaviour of the parts in such *emerging complexity* results from a correspondence with the whole. In contrast to a set of unrelated or dissociated elements, a self-similar, self-organized set evolves as a coordinated ensemble, oriented towards unity within diversity and to diversity within a range of similarities.

In communication theory (Shannon 1937, 1948), redundancy is essential for the consistency of a message. Moreover, the message's code implies some form of self-reference, in order to inscribe the message in a language context. Actually, redundancy and repetition are forms of self-reference. A language eluding such functional similarities in its repertoire and rules is inconceivable, without sufficient redundancy to provide its own structuration (see Moles 1963, Campbell 1982, Tiits 2002).

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[191] See Swedenborg's *Principia*, Stockholm, 1734. For an introduction to the intuition of self-similarity in Swedenborg see Baryshev and Teerikorpi (2002:54–65).

Although without using the expression 'self-similarity', Tzvetan Todorov (1968/2004:68) describes language redundancy in terms of elaborations within elaborations changing their surface appearance, but remaining invariant through different scales: he distinguishes between a *monovalent* discourse "which does not evoke previous 'ways of speaking'", and a *polyvalent* discourse, evoking various ways of speaking in a more or less explicit form. The relationship between monovalent and polyvalent discourse somehow resembles the link between Peirce's *firstness* and *secondness*, explained in subchapter 3.8. According to this viewpoint, redundancy seems evolving from singular to plural signification: from identity to similarity and/or difference.

Todorov (1968/2004:47) conceives literature as a sub-system within language; as a world within a world sharing functional similarities. He talks about a "figural degree" of discourse, conceiving that tropes operate as relationships *in absentia*, admitting a gestaltic theory of the figure: "[figure's] definition should not be sought on the relationship of figure to something other than itself, but in its own existence; a figure is that which can be described as a figure" (*op. cit.*:64–65).[192] This sort of tautological self-reference is also consistent with Hegel-Peirce's *firstness*, and with Peirce's phenomenological trichotomy. Congruently, Todorov (*ibid.*) appoints a specific figural trichotomy: *repetition*, *antithesis*, and *grading*, which can coexist in a self-similar discursive pattern, more or less as different categories can coexist in the same (inter)semiotic map (see chart ◊381 on page 106). Such figurations and categories are found, by analogy, in musical structures.

Many music theories foreground the issue of similarity between elements, sets, and systems, to approach the notions of coherence and functionality in music. Again, this scheme falls under the trichotomy suggested by Peirce (1903d), and refined by Todorov (1968) as linguistic and symbolic constructivism. This perspective includes music analytic theories based on the relationship between *singular identity* (or identity of a same element within its functional context), *form* (i.e. the structural organization between two or more identities, according to a symbolic convention), and *transformation* (both, of simple singularities and complex forms). For example, in

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[192] This coincides with the definition of *self-reference*, given in subchapter 3.6.

his study *On the Similarity of Twelve-tone Rows*—based on the concepts of vectorial and transformational similarity developed by Forte, Morris, and Lewin—Tuukka Ilomäki (2008) notes that formal similarity measures can be used to explain ways of 'being similar', whilst transformational similarity measures can be employed to explore the symmetries produced by sets of transformations in themselves. In addition, Ockelford (2005) proposes that it is possible to explain the generative relations of music as a basic process of symmetry and similarity in which minor relationships determine greater relationships.

The musical relationships studied by Lewin (1987), Ockelford (2005), and Ilomäki (2008), among other specialists interested in the functional similarities of music, belong only to a selective area of these categories. At the same time, it seems obvious that in the immensity of types of similarity, the relationships of self-similarity constitute a special group of structures which need a special conceptualization.

In the following pages a general classification of musical self-similarity by layers is proposed, in order to facilitate thematic research on its wider possibilities. Since the criteria for ordering these layers lie on a rudimentary notion going from the simple to the complex, and from the smaller to the bigger, first a subchapter presents the concept of a sonic 'elementary particle', with the aim to begin with a revision of the 'simplest' kind of musical self-similarity. Nevertheless, many of the notions assuming a *sonic quantum* are far from obvious; rather they require an explanatory effort before a discussion can be opened on self-similarity in fundamental acoustics, in a second subchapter.

4.1. In search of sound's elementary particle

Dahlhaus (1990:3–11) notes that where rationalist research seeks to explore musical foundations, two types of orientation usually appear: those that attribute music to culture, as an effect of history, and those that attribute music to an underlying physical order, with universal bases valid in all cases and for all relations.

The attribution of music to culture as an effect of history (e.g. Fétis 1832) often falls short in that it is unable to explain how the biological phenomena are related to

their mechanical and acoustic features. In contrast, the universalist or *physicalist* attribution (e.g. Riemann 1873) exceeds its possibilities to demonstrate a supposed 'musical order' for all things—the foundation of Platonic idealism. It is parallel with the Pythagorean dilemma (*qua* fractal) discussed in the Introduction, which recalls the apparent opposition between determinism and indeterminism; between the desire for absolute control on the one hand, and a need for flexibility to facilitate the control of what happens, on the other. In this context it is useful to review some of the key ideas in music theory, dealing with a 'fundamental particle' of sound, and attempt an argumentation to consider that the concept of *elementary particle* is not necessarily in opposition to the concepts of biology, or to phenomena pertaining to history and culture.

The 'atomic' notion of sound

The investigation of the fundamentals of music, which from the rationalistic view must be based on universal physical laws, culminates with the concept of the 'sonic atom'—i.e. the maximal reduction of sound as mechanical particle or wave (see Backus 1977:33–39). Since the beginning of its formulation, this concept was strongly influenced by physical determinism. Attracted by quantum mechanics' prestige in modern science, many composers and theorists ventured, in the twentieth century, their own ideas on how an atom should be at the basis of the fundamental relations of sound. Hindemith devotes the early chapters of his treatise on composition to explain what he calls "the nature of the atomic structure of music" (Hindemith 1937/1941:54). Stockhausen (1989:33, 37) explains the influence of this idea in his own imagination:

> When I started to compose, after the War, there were many different directions in musical research which had been prepared by the great masters [...] It fell to me to synthesize all these different trends for the second half of the century, perhaps in a similar way that Heisenberg, in the first half of the century, had the role of bringing together the discoveries of Planck and Einstein in atomic physics. [...]
> We could speak of the strong influence on musicians during the early fifties, of certain books for the general reader by Einstein, or Heisenberg, of biologists like [Carl Friedrich

von] Weizsäcker, or Norbert Wiener. There was similar thinking everywhere: reduction of the process of forming to the smallest possible element.

What is disconcerting from this expression is its lack of emphasis on the fact that this approach between music and science is metaphorical; it constitutes a figure marked by poetic intension. Stockhausen and some other composers of his generation and their disciples, using similar arguments, seem to have benefited from this ambiguous usage, and by the widespread prestige of the quantum postulates and the new scientific language.[193]

Rasband (1990) and Schroeder (1991) show points of convergence and divergence between a theory of elementary particles of sound and the standard quantum theory. They still have presenting remains of metaphorical description, but both authors clearly move towards a strict analogy of the tiniest particles of matter and respect the tiniest particles of acoustic energy.

From a perspective oriented to music theory, both Rocha Iturbide (1999) and Roads (2004) acknowledge the seminal work of Dennis Gabor (1947) as the foundations of the modern method of investigating the 'basic particles' of sound. Gabor argues—with a special focus on human ear sound perception—that any sound, including continuous tones, can be conceived as a succession of *phonons* or elementary particles of acoustic energy, either specifically from the molecular vibration, or from the excitement of elementary particles, transmitted to a system of molecules. This treatment does not differ from the description of an elementary particle by Stockhausen (1989), but draws on the meaningful quality of analogy: Gabor does not want to portray an exact parallel between acoustics and quantum mechanics; instead, he looks for an elementary 'grain' of acoustic energy.

The *Chladni plates* visualized as corpuscular patterns, and the *Fourier transform* implemented to analyze an acoustic signal as a set of frequential components, contribute to the notion that a fundamental sound system is composed of correlated 'grains' or 'points'.[194] But passing directly from this image to a parallel with quantum physics, requires extreme caution. In this context, Roads (2004:27–28) enters into a

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[193] For a systematic criticism of this trend in the social sciences and the humanities, see Alan Sokal, *Beyond the Hoax: Science, Philosophy and Culture*, Oxford University Press, Oxford, 2008.

[194] On the Chladni figures and the Fourier transform, see subchapter 4.2.

philosophical disquisition when asking if the 'sonic atom' does or does not exist, using a paradigmatic analogy with arithmetic oneness:

> Consider the whole number 5. This quantity may be seen as a sum of subquantities, for example $1 + 1 + 1 + 1 + 1$, or $2 + 3$, or $4 + 1$, and so on. If we take away one of the subquantities, the sum no longer is 5. Similarly, a continuous tone may be considered as a sum of subquantities—as a sequence of overlapping grains. [...] This argument can be extended to explain the decomposition of a sound into any one of an infinite collection of orthogonal functions, such as wavelets with different basis functions.

In a simple example of sums, Roads—based on the Gabor matrix—envisages a matrix of sound production. In this matrix, resembling a grid in which the basic spaces are 'basic sound grains', the elementary particles of computer sound synthesis have their place. This explanation is sufficient for Roads because the space he conceives for testing his hypothesis is the space for sound representation in the computer, which is characterized by a fundamental segmentation of information in a numerical (binary) sequence. Wherever a *numerical minimum* (i.e. a basic operational symbol) does exist, then there is also an elementary minimum for sound processed by a computer. From the cognitive view, this concept matches with the *gatom* (Gestalt atom) proposed by Cope (1987:36) for music theory.[195]

The ultimate answer to the question of whether the existence of the 'sonic atom' can be objectively analogous to the existence of the standard atom, lies in the descriptive nature of both concepts: whereas the standard atom is a basic model of the relation matter/energy, the sonic atom is a model for the acoustic energy at the level of molecular relations (i.e. intramolecular vibrations). As the study of molecular motion and friction, and the measurement and interpretation of pressure changes in fluids are linked to classical mechanics, it is clear that such a study involves many aspects of empiricism from classical physics (e.g. from Archimedes to Bernoulli), unlike quantum mechanics, whose theoretical subtleties are far too empirical. The 'sonic atom' has a more simple characterization: a wave given as the 'fundamental' frequency between two molecules, or between two sets of molecules statistically defined, that corresponds to the 'basic grain' of its sonic energy. There are, however, some deep implications to consider.

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[195] On Gestalt theory, see subchapter 3.5.

Sound results from the vibration of molecules in a space: if such a space is 'void' (that is to say, without molecules neighbouring massively), then sound cannot be transmitted (Backus 1977:32). However the atoms and the elementary particles also vibrate and oscillate in characteristic ways, even if their vibrations are completely inaudible. On this premise, it is possible to induce the vibration of tiny particles that, massively, can emit frequencies translatable into acoustic vibrations. Indeed, as Roads (2004:34) points out, "optical energy sources can induce or interfere with mechanical vibrations". An example of this is the induction of acoustic vibrations on the surface of a crystal by two lasers with a minimum gap in their wavelength, producing beats measurable in gigahertz (*ibid.*). In a few words, *everything* sounds or resonates, either by physical induction or by interference, which can eventually be interpreted as intersemiosis. Consistently, the sonic atom can operate as a rigorous (i.e. proportional) analogy of the physical quantum, in the context of an equally rigorous intersemiotic translation.

For music, the fact that 'everything resonates' reveals a deep bond between sound as sensation and the physical world, as it is understood and formalized for study. This idea is related—by the human capacities of understanding and representing—to the notions of finite and infinite. For instance, since Fourier analysis has no operative bounds for infra and supersonic frequencies, such analysis may extend to actual and potential infinity. Rocha Iturbide (1999) and Roads (2004) explain, with numerous examples, the techniques and possible results that can be achieved in music from this conceptualization, connecting the analogy of 'sound particle' with more general physical principles. This correlation has a cultural justification in Western traditions at least since the Middle Ages and the Renaissance, stretching to modern times, as summarized below.

The need for a transcendent analogy

The Neo-Pythagorean impetus relating music to science, and specifically to a meaningful concept of the 'fundamental particle', became enriched over the course of the twentieth century, in particular, as noted by Stockhausen (1989:37), under the influence of the *General theory of relativity*. Organicist prospects resulting from this model of thinking stimulated the interpretation of biological processes as musical

codes, directly extracting information under symbolic attribution—for example a genetic code—to adapt semantic-syntactic functions. As a matter of fact, the idea of 'organized sound' posed by Edgar Varèse (1883–1965) and materialized in his *Poème électronique* (1958), prepares the ground for the conceptualization of a self-organized music, inspired by sound synthesis in analogy with biochemical synthesis (see Tedman 1982).[196] More recently, the amount of initiatives employing this analogy increased with various forms of intersemiotic translation. Some representatives of this trend include Mark Pearson (see Pearson 1996), who uses 'cellular' models applied to a method of electronic composition; Peter Gena (e1999, e2006), who decodes the human genome and constructive codes of bacteria and viruses to create sound sequences; and Susan Alexjander (e2007), who takes the atomic frequencies of the four DNA nitrogenous bases and the six basic elements of biological structures (see table below), and translates them into musical pitches and gamuts.[197]

element	Larmor frequencies	equivalence	approximate tone
Hydrogen	42.5776×10^6 hz	42,577,600 hz	E
Phosphor	17.236×10^6 hz	17,236,000 hz	C–C#
Carbon	10.705×10^6 hz	10,705,000 hz	E
Oxygen	5.772×10^6 hz	5,772,000 hz	F–F#
Sulfur	3.266×10^6 hz	3,266,000 hz	G–G#
Nitrogen	3.076×10^6 hz	3,076,000 hz	F–G#

◊410. Simplified scheme with the six elements composing 99% of living organisms on Earth, including the magnetic resonance frequencies of their atomic nucleus (called Larmor frequencies) and their equivalence in hertz (which, for practical purposes can be bounded into a module and a compact pitch set), and their corresponding approximation in musical tones. Source: Alexjander e2007.

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[196] The idea of music as *organized sound* is very widespread in modern musicology. For instance, Blacking (1973:10) asserts that "Music is a product of the behavior of human groups, whether formal or informal: it is humanly organized sound." In addition, Scruton (1999:16) says that "Poetry too is organized sound: sound organized thrice over, first by the rules of syntax and semantics, secondly by the aesthetic intention of the poet, and thirdly by the reader or listener, as he recuperates the images and thoughts and holds them in suspension."

[197] Alexjander makes a paraphrase of the arrangement of the nitrogenous bases (adenine, guanine, thymine and cytosine), from which she develops a systematic harmonic structure by analogy with certain 'fundamental proportions' (see Alexjander and Deamer 1999; Alexjander e2007). This idea has antecedents in Voss (1992, 1993) and Josephson (1995).

The search for the association between musical foundations of sound and physical and chemical 'basic' structures is related to an evolutive history of human analytical intuition. According to the configuration of the cognitive processes, for human causation and causality *all is frequency* in the image of its own biological makeup. It is conjecturable, then, that the human faculties give meaning to a world of recurrences (see Chávez 1961:38–41; Bateson 1972:147, 421), through segmented systems of harmonic relations, in successive scales. This trend stands out in Alexjander (e2007) when she asserts that "Hydrogen and helium are in the relationship of a perfect fifth/fourth, almost exactly. Their tones are: hydrogen E; helium B-C. [...] It's a mirror image of the overtone series." In this fashion, the intuition of proportion pervades the generalized conception of music as a system of analogies.
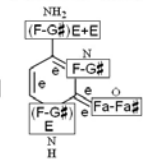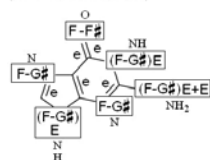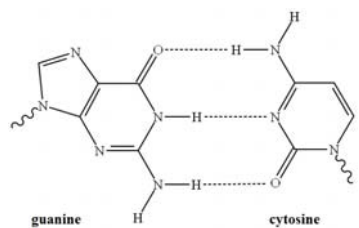
Alexjander   Tymoczko
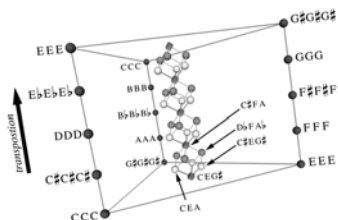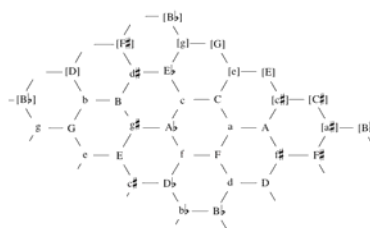
◊411. Schematic representation of two models of organicism, as interpretation of the analogy *biological structure – musical language*:

(a) *proportion* (Alexjander e2007), as chemical structure of two nitrogenous bases (top left) and their respective frequencies of resonance (below left); and

(b) *paradigm* (Tymoczko 2006), as organic parallelism in the geometric functions between tonal triads in the so-called 'tonal network' or Riemann *Tonnetz* (right top), and in families of major and minor chords as chained pitch classes (right below). Subchapter 6.4. interprets this network as a self-similar tessellation (see pages 406–409). Copyright by The American Association for the Advancement of Science. Reprinted with permission.

According to Kindermann (e1999), this kind of analogy is justified by the idea of a 'fundamental relationship' that links sound 'organization' with the self-organization of matter/energy. He puts as an example the self-organization of atoms in the configuration of crystals, attributing structural similarities to music: [198]

> We are all familiar with crystals, in which the individual atoms are arranged in periodic lattices. And we also know amorphous substances, such as ordinary glasses or most liquids, in which the atoms are randomly distributed. Until recently, few if any people suspected that there could be another state of matter sharing important aspects with both crystalline and amorphous substances. Yet, this is precisely what D[aniel] Shechtman discovered when they recorded electron diffraction patterns of a special aluminum-manganese alloy (Al_6Mn). The diffraction pattern, i.e., the two-dimensional Fourier transform, showed sharp peaks, just like those for a periodic crystal. But the pattern showed a five-fold symmetry that periodic crystals simply cannot have. However, as we know from the number-theoretic Morse-Thue sequence, sharp spectral peaks and aperiodicity are no contradiction, as long as self-similarity prevails. [199]

The fact that this justification appears in a musicological-oriented text is remarkable, enlisting discoveries made in the field of atomic symmetry to enrich music theory. The grounds for this conceptualization are, however, not entirely new. Pythagorism and Ptolemaicism are imbued with the same spirit of finding similarities in supporting universal harmony. Plato's criticism against the excessive interest in the numerical characteristics of tones (*Republic* vii: 531a–b) can be extended, in consequence, to new perspectives that reduce everything to numbers and geometries. This reminds of Aristotle's judgement that it is a misunderstanding to consider "that real things are numbers" or mathematical relationships (*Metaphysics* 985b32–986a2, 1090a20–23, 1093a28–b4; *De Caelo* 290b21–3). Nevertheless, the paradox remains: numbers and mathematical relationships may be 'things' so 'real' as things are 'real' for Aristotle—for instance the Aristotelian notion of the universe with its symbolic interpretations. It is necessary, therefore, to insist on the definition of a cultural framework for the concept of 'fundamental particle' of sound, in relation to music theory.

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[198] In this regard, it is important to consult subchapter 6.4.

[199] The Thue–Morse sequence is a simple set of binary digits alternately concatenated and complementary which begins 0, 1, 10, 1001, 10,010,110, 1001011001101001... This sequence, along with other sequences of numbers and geometric progressions, is self-similar and has applications in the investigation of music and speech, as discussed in Chapters 3 and 4. On the representation of the atomic bonds and the molecular systems in crystallography, in conjunction with music theory, it is advisable to observe what is stated in subchapter 6.4., especially on pages 416–419 (on the same subject, see also pages 123, 133, 148–149, 211–212, 320, 435, 438).

The concept of 'fundamental particle' in this study

The symbolic and systematic use of the point in modern music theory is profuse. Stockhausen (1989:35), who was originally in favour of a general conceptualization of the sonic atom, deemed point's formalization as the fundamental representation of sound, and as the 'minimum material' in a constructive basis:

> The description 'star music' was used casually by a music critic in Cologne, Herbert Eimert, after hearing these piano pieces of Messiaen, because the music sounded like stars in the firmament. The term 'punktuelle Musik' ('point music') was one I was using at the time.

Indeed, in 1952 Stockhausen began to develop his project *Punkte* for orchestra, based on this concept of music as a set of related points. Shortly afterwards, elaborating on this idea, he perfected his ideas of *mass* and *whole*, which he used in later works (see Maconie 1989:38–46).

This research prefers the term *point* over *atom* for a simple connection with musical tradition. In this context, it is clearer to refer to a point in a length-pitch coordinate, than to an 'atom'. Historically, the concept of point (from the medieval *punctum*) is related to such a coordinate in its visual representation, in the music staff or in other referential systems. This notion of point appears in many treatises on aesthetics, proportion and spatial distribution (see Kandinsky 1926; Ghyka 1927, 1931; Norden 1964, 1972; Howat 1983a–b; Ockelford 2005). In this work, Chapter 6 on proportion and musical self-similarity continues a similar approach. Consistently, and for reasons also observed by Rocha Iturbide (1999) and Roads (2004), the implementation of computer programs to produce complex self-similar structures through sound generation matrices, demands a systematic use of the point and its immediate consequences in classical geometry: the line and the cluster.

The examples given below, on mechanical self-similarity, use the same model of representation, which is also is common in the treatises on acoustics (e.g. Backus 1977). For all of these considerations, a correspondence is assumed between point, line, and cluster, as signs of the same self-similar system.[200]

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[200] A graph picturing this correspondence appears in the 'point made of points', by Kandinsky, in the scheme ◊611, with its translation as sonority in length-pitch coordinates, in turn represented graphically.

4.2. Mechanical self-similarity

The mechanical symmetry of an acoustic phenomenon can be explained as a result of the homogeneous behaviour of a medium under constant physical conditions, such as temperature or air pressure fluctuations. For example, if the material of a vibrating string has no irregularities in its length and its vibrational oscillation is periodic, a pattern of mechanical symmetry is produced around the direction of the string's vibration:
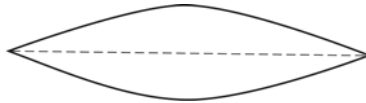
◊420.

The arc of this string, which represents the length of a stretched string put into vibration, corresponds to its lowest natural frequency or *fundamental frequency* with its partial vibrations. In the ideal case, when the string tensors are absolutely fixed, intersecting the vibration at right angles, the partial vibrations of the string (called *harmonics*) correspond to multiples of the fundamental. The string then vibrates as one over the integers of its aliquot parts: $^1/_2$, $^1/_3$, $^1/_4$ ... $^1/n$ of the string length, where n is the infinite series of natural numbers. (See Backus 1977, Cook 2007).
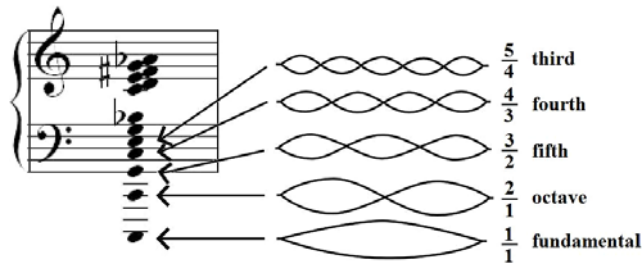
◊421. Harmonics of the fundamental C_1 (32.7 Hz) with the aliquot divisions of a corresponding string, and its equivalent tonal intervals, arranged hierarchically (right of the scheme). The eigenvalues 1, $^1/_2$, $^1/_3$,... from this series are (pre)self-similar characteristics of music harmony.

Harmony in a vibrating string or in an air column can be described as a simple self-similarity phenomenon, in which the whole or fundamental 'reproduces' at smaller scales, in the form of its harmonic components. Speaking roughly, one may say that the difference between *harmony* and *dissonance* equals to the relationship between similarity and dissimilarity in the parts of a whole (see ◊422 and ◊423).

a)
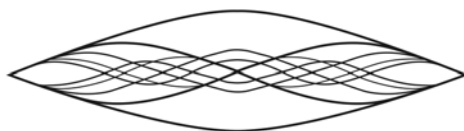
b)
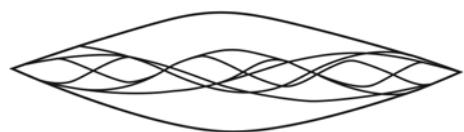
◊422. Idealized representation of two modes of vibration of a string. Amplitude appears increased for visual clarity:

    a) Convergent similarity of the harmonics in ◊421.
    b) Divergent similarity in a disturbed eighth, changing the relative phase of the harmonics (e.g. in a suddenly distempered string).

A vibrating string or an air column are just some of the best known cases of vibratory systems with varying degrees of self-similarity and harmony. There are many other cases in plates, membranes, bars and rods with special acoustic features, as happens with the *isospectral manifolds* and the *Chladni's figures* in emerging patterns of similarity and self-similarity, within a variety of harmonic relationships.[201]

Chladni's figures are geometric formations on the surface of vibrating thin plates whose nodal areas are made visible by the distribution of powdered material (see Chladni 1787, 1830; Tyndall 1867; Rossing 2000; Ashton 2001). The patterns on

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[201] For an instructive introduction to the symmetry of acoustic patterns, especially as intersemiosis between mechanic phenomena and their graphic representation, see Antony Ashton's breviary, *Harmonograph: A Visual Guide to the Mathematics of Music* (2001).

these surfaces reflect the relationship between amplitude and frequency, and respect the vibrating area and the tensed material quality. Some of these patterns show relations of symmetry at two or more structural levels. A trend to a similar kind of sediment distribution in different scales is observable in several examples, producing forms of statistical (pre-)self-similarity.[202]
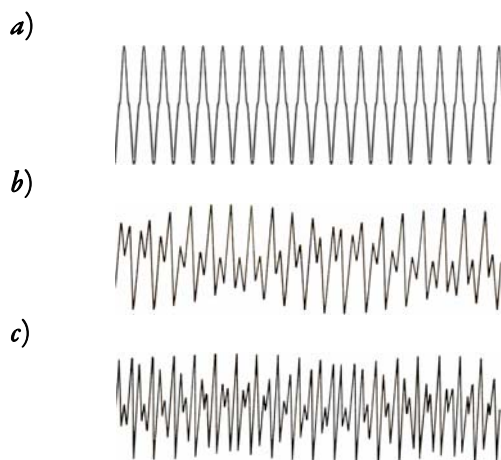
◊423. Waveforms in (*a*) *perfect* and (*b–c*) *imperfect* consonances.
    a) Full similarity (motif's repetition) in the vibrations of a fundamental
      tone with its first eighth (perfect octave).
    b) Similarities displacement in an eighth slightly out of tune.
    c) Similarities displacement in an fifth slightly out of tune.

    Examples *b* and *c* show pre-self-similar patterns; a more complete self-
    similarity can be traced in a full-harmonic array.

In addition to simple cases of consonances slightly out of phasing (see ◊ 423), another manifestation of simple mechanical (pre-)self-similarity in acoustic systems are the isospectral manifolds: physical configurations related to the problem of constructing two drums with very different shape that share the same harmonic spectrum or *eigenfrequency spectrum*. Since two drums built with the same material but with different symmetries and perimeters usually produce different frequential

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[202] In short, the parametric functions of the plates' modal frequencies, in Chladni's figures, reflect the same kind of consistency in different frequential scales (see Rossing 2000:89–92).

gamuts, Kac (1966) queried whether it was possible to build two drums with different shapes but with the same eigenfrequency spectrum (i.e. two drums of distinct physical appearance producing the same sound). The affirmative answer by Gordon *et al.* (1992) led to the identification of seven *polyaboloes* which produce isospectral manifolds with nodal areas of statistical self-similarity.[203] The result is that different drum varieties may produce, with the same number of polyaboloes, the same set of frequencies. This type of configuration of plates and membranes is particularly related to the topology—i.e. the forms of spatial extension of different geometries—of a vibrating system, independent of the scale of the system.[204]

Spectral self-similarity

In a combination of objective and subjective components, the *colour* of sound is the quality that makes distinguishable a sound of another (see Erickson 1975:4–5).[205] This includes the timbral differentiation of voices, and the qualitative distinction between two equal pitches in the same instrument produced with two different fingerings; for example an E_4 produced in the third string, or in the fourth string of a violin.

The spectral identity associable to a self-similar pattern is not unique to certain sound objects or certain musical instruments. Each acoustic phenomena with periodic motion—including the relatively periodic oscillations and waves—is characterized by a distinctive timbre. Precisely, what makes the sound of a verbal language unique within the community of speakers in that language (and the individual within a speech community), is the special collection that integrates every acoustic cluster of self-similarity in these wholes. As Boomsliter and Creel (1961:10) observe, "A vocal tone may contain as many as thirty partials, with the energy well

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[203] This subject is addressed again in subchapter 6.4., apropos of tilings in the harmonic spectrum. See especially pages 420–421, which include a graphical representation of the basic polyaboloes.

[204] In other words, two drums with the same area and the same set of polyaboloes share a common frequential identity, regardless of the built size. The affinity between two or more polyaboloes is invariant across physical scales.

[205] Erickson (*ibid.*) explains that "timbre depends primarily upon the spectrum of the stimulus, but it also depends upon the waveform, the sound pressure, the frequency location of the spectrum, and the temporal characteristics of the stimulus."

distributed among them, frequently with the fundamental very weak and the greatest amount of energy in high partials such as the 12th or 16th."

According to Fagarazzi (1988), harmonic spectra analysis and its synthesis in the form of hierarchies, relates timbre or sound colour with a harmonic power that can be used as a compositional grammar. The very point of this idea is that, whether a sound can be identified by its timbre as spectral configuration, the same configuration may establish the rules for the use of harmony in the same sound source, i.e. self-similarly associating timbre and harmony (see also Grisey 1987, and Waschka and Kurepa 1989). Furthermore, the same rules establishing the relationships of vertical intervals can also be used to determine metrical and rhythmic relationships (see Cowell 1930, Xenakis 1963/1992), which means that a *single sound* is enough to generate a complete grammar with its own context of use. This idea had been foreseen by Varèse (1936:26), directly addressing the distinction between paradigmatic analogy and proportional analogy—in this case a distinction between metaphor and synecdoche, respectively—under the scheme of a sonic map:

> The role of color or timbre would be completely changed from being incidental, anecdotal, sensual or picturesque; it would become an agent of delineation like the different colors on a map separating different areas, and an integral part of form.

From this conception of timbre—particularly for its importance in the design of the *forms* and *masses* of sound theorized by Varèse (*op. cit.*)—the traditional levels of musical structuring (e.g. melody, harmony, rhythm and intensity) are framed in a broader picture of congruence between partial and global layers of structure-meaning. Hence the 'minimal' details of sound, like the tiniest aspects of noise, register, or timbre, come to be considered as essential components of the musical form as a whole. The 'map' mentioned by Varèse in the quotation above corresponds consequently to the *map of the map* of a self-similar musical structure.[206]

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[206] The development of this intuition in Varèse's musical thought is dealt with extensively by Erickson (1975), especially in his chapter 'Some territory between timbre and pitch' (*op. cit.*, pages 18–57).

Fourier analysis

Fourier analysis consists of the study and identification of the parts of a vibratory system in homogeneous sets that give coherence to the same acoustic system.[207] Fourier analysis is named after Jean-Baptiste Joseph Fourier (1768–1830), a mathematician and physicist originally interested in the study of heat propagation in solids. Fourier found that heat flow behaves like a wave, and concluded that heat waves are periodic waves consisting of the same pattern repeated over and over again. This finding, congruent with the study of waves in mechanics and acoustics, culminates with the postulate that says that "No matter how complicated it is, a wave that is periodic—with a pattern that repeats itself—consists of the sum of many simple waves" (Gleason 1995:11). Fourier postulated that developing a function by a
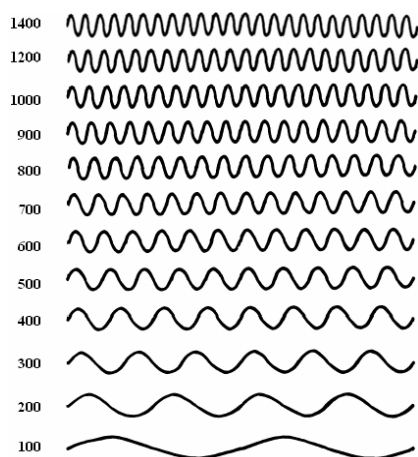
◊424. Fourier analysis of the first 12 partial components of a complex tone with its fundamental at 100 cycles. The long-term shape trend is the same as the one of the fundamental, composing a self-similar texture. Schematic view after Boomsliter and Creel (1961:11).

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[207] As shown here, Fourier analysis is of great relevance for the study of self-similarity in sound, especially considering that it can be employed in a wide variability of time and frequency domains and functions (discrete, continuous, periodic, aperiodic, finite, infinite). Several authors quoted in this study, including Gabor, Barlow, Beran, and Roads, assigned an important role to it in their own research. However, it may not be clear for the reader how to get a first approach to Fourier series and analysis. In this case it is highly recommended to consult the text *Who is Fourier?* (see Gleason 1995), an instructive general introduction to the subject which then departs into more specialized areas.

trigonometric series—afterwards known as a Fourier series—simplifies the mathematical representation of the whole process. The figure ◊424 illustrates this postulate with a typical analysis of a *fundamental* (wave segment showing two crests) and its partial components (added as arithmetic progression $n + 2$, keeping the same amplitude). The result is a wave succession reflecting the self-similarity of the series.

The sum of *Fourier series* corresponds to the formula for synthesizing a periodic signal, just as a self-similar set can be reconstructed by the predictability of its relations in a first set of self-similarity. Fourier theory states that the function of such a signal $= x(t)$, of a period T, can be represented by a sum of infinite series, as a sum of sinusoidal functions harmonically related to the frequency $\omega_n = n\omega = 2\pi n / T$:

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega t + \varphi_n)$$

In this equation, C_0 is the 'hinge' that pulls the waveform up or down. The first sinusoidal component C_1 is the fundamental frequency (represented by 2π as the whole basic, single waveform), and coincides with period T (i.e. $\omega = 2\pi / T$). The numeric variables C_n and φ_n give the magnitude and phase of each component (see Gleason 1995:360–368, 426; Roads 2004:244–245).

The Fourier series does not predict, however, how coefficients C_n and φ_n are grouped in an arbitrary sound input. For this the *Fourier transform* is required as a method to find the size of each complex frequency, in order to synthesize—i.e. rebuild—an original signal (Roads, *ibid.*). As Miramontes (1999:8) observes, this method also has a special significance for sound analysis: "As the prism is the material device that allows us to split light into its elementary components, the Fourier transform is the 'mathematical prism' with which any sound can be decomposed allowing us to represent a spectrum in a time series". Implementing the functional duality that characterizes Fourier analysis, its inverse operation can, instead of decomposing the source signal, recompose it by synthesis. Consistently, when Xenakis (1992:293) explores what he calls the *microscopic* construction of sound, extending its principles to a larger scale, he observes that there is only a

structural step in using the Fourier transform dualism from the self-similar analysis of sound, to the self-similar sound synthesis:

> This approach to sound synthesis represents a non-linear dynamic stochastic evolution which bypasses the habitual analysis and harmonic synthesis of Fourier since it is applied to the $f(t)$ part on the left of the equal sign of Fourier's transformation. This approach can be compared to current research on dynamic systems, deterministic chaoses or fractals. Therefore, we can say that it bears the seed of future exploration.

This approach, which promotes the use of Fourier analysis for the investigation of musical self-similarity, is also relevant, as suggested below, in the study of aperiodic textures, noises and complex timbres, connecting them to more general criteria of harmony.

Turbulence patterns

Among the more sophisticated forms of mechanical self-similarity in music (i.e. in its acoustical implications) there is a wide variety in patterns of turbulence. These patterns can be described roughly as the relationship between the constant instability and consistency of irregularities of a system of frequencies in an acoustic flow, at various scales.

Madden (2007:107–109) suggests that self-similar behaviour in turbulences may occur in a variety of phenomena of resonance, among which is the production of sound clusters (also called *multiphonics* or *multiple sonorities*) on various wind instruments (wood and brass with simple or composed embouchure) or bowed strings. Nonetheless, the researches of Karplus and Strong (1983), Roy (e1992), Cook (1997), and Bader (2006) explain that various forms of turbulence also occur in the initial moments of the attack of different percussive systems and plucked or strummed strings, most of which produce self-similar patterns.

Bader (2005) analyzes the sound production in flute-like musical instruments with beveled embouchures, in which the coupling between the performer's mouth and the embouchure is also a space for turbulence flow: "Here the flute tubes eigenfrequencies forces the self-sustained oscillation of the generator region at the blowing hole into the tubes resonance frequencies." (*op. cit.*:109). This brief description suffices to illuminate—at least at a first approach—the general

hypothetical frame for this kind of self-similarity: the turbulence in question has self-similar properties because of the form in which breath is projected to the embouchure, as 'self-sustained oscillation' by a system that is mechanically redundant and self-referential with respect to the cycles and components of its wavelets.

Bader demonstrates that only a small amount of the blowing wind actually enters into the tube, projecting the rest to the outer space of the instrument, where patterns of turbulence are produced. Finally he shows that changes in pressure, related to directional changes in the flow of air over the embouchure, affect the generation of turbulence complexes in the studied system. Wolfe *et al.* (e2001) explore in detail this kind of behaviour in transverse flutes from different epochs and with different tunings, concluding that the turbulent flow is a defining characteristic of the timbral qualities of these flutes.
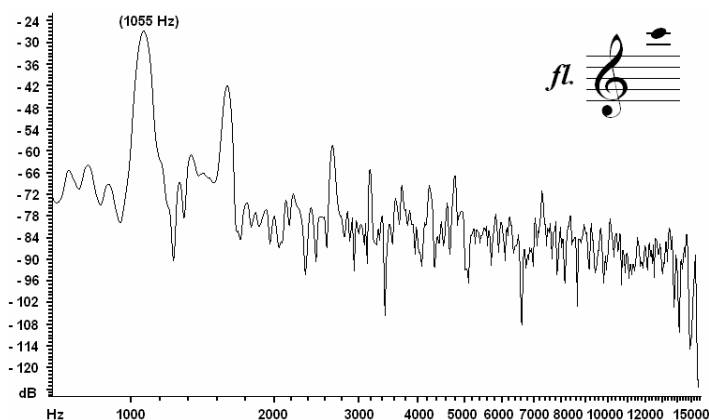
◇425. Harmonic spectrum (excluding the fundamental) of C_5 (~525 Hz) in a transverse bass flute. A turbulence pattern matches shortly before and after the first harmonic (1055Hz), a characteristic of acoustic behaviour in flute-like instruments. The C marked in the staff represents real notation with the approximate location of the first harmonic (i.e. when reading the staff, the performer should produce the fundamental tone an octave below the written C, since the writing of bass flute is transposed into a high octave).

Mathieu and Scott (2000:183–184, 300–305) identify modes of statistical self-similarity among deferred velocities in a turbulent system (e.g. in jets and wakes),[208] in which they observe a correlation between scale and velocity: "typical turbulent flows have apparently random velocity fluctuations with a wide range of different length and time scales [...] The size of the large scales of turbulence increase with time, while the velocity of fluctuations decay". Among these modes of self-similarity are *isotropic turbulence*, which involves statistical self-similarity in velocity gradients, and *intermittency*, a related concept which is explained in subchapter 6.2.[209]
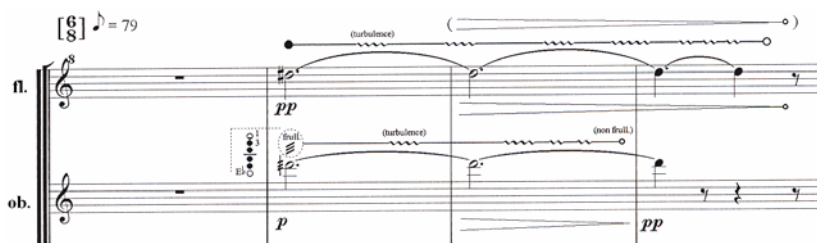
◊426. First measures of the score *'Na 'bäfi xi ñudi ga tuhu* (*Within a nesting of songs*) [2002], by Gabriel Pareyon, for instrumental ensemble. The sample shows the flute (one octave higher) and oboe parts. The horizontal line above the notes, with the word 'turbulence', graphically represents an unstable emission of sound, controlled as an amount of diffuse air in the flute embouchure, and a special fingering combined with gentle pressure in the oboe's double reed mouthpiece. The mechanical principle of sound instability in both instruments is completely different, as well as the obtained result: both instruments are related by a reciprocal imitation, into a bound between intrasemiotic and intersemiotic translation (i.e. between idiomatic affinity and symbolic approximation). Copyright by Jurgenson Editor, Moscow, 2003.

Bru (1996) isolates the radius of a first turbulent system in the mouthpiece of the flute, which, with a filter implemented, allows the precise measurement of the source spectrum distinguishing nodal bipolar regions involved in the turbulence formation.

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[208] These authors (*op. cit.*:301) make the following observation: "by statistically self-similar we mean that velocity and length scales can be found, which are solely functions of time, and that, if the scaled velocity is regarded as a function of the scaled position, then its statistical properties do not evolve with time."

[209] See page 352–353.

A practical use of turbulences can consider such relationships in electronic sound synthesis (Bru, *cit*.; Polotti and Evangelista 2001; Bader 2005), or just be suggested in traditional musical notation, specifying the mode of production (see example in ◊426).

Arneodo (1995) suggests 'fractal' nesting at different power rates and simultaneous acoustic substructures (*multifractals*), which are proven by Bigerelle and Iost (2000) and Su and Wu (2006) for a variety of musical examples.[210] This is obviously connected to Xenakis (1992:293) and his idea, already alluded to, on the sound components multiscaling in different parameters. Congruently, Vaggione (1996) suggests that the analysis of subharmonics is one of the keys to access turbulence modelling and to develop multiple self-similarity analysis or *multifractal* analysis, to use Schroeder (1991) and Arneodo's (1995) terminology. According to Vaggione (1996:35) "The somewhat artificial attempts made to date, to relate chaos theory to algorithmic music production can find here a significant bridge between different levels of description of time-varying sonic structures."

The quantum analogy of sound and its gestalt contents

By analogy with the periodic vibration of the undulatory light particles, or *photons*, the nodes of vibrating and pulsating energy between molecules are called *phonons* (see Schroeder 1992:43). To what extent phonons can be understood not as microwaves—of electromagnetic character—but as ultrasound, is a question of approach, depending on the analytical purpose. What is a fact is that the concept of phonon is generalized and common in Fourier analysis, to express vibration ranges in superconduction processes and temperature transitions in solid bodies, both in 'very low' temperatures to 'very high' ones (i.e. molecular cohesion thresholds).

According to Robert Erickson (1975:49), "Varèse's favorite image for musical organization was crystal structure". This has profound implications. After the analysis of structural symmetries in crystallography (Bravais 1848, Bernal 1926, Hilbert and Cohn-Vossen 1932/1952:52–53), it is possible to identify a typical

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[210] The concept of 'nesting' is further developed in subchapter 5.4.

relationship between atomic grouping, molecular symmetry, and a configuration of phonons on the energy state of crystals. Schroeder (1991:43–44) observes that many of these configurations tend to form self-similar complexes—in a deterministic and statistical sense—for example, in the massive vibration of thousands or millions of molecules associated within the same system. Schroeder (*ibid.*) calls *fractons* this type of phonons, and believes that they are "of growing importance in our understanding of vibratory nature", given that they can be studied for their mediation between the subatomic and the molecular—even audible—worlds.[211]

Since optical energy sources can induce mechanical vibrations (Roads 2004:34), and Fourier analysis has no operational implicit bounds—a periodic wave can always be analyzed in smaller formants—then there are similar arguments to describe acoustic and mechanical phenomena in all their vastness, with the same generalized representations and basically with the same symbolization and abductive operations. This is nicely summarized in Livio's (2002:149):

> Kepler's model had something in common with today's fundamental theory of the universe: Both theories are by their very nature *reductionistic*—they attempt to explain many phenomena in terms of a few fundamental laws. For example, Kepler's model deduced both the number of planets and the properties of their orbits from the Platonic solids. Similarly, modern theories known as string theories use basic entities (strings) which are extremely tiny to deduce the properties of all the elementary particles. Like a violin string, the strings can vibrate and produce a variety of 'tones', and all the known elementary particles simply represent these different [harmonic] tones.

In this description is evident the role of intersemiotic translation as a method to link a *known* domain (mechanical acoustics) with another, probably *knowable* (quantum mechanics). The picture of a vibrating string is the centre of a system of analogies, extending from the scope of elementary particles to *wider* relationships in the cosmos. Valid or not in its operational sense, it is clear that this physical description introduces, through abduction, a general understanding of fundamental self-similarity. Much remains to be explained about the meaning of this type of hypothesis, but a first step in this direction is the scrutiny of its comparative

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[211] The focus on self-similarity in the field of crystallography, as an analogous description in different types of musical information structuring is exposed at several points in this work: see pages 123, 133, 136, 211–212, 320, 416–419, 435, 458.

processes and the instruments of an implicated intersemiotic translation. In this context, Lehar (2002:159) remarks the Gestalt of physical processes such as the vibration of plates and membranes, with formal and structural consistency in their modes of representation, perception and intersemiotic translation. For example, a Chladni figure obtained by a vibrating plate can be represented by the recording of the vibrating plate; in turn this recording can be used to make a similar plate vibrate, producing the same Chladni figure. In consequence,

> [t]he audio tone can therefore be considered as an abstracted representation, or reduced-dimensionality encoding, of the spatial pattern on the plate. Thus, matching the tone generated by a vibrating plate to a tone stored in memory corresponds to a recognition of that spatial pattern, just as the activation of a cell body in a receptive field model represents a recognition of the spatial pattern present in its input field. The item in the resonance model corresponding to the cell body in the receptive model can be envisaged as some kind of tuned resonator, perhaps a cell with a natural tendency to spike a characteristic frequency.

From this perspective, Lehar (*loc. cit.*) draws attention to the Gestalt consistency among different physical and biological strata, involved in their own processes of representation as self-referential systems with self-similarity surfaces.

4.3. Biological self-similarity

Self-similarity in biological structures also occur in various strata: in the chemical bases of cellular processes, the genetic makeup of the cell nucleus, the organization and coordination of cellular tissues, the circulatory system and the nervous system distribution in both ventral and dorsal networks, cooperating as performative and cognitive systems preconditioning language (see Mandelbrot 1982:150). Internal symbolic communication or *endosemiosis* connecting the systems of an organism operates in synecdochic intersemiosis (see Sebeok 1977:1061).

Hess and Markus (1987) find that, in biochemistry, dynamic processes, electrical oscillations and functional periods tend to self-similar chaos, especially in typical behaviours of bifurcation. West (1990) focuses on this specific topic. Such an *organic self-similarity*, related to power laws that affect the fundamental organic structures, is

reflected in a systematic *coherence* along biological relationships and functions, pointing to some general principles of structural economy and self-organization, which also involve basic aspects of music.

Hoffmeyer (1998) and Brier (2001, 2004) acknowledge that the nervous system and subordinate systems such as the immune and hormonal ones, keep an electrochemical communication that stimulates states of volition, affect, perception and inference. According to these authors, such communication is a subsymbolic precedent of language, being influenced by endosymbiotic systems with their typical rhythms and hierarchies. Josephson (1995:280) shares this view of language, connecting it with his own concept of music:

> Previous papers [Josephson 1992, 1994] have criticised certain conventional views of music, such as the idea that music is intrinsic to a musical culture and therefore has no intrinsic meaning of its own, and argued instead that music constitutes a general symbol system, analogous in many ways to DNA [...] but in view of the existence of significant differences in detail between the characteristics of the two kinds of symbol system, especially in that music (like natural language) is processed by a mind that in some sense *understands* it, whereas DNA expresses its potential by a more passive process, a slightly different mode of attack is appropriate.

From the standpoint of the present study, there is no need for the opposition between 'passive' and 'active'—typical of structural rationalism—that Josephson privileges. The need is, rather, to understand the meaning of local and global tendencies in their long-term coordination as suggested by Prusinkiewicz (1990, 1992) and Meinhardt (1998). As in the case of a binary system of information, what is meaningful is not the relationship 0 = *passive* ↔ 1 = *active*, but how these symbols are organized in the long term, as messaging systems of systems in a richer context of information (see Eco 1962, 1968, Campbell 1982, Jensen 2008).[212]

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[212] There is blatant evidence supporting this hypothesis: this sort of long-term organization happens actually in internet, with the constant transformation of content based on initial conditions of information and distribution.
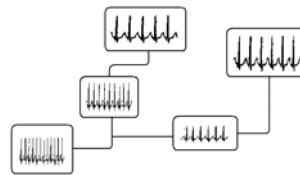
◊430a. Left: Cardiac pulsation represented as wave in an electrocardiogram. The waveform of a normal pulse is never perfectly regular, analogous to the perfect eighth represented by ◊423*a*, but similar to a wave 'slightly out of tune' and adaptable to a variety of amplitudes according to the metabolic and functional needs of the body.

Right: Taxonomy of five cardiac systems characterized as self-similar relationships in terms of an evolving network. This example suggests that combining the different pulse shapes in a large enough sample of heart rates, it is possible to describe the sample as a whole group made of smaller groups of individual cases with an invariant trend of frequency, i.e. an example of self-similar distribution. To continue this example it would be necessary to group the five groups of the scheme into a group inside another group functionally connected with other sub-groups. The latter is represented by the Verhulst diagram below (◊430b), in which the sub-groups from the global scheme encompasses other groups of extensions, in successive bifurcations.
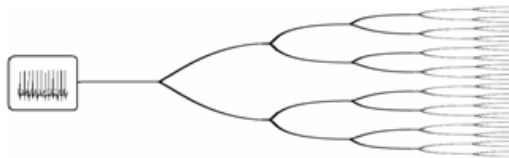
◊430*b*.

Acoustic systems in biology involve relationships between a certain physical level to other, more dense levels with autonomous categories based on physical properties: in a first physical level there is a chain of sequences, *binary codes* producing patterns of activation–inhibition, as happens in the vibrations of a string or a membrane. These patterns already present, however, self-similar chaos, as can be noticed in cardiac pulsation in individuals, groups of individuals, and taxonomies, as suggested in ◊ 430. The codes shaping these patterns cannot be explained as a linear physical interaction. Even at the level of the nitrogenous bases of organic chemistry, 'something new'

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occurs: series of operations succeed in forming emerging patterns, self-organizing and self-organized structures with a tendency for a codal sophistication and a continuous reconfiguration—replication, adaptation, translation, recreation—of messages. A relatively simple case of this is the heart's skill to adapt to different rhythms according to changes in metabolic rate and body performance (Weibel 1984:171–172, West 1990:67–76).[213]

Coherence between physics and biology

M. Garavaglia[214], supported by laboratory experiments, confirms the conjecture made by Voss and Clarke (1975), suggesting that self-similar patterns occur in the eardrum vibrating surface, comparable to $1/f$ noise (an issue developed in Chapter 5). This relationship reveals a deep connection between resonance surfaces in organisms and their perception of sound—assuming that not only the eardrum, but all the parts of the outer, middle and inner ear, as well as bone and dermal resonators and the vibration of body fluids, together constitute a complex system of acoustic systems—it is conjectured that such systems are configured in affinity with harmonic patterns linking the physical properties of the vibrating body with its own anatomical and perceptual characteristics.[215] This implies a certain coherence between the systems of acoustic perception, the systems of language articulation, and the form of each communicative organism.[216] From this supposition, the mechanist approaches on the phonetic– phonological principles remain open to explore the sense that some

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[213] Weibel (*op. cit.*:172) notes that "Cardiac output and heart beat frequency are proportional to the aerobic capacity of the body: heart frequency and cardiac output per unit body mass are proportional to $M_b^{-0.25}$ in mammals of varying size."

[214] Mario Garavaglia (1937), researcher and professor emeritus at the Department of Physics, National University of La Plata, Argentina. Personal communication during the Third International Congress of Form and Symmetry ISIS-SEMA, Buenos Aires, 2007.

[215] Akiyama *et al.* (1996) remark, as a method of diagnosis and analysis, the study of the relationships between acoustic self-similarity and the characteristic consistency of biological tissues: "Acoustical characteristics of biological tissue is dependent on pathological states of the tissue. [...] Since the tissue structure itself has a self-similarity characteristics, it is important to show how the echographic image of the tissue has the self-similarity characteristics and how it could be extracted from the image."

[216] Liberman *et al.* (1957), and Liberman and Mattingly (1985) present antecedents to this idea.

semiotists and linguists attribute to the membrane as the 'first instrument' of language (see Guerra Lisi and Stefani 2004:81).

Nelson and Cox (2000:3), introducing their *Principles of biochemistry*, make clear that "Living organisms are composed of lifeless molecules". This assumption, which has antecedents in Lamarck (1809),[217] suggests that the structural relationships of organic chemistry and the general physical principles are transmitted—becoming more intricate and selective—to the living organisms. Regarding the acoustic systems found in living organisms, such systems depend not only on a particular biological structure, but obviously, on general acoustic principles. The examples in the previous subchapter may provide support to explain this principle: the eardrum acts like a vibrating membrane subjected to general physical laws, like all the acoustic systems of an organism oriented by power laws.[218]

Coherence between biology and language

Josephson (1995) notes that, at the intracellular level, the functional relationship *mitochondria – cell nucleus* has a certain syntactic parallelism with the relationship *verb – subject*: one provides power, the other determines the meaning of the action. At the semantic level there is another parallelism: that of the compatibility and selectivity mechanisms in a limited number of terms, whose operation is comparable between the bases of the cell nucleus (DNA) and the lexicon integration. In this sense morphemes present some functional analogy with proteins. Furthermore, molecular biology often refers to the decoding of protein chains using the analogy of a grammatical process, alluding to an 'electrochemical grammar' to determine their association.

Niels Kaj Jerne (1984), in his speech "The generative grammar of the immune system", makes parallels between the generative description of language according to

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[217] Jean-Baptiste Lamarck (1744–1829), *Philosophie Zoologique, ou exposition des considérations relatives à l'histoire naturelle des Animaux; à la diversité de leur organisation et des facultés qu'ils en obtiennent; aux causes physiques qui maintiennent en eux la vie et donnent lieu aux mouvemens qu'ils exécutent; enfin, à celles qui produisent, les unes le sentiment, et les autres l'intelligence de ceux qui en sont doués*, Dentu, et l'Auteur, Paris, 1809.
[218] This notion has been introduced in section 3.9.5.

Chomsky (1957), and the mechanisms of recognition and association between antigens and antibodies. The latter would use semantic-syntactic relations, whose bonds "are similar to the functional bonds in the components of a sentence" (Lassagne 2004:147). This analogy can be extended to the observation of morphemes from their self-assembly after some functional principles, comparable to the self-assembly of proteins involved in genetic structuring. As Marcus (1974:85) suggests "Amino acids are like terminal symbols of a virtual genetic grammar". Marcus (*op. cit.*:85–88) even speaks of *morphological meaning, synonymy, homonymy* and *well-formedness rules,* as basic qualities for the stability and continuity of the genetic code, in a theorization that is clearly sympathetic to its parallel in generative music theory: the study of Lerdahl and Jackendoff (1983). At present the concepts of semantics and syntax in biological processes are refined in a variety of theories and applications (e.g. Sebeok 1977, Guerra Lisi and Stefani 1997, Jablonka 2002, Seddon 2002, Reygadas and Shanker 2007, and El-Hani *et al.* 2009, among many others), some of which begin to have adaptations in a new generation of *naturalist* postulates in music theory (e.g. Mâche 1983, Josephson and Carpenter 1994, 1996, Josephson 1995, Kaipainen 1995, Reybrouck 2001, Ockelford 2005, Martinelli 2002, 2007, Jensen 2008, Ojala 2009).

Music—which is akin to the functionality of biological forms and the distribution and consistency of genetic information in redundancy—uses precepts of use and meaning that define its basic grammars. Amozurrutia (1997:4–5) believes, in consequence, that

> On the trail of the musical idea, the composer gathers a bunch of cells and structures to build an 'organism' with a well-defined form. [...] Many musical creations are based on a fragment of life, so they are subjected to very specific forms and structures, guided by intuition or by a certainty from the unconsciousness.

There is, thus, a consistency between the *tissues* of the musical discourse, governed by codes and messages that are consolidated and renewed in an *Umwelt*. For the tradition of naturalist semiotics founded by Jakob von Uexküll (1864–1944) and Thomas A. Sebeok (1920–2001), the Umwelt is the 'world around' of a living organism, in which sign systems are produced characterizing each biological community. Accordingly, the biological tissues constituting the animals and their

ways of perception and communication, are directly associated with their Umwelt (see Kaspar 1983). Music—resulting from specific Umwelten—is also connected, in this fashion, with biological signs and relationships on a 'small' scale, with Umwelt–niche relationships on a larger scale.

According to Donald L. Hardesty (1972, 1975), a niche is a node of symbiotic relationships in an ecosystem. The present study suggests that both concepts, the former (Umwelt) borrowed from semiotics, and the latter (niche) from ecology, are complementary, and by no means they can be conceived as rigid ontologies, but rather as flexible, *self-creating spaces* where dynamic relationships are in interplay, including perceptual, motile and subjective features. As Reybrouck (2001:604) notes, "Listening to music is highly related to Umwelt-research. Music is not to be considered as an acoustic niche, but as a subjective environment." Thus, although sound *acts* in an acoustic space, music *lives* in a relatively subjective world.

Tension between musical analysis and biological description

A third 'organicist interpretation' of music, with different methods and goals, avoids direct descriptions from biology to explain musical relationships through geometric abstractions. This is the case for much of the statistical analysis of Forte (1973), Morris (1987), Huovinen (2002), and Tymoczko (2006), to name a few authors. Other analysts reconcile this model with theories adapted from linguistics, as with the Schenkerian analysis, which develops the concepts of *chord grammar* and *chord significance* (see Salzer 1952:10–11); the work of Lidov (2005), Lewin (2006) and Almén (2008), about the relationship between music, text and language; and the work of Lerdahl (2001), which puts an organic metaphor to the service of linguistic generativism.

The very different analytical tendencies observing music as a constructive process have, however, something in common: all of them reflect the weight of a tradition in a specific Umwelt–niche, within which the relationships of discourse and expression are operational processes sympathetic to organic development. Pitch-class set theory effectively neutralises the tradition that regards certain rows or scales, or certain pitch-class sets, as 'natural'. But this discrimination does not imply the abandonment

of the organic-functional model. On the contrary: the logical positivism and structuralism in the foundations of pitch-class set theory (Babbitt, Forte, Lewin, Morris) provides an analytical framework comparable to the scientific positivism that have explored the broader fields of biology and organic chemistry since the late nineteenth century.

Whether the rationalist-structuralist model has practical advantages of systematization over an intuitionism—limited to an emulation of observable causes—it is undeniable that the analysis of the similarities between patterns of preference and performance of the listener, and between patterns of biological relationships inherent in the same listener, provides clues to discern the typologies and universals of music and language.

Since energy and matter, relating electromagnetic and molecular vibration, are identified by frequencies and recurrence modes assimilated by analysis as part of an experience (i.e. the analytic process and conclusions), *everything* perceived can be translated as time series or as collections of intervals which can be implemented for musical analysis and composition, under different proportions and paradigms (see Coons and Kraehenbuehl 1958; Kraehenbuehl and Coons 1959). The levels of biological self-similarity and the levels of musical structure, in any case, are useful for detecting correlation between *inner* and *external*, between the endo- and exosemiosis that Sebeok (1977:1060–1061) notes:

> Clearly, man's semiotic systems are characterized by a definite bipolarity between the molecular code at the lower end of the scale and the verbal code at the upper. Amid these two uniquely powerful mechanisms there exists a whole array of others, ranging from those located in the interior of organisms (von Uexküll's *Innenwelt*) to those linking them to the external 'physical world' (his *Umwelt*), which of course includes biologically and/or sociologically 'interesting' other organisms.

The role of music as a mediator between levels of biological similarity, taking into account the healing traditions, provides a space for seeking a balance between sound, symbol and body. This was noted by Plato, for whom harmony and rhythm "can heal a broken spirit" (*Timaeus* 47c7–e2). Tarasti (2004)—alert to this tradition on sound as a dialogue between *interior* and *exterior*—conceives that a fundamental aspect of music is supporting the transition from the organic to the metaphysical: from the

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inner to the external world and then to another world in which the prior is contained. Tarasti refers particularly to a transition from the 'intraceptive' or endogenous to the 'exteroceptive' or exogenous in the transformation of the narrative modes of music in an organic way migrating to a metaphysical way; in a step between what the existentialist Jean Wahl conceives as 'trans-ascendence' and 'trans-descendence' (see Tarasti 2004:11). Moreover, as Reybrouck (2001:599, 626) states,

> Knowledge as an instrument of adaptation is not concerned with the representation of a 'real world' but is a tool in the pursuit of equilibrium and to steer clear of external perturbations and internal contradictions. [...] What really matters is not the representation of an ontological musical reality, but the generation of music knowledge as a tool for adaptation to the sonic world.

Musical knowledge—including memories, experiences, emotions and imaginations of music—is then, not a mere collection of objects and fixed ideas about the 'world', but rather a constant reorganization and invention of the Umwelt as environment.

The concept of abduction *in Bateson*

The concept of *abduction* coined by Ch. S. Peirce (1903a), which refers to the conjectural process for a hypothesis formulation, is refined in Bateson (1972), who uses it to describe the consistency of an organic process by comparisons between correlations and by a characteristic symmetry or asymmetry. This concept, applying its own methodology beside inductive and deductive methods, is fundamental to the holistic-qualitative traits with which Bateson explains biological relationships.

For Bateson 'small differences' in a whole of structural similarities are crucial to understand the major transformations in evolutionary processes, and to explain much of the creative thought processes. This notion is developed in subsequent chapters which consider *musical creativity* (composition, improvisation, interpretation, and appropriation by analysis, synthesis, paraphrase or recursion) performed by systems of self-similarity, sensitive to initial conditions and open to gradual processes of local differentiation. Systematic and significant accumulation of antisymmetries in music, resulting from the quasi-periodic organization of 'small differences', is also a central issue in the final subchapter, *6.6.*

Another important point in Bateson (*op. cit.*) is his emphasis conceiving the mind as not appart from the matter. Opening a line of investigation further developed by Damásio (1994, 2000), Guerra Lisi and Stefani (1997), Reybrouck (2001, 2005), and Reygadas and Shanker (2007), Bateson claims the fact that any observation and hypothesis on the physical world, necessarily involves the observing mind—and culture as product of the mind-body, in interplay within an Umwelt. A mind therefore creating the world. In consequence, music can be thought as a physical and biological form reifying the physical and biological world.[219]

4.4. Structural universalism: the linguistic model

General aspects of music and speech can be compared despite their functional and structural differences.[220] The feasibility of comparison is so realistic, that individuals with vascular damage in the brain's left hemisphere, which handles tasks involving syntax and words articulation, can recover some verbal skills using the brain's right hemisphere, 'singing' words rather than talking.[221]

Aiello (1994) lists a wide variety of similarities between music and speech. She remarks, for example, similarities in learning processes between children and teachers, as well as typical forms of intuition and practice in society. In this direction, Lidov (2005) acknowledges basic similarities of articulation, inflection, reference, segmentation, discourse and ideology. Bernstein (1976) conceives, besides, that the recurrent behaviour which characterizes the harmonic components of sound both in speech and in music, offers powerful evidence to recognize fundamental physical patterns as determinant for musical and verbal structuring. In any case, all these forms of analogy have a common feature: for all of them there are instances of recursion and self-similarity.

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[219] This is why the concept of self-reference, explained in subchapter 3.6., is so relevant in the study of music, and in the investigation of languages as self-similar forms.

[220] This requires reading the subchapter 1.2.3. on *cognitive domains*.

[221] This treatment, known as *Melodic Intonation Therapy*, is widely used in actual neuroscience, and is based on the fact that patients who have suffered an injury in the brain's left hemisphere are unable to utter words, but they can sing words (see Schlaug, Marchina and Norton 2008).

Since the general aspects of acoustic and mechanical self-similarity are discussed in subchapter 4.2., this subchapter focuses mainly on phonological and syntactic recursions typical of human language, along with their musical analogies. Other important forms of language recursion—pragmatic and contextual—are discussed in more detail in subchapters 2.5. and 2.6.

Various phenomena of phonological and syntactic recursion may be described as processes of homeomorphism (ὅμοιος, similar, and μορφή, form), in which a pattern reappears in one or more correlated layers. Many phonological structures have homeomorphisms in the functional cycles of a morpheme within a word; a word within a sentence; a sentence within a discursive construction, or a discursive construction within a rhetorical style. Typical cycles of sound patterns can be found at all these levels, forming part of a phonetic repertoire, of cycles of implication and subordination, and of more complex hierarchical relationships. Linguist Rush Rhees (1954/1971:69) suggests another kind of 'linkage of scales', depending on form and contextual use: {name→rule→correct→incorrect→concept→intention→understanding→ communication→language·institution→social·life}. Ballantine (1984:5) believes that these strings are consolidated into musical relationships as a self-similar process: "Social structures crystallize in musical structures [...] in various ways and with various degrees of critical awareness, the musical microcosm replicates the social macrocosm". Again, these forms of concatenation are presented as cycles made of cycles.[222]

Homeomorphism levels in language hierarchies are many, but according to linguistic functionalism, they often follow the same pattern of subordination. Functionalism specialists, including Tesnière, Chomsky, Dik, Jackendoff, and Givón, agree that this scheme can be represented as a continuous *chunking* in successive hierarchies, as suggested in ◊440. This type of hierarchical chunking, originated in a first single notion or *singularity*, owes to a priority for establishing frames of reference based on a central axis which is characteristic of the cognitive centrality of the individual, as Givón explains (2002:40–41):

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[222] This idea is developed in subchapter 4.8., under the concept *spiral of styles*.

> The most urgent adaptive pressure toward *automated processing* is the need to draw rapid conclusions about—and responses to—category membership, based on a relatively quick scan of few observable features. In a nut shell, *stereotyping*. The more prototypical members of a category, its bulk, tend to be processed automatically—fast and relatively error-free.

This practical function of the stereotype is also verified in the harmonic categories and the strata of musical metre, *summarizing* frequencies which are too long or too short, and adhering them to segments that can be identified more easily and quickly, as intervals of everyday use in speech or music listening. Using the name *phonemic restoration*, Aiello (1994:44) refers to a mental process, analogous to music and language, which permits the completion of sound structures under a categorical membership, strengthened by memory and contextualized use.

subordination level	hierarchical representation according to linguistic functionalism	representation as self-structuring symmetry
$0 \ (2^0 = 1)$		
$1 \ (2^1 = 2)$		
$2 \ (2^2 = 4)$		
$3 \ (2^3 = 8)$		
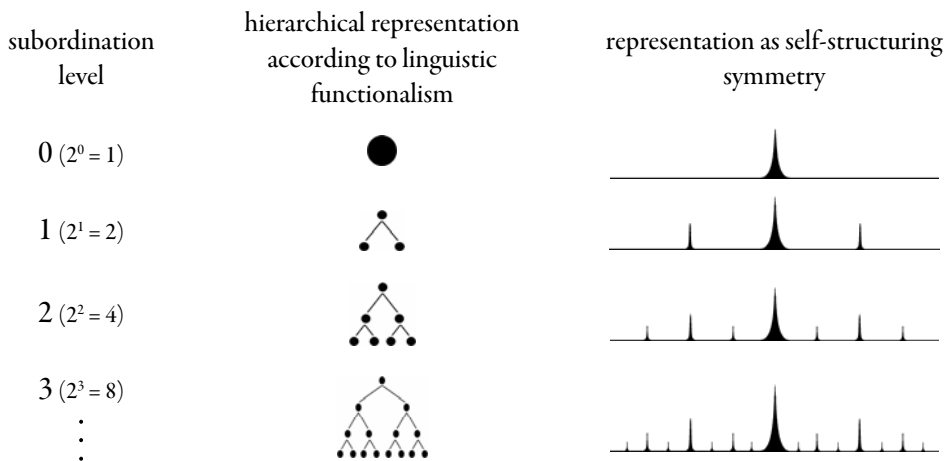
◊440. Comparison between subordination levels and dyadic implication in linguistic structures within a self-similar continuum. In the left column, the number 2 between brackets-left represents an element implicating another element or *subordinate*, followed by an exponent (indicating the level of subordination) and an equivalent (indicating the number of subordinates at the lowest level).

Josephson (1995:282) believes that the processes of musical generalization involve grammar structuring: "[In music], correspondingly in the case of language, rules and generalization procedures appear to relate to grammatical categorizations

rather than to semantic ones".[223] The cognitive strategies for generalization, stereotype and automated reconstruction, comparable across different aspects of music and language, ordinarily enable a *typing* of functional recursions to attribute sense to perceived relations in a system of signs, based on the intuition that such a system operates regularly within a categorical membership.

Language's self-organization

Recursion, defined in 3.7., provides functional openness for the combination of the parts of a motif or a phrase. This applies similarly to music and to language. For instance, in verbal language there are precise directions for the conjugation of the verb 'walk' in the third-person in singular simple present; but there is no specific limit to use the verb preceded by a proper name. The expression 'x walks', thus, can just as easily be expressed as 'x_1 walks', 'x_2 walks', 'x_3 walks', and so on, indefinitely. Also indefinitely these expressions can be followed by a complement, so that it is possible obtaining any grammatically permissible combination between verb, subject and object. No matter the realistic sense of the resulting expression. The point here is that the pragmatic and expressive use of language indefinitely allows a wide range of elaborations—as happens in poetic language, in humoristic ecolects, and in everyday speech—ensuring a potentially infinite richness by recursion. Gal and Irvine (1995:974) identify this feature as *recursiveness*, and associate to it self-similar structuring in subcategories and supercategories within 'creative' relations:

> *Recursiveness* involves the projection of an opposition, salient at some level of relationship, onto some other level. For example, intragroup oppositions might be projected outward onto intergroup relations, or vice versa. Thus, the dichotomizing and partitioning process that was involved in some understood opposition (between groups or between linguistic varieties) recurs at other levels, either creating subcategories on each side of a contrast or creating supercategories that include both sides but oppose them to something else. Reminiscent of fractals in geometry, and of the structures of segmentary kinship systems (as well as other phenomena involving segmentation), the myriad oppositions that can

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[223] Studying this tendency to stereotype as 'generalization procedures' in typical structures of tonal music, Lerdahl and Jackendoff (1983) explains how such a tendency leads to a widespread use of preference, formedness and correction rules.

create identity may be reproduced repeatedly, either within each side of a dichotomy or outside it.

Recursion, whether grammatical or pragmatic in its orientation (subjected to variables 'unforeseen' by the initial rule), is actually a property of self-similar structures of language in general. The stress that Gal and Irvine make on a quasi-fractal analogy regarding language seems also justifiable, thus, to define musical recursiveness.[224]

Lerdahl and Jackendoff (1983:214), based on the Chomskyan (1955, 1957) model of linguistic generativism, explain the logic of musical recursion as follows, referring to the tree presented in ◊441-*d*:

> An event e_i is a 'direct elaboration' of another event e_j if e_i's branch terminates on e_j's branch. An event e_i is a 'recursive elaboration' of another event e_j if it is a direct elaboration of e_j or if its branch leads upward through a sequence of direct elaborations to e_j's branch.

In this scheme (see ◊441-*d*) there are at least three levels of self-similarity: the lower bifurcations (from e_1 to e_6); the recursion's second level $\{e_3, e_4\}$ within a third one $\{e_1, e_2\}$; and the group $\{e_1, e_2, e_3, e_4\}$ within another group X. This sort of structure is infinite, given that X is a subset of U, where $U\{X_n\}_{n=1}^{\infty}$. This form of self-similarity is analogous to the cases of absolute self-similarity mentioned in subchapter 3.3., for the actual iterations happening in the recursive networks—i.e. not only for their potential features, related to limited self-similarity.

In tonal music grammar recursiveness is easily verifiable. For example, there are specific rules to form the triad of the E♭ chord, and to relate it to its degrees IV and V, to its inversions and to related tonalities. But there is not any rule, invariable or generalized, to sanction its exact length, nor an absolute rule that dictates the number of omissions or repetitions of its relations with other chords. So the preceptive rigidity to configure the E♭ chord with its main harmonic functions, combined with flexibility to use it in context, permits the use of that chord within a potentially infinite variety of tonal relations: with hierarchical functionality (such as the E♭ chord within its own tonality), with relative functionality (in sympathetic tonalities), or in a function completely subordinated to another chord (E♭ as subsidiary chord).

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[224] In order to clarify the subtle differences between 'recursion' and 'recursiveness', it is advisable to consider what is stated in subchapter 3.7.
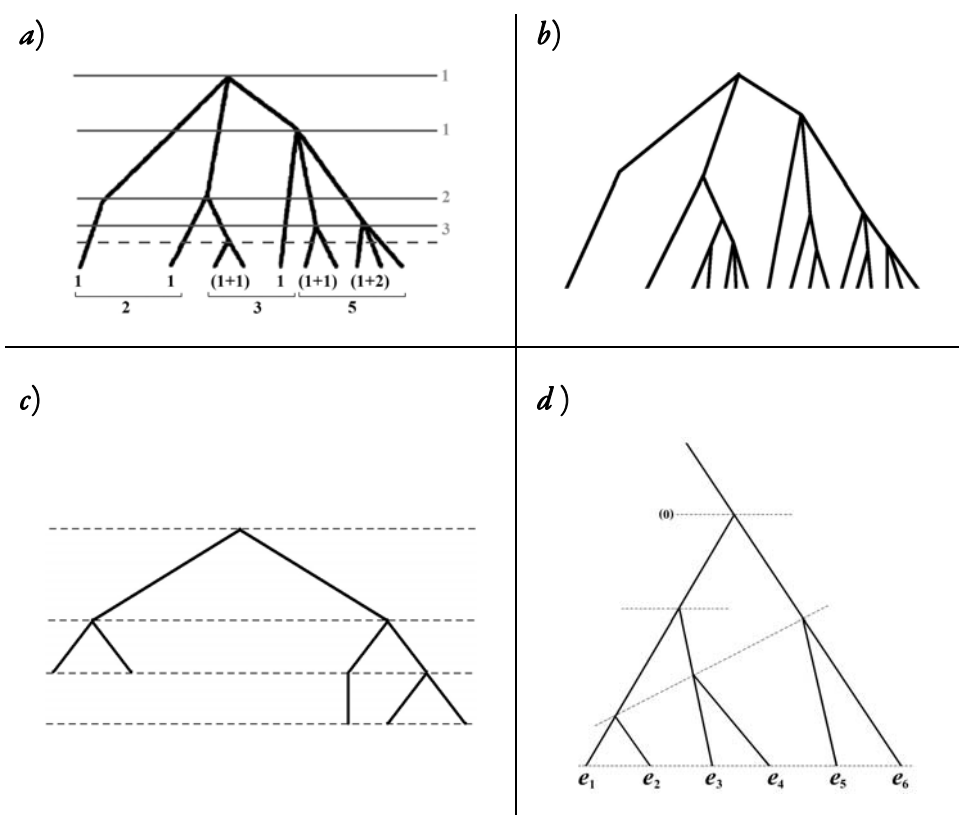
◊441. Derivative organization arising from language structures as self-referential processes:

a) 'Self-development' model of mental categories proposed by Charles S. Peirce (1903a); adapted and refined as self-referential structure in Pareyon (2010b). The horizontal lines indicate stages of recursion, using the Fibonacci sequence as a constructive indicator (not in Peirce's original text).

b) Development of the previous case (*a*) following the same constructive principle (from Pareyon 2010b).

c) Grammatical system for a sentence structuring, according to Chomsky's (1956:116–118) linguistic model.

d) Development of the previous case (*c*) adapted by Lerdahl and Jackendoff (1983:214) as a model for the analysis of musical hierarchy. This case presents five recursions derived from an initial recursion, labeled e_6. Dotted lines indicate stages of recursive self-similarity. In (*d*) the symbol (0) denotes the pre-self-similarity level.

Recursiveness is not unique to tonal music. It can be found in any system within a context of creativity, through a certain *pragmatic self-similarity* in which the infinite levels of subordination from a precise rule, depend on the election of an 'appropriate' function f regarding other $\{x_1, x_2, x_3...\}$ associated functions. That is: $f\{x_n\}_{n=1}^{\infty}$. An example of this may be a serial music row in which pitch repetition is determined by a rule of permutation, in combination with a flexible rule, which, for example, may affect length, loudness, or timbral association. In this sense, the richness of the serial repertoire is limited to the same principle of balance between flexibility and rigidity found in parallel contexts of creativity, as in psychological recursions or in the most general biological endorhythms. This is why integral serialism could not totally abandon its open criteria for selection, in combination with closed criteria. Absolute determinism exhausts the contingency of pragmatic self-similarity and fatigues musical discourse.

In an example using two simple instances, (1) a fixed rule f (*axiom*) and (2) a 'flexible' variation of the frequency (*recurrence*) of an element x (e.g. a point), is easy to observe how a series of complexities is produced: let f correspond to a sound grain and x correspond to its undetermined recursion in a series of time (*length*), representable as $x_1, x_2, x_3...$ in successive iterations of the function $f(x_n)_{n=1}^{\infty}$. A random segment of the series, horizontally represented, would be as follows:

$$\infty \longleftarrow\text{[illustration]}\longrightarrow \infty$$

◊442a.

where each point represents a fixed sound grain, succeeding in variable clusters or horizontal segments representing durations. After this case an infinite number of combinatorial operations can be computed: given that this sample is part of an infinite series, then the series contains infinite identities (i.e. points) x_a, x_b, x_c ... where subindices represent different consecutive lengths. Then, the ordered set of these lengths can be partially represented by the following subset (note that the leftmost column is made of 'consecutive lengths' represented as natural numbers 1,2,3,4,5... where each unit takes the place of a point):
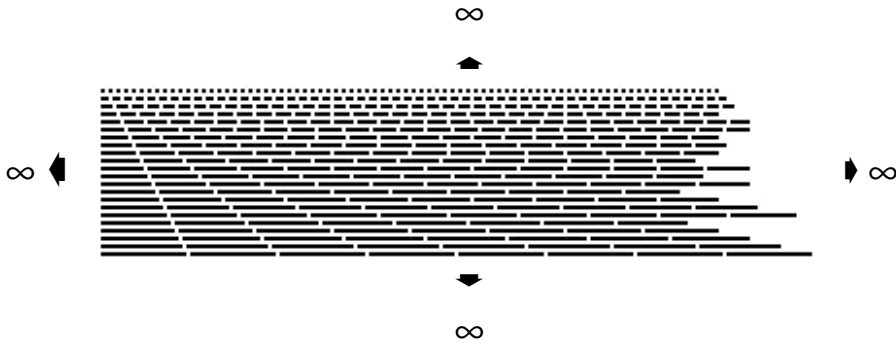
◊442b. *Aggregate pattern* segment as ordered set (◊442a represents a random unordered segment). The symbol ∞ represents infinite continuity of the pattern moving downwards and rightwards, and suggests an equally infinite symmetric reflection leftwards and upwards.

Because of its self-similar characteristics, its schematic construction and easiness to form complex designs with combinations and transformations of itself, this pattern is reintroduced in Chapter 6 (see graph ◊620, page 338). Within this section, it serves to illustrate the discussion started in the Introduction and continued in subchapters 3.3. and 4.4.–4.8., on language and its representation as actual infinity.

Language: an insight into its self-similar vastness

The question 'what kind of self-similar object is language' cannot be answered in a compact and unique form, because its self-similarities result *within* and *between* relationships in different layers, from basic coding sequences, to the elaboration, preserving, transmission and transformation of messages in cultural diversity. There is not a single form or a unique family of language self-similarities, but many. Moreover, there are language relationships which are analogous to examples of a *deterministic fractal*, as happens in language's infinite recursiveness; and *stochastic fractal*, as happens in the structural self-similarities between syllable, sentence,

discourse and style.[225] Language, in its turn resulting from a self-similar biological structure (the nervous system and the vascular arborescences), is a self-similar system branching from another self-similar system.[226]

There cannot be, however, a totally closed-in-itself conception of language just like a deterministic fractal, but through abduction (compare with Peirce 1903a). The opposed argument, implicating pre-programming language within all its details once it is known at all in its elements and functions, is a fallacy. This sort of positivist argument had been challenged before the advent of the fractal conceptualization, by Gödel's Second Incompleteness Theorem (1931), which states that "the consistency of a formal system cannot be proved within that system". This does not mean that it is impossible to create and develop artificial languages like those used in computing; rather it means that it is impossible to create a model representing all relations of language, because such a model would be much more complex than the language itself.

Notwithstanding anything stated in the previous paragraph, it seems obvious that the study of language through its self-similarities may help to understand some language relationships that could not be analyzed using traditional tools of linguistics. Contrary to the Saussurean description of language, the notion of language as self-similar complexity suggests that there are no arbitrary features of language, since even its accidents and irregularities come from deep structural relations (especially within a musicological context, see Monelle 2000:12, 66, 148). In contrast to a neo-positivistic 'fractalism', this concept of consistency or non-

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[225] However, this kind of relationship, according to what is expressed in section 1.3.4., is not properly a fractal, but a 'cuasifractal' system with statistical self-similarity.

[226] "A virtue of the fractal approach to anatomy is that it shows the above requirements to be perfectly compatible. A spatial variant of the Osgood construction described in the section before last fulfills all the requirements we impose upon the design of a vascular system." (Mandelbrot 1977:150). After Studdert-Kennedy *et al.* (1970), A. M. Liberman and Mattingly (1985), Studdert-Kennedy and Goldstein (2003), and other reviewers of the *Motor Speech Theory*, it is quite obvious that language cannot be reduced to an anatomical analysis. But the functional coherence between sensorimotor structure and language structuring cannot be disregarded. As E. Newton (1950) and Hall-Craggs (1969:368) realize, "Biologists will not disagree [that] *function creates form*". Thus, if there is adequate evidence on a correspondence between the evolution of anatomy and the development of language functions, the virtue proclaimed by Mandelbrot also has relevance to language.

arbitrariness between sign and form, and between code and message, promote the transition of a closed theory to an open one, in that symbolism is attached to the aesthetic and emotional loading of language. Within a so complex intertwining, such a load is preserved regardless of the scale in which language phenomena are observed.

Hřebíček's perspective

Luděk Hřebíček proposed, in the nineties, an update for theoretical linguistics, assuming scientific knowledge that could impel a recontextualization for the study of language in the light of new concepts of dynamical systems. Hřebíček (1994, 1997) pays special attention to the issue of linguistic self-similarity. Yet assuming the validity of the notion *natural fractal*, coined by Mandelbrot (1982), Hřebíček published his article "Fractals in Language" (1994) as an attempt to explain the Menzerath-Altmann law, with relevance to the theory of phrases extension or theory of *sentence aggregates*. This law applies to the discrete probability distribution in the frequency of data which can be syllables, words or phrases in a text.

An aggregate denotes a group of sentences in a text containing a word or *lexical unit* (Hřebíček 1997:104).[227] The Menzerath-Altmann law states that "The longer an aggregate (in the number of sentences) the shorter its sentences (in the number of words)";[228] this implies a tendency to concentrate the references structure of a text, towards a compact group of units. This law holds in the phrasing level and in the parameters for the length dependencies of sentences counted in words and syllables, contained in turn by the phrasing length counted in sentences. This theory has been applied to samples with different verbal languages in a variety of styles at different structural levels, "so that it can be accepted as a well-tested theory" (Hřebíček, *loc. cit.*).[229] Hřebíček (1995, 1997) suggests that the fact that the Menzerath-Altmann law is verified jointly in levels of syllables, words, sentences and text phrasing, permits

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[227] This definition of 'aggregate' is operatively affine to its homonym used in pitch-class set theory, as defined in subchapter 3.4. (see pages 81–82).

[228] See Altmann 1980; Altmann and Schwibbe 1989; Köhler 1986; abridged and reformulated in Hřebíček 1997.

[229] Later Kulacka (2008) found that this law is not verified in all texts, but this non-fulfillment may be attributable to a too segmented sampling of texts with recursive features, without giving special preference to ordinary verbal samples.

conceptualizing language through specific distribution features tending to self-similarity.[230] At the conclusions of his 1995 article, Hřebíček acknowledges that language can be a self-similar system, or an affine conglomerate whose self-similarity would be a simple consequence of the statistical approach to measuring the lengths of the constituents of language; results obtained by Kulacka (2008) point to this second direction.

In his 1997 article, which revises and expands some of his previous ideas about linguistic self-similarity, Hřebíček also takes into account another meaningful aspect: it is likely that each compound of aggregates does function following the scheme of initial axiom and application of rules; then language constituents and their levels of organization could be explained as emergent systems sensitive to initial conditions. As in dynamical systems with these characteristics, generation and phrasing extension would depend on stable initial conditions turning into chaotic behaviour.

Very probably, a corresponding version of the Menzerath-Altmann law is fulfilled in the sequentiation of musical motifs and phrases: the longer a string is obtained by a system of rules, there will also be a greater tendency to segmentation, by common rules of agglutination which are reflected by the statistical analysis. Therefore, syntax segmentation in music would be analogous to verbal language segmentation, following a common tendency to compact subsets of significant elements into larger sets, oriented by the same organizing pattern.[231]

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[230] According to Hřebíček (1997:103) "The so-called Menzerath-Altmann law represents an application of the power law of the type $y = Ax^b$, where x is the construct, y the respective mean constituent and A and b are parameters."

[231] This has been explained from the generativist perspective by Lerdahl and Jackendoff (1983) and Lerdahl (2001), under the concept of *well-formedness rules* (see subchapter 3.2.).

4.5. Stylistic endomorphisms

A musical endomorphism (ἔνδον, inside or within, and μορφή, form) implies a nestedness of objects within objects or processes within processes, as usually happens in a musical style, made of sets of sets.[232] There are many examples of such a nestedness within different musical traditions. As Meyer (1987:24) observes, "within the dialects of the Baroque and Classic styles, subdialects may be distinguished." Of course, this is not unique of the European tradition. Nettl (1954:46) identifies the same kind of nestedness in the autochthonous music of the Americas: "Musical areas [in the North American continent] could be identified at various degrees of homogeneity; [...] even each sub-style within a tribal style, could be considered equivalent to a musical area".

An example of musical self-similarity at this level is the statistical record of a specific tone within a scale; a scale within its traditional use in a specific repertoire; or a gesture repeated and expanded within a musical piece and perhaps within a large set of musical pieces within the same style. Musical endomorphisms are also found in many other aspects of musical recursion. It can therefore be made an account of the *pizzicati* in the violin technique after Tartini, as an stylistic trait from idiolectal origin. Other features can then be added, historically overlapped or transformed by Paganini, Sarasate, White, and other virtuosi from the nineteenth-century. A subsequent layer in this account can add the pizzicato Bartók as a modification of the original technique. At the end, an image of overlapping tendencies or endomorphisms will be obtained, defining a style on the violin, and within this style, other styles formed at the bases of other styles. Something similar can be stated on any other instrumental technique indicated in a score or suggested by practice, in relation to a historical *antecedent*; its respective *consequents* will produce structural layers gradually becoming dense in a context of relationships emerging in the musical discourse.[233]

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[232] A detailed definition of style is discussed along subchapter 4.7.

[233] Under this conceptualization, Segond-Genovesi (2009:247–260) proposes a "legitimation" of the statistical deviation (*légitimation de la déviance*) into two large semiotic bodies considering (1) the polysemies of the principle of repetition, and (2) musical syntax and paratexts.

Roughly speaking, musical endomorphisms behave like the alluvium of the bed of a river does, gradually or radically modifying its patterns depending on the fluid dynamics and sedimenting processes. In this paradigmatic analogy, the 'river' is the metaphorical representation of the 'style'.

Adapting the Saussurean structuralism, musical endomorphisms can obviously be analyzed as processes of language in synchronicity (e.g. the use of a tone in a specific harmonic function) and diachronicity (e.g. the use of an instrumental technique along the history of a style). In any case, they emerge with other kinds of self-similarity, as described above, of a mechanical, biological and ecological type, as precondition of music as a cultural phenomenon. There is, moreover, an analogy between a music endomorphism and the acoustic self-similarity of a resonance: any consequent may be associated with the resonance of its antecedent, so—for instance—the structural function of a chord is also a 'resonance' of prior *referential chords-layers* within a score, a repertoire, a tradition, a culture.

Parallels with spoken language

From the level of articulation, the self-similar features of language are more intuitive. For example, an endomorphism perceived as self-similarity between phonetic-prosodic relations of an idiom—its *timbral* unique features—involving the formation of styles within a certain verbal language.

Spoken language works as a self-similar network shaping the rhythms, intonations and cadences of speech. From specific recurrences of phonemes and accents, there is a corresponding sound 'image' of speech, not only by the sequence of lexical items with basic meaning (the typical semantic function of morphemes-phonemes), but particularly by the suprasegmental sound elements that do not have lexical meaning but are essential to meaning (see Hjelmslev 1928).[234] One may add to

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[234] In this context, after Hjelmslev (*cit.*), lexicology states that the sound x may lack a specific or isolated meaning. However, eliminating x from the verbal repertoire (also called *dictionary*) directly affects the meaning of many words. This occurs similarly in many cases of music, when asserting that an isolated element x does not mean or denote anything (Umberto Eco [1976:88] says that "The problem is 'what' the note C denotes, and 'whether' it denotes at all"); nonetheless, whether the same element is extracted from a contextualized system, the musical meaning may completely change.

this perspective, that the particular emission of a phoneme *x* in each speaker contributes to identifying the individual way of speaking of each individual (i.e. idiolect). In the long term, the sound image of speech reveals a consistent pattern of phonetics. At a second level there is a convergence of phonemes related within a vernacular style (i.e. *ecolect*), in which the individual speaker is in contact with a group of speakers. The phonetic traits of the individual are not identical to the local group—something that would eliminate basic aspects of originality and identity— but overall they share sufficient characteristics to integrate a certain phonic consistency. A following self-similar system emerges with the integration of regional dialects, articulated not necessarily by semantic values but by phonetic traits in common. One may add to this the successive cycles between local and global, as sound levels are distinguished in layers forming a social niche.

The whole these phonetic relationships make, constitutes a self-similar complex. Complexes alike, observing them from the surface of the dialect or the global language, to the depths of the individual expression, can be described as sets of frequencies defined by phonic endomorphisms: internal forms of individual soundings—idiolectal phonetics—weaving the 'phonic cell' of the verbal tissue as a whole.

By analogy, musical discourse also presents this kind of self-similarity, which in the long-term determines stylistic contour. The *idiolect-ecolect-idiom* string also happens in music as transmission and expansion of emotions and conceptualizing through sound-in-culture, with the ability to express, symbolize and represent within cyclical negotiations—that simultaneously are closed by rules, and open to chaos in their practice.[235]

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[235] Luhmann (1990:12) acknowledges that "The problem, then, becomes to see how autopoietic closure is possible in open systems. The new insight postulates closure as a condition of openness, and in this sense the theory formulates limiting conditions for the possibility of components of the system." From a linguistic (statistical) view, Hřebíček (1997:105–106) also considers a productive exchange between rule and variation, or between order and chaos, in the open elaboration of sentences based on a closed grammar.

Recursiveness and interpretation at the basis of musical creativity

For language in general and for music in particular, the notion of repetition is akin to that of absolute determinism, as it involves a purist idealization of temporal cycles and spatial forms, which can be replicated, for example, in an equilateral triangle or a circle, retaining their intrinsic abstract properties. Hence the idea of repetition is familiar to that of a perfect closed cycle, by contrast to the image of recursion, which, as Marrades Millet (1998:54–55) observes: "implies doing every time something different from what was originally shown, but in such a way that the result equals to act in each case under the same rule."

◊450. Idealized scheme describing the difference between *repetition*, visualized as a perfect cycle, and *recursion*, which, through interpretation, 'produces something new' from its self-reference. In this case the phenomenon of self-generation or autopoiesis is represented as asymmetric deviation or irregularity bounded by the cycle's regularity (i.e. the recursion, properly).

For there to be interpretation there must be recursion. In other words, for there to be music as 'living expression', music must express the fact that it occurs through the unstable capabilities of the individual and the conditions of context, unstable too. Recorded or completely automated music—as any fixed language—can inform, but is unable to express the unstable interactions and the emotional, pragmatic and intentional traits which characterize the richness of living language.[236] 'Living language' also means, in this case, recursiveness: an open system which is changing from an agreed, relatively closed and stable grammatical model.

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[236] This idea implies the opposition between variation and repetition. Lidov (2004:40) notes, in this context, that this opposition plays an important structural and creative role in music.

◊451. Five versions of the traditional Chinese song "Wu-a-hei-hei", performed by five different native singers. The subtle differences in the melodic-lyric gestures are typical examples of recursiveness as interpretative variation. The measurement of the slight, continuous differences, reminds the calculation of metric space in fractal geometry (see e.g. Hřebíček 1994, 1997). Original manuscript by Frank Kouwenhoven (2005:148). Copyright by the *Journal of the Department of Ethnomusicology*, Otto-Friedrich University of Bamberg. Reprinted with permission.

Paradoxically, deterministic self-similarity in a symbolic system—usually a sign of structural consistency—may inhibit information richness and communication.[237] For the projection of intentions and emotions of *individual language* (i.e. idiolect), the links of similarity in a musical structure must be 'not absolutely exact'. If a musical expression results identical to another, the boundaries for comparison, analogy and resemblance diminishes, giving up their places to monotony (i.e. low level of information).[238] By contrast, limited self-similarity allows the interplay of reference,

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[237] This notion is developed in subchapter 6.6., in the context of concepts such as asymmetry, antisymmetry and self-dissimilarity; and in subchapter 5.1., in the context of musical information.

[238] The successive copy of identical models is a procedure that impoverishes the representation of the 'real world' because real world pattern's irregularity is more complex than the exact copy. For each copy of an original pattern, the 'real world' inserts some type of interference which alters the exactness of the reproduction. This process creates, by a symmetric transformation, relationships of affinity and similarity, rather than sequences of equalities.

difference and comparison, enriching the discourse without losing it into the unlimited diversity.

Recursion of individual elements within a formal or prescriptive grammar is also related to the concept of 'authenticity' of language: an idiolect is authenticated not only by obedience to conventional rules, but especially with the contribution of the individual's own characteristics. This universal requisite of language is also evident in musical performance and creativity. The example in ◊451 shows a traditional Chinese song in five different versions by five singers, compiled and transcribed by Frank Kouwenhoven (2005:148). Despite the cultural specificity of this example, this sort of variety within similarity is common to any musical performance in any cultural context. This reveals musical self-similarity in two essential aspects: (1) direct relationship with recursion, which, adding features *in style* by the use of a musical code, uprises the codal information with the richness of the idiolectal message, and (2) consolidation of musical sense in that, adding unique features, each version produces a set of elements sharing statistical distribution of frequencies, lengths and loudnesses at different rates.

The perspective of Beran and Mazzola (1999a)

After computing the statistical analysis of scores of European composers from different eras, and comparing their results with an associated selection of recordings by various performers, Mazzola and Beran (1999a) claim that musical performance is a particularly complex process, inter alia, because of the abundance of ambiguous information contained in a musical score (including terms for change in tempo like *ritardando* or *accelerando*, or dynamic marks like *ppp* or *pp*, etc.), and because of 'hidden information' that is generally not explicitly included in the score, but still closely related to the articulation of figures, metre, accentuation, and the irregular 'details' variating in each performance, which are stylistically meaningful.

Beran and Mazzola, aware of the analytical difficulties for a general definition of style in Western classical music, restrict their research to the statistical analysis of metrical, melodic and harmonic values in the score, in order to determine these values' influence on musical performance, especially by their correlation with tempo

curves or deviations (*Agogik*) which characterize an essential aspect of authenticity for each performance.[239] Overall, their analytical approach aims to identify the specific set of symbols that make up each score chosen, separating them in functional categories, for obtaining, in place of the score, three correlated sets with metric, melodic and harmonic values. Finally, Mazzola and Beran allot specific 'weights' for each of the different profiles and levels considered as performance values.

This analysis, like other methods used to assess the interpretation of Western classical music, is based on hierarchical criteria: a note in the score is statistically significant if it is part of a local metrical subordination; a tone is melodically significant if it is part of motifs or phrases in a comparable way as in (many) other parts of the same piece; and a harmonic relation is especially significant if it coordinates or subordinates other harmonic relationships, in the sense of a prescriptive grammar (e.g. Rameau 1722, 1726; Riemann 1873; Schoenberg 1911; Hindemith 1949). However, the most original aspect of Beran and Mazzola's investigation is their proposal to examine how the distributions of 'weights' are correlated in the metrical, melodic and harmonic structures, regarding the tempo deviations of each performance. Interestingly, their results confirm the hypothesis of variability in music performance, as recursiveness of an open system providing solid empirical evidence for the correlation between self-similarity and musical consistency:

> A musical piece (and hence its performance) is musically 'coherent', if each or most of its parts are similar (or 'self- similar') in some sense to other parts, possibly on another time scale. In particular, what happens now should be related somewhat to the entire past (and future) at various levels of 'resolution'. The fractional differencing parameter *d* is directly related to the so-called self-similarity parameter of self-similar stochastic processes. The value of *d* is a measure for the type of self-similarity and can therefore also be interpreted musically. (Beran and Mazzola 1999b:226; brackets in the original text).

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[239] This conceptualization of 'curve' or 'deviation' needs to be understood here as the affinity between two or more behaviours tending to self-similarity. Actually, the definition of 'fractal' proposed by C.A. Pickover (1988:181) approaches intuitively to this notion: "fractals [are] intricate curves that exhibit increasing detail with increasing magnification". Several examples of self-similar curves in music are provided along chapters 5 and 6.

These results can hardly be considered unique and exclusive to Western classical music—as discussed in the previous section with Kouwenhoven's (2005) study on recursiveness in traditional Chinese songs. Accordingly, it is hypothesized that every time a performer displays her/his own recursiveness traits, an 'autopoietic exercise' completes a cycle of *minor* irregularities within the *major* regularities characterizing a repertoire *in style*, ensuring through practice the creative richness of musical tradition in the long term. Such an autopoietic exercise consists of the performance fitness, acting 'freely'—even chaotically—and creating meaning within a much more stable environment for grammar and stylistic orthodoxy, equal to previously created—even formalized—meaning.

4.6. Transcultural self-similarity

The recursiveness of cultural typologies appearing in *Umwelt*–niches imply certain relations of self-similarity.[240] For music, the semiotic modes of this self-similarity are related to the *music universals* in the context of generalized *universals of language*.[241] Within this framework self-similarities emerge for reasons related to those at biological, phonological, and pragmatic levels; and, as suggested in 4.4., they gain density through the phase *communication→ language institution→ social life* (see Rhees 1954/1971:69; Osgood 1964).

Haugen (1950), Weinreich (1953), Paradis and LaCharité (1997), Thomason (2001), and Good (2008), among other linguists specializing in the investigation of phonological borrowing, provide elements to consider that music and verbal language share, if not the same types of borrowing, indeed the same trends for borrowing in various strata, particularly visible on the pragmatic surfaces. The most prominent differences between musical and linguistic borrowing are found—as in other similar analogies—within relationships of meaning and use 'in social life', according to Rhees terminology (Rhees 1954/1971:69). These differences are already

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[240] For a definition of the coupling *Umwelt*–niche, see subchapter 4.3.

[241] On *music universals*, see Merriam 1964, Nattiez 1987, Padilla 1998, and Tagg e1999. On the *universals of language* see Trager 1958, 1961, Sebeok 1977, Diller 1981, and Martinelli 2007.

defined by several lines of research.[242] For purposes of this section it is sufficient to note that music shares with language a transfer of uses, symbols and styles in patterns that recur at different social levels and at different times and idiosyncratic circumstances.

Musical borrowing

In the case of music there are structures which reappear as 'self-borrowing'; this is fairly obvious for many scores of Vivaldi, Haendel, Haydn, and W.A. Mozart (see Stephan 1979, Escal 1981, Brown 1992). Other structures are repeated within the same tradition, from one context to another, for instance in the transition of folk songs and dances to the concert repertoire, as happened with the parts of the Baroque suite which originally were not intended for chamber music. Some cases of this are the gigue, the courante, the contra-dance, and the passacaglia. Some other musical structures move from one culture to another, as happened with the chaconne, a Mexican dance which around 1598 became a street dance in Seville and Naples, and later became a court dance in Rome and Florence, and ended up as part of the baroque instrumental suite in Germany and Denmark.[243]

Musical borrowing occurs in a variety of degrees and forms, ranging from direct copying to the transformation of the original. Burkholder (1985, 1994) identifies six functional variables of musical borrowing: 1. Context (genre, texture [monophonic, polyphonic], cultural source), 2. Relationship level (rhythm, harmony, melody, instrumentation, texture), 3. Relationship mode (between the borrowing and the new produce), 4. Alteration mode (complete, incomplete, minimal, recontextualized), 5. Musical function (basic structure, theme, phrase, motif, gesture), 6. Motivation (imitation, representation, symbolizing, association,

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[242] Among these lines of research the present study considers the following: Haugen 1950, Seeger 1960, Escal 1981, Lerdahl and Jackendoff 1983, Price 1988, Fiske 1990, Nattiez and Abbate 1990, Monelle 1992, Tarasti 1992, 1994, Aiello 1994, Burkholder 1994, Davies 1994, Lidov 2005, and Almén 2008. They all have antecedents and consequences that would be impossible to summarize here.

[243] Musical documentation on this subject is abundant. For a general approach, see Richard Hudson, *The Folía, the Saraband, the Passacaglia, and the Chaconne* (vol. IV. *The Chaconne*), Musicological Studies and Documents, no. 35, American Institute of Musicology, Hänssler-Verlag, Stuttgart, 1982.

comparison, variation, paraphrasis, metonymy). In short, when the same musical structure partially or fully returns across different contexts and traditions, it can be said that it occurs in various degrees of self-similarity in one or simultaneously in several of the typologies identified by Burkholder, for each of their six main variables. This connects directly with what is stated in subchapter 3.9., complementing the notion of the intrinsic self-referentiality of music.

Codal exchange between species (interspecies borrowing)

For bioacoustical codes partly shared by different animal species there is also a borrowing diversity under any of the variables identified by Burkholder. Evidence of this is the imitation of a rhythmic or melodic pattern made in a human cultural niche, from a bird's vocalization. In return, it also occurs that a bird (*Amazona aestiva* or *Psittacus erithacus*, just to cite two common examples) may attain to imitate a rhythmic or melodic pattern produced in a human cultural niche.[244]

Despite the anthropocentric tendency to adjudge music only to humans, at least since the fourteenth century several composers and music theorists have accepted —adding qualifiers—that some birds are able to create and perform music. This corresponds to the description of such music as *musica avicularis* (Schröder 1639), *musurgia* (Kircher 1650), *natural music* (Tiessen 1953), *protomusic* (Hartshorne 1973) or *micromusic* (Szöke, Gunn and Filip 1969; Messiaen 1994).

Josephson (1995) suggests that at least some subsymbolic aspects of human music are part of a 'pre-existing' system, so that an individual would have acquired the system from other individuals and other societies, with multiple sources in nonhuman sound codes—Josephson mentions the participation of domesticated animals in the modern societies' environment. Accordingly, an amount of the *music universals* would have been 'discovered' rather than created by humans—something already hinted by Hsü (1993) in the context of musical self-similarity. Asking the

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[244] According to Gelman and Brenneman (1994:374), this exchange of musical patterns between birds and humans is possible, primarily by the functional similarities in the dynamics of the interplay between an innate cognitive platform and common processes of cultural learning.

question, how do humans acquire these codes, Josephson (1995:283) answers that this should happen

> by the same means as those by which human beings acquire language from each other, or to take a more asymmetrical case, pets from human beings (this process also involving shared meaning, shared utterances, and processes for acquiring some comprehension of symbol systems).

The renderings of musical codes can occur, thus, in transcultural relationships, strictly speaking, providing or modifying rhythms, pitch intervals or sound gestures across different species. Recovering a notion formulated by Abraham Moles (1963), this type of transmission of bioacoustical codes can be *unilateral* (without feedback information system) or *bilateral* (with feedback information system).

Modern biologists conceives the musical culture of birds as a relevant issue to explain the processes of teaching, acquisition, variation and preservation of idiolectal and ecolectal identity in communities of different species. Catchpole and Slater (2008:49–84, 265–270), in their treatise on bird vocalizations, devote an entire chapter to aspects of mimicry, song learning and variation, and a special subchapter on what they recognize as "cultural change", based on statistical information gathered and processed by themselves and by other ornithologists such as Eens *et al.* (1991a–b), Seddon (2002), and Seddon and Tobias (2003, 2006), among others. Conclusions derived from this analysis suggest that musical culture and its modes of transmission are not uniquely human attributes. This issue is not just a curiosity for musicology, since—as Szöke, Gunn and Filip (1969), and Hsü (1993) acknowledge—it raises many questions about very basic concepts of music, aesthetics and ecology, which have not previously been satisfactorily answered:

> The mere fact that such highly structured human-*like* avian music *exists* as a *biological* (and at the same time physical, neurophysiological and audiopsychological) phenomenon *outside art and aesthetics*, raises cardinal problems concerning theory and specialized science to zoologists as well as neurophysiologists, to animal psychologists as well as theoreticians of human musicology and music as a 'world phenomenon' in general. (Szöke, Gunn and Filip 1969:432)

It is clear that this concern deserves a long-term and specialized study. For now what is most remarkable is to note that the transmission of musical patterns from one culture to another is possible with the participation of at least two distinct animal sources, taking for example the contact between birds and humans.

The symbiotic relationship between birds and humans is ancient. Many prehistoric representations carved in stone or drawn allude to bird hunting scenes or magic scenes showing birds in a first perspective. The beginnings of ideographic writing also reflect this relationship. René Guy Busnel (1963:88), impressed by these testimonies, in his compendium of bioacoustics includes a scene of the Egyptian tomb of Menna, dating from around 1200 BC, showing a young hunter holding a bird in his hands to attract his prey using bird vocalization.

Notwithstanding, the musical exchange between humans and other species is not just an old scenario. Guatemalan composer and musicologist Jesús Castillo (1877–1946) transcribed, around 1929, the vocalization of a tzentzontli (*Mimus polyglottos*) collected in the wild, in Huatal, department of Nariño, Colombia, which sang a melody "in the natural diatonic scale" (in A). This melody was very similar to that of the *zubak*, a Mayan tune to accompany the traditional dance of *La conquista*, which Castillo had collected in two versions, from Chicalahá, and from Almolonga, department of Quetzaltenango, Guatemala, about 1,300 miles to the Northwest of Huatal (see Mayer-Serra 1947:212, and Vela e1971:5).[245] Although Castillo was inclined to suppose that the local people took from the same bird (i.e. the same specimen) the constructive elements of their music, it is not inconceivable that the process had been inverted: that is, that the tzentzontli (i.e. several individuals from the same species), due to its imitative skills, had reproduced similar melodies in several places of the same area, after listening to human tunes—even the specific melody of the *zubak*. If this is true—or at least statistically plausible—along with the growing custom of incorporating singing birds to domestic life, the tzentzontli free or in captivity may had played a role as an information conduit from pre-Columbian

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[245] Vela (*loc. cit.*) reports that *zubak* is not only the name of the traditional tune but, interestingly, that is also a traditional Mayan flute to be performed with the homonymous dance: a bone small four-holed recorder which could be an ideal means to imitate a bird song.

times, modifying its repertoire as the human cultural trends had been changing in a common ecological context.

The acoustic contact between birds and humans also reveals self-similar recursion in 'unilateral transmission' (i.e. without *feedback information system*): Brandily (1982) documents a close relationship among four species of birds and the culture of the Teda people, in northern Chad. The researcher found that the Teda, precluded to develop systematic agriculture and livestock due to extremely poor soil, created a vocal communication system with birds, distinguishing four edible species (*Columba livia targia, Streptopelia turtur, Quelea quelea* and *Pterocles lichstensteini targius*). Crediting each species with a local name and a repertoire of songs and symbolic concepts, the Teda created thus a complex of cultural values permeating their musical tradition and poetic texts. Brandily (1982:373) notes that "Word-by-word translation of such texts cannot convey directly their meaning to anyone unable to place them in the same experiential context", and examines the functional relationship between the symbolic and phonetic elements of these materials, noting that in some cases there is a recreation of the onomatopoeic sound of the birds, but in others there is a stylization of their musical pitches and rhythms.[246] In short, this embraces the two typical operations of musical borrowing: *copy* and *transformation* (*theme* and *variation* in a melodic context; *repetition* and *recursion* in an informative one), which are fundamental for music as language. According to Class (1963:138), the whistled languages of many peoples of herds and hunters dispersed in the world, are "the informative skeletons of language [...] and constitute a remarkable intermediary between the signal systems such as suggested in the study of animal species, and the normal human languages."

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[246] In this context, it is impossible to ignore what Gérard Genette (1976:165, 177) says: "L'onomatopée est donc un mot forgé par imitation d'un bruit extérieur (y compris les cris d'animaux), le mimologisme, un mot forgé par imitation d'un cri, ou plus généralement d'un « bruit vocal » humain."; "Le principe d'analogie par transposition, qui fonde la métaphore, est donc « l'element facultatif de la création des langues », tout comme le principe d'onomatopée est son élément organique et mecanique."

◊460. A common blackbird vocalization (*Turdus merula*) compared with two different kinds of European music. Every sample has been written in the same octave and same key in C (see text for an explanation). Source: J. Hall-Craggs 1969:377.

Hall-Craggs (1969:376) further suggests that the transcultural acoustic self-similarities shared by humans and birds can also occur in a convergent mode. Thus, she compares the vocalization register of a common blackbird (*Turdus merula*), with the opening phrase of the *Gigue* from J.S. Bach's Suite no. 3 in D, and then with an entire sea shanty, from J.J. Terry's *The Shanty Book* (London 1921). In order to facilitate this comparison (see table ◊460) Hall-Craggs puts all the examples into the same octave (C_4 – C_5) and the same key signature (in C). Every example is divided into two sections, as the material is compressed in two sub-phrases in Bach and the bird song, appearing decompressed in the shanty. By this comparison Hall-Craggs identifies the following similarities: equality of proportions in sections 1 and 2; change of tonal and temporal patterning in section 2 (much more pronounced in Bach); section 1 ends at a point of anticipation whilst section 2 ends at a point of finality; melodic outlines are similar and incorporate a "suggestion of pentatonicism" (sections 1: C' E G A [Bach *omits* the E]. Sections 2: C" G E C' [the bird *omits* the final C]); and the climatic points are the same (sections 1: the single A high. Sections

2: the single C high) [see Hall-Craggs 1969:375–376]. It must be emphasized that this is not to suggest a direct borrowing from one source to another, but just to open the possibility of *universals of tonal language*—'tonal' in the phonetic-phonological sense—by specific forms of organizing information, codal redundancy, and gesture expression, demonstrating the role of transversal synecdoche in these types of relationship.[247]

Scholes (1938), Armstrong (1969), Hall-Craggs (1969), Brandily (1982), and Head (1997), from different viewpoints, agree that bird vocalizations adapted by human communities, are a basic traditional source for the latter. Moreover, Thorpe (1963:191) conceives that "The fundamental intervals of human and bird song are the same", and makes the following suggestion (*ibid*.): "Since man always has bird song all around, impinging on his ears, is it not reasonable to suppose that he developed a musical signal system by imitating the birds?" Hsü (1993) contributes to an answer, pointing out that the same kind of systematic recursiveness and resulting self-similarity are due to partially homologous relationships, commonly present in both human music and bird's music.[248]

In conclusion, there may be, in terms of adaptation, an *exaptation* of songs being transmitted from a species to another, modifying its usage modalities, its character and its causal attribution.[249] Such 'exaptation' would be an evidence that during

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[247] This is partially confirmed by Hsü (1993:36): "A symphony of two varieties of blackbirds, accompanied by cuckoos or great tits, can have enough combination of tones to make a music of [quasi-]fractal geometry" (prefix in brackets introduced by the author of the present study).

[248] Hsü (1993), in continuity with Voss and Clarke (1975, 1978) and Voss (1987), attempted to prove that the self-similar spectrum $1/f$ is a general property of music, including some bird vocalizations. His experiment found, however, major obstacles: (1) the amount of samples analyzed is insufficient to compute conclusive results concerning a species in particular; (2) the variety of samples is equally scarce to draw conclusions, defining or generalizing features to all species of singing birds, (3) the samples analyzed correspond to transcriptions to the diatonic scale, not to high fidelity recordings made in situ: this problem is greater, considering that many singing birds produce multi-frequential tones, rich in timbre, and with a speed that usually surpasses the transcription resources. On the one hand, the notion of timbre as self-similar pattern is neglected by Hsü; on the other hand, melodic transcription already provides a considerable amount of *noise*, as defined at the beginning of subchapter 5.4.

[249] The term 'exaptation', common in revisions to Darwinian evolutionism, refers to the use and development of a cellular, physiological or anatomical function, other than the performance by the same capacity in which it originated (see Gould and Vrba 1982).

◊461. Examples of human imitation of ornitho-acoustic behaviour:

(*a*) Hoopoe (*Upupa epops*) vocalization according to Aristophanes (from *Ornithes*, 237 ff., 414 BC; in Greek characters). The music notation corresponds to a free transcription based on a recorded vocalization of the same bird species.

(*b*) Rooster (*Gallus gallus*) vocalization according to Athanasius Kircher (*Musurgia universalis* 1650).

(*c*) Theme of the traditional 'son' *El perico* [The parrot] (Veracruz, Mexico, *ca.* 1750–1770).

(*d*) Fragment of the habanera dance for piano *Los pollos tepiqueños* [Chicks from Tepic] (1874) by Clemente Aguirre (source: Pareyon 1998).

(*e*) Fragment of the first page of IV. *Chants d'oiseaux* (*Livre d'orgue* 1952) by Olivier Messiaen, with a stylized version of a common blackbird (*Turdus merula*) vocalization.

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thousand of years musical patterns transmitted from birds to humans have gone through a continuous process of evolution, comparable to the transversal synechism found in other instances of ecological, symbolic systems.[250] Subchapter 4.8. extends this issue to the synecdoche of *the house of the house*, connecting it to the conceptualization of ecology, as conceived by Parsegian (1968:589).[251] Accordingly, the definition of music as a "world phenomenon in general", proposed by Szöke, Gunn and Filip (1969:432), is an issue that also extends to the following chapters.

4.7. Tension between grammar and style

Leonard B. Meyer (1989:3), basing his ideas on criteria established by Jan P. LaRue (1969) and Charles E. Osgood (1964), defines style as "a replication of patterning, whether in human behaviour or in the artefacts produced by human behaviour, that results from a series of choices made within some set of constraints." In this definition the concept of *choice* results from the negotiations between individual and group, whilst the concept of *constraint* corresponds to the need for consistency through a *law* or grammar associated with a tradition.[252]

Despite its rigidity, grammar usually leaves 'gaps' in the regulation of language, as open margins to fertilize language through creativity and style practicing. The prescriptive character of grammar, in tension with the styles—of flexible nature—, is modified over the periods of a language, by a continuous negotiation between *rigid correctness* and *flexible creativity*.[253] This is the core of Meyer's idea about coordination between choice and constraint.

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[250] The concept of *intersemiotic continuum* is introduced in sections 3.8.1.—3.8.3.

[251] "a vast complex of living worlds within worlds, levels below levels, life beneath life" (Parsegian, *loc. cit.*).

[252] Aarts (2006:113) offers a broad overview on the plural definitions of grammar, as a constructive or constrictive system of rules in a language, mentioning aspects of syntax and morphology, among many others.

[253] Classical thought is eloquent on this particular topic. Horace (*Ars*, 70–71) notes: "Multa renascentur quae iam cecidere, cadentque quae nunc sunt in honore vocabula, si volet usus" (Many words which are now obsolete will be reborn and those now honoured will fall, if usage so decrees). This concept brings associated images of *looping, spiral, recursion* and *self-reference*, related to the notion of language self-similarity.

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Jan P. LaRue (1969) conceives style as a phenomenon of recurrence in several layers that he calls "dimensions". Within these dimensions musical objects and processes are grouped in structures, being both *contained by* and *containing* other structures.[254] LaRue (*op. cit.*:449) acknowledges, then, the typical processes of musical self-reference ("one can hardly discuss sources of [musical] movement without citing some musical element in a specific function"), and identifies the difference between *repetition* and *recursion*, associating with the latter the functions of "interrelations, interactions, and interdependencies of the music itself".[255] Congruently (*op. cit.*:450–451), he makes explicit the analytical-descriptive processes of music, as synecdochic processes: "once we comprehend the wholeness, the parts fall into a proper perspective."

According to Werner Winter (1960:3), style is characterized as a pattern of recursive selections from an inventory of 'optional' features of a language: "Various types of selection can be found: complete exclusion of an optional element, obligatory inclusion of a feature optional elsewhere, varying degrees of inclusion of a specific variant without complete elimination of competing features." This idea includes the notions of *fixed* and *optional* in language, and *rigidity* and *flexibility* of its recursions. Meyer (1956:202) conceives that the planned deviation from the rigid, as negotiation with the rule, opens the necessary margin for *creativity*:

> The unlimited resources for vocal and instrumental expression lie in artistic deviation from the pure, the true, the exact, the perfect, the rigid, the even, and the precise. This deviation from the exact is, on the whole, the medium for the creation [...]

The sense of music emerges, largely, thanks to a proper tension between grammar and style, or—using the terms of Meyer—between *exactness* and *planned*

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[254] LaRue's (*op. cit.*) semiotic perspective is clearly akin to Ch. S. Peirce (*CP* 8.122) model of the *map of the map*. It is also familiar to Peirce regarding the concept of abduction and the idea of the analytic-deductive processes as self-correcting processes: "the process is actually self-correcting: any redundancy that crops up in the observational phases will automatically disappear in the course of selecting the most significant characteristics for final evaluation" (LaRue 1969:449).

[255] In the source: "The general classifications and subdivisions inevitably produce some overlapping. For example, one can hardly discuss sources of Movement without citing some musical element in a specific function. Interconnections such as these, however, do not in any way produce a wasteful repetition or duplication: quite the contrary, they accurately reflect the interrelations, interactions, and interdependencies of the music itself."

deviation. Schuller (1986:157–158) offers a case in point for understanding this relationship: he presents a musical fragment written in two modes, one (*a*) as a parody of a grammar badly assimilated in its representation of rhythm, and another one (*b*) with the same substance as the previous one, although showing a more clear style, rigorously fitted to a traditional grammar. Both cases are correctly written, but only the latter results from a necessary negotiation between what is desired to be represented through proper musical notation, and what is representable as part of a traditional, yet relatively flexible style. Ultimately, this example (the conflict between *a* and *b*) resembles the ambiguous usage of many cases of written representation of speech. However, Schuller (*ibid.*) is not concerned only with music orthography; rather, he highlights a problem of style and idiomatic symbolizing of music in 'new' repertoire:

> Whereas a few years ago composers were still legitimately involved in exploring new rhythmic possibilities, I doubt that any further *reasonable* rhythmic figures can be discovered, and it seems to me that composers today should concentrate on finding those complex or "irrational" rhythms that can be incorporated into a practical repertoire.

The example Schuller suggests is a hypothetical instance of a piece written in a 'free' atonal style, comparable to many fragments that can be drawn from musical literature in the last third of the twentieth century. In this example Schuller believes that the case (*a*) exceeds the margins of stylistic freedom, breaking the musical grammar associated with it. In contrast, the case (*b*) of the same example, having the same pitches and same gestures as the former, and a similar distribution of pulses, achieves a more consistent language and a greater attachment to grammatical tradition, without sacrificing the essence of the style in which it is written. In the second case, a slightly more relaxed articulation coincides with a greater clarity in the figures distributed within the measure; even with a more realistic result for the nuances of intensity.

Like Schuller (1986), Schoenberg (1967, 1975) is also interested in clarifying the relationship between conventional rule and freedom of style. Even for seemingly simple details such as the notation of the *appoggiature*, Schoenberg (1975:309) seeks a satisfactory explanation for the links between grammatical correctness and creativity. His case study—again, like in the text written by Schuller—represents not only a

problem of writing, but rather relationships involving a complex of subjective values, often hidden in the roots of the idiolect, and in the dynamics between idiolect, ecolect and musical grammar. As Schuller (1986:278) notes, "The character and mood of a piece of music often cannot be fully expressed by notation alone. It needs some descriptive clues, and an acoustical realization."

As Ockelford (2005) suggests, the 'descriptive clues' that help us understand the relationship between style and grammar, between usage and musical language context, are found in the modes of repetition and similarity—with their corresponding suspension and difference—in a universe of musical samples. The methodical search for these samples has a variety of possibilities. Ockelford (*op. cit.*:38–41) concurs with the basic idea of Doležel (1969), about 'style as a probabilistic phenomenon'. Thus he formulates his own notions of *intraopus imitation* and *interoperative organization* to describe similar relations and functions within a set of structures sharing statistical trends. Ockelford (2005) conceives these relationships as part of a probabilistic continuum in which style is intertwined in a margin of prototypical operations. Monro (1995), Beran and Mazzola (1999a–b), and Yadegari (1992, 2004), establish the grounds for evolving from the identification of statistical biases of resemblance, to the investigation of *intraopus* and *interoperative* self-similarities. Adopting Ockelford's (2005) terminology, such self-similarities can be seen as systems of consistency in the transmission of the musical message, through stylistic continuity. In other words, in the investigation of musical style, the criteria of *similarity measure* and *dissimilarity measure* (see Ilomäki 2008:35) have passed, as generalized qualitative referents, to the criteria of *self-similarity measure* and *self-dissimilarity measure*, as referents of style.[256]

Doležel's (1969:13–16) study brings to light significant aspects of musical style research: indices of random fluctuations in the samples; fluctuation vectors from a fixed initial model; context independence in a sequence of rules based on an axiom operating on a string of symbols; and idiolectal and ecolectal sensitivity respect to rule context. All these aspects establish the criteria for the study of generalized pragmatics. The corresponding study of an analogous pragmatics for music is

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[256] See subchapter 6.6.

developed by Lerdahl and Jackendoff (1983:309–311), and revised from a historicist approach by Meyer (1989). Ockelford (2005) contributes to this view, combining Ruwet's (1972) poetics with new methodologies for representing and classifying repetitions in music. Beran and Mazzola (1999a–b) and Beran (2004) pay more attention, in contrast, to stochastic aspects of music performance as the main subject in the study of probability in music styles.

Ruwet's (1972) poetic theory is particularly significant for the structural study of musical style, since, consistent with the general principles of Schenker (1932), of *Ursatz* and *Vordergrund*, it also matches Chomsky's (1972:62–119) notion of language as *deep structure* and *surface structure*. Thus, the metaphor of style as surface and grammar as profundity, allows us to describe music also as a flow in which some structures functionally 'slide' over others. Languages, at the surface, change with the societies practicing them: What is meant by the notion attributed to Hockett (1954:106), "Languages differ not so much as to what can be said in them, but rather as to what is relatively easy to say in them", is that, although some basic principles prevail in language, styles—like individuals—present the surface of a characterizing physiognomy that allows them to express and represent in their own, unique way, things common to 'other' *langues* (in Saussurean terms).

According to Tarasti (2000:126), "style can be conceived as a set of rules that function as a mechanism for producing new texts, in the same way as grammar generates language". Adler (1911:10) already proposed that musical style (*Stil*) "is equivalent" to the functional ordering of the 'phrase' or the 'part' (*Satz*).[257] Obviously it is impossible to put style and grammar in the same plane. In order to develop a coordinative sense, style and grammar need to keep appropriate distance and tension.

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[257] The source says, in its original context: "Indem die künstlerische Haltung des Ganzen in der technischen Ausführung übereinstimmend bewahrt wird, entstehe der Stil. In der Tat findet man für den Begriff des Stils oft schlankweg den des »Satzes« unterschoben. Man spricht vom schlichten, überladenen, reinen, fehlerhaften Satz, vom polyphonen, homophonen Satz—also eine vollkommene Gleichstellung von »Satz« und »Stil«". Adler includes the equation »Satz« = »Stil« in his table of contents (*op. cit.*, page v). The use of the term *Satz* in music theory is particularly rich. It can alternatively be translated as *phrase*, *subject, comes* (counterpoint, fugue), *set* or *nest* (rhythm, harmony), *movement* (section of a musical piece), or *composition* (musical work).

Grammar provides basic functional regulation, whilst style 'moves' in a plane adjacent to such regulation (see Sperber and Wilson 1986).[258]

At the relatively flexible surface of grammars is where the dynamics of style take place, playing a game between homogeneity (things common to a culture) and heterogeneity (relationships perceived in different ways, around things common to a culture). Echoing Hockett (1954:106), a musical repertoire *in style* is distinguished not so much for what its corresponding grammar can dictate, but rather by what is easier to associate with the flexibility of its grammar. In Doležel's theory this notion translates into 'what is more probable', assuming individual (i.e. idiolectic) and contextual (i.e. ecolectic) variances (see Doležel 1969).

Including a significant collection of musical examples from different traditions, it is clear that none of those examples constitute a grammar, strictly, but rather a *grammar-in-action*, i.e. a dynamic adaptation made by individuals and groups of individuals within a certain Umwelt-niche.[259] This form of flexible adaptation on the collective and the individual, reproduces—to a greater or lesser extent—symbolic and operational variations for continuity of style. As a matter of fact, Mauricio Kagel (see Padilla 1984:123) believes that the composer, through driving his idiolect, needs to sacrifice grammar and rigid notions of order to eventually ensure the successful renovation of grammar itself.[260] Similarly, other individuals make their own idiolectal contributions in shaping distinctive styles, with which they can be recognized, and with which they participate in developing ecolectal complexes, and finally to the enrichment and transforming of grammars, in a cyclical movement.[261]

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[258] According to Sperber and Wilson (1986:83–117) an *input* is relevant in a cognitive system, only if it overlaps to an *antecedent*, so that the input information can be associated and anchored to that antecedent; and if the input adds 'new' information from context. The alternation between *input* and *addition* functions is analogous to the coordination between grammar and style, in a musical system.

[259] See subchapter 4.3., for a definition of the Umwelt–niche coupling.

[260] In the source: "My role [as composer] is to influence cultural forms to reach a greater flexibility, a greater liberality. [...] My métier is a legacy of the nineteenth-century bourgeois culture. What can I do with it? I try breaking it slowly [since] they are actually very, very tight forms."

[261] Inasmuch as the grammatical tradition to which they belong has not been extinguished, as languages and traditional practices in community often go extinct, for a variety of reasons (see Thomason 2001).

4.8. Dynamics between idiolect, ecolect and grammar

A musical idiolect is the set of peculiarities that characterize an individual as *musical*. By extension, it is the way each individual experiences aspects of a musical tradition, and—also—the individual modes of musical creativity and expression. It is a particular way of musically 'speaking'; a fingerprint that can be identified with a specific psychological, cultural and ecological coordination. Since there are not two identical individuals, either there are two identical idiolects.

According to Reygadas and Shanker (2007), the idiolect emerges as expression of the centralized ego, in different stages and modalities: *pre-symbolic, relational, volitional, pre-verbal, creative* and *communicative*. From a structuralist perspective, its features are also due to interperceptual complexities during transmission, selection and assimilation of shared messages (see De Saussure 1916). This implies two types of idiolectal flow: from a *self-perceptive* source and from an *interperceptive* source, by analogy with the two reflective modes of translation according to Jakobson (1959): *intralinguistic* and *interlinguistic*. A link between two or more interperceptive idiolects constitutes a new complexity, a plot on a different level, which corresponds to *ecolect*, by analogy with the different levels involving intersemiosis.[262]

The ecolect is the way that codes and messages are shared in a common symbolic space: *niche* according to sociology (Hardesty 1972), or *Umwelt* according to semiotics (von Uexküll 1940, 1957; Sebeok 1977). Domínguez Ruiz (2007:17) conceives the ecolectal means within a social environment and, under the concept of a 'resonance', identifies it as a self-referential dynamics:[263]

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[262] This paragraph requires reading of subchapters 3.6. and 3.8.

[263] According to the hypothesis of B.F. Skinner (1957), the connection between idiolect and ecolect can also be defined as a series of 'echoic behaviours' in each individual. Such behaviours

> Sounding environment is a social resonance and, in consequence, a changing environment. [...] Changing sounds are translatable in different scenarios that, however, do not lose their systemic character since sound always refers to the social context from which it emanates.

Ecolect is characterized by a common set of rules, on the one hand, and by a variability of its typical traits, on the other. It is the mediation between idiolect, as externalized interiority of individuals, and the Saussurean *langue*, as coordination of a coordinated community. In fact, Schleiermacher (1813/2002:46) conceives the relationship between langue and idiolect, not as contradiction, but rather as complementarity: "all people [...] play their part in shaping their language." This leads to acknowledge a difference between the structuralistic view of language as a system of messages transmitted in society, and the post-structuralistic notion of meta-linguistic dynamics, in which a continuous movement of emerging symbols—and not the message *per se*—corresponds to the essential image of any language.

It is noteworthy, however, that the flows of idiolects forming ecolects and, subsequently, ecolects of ecolects (dialects and langues), introduces to what Gottlob Frege had noticed as paradox of impossibility of communication (see Frege, 1952).[264] Under this notion, an idiolect is a collection of individualistic preferences constantly changing; therefore the transmission of messages should be inconsistent and communication impossible. Exploring this problem, Mufwene (2002) suggests that language is based on a relationship analogous to how genetic cooperation takes place within the biological tissues, with the direct participation of subordinates (or *apprentices*), that provide correspondence, selectivity and modification of content for their coordinative processes (or *learning*).[265]

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become evident in simulated attitudes, repetitions, fixations, and in ways of expressing and solving a problem. In this case the ecolect is of great importance during learning process, since it constitutes the margin for transactions between individual preferences and needs, and the symbols and symbolic protocols established by a community.

[264] The links between the *Frege's paradox* and the *(pseudo)problem of idiolect* are explored by Carlo Penco in "Idiolect and Context" (R.E.Auxier and L.E. Hahn, eds.), *The Philosophy of Michael Dummett – Library of Living Philosophers*, vol. 31, Open Court Publishing Company, Chicago, 2007; pp. 567–590.

[265] Genetics also speaks of *replication* and *translation* of codes, using limited and specific molecular bases, as an alphabet (see Jerne 1955, 1984). On the parallels between genetic encoding and generative linguistics, see subchapter 4.3.

Mufwene's hypothesis—in which the individual of a *species* (comparable to language as a whole) is constituted by *organisms* (comparable to idiolects as particularities)—implies a metaphor for explaining evolutionary endosymbiosis: according to this concept, the history of the bodily organs corresponds to an association of individualisms coordinated by a central organ (nucleus in cells, brains in animals). This metaphor fits the image of idiolects grouped into systems (ecolects), coordinated in a larger system (language) with the prescriptive assistance of grammar as a general system of codes. Then an image of self-similarity and consistency of endosymbiotic structures converges at different levels, in its many transitions and operational steps within an environment. Moreover, from this perspective, the classical nature-culture opposition is replaced by an interactive reciprocity between individual and groups of individuals, and between individuals and their environment (an idea developed in Ingold 1992:31–32, and Reybrouck 2001:601).

As Lidov (2004:21) notes for the specific case of music, "Dialect and idiolect may reinforce each other or may resist each other".[266] Assuming that this image does not imply a break, but coordinative continuity, this time the concept of self-similarity —as gradual connection of systems of consistency-discrepancy, or information-entropy—contributes to understanding the *dyadic feature* of language recreation.[267] In this context, the apprentice's learning processes is a bond between idiolect and ecolect, strongly supporting the role of authenticity in the transmission of grammatical correctness and style (see Beebe *et al*. 1997, Fogel 2000, and Guerra Lisi and Stefani 1997, 2004, 2006, 2008).

Tarasti (2000:126) notes that certain schools of musical interpretation owe their existence to the authenticity of contents transmitted from teacher to student. The same notion can be extended to the teaching of composition, in which tradition highlights the patterns of authenticity and originality of the musical language, as

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[266] When Lidov (*loc. cit.*) refers to an ecolect, he uses the word 'dialect'. This subchapter explains why this equivalence should be avoided.

[267] The *dyadic feature* refers here to the close relationship between teachers and learners in language continuity and creativity, through recursiveness. On the latter concept, see subchapter 3.7.

conceptualized by Milton Babbitt (1970:12): "the technical traditions—even though the sources and traditions may be of recent origin—provide not only a point of entry but, eventually, the bases for determining the depth, extent, and genuineness of the [musical] work's originality". This form of linkage between individual and tradition—between idiolect and ecolect; between use and grammar—is evident in the processes of education in various forms of traditional music, including many aspects of musical performance. It is also noticeable in the teaching processes of other animal species, such as in the transmission of code 'singing' in blue whales, dolphins and numerous species of birds (see Payne and Payne 1985, Catchpole 1973, 1980, Eens *et al.* 1991a–b). It is clear, thus, that the relationship between message learning and message renewal is a key to self-organization in systems of expression and communication; however, from a post-structuralist vision, it is also evident that the continuous dynamics between symbolic emergences—a concept explained in subchapter 3.8. as *intersemiotic continuum*—is the most relevant feature of self-organization in meta- and subsymbolic contexts: here the nature of the replacement and survival of the original features, is the fundamental relationship between the *different* and the *similar*.

Gelman and Brenneman (1994:382) suggest that the lack of an identical model between apprentice and teacher ensures learning, both as content redundancy and transformation of knowledge gained through creativity. This perspective is based on the fact that, whether the redundancy law extends to learning, this makes it easier for learner and teacher not to share exactly the same interpretation of the contents, but the essential qualities of what is already known and what must be learned. This form of redundancy appears condensed in scheme ◊450 (see page 173), which distinguishes between *repetition* (a mere copy in learning) and *recursion* (with the possibility of correct interpretation, in turn flexible to other, new interpretation). Under this principle of redundancy and recursion—obviously not a simple repetition—the concept of authenticity in musical styles makes sense.

According to Eco's (1968:117–118) perspective on language dynamics, and its adaptation to musical language made by Lidov (2004:21), the 'forces' at play within ecolects can be characterized as *attraction* and *repulsion* between idiolects. Attraction influences the individual to selectively absorb elements from the context, whilst repulsion serves as an identity preserver, facing the vastness the ecolectal resources. Thus, the relationship *grammar-idiolect-ecolect-style* forms a functional cycle between simplicity and complexity; between structure (i.e. musical form and content) and meta-structure (i.e. multi-layer self-referentiality and meta-symbolic organization of musical dynamics). Musical precept and interpretation, structure and meta-structure, cooperate as bonds between order and discrepancy (or similarity and difference). According to Campbell (1982:264) this is clear for information theory, since "Structure and freedom, like entropy and redundancy, are not warring opposites but complementary forces". An analogous form of cooperation is found throughout the arrangements between cognitive platform and cultural transformation, as Gelman and Brenneman (1994:371) observe: "innateness and cultural creativity are not opposites and indeed work together to guide the acquisition of human knowledge".[268]

Grammatical correctness is indispensable for communication. Basic aspects of information and the message are lost in its absence. Grammatical correctness is passed from one generation to another, and constitutes the rigid basis of language through its different cognitive domains.[269] However, grammar cannot constitute alone the full extent and complexity of language. As Meyer (1956:202) notes, the features of musical sense in recreation and recursion, "lie in the deviation from the exact". Congruently, Papadopoulos and Wiggins (1999:2) acknowledge that musical grammars present "some important problems" of structuration, namely: contextual inadaptability, semantic weakness, and intolerance of ambiguity, needed for symbolic and pragmatic nuances. Thus, in order to propel the *spirals of language* (see graphics ◊480 and ◊481 in following pages), grammar requires its own infringement through idiolectal preferences. This characterization of the *global language*—embracing the musical languages—obviously differs from the merely mechanical metaphor, since it

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[268] Gelman and Brenneman (*op. cit.*) make this note in the context of learning elementary numerical and musical relations, within its social acquisition and transformation.

[269] Understanding this subchapter requires to consider what was stated in subchapter 1.2.3.

rather highlights the need for vulnerability of the systems of symbols, instead of fixing them as rigid models. This is what Campbell (1982:264) defines as 'freedom', and Meyer (1956:71, 197–202) identifies as 'deviation', i.e. the ability to transform a prescriptive or descriptive grammar, into a *grammar-in-action*.
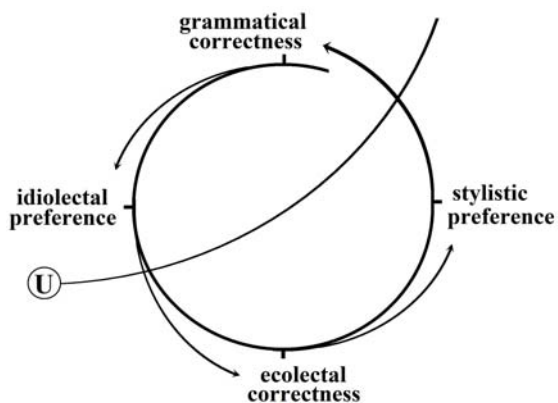
◊480. Schematic view of the grammatical cycles, characterized as looping of stages of an open system of symbolic recursion and creativity. The picture summarizes the typical functional relationships of systems of musical elaboration (including compositional and performance styles). The arc crossing the figure, with the label U, denotes larger cycles of influences from the Umwelten-niches level, described in subchapter 4.3.

Lidov (2004) proposes to develop Ruwet's (1972) combination of *deep structure* and *surface structure*.[270] To this end, he connects the concept of *grammar* with *dialect*, and *design* with *idiolect*, in a schematic adaptation that is reminiscent of the Saussurean structuralism (see Lidov *op. cit.*:41–58). Within this frame, Lidov's (*op. cit.*:19) usage of the term *dialect* is excessively general: "Dialect, then, is simply a shared language".[271] Even from a conservative viewpoint, De Saussure (1916:294) notes that "It is difficult to say what the difference is between a language and a dialect. Often a dialect is called a language because it has literature." It is unclear, therefore, how such a concept can be directly transferred into music: how a net difference between musical language and musical dialect could be definitely established by a written tradition? At least in linguistics, this separation contributes to segregation and a lack of understanding of languages that are marginalized by a dominant culture, losing all orientation between idiolect and ecolect, and between autochthonous and allochthonous forms of minority—that, in their musical analogies, enrich a vast plurality of cultures.

De Saussure (1916:112) distinguishes linguistic structure (*langue*) and speech (*parole*) as two basic traits of language in which the former is "the whole set of linguistic habits which enables the speaker to understand and to make himself understood"; whilst the latter is the totality of what is said through individual combinations depending on a massive usage of speakers (the connection with idiolect and ecolect, in this order, is obvious). Then, De Saussure adds a temporal vector to show the relationship of language in its diachronic and synchronic axes, using the

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[270] Such a conceptualization is based on the Chomskyan principles of generativism (see Chomsky 1972:62–119). For more details see pages 20–23, 155 and 190 of the present study.

[271] Lidov (2004) does not make clear either why *grammar-dialect* constitutes a polarity opposed to *idiolect-design*. Why does he not associate grammar with design, too? Such a duality of dualities is not very effective in binding *design*, basically a concept of spatial perception, with the three other traditional concepts of linguistics. Lidov (*op. cit.*:18) seeks how to justify this descriptive terminology, but he is dissatisfied with his own motivation: "For someone with my pretensions to announce *general* principles for *comparative* semiotics, it was most disconcerting not to have portable terminology!" He (*ibid.*) even acknowledges his inconsistent usage of the term *design* associated with *pattern*. It is noteworthy that, conversely, Seeger (1960:226–228) proposes the basis for investigating the relationship between musical design and musical grammar, not in terms of opposition or polarity, but in terms of "music-order as design and as logic".

Cartesian model with rectilinear relations and structural rigidity. In contrast, in order to explain the dynamics of linguistic processes, Eco (1976:121–125) introduces systems of 'attractors' and 'repulsors' in functional contiguity. These systems are cyclically placed in ◊480 and are defined as *grammatical correctness, idiolectal preference, ecolectal correctness* and *stylistic preference*. According to this picture, the production of dynamic structures of language (the Saussurean *langue*) moves between rigid and flexible states, combining correctnesses with preferences from one point to another. Such a movement does not need to be described as a circle completing a closed program—since its orientations are opposite and complementary at a time—but as an open sequence, according to *migrations* from one step to another, in recursive loopings. This scheme is fully compatible with a dynamic theory of musical styles, and helps to understand how "musical traditions may change with time", and music "will perhaps not remain unaltered" (Kurkela 1986:35).

The movement between 'correctness' and 'preference'—an oscillation in the grammar cycles—is analogous to the movement between stages of an epistemic system (Pareyon 2009). Accordingly, in his theory of revolutions, Thomas S. Kuhn (1962) identifies this cyclical movement in the history of ideas, as a periodic oscillation of the knowledge systems, between accumulation (*paradigm*) and rupture (*revolution*). Kuhn conceives these fluctuations in the long term, but also explains that the major transformations of thought occur after numerous oscillations on a smaller scale, consistent with the pattern of cycles represented in ◊481. These cycles are far from being unique, stable and bidirectional relationships—as suggested by the Saussurean structuralism. Rather they are multiple and unstable features reflecting human behaviour in the margin between individual expresiveness and the collective flows of culture.
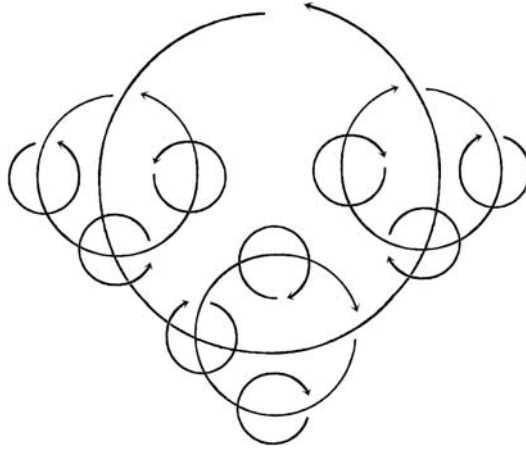
◊481. Self-similar pattern representing the spirals that emerge within the negotiations between *idiolect, ecolect, stylistic preference* and *grammatical correctness*. The scheme suggests that the cycle shown in ◊480 does not occur in a straightforward manner, given that the spirals are not free of interferences from the behaviour of other cycles occurring at different strata. This scheme is also related to figure ◊381 and to the *intersemiotic continuum* (IC) theorized in subchapter 3.8. The regularity of the figure is idealized; the cycles can occur, of course, in a variety of irregularities.

Style tends to soften the language structures through an intentional and expressive deviation: its character is a *moving-to*. Grammar operates, in contrast, at a relatively fixed structure that provides orientation: its character is a *staying-at*. Grammar-structural modifications are the result of a large number of small changes in a certain direction, over a long period of time. Eventually, semantics can operate as a *moving-to*, both in its referential level (e.g. as means of synthesis or comparison), and with the cycles of invention and renewal of lexicon. There are also figures and tropes with both characters of *moving-to* and *staying-at*, such as chiasmus and synecdoche. In practice—and from a general perspective—music corresponds with this formula. Like other language domains, music '*stays-at* a set of axioms and rules', and '*moves-to* a significant creativity'. Thus, musical practice depends not only on a predetermined program or analytical determinism, but depends, in particular, on a degree of interpretative flexibility. The musician, based on a method, *stays-at* principles and

protocols in a limited range of structural functionalism. But transformations occur since the first exchange between idiolect and ecolect, and lead to systematic deviations through the emergence of styles. Structural changes in the history of music would then be similar to the revolutions studied by Kuhn (1962), in which a symbolic cycle gradually gives way to a self-organized meta-symbolism.[272]

It is also possible to study the changes of musical style in a manner similar to the changes in language style. So, it is possible to study how *music(s) in contact* influence each other in a way comparable to how the interactions of *languages in contact* are studied. For Weinreich (1953) languages are not isolated systems, but varying worlds of relationships that, in contact with different idiolects, ecolects and sociolects, perform biases to negotiation, imposition, adoption and selection, depending on the role played by each participant or groups of participants, within intersemiotic cycles. In addition, Thomason (2001) notes some typical stages of these biases, identifying them as "mechanisms of interference", and classifying them by the mode of operations involved, in *symbolic generation, mutual influence, unilateral influence, passive familiarity, fatigue*, and "linguistic death" by *attrition* (i.e. reduction of semiotic effectiveness), *grammar replacement* and *endogamy*. Parallels with musical language can be identified in a variety of possibilities.

Other specialists in sociolinguistics and ecolinguistics, including Stewart (1965) and Nelde (2002), recognize not only 'contact languages' but also contact layers separated by tendencies of practice. Nelde identifies *autochthonous* and *allochthonous* varieties, whilst Stewart suggests a gradation between *acrolects* and *basilects*—'high' and 'low' languages according to social strata with different usages and practices. Social class, workplace, educational level, gender and ethnicity involved in different modes of speech, also determine musical emotions, practices and preferences.[273] Overall, the distinct forms of musical practice, and their pragmatic and intentional

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[272] In music, this relationship is also clear with the functional, diachronic connection of the synthesis and analysis of musical ideas and concepts (e.g. in the 'symbiosis' between composition and musicology).

[273] See Blacking (1973), Ballantine (1984), Tagg (e1999), Kieran (1996), Vieira de Carvahlo (1999).

strata, constitute self-similar systems, somehow approximated to what Pegg *et al.* (e2008) suggest:

> In a fractal landscape of potentially infinite regress, global forces produce 'micromusics', that is, endlessly varied local and localizing particularities. The term 'culture', unhitched from its national moorings, assumes different forms. Culture is, in this context, provisional, reflexive and mediated. It is no longer the semi-invisible ground of being and belonging, but a site of manipulable and malleable self-fashioning, in which the boundaries of this self are constantly open to question and negotiation. Hybridity and creolism are crucial aspects of global cultural consciousness, not in the sense that their origins are 'no longer' pure (since no culture's origins are or can be), but in the sense that they engender new forms of relativizing self-consciousness, of being neither here nor there, 'us' or 'them', but being in-between, in a 'third space'.

According to what is explained in the present study, the introduction of the fractal notion in this quote involves risks of inaccuracy and creates expectations that cannot be satisfied.[274] Suffice it to replace such a notion by the concept of *self-similarity*, to gain clarity and analytical-deductive precision, especially if combined with the concepts of *intersemiosis* and *synecdochic function*. This observation aside, the quote above interestingly reflects a paradigm shift in the way of conceiving the links between individual and community through their musical expressions: the very idea of 'third space', inspired by Bhabha (1994), is the one advocated in this investigation, as 'mediating text', typical in the processes of intersemiosis that permeate the symbolic negotiations of music.[275] This notion fits with the idea of Ballantine (1984:5) in that "in various ways and with various degrees of critical awareness, the musical microcosm replicates the social macrocosm." It is evident that, reciprocally, the social macrocosm also 'replicates' the musical microcosm.

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[274] See subchapter 5.5.

[275] On the concept of 'mediating text' in a semiotic and hermeneutic context, see Ricoeur (2004:13–14): "phènoménes d'intertextualité dissimulés dans la frappe même du mot. […] les deux textes de départ et d'arrivée devraient, dans une bonne traduction, être mesurés par un troisième texte inexistant".

Self-similarity in musical information and proportion:

From Simple Synecdoche to Complex Intersemiosis

Self-similarity as information

Structural economy is a powerful connection between musical constructivism and self-organizing processes in general. Such economy, however, contradicts other fundamental aspects of music as repetition. How, then, can two basic and divergent aspects of music cooperate since one seems to discard the other?

This chapter offers a variety of examples to explain how the structural oppositions of music, such as economy *vs*. repetition, simplicity *vs*. complexity, and similarity *vs*. variation, are coordinative relationships rather than only additive-subtractive forms. This idea appears, in germinal state, in José Vasconcelos' (1951:20, 52–53, 56–57, 137–138) aesthetic philosophy:[276]

> Aesthetics is the philosophy of quality, of the undivided universe; its truth must be sought through coordinative thinking. [...] Coordinate is to harmonize. [...] Full harmony is not additive but heterogeneous and coherent. [...] Modern mathematics, which now is far from Pythagorean arithmetic, expands and enriches the original intuition of rhythm, number and harmony. In reality, the relationship of music and mathematics aims to bring philosophy to the criterion of harmony.

The search for harmony among its discrepancies, and its coordinative congruency, sublimate in the concept of proportion, discussed in Chapter 6. This chapter assumes, more generally, that the coordinative relations of music are complementary and *multi-layered* (see Koblyakov 1995:299). This means that such relations can be cooperating in parallel or in disjunction, at different semiotic levels simultaneously, without necessarily interfering, or with selective interference. Beran (2004:83) simplifies this relationship as reciprocity between information and uncertainty: "the term 'information' can be used synonymously for 'uncertainty': the information obtained from a random experiment diminishes uncertainty by the same amount."

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[276] For an introduction to Vasconcelos' philosophical and musical thought, see Pareyon (2005).

The basis of this criterion are settled upon the foundations of information theory: Shannon (1937, 1948), Shannon and Weaver (1949), Wiener (1948), Moles (1952, 1958, 1963, 1964), and Cherry (1957).

The use of information theory in music goes back to the origins of that theory itself. A minimal account of this relationship should include the studies of Pinkerton (1956), which reconstruct 'melodic probabilities' after classifying samples of traditional repertoire from diverse sources; Youngblood (1958), who analyzes structural trends in samples of Gregorian chants and melodies of Mendelssohn, Schubert and Schumann; Hiller and Isaacson (1959) and Hiller (1981), who use information theory to develop compositional sequences by generating a random code with high content of information and low redundancy; Knopoff and Hutchinson (1981), who introduce a 'dissonance factor' to classify tonal subsets, proposing models of entropy to reflect the diversity of a 'musical alphabet' (in probabilistic terms); and Harley (1995), who enriches the concept of 'music as information', bringing it to the notion of "music as non-linear process", and to "mapping processes to translate the generative numerical data into musical data in ways which take into account the particular characteristics of each musical parameter or procedure" (Harley, *op. cit.*:223).

This chapter is limited to an overview of this selection of approaches, keeping the focus on the main subject of this study. There is also a summary of Shannon's theory (1937, 1948) and an exposition on some ideas of Moles (1958, 1963, 1964) and Cherry (1957), paying attention to their concepts of *redundancy, consistency* and *uncertainty*, involving the notion of self-similarity as musical information. This exposition critically analyzes the thesis of mechanistic structuralism, and studies some typical problems of open systems and the dynamics of message in order to continue the discussion on the relationship between determinism and indeterminism in the context of musical self-similarity. The main guides for this inquiry are Eco (1962, 1968), Campbell (1982), and Hayles (1990), which although not musicological texts, connect aspects of information theory to aesthetics and to related subjects such as narrativity and expectation, code design, and the efficiency of message transmission. The articles of Youngblood (1958) and Knopoff and Hutchinson (1981), and the books of Tiits (2002) and Beran (2005) are, in addition, technical guides linking music to information theory from different perspectives.

5.1. On the concept of musical information

For information theory, 'information' means possibility of deviation in the sequence of a *probabilistic tree* (see ◊510). Many authors, including Youngblood (1958:25) and Eco (1968:25) put this possibility in parlance terms as 'freedom of choice', for instance in the series of preferences of a listener to respond to a melody, or in the preferences of a performer to improvise on a given theme. In this scheme, 'low information' means high predictability and, conversely, high predictability means 'low information'.
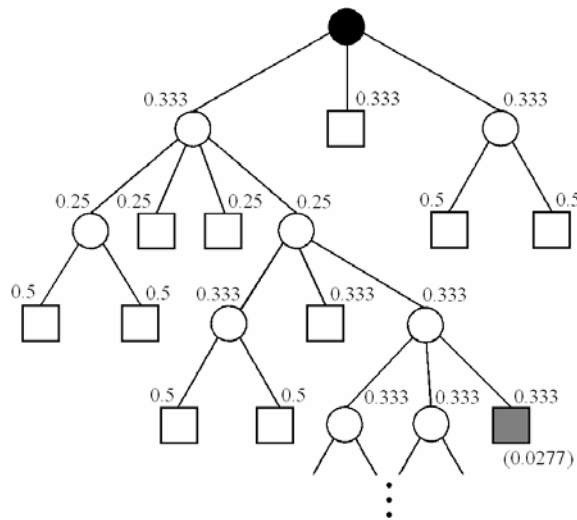
◊510. Probabilistic decision tree. The solid circle represents the starting point of decision-making; the filled box (bottom right) represents a provisional point of arrival. Squares represent closed decisions or invariable states; circles represent an alternative for each level. Fractions indicate the likelihood of each object to be elected at each step, and the number in brackets, below, indicates the overall probability to move from the starting point to the destination. The ellipsis points represent vertical continuity.

This description can be complemented by what Eco (*op. cit.*:25) states: "information means the value of probabilistic equality between several elements combined, which is higher as the selection of possibilities increase". This is akin to Beran's (2005:82) notion of a *vocabulary* (V) consisting of only one element $n = 0$, where information

content equals zero, "because we know which element of V will be contained in the message even before receiving it". In other words, the greater a bound for the code choice in message elaboration, also the greater the chances for a meaningful message in an enriched source. At the same time, the probabilities of decoding the message depend on the precondition of knowing its statistical properties, starting from a 'source'. In statistical terms, to know the code's behaviour equals to decipher it.

Need for the ordering function of the code

The problem of using a code which is 'too' rich is that whilst this may form a very detailed message with a huge amount of information, the energy and time needed to decipher the message also become huge and, therefore, impractical. The next example illustrates this difficulty:

Let $^4/_4$ be a measure in which the register of a monodic instrument can be inscribed in its hypothetical extension of three octaves, ranging from C_3 to C_6. Assuming that this instrument can only play the pitches of the chromatic scale, and the measure only supports wholes, halves, quarters and eighth notes (including pauses with equivalent values), then the 'potential measure' containing all combinations of these elements will consist of 36 pitches raised to the power of 4 (symbols available to represent the durations of the tones) + 4 (same amount of symbols for silences), this means: 36^8 – 34 (i.e. deducting the amount of *void* or silent measures) which equals to 2821109907422. If this result is combined with a potential (discrete) variety of loudness and colouration, the number of possible combinations is extremely high, even for this example with a rather rudimentary instrument. As Eco (1968:26–27) acknowledges:

> The information from the source, as freedom of election, is very rich, but the possibilities to transmit it individuating a whole message result too difficult. [...] At this point, there comes into play the ordering function of the code. The code represents a system of probability superimposed on the equiprobability of the source in order to ensure communication. It is not 'information' as statistical measure that requires this element of order, but its transmissibility. Thus the very numerous messages considered as possibility, are reduced to the lesser number of messages allowed by the code. In a situation of equal probabilities from the source, a system of probabilities is introduced: some combinations

> are possible, some others are not. Therefore, the information from the source decreases and the possibility of transmitting messages increases.

What musical traditions have been doing for centuries, and apart from an information theory (similarly to what has happened outside the formalization of verbal language) is to limit messages to grammatical conventions in order to bind the source information and increase the communicative efficiency. Most historical criticism, adverse to new forms of music or language, spends its forces against what is considered an attack on the ordering function of the code, which is present in collective usage. This opposition is often given an ethical character, analogous to the aesthetic requirements that dictate what to do and what to avoid in the continuity of a stylistic tendency.[277] It can be hypothesized that such empathy between ethical and aesthetic canons are due—at least in a considerable proportion—to a strong intuition on such an ordering function, typical of communicative societies, and of biological codes in general, characterized by operational economy and a search for efficiency in message transmission. Music could, in this fashion, be a realization of complex aspects of this intuition, as Josephson and Carpenter (1994:3) suggest:

> By virtue of the fact that music can be considered as information, it follows that information plays a significant role in the functioning of [an] aesthetic subsystem. In listening, information fed in determines the state of the aesthetic subsystem. Conversely, in composition the aesthetic subsystem generates information. The importance of information is one of a number of aspects in which aesthetic processes parallel life, where information (e.g. DNA) plays a similarly important role.

Whether the economy of the code and the gradual sophistication of the message are biological characteristics in general, then the ordering function of the code could be a powerful reason to explain why—as shown throughout the previous chapter—self-similarity relations are found in a vast range of diversities. Self-similarity can be

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[277] José Vasconcelos (1882–1959), who was attending the premiere of Stravinsky's *Le Sacre du Printemps* (1913) at the Nouveau Théâtre des Champs-Élysées of Paris, mentions details about the scandalous reception of this work by the public, and speaks of the "moral defense" made of it by some French nationalists in favour of the strategic alliance with Russia, in the political situation of that time (see Vasconcelos, *La tormenta* [The Storm], Botas, Mexico, DF, 1934:41). Apparently, the scandal was stimulated by the use of dissonances and rhythmic and metrical contrasts, unacceptable as a musical code by some part of the public. With the passing of the twentieth century, Stravinsky's work was accepted as a new structural paradigm when its own principles of ordering function were understood.

understood, then, as a mechanism preserving information at low cost—structural and energetic.[278]

The always-rich source

The source marked as a solid point in the probabilistic tree ◊510 represents a *positive* initial state from which two or more choices can be taken in a finite set, in their turn leading to one, two, or more subsequent elections. This source represents an absolute starting point with specific values and possibilities for well defined relations and functions. However, the source in a musical system is never an absolute starting point, but a complex continuity of physical, biological, aesthetic, psychological and idiosyncratic tendencies, as summarized in Chapter 4. According to cultural context, this continuity is justified under different symbols and invocations. For instance, Toshio Hosokawa (1955–), following an idea of Tōru Takemitsu (1930–1996), justifies the first sound of a musical piece as "fruit of silence [...] Any initial sound comes from silence".[279] This *silence* is far from being void or emptiness; it is, rather, a seed. On the other hand, imitation and learning processes are fundamental to the 'authentication' of musical traditions (see Tarasti 2000:126); so in the style of composition, improvisation or musical performance, the probabilistic source is always situated 'somewhere' within a cultural corpus.[280]

Yadegari (2004:48–50) investigates the problem of measuring music from a deconstructionist approach. He sees the conflict between scientific structuralism and use of language as a relationship between culture and nature, and concludes that the interpretation of 'information at source' is inadequate to describe the dynamic relations of music and language. It is, then, necessary to rethink the "question of origins"—in its probabilistic sense and within its cultural implications—assuming self-referentiality as a characteristic of discourse:

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[278] This is explained in section 3.9.5., in terms of power laws.

[279] This notion was transmitted verbally (Malaga, Spain, 2001). Hosokawa, himself a music theorist and composer, acknowledges that this idea originates in a traditional conception of sound 'within' silence, in a manner similar to a flower within a specific space, according to the Japanese tradition of *ikebana*.

[280] As John Blacking (1973:x) notes, "music cannot be transmitted or have meaning without associations between people".

> In the same way that linear mathematics was a basis for traditional epistemology, one can argue that non-linear dynamics, in which self-referentiality plays a structural role, can be a defining basis for a different form of reasoning, scholarship, and paradigm of thought. Similar to a deconstructive model, within a model that accepts self-referentiality as an innate quality of discourse, the question of "origins" transforms. (Yadegari, *op. cit.*:49)

This refreshes the idea to assimilate music data using information theory: it is no longer sufficient to obtain pure units, ratios, coefficients or products, nor to pursuit the automatic application of formulae. There is a need, rather, to transform the overview of the message as a vehicle serving a fixed notion of information, into a coordinative conceptualization harmonizing code, message and culture. The 'seed', 'vanishing point' or 'generating entity' of a musical system or process can no longer be read unilaterally or unidirectionally. The structuralist issue of origins and generativisms applies—mutatis mutandis—to the investigation of the dynamics between global and local defining the same system or process.

Balance between economy and operational expense

The relationship of equilibrium between structural economy and energy expense is present on a variety of levels, from fundamental physical interactions and general biological principles, to the use and transformation of grammars.[281] This topic is too broad, but there may be some examples which open a more specific discussion in the field of musicology and music aesthetics.

In crystallography, for instance, *conservation laws* define the consistency of molecular relationships in magma at high temperatures and high pressure, by the rhythm of temperature and pressure variations.[282] The atoms of minerals fail to organize in an accelerated process of cooling and depressurization of magma, forming igneous rocks and pumice at the earth's surface. However, if cooling is slower and takes place inside the earth's crust, then the atoms may develop ordered structures in form of crystals. This means that the same atoms—even the same molecules—may

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[281] An operative notion of 'grammar' is introduced in subchapters 4.5.–4.8.

[282] The subject of crystallography, for its affinity with the classification of symmetrical patterns of music, is introduced in section 3.9.5. On the analogy between crystallography and musical information self-structuring, see also pages 133, 136, 148–149, 320, 435, 438. Within the context of self-similar tessellations, the issue is developed in subchapter 6.4. (see pages 416–419).

form different types of structures with varying degrees of consistency, depending on the rate of their generalized correlations (Gleick 1988:iv–vii; see general introduction in Barenblatt, Entov and Ryzhik 1990). Quan and Song (2002:8247) consider that such consistency determines the relationship between symmetry and flow properties "at all scales" in a mechanical process.

Prusinkiewicz and Lindenmayer (1990), Prusinkiewicz and Hanan (1992), and Meinhardt (1998) explain that, for all plants and animals, the relationship between their fundamental processes configured as stimulus–inhibition, or action–inaction sequences, also constitutes a balance between structural economy and operational expense, linked to the processes of growth and regulation through the operation of simple mechanical and chemical processes. In cardiovascular systems, for example, duct bifurcations are determined by the relationship between body mass and metabolic rate; something known as *allometric relationship* (see Weibel 1984:172, and McNeill-Alexander 1999:26–33). Vascular mechanics postulates that the shape of a vessel network is determined by the relationship between the properties of the substance that composes it, and the performance of energy required to sustain its metabolism (McNeill-Alexander, *ibid.*).

In perceptual systems of animals the Weber-Fechner law states that the smallest discernible change in the magnitude of a stimulus is proportional to the magnitude of the stimulus (see Moles 1954).[283] This means that there is a coordination between 'economy of reception' and 'expense of reaction'. This law is also related to Zipf's law or *law of least effort* in the processes of language, which requires a separate explanation for its meaning in music.[284]

The study of information in cognitive processes in music also provides means for exploring the coordination between quantity and operability: Chai (2005) wonders how the human mind retains and withdraws large amounts of musical information, based on little evidence of stimuli that trigger information networks. His general

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[283] Moles (1954:241), in his descriptive reduction of the concept of music, believes that the perception of 'musical sound' is due, necessarily, the Weber-Fechner law: "A musical sound is a quasi-periodic phenomenon, best expressed by a three-dimensional representation in terms of level, pitch, and duration; its perception follows the Weber-Fechner law."

[284] See following section.

hypothesis suggests that information systems, including the operating model of the mind/brain, require not so much data masses, but rather segments that are useful for the reconstruction of 'tracks' using general references. This function allows people to reconstruct musical patterns in memory and, hence, also allows music interpretation.[285] Chai (*ibid.*) emphasizes that self-similarity plays a fundamental role in this process, since it reflects the way in which the tracks or referential stimuli are related to segmental reconstruction in the recursive cycles of musical grammars.[286]

The examples given, among many others akin to them, support an idea of 'organicist music', in which characteristic information reflects a coordination between economy and operative expense. In this manner it may be understood how a simple relationship in music can branch into clusters of complex relations, through self-reference and self-organization processes coordinating the spending and performance of its recursions. The open nature of musical systems developing from a relationship or a group of simple relations explains how self-similarity emerges in the generation of symbolic and operational complexity, beginning with the use of a few elements, as schematically suggested in the tree in ◊510.

The organicist notion of music can be widely documented through the intuition of the relationship between economy and structural expense. It may suffice to note some classical references from the existing literature: Edward C. Bairstow, in his book *The evolution of musical form* (1943:44), claims that instrumental forms "are born and develop." Felix Salzer devotes the a whole chapter of his *Structural Hearing* (1952, part II, Chapter 2) to a 'demonstration' of the organic functioning of music, using comparisons with anatomy and describing musical movement as a biological process. Sanders (1975) and Sanders and Lefferts (2001) consider that the use of 'organised' patterns in the Ars Nova supported a change in formal settings:

> [B]y means of numerical coordination of heterogeneous, hierarchically ordered durational components, in which melodic considerations are of no structural importance, to the

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[285] So, for example, giving a cue for a song, melody or chord progression, a wide range of associative chances is displayed into a repertoire constricted by style. This model of connectivity helps to explain the processes of segmentation, stereotype and continuity, typical of music in many aspects of perception, improvisation, analysis and composition.

[286] Givón (2002) develops a similar notion for a general cognitive framework. See 4.4. on structural universalism, especially pages 160–161.

creation of a musically and textually homogeneous contrapuntal fabric from one congenial set of melodic cells.

The idea of development after a minimal group of germinal relationships is also a central notion of the Viennese classicism. Haydn and W.A. Mozart consistently employ a technique with which they reuse melodic cells from other compositions, of their own and of others, to elaborate musical combinations gradually more stylized. By this technique Mozart reconstructs his Mass in C minor K247/417b into the oratorio *Davidde penitente* K469 (1785), and transforms several of his serenades into symphonies and vice versa (see Stephan 1979, Escal 1981, Brown 1992; authors who deliberately use the concept of 'melodic cell' in an organicist context). The harmonic-melodic constructivism from the nineteenth century extended to the twentieth, also using procedures developed from simple elements that 'flourish' in large instrumental forms. Many scores of Beethoven, including the fifth and seventh symphonies, and string quartets, employ these procedures with rigor and imagination. In fact, much of the musical form theories of the nineteenth-century —usually in adaptations of the naturalistic concepts of Hegel and Goethe—are based on comparisons with organic tissues. Moreover, Adolph Bernhard Marx (1795–1866) theories suggest that the derivative relationships of the sonata form are 'derivative cellular processes', rather than merely additive processes (see Burnham 1989:253–254).

Schoenberg (1967, 1975) conceives *Grundgestalten* or 'basic-figures' as an organic principle for compositional processes,[287] and Anton Webern, in *Der Weg zur neuen Musik* (1960) repeatedly refers to Goethe's organic metaphors.[288] However, this notion is not absent in other contemporary styles and constructive techniques. For instance, Hepokoski (1993, summarized in 2001) believes that the symphonic music of Sibelius concentrates the "germination and metamorphosis of motivic cells [...] within the almost imperceptible mechanisms of tempo change and texture change, and the uncanny interrelatedness of the themes." This concept of cell is explicitly organic and comes back in texts that investigate the transformations of rhythmic and

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[287] See *Gestalt* definition in subchapter 3.5.
[288] These metaphors are treated in detail at subchapter 6.5.

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harmonic-melodic materials in the modern atonal and post-tonal repertoire. Kooij (2004) talks explicitly in these terms, in the context of polytonality, presenting as a case study the music of Willem Pijper (1894–1947), who implemented a technique of plant modelling to create music. The work of Charles Wuorinen—adhering to the integral serialism of Schoenberg and Webern—is also remarkable in the sense that, as Carey (2005:38) notes: "Influenced by Mandelbrot and fractal theory, his formal designs emphasize organic self-similarity, relating the largest aspects of a musical composition to its smallest gestures."

Nevertheless, the notion of organicity as information is not limited to the study of Western music. It is also found, for instance, in investigations about the traditional music of Uto-Aztecan groups and other native peoples from the North American continent: e.g. Tomlinson (2007:68) notes that bodily symbolism and psychophysiological processes give shape to the *Cantares Mexicanos* compiled by Bernardino de Sahagún between 1530 and 1580. Such a notion of organicity takes into account operative cycles in semantic, syntactic and instrumental constructive layers, combining aspects of a fixed grammar with a relatively flexible implementation. Similarly, Lewis Rowell (1983:191), refers to "skin, flesh, and bones" as essential symbols for the organic-metaphorical structuring of music in the Nō theatre of Japan; and William J. Jackson (1993) conceives an empathic approach exploring self-similar properties of the song repertoire in the Bhakti tradition, widespread in different regions of India, taking as case study the work of the Hindu Brahmin and composer Tyāgarāja (1767–1847), whose songs with traditional Sanskrit texts and "colloquial" texts of classical authors (Annamacharya, Kanakadasa, Purandaradasa, Ramadasa), "reiterate basic Bhakti themes in similar word patterns and sometimes [with] the same ragas" (Jackson 1993:71). All these traditions depend—at least partially—on an informational context based on a certain notion of 'organicity'.

It is possible, therefore, that each culture finds its way to observe, emulate and transform its own notions of 'organicity', 'nature' or 'life'—the symbolic viariation of these concepts is evidence of the same refinement—reflecting itself through its own interpretation of a musical repertoire maintained by tradition. Edward Sapir's thesis

(1921) can be revised from this perspective, inasmuch as language affects the culture of societies, and the societies influence language through preferences by selection of information. So, Merriam (1964:145) believes that music can be conceived "as the end result of a dynamic process"—i.e. information exchange and transformation—as culture is "carried by individuals and groups of individuals" (*op. cit.*:27).

Zipf's law

Zipf's law is an empirical law employed in probability and statistics, which reflects an approach to a probability distribution applicable to a variety of samples in many different fields of physics, biology and social sciences. The law, originally proposed by linguist George K. Zipf (1902–1950), states that in a generalized sample of verbal expressions, the most frequent word will occur approximately twice as often as the second most frequent word, which occurs twice as often as the fourth most frequent word, and so on (see Zipf 1935, 1949). In the end only a few words are used very frequently, whilst most of the words are little used. This principle is summarized in the formula

$$P_i \sim P_1 / i,$$

where P_i is the probability of selecting an object i, and P_1 is the probability which a first term of a series has to appear in a repertoire of objects. Given that the successive probabilities are positive and ordered ($p_1 \geq p_2 \geq ....$), and for all i, $p_i \leq 1/i$, it is suggested that a second term occurs approximately $1/2$ as often as the first term does, whilst the third term occurs approximately $1/3$ as often as the first, and so on. From this law it is also conjectured that the most common words tend to be shorter, and that when they tend to be too short, then they are replaced by longer words (Sommerfelt 1961:27).

Diederich *et al.* (2003) assume that under Zipf's law, the number of elements $\alpha(f)$ in a text, occurring exactly f–times, is determined by $\alpha(f) = f^\gamma$, where $\gamma \sim 2$. In this equation γ depends on a "cultural index", determined by experience and language competence, which makes possible to isolate related sets of elements to a greater or lesser approximation to $\gamma \sim 2$. The statistical meaning of this conjecture is so important for stylometry, that, according to Diederich *et al.* (*op. cit.*:110), permits an

objective calculation that can answer a question such as "How many new words would Shakespeare use if he were to write another play?"

Wheeler (1929) and Tolman (1932)—experimental psychologists quoted in the investigations by Thorpe (1966:311) and Hall-Craggs (1969:373)—anticipated the Zipf's law (or *Principle of least effort* in Zipf 1949), linking statistical experiments with pattern codification of animal behaviour. Accordingly, an animal tends to minimize the effort required to achieve a goal, following a principle of hyperbolic distribution and entropy loss. Hall-Craggs (1969:373) connects this principle to music theory:

> These laws failed to come up to expectations as instruments for exact prediction. However, in music, whether it is considered as an art or as a long range communication system, it is the rhythm which may be held to account for the curiously predictable nature of the phenomenon and which led [Ethel Dench] Puffer (1905) to define music as 'the art of auditory implications'.

Further analysis in continuity with this idea demonstrates that not only rhythm, but the constructive aspects of music in general can be measured with Zipf's law applications (see Daugherty *et al*. e2003) inasmuch as the same power laws and generalized *well-formedness* rules govern most of the functional and structural relationships of music (see Lerdahl and Jackendoff 1983, Carey and Clampitt 1996, Carey 2007).

Regarding the absolute satisfaction of the "generated expectations" which Hall-Craggs (1969:373) points out, it must be considered that Zipf's law is a conjectural, untested principle under empirical bases, and without mathematical proof. As other distributive principles of verbal language and well-formedness rules in musical language—as happens with the conventions for employing harmonic proportions—Zipf's law reflects a probabilistic trend rather than an absolute physical reality.

J.R. Pierce's (1961:238–249) description of Zipf's law, focused on the idea of efficiency in terms of an ability to "emphasize some choices at the expense of others", has a special significance for music, assuming that many musical strategies for consistency are based on a same kind of efficiency. As a matter of fact, the structuring form of Zipf's equation, as the series

$$1 \sim 1, 2 \sim {}^1\!/_2, 3 \sim {}^1\!/_3, 4 \sim {}^1\!/_4, 5 \sim {}^1\!/_5 \ldots$$

constitutes a self-referential sequence, comparable to the aliquot division of an acoustic system, with a fundamental frequency and its natural harmonics, as shown in ◊421. Such a self-referentiality can also be interpreted as a convergent sequence with statistical self-similarity, akin to a self-structuring Lindenmayer system, for example:
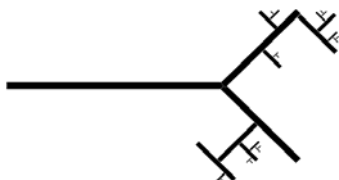
◊511. Self-similar system developed as a projection of Zipf's probability sequence: to an initial straight line segment or *source* = 1, two additional segments are coupled with the length $^1/_2$ of the source. For each of these two added segments, three more segments are coupled with the length $^1/_3$ of the source, and so on. Starting from the origin, which represents a *fundamental hierarchy*, the subsequent segments move by 45° and are coupled in a 'T' shape at their midpoints, with random distribution.

Whether this sort of series occurs in a profusion of cases following consecutive sequences of natural numbers, it is not less interesting to music that in both kinds of examples (represented by ◊421 and ◊511) there is a hierarchical trend suggested by the series itself: 1 ~ 1 refers to a correlative influence over the other terms; 2 ~ $^1/_2$ refers to a subsequent range; 3 ~ $^1/_3$ to a following subsequent range, etc.

Zipf's law seems to be fulfilled in terms of the more general requirements, including functional harmony, in which 'a few' basic relations prevail throughout a tonal system, whilst 'some' secondary relations prevail only in certain cases, and most of the other possible relations are practically eliminated or are very infrequent. This relationship is also found in the configuration of scales in the harmonic-melodic systems in which there is a tendency to form functional structures with two types of steps between the elements of the scale; with three types of steps—less frequently; and with four kinds of steps—even less frequently. Carey (2007:97) concludes, consistently, that "for cognitive reasons, but also for structural ones, we are not likely

to find musically useful scales with four or more step sizes".[289] This conceptualization is also found in Beran (2004:64), for the specific case of the twelve-tone rows: "In music that is based on scales, pitch (modulo 12) is usually not equally distributed. Notes that belong to the main scale are more likely to occur, and within these, there are certain prefered notes as well." Lerdahl and Jackendoff (1983:308) distinguish, however, trends and preferences as regulatory processes under the concepts of *well-formedness* and *preference rules*, respectively. In particular, the principle of well-formedness,[290] closely related to the functions of a generative grammar, develops consistency rules in which recurrent employment produces self-similar structures in language and music.

Another example, among many others, with a main structure typically oriented by Zipf's law, is in the music for the Zande dance-song *kpóníngbó*, traditional of Central Africa, which is played with a drum, two cowbells and a xylophone that follows the vocal pattern.[291] According to Arom (1991:630–631), the ensemble articulation follows the proportion 2:1 (vocals and xylophone), 4:1 (drum), and 8:1 (cowbells). The ubiquity of this kind of proportional hierarchization leads to hypothesize that Zipf's law is a universal feature in music and language.

Zipf's equation is also used for music synthesis. Hiller (1981:12–14), for example, proposes its structural implementation in a 'flow chart' with algorithms which create musical sequences. This notion comes from Voss and Clarke (1975), who proposed to correlate the subjective sensation of music, generalized as a psychoacoustic system, with the typical distribution of fractional $1/f$ noise.[292] They find that fluctuations in pitch and amplitude in music also tend to a statistical Zipf distribution ($\sim 1/f$).

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[289] This subject is developed in subchapter 5.4., with a variety of examples. The topic is also connected to the discussion on the devil's staircase, in subchapter 6.2. (see pages 354–363).

[290] As stated in subchapter 3.4., a musical system is *well-formed* "if its generator always spans the same number of step intervals" (Carey and Clampitt 1996:63). For a detailed description of the concepts *well-formedness* and *well-formedness rules*, see Lerdahl and Jackendoff (1983:36–39, 55–62, 312–314).

[291] Another typical case in this context is the gamelan metre in the traditional music of Indonesia. On this particular subject see subchapter 6.3. (page 382).

[292] Subchapter 5.3. provides a detailed definition of $1/f$ noise, together with examples of its typical patterns and applications in music analysis and synthesis.

Using this distribution as a computational principle, they produced a repertoire of pieces which have 'musical qualities'.[293] Voss and Clarke (1975:258) conclude that

> the sophistication of this '$1/f$ music' (which was 'just right') extends far beyond what one might expect from such a simple algorithm, suggesting that a '$1/f$ noise' (perhaps that in nerve membranes?) may have an essential role in the creative process [of music].

Taking these results into account, Daugherty *et al.* (e2003), and Manaris *et al.* (e2003), find a group of musical relations measurable by Zipf's distribution, including criteria of pitch, amplitude, duration of notes, melodic intervals, and harmonic and timbral patterns. These investigations confirm the relationship suggested by Voss and Clarke (1975, 1978), between a self-similar configuration of sound ($\sim 1/f$) and the subjective sound sensation considered also by its function in preferential bias. Note, as well, that the consistency of what Voss and Clarke (quote above) suspect as the role of 'nerve membranes' coincides with what Garavaglia identifies as "self-similar patterns in the vibrating surface of the eardrum".[294]

5.2. Structure and randomness

All processes of control and choice in musical creativity—in its broadest sense—are bound to a necessary exchange between precision and uncertainty. This exchange is closely related to the constructive role played by a fixed (or exact) grammatical rule and its flexible (or uncertain) interpretation. In this context, the concepts of *randomness* and *entropy* are required for the scaling and statistical projection of process with relative uncertainty. Obviously, randomness does not refer to a mere subjective impression of chance, but to the rational estimation of an unknown behaviour based on the observation of a known state. Mandelbrot (1956:190) conceives that "Randomness is introduced by following the modern statistical theory of the estimation of non-directly observable intensive variables of state".

Theoretically, probability trees under the same general conditions tend to infinite self-similarity because for each level of probability there is the same pattern of

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[293] The aesthetic and idiomatic issues arisen from such procedures are discussed in a general framework, in subchapters 4.7. and 4.8.

[294] See 'Coherence between physics and biology' in subchapter 4.3.

possible deviations, and probabilities decrease by the same proportion. Correspondingly, information increases. In a random experiment, however, the conditions of probability systems vary, so the local conditions, not the global, determine the subsequent state of relations at some point of the deviations. It is possible that the overall behaviour of this experiment tends towards chaos or generalised disorder, which can be measured as entropy.

The concept of entropy

Eco (1968:25–26), using words and descriptions that he also uses to explain the concept of *information*, tries to explain the concept of *entropy*. Actually both concepts can be characterized as 'two sides of the same coin', a relationship that can be summarized as the reciprocity between information and uncertainty:

> Information gives the measure of a condition of equal probabilities, from a statistical distribution existing at the origin. Information theorists call this statistical *entropy*, by analogy with thermodynamics (Wiener 1948, Shannon 1948, Cherry 1957). The entropy of a system is the state of equiprobability to which the parts of the system tend. Entropy is identified with a state of disorder in the sense that an *order* is a system of probabilities introduced into the system to predict its evolution.

Shannon and Weaver (1949:20) introduce the term 'quantity of entropy' (H), relating the value of H to the 'quantity of information'. As Whittaker (1949:78) observes, "Every pound of matter has a definite quantity of entropy, depending only on the state of the matter; and the entropy of a compound system is equal to the sum of the entropies of its constituent parts."

According to Shannon (1948:389–390) entropy can be calculated by multiplying the logarithm of each state's probability by the same probability, summing the results consecutively. This statement can be expressed mathematically as:

$$H = - (p_1 \log p_1 + p_2 \log p_2 + \ldots + p_n \log p_n), \text{ or:}$$

$$H = -\sum p_i \log p_i \, ,$$

where p_i represents each of the consecutive probabilities. As probabilities correspond always to numbers less than 1 (i.e. this is to say they represent a fraction of the unit,

which represents absolute certainty), their logarithms shall always be negative, hence Shannon and Weaver (1949) introduce the negative sign to yield a positive result. The result obtained by comparing the entropy of a random origin with that which it would be if all the choices were equal is known as 'relative entropy' (*Hr*) and is used to know the potential degree of disorder in a chaotic process.

Because the niche from which the composers select their materials determines general features of the compositions elaborated with typical relations from a certain probabilistic origin, Knopoff and Hutchinson (1981:20–24) consider that the measurement of entropy in each piece of music reflects the characteristic entropy of a musical style. So the relative entropies reflect individual choices from more general preferences.

> The notion of entropy has been devised to provide a measure of the diversity of possible messages that can be formed out of a random but biased alphabet of characters. The bias arises because of cultural and individual practices. [...] A description of entropy is therefore pertinent to the discrimination of stylistic differences among composers, since we may hope to detect differences in usage from among the symbols available to the composer

Although these authors emphasize the measurement of entropy as a resource for classifying different modes of written music, calculation of entropy in all parameters of music—for example, using a recording to classify the style of an interpreter, or using a set of recordings to classify more general interpretational biases from the same piece of music—can be very useful for musical stylometry (see Beran and Mazzola 1999a) and for the classification of self-similarities revealing a stylistic process (see Bigerelle and Iost 2000).

Stylometry

Leonard B. Meyer (1989:3) defines style as "imitation of patterns" in a process of restriction and election.[295] Preceding this definition, Ingarden (1962:11–12) believes that style is a distinctive aspect of musical performance, under the guise of a self-similar sequence:

> Each separate performance of a musical work is, as an individual object, completely univocally determined in every respect possible for that object and is so determined ultimately by qualities that admit of no further differentiation. [...] While the separate parts of a performance follow one another *in reality*, in definite phases, the parts of the musical composition itself exist simultaneously as soon as it has been completed [in time].[296]

This notion of interpretation can be extended, however, to composition and musical analysis, assuming that in any case it involves a phenomenon of interpretation of style as "imitation of patterns" within a tradition. Therefore, under the Ingardean (*cit.*) argument, self-similar relationships in music are not unique to instrumental performance, but can be found in virtually all aspects of music based on imitation and recursion, being measurable as a time series (see Bigerelle and Iost 2000, Su and Wu 2006, Das and Das 2006, and Dagdug *et. al.* 2007).

Doležel (1969:10) conceptualization on frequencies counting within a language style can equally apply to music: "The foundations of the statistical theory of style can be summarized in a simple statement: *style is a probabilistic concept*". In fact, Pinkerton (1956) and Youngblood (1958) conceive the same idea as part of their adaptation of

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[295] This definition is discussed and developed in 4.7., in the context of the overall relationships between grammar and style. It is advisable to consider what is stated there, for an accurate explanation of style as coordination between constraint and choice in a musical system.

[296] In the original text [*op. cit.*:11]: "Jede einzelne Ausführung eines Musikwerkes ist, als ein individueller Gegenstand, in jeder für den betreffenden Gegenstand möglichen Hinsicht vollkommen eindeutig bestimmt, und zwar letzten Endes durch Qualitäten, die sich nicht weiter differenzieren lassen." (*in situ* Ingarden announces that this notion is developed in his work *Der Streit um die Existenz der Welt*, Max Niemeyer, Tübingen, 1964; vol. II, Ch. 9). He then continues [1962:11–12]: "Während die einzelnen Teile einer Ausführung in ganz bestimmten Zeitphasen realiter aufeinanderfolgen, existieren alle Teile des Musikwerkes selbst, sobald es natürlich fertig ist, zugelich." It is clear that Ingarden does not use the term "self-similar", but his description is fully compatible with the concept defined here (see subchapter 3.3.)

information theory to musicology.[297] Doležel (1969:10) specifies that the probabilistic concept of style has two fundamental features:

> 1. *Probability Distribution*. In a probabilistic 'world', the occurrence of phenomenon A is not unequivocally predetermined by the existence of condition X. Under condition X, the phenomenon A will occur with a certain probability only–$Px(A)$; with a probability $Px(B)$, phenomenon B can occur; with a probability $Px(C)$, phenomenon C, and so on. Even if the probability of A under condition X is high–that is, when $Px(A)$ approximates 1–the possibility of non-occurrence of A, or the possibility of occurrence of B, C... cannot be excluded. The expectancy of each phenomenon of the set A, B, C... under the condition X is given by the *probability distribution $Px(A)$, $Px(B)$, $Px(C)$*.

> 2. *Frequency Distribution.* The probability distribution describes the expectancy of phenomena A, B, C... in a complete or ideal ensemble of occurrences–in the so-called population. Empirically, however, phenomena A, B, C... occurr or can be observed in rather limited and not-ideal ensembles, in the so-called samples. In samples, values of the probability distribution are liable to random fluctuations. In other words, one cannot expect that the values $Px(A)$, $Px(B)$, $Px(C)$... will be stable (identical) in various samples; rather, probability distribution is reflected in sample *frequency distributions*: $p^{(1)}x(A)$, $p^{(1)}x(B)$, $p^{(1)}x(C)$...; $p^{(2)}x(A)$, $p^{(2)}x(B)$, $p^{(2)}x(C)$... where the index numbers *1, 2*... symbolize various samples. The values of the frequencies– $p^{(i)}x(A)$... fluctuate in a certain admissible (statistically insignificant) interval around the values of probabilities $px(A)$.

Doležel (*ibid.*) emphasizes the fact that language, or at least the samples that can be extracted thereof from a variety of texts, presents "strong evidence that style properties of texts display both features of probability distribution and frequency distribution." The stylistic features of music also provide evidence to be considered as characteristics of probability and frequency distribution. Waugh (1996), Bigerelle and Iost (2000), Beran (2004), Ockelford (2005), and Ilomäki (2008), among other sources, explicitly target this evidence with a wide variety of approaches. Ilomäki (2008:35), for example, points out structural difference and similarity trends as a process that defines musical form and style:

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[297] The original idea of Pinkerton (1956) and Youngblood (1958), of measuring the entropy of a melodic contour in order to identify a musical style, is developed by Beran and Mazzola (1999a–b) and Beran (2004:93–105), expanded to spectral entropy analysis, and to the 'weights' of metre, melody, and harmony as related functions.

> Similarity and dissimilarity are seen here as two sides of the same coin, only the focus is different: a similarity measure evaluates the number and significance of the shared features between two musical objects, and a dissimilarity measure evaluates the number and significance of the differentiating features. Nevertheless, they both represent the same continuum: at the one end is similarity and at the other end is dissimilarity.

Doležel's original proposal does not put aside grammatical functionalism to favour a theory of style as probability. Conversely, he conceives that *grammatical competence* "is a necessary background for any theory of style" (Doležel *op. cit.*:12), and he stresses the relevance of clarifying the definitions of grammaticality, ambiguity, and similarity to measure a stylistic trend. On the ground of these concepts and with their dynamic relations in pragmatics and context, some *frequency vectors* can be charted in order to identify a musical style, by comparison with other statistical samplings.

The statistical analysis of style, or *stylometry*, is based on the notion that each idiolectal source (author) has an authenticity or 'identity footprint' that consists of a unique convergence of relationships. This trace of identity is as a typical pattern in the idiolectal output and has representative value for each sample in a detailed statistical analysis of music (see Beran 2004:92–93).

Since "style is a probabilistic concept", the classification of an identity footprint within an array of idiolectal samples can be characterized as a categorization concern. Diederich *et al.* (2003) acknowledge that an obstacle in this concern is that it is not entirely clear which features within a sample are to be taken into account to classify the so-called identity footprint (in a recorded conversation, a text, a piece of music); thus, they look into a variety of methods for obtaining satisfactory results linking the analyzed samples with their sources of identity. Diederich *et al.* devote their work to identify the author of a text, focusing on aspects of intellectual property, but also delve into issues that involve the study of style in a broader perspective, including the adaptation of Zipf's equation (*vid. supra*).

The operational domain of a semantic-syntactic function may change significantly from one paradigm to another within the same culture; it also may change within the same idiolectal source (i.e. in the same author), as the difference between two samples from the same *text*—e.g. between two identical operations that have different meaning depending on contextual usage. For this reason Diederich *et*

al. remark that the traditional methods of stylometry yield data that can be considered only partially. On the other hand, however, Diederich *et al.* reach a more thorough description of the statistical features that determine an idiolect, matching a larger number of samples in data processing, than in previous research.

A detailed insight into the work of Diederich *et al.* is a goal that falls outside the scope of this research. However it is noteworthy that some relevant issues of musical stylometry have been developed by Pinkerton (1956), Youngblood (1958), Knopoff and Hutchinson (1981), Harley (1995), Beran and Mazzola (1999a–b), Bigerelle and Iost (2000), Beran (2004), and Su and Wu (2006), mentioned at different points in this study.[298]

Markov chains as degrees of entropy

Markov chains are named after mathematician Andrei Andreyevich Markov (1856–1922), who first defined them in 1906 in a paper dealing with the *law of large numbers*, and with the purpose of studying the transition probability of the original text of *Eugene Onegin*, a novel in verse by Alexander Pushkin (1799–1837). This novel is almost wholly written in verses of iambic tetrameter with the unusual rhyme scheme *aBaBccDDeFFeGG* where the lowercase letters represent feminine rhymes (i.e. rhymed on the penultimate syllable) and the uppercase represent masculine rhymes (on the final syllable). It is worth noting that this limited scheme reduces the probabilities of a well-structured composition, so its study had both functional-linguistic and statistical interest in the times of Markov. He calculated the transition probability by counting consecutive pairs of letters in the Cyrillic alphabet; he then divided these counts into two groups based on initial letters (e.g. a group of counts for pairs of letters starting with *A*, another for pairs starting with B, etc.; placing each of these groups in series within a transition matrix); and finally he 'normalized' the transition probabilities by dividing each individual count by the total number of counts of the same group. Markov then noted that his method was statistical, rather

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[298] In particular, see subchapters 4.5. (stylistic endomorphisms), 5.5. (pertinence of fractal dimension), and 6.2. (noise and isotropy, chaotic functions).

than probabilistic, producing only a partial description of a non-random behaviour, rather than the general prediction he had first wanted.

Shannon's theory (1937, 1948) adapted Markov's experiment by attributing a probabilistic state to each symbol used within a code, in message transmission. Under this scheme, when 'information content' is high, the probabilistic transition will be close to $1/n$, where n is the size of the 'alphabet' used in the transition matrix. So when redundancy is high, transitions are strongly oriented toward certain patterns of recurrence. With this method, Shannon managed to decode messages by filtering a set of samples perturbed by noise (see Campbell 1982:67–73).

In short, a Markov chain is a simple model of probabilities in which the state of an event is determined only by its immediate antecedent (see Jones 1981:46, Roads 1999:878, Beran 2004:169). In this context, Youngblood's (1958:27) clarifies the relationship between part (moment, event) and whole (final effect, entire sequence of partial states):

> One type of stochastic process, the Markov chain, is a sequence of events in time in which each event has a calculable probability. Furthermore, this probability is the result not only of the frequency of the event in the set of events under consideration but also of the effect of those events which immediately preceded it on the event of the moment.

To give an idea of how this series of processes approaches the probabilistic states of a code forming a text, Shannon (1948:I,3) provides an example assuming a 27-symbol "alphabet" (26 letters and a space) in six levels of structural ordering: in the first one, called *zero-order* approximation, symbols are independent and equiprobable, forming a line with a general result that reflects a highly random process; in the second level, called *first-order* approximation, symbols still in an independent arrangement but with frequencies vaguely representative of an English text chosen by Shannon; in the third level or *second-order* approximation, a 'digram structure' (i.e. with clusters of letters in long and short groups) appears more closely to English; in the fourth level or *third-order* approximation a 'trigram structure' (i.e. with clusters of letters in long, medium and short groups) appears to apporximate even more closely to English. In a fifth level, called by Shannon as *first-order word approximation*, rather than continuing with a tetragram... *n*-gram structures, "it is easier and better to jump at this point to

word units." Here words are chosen independently but with more appropriate frequencies. In the sixth level or *second-order word approximation*, "the word transition probabilities are correct but no further structure is included." At this level the resemblance of the words to common English is very clear, or at least, as Shannon notes, "have reasonably good structure out to about twice the range that is taken into account in their construction."

Shannon (*ibid.*) mentions the empirical method from which he developed his analysis of order/disorder in a data set: for instance, the first two samples were constructed by the use of a book of random numbers in conjunction with a table of letter frequencies. In the levels which follows he opens a book at random and selects a letter at random on a page. This letter is recorded. The book is then open to another page and he reads until this letter is encountered. The succeeding letter is then recorded. Turning to another page this second letter is searched for and the succeeding letter recorded, etc. A similar process was used for the following levels. Shannon then formalises his method. It is obvious that a similar procedure can be performed with a collection of scores or musical recordings, to analyse various aspects of redundancy and consistency of structural information.

Currently, the vast majority of software used for information reconstruction, for interlinguistic translation, and for a wide variety of word processing and searching on the web, uses algorithms based on Shannon's (1948) formalization. This application also involves various aspects related to music, from developing melodies using computer software, to the investigation of the borrowing, change and transformation of codes studied by stylometry, as mentioned in the previous section, after Pinkerton (1956), Coons and Kraehenbuehl (1958), and Youngblood (1958). It is used also in structural planning and synthesis for composition, as explained by Ames (1989).

By strict analogy with Shannon's probabilistic model, the same levels of order —or disorder if one conceives the reversible case—can be laid out for a musical motif, phrase, or section; it can be even used with a complete musical piece—which may require too much work, whilst not implemented as computer algorithm. This procedure (analytical or synthetic) can be applied to any sample of music independently of style and tradition, keeping the gradation of stochastic relations

from an original chain. Especially with long chains, extending the levels of order/disorder is possible, so Shannon's six levels is not necessarily a definite limit.

◊520. Example of *degradation* of the chromatic scale (n_0) by seven levels of entropy (n_1–n_7). In the left column (*a*) there are three degrees (n_1–n_3) as a process of symmetric inversion, starting with the series endings. From n_4, the process forms an increasingly segmented series with a lower contiguity degree, breaking the original symmetry.

Barlow (2002, final section) suggests that the order/disorder scale in Markov chains, usually with six or seven levels, can have intermediate levels, which he uses to elaborate sequences of musical structuring in a wide range of variation between order and disorder. Beyond the meaning of this task on information theory, it is quite clear that this extension of consecutive rulers for measuring a code is compatible with what Wacław Sierpiński (1934) proposed in his chapter on "Applications to the relationships between category and measure", which Mandelbrot (1967) confirms by demonstrating that the magnitude of a measurement depends on the used scale. Thus, levels of order/disorder in Markov chains can be variated, depending on the needs for measurement and the characteristics of the measured object. In ◊520, for example, eight levels of order are established as structural degradation of the chromatic scale, with twelve elements. In this case the transition probability is confined to the consecutive permutation of the row parts, seeking a higher degree of

the original series decomposition: in n_1 the probabilities of repeating the first element (0) are necessarily minimum, so it moves to the series ending; in n_2 the probabilities of repeating the second element (1) are necessarily minimum, so it moves to the series ending; and so on. It is obvious that following this displacement without return to a previous state, at some point (represented here in n_7) the original contiguity between any element in the series, is reduced to a minimum. The levels of order/disorder in this example are of theoretical and compositional interest (analytical and synthetic), as they contribute to investigation of the properties of the series.

Synecdochic function

The 'synecdochic function', a term commonly used in linguistics and literary studies (see Freedman 1971) to explain the correlation of synecdoche in a system of structural and symbolic transitions of language, can be implemented in any system of musical relations having analogous conditions to a symbolic structuring—admitting, for instance, that the systematic correlation duration/intonation can constitute or produce an operational set of musical symbols.

In the context of information theory applied to music, synecdochic function (σ) shows the degree of self-similarity across structuring levels as information increases and redundancy decreases. For instance, $\lozenge 521$ suggests that, starting from an initial state of maximum redundancy (Ω) comparable to $1/f^0$ noise,[299] the relations of similarity are null within a bound that narrows towards a generalized probabilistic transition point: what Shannon (1948) conceives as *first-order word approximation*, i.e. the limit where the relations of similarity emerge, getting closer to a potential self-similarity in the threshold of *second-order word approximation*. At this point the relations of self-similarity are likely to bifurcate into consecutive levels. In this example, proliferation of σ is attributable to a development of the system consistency (i.e. grammatical recursiveness), as specified in subchapter 3.7.

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[299] See the definition of $1/f^0$ noise on pages 241.
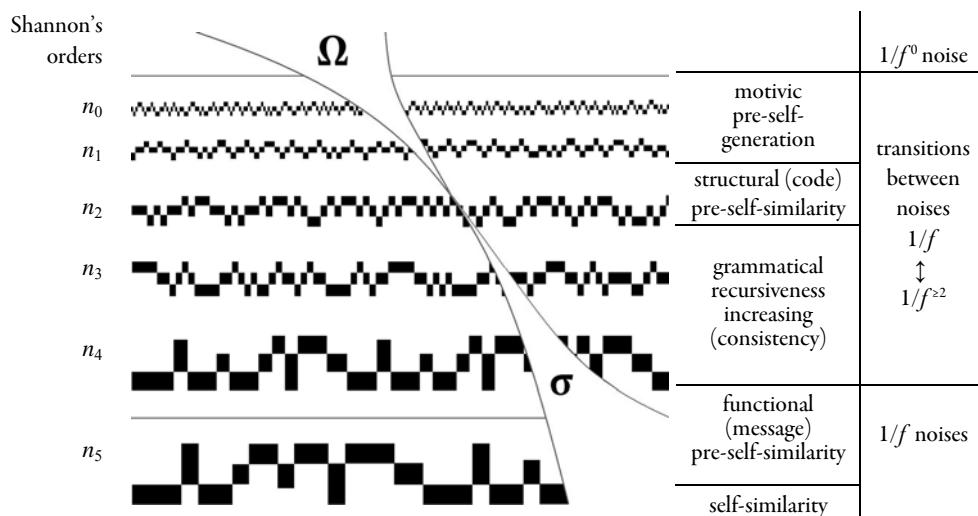
◊521. Schematic view of levels of order/disorder in a musical process with parameters of length (horizontal) and pitch (vertical). The leftmost column displays the levels of entropy suggested by Shannon (1948). The far right column associates fractional noises (see Mandelbrot and Van Ness 1968), whilst the next column suggests a grammatical interpretation of music as recursiveness with self-similar trend. Whereas the symbol Ω on the top represents the highest level of disorder or chaos, the symbol σ represents the synecdochic function that allows logical and structural association between the parts and their total, statistically self-organized under the power laws, involving different modes of self-similarity. The continuum connecting Ω with σ is semiotically related to Peirce's *synechism* and to the intersemiotic theory (IC) detailed in subchapters 3.8. and 3.9.

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example, proliferation of σ is attributable to a development of the system consistency (i.e. grammatical recursiveness), as specified in subchapter 3.7.

In a process of musical information, the synecdochic function is the fundamental relationship with which the message prescinds from the code's completeness, to nevertheless be able to guess the total amount of information by the interpretation of its parts (*abduction* in Peirce 1903a/1998:162). This relationship, basic to any form of musical interpretation, may be unaware (e.g. listening to a signal whose frequency spectrum is completed by the auditory cortex);[300] partially conscious (e.g. in the musical performance of a score), or conscious (e.g. music analysis).

This theorizing also may contribute to better understand the concept of *motif*, as explained in 3.3. (see in particular scheme ◊333), and its constructive role as seed for development and pre-contingency of musical structuring.[301]

Random constructionism

Markov chains are among the most popular tools used in algorithmic composition in the last sixty years,[302] due to their intuitive handling and adaptability to a variety of gradients controlling different parameters of music. The probabilistic properties of a Markov chain can be represented by a grid or matrix reflecting a transitional state from an initial relationship (see Roads 1999:878–880). A matrix for the elaboration of a pitch sequence is presented in ◊522. The letters in the left column represent *output tones*, whilst the letters of the top line represent *incoming tones*. In this example

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[300] Because of this, the synecdochic function operates in the construction of numerous psychological effects giving sense to music by auto-completing (*i.g.* conjecturing) the missing parts in a code. An example is a 'complete' perception of a compressed code—such as MP3 format—whilst listening to any recording. The synecdochic function extends, however, to a great number of forms of psychological sound processing, including normal conversation.

[301] In fact the conceptualization of the synecdochic function in music, regarding the structural notion of 'motif', satisfies what William Freedman (1971:129) intends as such in literary theory: "Since the symbolic motif is basically microcosmic, since it is a part of a literary work that may often stand for the whole, it performs, I think, a synecdochic function." This idea, which enriches the research of music as intersemiosis and analogy with the discursive language, is here developed in subchapters 4.4. to 4.8.

[302] Around 1950 Markov chains were included as a significant issue in educational programs for composers, in countries of Northern Europe. Karlheinz Stockhausen (1989:50) recalls that "Professor Meyer-Eppler, a teacher who had come from physics and phonetics, […] would give us exercises demonstrating the principle of the Markoff series."

the constructive process begins with the upper cell of the leftmost column (bold C), which initially has two options for succession: C# or E. Numbers in each box represent the probabilities of a pitch to be related to another pitch, in order to start the development of a sequence. Henceforth, the first relationship has two options with $^1/_2$ probabilities each to be chosen. The second relationship can be originated in C# or E read as outcomes. For both cases pitches C, D, and F have $^1/_3$ probabilities each. The next step repeats a similar operation, depending on the chosen outcome, and so on. The row in ◊522b shows a sequence of 12 pitches produced with the matrix. Obviously, this sequence is just one of many possible, since for each step towards the income pitches, each election draws one of several possible continuations. Another more complex option is the production of other sequences with the same matrix or other matrices, in order to develop multiple pitch-rows with simultaneous relationships, something useful in a composition plan that elaborates polyphonic and/or multi-textural lines (see Xenakis 1992:87–88, Steinitz 1996a:19).

	C	C#	D	E	F
C	0	$^1/_2$	0	$^1/_2$	0
C#	$^1/_3$	0	$^1/_3$	0	$^1/_3$
D	0	$^1/_4$	$^1/_4$	$^1/_4$	$^1/_4$
E	$^1/_3$	0	$^1/_3$	0	$^1/_3$
F	0	$^1/_2$	0	$^1/_2$	0

◊522a. Matrix for production of pitch sequences.

◊522b. 12-tone row produced by the matrix in the upper scheme. This sequence is just one of many possible, since for each step towards the incoming symbols, each election draws one of several possible ways.

◊522c. A second option of a 12-tone row produced by the same matrix.

The same procedure, using a similar matrix, can be used to produce sequences of lengths. By using this second option in combination with the previous one then one can get a third kind of sequences with variable lengths and pitches altogether (as suggested in ◊522c). A $^1/_4$ rest is included in ◊523a, which can get a *punctuation mark* function, useful for gestural phrasing or motivic elaboration such as those shown in ◊523c. It is obvious that such values can be adapted into an almost unlimited variety of compositional requirements. Pitch and length values can also be recombined again by matrices representing other values within a specific scale, as well as by matrices representing sets with other sort of items or instructions.[303]

	♪	♪	♪.	♩	𝄽
♪	0	$^1/_2$	0	$^1/_2$	0
♪	$^1/_3$	0	$^1/_3$	0	$^1/_3$
♪.	0	$^1/_4$	$^1/_4$	$^1/_4$	$^1/_4$
♩	$^1/_3$	0	$^1/_3$	0	$^1/_3$
𝄽	0	$^1/_2$	0	$^1/_2$	0

◊523a. Matrix for production of note sequences, analogous to the previous 522a. The generative process is the same, although in this example the general output determines musical lengths.

◊523b. 12-durations sequence produced by the matrix in ◊522a.

◊523c. Musical sequence obtained as a combination of ◊522a and ◊523b.

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[303] For a detailed description of a systematic use of Markov chains in algorithmic composition, see Jones 1981.

The use of Markov chains as shown in the two previous examples (◊522, ◊523) corresponds to a stochastic generation in which, in the long run, the structures produced tend to a self-similar distribution since the generator matrix consists of a set of elements whose replication is limited in each recursion, to the same number of possibilities. At the same time, the use of a finite matrix such as the one used for these examples, is a case of self-reference since its transitions are derived from rules contained by the same matrix.[304] It can be argued, consistently with this relationship, that the greater the variety of generative relations (i.e. implementing a larger and more complex matrix), the more complex the (sub)products of a self-similar set in the long-term. In this case the synecdochic function progressively extends between the initial point of similarity to the point σ (see ◊ 521).

Music microvariation and chaos

Many musical styles are based on the continuous variation of a figure given by tradition or suggested by practice. This commonly happens in improvisation as found in a wide variety of forms, as well as in the *gloss* or figural variations of a musical segment using grammatical rules. In general terms, musical borrowing, imitation, change, variation and transformation follow relatively open rules within a more or less flexible limit, although almost always with a fixed grammatical part providing basic organization. These musical relationships operate through 'macrovariations' in which most of the (re)creative process occurs in transitions among *orthodoxy, effect*, and *election*, using Doležel (1969) terminology for his theory of style; and among *grammaticality*, *ambiguity*, and *similarity*, according to Fodor and Garret (1966) terminology on syntactic competence.

In contrast, one finds arrangements of 'microvariations' in which the (re)creative process occurs through semi-hidden or hidden operations. An example of this are the vocal *glissandi* in ◊451, in which the gross pitch-length relations are established, but the timbral qualities and the loudness and intonational subtleties remain hidden or vaguely suggested. This sort of microvariation seems to be a universal trait linked to

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[304] Cabrelli, Heil and Molter (2004:vi) define a self-similar matrix as "an $n \times n$ expansive matrix that maps a full-rank lattice into itself."

the diffuse nature of the thresholds of musical practices, sometimes denoted by culture (e.g. a musical tradition may stimulate the recognition of some intervals that other traditions do not recognize), sometimes fixed by the physiological limits of perception. Microvariation operates near to these thresholds.
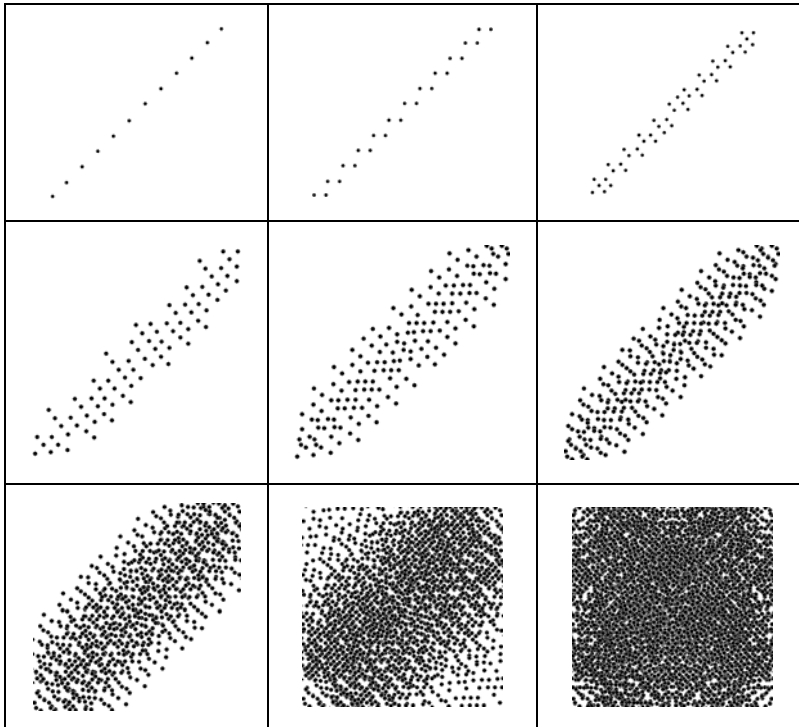
◊524. Schematic representation of the chromatic pitch-space and its microtonal divisions by halves. Ordered from top to bottom and from left to right, the sequence represents the tendency to chaos in the micro-intervallic human (i.e. non-automated) articulation of sound, in a continuum from an attractor at the diagonal axis. The symmetrical arrangement in the first three boxes (i.e. semitones, quarter-tones, and eighths of tone) reflects a preservation of relative equidistances. After the fourth box (middle left) a progressive imbalance is noted affecting homogeneous distribution towards increasing entropy. This scheme is analogous to the diagonalization of musical durations by halves of time units. It is important referring to subchapter 5.4., for further explanation.

Graph ◊524 presents a scattering pattern which begins (left box, top) with the diagonal representation of the chromatic row as equal consecutive intervals. The equidistance between points (representing pitches) indicates equal intervals between semitones (the example is based on this equivalence; thus it does not consider a Pythagorean coma between semitones). The top-centre box shows the diagonal with a 'correct' articulated symmetry between quarter-tones. The box that follows to the right shows a lower exactness for the eighth-tones distribution, reflected in a slightly asymmetrical plot. Following this development, the subsequent boxes show a growing trend to disorder in which symmetry blurring intends to mean a gradual weakening with respect to the initial references.

In an empirical testing of the same process, one can say that a continuous, too fast articulation of voice or of a musical instrument, tends to monotony (i.e. Gestaltic or perceived monotony). The correlation between these two examples—the former focused on pitch location; the latter on durations—shows a same bias to disorder at the threshold of microvariations, and a tendency to organization from indeterministic chaos at the basis of musical performance.

Chapter 6 examines in detail the generalized transition observed in many dynamical systems in the form *proportion → self-similarity → chaos* (arrow denotes tendency).[305] Suffice it to say, for now, that in Western tradition, the same transition may appear intuitively in the form *harmony → similarity → noise*, as suggested e.g. in ◊624, conceiving a specific tonal scale—like the natural diatonic scale—as a structural antecedent of the chromatic scale. In other words, this transition reflects a shift from a general scaling measure to another, less generalized scale. The latter will need, thus, a more detailed regulation in order to adjust its subsidiary rules to the general rules of the pitch system. In consequence, one may assume that a musical grammar may perform the transition *repetition → recursion → elaboration on the recursion*, as an expression of the synecdochic function. Carey and Clampitt (1996:65) associate this relationship with the concept of *well-formedness* (introduced here on pages 60–62), with its essential role in conceiving structural rules in music as well-structured grammars.

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[305] This conceptualization require reading the section 3.8.3.

5.3. *Music* in noise

From its beginning this chapter emphasizes the equiprobability between information and uncertainty. As a result of this relationship, this study assumes that *all music has noise* and *all noise has music*. Voss and Clarke (1975, 1978) find that $1/f$ noise is ubiquitous in speech and music: this has a marked influence in musical acoustics and in the investigation of its functional relationships, as explained below.

The concept of noise as a total complex of tiny events is more significant than that of a set of isolated events in large scale. Mandelbrot and Wallis (1968:909) use the term *Noah effect* to refer to events of great magnitude, extremely rare in a quasi-periodic system. According to the statistical observation of these systems, a massive set of tiny events contains much more information than a single large isolated event. On the other hand, to find a set whose small parts are completely unrelated is difficult in practice and contradictory in logic, as its dissociation would be opposed to the notion of set. Indeed, the richness of $1/f$ noise as a set is significant in that all its parts are correlated by the same kind of energy distribution (see Mandelbrot and Van Ness 1968).

$1/f$ noise has a power spectrum in which each time its frequency doubles its density decays 3dB. In other words, the probability of a specific frequency decreases in the same proportion as the frequency increases. In musical terms, in $1/f$ noise each octave carries an equal amount of noise power, inasmuch as what happens in the lower frequencies also happens proportionally in the higher frequencies, with an amplitude exponentially lower. $1/f$ noise wavelets and signals, in scalar invariance, are related by their generalized self-similarity.[306] This becomes evident when applying the logarithmic subdivision of its frequency axis: comparing the segments that make up the frequency spectrum with the signal as a whole, one finds the same energy

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[306] For a general introduction to the concept of 'invariance', see subchapter 3.4.

distribution. Mandelbrot (1982, 2002), Hsü and Hsü (1990, 1991), and Polotti and Evangelista (2001:28), among many others, acknowledge these features as 'fractal properties' of $1/f$ noise.[307]

Many energy fluctuations studied by Fourier analysis in solid media and in fluids, are called 'fractional noises' and represented in the form $1/f^k$, as their spectral density in a sample takes the form λ^{1-2H}, with λ as frequency (f), and $\frac{1}{2} < H < 1$, being H a parameter that generally tends towards 1. Under this definition $1/f^0$, $1/f^1$, $1/f^2$ noises and other using a similar representation are considered 'fractional noises' (see Mandelbrot and Van Ness 1968:422).[308]

According to Voss and Clarke (1975, 1978), the power spectrum of $1/f$ noise is similar to the average spectrum accumulated in the resonance of a tonal harmonic repertoire, and the outline of its typical sonogram is analogous to that of a piece of music or a chain of verbal utterances.[309] In fact its distribution is equivalent to Zipf's distribution described in subchapter 5.1. as a general property of speech. Roads (1999:881) explains, however, that whereas in the latter sense f refers to a 'mode of change' of a system—this is to say the mode in which its elements or substructures are repeated—in $1/f$ noise parameter f refers to acoustic frequency. In any case f represents a mode of information distribution that in $1/f$ tends towards self-similarity.

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[307] $1/f$ noise is also called 'pink noise' because its power spectrum density and frequency distribution are typically characterized as intermediating the general features of 'white noise' or Johnson noise ($1/f^0$), and the so-called 'red' or 'brown noise' ($1/f^2$). Since spectroscopy considers that pink is an intermediate color between white and red, in a metaphorical way this description takes the same meaning for the acoustic signals. Unfortunately this metaphor leads to several misunderstandings. In fact, white noise takes its name from white light in which the power spectral density is distributed over the visible band in such a way that the human eye's three color receptors (cones) are approximately equally stimulated. In its turn, 'brown' or Brownian noise is named after scientist Robert Brown (1773–1858), who was the first to describe the chaotic behaviour of the so-called Brownian motion, similar to the characteristic pattern of $1/f^2$ noise. Instead of following this metaphorical usage (using adjectives such as white, pink or brown), this study prefers the proportional analogies $1/f^0$, $1/f$ and $1/f^2$.

[308] In this formalization H means Hurst exponent, which Beran (2004:92–93) defines in a context of statistical analysis of music. Mandelbrot and Van Ness (1968:422) consider that the term 'fractional noises' is more correct than '$1/f$' noises, since the latter strictly corresponds to the specific noise having such a spectral relationship.

[309] As examples see sonograms in ◊560 h, k (pages 292, 294).

Like music, one may consider $1/f$ noise substantially as a temporal phenomenon—without necessarily ignoring its aspects of simultaneous complexity. When Dodge (1988:11) says that $1/f$ noise has "memory", he conceives $1/f$ noise as time series. Consistently, Roads (1999:881) notes that $1/f$ noise properties are logarithmically correlated to their past: "Thus the averaged activity of the last ten events has much influence on the current value as the last hundred events, and the last thousand (using logarithms of ten)." Therefore one may say that $1/f$ noise has long-term memory inasmuch as its events recur the same configuration for each period of time. This periodic behaviour also suggests an analogy with music, especially with a repertoire made after a hierarchical organization in which specific relationships return in time, giving a structural sense to the whole (see Bartlett 1979).
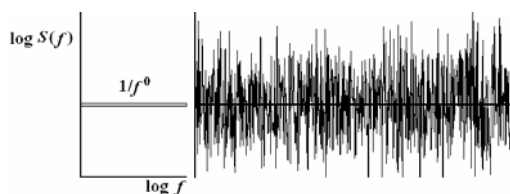
a) $1/f^0$ noise (or *white noise*)
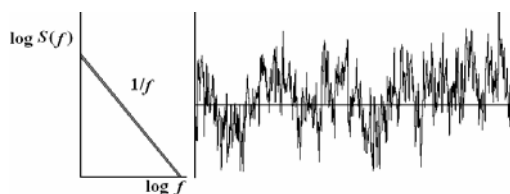
b) $1/f$ noise (or *pink noise*)
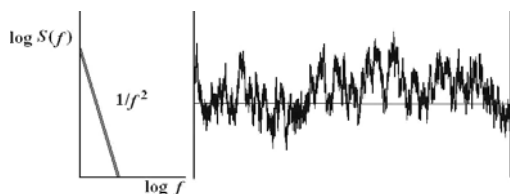
c) $1/f^2$ (or *Brownian noise*).

◊530. Typical waveforms of $1/f^0$, $1/f$, and $1/f^2$ noises. Power spectral density
is pictured by the boxes left to the sonograms with usual intensity (vertical)
and length (horizontal) representation. Summarized from Voss (1993:9).

$1/f^0$ noise, a phenomenon significantly different from $1/f$ noise, has a flat power spectral density (see ◊530-*a*, left box). Nominally it contains 'all' frequencies, and all of them have the same power in a characteristic random distribution without any specific trend to self-similarity. It is known as *white noise* because, by paradigmatic analogy with white light, does not show a greater liking to any frequency or colour in particular. In this context the simile employed by Miramontes (1999:6) is quite descriptive: "In music white noise should be achieved if all the instruments of an orchestra would play different notes [i.e. tones] at once without any coherence or coordination."

A third kind of noise is Brownian noise which power spectral density is directly proportional to $1/f^2$, which implies that density decays 6dB by each frequency duplication. Its sonogram tends to be analogous to Brownian motion observed in the physical trajectory of some particles suspended in viscous fluids, for example of a pollen grain suspended in a water drop (see Brown 1828). Brownian motion has in common with other stochastic processes—like Markov chains—the fact that any of its events in a moment z is determined by its immediate antecedent in a moment y. This corresponds to a *random walk* pattern in which each point 'moves' in a random Gaussian distribution with respect to the previous point. Gaussian distribution implies a bounding distributive probability, so that points become more or less contiguous in their erratic movement, as shown in the downward and upward movements in the Brownian series shown in ◊530-*c*. Mandelbrot (1984:5) conceives that "Brownian motion is a natural fractal", suggesting that like $1/f$ noise, $1/f^2$ noise also reflects a self-similar behaviour—although in its own random way.[310]

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[310] The auditory sensation produced by $1/f^2$ noise on the human ear approaches that of $1/f^0$ noise, but at a lower apparent frequency (see Gardner 1978). Some examples that can be intuitively associated with it are the constant sound of raindrops hitting the floor in a storm; the sound of breaking waves on shore and on reefs, or the sound of tree tops swaying in the wind. These comparisons may be analogous, in the strict sense of proportion (ἀναλογία) as explained in 3.1. On the accuracy of such analogies as synthesis or analysis, see subchapter 5.5.

Noise's musical scaling

A relatively recent method of classifying music is by comparing its patterns of recurrence with fractional noise models, particularly $1/f^0$, $1/f$, and $1/f^2$ noises. This method favours a description of intrinsic acoustic relationships in a musical system: for instance, the pattern $1/f^0$ contains monotony with high and low information redundancy, comparable to granulated timbres and pulse sequences without a specific trend of order. The pattern $1/f$, in contrast, can be conceived as an image of self-similar structures with harmonic hierarchies, softened timbres, and organized phrases comparable to those of verbal language. The pattern $1/f^2$ can be musically interpreted as a collection of structures made by contiguous relations in which only the immediate past determines the actual state of a musical event. It can also be associated with less harsh or less diffuse timbres than in $1/f^0$, but less smooth than in $1/f$. Interestingly many descriptive combinations can be made after these notions, scaling some of them to serve as further reference as suggested in ◊ 531.

Analogue adaptation in ◊531 shows that pattern $1/f^2$ differs from $1/f$, elaborating 'gestures' more differentiated between continuity and discontinuity, with fewer skips per period and an apparent middle-term seesaw behaviour, and a lower musical affinity in the sense that contours and spaces become less attractive inasmuch as they reduce their expectancy degree—although this depends largely on the way of approaching to the pattern. The combination of these textures or 'colours'—timbral, rhythmic, harmonic, and durational at the same time—as part of a synthetic or analytic strategy, suggests also an application of several and diferred indices of musical self-similarity.

Whereas ◊531 pursues an intuitive, direct emulation, ◊532 shows a procedure for abstracting the different patterns of noise analogized as 'grammaticalities'. Example ◊531 is limited to a subjective imitation of the three different contours of noise, whilst in ◊532 there is a functional specificity in which, with the systematic use of a matrix, more musical parameters are constrained as in a generative matrix using Markovian constructionism. The musical result in ◊532-*b* conveys, therefore, some of the most inherent properties of the source: whether $1/f^0$ has a 'musical surface', a dispersion of gestures and a lack of overall cohesion is evident, too. Rather, it

a)
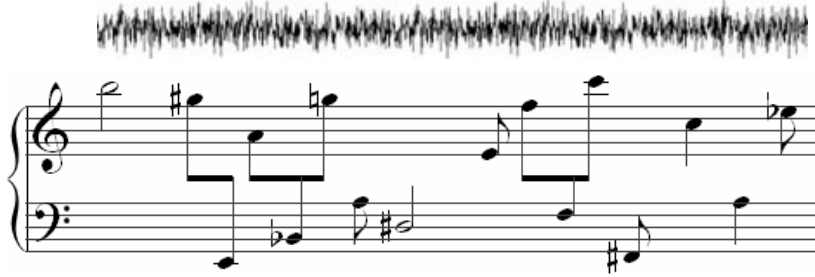
Analogue emulation of $1/f^0$ noise.

b)

Analogue emulation of $1/f$ noise.

c)

Analogue emulation of $1/f^2$ noise.

◊531. Segments of analogue emulations of fractional noises corresponding to the previous table, as an adaptation to a one-voice keyboard in a measure of 24 pulses. In all three cases the tonal range is bounded to the index $C^3 - C^7$, with durations limited to $^1/_2$, $^1/_4$, $^1/_8$ by random choice.

$1/f^0$
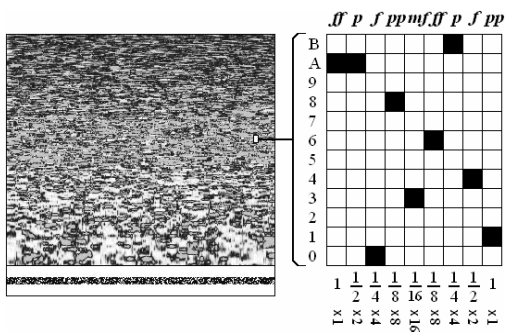
$1/f$
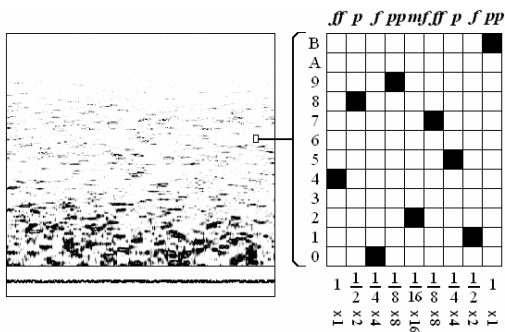
$1/f^2$
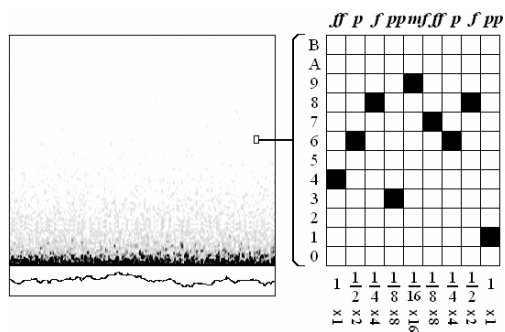
◊532a. Typical spectrograms of $1/f^0$, $1/f$, and $1/f^2$ noises. Each spectrogram is associated with a matrix (right) symbolizing a random micro-segment of each sample. Verticality represents pitches of the chromatic row (elements equally spaced in alphanumeric code ordered from the bottom to the top, representing sounds from low to high). Horizontality represents a variable loudnes scale (from *pp* to *ff*) as well as durations divided by halves, after the unit (1); the symbol x indicates repetitions of notes. The following chart presents a possible interpretation of these patterns as musical grammars.

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1/*f*⁰

1/*f*

1/*f*²

◊532b. Musical implementation of the previous scheme as a grammatical analogy adapted to the idiom of classical contrabass. Pitch space is bounded to an eighth (based on low E) and phrasing is adapted to the bow of the instrument. Tempo and common time ($^4/_4$) are stereotyped to facilitate the example.

*NB: For the tonal scale, the 0 in the matrix corresponds to the pitch E (so F = 1, F# = 2, and so on). In all three examples, accidentals affect all equal pitches within a bar (by convention, the accidental extends its value to the following measure only when a tie connects it to an immediate, same pitch).

seems that the pitches and their distributions are determined completely at random—as typically occurs in $1/f^0$ noise—and that the musical aspect depends on the musical parameters and identities to be associated. The result with $1/f$ (also ◊532-b) is much more satisfactory for the simple fact that the internal symmetries are related to a higher periodic symmetry, which is a characteristic of this type of noise, as well as of Zipf's distribution. Furthermore, one can assume that the sample consists of a ground generator, an intermediate exposition, and a closure. Very probably in an example with two instruments the sequence *word → dialogue* or *discussion → resolution* could be suggested. In contrast, for the case with $1/f^2$ the notions of motif, phrase and period—if any—are unintelligible. Neither seems to exist a clear connection between beginning and end, but hesitation follows a state of rest, and then another pace of relative rest, without obvious consistency—a feature that is generally perceived in Brownian motion.

The method of Voss and Clarke

Voss and Clarke (1978), after having analyzed the amplitude of electrical impulses of noise (especially $1/f$ noise) as time series, developed an original methodology to interpret such impulses as intervals of musical information.

In short, the method of Voss and Clarke characterizes time fluctuations in two acoustic parameters: *audio power*, related to music loudness, and *zero crossing rate*, related to the distribution of sound in time. Using this deterministic coordinate and analyzing a large set of samples with music from different styles, Voss and Clarke found that the set spectrum corresponds to the proportion of $1/f$ in a fixed range at very low frequencies. This reflects a general stochastic trend toward $1/f$, rather than overall dispersion in a $1/f^0$ spectrum.

Voss and Clarke's theoretical frame and implementation is also based on crossing relations of voltage, amplitude and harmonic spectrum within a signal. The inclusion of voltage—an uncommon parameter in music theory—is not unjustified in this frame as this comparison of $1/f$ noise with music was done as a statistical projection of music radio broadcast in combination with generalized analysis of recorded musical performances. Using the random fluctuation of voltage in an electronic

synthesis experiment, Voss—himself not being a composer—created his own music in which pitch scales and time span totally depended on an oscillator governed by voltage.[311]

Bolognesi's adaptation

Bolognesi's work (1983) is particularly relevant because, as Roads (1999:885) noted, converts Voss and Clarke's (1978) deterministic generator into a logical system of stochastic selection.[312] Bolognesi understood the significance of restraining a rigid grammar through flexible implementation. This proposal demands, since the first stage of research on musical self-similarity, to complement the arguments of music engineering with the notion of music recursiveness as a problem of language.

Bolognesi's adaptation focous on the production of stochastic self-similar sequences, starting from a random binary sound output. Henceforth, such sequences can be divided into other sequences, by analogy with what happens in a language consisting of units, structures and aggregates following Zipf's distribution. Another adjustment implemented by Bolognesi, according to Roads (1999:886), was the substitution of Voss and Clarke' generator, using several hierarchical generators coordinating different production algorithms. This adjustment follows the general criteria of consistency, according to an initial plan included within the algorithms themselves. Like authors reporting self-structuring behaviour in generative grammar (see subchapter 4.4.), Bolognesi (1983:26) emphasizes the hierarchical relationships: "the stochastic melodies generated by Voss and Clarke's algorithm and by variants of it [introduced by Bolognesi and Fagarazzi] show what may probably be considered the most primitive form of hierarchy: *self-similarity*."

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[311] Madden (1999/2007:116–141) summarizes Voss and Clarke's technique mentioning also other approaches to noise as a source for music. His study is completed with the analysis of a selection of musical pieces, displaying an approach towards specific noise patterns using statistical data and scattering plots. This selection includes music by Xenakis, Schubert, Dodge, Ian Stewart, Babbit, Webern, J.S. Bach, Chopin, Pärt, Schoenberg, and Ligeti, and excerpts of Gregorian chant and two traditional tunes in English.

[312] $1/f$ noise can be generated by iterations of a *fractal equation*, and has specific applications in the production of sound patterns and musical scales (for a summary see Voss and Clarke 1978, Dodge 1988, Nelson 1992, 1994, Madden 1999/2007, Yadegari 2004). The aesthetic implications of this topic are discussed in Chapter 5.

Polotti and Evangelista (2001:27–30) report "experimental evidence" which reveals quasi-periodic behaviour of approximately $1/f$ noise in the harmonic spectrum of *sidebands* in music repertoire including voices.[313] This not only confirms Voss and Clarke's (1975, 1978) suggestions, but also draws a more complete description of the acoustic patterns of the human voice in connection with the use of musical instruments.

Polotti and Evangelista (2001), after developing a rigorous definition of quasi-periodic (they call it 'pseudo-periodic') $1/f$ noise, explore self-similarity properties in $1/f$ wavelets and signals to implement them for additive synthesis techniques. The term 'additive' here refers to a separate treatment of each wavelet, before it is added to a carrier signal. Unlike other common techniques for electronic sound synthesis, in this case not only are sinusoidal components added to the signal, but noise components are also inserted with the purpose of "maintaining a real-life 'color' in sounds" (*ibid.*:36).

Random walk and compositional methods

A typical form of Brownian motion is the *random walk*, i.e. the movement of a point in a space, whose position at any moment depends on its position in a previous moment, and on some random variable which determines its direction and span, since the 'walk' tends to have the form of continuous lengths with discrete nodes. A special form of random walk is the Lévy flight, described as a noise pattern in the following section.

Xenakis (1963/1992:289) suggests that, whereas the square waveform is the "most economical" plane wave to cover a $1/f^0$ noise spectrum, the most economical waveform able to represent "melodies, symphonies, and natural sounds" is that which offers certain intelligibility by its temporal periodicity and symmetry of the curves:

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[313] In an electroacoustic signal a sideband is a band of frequencies higher or lower than the carrier frequency, containing power as a result of the modulation process. The sidebands consist of all the Fourier components of the modulated signal except the carrier. All forms of modulation produce sidebands. A single frequency in a sideband is a side frequency. (From *Encyclopaedia of Information Technology*, Atlantic, New Delhi, 2007:449).

> An attempt at musical synthesis according to this orientation is to begin from a probabilistic wave form (random walk or Brownian movement) constructed from varied distributions in the two dimensions, amplitude and time, all while injecting periodicities in time and symmetries in amplitude. If the symmetries and periodicities are weak or infrequent, we will obtain something close to white noise. On the other hand, the more numerous and complex (rich) the symmetries and periodicities are, the closer the resulting music will resemble a simple held note. Following these principles, the whole gamut of music past and to come can be approached. Furthermore, the relationship between the macroscopic or microscopic levels of these injections plays a fundamental role.

In short, what Xenakis means by these words, is that music can be conceived as 'inherently significant' inasmuch as its probabilistic wave moves from a random walk (e.g. $1/f^2$ noise), into a consistent and relatively self-similar waveform ($1/f$ noise); and 'less significant' as its probabilistic wave tends to aperiodicity and to continuous self-dissimilarity (e.g. $1/f^0$ noise).[314] Xenakis (*op. cit.*:290–293) also proposes a strategy which puts into practice what he discussed in theory, and which begins by implementing a stochastic pitch distribution in relation to a time abscissa. In parallel with the axis of amplitude, he assigns specific values for each boundary, until they form a polygon inscribed in a sine wave, or a rectangular waveform, or a waveform produced by a stochastic function such as Gaussian, Cauchy, logistic, or other *multifractal* functions.[315] Xenakis correlates 'microscopic' audio segments produced by this method, with 'macroscopic' sets of self-affine generalized features, creating a complex consistency between the whole and the part.

Nelson (1992) also uses a method based on Brownian motion, to generate the displacement of a middle point in a coordinate of amplitude and duration. This goal is achieved by taking the midpoint of a straight line segment, then moving it up or down at midpoints forming subsequent segments, and continuing this movement by the same method in an iterative process. By repeating this segmentation over and over again, the result obtained is a highly irregular shape comparable to that of Brownian

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[314] Murphy (e2007) introduces the concept of self-dissimilarity into music theory, but restricted to the relationship between network and hyper-network in the context of musical recursion.

[315] The concept 'multi-fractal' is introduced in subchapter 4.2. (pages 147–148). The notion of mathematical function is explained on pages 346–348.

motion, and which produces a structured, continuous subdivision of time, pitch and amplitude.

Nelson's final purpose is to generate an electronic accompaniment using a fractal algorithm, on which he improvises a melody. His first work created with this method, *Fractal Mountains* (1989), employs MIDI horn and computer. This title reflects the analogy the composer makes between the musical result obtained and the form of a mountain range that characterizes the two-dimensional representation of Brownian motion (see for instance ◊531-*c*, or ◊532-*a* bottom).

Other timbres of noise

If all noise models were restricted to the examples mentioned above, musical analysis and synthesis by these means would be rather poor. Evidence suggests something very different: noise is widely varied as its actual and potential sources and transformations are.[316]

Schroeder (1991) defines 'black noise' as $1/f^{\beta}$, where $\beta > 2$. Kuo (1996), for his part, refers to 'gray noise' as an intermediate between $1/f^2$ and $1/f^{\beta}$ noises. The inversion of the $1/f^2$ noise power spectrum, increasing its energy as its frequency grows up proportionally is either labeled 'blue noise' (Wang 2008) or 'purple noise' (Zhang and Schwartz 1996). In this case each time the frequency doubles, its power spectrum density increases by 6dB, which means that the energy is proportional to f^2: the higher the frequency the greater the energy.

In his article "El color del ruido" [The Colour of Noise], Miramontes (1999:5) suggests that: "If light can be represented in a spectral graph, why not sounds? Indeed, also the sound features can be perceived in a spectral density." Although light frequencies cannot be directly converted into sound because the particular characteristics of their waves are different,[317] it is a fact that all spectral densities and

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[316] Hence the relevance of the epigraph (Russolo 1913) heading this subchapter (see page 238).

[317] Otherwise the human eye could see the sound and light could be perceived by the ear. The structural differences between these perception organs also reflect a difference in the nature of the perceived phenomena. Whereas sound results primarily from air—or other fluids'—pressure fluctuations, light is primarily an effect of electromagnetic radiation, not after contact between molecules, like sound, but after elementary particles (photons) behaviour. In the case of

ratios between power and frequency can be objectively measured and compared by proportional analogy. Moreover, any frequency spectrum can be interpreted as forms of noise, including patterns of climate change, statistics, economics, and financial charts, the cycles of solar activity, the rhythms of floods in hydrological systems, cardiovascular pulsations, the tides and the rhythm of the waves, earthquakes, the Larmor frequencies (i.e. the magnetic resonance frequencies of the atomic nuclei), the global dynamics of cities... all or almost all periodic or quasi-periodic cyclical phenomena can have an analogous interpretation within a modular bounding for sound production (see Miramontes *op. cit.*).[318] The general basis of this concept in relation to music is discussed at other subchapters in this study, so here there is no need to repeat information.[319] Instead, a selection of noises having self-similar properties is explained below, which can be useful for music analysis and synthesis, as happens in the cases of $1/f^0$, $1/f$, $1/f^2$, $1/f^\beta$ and their inversions:

After Van der Ziel (1950) *flicker noise* is commonly classified as a $1/f$ noise characterizing electronic circuits. Grebennikov (2007:196) notes also that its power spectrum density is proportional to $f^{-\gamma}$, where $\gamma = 1.0 \pm 0.1$ "in a wide range of frequencies". Motchenbacher and Flitchen (1973:172) explain that this noise appears in carbon resistors, during the continuous forming and extinction of carbon particles' micro-arcs randomly distributed in space. This is confirmed by T.H. Lee (2003:345), who reports that resistors made with semiconducting carbon thin film—very different from the previous ones—produce less noise than the latter. Flicker noise surplus then depends on the material source with which it is produced and propagated. This notion contributes to the conjecture stated in 2.3., suggesting that the organic carbon-based tissues involving bioacoustical systems contain the same self-similar 'footprint' that characterizes music and language, correlated by power laws.[320]

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psychological associations between color and sound frequency, one may refer to two types of experiences, known as *induced* (or *weak*) *synaesthesia*, and *true* (or *strong*) *synaesthesia*.

[318] The concept of modular bounding is explained on pages 393–395.

[319] See pages 55–58 (analogy); 102–109 (intersemiotic translation); 123–124, 134–135, 150–153, 171, 192–193, 360 (resonance as metaphor); 366, 437 (resonance as Gestalt).

[320] Grebennikov (2007) also notes the participation of silicon in electronic devices, related to a hypothetical source of flicker $1/f$ noise. It is worth to mention that organosilicon (i.e. organic

Another aspect of flicker noise is its association with the phenomenon of *attack*, or residual cluster at the beginning of sound production with energy 'excess' in an acoustic system; for example the attack on a plucked string, or on the beveled mouthpiece of a flute.[321] This 'attack noise' is treated by analogy as flicker noise in a process of electronic synthesis, as proposed by Karplus and Strong (1983), Roy (e1992), and Polotti and Evangelista (2001).

Burst noise or *popcorn noise*, also known as *bistable noise* or *random telegraph signals*, has a general behaviour similar to $1/f$ but its waveform fluctuates, in short and unpredictable cycles, to the square shape. It is the sudden noise found in electrical semiconductors in the form of short random step transitions between two or more discrete voltages. T. H. Lee (2003:347) notes that "it is understood even more poorly than $1/f$ noise, and it shares with $1/f$ noise a sensitivity to contamination (from other noises) [...] is characterized by its multimodal—most often bimodal—and hence non-Gaussian amplitude distribution."

Bursting patterns (or *contact noise*) are used as an effect of distortion in electric and electronic music, and are associated with techniques of sound granulation (see Clarke *et al.* 1996, Rocha Iturbide 1999, Roads 2004). For analytical and synthetic purposes, its irregular spectrum can also be compared with a random percussion repertoire, or with some percussive aspects of string and wind instruments, especially in the case of peaks in constant noise or sound leaps in unpredictable periods.

Lévy flight. This is not properly a noise, but rather a form of random walk or Brownian motion with statistical self-similarity and other features comparable to $1/f^2$, that can be interpreted as noise. Its frequency spectrum is distributed in the form $y = x^{-\alpha}$, where $1 < \alpha < 3$. After a very large number of steps, the distance from the origin of the random walk tends towards a stable distribution with scale invariance.[322] Unlike typical Brownian motion, Lévy flight commonly intersperses very long steps with clusters of hierarchical nestings. It is useful for music as its behaviour can be described as a Markovian process in which the statistical values and

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compounds containing carbon silicon bonds) also plays a basic role in some essential aspects of biological self-structuring.

[321] See pages 145–146.

[322] See subchapter 3.4., on invariance.

nestings can be adapted to any type of musical parameters (e.g. duration, rhythm, timbre, loudness, and pitch distribution by clusters and sequences).

Bolognesi (1983:31–36) characterizes Lévy flight as an infinite sequence of isotropic and statistically independent jumps performed by a mobile point, satisfying the distribution

$$P_r(x) \equiv \text{Prob}\,(r > x) = \begin{cases} x^{-D} & \text{if } x > 1 \ (D > 0, \text{real}) , \\ 1 & \text{if } 0 \le x \le 1 , \end{cases}$$

where r is the random variable representing the length of the jump, and the exponent D corresponds to the fractal dimension of the space in which is found the mobile point, determining its distribution. Provided the scalar independence of the whole process, the control of parameter D leads to controlling the whole distribution: "If we apply a Lévy flight to music generation, we may control the degree of clustering of our hierarchical structures by means of a single parameter D" (Bolognesi, *ibid.*:31).

Fagarazzi (1988) uses Lévy flight to associate the harmonic spectrum of sound with a prescriptive harmony, virtually creating a self-generating musical grammar. Moore (1990) uses it to generate random sequences of arbitrary length whose spectrum approximates to $1/f^{n \ge 0}$. Conversely, Beran (2004:93) employs fractal dimension, Hurst exponent and other parameters related to the Lévy flight, for analytical purposes in music samples with stochastic distribution.

Weierstrass function. This mathematical function has the property of being continuous on $\mathbb{R}$ but not differentiable at any point of $\mathbb{R}$ ('not differentiable' means that the function curves are so irregular that no tangent line could fit into them without being intersected).[323] This property is related to the self-similarity of the function, which amplified in a segment reflects the whole of its typical relations. Its formalization is due to Karl Weierstrass (1815–1897), who refuted the misconception that every continuous function must be differentiable except on a set of isolated points (see Weierstrass, 1870). There are applications of Weierstrass function in electronic music, based on its scale invariance property. For instance, Schroeder (1991:97) mentions that

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[323] For a detailed explanation of these concepts see subchapter 6.2. (especially pages 354–363).

> An example of a musical chord patterned after a Weierstrass function can have the following weird property. If recorded on magnetic tape and replayed at 'twice' the recording speed, the chord will not sound an octave higher in pitch, as every well-behaved recorded sound would, but a semitone lower.

Miramontes (1999:10) uses Schroeder's mathematical argumentation to express that "if music loudness increases twice ($1/b = 1/1/2 = 2$), music still sounds the same". Beyond this curious effect, Weierstrass function can be used in musical synthesis and sound modelling. Monro (1995:89) characterizes it as an ideal model for 'fractal interpolation': "Fractal interpolation is a method of generating functions that pass through given points", and notes that this is useful "to generate melodic material and to control the large-scale structure of a piece" (*ibid.*:91).

Numerical generators. There is an immense variety of methods for generating stochastic and random sequences of numbers.[324] This includes infinite sequences of digits in irrational numbers like $\sqrt{2}$, φ, e, and π, in which number distribution has a random outlook that usually increases with the size of the sample. This behaviour can be exploited in the development of matrices for $1/f^0$ noise generation. Nonetheless, instead of using a massive-sampling divergent methodology to remark randomness, the numerical sources can also be used in convergence methods with small samples suggesting a local stochastic behaviour, intuitively more similar to $1/f^2$ than to $1/f^0$. This is the case of Pareyon's (2001) proposal to use π for a quasi-harmonic modelling of tones, rhythm, and phrasing, using the first one thousand digits of the decimal sequence and musically noting the repetitions of dyads, triads, tetrads... until they form a soundscape consisting of a careful intersemiotic translation of the numerical sample. Hence, for example, the Feynman point (a sequence of six 9s that begins at the 762nd decimal place of the decimal representation of π) become noticeable in an analogous way that a major summit could be noticeable in a landscape of lower ridges (i.e. repetitions of two and three digits within the same sample).[325]

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[324] For instance, Roads (1999:881–889) and Kindermann (e1999) invoke the use of pseudo-random number generators with a uniform outcoming signal for noise synthesis; and Tiits (2002:156) and Salter (2009:40) emphasize the use of similar generators shaping the 'weight-vector' components mapping a partially random function. On the same topic, see Fripertinger (1999).

[325] A first intuitive approach suggests a self-similar surface in a 'big' sample (e.g. the first billion digits) of π, in the sense that groups of five, six, seven... adjacent digits have a similar (i.e.

There is also a wide variety of numerical methods which, obtained from a cyclical self-reference, tend to a self-similar surface. This is the case for sequences of numbers in Pascal's triangle, Farey trees, and the Thue-Morse sequence, all of them linked to the irrational number φ and the golden mean, which are dealt within subchapter 6.3.

In the analytical context infinite sequences with irrational numbers, as in the specific case of π, have also general interest as they reflect the typical transitions (e.g. *harmony → similarity → noise*) between states of musical structures acting as dynamical systems. This can also be interpreted as a transition between σ and Ω in an information process (according to ◊521).

Fractal generators. Many authors (see for example Al-Akaidi 2004:96, 115–123; Lowen and Teich 2005:115) associate $1/f$ noise with the concept 'fractal noise'. However, endlessly self-similar pure $1/f$ noise—a theoretical elaboration—is not the only noise with absolute self-similarity. Actually 'all' infinite functions which are continuous everywhere but differentiable nowhere—as in the above mentioned Weierstrass function—also happen to be 'fractal noises', at least as a mathematical analogy.[326]

Direct generation of noise from a fractal algorithm can be done using different approaches. One of them is to convert the fractal's overall symmetries into an affine time-frequency contractive mapping. Obviously this is a technicality to express an intersemiotic translation by strict analogy. Using the same method, other fractals, like the triadic Cantor set produce a similar result in which the tiniest parts—Cantor dust—are recursively distributed towards infinity, whilst self-similar relationships are destroyed in the *n*-th iteration. Cole and Schieve (1993) confirm this transition to $1/f^0$ noise in the Koch curve and Cantor triadic set contracted into one dimension, where each "interval merges or collapses with its neighbour" (*ibid.*:315).

As can be expected from the differences among their structural features, neither do all (pseudo)fractals produce the same kind of noise at their first iterations. For

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comparable but not identical) distribution in smaller groups (made of two, three, four... adjacent digits). In the long term, however, the numerical plot should trend to $1/f^0$, as π is conjectured to be a *normal number* (a sequence of numbers is statistically normal if all strings of equal length occur with equal asymptotic frequency).

[326] On the special use of the word 'fractal' in this study, see section 1.3.4.

instance, whereas Koch's curve, with its typical distribution to $^4/_3$, generates a pseudo-harmonic pattern with noise growing up in its smaller intervals. The H-fractal or Mandelbrot tree also produces a pseudo-harmonic pattern, although with a widespread distribution completely different, performing a mechanical rhythm that matches its visual and auditory representations.

5.4. *Noise* in music

One way to interpret musical self-similarity is to consider disorder *nesting* within order at different scales and proportions. In mathematics and music theory the concept of 'nesting' can have different interpretations and applications, but in general they all share the notion of holding sets of objects, or groups of relations, within sets or groups of larger sets or groups, whose basic references are comparable to the relationships of their smaller parts.[327]

Fitzsimmons *et al.* (1994:595) and Xiao (2004:277–278) give examples of a type of *finite branching* in self-similar objects, with scalar invariance and symmetric connectivity, under the label *affine nested fractals*. In addition, Xiao (*op. cit.*:278) demands, for a *nested fractal*, the condition that all similarities involved with a self-similar system have the same proportion of contraction. By analogy, this concept is useful to explore the notion 'noise in music', related to those of *proportion* and *contrast of a proportion*,[328] and opposed to the rationalist dogma that oversimplifies noise as 'error' or 'equivocation' (e.g. Shannon 1948, Weisstein e2008).[329]

In this study, the concept of *noise* is interpreted as interference and reciprocity with respect to a certain *sound*, or as information and code perturbation, assuming that such perturbation can be nested within a musical structure: for example, within a

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[327] This notion falls into the third definition of the verb 'nest' in the Oxford English Dictionary (2009): "nest. [...] 3. a set of similar objects of graduated sizes, made so that each smaller one fits into the next in size."

[328] See subchapter 6.6.

[329] The definition of 'noise' in Weisstein (e2008) is symptomatic in this sense, especially by his concept of logical opposition to 'true': "Noise. An error which is superimposed on top of a true signal." This concept is valid as operative principle in Shannon's (1937, 1948) theory, but is insufficient in a broader framework in which noise has aesthetic interpretation and serves as a complex code enriching message (see Attali 1977, Hegarty 2007).

rigid organizing framework—an invariant group of constructive relationships—holding a flexible condition inherent in the organization itself. This perspective on the concept of nesting is developed in a separate section below.[330]

Simultaneously, in music theory there are special notions of the concept nesting. For instance, in pitch class sets analysis the term 'nesting' appears after Lewin (1962:99) as potential for symmetric structuring within a pitch class segment. This concept is described in a later section.[331] A reason for avoiding it as a referential point from the beginning of this subchapter, is the many ways and possibilities of nesting in music, requiring different approaches. Lewin (1987), in his general theory of intervals and transformations, observes a dichotomy between the Cartesian schematization of music, and what he characterizes as "transformational approach"; this dichotomy is related—in a much broader sense than the one assumed by Lewin—to the nesting of a flexible condition within a rigid scheme.[332] From this open perspective, the dichotomy between Cartesian scheme and transformational attitude may be associated with a broader analytical-constructive dialectics, such as musical axiologies between *the Pythagorean* and *the discursive*, or between *the symbolic* and *the pragmatic*. Finally this subchapter wants to make evident that these 'oppositions' are not absolute, but operate in coordinated modalities.

A chief purpose of this subchapter is to develop the concept of 'noise in music' analyzing the inherent monotony within isometry and music measurement rules in general: [333] fixed rules and rigid derivative vectors are 'in interference' with music —which yet make music possible owing to an operative game between measurement rule and interpretation. Measurements introduce fuzzy edges into musical relations; rational sequences—and rationalisms—aggregate *noise* into *pure signal* sounds. This phenomenology can be connected with what Reybrouck (2001:621–622; based on Meystel 1998:349) identifies as 'infinite nestedness of swarms', in a musical context:

> [The] construction of infinite nestedness of swarms within swarms is primarily object-oriented and has the advantages that come with multiple granularities and nestedness. It

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[330] See 'Broad sense of the concept of nesting', page 284.

[331] See 'The concept of nesting in Lewin', pages 281.

[332] Subchapters 4.7. and 4.8. explain, in a correlated context, how the rigid framework comparable to a grammar, requires for its own functional continuity, a pragmatic flexibility comparable to an idiolectal trait.

[333] An independent section, on pages 45–47, introduces the concept of isometry.

has an enormous power of interpretation, but at the cost of high resolution knowledge: the more generalized the knowledge is, the more details seem to disappear within generalized entities.

Adopting the notion *aliquid metitur pro aliquo*,[334] any music has implicit its own noise, and any noise—being measured—has implicit its own music. Extending the information theory postulate on reciprocity between information and entropy, it is clear that this correspondence operates as synecdoche, which is essential for building typical relationships of music.[335]

The scheme shown in ◊540 is a first example of noise produced by music rulers interfering with music itself. This example—in which the representation of musical temporality and spatiality directly affects the time and space of music in its cognitive, aesthetic and cultural domains—recalls other comparable cases in the history of Western musical thought, such as Alphonse Allais' (1854–1905) *Marche funèbre composée pour les funérailles d'un grand homme sourd* (1897); or John Cage's (1912–1992) three-movement composition *4'33"* (1952), "for any instrument or combination of instruments", in which the performer(s) stay 'silently' with their instruments during a period of four minutes and thirty-three seconds. The example in ◊540, however, highlights noise's implication, not over silence that denotes the absence of something—musical symbols in this case—but in the contingency of sound in the *physical world* as cultural, human environment; an aspect that, without being explicit, is latent in the examples of Allais and Cage.

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[334] The classical Latin sentence, *Aliquid stat pro aliquo*, attributed to St. Augustine (*De doctrina christiana*, II:1), has been taken as a fundamental motto in semiotics after Charles S. Peirce [1898] (*CP* 2:257); it means "something stands for something else" [to be interpreted]. Hereby I allow myself to replace *stat* by *metitur*, for the sake of sense in this chapter, assuming that "something stands measuring for something else"; in other words, something stands for something else, by analogy.

[335] This idea is developed in subchapters 5.5. and 6.6., in their respective contexts. The examples given throughout this subchapter are combined with specific cases showing this reciprocity.
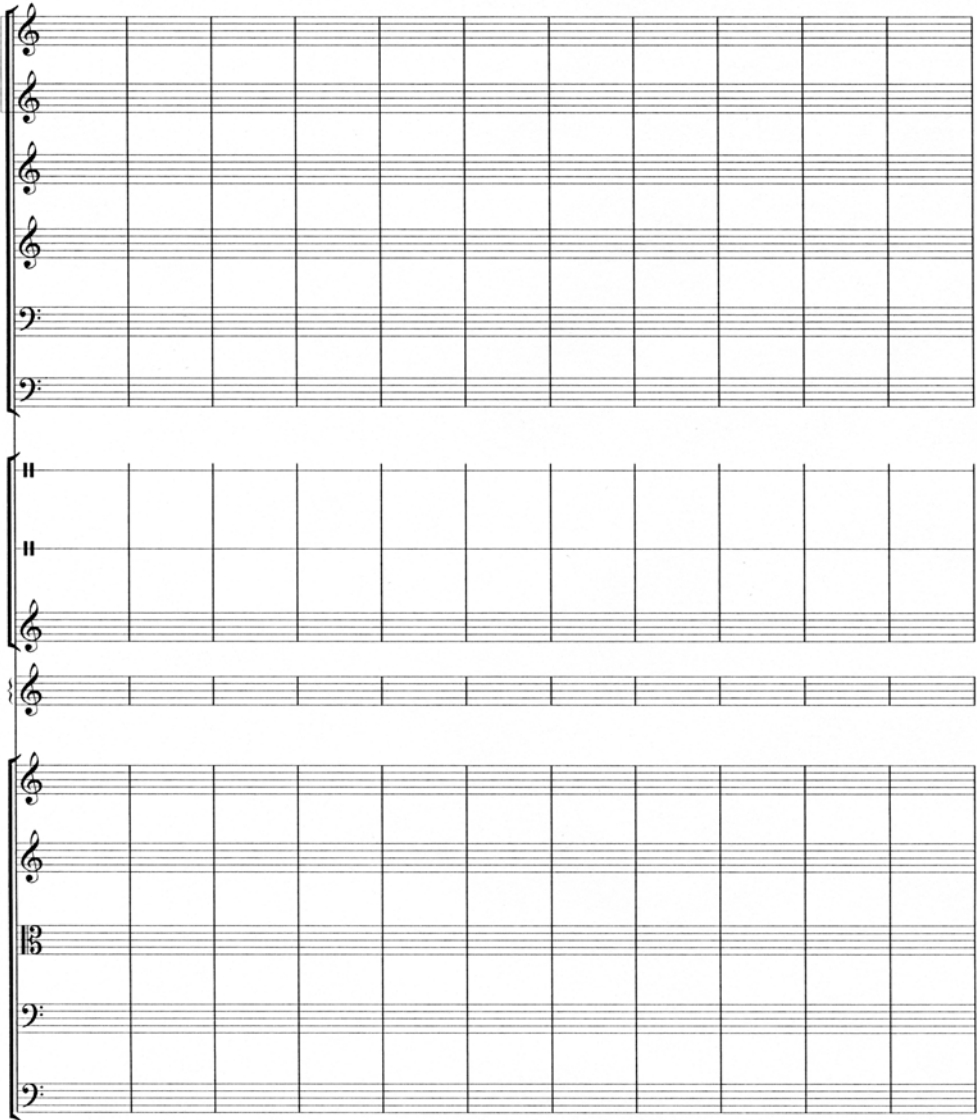
◊540. Example of *noise* in music with two cases of analogy under the same symbolic representation (although with different semiotic domains):

(*a*) staff design in interference with blank space;

(*b*) interference from the analytical-cultural preconception of space/time, regarding the potential organization of music. Furthermore, the relationship *a* → *b*, as well as *b* → *a*, is of an intersemiotic kind.

Another form of ruler interference—or of a regulatory system that introduces complexity into a relationship which is simple at its origin—is the link of one element with another and with each one of the elements of the set that contains them. This relationship, by measurements and fixed proportions in successive iterations, creates a map of the mathematical properties of the measured whole. All possible transformations in such a set correspond to the operational map of the whole. From this point of view, basic structural functions such as the Cartesian set in a musical system, hold a potential tendency toward (pre)self-similarity,[336] inasmuch as they are recursive engines of self-reference. As a matter of fact, the following section shows that a simple set (after it contains three elements, and conceiving that the parts of the set are related to its properties), tends towards self-organization in simple (pre)self-similar forms.

Nestings within series

Within some set, such as the pitch-class sets of the previous section, there may exist various subsets which are particularly important to our understanding. We may speak of these subsets as *nesting* within the larger set. A useful nesting of this kind, within a series of pitches, for example, implies that these pitches are related in some way, such as their harmonic or modal relationships. To illustrate this, we consider the pairs of pitches that break down the harmonic relationships of a chord in a useful way. Such useful pairings may appear in succession, simultaneously, or overlapping in a piece of music, in accordance with the forms of distribution implied by the given set (i.e. the chord). Thus we are led to adopt the *Cartesian product* of a set with itself as the natural self-relation for nestings of this kind.

A Cartesian product of two sets is the set of all ordered pairs of elements, (a,b), where a is taken from the first set and b is taken from the second. A Cartesian product of a set with itself is therefore possible if we take the second set to be a copy of the first. The size of a set is just the total number of its elements so, for instance, the Cartesian product of a set of size n with itself has a size, $n \times n$. The number of times the elements of a set can relate to each other (i.e. be paired together), is just the

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[336] The concept of pre-self-similarity is defined on pages 75–78.

size of the Cartesian product of the set with itself minus the number of pairs of the form (a,a). For example, for the word 'four' this number is 6 as the following figure makes clear:
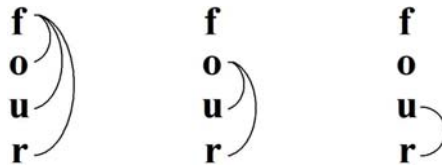
If we denote this number by Cp, then, by observing that it is just the number of ways 2 items can be selected from n different elements, we can obtain the following formula from the binomial coefficient:

$$Cp = n \cdot \frac{n-1}{2} \,,$$

This information can be used to determine the potential relationship of an alphabet, using letters, or the potential relationship of phonetics in a language, using phonemes. It can also be used in a musical context, for instance, to characterize the number of combinations of a certain pitch with the pitches of the chord to which it belongs or with the system of melodic lines and chords of an entire composition. For example, a triad's Cp is 3, represented by three arcs in this scheme:
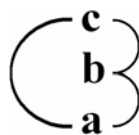
Accordingly, Cp in the diatonic scale (7 elements) is 21. Cp in the chromatic set (12 elements) is 66, and in a quarter-tone scale (12 × 2) is 276.

A *Cp*-relation (i.e. a subset of the Cartesian Product) associated with a group of intervals (e.g. the arcs drawn in the previous examples) reflects the possible combinations of ordered pairs formed with the elements of a set, but it may also show a similarity trend in pitch or pitch-class recursion. The example ◊541 displays, for instance, a degradation of similarities in pitch starting from the shorter interval between the lowest level and the next closest. Thus, a useful *Cp*-relation may serve to capture the various notions of self-similarity in a piece of music.

Source (generating chord):

Useful subset of the Cartesian Product:

Transformation of similarities ⟶

◊541. Example of an intervallic series in the form of ordered pairs
extracted as *Cp* from a generating chord.

Pair combinatorics is relevant in music insofar as it is concerned with pitch intervals, but metrical, dynamic, and gestural references are better elaborated from simple binary relations, such as the *Cp*-relations discussed above. For instance, Forte's (1973) *similarity relation* (called R_p) is a binary relation on the set of set classes; in other words, a relationship of invariance between sets within sets, since 'class' is another word for a set.

In Lewin's theory (1987), the abstract algebraic idea of a *group* is used to great effect. A group is not simply a set of objects, but a set of objects combined with a *law of composition* representing in this case the musical relations operating through the pitch-family. This law predicts what happens when "multiplying" an element of the set by another, thereby capturing the particular relationship of interest between these two elements. Lewin (1987:6–14) defines this law as *binary composition* based on *Cp-*

functions (see also Satyendra 2004:105), where a function is just a special type of Cartesian-Product-subset.

In this context, Ilomäki (2008:33) remarks that "[the so-]called similarity relations are typically not relations on a set, but are functions from the Cartesian product of the set of set classes to some range of values."—i.e. a binary operation. With respect to an analogous relationship in binary musical gestures, Mazzola and Andreatta (2007:37) postulate that the classical twelve-elements pitch-class groups can be expressed as a Cartesian product of a topological space: the gestural space of the classical twelve-tone music.

Consistently, *Cp*-relations reflect the self-referential structuring of the elements of a set, and of groups of relations in a functional distribution. Peles (2004:83), in his analysis of Schoenberg's serial music, characterizes this quality in the pitch-class aggregates:

> [A]n aggregate in pitch-class space is perfectly balanced and complete. Its set of relationships between pairs of elements is precisely the Cartesian product of the aggregate with itself, and each element stands in exactly the same relation to every other: one each of each of the twelve intervals (and thus transpositions) and one each of each of the twelve inversional indices.

The significance of these functions in the context of this subchapter, does not proceed from the calculation of relations or the estimation of probabilities in a musical system. Clearly, *Cp*-relations in music do not always happen in an integral or a consecutive manner; their direct occurrence is rather uncommon. Significance proceeds, then, from the fact that a variety of self-referential bonds are implicit in pre-musical bases, as seeds of self-organization; in other words, as forms of nested self-similarity.[337] Part of this variety is explored below.

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[337] In this sense 'pre-self-similarity' and 'nested self-similarity' are equivalent expressions. In a specific analytical context, the second one deals with a higher theoretical development, as suggested at the beginning of this subchapter.

Self-similarity implicit in musical rulers

The analysis of harmonic intervals as exemplified in ◊541 is not restricted to dyads. The identification of groups of relationships between more than two pitches is obviously possible, and indeed combinations of three elements are useful. Without inversion, a triad (e.g. 0,1,2) has only one form of combination with its own elements; a scale of four pitches (e.g. 0,1,2,3) can form four triad-combinations without repetition; one of five (0,1,2,3,4) can have ten triad-combinations without repetition, and so on (see ◊542). Again, the binomial coefficient (henceforth *Bc*) is used to enumerate the triad-combinations from a set of *n* pitches by substituting $k = 3$:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where ! indicates the factorial of each term. For example, the number of similar subsets of triads (i.e. 3 elements) from a hexatonic scale (i.e. 6 elements) is 6!/(3!(6 − 3)!) = 720/ 36 = 20, which is the number of combinations of the six pitches clustered in triads. Following the same procedure it can be established that the diatonic scale (7 elements) has a *Bc* of 35 triads, and the chromatic scale (12 elements) has a *Bc* of 220 triads.

Figure ◊542 illustrates the usefulness of this by forming groupings of triads from scales of three, four, five and six pitches, with respect to some arbitrary musical ruler. These triad-combinations are labeled (1×3), (3×3), (6×3), (10×3). They comprise the elements of a set which we call U (see ◊543a). This means that for each scale there is a limited number of triads that reflects the Universe of possibility in terms of what can be played together.

Obviously, if the triads of each subset are interpreted as ordered intervals, it cannot be understood that the triadic configurations are repeated in additive segments. To understand this configuration it is necessary to stop thinking of a specific tonality and to abandon the notion of relative values between pitches.[338] What matters here is the abstract shape of the triads with respect to the musical ruler.

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[338] Later (on pages 280–282) the notion of *nesting of similarities* is introduced within the frame of self-similarity in pitch-class set theory.

For this reason the examples in ◊542 and ◊543a lack clef and tonal index: the actual spacing of adjacent pitches is unimportant in our analysis, so that we might choose to define a purely abstract sense of distance between our elements, $dist(x, y) = |y - x|$, where x and y are given by their natural number in the scale (i.e. 0,1,2,3...).
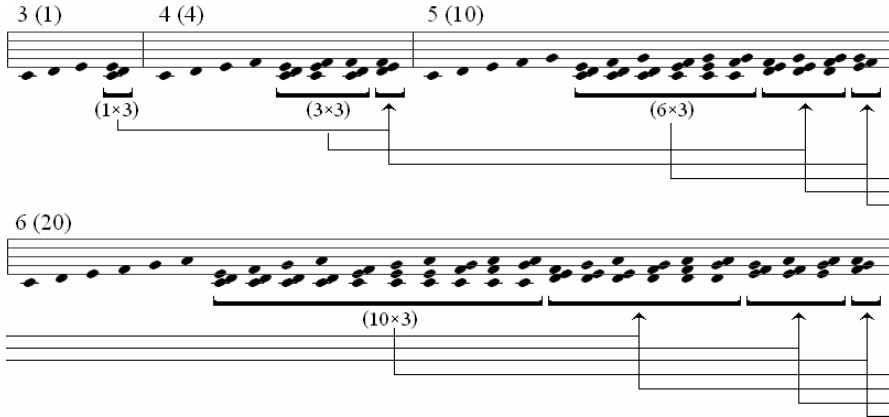
◊542. Binomial coefficients (in brackets) of triad-combinations from 3, 4, 5, and 6 elements. Arrows indicates abstract structural recursion, and horizontal thick lines indicate segmental repetitions.

The classification of *Bc*-triads forming the scale of six tones by unordered intervals (i.e. taking into account only this abstract distance between pitches), shows that the first segment is a subset of abstract intervals that "span" the whole—provided we take note of only their shape and spacing with respect to the ruler—and may therefore be thought of as the "Universe" U (see ◊542).
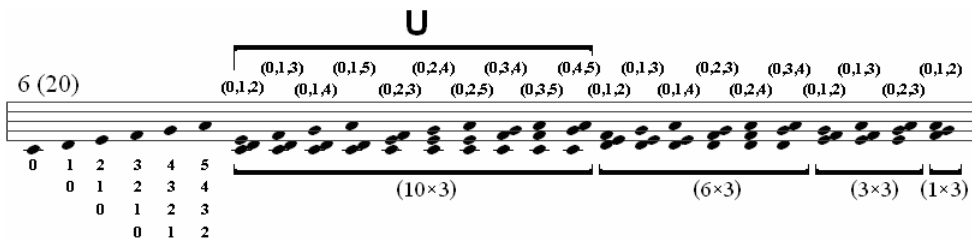
◊543a

For example, ◊543a shows an equal six-tone scale with a *Bc* for triad selections of 20. However, the elements composing U are not 20, but 10. The three additive segments (6×3), (3×3), and (1×3), contain triads like (0,1,2) = |0,1,2|, already existing in U. Furthermore, these segments do not simply repeat all the triads already included in U —for instance (6×3) does not contain |0,4,5|. Rather they sample from the full set triads whilst combinations from the initial segment appear gradually less frequently. One can classify, then, the hierarchical frequencies (i.e. repetitions) which constitute the whole system (see ◊543b).
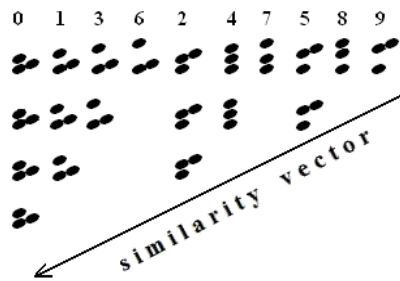
<div align="right">◊543b</div>

Graphic ◊543b shows how the elements in the set do not appear with the same frequency. The first shape is more frequent than the second, and the second is more frequent than the third. This hierarchical series is not consecutive, however, and in order to see this a natural number is assigned to each shape in the series—the lowest number (0) representing the highest hierarchy. Consequently, the highest number (9) will represent the lowest and 'less relevant' frequency. Thus the numbers placed on top of each element reflect its 'weight' within the series. This ordering follows two criteria: (1) the number of times the shape is found within the groupings, and (2) the ordinality in the series elements. In consequence, the fourth shape occupies a higher hierarchy than the seventh, and in turn the seventh has a higher hierarchy than the last.
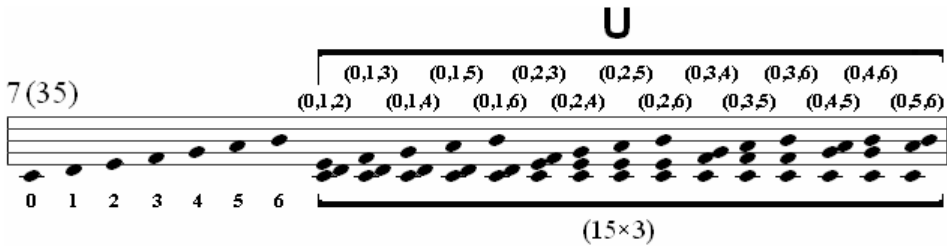
◊544a

According to ◊543a, the distributive properties of the 20 triad combinations are not unique to this scale; rather, they are general patterns of the groupingss obtained from associations of the scales. In this case, for a scale with 7 elements in which the *Bc*-triads number 35, the set U is composed of fifteen shapes that are hierarchically grouped as follows:
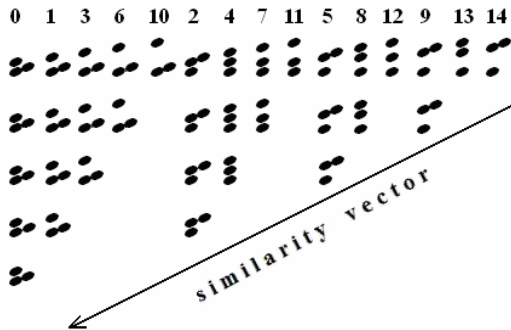
◊544b

Comparing ◊543b with ◊544b, presents a relationship of similarity in which, from four collections of contiguous shapes the set has increased to five. This similarity generalizes to larger *Bc*-triad selections when the number of elements in the scales increase. Furthermore, ◊544b represents a relationship of self-similarity, typical of U in *Bc*-triad of 7 elements. But one cannot say that this relationship is unique to U in *Bc*-triad of 7 elements. It is, again, a typical relationship of groupings derived from associations of the series, including uneven tone series.

The hierarchies of groupings are thus splitting, forming new spaces between them, occupied by new relations of subordination as one moves forward in the series of elements. These spaces, continually depending on the original association rules, are

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potentially associated with the process determining the relative self-similarity of a set.[339] This notion corresponds with the theoretical foundations of David Hartley's (1705–1757) 'hyperchord' or 'chord of chords', originally conceived as a generalized harmonic system ("Hyperchord [...] a harmony that contains all possible harmonies within itself", Allen 1999:386), as well as with Moritz Hauptmann's (1792–1868) concept of 'triad of triads' (see Hauptmann 1853), which has recently been developed by Engebretsen (2008) in the context of a reinterpretation of Hugo Riemann's (1849–1919) theories. The metrical analogy of the same principle also motivated E.T. Cone's theory (see Cone 1968) on pulse strata successively contained by larger strata.[340]
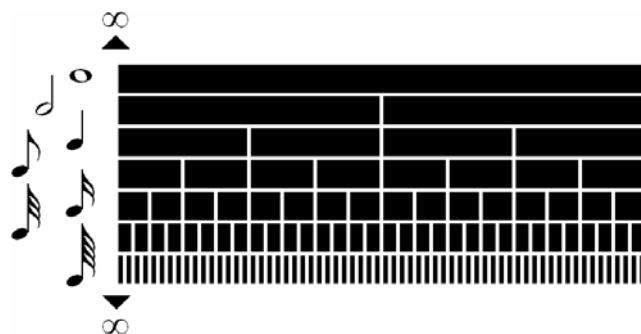
◊545. Music metre subdivision as dense self-similar set, in which between each of its parts there is always a fractional part of the whole. The length of each segment is divided into two equal halves. The parts of the scheme are limited to the sequence of single note (1), half-note ($^1/_2$), crotchet ($^1/_4$), quaver ($^1/_8$), semiquaver ($^1/_{16}$), demisemiquaver ($^1/_{32}$), and hemidemisemiquaver ($^1/_{64}$). However, following the same relationship one can add larger segments (above the single note) or shorter (under the sixty-fourth note), endlessly. The width of bars is trivial, since it is determined by the space allocated for this example.

[339] Examples in ◊541–◊544 are a precedent of what is discussed in chapter 6.4. on brocades and tessellations. They represent only a layer of rigid organization of musical space, comparable to other spaces of grammatical rigidity on which flexible structures move.

[340] See also pages 332, 350–351.

Self-similarity implicit in metrical subdivision

The binary system of musical metre subdivision constitutes an absolutely self-similar set, since the distribution of its parts correspond to the same proportion (1:2) at all levels of an infinite set. This system is a self-similar dense set of dimension 1. It is 'dense' because no matter what the level of its subdivisions, there is always a smaller part between two segments. Example ◊545, analogous to this definition, does not continuously cover the horizontal line in the complete sense of the *arithmetic continuum*, however, it is a system which for all practical purposes will suffice to cover the metrical possibilities as closely as we could ever require, just as the rationals approximate the reals.

Metrical binary subdivision is equivalent to binary notation for the rationals where, in contrast to the decimal notation, the unit fractions used take the form $1/(2)^k$. As is customary in analysis, if we admit the limits—i.e. the convergent series— then we can 'complete' the system and handle the so-called irrational quantities confidently. Of particular importance to music is the measure, or unit, which we are subdividing. In analogy with the limit of the series $^1/_2 + {}^1/_4 + {}^1/_8, {}^1/_{16} + ... = 1$, it follows that, in absolute terms, such a subdivision completely fills the whole measure and is, in some sense, equal to the unit itself:
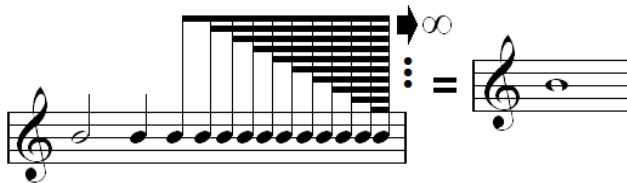
◊546. Binary prolatio embedded into musical measure as a normal
sequence, convergent to 1.

The metrical subdivision's ternary system follows the same principle of convergence, although with a $1(^1/_3)$ proportion at all levels. Unlike the Cantor set (see ◊332), which is a fractal, the continuous metrical subdivision, binary or ternary, is not a

fractal; the projection of its infinitesimal parts is analogous to the infinite sequence of points forming a straight line.

Note also that the relationship between binary and ternary metrical subdivisions corresponds to the ratio 3:2, otherwise denoted by the fraction $^3/_2$. It follows, therefore, that the *sesquialtera* (i.e. $^3/_2$, or 1 + 0.5) relationship, generalized for musical metre and harmony, appears as a basic self-reference which (pre)determines fundamental aspects of recursion and order in music systems.[341]

The fact that the wholeness of music metre can be characterized as a quasi-fractal set, requires a counterbalancing with the needs of metrical contrast and rhythmic change and the non-repetition principles of composition (see Chávez 1961, Schoenberg 1967, Ruwet 1987), in equilibrium with grammatical recursiveness. Monotony of pure metre is contrary to the rich meaning of musical self-reference. From this perspective, the nestings of metric contrast stimulate musical coherence via the stratification of metre, so that hierarchical arrangements of metre and rhythm provide varying structural levels which enrich the monotony of pure metre.[342]

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[341] This issue, including the mention of sesquialtera, is discussed in subchapter 2.3., on symmetry. Here the self-reference of the series $^0/_2$, $^1/_2$, $^2/_2$, $^3/_2$, $^4/_2$… (equals to 0, 0.5, 1, 1.5, 2…) is assumed as related to the naturals' self-reference: 1, 1+1, 1+1+1, 1+1+1+1… (equals to 1, 2, 3, 4…).

[342] Metre's contrast and hierarchization criteria are based on an intuition of metre's pulse distribution, studied as a verbal phenomenon by Lotz (1960), Liberman and Prince (1977), Halle and Vergnaud (1990), and Hayes (1995); as a gestural phenomenon by Stokoe (1972), Brentari and Crossley (2002), and Mazzola and Andreatta (2007), and as a musical phenomenon by Yeston (1976), Lerdahl and Jackendoff (1983), and Barlow (2001). There is no consensus on a general regulation, because each of these authors adapted a general model to explain different particulars, some of which are not analogous in verbal language and in music. However, there is a broad acceptance of some basic principles of metrical structuring, corresponding to specific traditions and styles.

Metrical recursion

Metre operates as an explicit system of recursion, consisting of a stereotype of space-time in which typical repetitions of pre-established forms are nested. However, metrical recursion does not occur as mere repetition of pulse sequences, but as a symbolization of a 'fixed space' on which varying relations move cyclically. Such a recursion consists, in turn, of layers of subsequent fixed spaces, nesting successive displacement variables. Herein lies the theoretical relevance of the *hypermeasure*, a concept proposed by Edward T. Cone (1968:79) together with its conceptual derivations *hypermeter*, *hyperbeat*, and *hyperdownbeat*, refined by Kramer's (1988:86–93) analytic method, as well as implemented for the study of grammatical functions in Lerdahl and Jackendoff's (1983:20) generativism.[343]

The functional relationship between symbolic space and displacement variables is characterized by the complementarity between metre and prosody. Whereas metre is the schematic part of a body-movement analogy, prosody is the actual part of the scheme's interpretation, converted into a space for negotiation between idiolect and grammar.[344]

In metre, as in other processes involving statistical and functional self-similarity, the latter emerges from the interaction between rigid and flexible parts,[345] which can also be interpreted as an interaction between *actual* and *potential*—where by potential we might mean the musical score and the metre signature written in it, and by actual, its interpretation by a player.[346] A typical proof of this relationship is the characterization of metre as dance scheme, affected by a need to differentiate hierarchies of pulses; or as a template to regulate and orient the cyclical movement of a verse-reading, affected by breathing. This characterization extends to singing and instrumental performance.

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[343] This issue is discussed in subchapters 3.7. and 4.4.

[344] See subchapter 4.8.

[345] Swift (1998:28), for example, identifies criteria of metric and agogic flexibility common to the music of Arnold Schoenberg and the poetry of William Carlos Williams.

[346] Also in the context of musical metre, Volk (2007) proposes an interpretation of the tension between rigid and flexible, that is compatible with this perspective: he suggests that metre is a structural frame in which "poles of permanence and change" interact determining statistical relations at local and global layers, in a piece of music.

Melodic span

The measurement of *type intervals* is of great significance in melodic analysis. In a similar way, measuring *metric relations* is important in order to know how they determine the shape of periods and sections within larger structures. This measuring is also a usual method in the compositional process, determining as it does the relationships between musical events and objects.

The example in ◊547 suggests a group of basic relations among *uneven* (i.e. non-monotonic) pitches, which can be considered as *constructive typologies* within a melodic context.[347] This scheme follows from the assumtion that every melodic interval can be described as a sequence of relationships between just two elements, (*a* → *b*), where brackets denote the interval, arrow denotes relation (i.e. 'higher' or 'lower in pitch than'), and letters *a* and *b* denote two different pitches in a melodic segment. The generalization of this principle permits associating these prototypical relations with a wide variety of cases as every melody is composed of steps or jumps in a scale —in Copland's (1939:51) words, "all melodies exist within the limits of some scale system." Hindemith (1937/1941:57) goes beyond this description, observing that the motion itself generates 'melodic tension' with respect to harmony: "The motion from one tone to another produces melodic tension, and the bridging of a gap in space by the simultaneous juxtaposition of two tones produces harmony".

[347] The term *constructive typology* refers to a group of functional relationships between elements in a musical grammar. The representation of these typologies, as points and lines according to example ◊547, is based on the formalization of "symbolic spaces" proposed by Lewin (1987).

0	origin (head note position)	alternative notation
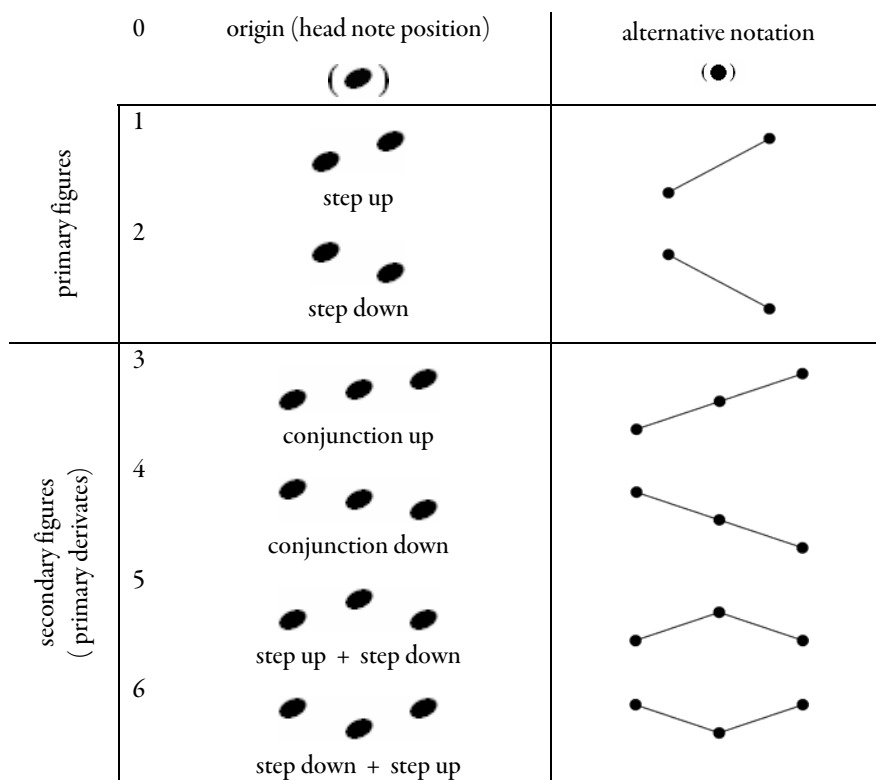
◊547. Chart including the basic step typologies between two pitches (figures 1 and 2) and three pitches (figures 3, 4, 5 and 6), starting from an origin (level 0). The left-hand column suggests traditional notation. The right-hand column represents more abstract figures that are used as an absolute notation in the following diagrams (◊548–◊551), characterizing motivic-melodic movement. Points symbolize 'elements', i.e. pitches from a pitch-set class, and lines symbolize the 'type interval', i.e. melodic orientation or relationship between two pitches. This scheme simplifies Edward T. Cone's (1968:26–27) description of musical movement as an analogy of an 'object' (●) 'moving' upwards or downwards (/, \).

From this schematization, one can perform a variety of combinations thereby building up an emergent sequence (e.g. a melody), where we may chose from all possible combinations *ad infinitum*.[348] Under the scheme ◊510, the relationships

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[348] Chapter 6 explores in more detail this self-structuring sequentiation, in the specific context of L-systems and the Thue-Morse sequence.

between these sets can be represented as election probabilities in a system with known origin and distribution rules. The potential organization of these typologies as emergent patterns, gains structural density in a way analogous to the continuous self-structuring of verbal intonation, following a rough approach to 'high' or 'low', starting from the initial range $[a, b]$.
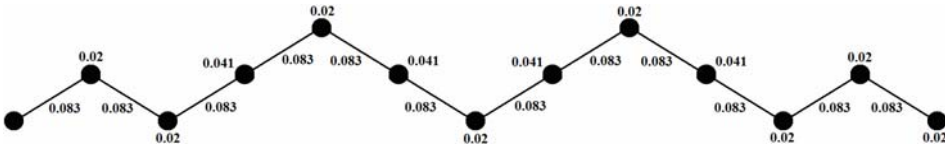
◊548. Symmetrical pattern of twelve steps as a system of probabilities in an abstract melodic context. Primary figures (1 and 2 according to the previous chart) have all probabilities of occurrence in each step (0.0833... × 12 = 1); thus they occur twelve times in this example. Figures 3 and 4 are half probable in respect to the primary ones (0.04166), and figures 5 and 6 are half probable (0.02) in respect to the latter. The whole pattern matches Zipf's distribution and summarizes typical relationships of melody as a succession of steps.

The recursive functions implied by a finite initial set of melodic steps is directly relevant to the self-structuring of melodic typologies, such as, for example, in the composing process of J.S. Bach's *Prelude*, from his Suite no. 1 for cello, wherein a compact system sets the self-referential context for the whole piece. In this example, the generating motif evolves along with the *Prelude*. Nonetheless, within the piece as a whole, the motif's original orientation prevails; each measure reflects, together with local variations, the overall structural identity. This is observable even in the *Prelude*'s final motif, which operates symmetrically with respect to the generating motif (see ◊549b).

These examples, despite their reductionism, cogently summarize the coherence and self-structuring tendencies of melodic movement—in terms of a small set of simple initial relationships between a few elements. These tendencies appear within both in tonal and atonal modern repertoire, by a same need for consistency across intrastructural similarities. Taking as an example the measures 52 and 53 of Edison Denisov's (1929–1996) *Fifth Study* (1983) for bassoon (see ◊550), one observes the

same tendency to reuse and consolidate typologies that are present from the beginning of the piece, in a form of discursive statement. In fact, the structural (rhythmic and melodic) basis of the entire piece relies on typologies 1 and 2, transformed in upward movements that determine the global surface of this work. This typical behaviour can be extended to a wide variety of repertoire that—as in the specific case of instrumental studies—performs a consistent recursion of its constructive relationships.

◊549a. The first measure and generating motif of J.S. Bach's *Prelude*, Suite no. 1 for cello. The sequence below the musical staff represents the articulation of typologies according to chart ◊547. The numbers correspond to the 'weight' of different step typologies (from 1 to 6). For instance, the number 0 represents the source, common to every step, and the numbers in brackets indicate the secondary figures completing the system. The rows of numbers do not reflect the quality of tonal intervals—something that can be done with conventional analysis—but stereotype the steps in the sequence, showing the skeleton of the melodic movement.

◊549b. Left: conclusive bar of the *Prelude*, with its melodic typologies. Right: condensation of the similarity of the melodic intervals at the beginning (measures 1–4) and the end (measures 39–41) of the piece. The structure repeats exactly at the indicated measures; the derivative measures which make the rest of the piece result from this simple gestural similarity.

Obviously, intrastructural features of similarity cannot be of the same type in all pieces of music. However, it is noteworthy that in a vast array of melodic repertoire from different eras and traditions, melodic typologies indicate recursion formulae that, on the basis of simple relations, trend to a higher organization.[349] In this fashion, the part within the whole reflects the whole in the part, whilst the most complex polyphonies are made of the simplest melodic typologies.

◊550. Measures 52–53 of Edison Denisov *Fifth Study* for bassoon (1983). Copyright by Deutscher Verlag für Musik, Leipzig. Fragment reprinted with the kind permission of the publisher. The step sequence below stereotypes the whole sequentiation of the study. The symbol *P* has been included to indicate 'pause', which also follows Zipf's distribution in a massive collection of similar pieces of music.

This description of melodic intervals as sequences of basic steps or linkage typologies, bolsters the hypothesis of a close relationship with Zipf's law: primary figures 1 and 2 (according to ◊547) are absolutely repeated in all melodic patterns (i.e. in all sequences of non-monotonic pitches); figures 3 to 6 are less often (with lower probabilities, although still very common in a very large collection of samples); a set of secondary derivates (direct ascents or descents by four steps in any scale) becomes less likely; and a set of tertiary derivates, becomes even less likely. This

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[349] This notion is confirmed by statistical evidence provided by Beran and Mazzola (1999a–b), Bigerelle and Iost (2000), Foote and Cooper (2001), and Paulus and Klapuri (2006), that reflects a systematic use, in music, of principles of symmetry and repetition, and coordination between similarities and differences, as a means of structural and discursive sense.

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means that, following this principle, there should be very scarce examples of tunes consisting of continuous ascent or descent segments with 'many' steps (i.e. 7, 8, 9 or more) within the same basic scale.
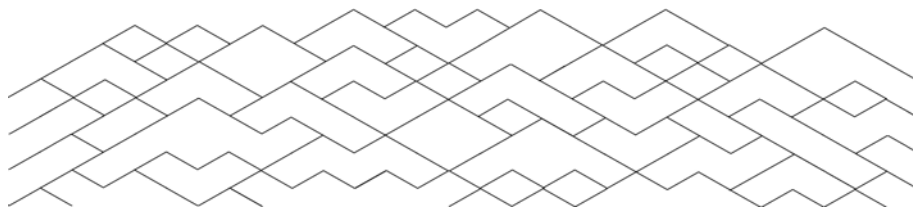
◊551. Hyper-melodic pattern with an abstract sample of overlapping *type-intervals* compiled from a collection of monophonic pieces. In this example, one can see that short steps are more common than medium steps, and the latter are more common than larger steps, which reminds Zipf's distribution as described in subchapter 5.1.

Due to its symmetry, example ◊549 provides a description of the development of melodic consistency with the use of the same basic set of typologies. However, this consistency does not only depend on the basic relations of symmetry (see ◊232, ◊233), but also—more generally—on the bulk of distributive relations following a similar pattern of simple steps in the short term, which are combined in a multitude of possibilities in the long term.[350] The boundaries of this form of organization tends to approximate self-similarity. Accordingly, Liu (2008:99) concludes that in a maximum of asymmetrical relations sharing the same basic behaviour, these relations tend to self-similarity: "Maximal non-symmetric entropy leads naturally to Zipf's law [...] Equally, if we add other auxiliar parameters, we will obtain other distribution laws." Consistent with this idea, Brothers (2007) finds that Zipf's distribution governs, for instance, the *Bourrée* – first part of J.S. Bach cello Suite no. 3.

The extension of this constructive principle in a broader range of musical cases reveals a common behaviour behind the development of musical consistency. Beran (2004:93) suggests, however, that no special importance should be given to self-similarity patterns in musical sequences in general. The significance of these patterns should be judged rather by their meaning in the context of convergent self-similarity,

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[350] This invokes the coordination between contingency and dependence, referred to in subchapters 3.3. and 3.4. (see pages 68 and 83).

by the systematic functions of consistency in a whole. Therefore, according to Beran, such consistency would be attributable to the self-similarity of a system, only if the same kind of self-similarity is present at different levels of the musical structure.

When combined with other analytical techniques, the analysis of basic typologies of a melodic sequence can provide a richer comprehension of the processes of self-similarity of melodies in simultaneity with harmony and metre. In this direction, Brinkman and Mesiti (1991) emphasize the comparison of local and global measures by overlapping two-dimensional maps—in this case representing pitch and duration. Stephenson (1988) also proposes the use of lines and angles in four types of 'coordinates': pitch-duration, scale-duration, tonal cycles-duration, and pitch-metre; whilst Beran (2004:59–61) devises the notion of 'melodic weight', looking at the correlation between tempo and loudness.

Rhythm as rules of recursiveness

In contrast to duration and intonation as typical parameters of melody, the structural 'axes' of rhythm are duration and stress (see Yeston 1976:4, Lidov 2005:154). The primary figures of rhythm operate similarly to primary figures in the scheme ◊547, as they consist, in principle, of simple relations between two basic elements—this time of a pulsing and prosodic character.

Pulse and tone are quite distinct entities in music theory, but their basic organization still follows common rules of reflection and reciprocity. That is, we can classify rhythmic acts into those which are equivalent in terms of both duration and stress. Such equivalence defines a relation between rhythmic elements that is *reflexive* (i.e. self-related), *symmetric* (if x is equivalent to y then y is equivalent to x) and *transitive* (if x is equivalent to y and y is equivalent to z then x is equivalent to z):

- Reflexivity: $x \sim x$
- Symmetry: $x \sim y \Rightarrow y \sim x$
- Transitivity: $x \sim y \wedge y \sim z \Rightarrow x \sim z$

which serves as a starting point for a self-structuring recursiveness, and which can be structurally connected with the properties of symmetry as explained in 2.3.

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Just as the analysis of a genetic code is not enough to understanding all the peculiar traits of individual behaviour, the analysis of a rhythmic-intonational code in music—as a result of an alternation of elements—does not lead automatically to an understanding of the deepest aspects of musical style and the context in which it develops (see Tiits 2002:28–33). This is why Koblyakov (1995:299) emphasizes that "In a musical model the system of coordinates is not set but created". This means that for each recursion cycle, the musical system is open to functional and structural changes, and—by analogy with speech—the simplest recursion in musical structuring can be fully meaningful as a 'making sense' information process.

Yeston (1976:4) suggests that there are two kinds of 'functional statements' in rhythmic recursions: those that proceed from *pitch to rhythm* and those that proceed from *rhythm to pitch*. Each of one requires a specific grammar with its own rules of recursiveness. This dichotomy seems to be nested itself within human physiology and evolution. As in lyrical drama, the relationship *pitch to rhythm* assimilates the subordination of body movement to the voice's movement. In some instrumental genres the relationship *rhythm to pitch* is reminiscent of the voice subordinate to body; as happens in dance music, or as happens when moving the body in ordinary actions. As Chávez (1961:39) notes, "Dancing seems at first to have been nothing but a form of walking. [...] the arts of rhythm—poetry, music, and dance—were born the day man first walked rhythmically."

The relationship between pitch and rhythm can be represented, therefore, by a functional reciprocity. Starting from an original point, intonation and rhythm follow the same operating principle, not based on a mere duality, but rather in a sequence of simple elements that make up a complex code in a medium to long term: *pitch subject to rhythm* and *rhythm subject to pitch* are relations that can operate in a single musical expression in a similar way to how—simultaneously and without conflict—certain functions of prosody and intonation operate in language.

Based on Peitgen and Richter (1986), Koblyakov (1995:297–299) synthesizes this concept in the formula $x_{n+1} = x_n^2 + C$, and which is directly related to rhythm as proportion and self-structuration, as analyzed in Chapter 6. In this context, the mechanisms of rhythm as proportion and self-structuration operate as an *ordering*

function of the code,[351] and—according to Koblyakov (*op. cit.*: 297)—constitute a principle of self-similarity for rhythmic, melodic, and harmonic structures, and in general for all musical complexities formed with a constructive relationship between syntagma and paradigm.

Lewin's transformational model (1987)

Much of the analytical work of David Lewin (e.g. 1982, 1987) is based on observations on music functional recursiveness, classifying its symmetries, repetitions and intersections. Heinrich Schenker (1868–1935) had already attempted to explain general and particular relations of music by the way some segments comprise and intersect others, considered as subordinates. In his analytic–generativist theory Schenker (1932) conceives that every tonal structure is elaborated as an *Ursatz* complex (see Salzer 1952, Pankhurst 2008). The effects of this complex are thus embedded in the intrinsic properties of the fundamental relations of the *Ursatz* (which is a basic *musical idea*) and in its prescriptive orientations. Extending Schenker's theory, Salzer (1952:32–36) emphasizes the study of two sorts of categories: the music strata (*Schichten*), and the *prolongation* and *functional movement* of music. His global vision of a musical structure tends to identify hierarchies, always observing the nesting of one structure within another, which the first determines. Therefore, he uses the image of "chord *within* the chord" (see Salzer, *op. cit.*:37), consistently extended to a system of strata.

Whereas Salzer devotes special attention to the study of fundamental chords with respect to higher hierarchies and their subordinates, Lewin (1987) looks for an explanation of these relations in a kind of germ latent in tonal structures (extensible to post-tonal), starting from a basic hierarchy with a typical symmetry. Firstly, Lewin conceives a universe of 'objects' (pitches, chords, scales) and 'relations' (inversions, transpositions, transformations) between such objects; consequently, he uses the concepts of *space*, referring to objects and their sets, and *group*, referring to actions and relationships between objects. In short, as Satyendra (2004:101) suggests, "It is

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[351] See pages 208–210.

useful to liken the difference between space–elements and group–elements to the difference between nouns and verbs." Under this consideration, transformational theory deals with the contingency of relations and functions between groups and tonal space, rather than—in contrast to Salzer, for instance—establishing relationships between objects, functions and context. According to Lewin (1987:159), the transformational attitude "does not ask for some observed measure of extension between reified 'points'; rather it asks: 'If I am at [a space] *s* and wish to get to [a transformation] *t*, what characteristic gesture should I perform in order to arrive there?" This criterion is substantially different from the criteria used in examples ◊541–◊544, insofar as the objects ordered from certain rules of measurement are not the ones to be found in a relation of similarity or (pre)self-similarity. The transformational perspective opens the possibility of observing such relations as variable functions of groups and spaces.[352] In consequence, the image of "self-contained" object (Salzer 1952:112) gets enriched with the possibilities of "space within space", "group within group" or "transformation within transformation". As a matter of fact, the formal definition of transformation is based on the principle of recursion, a potential motor of self-similarity in *musical systems* (that is, sets of objects and groups of relations interacting): "A transformation is formally defined as a mapping from a set to itself." (Satyendra 2004:104; see also Lewin 1987: xxx, 88, 124–127, 152).

The concept of nesting in Lewin (1962)

Lewin (1962:99–100) uses the concept of 'nesting' in an intuitive way, to classify unordered overlapped segments, common between pitch rows from the same chromatic whole. This notion emphasizes the interval content shared by two rows, so that it is possible to extract segments by permutation and inversion operations. Equally it is possible to extract the interval content shared by two segments, in consecutive layers. From this idea, Ilomäki (2008:196–222) proposes measurements

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[352] This change in perspective is stated from the very definition of musical self-similarity, at the beginning of subchapter 3.3.: "there are structures […] in which a relative self-similarity does not appear as a kind of 'scale-bound object within a similar object', but as a statistical feature: distributions, and not shapes are found in a self-similar relationship."

of functional similarity, based on features of nested subsets and their combinations. So far Lewin's (1962) notion of 'nesting' is consistent with the concept of 'pre-self-similarity'[353]—since it presents an original set from which subsidiary relations are extracted, with the possibility of iterating recursions in successive layers—in Ilomäki (2008) this notion extends to a relative self-similarity, assuming segmental deviations in a much larger number of layers. Actually a finer point is worth pursuing: that this self-similarity does not appear simply in the repetition of common elements along deviations, but in the iteration of operations with which segments are obtained. This sort of relationships of functional similarity among pitch segments—eventually instances of self-similarity—is closely related to Klumpenhouwer nets in their self-similarity cases, as explained below.[354]

Recursion of networks and hyper-networks

In pitch-class set theory, *Klumpenhouwer* networks allow visualization of links between pitch-class sets as similarity systems and as modes of identical representation or *isographies* at different functional relationships. A Klumpenhouwer network is any chord network using transposition or inversion operations to interpret relationships between pitch-class sets (Lewin 1990:84).[355] In other words, Klumpenhouwer networks characterize the form with which some simple functions of transposition and inversion, 'orbit' around the same object (for example, a triad). Under this analysis, different pitch-class sets can be represented by networks that have the same appearance, but at the same time present different groups of relations, alternating forms of transposition and inversion. In this context, and evaluating the relations of self-similarity in transformational terms, Buchler (e2007) postulates that there are cases of structural recursion in which "networks of networks" emerge. Such emerging

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[353] This notion is defined in subchapter 3.3.

[354] According to its initial statement, this subchapter is restricted to the operational interference of 'rigid' schemata with respect to the 'flexible' relations of music. It would be impossible here to give an analytical summary of the theories of Lewin (1962, 1990) and their development in Murphy (e2007) and Ilomäki (2008), among other authors who propose methods for the study of similarities in the context of pitch-class set theory, which eventually could lead to a specific theorization of musical self-similarity from this perspective.

[355] See subchapter 3.4., directly related to this topic.

structures correspond to Klumpenhouwer networks intertwined so that changes in them become similar to the transformations of a network group. In addition, Murphy (e2007) suggests using the "term *self-similar* for the situation in which a hyper-network matches, to some degree, one of its constituent networks, and the term *self-dissimilar* for matches that are not self-similar." However, the notion of self-dissimilarity, related to self-similarity in a manifold way, requires a separate treatment in subchapter 6.6.

It is worth to mention, as a complement to what is stated above, that the idea of harmonic nesting in networks and hyper-networks is compatible with the dynamic concept of *fractal nesting* suggested by Koblyakov (1995:299): since "the spectrum of similar passages stipulated by [a functional] similar conjunction at different scale-hierarchical levels, [one may say that] a system of 'inserted fractals' occurs with smoothly changing dimensions in real time".[356] Koblyakov (*ibid.*) claims this *transdimensional continuum* "gives birth to time or, to be exact, to the sense of time as a correlation of different processes in different metrics and scales".[357]

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[356] In this conceptualization, the notion of 'fractal' is attached to what is defined in section 1.3.4. It is obvious that what Koblyakov intends by 'inserted fractals' actually means 'nested fractals'.

[357] Several years before I became aware of Koblyakov's (1995) notes, I attempted to express this idea in a draft on the genealogy of musical time in a transdimensional continuum, mixing concepts from Georg Cantor (1845–1918) to Julian Barbour (1937–), with Peircean *synechism*. Unfortunately, the text was published with many typographical errors and obvious conceptual infelicities (G. Pareyon, "Transfinitud y forma: reflexiones en torno a la música contemporánea" in [Hebert Vázquez, coord.] proceedings of the *Primer encuentro transdisciplinario en torno a la música*, Universidad de San Nicolás de Hidalgo, Morelia, 2000). I would like to put forward a thorough reelaboration of this writing, nevertheless, as further research.

Broad sense of the concept of nesting

The term 'nesting'—restricted until this point to Lewin's (1962) theory in its Cartesian and transformational perspectives—can also be understood in a broad sense, encompassing any kind of self-similar process branching in finite layers.[358]

Introduced by Lindstrøm (1990) in the study of Brownian motion, this 'broad sense' refers to the structural relationship between potential and actual features in the states of a dynamic process and to all its objects.[359] Within the same context, Xenakis (1992:289–293) calls 'nesting' the samples of noise in stochastic functions producing $1/f^0$ and $1/f^2$ noises, with special interest in the (a)symmetries and (a)periodicities of sound synthesis.[360] By contrast, Kramer (1995) associates this term to the power of structural durations nested in ratios, proportions, and series in a spectral sound sample.

Congruence between segment and self-similar whole can be detected within an overall evaluation of nested sets within larger sets with more recursions. As explained below, the analytical arcs system proposed by Wattenberg (2002) is a useful option for representing the same kind of self-similarity in one or two functional layers, but at the same time introducing fuzziness between detail and generality. On the other hand, Visual Recurrence Analysis (Bourke e1998, Kononov e1998) is able to penetrate in both directions, with sufficient transparency and statistical products which may also be useful, as explained below.

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[358] In support for this use, see Fitzsimmons *et al*. 1994:595, Kigami 2001:70, and Xiao 2004:277–278.

[359] As part of a compositional technique, I have employed such a broad or intuitive sense of nesting (see Pareyon 2003), referring to a simultaneous multi-layered self-similar music from a basic set of rules defined for each instrument (minimum measurements of texture, duration, tempo, and local-absolute loudness), developing similar relations on a larger scale (metre, harmony, melody, and global-relative dynamics and agogic).

[360] Interestingly, Xenakis (1992:183) also introduces the word 'nesting' for a synecdochic conception of music, although he does not provide a larger development of it: "[T]he Greeks used in their music a hierarchic structure whose complexity proceeded by successive 'nesting', […] by inclusions and intersections from the particular to the general". This matches with the concepts developed here, in subchapters 4.4. to 4.6.

Holistic sound visualization: 1. *Arc plots*

The study of acoustic nestings is accessible using statistics. Anticipating aspects of research done by Bourke (e1998), Kononov (e1998), Foote and Cooper (2001), and Paulus and Klapuri (2006), the proposal of Church and Helfman (1992) focuses on nesting of information in general, within a function space. Like other exploration and emulation tools derived from biology and organic chemistry,[361] the *dotplot* developed by Church and Helfman is based on a relatively simple statistical method, whose operation can be systematized in complex dynamics:

> Dotplot has been developed for browsing millions of lines of text and source code, using an approach borrowed from biology for studying self-similarity in DNA sequences. With conventional browsing tools such as a screen editor, it is difficult to identify structures that are too big to fit on the screen. In contrast, with dotplots we find that many of these structures show up as diagonals, squares, textures and other visually recognizable features (Church and Helfman, *op. cit.*:58)

The dotplot is obtained from a simple principle of data distribution: for instance, if the analyzed information consists of twelve elements with similar distributional properties (e.g. the chromatic set), such elements must be allotted by intervals reflecting a reciprocal affinity on a diagonal dividing the analytic square viewer. Perfect symmetry drawn along the diagonal reflects the identity of the overall relationship.[362] If, in particular, what is analyzed is an alphabet (i.e. a finite repertoire of symbols), then the more the analytical sequences are coordinated beside or on the diagonal, the greater their distributive–qualitative affinity; or—conversely—the more they are in asymmetry with the diagonal, the greater their mutual differences.

The first results using dotplots were displayed in a monochrome grid; later colours were added, associated with data distribution patterns. A detailed explanation of this kind of analysis occupies the following section, devoted to Visual Recurrence Analysis (VRA). Rather, this section points out how Wattenberg (2002) modified the dotplot technique using a simple comparison by rows. This modification ignores textures that are statistically useful in VRA, but instead it

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[361] These tools include projection of population in a logistic map, and L-systems. Due to their musical use, these concepts are explained in subchapters 6.2. and 6.5., respectively.

[362] This example matches with what is shown in the chart ◊524 (top left), on page 236.

helps—in a simple fashion—to visualize hierarchies and the overall strata in a text or a data sample with musical attributes. Wattenberg's method consists basically of connecting elements, or related data chains, showing their frequencies (i.e. repetitions) by the use of arcs. The result of this analysis with simple Markov chains, such as those shown in ◊520 or ◊522 is not very useful, as they only show what is already clear: that original order is gradually lost for each recursion of a chain. However, on the other hand, if it is used to describe what occurs in a text with allocated repetitions, then the arcs system is eloquent about the quality of what is repeated (see ◊541). The same method is useful for analyzing a sequence or group of musical relations, in order to see how functional hierarchies and strata—if any—are organized. In essence this method of analysis is prior to Wattenberg. For example, François-Bernard Mâche (1983:179–180) uses it to indicate inner coherence of a text or a musical segment, as well as to summarize the overall relationship in each case study.

The arcs diagram in ◊552 shows not only the inner organization of the text and its strategic repetitions, but also the actions related to it, and the images produced in the reader's memory. The example in ◊553 suggests an analogous diagram showing the typical structural consistency of a musical piece (in Wattenberg 2002, there are more specific musical examples). A system becomes more visible, thus implementing direct comparisons of functional relationships between music and verbal language. Nesting in music samples occurs, however, through a consistency of layers within layers that are more complex than in a sample of written language. As suggested in Chapter 4, the surface of a musical system is actually tied to deeper relationships of self-similarity at various levels. This happens, for example, when the distribution of the harmonic components of timbre is connected with a likened distribution at the metric–harmonic macrostructure; or—even in a more subtle way—when a physiological pattern (e.g. heart beat, breathing periodicity) affects and determines crucial aspects of the instrumental performance.
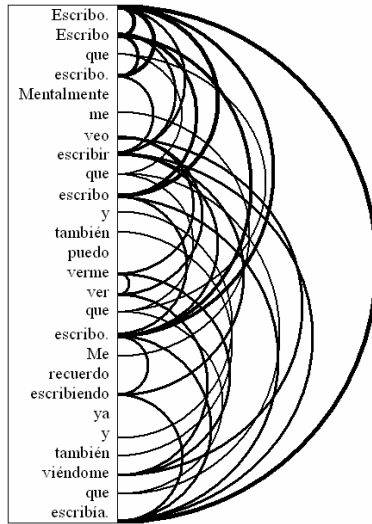
◊552. Arcs diagram encompassing the first sentences of Salvador Elizondo's poem *The Graphographer* (1972)—the same segment appears translated into English at the beginning of subchapter 3.6., as an example of literary self-reference. In this diagram the arcs' thickness denotes syntactic hierarchy. Amplitude reflects constructive strata, in which the conjugation of the verb *to write* in first person singular, is very noticeable. Isolated words like *Mentalmente, puedo, recuerdo, ya*, are also easily seen, playing their own structural function within the text's varying tensions.
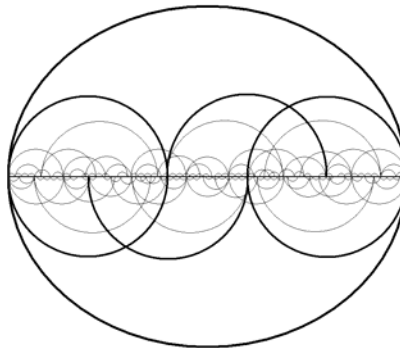
◊553. Arcs diagram typical of a harmonic recurrence analysis. The shorter arcs represent basic cycles of a same function. Different sizes reflect segmental hierarchical structuring. When extended, the higher hierarchies take up a broader space and become more visible. The lower half mirrors the top half in inverted symmetry, in order to make obvious a correspondence among temporal relations as a whole (horizontal *displacement*). The notional grounds of this analysis, developed by Wattenberg (2002), can actually be traced to Schenker's (1932) and Salzer's (1952) strata classification and prolongational analysis.

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This kind of analysis and visual representation of music is compatible with Schenker's (1932) theory of strata and structural prolongations. Moreover, for both examples in 541 and ◊542, the classical topics of Schenkerian analysis can be implemented as layer or level Schicht (*Schicht*), fundamental structure (*Ursatz*), interruption (*Unterbrechung*), surface or foreground (*Vordergrund*), and hidden repetitions (*verborgene Wiederholung*), among others. Accordingly—as Wattenberg suggests—Schenkerian analysis can be complemented using arc diagrams.

Holistic sound visualization: 2. *Visual Recurrence Analysis* (VRA)

VRA is a software for the qualitative examination of frequencies and non-parametric prediction of non-linear/chaotic time series. Recurrence patterns in VRA show the behaviour of a nonstationary series. In a stationary system the recurrence plot tends to be homogeneous along the diagonal, indicating levels of aperiodicity which can also be interpreted as degrees of self-similarity, i.e. degrees of relative periodicity within other consecutive periodicities. Its use was originally proposed by Eckmann *et al.* (1987:973), as a "A new graphical tool for measuring the time constancy of dynamical systems".

VRA allows detection of hidden patterns and deterministic structures in time series by using the 'recurrence plot' tool, which is essentially a graphical representation of the integral correlation, so that dependence on time in the system under study is prevalent, appearing in a multidimensional scaling. VRA first expands a given series of unidimensional time into a broader space within which the underlying generator develops (see Bourke e1998).

As a referential scale, VRA emphasizes the data orientation, instead of a specific location. If data time transformation can be decomposed into isometries through a diagonal matrix, then a differential directional scaling is obtained on the diagonal values, which are the scale factors in perpendicular directions. Bourke's (*ibid.*) formalization is actually useful in completing this explanation in the following terms.

Consider a series x with n terms:

$$x_0, x_1, x_2, x_3, x_4, x_5, \ldots x_i, \ldots x_{n-1},$$

for all vectors y_i of dimension (length) D with lag (delay) d, so that

$$y_i = (\, x_i, x_{i+d}, x_{i+2d}, \ldots\,)$$
$$D \geq 2, d \geq 1.$$

According to Bourke (*ibid.*) this is commonly referred to as embedding x in dimension D with lag d. The recurrence plot is formed by comparing all embedded vectors with each other, and drawing points when the distance between two vectors is below some threshold. A coordinate (i, j) displays a point where and if the embedding of vectors of cardinality i and cardinality j are less than distance r. For example, a point emerges if

$$\left\| y_i - y_j \right\| < r.$$

Then i is plotted along the horizontal axis, and j on the vertical axis. Both (i, j) represent 'time series': "therefore [the] recurrence plot describes natural *time correlation* information" (Eckmann *et al.*, 1987:973). If the recurrence plot presents at the same time a homogenous and irregular distribution of points then the series is 'more' stochastic: for a random series of $1/f^0$ noise with mean $= 0$, and standard deviation $= 1$, the recurrence plot is as shown in ◊560a, corresponding to $d = 2, h = 1$, $r = 0.05$.

The meaning of VRA plots lies not only in their statistical projection; most notably it lies in their ability to show aesthetic aspects of the contents analyzed. Their geometric patterns are a product of the information qualities processed. In this context, colour options are also implemented to make visible the relationships between frequencies analyzed in several dimensions, integrating an intersemiotic complex made of direct analogies between sound and image. Taking these qualities into account, Kononov (e1998) characterizes VRA as a tool for comparative studies in musicology and linguistics.[363]

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[363] The examples included in ◊560 were created by the author of this study, using Kononov's (e1998) free software. Exceptions are: $1/f^0$ noise example (*a*), adapted from Bourke (e1998); example (*f*) corresponds to number 3 in Dual-tone multi-frequency signaling (DTMF), used for

All examples, from (a) to (l), are accompanied by a header that represents their corresponding waveform, indicating frequency, length, and amplitude in the usual way. Noise samples (a) and (b) visually confirm what is stated in theory: that $1/f^0$ noise is a random process of signals with equal power within a fixed bandwidth at any centre frequency, and without tendency to self-similarity in any of its parts.[364] In this context, according to Molchanov (2005:248, 255):

> If X does not contain fixed points, then X cannot be self-similar, which immediately simplifies the arguments used to characterise union-stable sets. [...] The characterisation theorem of union-stable sets relies on the fact that a union-stable random set cannot be self-similar.

Examples (c) and (d) are useful for explaining VRA aesthetic-informative value. In both cases the plotted signal appears in the VRA box as a regular texture. Whereas the bidimensional heading can be characterized as a pattern along a horizon of frequencies (ordinate) and durations (abscissa), VRA reveals patterns of similarity and relative consistency in a multidimensional chart. Differences between (a)–(b) and (c)–(d) are evident. In the first two cases the signal is randomly scattered with a null code content (irrelevant information as a system). In contrast, in (c)–(d) information is somehow organized: it tends towards stability in (c), and stabilizes in (d) with a codifiable signal.

In (e) there is a slight modification of harmonic components of the original stable signal (d). Quite differently, (f) shows a DTMF signal corresponding to the harmonic tone of number three in digital telephony, as a texture that clearly shows the dual composition of the multi-frequency. The regular tiling of this pattern reflects timbre homogeneity, with an emission of its harmonic components distributed symmetrically.

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telecommunication signaling over analogue telephone lines in the voice-frequency band; and, as indicated *in situ*, the beginning of Beethoven's Fifth Symphony, which forms part of the original sample collection created by Kononov (e1998). This last example was included because of the easy access to this music as a commonplace in Western culture's collective memory.

[364] See the definition of $1/f^0$ noise on pages 241.

◊560. Visual Recurrence Analysis (VRA) examples.

a) **1/f^0 noise**
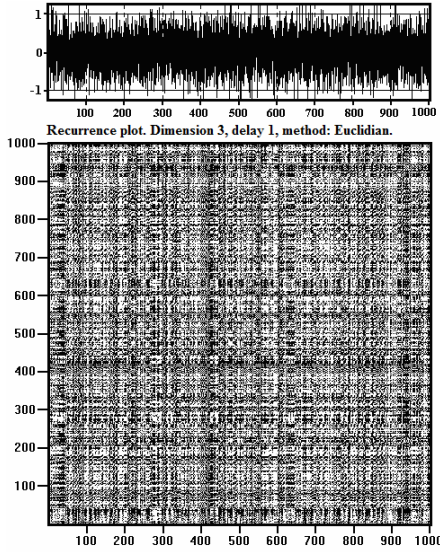
Recurrence plot. Dimension 3, delay 1, method: Euclidian.

b) **1/f^0 noise** (filtered)

Recurrence plot. Dimension 3, delay 98, method: Euclidian.

c) **Sinusoid with noise**

Recurrence plot. Dimension 3, delay 1, method: Euclidian.
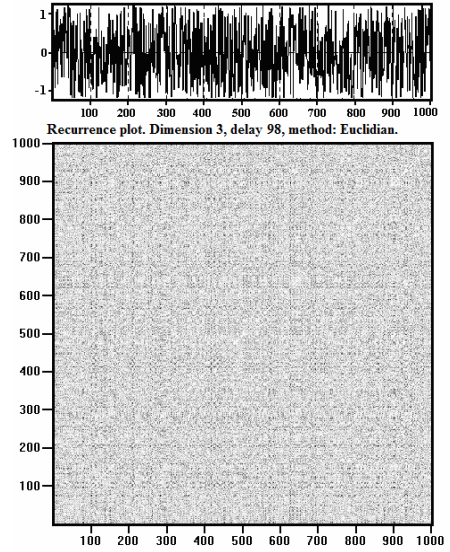
d) **Sinusoid** (pure tone)
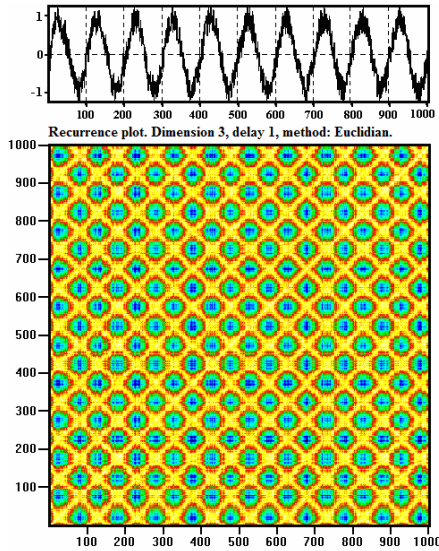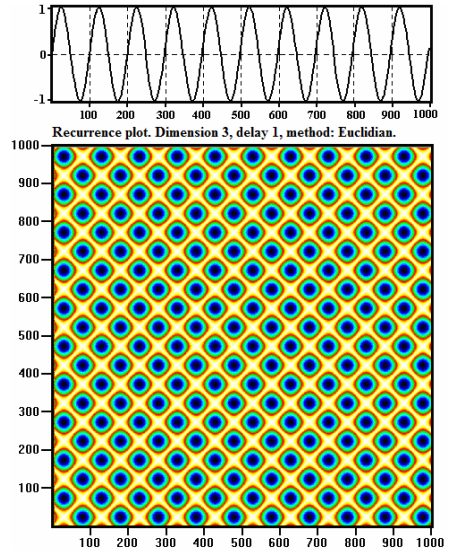
Recurrence plot. Dimension 3, delay 1, method: Euclidian.

e) **Sinusoid** (filtered pure tone)

Recurrence plot. Dimension 3, delay 42, method: minimum.

f) **DTMF 3 Signal**

Recurrence plot. Dimension 3, delay 1, method: Euclidian.
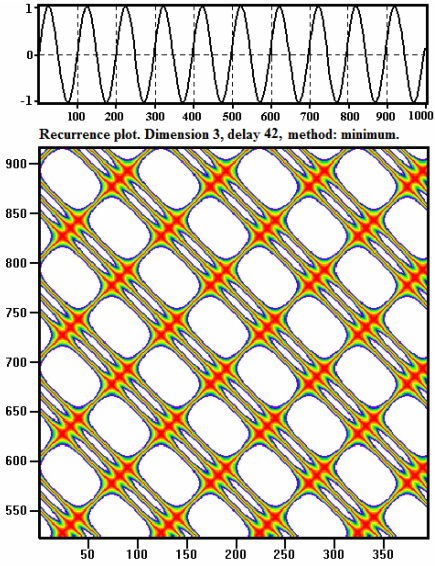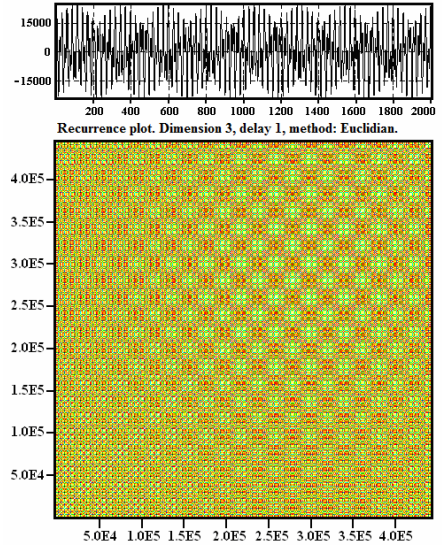
g) **A 220.00 Hz with harmonics 1-4**
(electronic synthesis)

Recurrence plot. Dimension 3, delay 1, method: Euclidian.
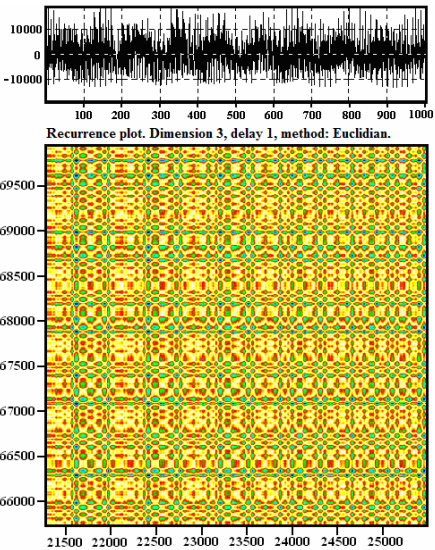
h) **Verbal expression**
(Linus Torvalds voice)

Recurrence plot. Dimension 3, delay 1, method: Euclidian.
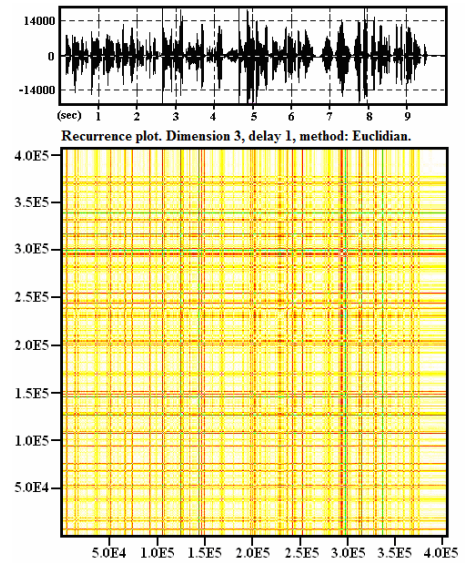
◊560. VRA: Exploring self-similar patterns.

i) D# (155.56 Hz), *crescendo*
(bass flute)
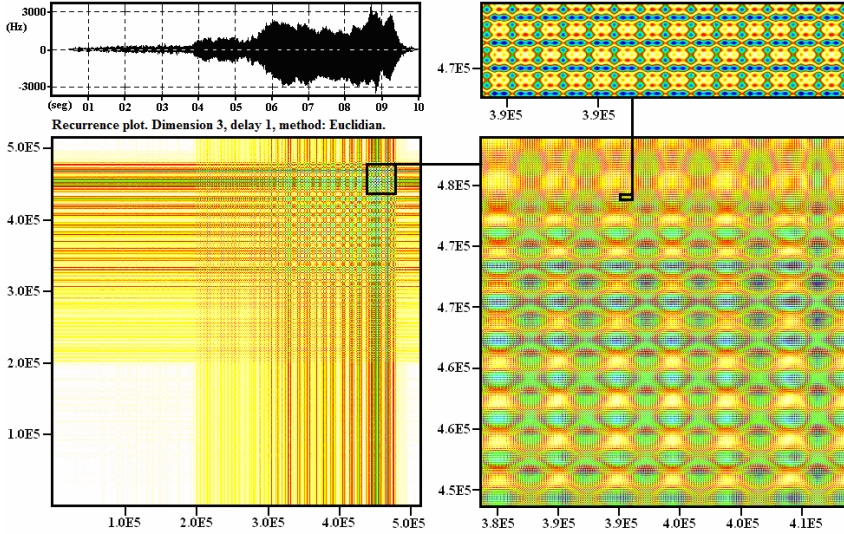
j) Bell beat
(electronic synthesis with Lyapunov's function)
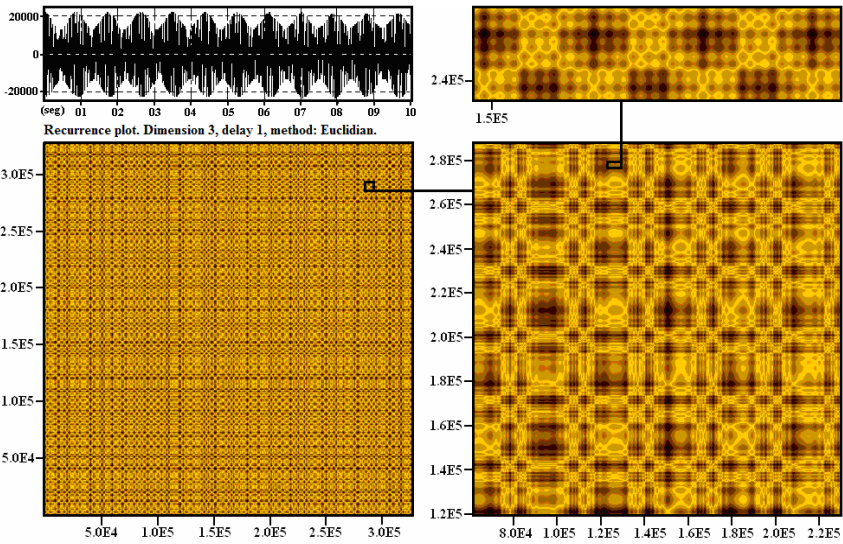
◊560. VRA: Exploring self-similar patterns (continuation).

k) Threnody of Manuela Gómez

(ritual mourning from the Batsil Winik'Otik people, Chiapas, Mexico)
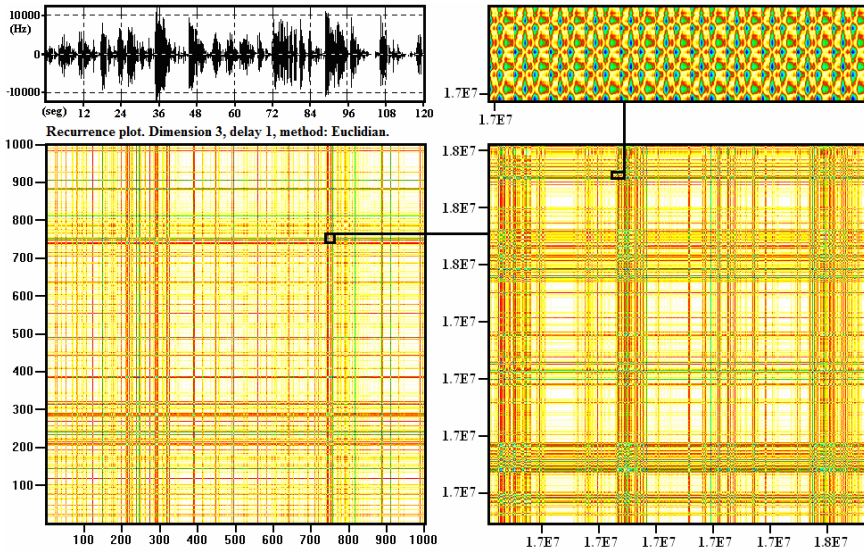
l) Beethoven, Fifth Symphony

(bars 01–58)
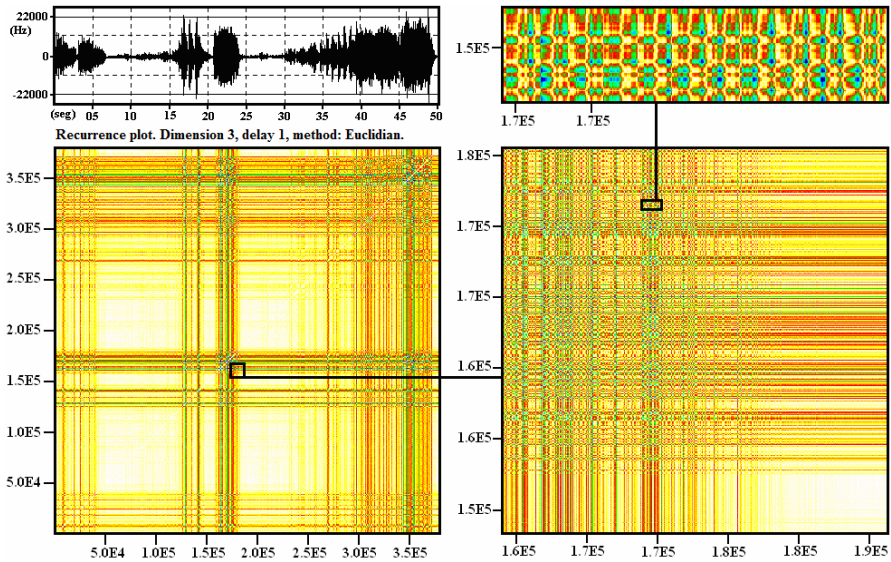
In (*g*), for the same reasons which apply to the previous example, timbre homogeneity is also evident. There are differences from (*f*), however, in the abundance of harmonics with a more 'hierarchical' distribution: timbre gets richer as it is reflected by an increasing pattern of correlation.[365]

Example (*h*) corresponds to a segment of a recorded interview with Linus Torvalds.[366] Specificity of the source is trivial in this case. What is meaningful in this example, is the fact that the verbal utterance is built by signals variating in frequency and amplitude, distributed along the duration parameter. This is radically different in comparison with the previous examples. Both the waveform and the adjacent VRA show this distribution as a rhythm—the rhythm of speech—with segments of varying intensity and intonational richness, corresponding to syntactic structure units (words). Verbal language information is encapsulated in physically differentiated acoustic chunks, in irregular recursive patterns that nourish the expressive and symbolic variety of speech. A little later in this section, this case will be summarized, by comparison with example (*k*).

Examples (*i*) to (*l*) consist of two VRA levels of enlargement: one at 'medium' scale (box to the right) and another at 'microscopic' scale (top right rectangle). The waveform in (*i*) corresponds to ten seconds of a D# (approximately 155 Hz) recorded from a bass flute. An increasingly richer production of harmonics results from the instrument's form and materials, and from the amplitude growing. In the box to the right of the example (intermediate level), round symmetry simultaneously recalls the production of stable tonal frequencies (compare with examples *c* and *d*), and similarity—more a congruence than a coincidence—associating the sound wave with the instrument's mouthpiece. Waveform similarity in example *d* may be compared, by proportional analogy, with the periodic vibration of a string.

Example (*j*) corresponds to the electronic synthesis of a bronze-bell sound, showing a tone generator with a large amount of harmonics. The apparently homogeneous plot at the first level of VRA actually contains complex patterns in its

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[365] This autocorrelation is observable in the image of a deterministic periodicity made of sub-periodicities with structural features that are similar to the general pattern.

[366] Digital recording available in http://www.freeinfosociety.com/media.php?id=50, file "Torvalds, Linus: Operating Systems" (website consulted on September 1, 2007). Torvalds is a software engineer best known for being the chief architect of Linux, a free and open source software. The selection of the audio file is arbitrary.

medium and micro levels, with a self-similar behaviour comparable to Lindenmayer system self-organizing curves within a Hilbert space. This example contrasts with (*i*), which has a more limited harmonic composition and lesser interferences from resonances. This comparison suggests the variety of harmonic textures (i.e. timbre) of various musical instruments and other objects of sound production, as isospectral manifolds in Hilbert spaces. Subchapter 6.4., on tessellations and brocades, resumes this issue within a context of constructive symmetry.

The following example (*k*) comes from a field recording with the voice of Manuela Gómez, a mourner woman and healer from the Batsil Winik'Otik people (Chiapas, Mexico).[367] There are some obvious similarities with the previous verbal example (*h*), although here the enlargement boxes show the similarity plots in the voice harmonic spectrum. Both vocalisation examples (*h* and *k*), extracted from cultural contexts which are quite different, and articulated by two individuals of different age, sex and ethnical origin, reveals coherent affinity under the same principle of harmonic and rhythmic generation. This feature reinforces the notion suggested in this study, of the relative homogeneity of human voice forming the same *family of musical instruments*, with timbral and prosodic variety originating in psychophysiological and cultural variety.[368] Whereas human voice (sound) can be described by analogy with a family of musical instruments, speech (language) can be described as the articulated natural repertoire of the same family.[369]

Example (*l*), like (*c*) and (*d*), is adapted from the files included in Kononov's (e1998) software package. (*l*) represents the starting segment of Beethoven's Fifth Symphony (bars 01 to 58), and suggests some similarities with respect to verbal

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[367] Recording included in the compact disc *Sistema de Radiodifusoras Culturales Indigenistas. Testimonio musical del trabajo radiofónico*, Instituto Nacional Indigenista, INI-RAD-I-5, Mexico, DF, 1995. Also kown as Tzotzil, the Batsil Winik'Otik are an indigenous Maya people of the central Chiapas highlands in southern Mexico.

[368] This notion is expressed in subchapters 4.4. to 4.6. More precisely, Pareyon (2006) suggests conceiving culture as a *choir* and language as phonological *texture*: a linkage of individual *songs*. Following this game of paradigmatic analogies, the history of culture can be described as a *choir made of choirs*, forming an overall noise at the macro level, with *songs* and *textures* articulating patterns defined at the micro level.

[369] This relationship of acoustic aspects of music and speech cannot be extended, however, to all theoretical perspectives. It is quite obvious that music and speech differ greatly in their uses and their ways of encoding and representation. This issue, which needs further discussion, is developed in subchapters 4.7. and 4.8., especially for its involvement with the concepts of grammar, style, idiolect, ecolect, and translatability.

language: the whole organization of information shows segmental accumulation of acoustic material in a form comparable to that observed in the verbal examples (*h*) and (*k*), with a correlated outline of semantic and syntactic value (although not lexical, in the case of music). The waveform of (*l*) represents a minor segment of the Symphony first movement. The complete movement waveform can be compared with segments of the vocal waveforms in (*h*) and (*k*): in the waveform of (*l*) there are five large sub-segments, comparable to five chunks of a continuous verbal expression, or to five groups of sentences or words contextually agglutinated. This encapsulation, common among systems of acoustic information, represents one of the greatest similarities between music and verbal language.[370]

Example (*k*), with Gómez's voice, corresponds to the utterance "oh, jt'tzön mbal mbe", in Tzotzil language. Torvlads' voice corresponds to the utterance "to kind of explain", in English. By contrast, example (*l*) belongs to a piece of symphonic music. Nonetheless, despite their obvious contextual differences, all three cases have in common the same kind of information encapsulation.[371] All of them show tendencies to a similar type of agglutination, forming nodes separated by quietening intervals, in a spread distribution of intensities, with acoustic contours that are not found in samples of random distribution of frequencies—$1/f^0$ noise, for instance; nor in relatively stable periodic signals, with simple or complex harmonic textures, as in examples (*i*) and (*j*). In these cases (*h*, *k*, *l*) there is a recursive coordination taking place in an intermediate state between random distribution and periodic repetition of similarities. Whereas randomization tends to a widespread difference, and continuous periodicity produces a monotony of similarities, the examples of music and verbal language tend to self-similarity, mirroring patterns of recursion and self-reference as signs of variation and operational consistency.[372]

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[370] On the structural similarities and differences between music and speech, see subchapter 2.4.

[371] Since examples (*h*), (*k*) and (*l*) correspond to systems of sound organization in a temporal and hierarchical distribution of codes, it is obvious that they keep generalized aspects of structural similarity, like information encapsulation in the forms of phrasing and hierarchical chunking. For an explanation regarding the latter, see subchapter 2.5.

[372] Other statistical methods used for the measurement of musical self-similarity are widely discussed in Beran and Mazzola (1999a–b), Bigerelle and Iost (2000), and Beran (2004). The Gabor function, associated with the Fast Fourier Transform, and which proceeds from a Gaussian frequency and a wave in the complex plane, is also useful for analyzing self-similar

5.5. Determinism and indeterminism in cooperation

There is a practical implementation for the opposition *absolute fractal* versus *natural fractal*. This includes the distinction between *deterministic fractal* (strictly a self-similar object or process in which fractal dimension is the same for a part than for its whole), and *stochastic fractal* (for instance, Brownian motion; or noises from plasmas and diffusion patterns in which fractal dimension is not the same for the part than for the whole). This view allows a finer scrutiny of the causal arguments for self-similarity, beyond the obscurity—and excessive generality—of the concepts *absolute fractal* and *natural fractal*.

The *deterministic fractal* can be roughly defined as the self-similar set whose axioms, initial conditions, and rules of operation and production are objectively representable (i.e. Cartesianly acceptable as extensions of a system), with general relationships representative and valid for all the parts of the whole. By contrast, the *stochastic fractal* can be defined as the self-similar set whose axioms, initial conditions, and rules are partially representable, with relations that characterize only a statistical sample, and which are valid for a specific segment of the whole (e.g. a Lévy flight segment). Both types of axiomatics can be implemented as compositional systems within different layers of recurrence and self-similarity, with grammatical or pragmatic character (see Xenakis 1992:266, 292–293; Bolognesi 1983:31–36). This implementation is also possible thanks to a negotiation between determinism and indeterminism. From this viewpoint, Biles (e1998) notes that:

> Some composers use algorithmic techniques only for generating low-level details, like
> pitches and/or lengths of specific notes, within tight constraints that they set. These

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patterns in the study of sound granulation, as suggested by Rocha Iturbide (1999) and Roads (2004).

composers determine the deep structure of the piece, make most of the larger-scale decisions, and use the algorithm only to generate the surface structure. Other composers may use algorithms that generate sequences that exhibit their own deep structure, which can allow the algorithm a more comprehensive role in creating the composition. The composer still makes plenty of compositional decisions, but these decisions are more collaborative in nature and often serve to emphasize the deep structure inherent in the sequence produced by the algorithm.

In this context, consideration of a functional co-presence of deterministic and stochastic (pseudo)fractals in music, may contribute to understanding in each case the boundaries of rigidity and flexibility of a musical system. Assuming this negotiation, Oliver's (1992:172) remark becomes relevant:

> Fractals do not necessarily offer hope that we can control elusive phenomena [observable in nature]. On the contrary, we are just beginning to understand that chaos and unpredictability are more deeply embedded in nature than we ever imagined. Fractals do, however, provide powerful tools for modelling and visualizing nonlinear systems.

The chaos referred to by Oliver corresponds with the traditional concept of chaos laid out by Henri Poincaré in 1890, studying the *Three-body problem*. This problem is related to the difficulty of determining the relative position of three bodies in their mutual attraction, with known initial positions and velocities. The major complication in this problem arises from the fact that a minimum variable in the system's initial conditions can quickly produce huge differences. Let's consider the case of a Lindenmayer system whose initial conditions are influenced by another system; the first data strings obtained will reflect a disturbance that grows logarithmically to infinity, in very few steps. Other similar examples can be excerpted from Chapters 5 and 6, as the chaotic fluctuations in the logistic mappings. This notion of chaos is also explained by Vázquez (2006:403), within the frame of musical analysis, referring specifically to Ligeti's *Étude 1: Désordre* (1986), for the piano:

> 'Chaos theory deals not with the completely random, but with systems displaying apparently irregular or unpredictable behaviour yet which obey hidden, purely deterministic laws.' [...] The different mechanisms that introduced 'chaos' in the work [*Désordre*] display a highly organized behaviour, most of them operating within a framework of stable patterns in sequence. The three main sections [...] for the metric-rhythmic organization of the piece reveal a highly organized structure.

In *Désordre*, as in the movement IV of Ligeti's *Piano Concerto* (1985–88), musical structures do not constitute fractals but "highly organized" processes with relative self-similarity. In this context Beran (2004) prefers the concept *stochastic fractal*, to interpret that the relationships of a musical system (in any of its parameters) can start from specific conditions (order model) and derivate in a set of relatively or absolutely unpredictable variables (by entropy or tendency to disorder in discrepancy with initial conditions). Each scheme of entropy in a set of intervals characterizes, thus, specific information content that can be classified by local variability. This means that information from entropy may also reflect a process of self-similarity on a specific identity, by tonality and modality; form of sequence or linkage of chords; form of rhythmic and metrical changes and transitions; idiomatic gestures of the instrument or voices; and agogic or tempo curves in performance. In all these cases, a higher entropy indicates a more uniform mixture in a local set of relationships, and a more homogeneously distributed self-similarity (see Beran *op. cit.*:93–96).[373]

Fractal dimension: An issue of pertinence

Fractal dimension, usually represented by capital letter D, is a statistical index that reflects the way an object or process fills its space. With integers, the topological dimension of a tridimensional object is 3; that of a bidimensional object is 2; that of an object with one dimension is 1. According to the Euclidean tradition, the point has null dimension, 0. Fractal dimension, instead, refers to *filling* the space in a 'fractured' mode, that is, not using an intuitive base of integers (1, 2, 3 dimensions), but using a base according to the intrinsic properties of the object or process that can be described with a fractal dimension, for instance, between 2 and 3, or between 1 and 2.[374] In the context of sound as information, Bigerelle and Iost (2000:2191) summarize the criteria of fractal dimension in the following terms: "fractal dimension becomes a number that quantifies the acoustic space occupation; when fractal

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[373] On the definition of distributive self-similarity in a stochastic process, see subchapter 5.2.

[374] Schroeder's (1991) study on power laws and self-similarity is also a didactic introduction to fractal dimension, including a variety of statistical applications, many of them of interest to music theory.

dimension increases, the sound power versus time becomes more chaotic." In other words, a signal's entropy is proportional to the increasing of D.

There is a variety of theoretical approaches to D, including the dimensions of Minkowski/Bouligand, of Rényi, and of Hausdorff or Hausdorff/Besicovitch, among which the latter is the most common for the fractal dimension.[375] Hausdorff dimension is not only useful for obtaining the fractal dimension of specific geometric objects, but is also used as a statistical index of a process distributed in time, which can be represented as geometric development or as *time series*.[376] Since music fits this serial analysis well, it is relevant to investigate its fractal dimension by (sub)sets of a given universe, in order to know how can music *fill its time and space*.

Methods to approximate the fractal dimension of a set of musical relationships are subtle tools that require combination with other statistical resources, in order to obtain, beyond a data bulk, meaningful results as a complement—for or against—of what classical theories of music and aesthetics suggest. As Beran (2004:92–93) acknowledges, the correlation and convergence of statistical values in music has much greater significance than the sole fractal dimension in a music sample.

Dagdug *et al.* (2007) provide an example of how to employ the fractal dimension to investigate degrees of correlation and consistency between the sections of a score. Their exploration finds out how a specific piece of music, the *Scherzo-Duetto*, K-73x, of W.A. Mozart, for two violins, tends towards structural consistency and statistical self-similarity in hidden correlations. The score was chosen for its evident features of functional correlation in its motivic, melodic and harmonic layers. Dagdug *et al.* draw special attention to a variety of symmetrical relationships between the two instrumental parts, some of which function as palindromes. They note that for relatively short distances between neighbouring notes, the overall correlation approaches $1/f$ noise. As the distance between neighbouring notes decreases, the long-term correlations decrease as well. They also note that, for some intersections of

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[375] For a detailed methodology to obtain the fractal dimension of a musical segment, see the quoted research of Bigerelle and Iost (2000).

[376] Any point x of a metric space E can be associated with an integer or with ∞, called dimension of E in x ($\dim_x E$). At the same time E can be related to another integer or ∞, called its dimension ($\dim E$). In this representation, a time series (T) appear as a points relationship (x) within a metric space. The series corresponds to the universe of points that integrate T.

the piece, the correlation tends towards zero, following a behaviour comparable to $1/f^0$ noise.[377] This form of combination of consistency and structural irregularity, typical of self-similar relations systems, is complemented by the result obtained with a fractal dimension.

Following a similar methodology, Su and Wu (2006) represent the melody and the rhythm of a piece of music in individual sets distributed along a straight line. The structure of the musical composition is expressed as the plot of local segments of a points sequence measured by its fractal dimension, as fluctuations of the Hurst exponent.[378] Su and Wu (2006) suggest that the shape and opening width of the multifractal spectrum plot can be used to distinguish different styles of music.[379] In addition, a characteristic curve is obtained by mapping the point sequences converted from the melody and rhythm of a musical work into a two-dimensional graph. According to these authors, each piece of music has its own unique characteristic curve that exhibits a fractal trait, "unveiling the intrinsic structure of music." [380] This confirms Bigerelle and Iost's (2000:2179) notion, on what "musics could be classified by their fractal dimensions", transforming the traditional classification of musical repertoire by styles. It must be stressed, however, in agreement with what is stated in section 1.3.4., that the fractal dimension of a musical object or process, does not constitute 'fractal music' itself.

As rule of measurement, in counterpart, fractal dimension is another way to introduce *noise* into the music that is analyzed: any analysis involves segmentation

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[377] Chapters 5.1. to 5.4. define both types of noise, with examples.

[378] Hurst exponent or Hurst-Hölder exponent, represented by *H*, indicates the relative tendency of a time series, or a regression to the mean or a cluster in any direction. The exponent *H* is closely related to fractal dimension *D*.

[379] On the concept 'multifractal' see subchapter 4.2. (especially pages 147–148).

[380] Pareyon (2007c:1302) suggests that such a (multi)fractal trait is also a feature of speech, whose correlation dimension is constantly changing: "As every linguistic complex in use has an irregular continuum surface, there is a particular dimension for every complex, that should be measurable considering the different constructive parameters of each linguistic object. As the linguistic matter is dynamic, it is very plausible that the fractal dimension of languages is constantly changing." Hypothetically, random samples of recorded verbal utterances, may fall into a space that ranges from $1/f^0$ noise (random distribution) to $1/f$ noise (self-structuring recursiveness), with a variety of fractal dimensions along this range. In addition, Das and Das (2006) suggest that instrumental music tends to a higher fractal dimension than songs and inflected speech forms, at least for the samples of Indian music they examine.

and interference from rulers on the measured objects or processes, as described at the beginning of subchapter 5.4. The analysis that Bigerelle and Iost (2000) conceive thus illuminates a local aspect of information distribution, whilst other aspects remain opaque, such as those related to (trans)cultural self-similarity, or those related to recursiveness in idiolects, ecolects and grammars.[381]

Bigerelle and Iost (2000:2180) state that "music has to be analyzed as a whole and not in parts." This idea, however, raises many questions about the concepts and methodology to be used, which fully complies with solving some aspects of sound engineering, but also leaves many gaps in the theory and philosophy of music: How can a whole be analyzed without distinguishing its parts? (indeed, the etymology of the word *analysis* relates to the segmentation or decomposition of a total). What is intended here by 'whole'? Does it mean the globality of the parameters of a musical system as implemented in a pre-established pattern of analysis? If so, how does this scheme include aspects such as the subtleties of musical interpretation, as a manifestation of culture? Is the fractal dimension of the recording of a piece of music equal to the fractal dimension of the 'same' piece displayed in a score? Or is it equal to a second or an *n*-th interpretation of the same piece, by the same performer? Is the 'wholeness' of music the wholeness of just one performance, or of just one score, or of all the musical pieces belonging to a historical period and a regional style? The interplay between these questions and the several answers to them, reveals that the fractal dimension, as theorized by Hsü and Hsü (1991), Hsü (1993), Bigerelle and Iost (2000), Su and Wu (2006), and Dagdug *et al.* (2007), is useful as a first statistical approach to a certain measurable phenomenon, but it says very little—or nothing—about the other modes of simultaneous musical self-similarity, referred to throughout Chapters 4 to 6.

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[381] See subchapter 4.6.

Common misconceptions

Many modern texts on music theory devote paragraphs or entire chapters to the relationship between music and mathematics. It is difficult to find a view that denies such a relationship in an absolute way. On the contrary, literature contains all sorts of mathematical justifications of music. Some of them are indefensible: "Nature is amazingly mathematical [...] Mathematics is the basis of sound" (Smith Brindle 1987:42–43). These assumptions are due to a misunderstanding: mathematics is not 'reality' in itself, but just a part of human reality. Mathematics is neither *Nature*, and nor does it *live* independently from nature, but instead it reflects a human-readable nature. Mathematics is real insofar as it is intelligible and it can be used and transformed in a way similar to how language participates modelling reality. Mathematical symbols, functions and algebraic representations, much narrower and exact than words from common vocabulary, are not exempt, however, from requirements of use, context, congruency, correlation and flexibility (see O'Halloran 2008).

Hsü and Hsü (1991), in the abstract of their article on the comparison of $1/f$ noise with music, note that "Suggestions have been made that computer musicians should attempt to compose fractal music, and questions have been raised whether there is such a thing as fractal music." The work of Hsü and Hsü has a strictly statistical approach to its subject matter, and its purpose is "to demonstrate the self-similarity of music and to explore its implications" (*ibid.*). Although the main goal is achieved, the authors avoid a straight standardization of the concept 'fractal music'. They only suggest—correctly, from this study's viewpoint—that fractal elaborations can contribute to a method to approximate a range of aspects of musical synthesis and analysis. This mathematical approach to music occurs—as in other similar cases of functional and symbolic adaptation—not as an absolute and effective substitution of relations, but within the frame of an intersemiotic translation.[382]

Confusion and disorientation on the fractal concept usage in combination with music theory has been stimulated by Mandelbrot himself (2002:28,193), in his desire

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[382] See section 3.8.1.

to bring his ideas to all disciplines, making a direct transfer from mathematics. To this end, Mandelbrot (2002:193) refers to Wuorinen and Ligeti's musical intuition, without giving any explanation of the compositional context:

> [I]ndependently of each other and of Voss and Clarke, at least two composers reported that reading about fractals led to identify fractality in music. Both Charles Wuorinen and György Ligeti concluded that a sequence of sounds that is not fractal is 'plain noise'; only fractal sound sequences can be perceived as musical. In a way, fractality lies more deeply than musicality because it does not distinguish good music from bad.

This quotation contains several errors and ambiguities that, instead of promoting a better understanding of the topic in question, introduces obscurity, making almost unintelligible the relationship between fractals and music. Findings by Voss and Clarke (1975, 1978), enriched by results obtained by Hsü and Hsü (1991), Carey and Clampitt (1996), and Bigerelle and Iost (2000), provide sufficient information to consider that the relationships of musical coherence and structuring are deeply influenced by physical relations of periodicity, correlation and self-similarity, as well as by biological relationships of recursion and self-reference. They do not conclude, however, that music consists of a specific set of fractal objects or fractal processes, neither of a particular fractal dimension, necessary for the existence of music. The $1/f$ self-similarity feature is, indeed, a general trait in a wide range of music samples, as Zipf's law is pervasive in a broad range of speech samples.[383] What this indicates, in principle, is that selective repetition—the economy of iteration and recursive poiesis, are functional and operational aspects, basic for music and language in general. But repetition and its economies are not music and language themselves: it is necessary therefore to consider aspects such as context, intentionality, idiosyncrasy and idiolectal traits, in order to obtain a more complete picture of music as language.

Mandelbrot's statement "only fractal sound sequences can be perceived as musical" is inaccurate and excessive: a pure $1/f$ noise sample, even if considered as 'beautiful' in a subjective experience, should not necessarily be recognized as 'music' within any cultural context. Obviously, music appreciation requires the concerted participation of culture, including relationships between collective assessment, apprenticeship, and individual appropriation.

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[383] See subchapter 5.1.

Correspondingly, a cultural contextualization of $1/f^0$ or $1/f^2$ noises, or of any form of non-self-similar noise, can be introduced within a musical idiom as filter, synthesis, generative algorithm, or instrumental resonance, which, in fact, occurs in a variety of musical instruments with residual noise; in the listening experience in a concert hall with a public; as well as in the human voice.[384] In agreement with Hegarty (2007), in all human manifestations, noise, like silence, can have or not aesthetic and musical attributes in different human communities.

Mandelbrot's quotation (2002:193) ends with a major misunderstanding, asserting that "fractality lies more deeply than musicality because it does not distinguish good music from bad." This amounts to saying that geometry is superior to music "because it does not distinguish between good and bad." This criterion —with an almost religious intentionality—permits to trace within the Pythagorean doctrine, the ethical and aesthetic gaps of fractal theory, as anticipated in the Introduction. The alleged purity and neutrality that Mandelbrot assumes for fractal geometry, reminds us, at the same time, of the purist axiology in Plato's moral geometry (*Timaeus* and *Republic*), and Spinoza's *Ethics* (1677), which conceives the Euclidean argument as the *tabula rasa* for ethics and aesthetics. Mandelbrot implies that such 'neutrality' can serve to give music its site, regardless of culture and experiential individuation. No matter the interpretations or idiosyncratic discrepancies; the predictable order of data and codes, and the structural consistency under a deterministic plan, is the privileged issue.

The expression "fractality lies more deeply than musicality" reminds us also of the myth of *pure language*, which Eco (2003:173–175) refers to as an essential aspect of the Pentecost. In this myth—comparable to the modern myth of *fractal language*— an enlightened congregation receives the pure, universal language lost in Babel (see also Eco 1971). The analogous concept of *pure music*, defended by thinkers such as Vasconcelos (1951), and by musicians such as Stravinsky (see Maine 1922:93), is pondered by Dahlhaus (1978:7–23) who, first of all, understands it as *ästhetisches*

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[384] This ubiquity of noise in music draws the attention of Ingarden (1962:47): "Every individual musical work is a formation consisting of various phases, in which tonal or sonic qualities, as well as noise qualities, constitute the fundamental moments of the work."

Paradigma—precisely, a paradigmatic analogy. In this context, Ingarden (1962:12–22, 83–85) acknowledges a necessary coordination between 'objective' and 'subjective' values, evident in a breadth of basic musical aspects as well as in aesthetic values that are also present in literature, architecture, painting and film (see Ingarden, *op. cit.*). As 'pure' music, the alleged 'fractal' music becomes reduced in any case to an imaginary product from the human bias to abstract key references between model and comparison—again, a basic process of paradigmatic analogy, also directly related to Peircean *abduction* (Peirce *CP* 7:219).[385] Thus it is possible to imagine the circle or the perfect equilateral triangle, and the axioms that define them. However, the absolute perfection of these imaginary objects—such as the perfection of fractals—becomes irrelevant to their musical implementation as a language.[386]

A (chimerical) fractal language

There are essentially two ways of understanding fractal geometry as a language: as a prescriptive-descriptive system reading *nature*, and as a *pure language*, as mentioned in the previous section. The former is of an open kind—that is, it accepts unlimited modifications—according to what Jürgens, Peitgen and Saupe (1990) suggest by the metaphor "The Language of Fractals". The latter, in contrast, is of a closed kind—as it does not allow modifications or adaptations; instead of it, the fractal concept is placed in the centre of all references. In short, the first kind constitutes a system of analogies and comparisons with the *world*, whilst absolute 'fractality' is presumably intended to constitute the world.[387]

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[385] On the definition of paradigmatic analogy, see subchapter 3.1. On the 'central references' of cognitive processes, see Introduction (pages 16–17, Lakoff and Fauconnier's *mental spaces*). See also subchapters 2.1. and 3.5., on *the perceived* as grouped into categories that facilitate the classification of relations by affinity or equality. The adaptation of Peircean semiotics within this framework is explained in subchapters 3.8. and 3.9.

[386] It is necessary, however, to take into account what is discussed in subchapter 3.3., on the relation of actual and potential in a musical context; and in subchapter 2.3., on basic concepts of musical symmetry.

[387] Unfortunately, this notion was adopted by Devaney, at least in an informal statement (see Devaney 2004:39): "It now seems to be painfully obvious that just about everything around me—in nature as well as dynamical systems—is a subset of fractal geometry."

The study by Hsü (1993), on the "fractal geometry of music", suggest a neo-Darwinian evolution of music oriented by a fractal guideline.[388] Accordingly, Benny Shanon (1993:105–106) believes that fractal geometry of language is latent in the correlation of different physical layers: "Fractal structures are noted in natural language [...] patterns similar to those encountered in the physical domain are also noted in language." And Wu and Sun (2008:232) conclude that "it is very convenient and feasible to use the fractal language to describe the rich and colorful appearance of nature." Roughly speaking, this corresponds with chapters 4–6 of this study, suggesting an empathy between musical form and (deep) content. However, fractal geometry *as pure language*—assuming that fractals constitute a language by themselves—creates too high expectations; e.g. Moisset de Espanés (1999:122) asserts that "Language generated by fractals allows us to think new thoughts and new ideas: complexity, infinity, growth, movement, expansion, articulation, totality, spatial richness, ambiguity." According to this belief, a sort of *newer* or *more perfect* musical language would be produced by simply converting the Mandelbrot set into sounding parameters. In practice, this is not the case even in Robert Greenhouse (e1995), Phil Thompson (e1997) or Paul Whalley's (e1999) attempts to create music implementing fractal algorithms.

In contrast, fractal geometry *as open language*, permits to consider that mathematical schemata are also product of human language, emotion, experience and culture, which, in search of 'new' possibilities, scroll through a whirling succession of models, trends and styles (suggested as Peircean *synechism* in subchapters 3.8.–3.9. and 4.7.–4.8.). From this perspective, what generates the *languages of self-similarity* is not fractals, but a much broader framework in relation to a symbolic, dynamic context. Thus, musical self-similarity is not *the* meaning of any musical expression by itself; rather, musical self-similarity is a sign of *search-for-searching*, i.e. building coherence through relationships of self-reference, recursion and synecdochic intersemiosis. Roads (1999:879) offers some valuable clues about the nature of this coherence, as a "probabilistic tendency" in an open system that is influenced by culture, idiolect, perception and emotion. Equally, Amozurrutia

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[388] See especially Hsü, 1993:34–38.

(1997:14) acknowledges that the apparently new fractal geometry, for its better understanding, must be valued and compared with the ancient myths and their implicated intersemiosis.[389]

Dodge and Bahn's (1986:185) assertion, on that "mathematical formulas can produce musical as well as graphic fractals", transmits the enthusiasm of a discovery, but lacks the rigor to face the consequences of an alleged *pure music*.[390] For the present study it is quite clear that a mathematical formula does not suffice to create music—and would not anyway constitute an intersemiosis between mathematics and music, but a mere homology. Instead, it seems more appropriate to note that the fractal equations and graphics contribute to the theory and practice of music, as do other tools of mathematics. According to what Borthwick (2000:662) observes in his critical review of Madden (1999),

> [T]he metaphorical use of mathematics must once again be stressed. Although there is nothing in principle to prevent any kind of mathematics from being used to generate some sort of music, very little music (if any) that currently exists can be explained entirely in terms of mathematics. The presence of the culturally-situated mind guarantees this non-commutative relationship.

Koblyakov (1995:297) concurs with this criticism, noting that, regarding music, "methods of mathematics state only the result, fixing its quantitative side. [...] But modern music theory does not either describe the sense and semantics of these processes [of musical self-similarity]". Hsü and Hsü (1991), at least partially aware of this complexity, use a descriptive subtlety when referring to "$1/f$ noise called music": they do not make an equivalence between $1/f$ fractal spectrum, and music, in its most general sense. They do not assume that $1/f$ is equivalent to music, but conceive it as an analogy. Without paying sufficient attention to this subtlety, Bigerelle and Iost (2000) assert that "fractal music can be created using this fractal spectra: $1/f$ spectrum is generated and the audio signal is obtained using the Inverse Fast Fourier

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[389] Amozurrutia (*ibid.*) notes that "The conscious use of myths and fractals allows for a broader dialogue between two or more creators or 'doers', a broader conjunction and harmony of the contents, whatever their field of work or instrument of creation, and therefore a greater impact on the attentive spectator." This interdiscipline involves the theories of self-similarity and intersemiotic translation, addressed during the present study.

[390] This concerns the recent discussion, on pages 306–307.

Transform". This is correct, except that the spectrum of $1/f$ noise or other fractional noises related, do not constitute music: they do not create, truly, a musical language. A musical language—unlike $1/f$ spectrum or a fractal object—does not require much of automated-iterative generation, as emphatically do cycles of interpretation with deviations and corrections in different functional layers, as explained in subchapters 4.7. and 4.8.

Therefore, the rigor of the *fractal music* concept is trivial, for there is not any music able to be associated with the strict definition of fractal (see section 1.3.4.). This observation also encompasses Amiot's (2008:158) "musically interesting notion of autosimilarity, just like the famous fractals". Such a notion can only be obtained by Peircean abduction and cognitive domain criss-crossing. So, music is never "just like the famous fractals".

The equation proposed by Miramontes (1999:10) is also subject to such condition:

$$g\left(bf\right)\frac{1}{\left(bf\right)^{a}} \;=\; \frac{1}{b^{a}}\frac{1}{f^{a}} \;=\; \frac{1}{b^{a}}\,g\left(f\right)$$

Inspired on Schroeder's (1991:96–99) formalization, this equation is based on the fact that power spectral density, directly proportional to $1/f^{\beta}$, can reproduce the same distribution of information by frequency doubling (something related to the scale invariance property of the Weierstrass function, mentioned in subchapter 5.3.).[391] According to Miramontes (1999:10):

> Multiplying the abscissæ by b is equivalent to stretching them by this factor. The previous equalities [see equation above] is telling us that, if we stretch the ordinate by a factor $1/b^{a}$, then the plotting of the function $g\ (f)$ looks exactly like that of $g\ (bf)$. This, however simple it may seem, is extraordinary. Since we have time represented in the horizontal axis, and the vertical axis represents pitch, then, if we play a disc at twice its normal speed (b = ½) enough to double up the volume ($1/b$ = $1/1/2$ = 2) to hear exactly the same music! The musical pieces having this virtue are called *fractal songs*.

However, what Miramontes calls 'fractal songs' is limited to $1/f$ noise in an electronic emulation. In practice, singing or playing on the piano a translation of this form of

[391] See pages 253–254.

noise does not make a 'musical fractal'. If it was, it would be the articulation of a musical idea from an imaginary fractal. This happens also when playing Roger N. Shepard's (1964) 'infinite scale' on a musical instrument.[392] The ultimate sense of this act is oriented, in any case, by the sorts of negotiation explained in the previous subchapter (5.4.): a negotiation in which the absolute determinism of fractals is subordinate to intuitive and aesthetic priorities. Thus the fractal object, in synecdochic intersemiosis, becomes musical analogy; or "poetic metaphor" according to Borthwick (2000:662):

> Self-similarity—the property of objects to contain copies of themselves, just as a cauliflower head contains a number of self-similar florets—provides a useful mathematical tool for analysing the relationships between structural levels conceived in Schenkerian terms. However, whether this amounts to much more than the kind of poetic metaphor familiar to nineteenth-century musicologists immersed in a world evolutionary theory, flora and fauna remains to be seen. […] The intervention of the human mind can always disrupt a pattern that nature itself is often powerless to resist.

Echoing this critical view, the two-dimensional pictures representing a fractal in a book or a screen are rather analogies promoted by a necessity—shared by physiology and culture—of stereotyping (see Popper 1979, 1988). Such pictures are aesthetic descriptions, in the sense that they are meant to be perceived as an imaginary representation of an elusive, although conjecturable object.[393] The "magnificent computer-generated images" (Steinitz 1996a:14) considered as fractals, are not actually fractals, as the drawing of a sphere is not the sphere, but just a representation of a geometric concept. The specific case of Steinitz's (1996a–b) analysis of the music of Ligeti, is an example among others in which the word fractal occupies a space that should be fulfilled by the concepts of self-similarity and synecdochic intersemiosis,

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[392] See example ◊642*a*, page 412.

[393] This idea corresponds to the notion of *picture* or *spatial imitation* that Foucault (1966:19–22) points out in his operative definition of similarity. Congruently, this concept of *picture* must be taken in a philosophical sense, as found in *Première communion de jeunes filles chlorotiques par un temps de neige* (1893), by Alphonse Allais (1854–1905), which graphically represents—with a blank paper—a piece of the culture of the viewer through her/his own expectations. This is the case, as well, with René Magritte's (1898–1967) painting *La trahison des images* (1929), with the inscription "Ceci n'est pas une pipe". Magritte himself launches the stereotypical mental operation of self-similarity, explicitly, with a later painting: *Les deux mystères* (1966). This issue is also analyzed in detail by Foucault (1973).

much more relevant to the case study. It is clear that in this example, as in any other similar cases concerning musical analysis, the concept of self-similar object or process is not equivalent to that of fractal. From this viewpoint, it is interesting that David Lewin (1987) was sufficiently careful to avoid associating his transformational theory, with fractals—even though he widely uses mathematical terminology, intimately linked to the study of self-similar relationships (e.g. nesting, recursion, invariance, and mapping from a set to itself).

However, Steinitz's (1996a–b) descriptive research on Ligeti's music particularly contributes to unveiling an idealism in which mathematics, *science* and *art*, replace the magic values of an allegorical tradition.[394] There is no sufficiently clear difference between this modern exercise of musical imagination, in respect of the Pythagorean doctrine and ancient numerology. "The unity of science and art" (Steinitz, *op. cit.*:14) finds, for the same reason, a functional obstruction: *scientific* determinism is of a different quality than *artistic* determinism (see Backus 1960, Borthwick 2000). The former looks for absolute proofs and formulates universal laws; for the second the search, and not the definitive finding, is the essential factor. Indeterminism in science can—and often must—be reduced to a minimum. For music, instead, indeterminism is necessary, desirable and useful: "musical practice shows [that] deviations are accepted and desired expressive properties of music" (Knopoff and Hutchinson 1981:19).[395]

Music requires the establishment of rules, usages and hierarchies, in a similar way to grammars for languages. But it is impossible to achieve this through comprehensive deterministic procedures, as with physics. Langer (1953:105–107) states—this time in agreement with current theories on music cognition—that music cannot abandon the sensible and immediate, ignoring its many aspects of perception and performance. Musical sound, silence, loudness, pulsations, countings, intervals in

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[394] This view coincides with the anthropological notion of language as "magic power", present throughout the investigations of Lévi-Strauss (1964, 1971). It is found also, mixed with music and song, in Carpentier's 'musicological novel' *The Lost Steps* (*Los pasos perdidos*, 1953).

[395] A brief outline of the history of this concept in Western cultures should also include the arguments of Meyer (1956:202), on the function of the "artistic deviation from the pure, the true, the exact…", and the concepts of Ingarden (1962:13–15) on the relevance of the *differences* in the interpretation of a 'musical work'.

general, have fuzzy edges and, at the same time, characteristic thresholds. Unlike general physics or fractal objects, musical practice does not have qualities that are not available to or connected to the senses and intuition. In this context Meyer (1967:246) notes that:

> It is an inexcusable error to equate acoustical phenomena with qualitative experiences. The former are abstract scientific concepts, the latter are psychological representations. One measures frequency [for example] but one perceives pitch.

Meyer (*ibid.*) particularly targets fetishism of the *concept-in-itself*, as a tautology that overshadows its musical significance. But, on the other hand, restricting music only to what is immediately or easily apparent, brings an incomplete picture of music and the human experiencing involved. Subchapter 3.3. of this study explains how finite and infinite are not necessarily opposed or in contradiction, within a same musical context.[396] Similarly—as stated in the Introduction, theory and imagination, schemata and practice, are not oppositions that require to be in irreconcilable conflict. In Meyer's (1967:246) conceptualizing, neither the "abstract scientific concepts" nor their "psychological representations" are total oppositions rejecting any possible coexistence or cooperation. Nor do their mutual association constitute "an inexcusable error", but a probability's *error* depending on the descriptive or prescriptive system adopted. In short—and contradicting Meyer—psychological representations can lead to scientific concepts, and scientific concepts can lead to psychological representations. They are, indeed, functionally interrelated. As the very qualities of music, neither do they consist of an absolute self-reference, nor are they a set of objective values that can fully explain their own relationships. For this they need unstable interpretation—such as language and aesthetic manifestations do—through exchange between idiosyncrasies and cultures *in fertile opposition*, i.e. within a creative contrast.

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[396] See pages 66–69.

Relativity of the self-reference

Unlike a fractal abstraction, musical self-referential systems are never completely devoid of correlation and influence to and from other systems. This is clear, for example, for the idiosyncratic developments and the contacts between idiolectal and ecolectal strata.[397] Similarly to what happens in speech, music's self-similar features are subject to a condition of functional openness.[398] In music as in language, an ontology that is *absolutely significant in-itself*, escapes to the elementary relations of music and language. The following quotation from Rhees (1968:274), that includes notions of self-reference, measurement and consistency—common to music and language—helps to explain the principle of 'non-absolute self-referentiality':

> I could not add [say]: "If I *say* there is such a function—if I write this down—then there must *be* such a function, for I have written it as a paradigm in my expression." The paradigm shows what I am saying. But here I seem to be treating it both as a paradigm and as something for which it is a paradigm. It is like saying that the presence of the standard meter rod in Paris shows that there is something which is exactly one meter long. It is the same with confusions about the grammar of ostensive definition when we speak of private experiences and sense data.

This quotation is especially meaningful in that—in contrast to the operation of analogy as proportion—it clarifies the operation mode of analogy as paradigm: Rhees provides the case of writing a mathematical function. The same can be done, however, writing a melody or a chord sequence in a score, followed by its interpretation with a musical instrument. In this operation the elaboration of a paradigm prevails as an automatic stereotype of a cognitive system (see Givón 2002:40–41). Such a paradigm consists of an imaginary, rigid model—ideally perfect—on which processes derived from conjecture and interpretation are developed. In this fashion, paraphrasing Ockham (see Leff 1975:355), practical conclusions can be resolved into speculative principles.[399] Conjectures that are

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[397] This notion complements what is stated in subchapter 5.1. on the *always rich source* of music as a system of probabilities.

[398] Such condition of 'openness' is explained in subchapter 3.7. as a condition for creative recursion.

[399] See page 326.

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generalized from the use of a paradigm, and in general, the processes involving Peircean abduction as the intuition of self-similarity, heavily depend on this notion.[400]

Avoiding the fallacy of a 'fractal music'—comparable to the fallacy of the standard measurement that Rhees notes in the quote above—Beran (2004:92) states that "In music, the idea of fractals was used by some contemporary composers, though mainly as a conceptual inspiration rather than an exact algorithm (e.g. Harri Vuori, György Ligeti)". Beran consequently offers a methodology to pay more attention to stochastic self-similarity relations within music, than to fractal objects.[401]

The characterization of the (mis)called 'fractal music' as descriptive music, matches what Ingarden (1962:51) identifies as 'pure music' with that which cannot lead to anything but "essentially musical". Since certain relationships are fundamental to fractal geometry, and cannot be introduced directly into music—strict scaling and absolute-simultaneous self-reference between detail and wholeness; and since for music there are essential relations that do not necessarily transpose into fractal geometry—e.g. feelings and memories directly associated with sound, then any alleged 'fractal music' serves as descriptive music or *program music*: as analogy or metaphor of an object taken for intersemiotic translation, in which necessarily something essential "is lost" or "altogether transformed"; although "something remains" (see Jakobson 1980:89–90).[402] In this sense, Stockhausen's (see Felder 1977:89) observations on intersemiotic translation seem relevant:

> Xenakis is a complete latecomer as a musician, and in a true sense he is no musician at all—I really doubt what he can hear, not only with the inner ear. Nevertheless, he is able to contribute something we find in all the sciences and arts; by transposition from one field to another, you transpose something from architecture to music, you learn the respective parameters, the limits of the instruments, and translate points on the paper to sound. Certainly something interesting comes out of it. If the method is quite unusual in the field of music you can be sure that something new, something that hasn't been done the same way before, will occur.

Regarding the concept of 'pure music', defended by Ingarden (1962:48–51), the same principle of 'non-absolute self-reference' is applicable. From this view, 'pure

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[400] On the concept of *abduction*, formulated by Ch. S. Peirce, see pages 23 and 459.
[401] See section 'Fractal dimension: An issue of pertinence', pages 300–303.
[402] For a discussion of this quotation from Jakobson, see pages 104–105.

music' is fallacious just as 'fractal music' is fallacious: both can be explained as imaginary models, paradigmatic and cultural sources within a mental space adapted for musical elaborations—that, *sensu stricto*, are neither 'pure', nor they are 'fractal'. Ingarden's (*op. cit.*) arguments on a pure music, are based on the Cartesian ideal of separation between mind and brain, characterized as the divorce between "real world" (*realen Welt*) and "not belonging to this world" (*Nicht-zu-dieser-Welt-Gehörens*). Ingarden (*ibid.*) believes that the "connection" with the real world is only possible through the tangible and spatial qualities of matter: "The connection of the architectural work with the real world is much more profound [in comparison to music]; it follows from its 'inner' (essential) structure. It has, for example, 'foundations' that connect it with the ground".[403] This argument is untenable: it is clear that music, existing as sound, has physical 'foundations' similar to those of a building, subject to the same power laws.[404] Neither can music from this perspective be an absolute self-reference, but a self-reference in context.

Prolongation of structuralism

Among the prominent features identifying the structuralist thought are the binary oppositions, that—as in De Saussure's doctrine—serve to characterize a system of meaning construction. Such a binarism still exists in the deterministic-rationalist discourse on self-similarity, as well as in its conceptual apparatus, supporting its function by pairs: fractal set/natural fractal, deterministic fractal/stochastic fractal, chaotic process/random process, real infinite/potential infinite, and—as pragmatic implementation—digital/analogous. The Cartesian opposition infinite/indefinite can be added to make even more obvious the links of rationalism with structuralist binarism. Nevertheless, these hinges can simply be surpassed by the interpretation that some of these concepts are not true oppositions, but rather partial states of an epistemological continuity, through systems of analogies (as suggested by the IC theory, introduced in sections 3.8.1. to 3.8.3.).

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[403] In the original text (Ingarden, 1962:49): "Der Zusammenhang des architektonischen Werkes mit der realen Welt ist ein viel tiefergreifender, es fließt aus seinem „inneren" (wesensmäßigen) Bau. Es hat z.B. „Fundamente", die es mit dem Boden verbinden."

[404] See section 3.9.5., pages 122–124.

The continuity of concepts such as *stochastic fractal–deterministic fractal* or *random process–chaotic process*, contributes to understanding that there are gradations in the construction of knowledge about a dynamical system behaviour, insofar as it constitutes a self-sufficient metalanguage that describes this relationship. Poincaré (1886/1956:1380) characterizes chance as "only the measure of our ignorance". The discrepancy between order and chaos, and between similarity and difference, is attached to this notion.

On the other hand, as Campbell (1982:219–220) notes, human knowledge requires a structure to be developed and transformed. If the primary system making knowledge possible (the brain and nervous system in general, assumed within its *Umwelt*-niche contextualization) has the characteristics of a specific structure, it is not surprising that knowledge itself also exhibits the characteristics of a related, consistent structure—at least in a primitive stage of consciousness, during the formation and linkage of ideas and insights. Therefore it is doubtful that post-structuralism has made a substantial progress in understanding the configuration of language (and music), regardless of structure. The most significant advance of post-structuralism lies rather in its ability to assimilate, within the structure, the transformational vulnerability of the structure itself.

In short, modern analytical thinking, but also the creative process that involve the usage of codes and message processing, are linked to the structure, as the message is linked to the form.[405] Consequently, both the discourse on musical self-similarity (including its terminology), and attitudes regarding music as a self-similar process (including its methodology), are extensions of structuralism.

Taking into account the arguments discussed in this study, one may conclude that music cannot prescind from structure. But music is able to moderate the structure for a more complete understanding of music as a 'living language', constantly changing. This implies that there is no a *pure* structure containing, like a

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[405] For an introduction on this concept, see subchapter 3.5.

perfect seed, the quiddity of the musical substance.[406] Karl R. Popper's (1988:127) argumentation against Laplacian determinism must, therefore, be considered:

> [T]here is every reason to regard man at least partly free. The opposite view—that of Laplace—leads to predestination. It leads to the view that billions of years ago, the elementary particles of [a first] World contained the poetry of Homer, the philosophy of Plato, and the symphonies of Beethoven as a seed contains a plant.

Rather, there are fixed structures, humanly conceived as deterministic forms, around to which analogies continuously vary. The continuous interaction between these forms and their relatively flexible analogies produces the *minimum instability* necessary for the multiplicity of interpretations and—finally—for removing and replacing the so-called fixed structures. This interaction allows us to recognize, within the notions of self-similarity and self-structuring, a renewal in which the iteration of simple relations can trigger an immense variety of structures. From this perspective, the self-similar musical discourse is an open and indeterminate form, flexible within the openness that allows its variety. This holds true especially if we accept that music self-similarity—embedded in synecdochic semiosis *in general*—tends to the plurality of forms.[407]

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[406] This notion contrasts with Goethe's conceptualization of the 'Primeval Plant' as "the most wonderful creature in the world" (see page 432). Thus, such an *Urpflanze* has a potential meaning as musical implementation (here the superlative determiner 'most' is rather undetermined, with a metaphorical function). Such a plant would be, in any case, an operative model: the paradigm that allows establishment of a starting point for understanding, as a system of references.

[407] Ingarden (1962:217) conceptualizes that, within the intersemiotic relations of the *arts*, a system of akin relationships amidst diversity (with its characteristic plurality) is much more relevant, rather than a set of processes or objects perceived by their similarly and equality.

Self-similarity as proportion

This chapter's main goal is to analyze the major implications of what Prusinkiewicz (1986, 1992) identifies in terms of proportion in self-organizing systems—the *golden ratio* in particular—as self-referential relationships. For this purpose the notions of *point, line*, and *mass* are revised, together with their constructive usage.[408]

Prusinkiewicz (*op. cit.*) and Prusinkiewicz and Lindenmayer (1990) consider the concept of proportion as self-similarity nesting. They propose that the recursion of a set of axioms and simple rules, including basic rules of proportion, can generate medium to long term highly complex self-similar structures, analogous to self organized structures observed in natural phenomena. Studying statistical correlations in a variety of chemical and biological processes, Mandelbrot (1977, 1982) notes the close link between self-organization and structural economy based on the recursion of proportional relationships. Employing statistical approximation to describe these relationships, Mandelbrot formulates the concept of *fractal dimension*, which represents the set that it refers to as a logarithmic characterization.[409] The proportion $^{\log 4}/_{\log 3}$, for example, represents Koch's curve (see ◊333) consistency. In short, a proportion implies the consistency of a relationship into another one. Accordingly, the concept of proportion is extremely useful for the intuitive approach to self-similarity as consistency of relationships in aesthetic phenomena.[410]

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[408] This subchapter considers the distinction made in subchapter 5.4., between *set* and *group*: the former made of *elements*, and the latter made of *relations*. However, this distinction is attenuated in a general context of analogies. For example, Stockhausen (1989:38,48) does not emphasize this distinction, but assumes a direct analogy between set and group.

[409] On the concept of 'fractal dimension', see the independent section on pages 300–303.

[410] Bigerelle and Iost (2000:2191) come to the astonishing conclusion that "the fractal dimension of a randomly construct triadic Cantor set is equal to the Golden Mean". This notion could contribute to a deeper explanation of the relationship between stochastic distribution and

In general terms, structural proportion operates as synecdoche, provided that the part can be defined or recognized in relation to the whole. For instance, the *golden segment*—explained at 4.3.—only makes sense for its relation to a larger body of geometry, with which it forms its distinctive proportion. As defined in 3.1., in the strict sense of the term ἀναλογία, proportion means analogy. Szabó (1978:151) also notes that the word ἀνάλογον refers to equality of arithmetic ratios. In this context the term λόγος does not refer so much to the concept of *word*, but it refers more exactly to the concept of *ratio*—as it is found in the origin of the term *logarithm*. It is also noteworthy that in this case the preposition ἀνά means *equality*. Ἀναλογία, translated into Latin as *proportio*, is nothing else but the equality of numerical ratios or geometric proportions (see Szabó, *ibid.*).

Proportion as aesthetic trend

The proportion $a : b = \varphi$ is one of the best characterized phenomena in *natural geometry*, because of the universality of its ratio as $^a/_b = \varphi$ ($\approx 1.6180339...$), and is found as general trend, for instance, in the anatomy of animals—including humans; as well as in many plants, especially in a large variety of phanerogams. The golden ratio appears in an enormous diversity of human expressions—in conscious or unconscious usages—through many forms of written language, visual design, painting, sculpture, architecture and music. Most of authors who have studied this topic, including Ghyka (1927, 1931), Huntley (1970), Doczi (1981), Schroeder (1991) and Livio (2002), also explain aspects of structural proportion in terms of characteristic constructivism in organic and inorganic forms, and even as a core issue in crystallography, after one of the first modern treatises on the subject, published by J.D. Bernal (1926).

According to the classical literature on architecture and design, from ancient Greece to Le Corbusier (1950, 1955), the balanced proportion of the forms is the means to achieve harmony in space and in the human use of such space. As Borwick (1925:12) notes, "The word proportion [...] speaks of measure and fair spacing, of

Golden ratio, as an aesthetic trait of a universe of musical objects and processes, as outlined in this chapter.

order, correspondence, harmony, of the relation of part to part." Moreover, Borwick suggests that the notion of rhythm depends on a certain proportion in time: "Rhythm can come to being only if time-values are absolute and time-elements strictly related and proportioned" (*op. cit.*:13). Stockhausen (1989:37) acknowledges that for music as for the general relationships between time and space, proportion generates the notion of *series* as basic principle of constructivism:

> [A] lot could be said about relationships between our [musical] work and the Modulor of Le Corbusier, who tried to lay the foundation of a new method of architecture based on the blue and red series of proportional measures. The word 'series' in a context of structural design comes up again in architecture and other fields of constructivist art.

The constructive implementation of a harmonic series—as happen for instance with the serial techniques of music composition—co-participates in the development of a prescriptive grammar and, consequently, of a grammatical logic. The logic of series and proportions is connected to the notion of musical grammar. As Beran (2004:1) notes, "*logical* construction is an inherent part of composition. For instance, the forms of sonata and symphony were developed based on reflections about well balanced proportions." From a statistical perspective, this notion corresponds with the traditional concept of harmony as accepted by Copland (1939:50): "A beautiful melody, like a piece of music in its entirety, should be of satisfying proportions."

The question here is how the concepts of harmony and proportion can be mutually related within such different worlds, one in the light and space, as referred to by Ghyka (1927, 1931), Le Corbusier (1950, 1955) and Huntley (1970), and the other in sound and time, as referred to by Copland (1939), Stockhausen (1989) and Beran (2004). Borissavlievitch (1958) suggests that visual preference for golden ratio is due to the fact that the visual field is not parallel, but intersects around a golden segment. This does not explain, however, why the golden ratio is found analogously in music, in poetry or in drama.[411]

The answer, according to authors such as Huntley (1970), Hofstadter (1979, 1995) and Livio (2002), must be sought in the human cognitive qualities; in the

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[411] Howat (1977:292) suggests, for instance, that "The idea of important events being strategically placed with relation to the development of a plot and sub-plots is an underlying principle of any drama."

universal modes of cultural elaboration, and in the relationships between body and context.[412] A hypothetical cultural universalism of the golden ratio should be linked, for these reasons, to other cognitive principles, such as those allegedly found in the configuration of language from Zipf's law, as stated in the previous chapter. Conceiving the harmonic proportion and the golden ratio as self-referential phenomena, correlated to power laws and Zipf's law, it can be understood that such phenomena are part of human physiology, which is not isolated from other relationships of functional self-similarity and recursion, common to basic organic principles.

Obviously, proportion is not *all* in musical structure, neither in its formal precepts. By itself, there is no structural proportion that constitutes the sum of a musical tradition. On the contrary, traditions, in their diversity, adopt proportions as part of their methods for recreating musical and aesthetic relationships in a broad sense (i.e. not really in the 'absolute' form claimed by Borwick, 1925:13). This equally applies to verbal and written language: no synecdoche is enough to create literature or oral culture; rather formulae recursion in context, within which the synecdoche has a very important role, contribute to language consistency—but also alters such a consistency through diversity, according to what is explained in subchapter 4.8. This note is valid for almost all authors studying the significance of proportion in music. As Howat (1983b:21) admits:

> Proportional structure is only one of many ways of ensuring good formal balance, and even then only if it is well matched to the musical content; it could do little to help music that is deficient in its basic material or other forming processes.

The first modern authors that produced texts on golden ratio in music, include Webster (1950), Norden (1964, 1972), Nørgård (1970), Lendvai (1971) and Kramer (1973). They recover the Pythagorean notion of harmony without involving the concepts of self-similarity and self-reference. Thus, when the initial findings of authors such as Prusinkiewicz (1986), Hsü (1993) and Koblyakov (1995) were

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[412] This issue is discussed more extensively in other sections of the present study. See especially subchapters 3.6. and 4.6.

published, the association *proportion–self-referentiality–recursiveness–self-organization–self-similarity*, barely pointed to a general outline for a new theory.

Roads' (1999:878–893) treatise on electronic music includes a section that encompasses the concepts of *Markov chains, noise, fractals, chaos, self-similarity* and *grammar*; it omits, however, the concepts of *proportion, self-reference* and *Lindenmayer systems*. Beran's (2004) study on music statistics does not include the concept of proportion; nor does it offer anything particular on Lindenmayer systems. This omission also occurs in Yadegari's (2004) dissertation. Probably the first text that stresses the musical connection between golden ratio, recursiveness and self-similarity, is the corresponding chapter in Madden (2007:67–96); however, it does not devote special attention to Lindenmayer systems either, ignoring their ties with systems of constructive self-similarity.[413]

The relationship between proportion and structural self-similarity, as can be seen in this introduction, are still far from being fully covered by musicology, in what is a relatively new field of study with many aspects open to exploration and discussion. This chapter provides only a summary of the current thinking on the issue, as well as some progress and possible directions for further study.

Proportion as mediation

Ghyka (1927), when defining the concept of proportion at the beginning of his *Esthétique des proportions* (1927), recalls Plato's *Timaeus*: "two things cannot be rightly put together without a third; there must be some bond union between them. And the fairest bond is that which makes the most complete fusion of itself and the things which it combines; and proportion is best adapted to effect such a union." This definition is similar in substance to the definition of translation that Ricoeur makes (2004:14) on the figure of *mediation*: "les deux textes de départ et d'arrivée devraient, dans une bonne traduction, être mesurés par un troisième texte". Benjamin

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[413] In contrast, the first work concentrating on musical Lindenmayer systems, by Manousakis (2006), focuses on their operational implementation, without going deeper into the relationship between music grammar and aesthetics, and without a general theory on musical self-similarity. Manousakis' objectives are, thus, the 'engineering' of music and the 'resolution' of problems, as outlined in the Introduction of the present study.

(1923/2000: 82–83) calls this mediation as "interlinear version" between two texts or between two symbols; he compares such a mediation with an "infinitely small point" in which two diverging symbols make contact; and he concludes that "Where a text is identical with truth or dogma, the text is unconditionally translatable." Proportion, as translation and analogy, depends on the consistency of one thing into another: this is the elementary connection between *firstness* and *secondness*, that results into generalized derivation or abstraction as *thirdness*, according to Peircean semiotics (Peirce 1903b, 1903d).[414]

In short, proportion amounts to an intrinsic combination of a whole for its parts. This description makes clear that proportion is a form of self-reference—of one thing mirrored by another. Ghyka (1927) analyses the relationships between points in a line that bring them together. Using this idea he defines the line itself and the concepts of continuous and discontinuous. The 'link' of this rule should be, according to Ghyka, a line, whilst the 'things'—the minimal ones—are points. Such a relationship is ultimately characterized, as observed by Wittgenstein (1953), as a 'game' of representations; or as suggested by Bateson (1972), Lakoff and Johnson (1980), and Fauconnier (1985, 1997), as an environmental system of mappings of the mind/brain, embedded in culture. In music, such a system provides both, 'representational games' and 'environmental mappings' for knowledge and reality creation. The latter is related to the general notion of 'what happens' (see Damásio 2000) between references, as Stockhausen (1989:38,48) notes:

> I use the words 'point', 'group', and 'mass' in order to generalize what is happening in music, and to make it clear that each is a particular manifestation of a larger trend. [...] In most of my works I have composed points in a determinate way, or groups, or masses. What does that mean? It means that one can hear very clearly the intervals which make the proportions, the duration of the individual points, the shapes of the individual groups and masses.

Selecting the notions of point, group and proportion, one following the other, is part of an analytical conception of space, common to geometry, architecture, the visual arts and music. When talking about these concepts, Stockhausen perceives the idea

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[414] See section 3.8.3.

of self-similarity by referring to consistency as a particular tendency within "a larger trend".

The notions of *point, group* and *mass*, listed in that order by Stockhausen, also correspond to cognitive foundations associated with the basic abstractions of *one, various* and *many*, present in all human languages (see Sapir 1921, Swadesh 1966, Gelman and Brenneman 1994). On these simple notions more complex concepts are progressively developed. In music there are many examples of this development, for example, between tone, chord and noise; or between pulse, rhythm and texture; or between acoustic grain, clustering and cloud—Xenakis (1963/1992:12) and Roads (2004:14–16) use the concept of cloud in this context. In all these cases one can find modalities of proportion having a function of mediation, e.g. in a chord, a rhythm, a motif, a melody, a pitch interval, a sound texture, or a musical form.

6.1. Dot and plot: basic analogies

The basic statement on the point as a representation of the elementary sound particle is made in this investigation in subchapter 4.1. What is said there must be taken into account in the context of the present subchapter. In particular, this subchapter emphasizes the analogous nature of the concepts of *point* and *set of points*.

The formalizing of the debate about the quiddity of the point is at least as old as the Pythagoreans, and evolves into a first period of discussions after Euclid's *Elements*, whose initial definition (ὅροι) in Book I states that "A point is that which has no part" (Σημεῖόν ἐστιν, οὗ μέρος οὐθέν.). Also in the *Elements*, the fifth of the 'common notions' (κοιναὶ ἔννοιαι) asserts that "the whole is bigger than the part" (τὸ ὅλον τοῦ μέρους μεῖζόν [ἐστιν].)

The controversy over the Euclidean notions of *point* as undivided integrity, and of *total* as parts' wholeness, reached its peak in the treatises of medieval philosophers. Roger Bacon, in *Opus Majus* (1267, part V), under the aegis of the *Elements* and the Platonic tradition, denies the infinity of an infinite atomic universe, arguing that the acceptance of an infinity of successive smaller scales leads to the assertion that the part is greater than the whole—which goes against the basic intuition of space.

However William of Ockham, in his *Questiones in quator libros sententiarum* (Lyons 1495) reveals that:

> It is not incompatible that the part is equal or not minor than its whole; this happens every time that a part of the whole is infinite. This is verifiable also in a discrete quantity or in any multiplicity whose part has units not minor than those contained in the whole. So in the whole Universe there are not more points than in a bean, as a bean is made of infinite parts. So the principle that the whole is greater than its parts is valid only for the things composed of finite integral parts.

This reasoning, which transcends Platonic atomism that indicates the existence of an indivisible minimal unit from which the universe is formed,[415] settles the idea of an endless universe, both in the colossal as in the smaller, at scales in which, the more the distance extends between its edges, the more evident its correspondences and similarities. This idea can be traced in the history of thought, and specifically in the history of the concept of self-similarity, abridged here in the Introduction, and explicitly defended by thinkers as diverse as Swedenborg (1734), Parsegian (1968) or Mandelbrot (1982).

This debate cannot be ignored in music, especially because of its relationship with the theory of proportions. The quest for universal harmony thus becomes a central issue for theorists who seek a logical and constructive dialogue between the parts and the whole. In this context, Jacob of Liège (*ca*.1340) combines Pythagorean harmony with its application to Aristotelian-Ptolemaic stellar mechanics, in order to explain the harmonic proportions of the monochord; and Johannes Kepler (1619) arrives at the study of the Platonic solids' proportions—placing them one within another—to make a descriptive analogy of what he considers a set of proportions, common to music and to the distribution of the celestial bodies.

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[415] Notice that the precise word that Euclid uses to refer to 'point', is σημεῖον, actually related to the concepts of 'sign' or 'trace'. The ideal 'minimal point' or *atom* in Aristotelian scholasticism is equal to such an indivisible minimal unity, a 'milestone' (Demosthenes 932.14) from which the universe is formed.
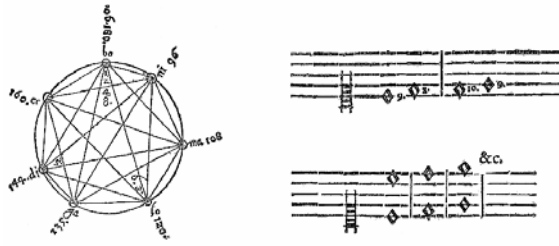
◊610. (Above). Proportion between dissonance and consonance in Jacob of Liège's treatise, *Speculum Musicae* (*ca*.1340), characterizing ratios and distributions in a circular orbit, followed on the right by their musical representation. This approach was made based on the study of the monochord's seven harmonic intervals.

(Below). Harmonic proportions of the celestial bodies as musical symbolization, according to Kepler's treatise *Harmonice Mundi* (1619), including Saturn, Jupiter, Mars, Earth, Venus, Mercury and the Moon (the Latin utterance *Hic locum habet etiam*, means that the Moon always 'has the same position').
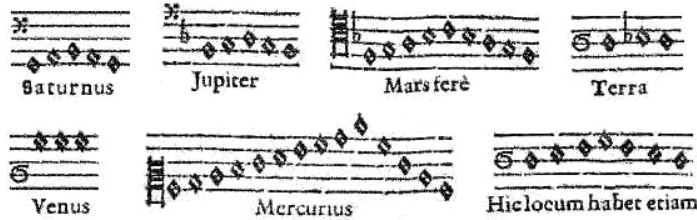
In the heyday of rationalism, Leibniz's monadology (1714), centred on a theory of maximum and minimum, provides the basis for the modern calculus of variations in a stable system. In this doctrine, monads are nothing but the characteristic essences of a set in equilibrium, between the homogeneous and heterogeneous. What Leibniz proposes, in short, is the estimate of the divergent properties of a system, through the general features of the whole in which they are embedded. Gustav Fechner (1801–1887) carries this principle to the realm of aesthetics, although without formalizing its study e.g. by investigating the nature of the repetitions, rhythms and proportions in an aesthetic system (see Fechner 1876). Ossowski (1966/1978: 60) claims the failure of this theory:

The old principle which Fechner, following in the steps of Aristotle, Descartes, Hemsterhuis and Leibniz, called the principle of joining divergences into unity (*Prinzip der einheitlichen Verknüpfung des Mannigfaltigen*), a principle which has become widespread in many versions in all of aesthetic literature, does not provide any criteria in its generalized formulation which would make it possible to test it empirically.

This methodological objection against Fechner will be discussed few paragraphs below. For now, it is necessary to highlight that Platonic atomism gained credibility not only with Leibniz's monadology, but especially with Kant's observation on that "Every composite substance in the world consists of simple parts".[416] This statement consolidates the analytical (mis)belief that "there exists nothing that is not either itself simple, or composed of simple parts" (*ibid.*). However, Kant himself acknowledges that 'parts' cannot exist out of a self-referential space: "space does not consist of simple parts, but of spaces" (*ibid.*). How to explain, then, consistency between part and whole, and coordination between simplicity and complexity, that seem pervasive in the universe?

Actualizing Platonic-Kantian atomism, Russell (1900) rejects the logic of the monads, judging that Leibniz' theory is grounded in the wrong idea that reality is a property attributed to a substance. Russell argues—in a critique that recalls Voltaire's objections against monadology—that Leibniz' doctrine leads to an absurd idealism. In such a way Russell formulates his theory of "external relationships" with which he tries, unsuccessfully, to sustain a claim about the pluralism of things, and about the Kantian assertion that wholes consist of relationships between simple things. From this was born the first phase of Russell's philosophy called "logical atomism", which eventually overextended its assumptions about language generalizations.

The next *tour de force* in this account is the emergence of fractal geometry,[417] a new paradigm that facilitates the conception of a specific identity for each point of an

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[416] *Kritik der reinen Vernunft* (1781), "Zweite Abteilung. Die transzendentale Dialektik" (Zweites Buch. 2: Zweiter Widerstreit der transzendentalen Ideen).

[417] This summary does not diminish the relevance of what Searle (1972) calls "Chomskyan revolution", referring to the paradigm shift in the study of language. What is at issue in this summary is to characterize the larger steps towards documenting a theory that involves the basic notions of proportion and self-similarity, and its role in music. On the influence of Chomskyan models in musicology, see subchapter 4.4. For exhaustive analysis on this subject, see Lerdahl and Jackendoff (1983).

infinite set, instead of assuming an abstract minimum or *simple particle*, as in Leibniz and Kant-Russell's different versions of atomism. This perspective reopens the discussion on the alleged elementary constituent of a self-referring set, vaguely suggested by the medieval speculations on the correlation between infinite scales, and somehow recovered by Fechner's principle. On the one hand, fractal geometry *makes real* Ockham's speculation, and transcends the Euclidean intuition on that 'the whole is greater than its part'; indeed, in fractal geometry the part is equal to or greater than the whole. On the other hand it offers—for the first time—logical and aesthetic arguments to counterbalance what Ossowski (cited above) sees as lack of criteria "in [a] generalized formulation".

Obviously, it is absurd to extend a sense of absolute self-similar minimalism to all relationships; nor is it possible to extend the relation of proportion to all sorts of relationships. In this context, the assertion of Devaney—an advocate of a rigorous notion of the fractal concept—on that "everything" is a subset of fractal geometry, is mysterious: "It now seems to be painfully obvious that just about everything around me—in nature as well as dynamical systems—is a subset of fractal geometry" (Devaney 2004:39). This statement equals to a monism or a generalized and extreme Pythagoreanism, suggesting that "everything is a set of points".[418] Such a conceptualization rather follows a system of analogies drawn by intersemiotic mappings featuring self-referential cognitive circuits (see subchapters 3.5.–3.8.). Accordingly, Ossowski (1966/1978:24) perceives a variety of configurations in these mappings, interpreting them as aesthetic configurations, and anticipates the idea of self-similar correlation between the notions of point, set and harmonious whole, that he calls 'organized whole':

> Spatial configurations with which aesthetics deals are configurations of the most varied
> types. These may be simple forms, i.e., when such whole elements are only geometrical
> points; and they may be complex configurations, configuration of the second or third
> degree, i.e., such in which the elements are configurations of simpler elements. In this case
> we are dealing with the organization of organized wholes. These may be closed

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[418] This criticism can be extended to other authors who consider the so-called 'fractals' as universal ubiquity. For example, from a mathematical perspective in Michael Barnsley (1988), or from a musical perspective in Charles Madden (1999/2007) and Hwakyu Lee (2004). An extended development of this criticism is found in subchapter 5.5.

> configurations, for example, the configuration of forms on a Persian carpet, or they may be unlimited configurations in two directions, or they may be also unlimited configurations in all directions.

In cognitive terms, such an 'organized whole' is available as aesthetic experience and logical structure, through the mechanisms of stereotype and abduction. In this sort of experience, as happens in music, self-similar infiniteness is not essential. What is essential is merely its conjecturability. A musical process-mapping analogous to a cartographic projection based on spatial conjectures, "implies a choice, that retains some relationships and excludes others; expressing in a reflected idea the results of a research that is not independent of hypotheses and the interpretation of facts" (Dainville 1986:394).[419]

Point made of points; line made of lines

According to the dialectical tradition, to analyze is to segment a whole into parts for its description and systematic study. This tradition, rooted in the teachings of Democritus, Euclid and Plato, indoctrinates how the world can be understood by the ways in which the atoms integrate matter, how points produce forms, or how spherical shapes make up the cosmos. Modern philosophy inherits and transforms these old ideas. For example, in Kantian or Russellian doctrines on a self-referential universe (mentioned in the previous section); or in the contrasting denial of the absoluteness of the simple part, in favour of the *implicate order* (see Bohm 1980), the *rapports de coordination* (Foucault 1966:130, 156), or the *rhizome* (see Deleuze and Guattari 1980:13).

From the view of aesthetic philosophy, before speaking of lines, planes and bodies, Kandinsky (1926:29–46) undertakes the analysis of the 'most simple' forms by studying the point, from a Kantian perspective:

> [T]he whole 'world' can be looked upon as a self-contained cosmic composition which, in turn, is composed of endless number of independent compositions, always self-contained

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[419] Dainville (*loc. cit.*) refers to a map of synthesis and correlation: "Une telle carte implique un choix, qui retient certaines relations et en écarte d'autres. Elle exprime une idée réfléchie, les résultats d'une recherche qui n'est pas indépendante d'hypothèses; elle est l'interpretations des faits […]".

even when getting smaller and smaller. In the final analysis, all of these—large or small, have been originated from points, to which point—in its original geometric essence—everything returns. (Kandinsky *op. cit.*:30–31).

According to the Platonic-Kantian tradition, the masses, clouds, or clusters of clusters, can form dense points, thus forming an infinite connection between minor and major, composing the 'map within the map' as suggested by Peirce (*CP* 8.122; see also Pareyon 2010b:35). The best illustration of such a point made of points—in turn made of points, again and again—is provided by Kandinsky himself (*op. cit.*: 45) [◊611, left.], denoting correspondence and self-similarity between physical universe and physical quantum. In this conceptualization, Kandinsky exchanges influences with the scientific and philosophical theories of his time. He also ventures into the analogy of a 'basic point' as a generator of musical sound, providing several graphic examples included in his book *Punkt und Linie zu Fläche* (1926).
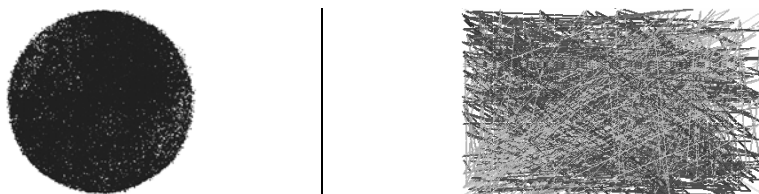
◊611. Left: Wassily Kandinsky (1926:45), "Large point made of small points". Along with this design, the author presents other analogous figures: a photograph of a stellar nebula, and a microscopic photograph picturing the formation of a nitrite, resembling sets made of points.

Right: Phase analysis of Kandinsky's drawing (left), scanned and processed as a system of acoustic relationships. This analysis shows local (probable) relations as a whole, in which lines *are made of lines* (compare with figure ◊658). The pattern obtained is comparable to a $1/f^0$ Gaussian noise sampling, and is a typical example of intersemiotic translation between image and sound. The rectangular shape (essentially irrelevant in terms of the analysis) corresponds to the computer screen used for obtaining the result.

Even today, the notion of point as the origin of a minimal relationship continues to be used in music theory. Typically, Ockelford uses two points to refer to a similarity between two musical minimums. When he theorizes about E.T. Cone's classical scheme, in which two basic elements are represented as *x*, *y*, where *y* derives from x as a primal musical similarity, Ockelford summarizes the relationship *x → y* as a relationship between two points, one succeeding the other (see Cone 1968, 1987:237; Ockelford 2005:21). Moreover, this relationship between points is usually represented by a second element: a line segment that, according to the Euclidean tradition, forms "a point continuum" (Euclid, *Elements*, Book 1, defs.3,4). This idea draws on a first level of the concept of line as isomorphous structure made of repetitions. According to Scholfield's (1958) aesthetic theory, proportions contribute to create order through repetition, or by the sum of "dominant shapes" and by "patterns of mathematical relationships" (see Bucher 1959:526).

By a Euclidean logic, after the point of dimension 0, follows the line of dimension 1. This seems to be valid for music, as for a variety of abstract expressions: "The straight line segment, determined by two points is, in geometry, mechanics and architecture, the simplest element to which the ideas of measurement, comparison and relationship can be implemented" (Ghyka 1927:24). In this view, with which Ghyka opens his text on proportion, he assumes the Euclidean principle stating that a line *is formed of points*. As it is also usual in the analytical tradition of music, Ghyka uses the straight line as a continuum of points, in order to develop a theory of measure and similarity, representing the relationships that characterize rhythm and harmony. In contrast to this conceptualization, and according to a *self-similar logic*, one finds the notion that a line is not made of points, but of lines, by analogy with the concept of point made of points, as suggested in graphic ◊611. Precisely, Peirce (1903a, 1903c) promotes this self-similar assumption,[420] musically articulated in Iannis Xenakis' *Pithoprakta* (1956).[421]

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[420] If in some aspect Ch. S. Peirce's mathematics revolutionizes Euclidean geometry it is in this one: the point is not anymore the generator of all things, but rather *things* come out from the recursion of their own self-structuring parts: "On a continuous line there are not really any points at all" (*CP* 3.388); "Breaking grains of sand more and more will only make the sand more broken. It will not weld the grains into unbroken continuity." (*CP* 6.168). This notion is developed, clearly, upon the Aristotelian *De Lineis Insecabilibus* (Book II, in particular);

Two complementary concepts of harmony

In music theory there are two interrelated concepts of harmony. One refers to the physical, *hierarchical*, configuration of pitches, by contrast to the homogeneous distribution of frequencies in $1/f^0$ noise, or in stochastic distribution in different forms of noise. The second one refers to the well-formedness function in the implementation of a musical grammar, based on the physics of sound structuring. Examples of the latter are the *methods of harmony*, such as the *Tratado de glosas* (1553) by Diego Ortiz; *Traité de l'harmonie reduite à ses principes naturels* (1722) by Jean-Philippe Rameau; Учебник гармоний (1886) by Nikolai Rimsky-Korsakov; *Harmonielehre* (1911) by Arnold Schoenberg; or *Aufgaben für Harmonieschüler* (1949) by Paul Hindemith. Rameau's treatise is particularly relevant, since it introduces the foundations of modern musical harmony as proportional theory. A subsequent treatise by Rameau is his *Nouveau système de musique théorique et pratique* (1726), in which he coins the term 'subdominant', and describes a proportional reciprocity in the tonal system: the root of the subdominant is to the root of the tonic the same as the root of the tonic is to the root of the dominant. Under this proportional conceptualization Rameau develops his constructive theory of triads—typical of the Western tonal system—and establishes the organization of the system of diatonic scales.

Both concepts, or rather both sets of concepts referred to as *harmony of sinusoids*, and *harmony of proportions*, can be summarized as follows:

(1) **Harmony of sinusoids** forming a frequency (pitch or tone). It is traditionally defined as the relationship between a generator tone and the degrees of its associated series. After Fourier analysis, which decomposes a function into a series of overlapping frequency sinusoids from a 'fundamental', it is possible to extend this relationship to smaller harmonic intervals completing the sound frequency spectrum setting both, tone and timbre. Under this consideration, the usual term of 'harmonic oscillation' denotes the sinusoids related at a frequency, as Fourier series. This concept of

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nonetheless, Peirce's conclusions are closer to the concept of topologic continuity, than to Aristotelian algebra and arithmetic.

[421] See graphic ◊656, on page 452.

harmony is the one used in the extraction of a tone's harmonic spectrum by electronic synthesis (as summarized in subchapter 4.2.). As suggested by Fagarazzi (1988), this method also permits the implementation of timbral, textural and durational functions.
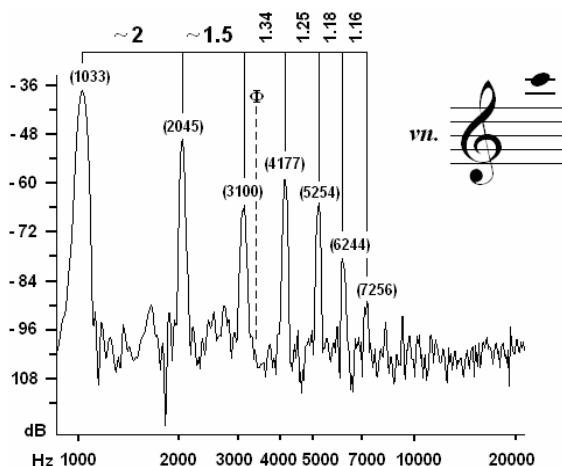
◊612. Harmonic spectrum of C_6 played in a violin (1st string). The staff represents traditional pitch notation, whilst the frequency graph shows its complexity as vibrating system. The highest peak corresponds to the 'fundamental' frequency of C_6 (in this case tuned to 1033 hertz), and the following hierarchies represent harmonic tones associated with the harmonic complex (numbers in brackets represent their frequencies). A relatively stable region is noticeable at the system's beginning (i.e. leftmost area in the waveform), whose maximum hierarchies ratios are close to 2 and 1.5 (see upper horizontal line), followed by a less stable region. Stability decreases as ratios approach 1, in a region that tends to noise at higher frequencies (approximately after 10kHertz), to a lesser amplitude. The symbol φ denotes an ideal point of the process bifurcation between hierarchical order and the same order fading out.

(2) *Harmony of proportions.* This concept, as a modern version of the Pythagorean and Platonic notions of harmony, extends into stereotyped geometric relationships in pitch sets and intervals selection (proportions of scales and chords), whilst associated with durations. This stereotyped concept of harmony is usual in the analysis of structural patterns in music, especially in durations of medium and long range (such as phrases, sections, movements or whole pieces).

Harmony of proportions is usually represented in a projective line, through the so-called *harmonic range*. Also known under the name of *harmonic system of points* (Hardy 1908:99,106) the harmonic range forms on the relationship between points *A, B, C, D*:
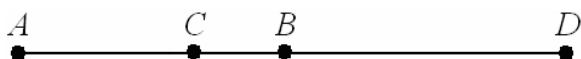
where $AD = 1$, $AB = {}^1/_2$, $AC = {}^1/_3$ and $CB = {}^1/_6$. So that $AD:AB = AC:CB$. It is then said that *ABCD* form a harmonic range (see Durell 1928:65–67). This is useful to represent, in a musical segment, the harmony of proportions in the horizontal coordinate of durations, and to represent their vertical distribution in simultaneous harmonic functions, whilst enforcing a relationship analogous to $1:{}^1/_2 \boxtimes {}^1/_3:{}^1/_6$ (read the symbol $\boxtimes$ as "proportional to") or, more generally, whilst the overall proportional relationship follows the so-called *harmonic progression*:

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

This infinite series corresponds to the progression of the aliquot divisions of a system of periodic oscillations, as shown in ◊421, from a fundamental frequency with perfect harmonic series, represented by ${}^1/_2, {}^1/_3, {}^1/_4 \ldots$ as regular periods of a vibrating system.

In short, one may say that a proportion does not depend on points or lines, but on the relationship between two variables: given two variables x and y, where y is directly proportional to x if there is a constant k not being equal to zero, so that $y = kx$. This relationship is represented as $y \propto x$. The constant ratio $k = y:x$ is called *constant proportionality* or proportionality of the relationship of proportion. Constant proportionality is a generalized phenomenon in music;[422] it often appears together

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[422] Fractions used in musical metre represent a proportion between metric unit and music measure. Fractions expressing harmonic intervals also represent a proportion of frequencies, corresponding to the degrees of a scale or the pitches of a tuning. In each of these cases, numerator (x) and denominator (y) are related as $y \propto x$. There are, however, other types of relationships that do not host any proportion and cannot therefore be represented as a simple

with self-referential operations in a system whose total exists as a function of its parts, whilst the parts are correlated as a function of the total, laying the foundations for a general notion of consistency. This mode of self-reference incorporates such a notion of consistency or harmony within a larger system in which the same fundamental principle applies to self-similar distribution of language systems in general, following Zipf's law (see Chapter 5). This includes generalized distribution of $1/f$ noise, and proportional self-reference as constructive operation. Such forms of reflexive constructivism obey the same basic principle of self-reference, since its development corresponds to a consecutive row from the unit, as the series $1, {}^1/_2, {}^1/_3, {}^1/_4...$ This self-referentiality involves, as well, self-organization processes in organic patterns, and aesthetic operations in a wide variety of modes of self-structuring proportions, as described in this chapter.

6.2. Transition: from simplicity to complexity

Hodges (2003:105) analyzes the cognitive operation that allows the transformation of a point into a basic sequence of points, and such a sequence into an infinite system. He characterizes this idea in the following manner: ● = finite unit, ●● = replication or basic symmetry of the finite unit, ●●● = basic sequence, extendable to an infinite isometry, with infinite relationships of symmetry.[423] Hodges notes that this simple process is sufficient to lay out an infinite pattern, and provides several examples of musical literature, in which this pattern works as an analogy of ellipsis in written language. Hodges wonders if such horizontal representation of this mental process is necessary, and he answers "yes and no": yes, because this representation corresponds with general intuition and a convention of writing; no, because it is necessary to include the feasibility of *affine transformations* that change the spatial relationships of a system, without changing the intrinsic relations of the system itself.

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relationship between numerator and denominator. These relationships, the so-called *irrational intervals*, coexist in music with simple notions of proportion.

[423] Subchapter 2.3. introduces the notion of 'isometry' (see pages 45–47).

The idea of developing complexity from a simple 'starting point', is stimulated by the observation of biological processes in which, from a sign (the Euclidean σημεῖόν) carrying a basic material with its intrinsic rules, a major structure is elaborated, with increasingly specialized polymorphisms and functions. Subchapter 6.5., dealing with self-replacement strings, presents this idea in the context of Lindenmayer systems.

The notion of 'starting point' in a complex system also appears in stellar physics, under the concept of 'singularity', whereby the entire universe would be formed from a first originating point, in which all matter/energy was initially concentrated (see Friedmann 1922). By the mid-twentieth century, most branches of physics opened the discussion on fundamental particles in a great variety of possible ways. Gabor (1947), for example, suggests an acoustic world made from a basic 'grain' of vibration: an infinitesimal point vibrating in space by a self-referential expansion, originating a self-similar sound complexity. Stockhausen (1989:37) summarizes this idea, also as part of an aesthetic debate: "There was similar thinking everywhere: reduction of the process of forming to the smallest possible element". This reductionism, but also its pragmatic implications in sound synthesis, are evident in modern theories of sound granulation (Ames 1989, Clarke *et al*. 1996, Rocha Iturbide 1999, Roads 2004), as well as in theoretical revisions on musical cognition and generativism (see especially Ockelford 2005). One of the central aspects of generativism in music theory is precisely the investigation of axioms and basic rules in musical constructivism, in order to describe how the more complex structures of music are originated in 'starting points' basis.[424] Clearly, this topic is too broad, so it must be constricted within the main subject of the present study, to a specific case; for instance, in the example of the *aggregate patterns* as explained below.

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[424] This conceptualization is schematically related to subchapter 4.4. See especially pages 165–166.

Aa)
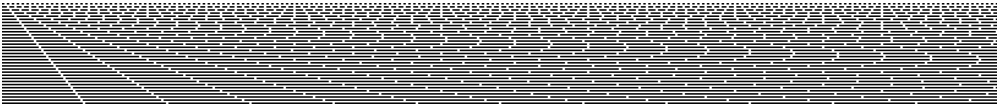
Ab)
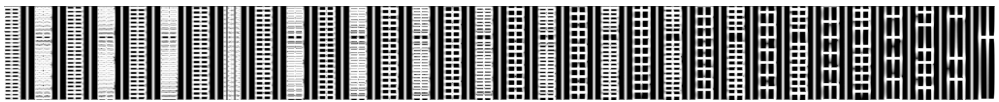
Ba)
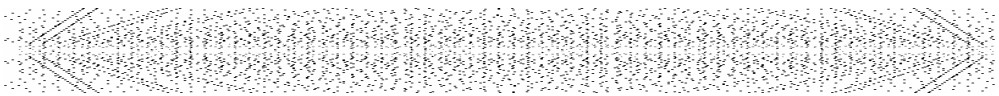
Bb)
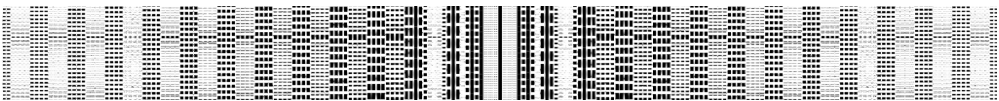
Ca)
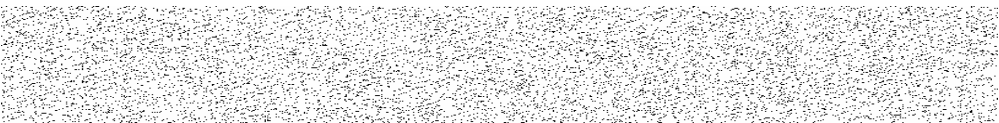
Cb)
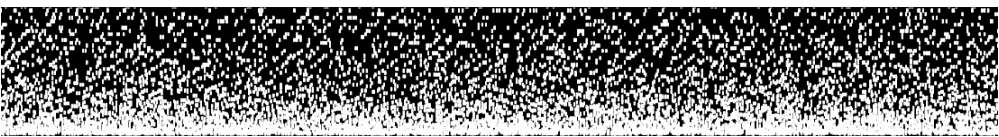
◊620. Aggregate pattern developed as audible forms. The six bars of this scheme (*Aa – Cb*) represent transformations of the same pattern. Suffixes *a* denote the numeric-geometric form of the pattern. Suffixes *b* denote the bit-maps rendered for each example (*a*) as harmonic spectrum, where the abscissa (x) represents time series, and the ordinate (y) represents pitches in Hertz (20 Hz to 20 kHz). *Aa* represents the original pattern, whose vertical relationships are growing as a series of natural numbers starting from 1 (= point). *Ba* is a multiplication and transformation of *Aa*; and *Ca* is a multiplication and transformation of *Ba*.

Aggregate patterns

Subchapter 4.4., which introduces the basic patterns of pragmatic self-similarity in language, introduces an *aggregate pattern* set by the ordering of the infinite sequence 1010101010… : 1201201201… : 1230123012… A segment of this sequence is represented in ◊442b by a first dashed line made of equal dots, according to the organizing principle employed by Hodges (2003:105), where the figure ●●● symbolizes a basic sequence extendable to an infinite succession. This structure reappears in ◊620*Aa*, accompanied by the acoustic rendering of its bitmaps, represented by its harmonic spectrum (*Ab*).

This scheme highlights the transition, in a few operations of affine transformation, from the basic sequence (1010101010…) to a complex plot (see ◊620*Ca–Cb*). A following, consistent overlapping, necessarily approaches—because of the many clustered similarities—a noise pattern. As a whole, these relationship follow the transition *proportion → self-similarity → chaos*,[425] due to the accumulation of affine transformations, as continuous recursion.

Noise and isotropy

Isotropy is the statistical uniformity in all parts of a system. As stated in Chapter 5, measures such as $1/f^0$, $1/f$, $1/f^2$, $1/f^3$… represent continuous proportions of energy distribution in a frequency spectrum. Some of these proportions contain inherent relationships of self-similarity and correlation (specifically $1/f$, and relatively $1/f^2$), whilst others just have a tendency to isotropic disorder: *lack of order* under the same tendency in all its parts, as happens in the stochastic signal of $1/f^0$ noise, or in Lévy flight (see Bolognesi 1983:31). Nevertheless, this disorder can also be 'organized' by adjacent hierarchies with self-similar trends in layered isotropies.

Xenakis (1963, 1992) allocates hierarchies to the ways of linking sound events in *minor* sets, with sound events in *major* sets; such as the immersion of subsets within sets with structural similarities. Harley (1995:221) notes that Xenakis uses stochastic

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[425] This transition, typical of dynamical systems, has been introduced in subchapters 3.3. and 5.2. A connection with the Peircean trichotomy is also explained in section 3.8.3.

functions to organize, through probability distributions, the general characteristics of sound 'masses', connecting them to their own *microelements*. This principle of design is based on proportion and functional self-similarity as sound features, implementing generalized isotropies as systems of scalar correlation. The crucial role of isotropies in Xenakis' constructive strategies is due precisely to the use of homogeneity of sound events, in a stochastic system that follows the same type of motion at various scales, simultaneously (see Xenakis 1992:13–14).

In broad terms one can say that the different modes of noise with self-similar patterns are characterized by their different layered isotropies; by their own way of repeating similar intervals and distributions at different scales. This includes patterns as diverse as the local perturbations in a turbulent system in a wind instrument; in self-organizing biases in systems related to $1/f$ noise; or in pseudofractal sound synthesis from systems such as the Weierstrass function.[426]

Besides self-similar distribution of energy in a frequency spectrum—which in musical terms can be translated as a correlation between pitch, amplitude and form of encapsulation (i.e. point, line, mass; tone, chord, noise; gesture, motif, phrase; etc.)—Xenakis (1992:13–14) also conceives the distribution of speeds between articulated points of sound, and defines three "hypotheses" of isotropy for the speeds within a homogeneous mass: (1) the density of speed-animated sound is constant, which means that the sound proportion tends to be the same for each segment of the pitch range; (2) the mean quadratic speed of mobile sounds is the same in the different registers; and (3) the absolute value of the speeds is uniformly distributed, which means that there is an equal number of sounds ascending and descending within a group of *glissandi*.

This notion of isotropy is consistent with the examples displayed in graphic ◊611, with Kandinsky's image of a point made of points, and with its intersemiotic translation as Gaussian noise. Xenakis (1992:13) himself validates this conversion with "arguments" about "*logical poems* which the human intelligence creates in order to trap the superficial incoherencies of physical phenomena, and which can serve, on

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[426] The Weierstrass function, with an introduction to its musical applications, is mentioned in subchapter 5.3. (see pages 253–255). The issue of turbulences in wind instruments is presented in subchapter 4.2. (see pages 145–148).

the rebound, as a point of departure for building abstract entities, and then incarnations of these entities in sound or light." Thus, Xenakis conceives isotropy and scalebound as means for musical coherence, contributing to structural sense by the systematization of self-similarity.

Harmony and noise in prime numbers

The connection between the sieve of Eratosthenes and Riemann zeta function is one of the most eloquent examples in mathematics, as a transition from simplicity to complexity.[427] In other words, obtaining the first numbers that are divided only by 1 and by themselves, is a relatively simple task; but comprehending the overall behaviour of these numbers represents a major conundrum for mathematics.[428]

In the mathematical study of dynamical systems, short-term behaviour is much less relevant than long-term behaviour. In contrast, for music, short-term tends to be more relevant than long-term behaviour—a trait conditioned by human short-term memory, particularly influential in music cognition.[429] For different reasons, this is also true in isolating a musical appearance in the *initial* sequence of prime numbers. For example in the bounded sequence:

$$\{ \wp \mid \wp \in [\wp_1, \wp_{22}]\},$$

where 1 corresponds to the first prime number and 22 corresponds to the twenty-second prime number, in consecutive order. Assuming this subset as a series of adjacent factors, $^b/_a$, $^c/_b$, $^d/_c$..., the first division is $^3/_2$ (i.e. the sesquialtera value, which

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[427] Riemann zeta function is named in honour of mathematician Bernhard Riemann (1826–1866), not to be confused with music theorist Hugo Riemann (1849–1919), several times quoted in this study.

[428] Literature on this subject is abundant, ranging from the discussion of strictly analytical issues, as in G.J.O. Jameson (*The Prime Number Theorem*, Cambridge University Press, Cambridge, 2003), and its aesthetic-mathematical view in Du Sautoy (*The Music of the Primes*, Harper, New York, 2004), to its poetic and philosophical interpretation in a narrative perspective (A. Doxiadis, *Uncle Petros and Goldbach's Conjecture*, Bloomsbury, 2001).

[429] However, the growing influence of mass data processing, from a statistical argument that extends in current musicology, should not be overlooked. From this perspective, quantitative analysis determines the qualitative interpretation, as happens for example in stylometry, as explained in subchapter 5.2.

is the most common ratio in musical metre and harmony), the second division is $^5/_3$, the third division is $^7/_5$, and so on. Thus, the following graph of ratios is obtained:
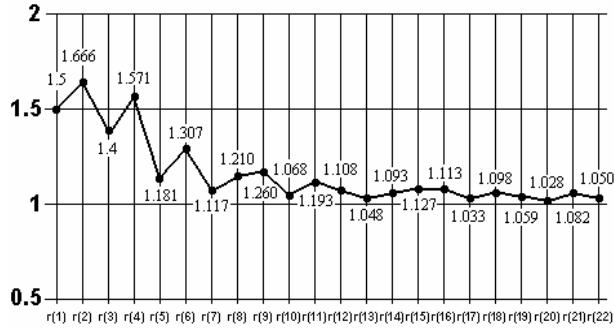
◊621a.

If the series of divisions are made with consecutive terms $^a/_b$, $^b/_c$, $^c/_d$, and so on, then the following ratios are obtained:
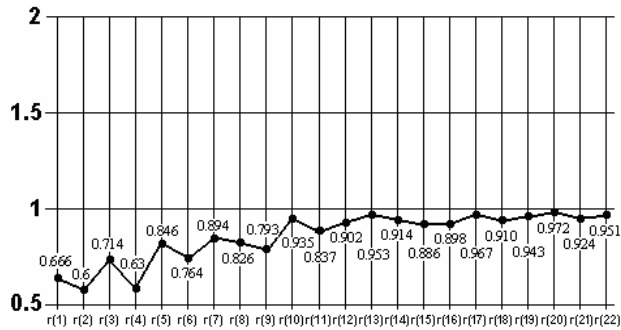
◊621b.

The latter graph, seen as a series of frequencies associated with the previous figure (◊621a), remains as follows:
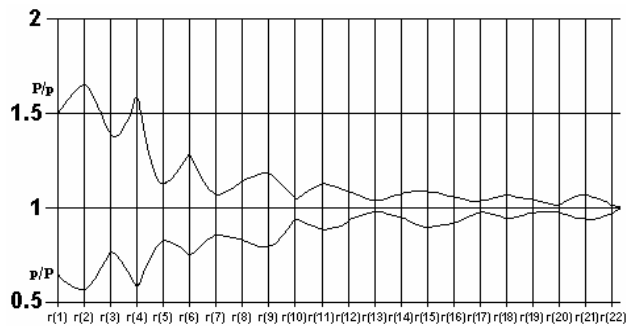
◊621c.

In order to emphasize symmetry in this representation, the same data can be displayed together with their inversion, as horizontal radial symmetry:
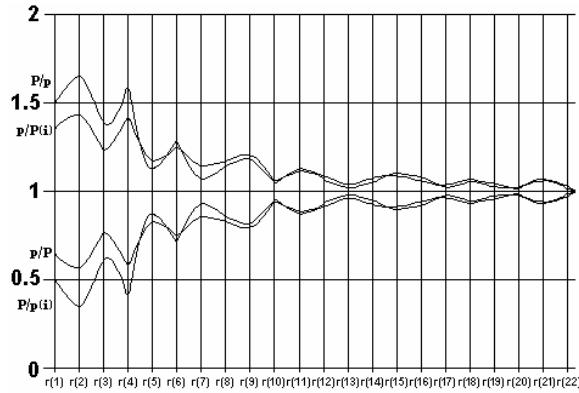
◊621d.

This sinusoidal structure can accept attributions of harmony in both senses, geometric and musical: geometric if one conceives a golden segment formed approximately between $r(1,9)$ and $r(10,22)$; specifically divided by $^{22}/_\phi$. This segment also keeps outstanding relationships of harmony between its first ten ratios. At the same time, they may be of musical interest if one assumes that the ratios correspond to an ordered pitch scale:

3/2	5/3	7/5	11/7	13/11	17/13	19/17	23/19	29/23	31/29
1.5	1.666	1.4	1.5714	1.1818	1.3076	1.1176	1.2105	1.2608	1.0689

The same, ordered by quotients:

I	II	III	IV	V	VI	VII	VIII	IX	X
31/29	19/17	13/11	23/19	29/23	17/13	7/5	3/2	11/7	5/3
1.0689	1.1176	1.1818	1.2105	1.2608	1.3076	1.4	1.5	1.5714	1.666
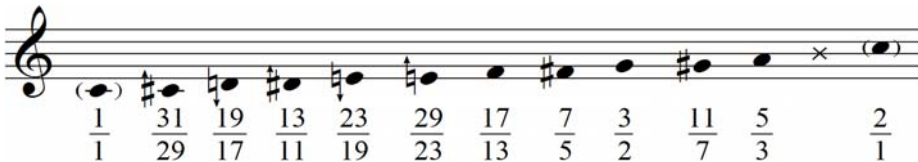
◊622. Analogies of $\{\wp \mid \wp \in [\wp_1, \wp_{10}]\}$ displayed as pitch scale.

harmony · similarity · noise
(proportion) · (approximate self-similarity)
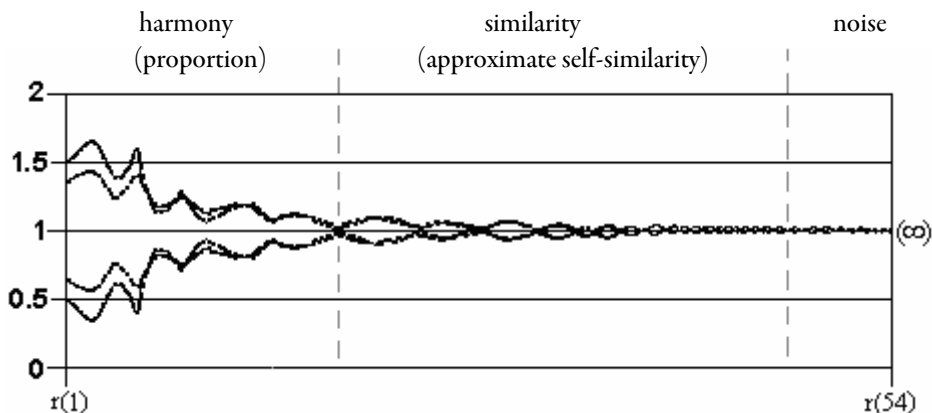
◊623. Above: visualization of $\{\, \wp \mid \wp \in [\, \wp_1,\ \wp_{54}]\}$ as sinusoid in transition: from a harmonic spectrum (*tone*) to a stochastic distribution of intervals closing to 1 (*noise*) with relative self-similarity in an intermediate segment. In musical terms, this initial segment of the prime numbers series represents a 'harmonic' region, directly translatable into scales and chords and even into a prescriptive grammar. A subsequent region can be embedded as intervallic correlation within the former (i.e. relative self-similarity); whilst a third, 'chaotic' region (i.e. gradually less suitable for musical practice) is comparable to an infinite noisy coda.

Below: visualization of the same set as multidimensional projection with radial symmetry, as quasi-harmonic system based on data from the previous example. The result is a sound object with relative self-similarity.

Whether the prime numbers series has harmonic features is, hence, an intriguing question. The answer cannot be univocal, since the example given is part of an infinite set, progressively less related to a paradigm of proportions. Such a series, apparently proportional and self-similar at the *beginning*, increasingly departs from any notion of consistency, proportion and self-similarity, getting lost amidst a growing irregularity (or, from the opposite point of view, a growing regularity

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tending to 1). In a larger sample of frequencies $\{ \wp \mid \wp \in [\wp_1, \wp_{54}]\}$, the limits of this model also become ambiguous between harmonic proportion, self-similarity, and distribution of less predictable events, comparable—under a first glance—to the stochastic distribution of frequencies in generalized noise (see ◊623).

The essay written by Du Sautoy (2004), on the history and mathematical meaning of prime numbers, employs the concept of music as a metaphor to suggest that a harmonious order exists in the prime numbers infinite series; but, even when he uses an explicit and promising title such as *The Music of the Primes*, he does not pay attention to the harmony of the *initial segment* (characterized in ◊622), nor to its *transition to noise* (pictured in ◊623). Nonetheless, his mathematical approach is of interest to music theory because it delves into the long-term pattern of prime numbers and their absolute behaviour, which, according to the Riemann zeta function, should reflect a general harmony in which the intervals are balanced as global form, although with clear differences among local forms.

Du Sautoy's (2004) central argumentation is that Riemann zeta function operates on the infinite harmonic series (i.e. the harmonic progression is described in 6.1.). In fact, the mathematical principle that gives rise to the Riemann zeta function *is* the Euler function, defined in real numbers as harmonic infinite recursion. Synthesizing the infinite sum of its terms, this function allows characterization of such terms on the conjectural base that, for all prime numbers, every even number greater than 2 is the sum of two primes.[430]

Du Sautoy (*op. cit.*:93–96) also notes that Fourier analysis, employed for the decomposition of any wave in an infinite series of harmonics,[431] is compatible with the Riemann zeta function, since each of the segments of the Euler function on real numbers can be decomposed into primes by the Riemann zeta function. In short, the Riemann zeta function converts prime numbers into wave functions: "these waves are not just abstract music, but can be translated into physical sounds that anyone can listen" (Du Sautoy, *op. cit.*:278). The question arises, therefore, whether this 'music'

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[430] This hypothesis is known as Goldbach's conjecture. In 1923, Hardy and Littlewood showed that, assuming the generalized Riemann hypothesis, Goldbach's conjecture is valid for all odd numbers that are sufficiently large.

[431] The concept of Fourier analysis is introduced in a special section, on pages 143–145.

or infinite set of frequencies tends towards $1/f^0$ noise as random distribution, or it rather tends (at least partially) towards $1/f$ noise and self-similar distribution. This question, deeply connecting music with the aesthetics of mathematics, and with acoustics and general physics (given the nature of fractional noises involved), is crucial to understanding the limits—if they indeed exist—between harmony, self-similarity and noise in the prime numbers set.

Besides prime numbers, a wide variety of dynamical phenomena present the transition *harmony → similarity → noise*.[432] In this general sense, the concept of self-similarity can be understood as the group of (a)symmetrical transitions in a dynamical system. For instance, Chapter 5 shows the diagonalization of frequencies and durations (see ◊525), following this behaviour. Another example is the bifurcation diagram of the *logistic map*, as described below.

Bifurcation diagram of the logistic map

The mathematical concept of *map*—also called *function*—can be particularly useful in music theory if we conceive a numerical value on $\mathbb{R}$ (i.e. in one dimension), that can be musically mapped into two dimensions (e.g. pitch and length).[433] The purpose of this analogy is to display some similarities between music theory and the dynamics in the *real map* (i.e. $\mathbb{R}$), as suggested in the following pages.

A bifurcation diagram of possible long-term values in a group of periodic orbits in a map, characterizes a dynamical system in terms of a bifurcation parameter inherent in the system. The bifurcation diagram is common to represent stable solutions with a solid line and unstable solutions as dotted lines. A specific case is the logistic function:

$$x_{n+1} = rx_n(1-x_n)$$

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[432] This concept has been introduced in subchapters 3.3. and 5.2. A connection with the Peircean trichotomy, as an epistemological model, is also explained in section 3.8.3.

[433] In mathematics, the term *map* or *mapping* is a synonym for *mathematical function*, denoted by f. Maps are systems of correspondences between specific objects within a set. Thus, a map $f: A \rightarrow B$ from A to B is a function f such that for every a in A, there is a specific object $f(a)$ in B. Historically, most of function theory in dynamical systems is based on Weierstrass (1880) and Poincaré (1886). For an introduction to modern function theory, related with topics mentioned in this subchapter, see Bak (1982), Klebanoff and Rickert (1998) and Demidov (e2006).

where $0 < r \leq 4$. By iterating the equation, for each new value of x, the previous value of x is multiplied by its scaling factor. For $0 \leq r \leq 1$ all iterations converge to fixed point 0. For $1 < r < 3$, a fixed limit of $1 - 1r$ attracts all initial values of x. For $r \geq 3$, the fixed point continues in principle, but its displacement becomes unstable, splitting in a first bifurcation that corresponds to a first alternation of values. In $r = 3.449499$ the bifurcation in two cycles splits in a bifurcation in four cycles. From $r = 3.54409$ to $r = 3.569946$, the four cycles bifurcate in eight, then sixteen, and so on.
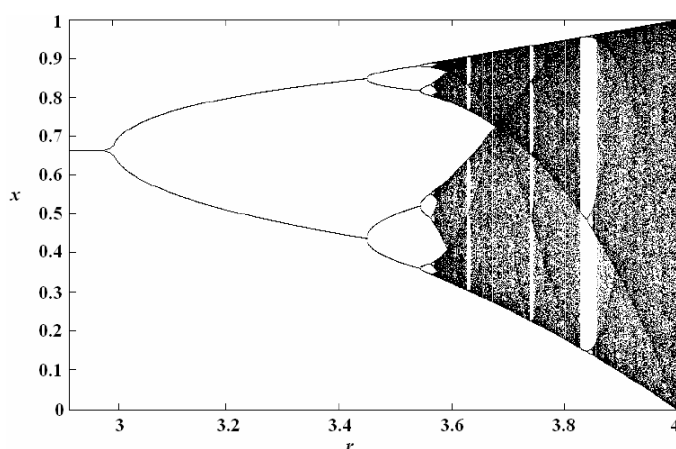
◊624. Bifurcation diagram of the logistic map for the values r from 2.9 to 4. Originally described by R.M. May (1974, 1976), this diagram corresponds to the function of r as series of values for x^n starting with a random value x^0 and iterating it many times. Numbers in the left column (x) represent the final states of the function. Gleick (1988:71) explains that bifurcations starting between 3.0 and 3.45 occur in the same way between 3.8 and 3.9, and then extend into the entire plot, and densely within the graph's chaotic regions.

In graph ◊624 the bifurcation parameter r is shown on the abscissa, whilst the ordinate shows the possible long-term value of the population of the logistic function. The bifurcation diagram shows the bifurcation of the possible periods of stable orbits 1 to 2 to 4 to 8, etc. Each of these points corresponds to a period doubling bifurcation. The length relationship of successive intervals between the values of r, for which the bifurcation occurs, converges to the first *Feigenbaum constant* (explained below).

Interpreting this function as an audio signal unveils the transition from periodic behaviour (r [2.9, 3.44]), to relatively periodic behaviour (r [3.45, 3.54]), to a quasi-periodic chaos (r [3.54, 4]), and finally to infinite chaos (r [4, $+\infty$]). This process is analogous to a transition from echo (long length and broad pitch intervals) to reverberation (progressively shorter delays and closer pitch intervals). Slater (1998:13–17) implements a modular music synthesizer coupled to an oscilloscope with an output impulse that produces this chaotic pattern; the spectrum and the impulse response of this signal systematically splits to set up a doubling periods torrent, resulting in a cascade of harmonics that can have multiple usages in music. Jedrzejewski (2006:172) employs a similar procedure for scalar design, using the first Feigenbaum constant as a source for musical tuning with self-similar features.

The bifurcation diagram of the logistic map is analogous to the periodic growth transition of a population, until its relative stability, and then, until absolute instability occurs (May 1974, 1976). This scheme is also analogous to allometric configuration in example ◊430b, and is the most studied model of chaos in systems of proportion and functional self-similarity in organic relationships. In this context, it is also related to Fibonacci sequences, golden ratio, and Lindenmayer systems, all of them currently used as resources for musical structuring.[434]

Slater (1998:16) suggests that any modular analogue synthesizer can be used to produce chaotic synthesis for musical purposes. This is proven by Polotti and Evangelista (2001), Sapp (e2003), Milotti (e2007), and Brothers (e2009), among many others. J.R. Salter (2009) devotes much of his doctoral dissertation to the study of the logistic function by its possible applications to music; moreover, his study concludes with a piece entitled *Chaos drumming*, for four percussionists, as evidence of his proposal. Madden (2007:52, 133) also reports a musical piece composed by Ian Stewart (1944–), using the logistic function as an example of "a deterministic chaotic process" implemented for sound production. This is also an example in which self-similarity of all intervals is not correlated to local figurative self-similarity; the parts of the whole are self-similar only in terms of their generalized distribution, but not by the rigorous scaling of a fundamental structure.

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[434] See subchapters 6.3.–6.5.

Feigenbaum constant

The (first) Feigenbaum constant is a universal constant for functions approaching chaos by periodic doublings. It was discovered by physicist M.J. Feigenbaum in 1975 (formalized in Feigenbaum 1979), when investigating the fixed points for the iterated function

$$f(x) = 1 - r\,|x|^r,$$

where f is a real function, defined positive and three times differentiable on $[0,1]$, without relative maximum value over this interval, and which characterizes the approximation of the bifurcation parameter whilst its limit value r increases for the fixed point x. The diagram shown in $\lozenge 624$ corresponds to several thousand iterations of this function in $r = 2$ for a discrete series, but with spaced r values, discarding the first hundred points or less, before the iteration defined for its fixed points, and then plotting the remaining points. After obtaining this plot, Feigenbaum calculated the increase (i) of parameter (Δ) for each bifurcation in r, as division of the following terms:

$$\frac{\Delta i}{\Delta i + 1} = \frac{r_i - r_{i-1}}{r_{i+1} - r_i}$$

Whilst i increases, the division converges to a specific value, known as the *first Feigenbaum constant*, which is represented as

$$\delta = \lim_{i \to \infty} \frac{\Delta i}{\Delta i + 1} = 4.669201609102...$$

The value of δ corresponds to the convergence rate of the bifurcations of nonlinear quadratic equations, so it is found in a variety of chaotic dynamical systems from an initial period of bifurcations. Klebanoff and Rickert (1998) associate it, for example, with diagrams of divergence in *tent maps* and the Cantor set, whose transitions are similar. In many cases, especially with emerging features of turbulence, the rate of convergence of the bifurcations is not constant, but quickly approximates to the

Feigenbaum constant as limit. On this basis, the prediction of the first bifurcations is beyond the limit by a statistically insignificant margin; the rest of the iterative behaviour of the function converges to the same rate limit δ, provided that the immediate proximity of the absolute maximum of the original function is a quadratic relationship—what Klebanoff and Rickert (*cit.*) explain in detail, for the cited cases. This requirement is met, for instance, in the arc at the edge of a circle; in the top of a parabola, or in the crest of a sine wave. Some illustrative examples of this are given in graphs ◊625 and ◊626, in a following section (see page 353).

About the concepts 'attraction' and 'attractor' in music

An attractor is a pole, a point, or a set of points in which the relationships of a dynamical system converge, partially or totally. Behaviour of parts within this system also reflects, a trend biased toward the attractor. Consequently, and according to the Riemann conjecture, 1 is the characteristic attractor in the series of adjacent divisors in the prime numbers set, as φ is the characteristic attractor in the series of adjacent divisors in the set of numbers forming the Fibonacci sequence.

The Feigenbaum attractor (i.e. the set of points generated by successive iterations of the logistic function for the critical value of parameter $r = 3.57$, where the doubling period is infinite) is a well-studied case of converging self-similarity, with many applications in different areas. Other dynamic processes based on self-reference and iteration of functions, with a variety of structural attractors, chaotic or not, are also of interest for music, as explained in the following pages.

The notions of 'attraction' and 'attractor', as physical processes, are noticeable in modern musical thought. For instance, Hindemith (1941:57) and Stravinsky (1947:31–37) agree that tonal relations in general can be described as relations of attraction and repulsion, similar to the relationships that occur in electromagnetic and gravitational systems. According to Edward T. Cone (1968:26–27), the transition from one pitch to another, in a melody or chord, within a system of musical harmony, is analogous to the displacement of an object in space, between a point of origin and a point of attraction, under specific laws of action, movement and rest, comparable to the physical laws of attraction and repulsion between two bodies.

For Steve Larson (1997:101–102) this analogy is also useful in explaining relationships of 'atonal' music, in which attractors correspond to statistical biases of repetition, symmetry, proportion, convergence, correlation and self-similarity. In a context of 'expressive meaning', i.e. the "quality experienced in music that allows it to suggest feelings, actions, or motion", Larson explores the relationships between what he conceives as *musical forces*. According to him (*op. cit.*:102), such musical forces are: *gravity*, or the tendency of an unstable note to descend; *magnetism*, or the tendency of an unstable note to move to the nearest stable pitch, "a tendency that grows stronger the closer we get to a goal", and *inertia*, or the tendency of a pattern of musical motion to continue in the 'same' fashion, strongly depending upon the Gestalt interpretation of a musical pattern.[435] In short (Larson, *ibid.*), "Musical motion is thus heard as a mapping of physical gesture onto musical space—as purposeful action within a dynamic field of these musical forces."

'Mapping' as described by Larson is intuitively useful because it justifies a deterministic method for intersemiotic translation, between *perception* of musical relationships, and *recursiveness* of what he calls 'musical forces', attached to a system consolidated by the ordering function of the code,[436] and the power laws governing analogies of *gravity, magnetism* and *inertia*. This form of structural validation applies also to the analogy of chaotic systems, with respect to the dynamic processes between 'musical forces'. Acoustic turbulence patterns described in subchapter 4.2. are an example of this relationship, well-documented after Feigenbaum (1980).[437] Methods for implementing the Duffing equation and other dynamical systems and chaotic attractors, as quasi-fractal generators, organizational models for timbre, and musical tessellations, also fit this description.[438] Relative self-similarity and chaos also occur at musical micro-levels of sound production: as Popp and Stelter (1990) note, self-

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[435] See subchapter 3.5.

[436] This concept is introduced on pages 208–210.

[437] Bader (2005) analyzes chaotic turbulences in flute-like musical instruments with beveled embouchuresa. This topic is part of subchapter 4.2.

[438] The Duffing equation is a non-linear second-order differential equation. It is one of the more in-detail studied cases of dynamical systems, and its solution is a typical example of chaotic behaviour (see Bourke e1998). The equation describes the motion of a forced oscillator with periodicity more complicated than in a harmonic simple system. It can model e.g. patterns of quasi-periodicity in a perturbed vibrating string.

similar processes tending to chaos are found in bowed instruments' noise, particularly when playing *molto sul ponticello*. This phenomenon is directly linked with forced oscillators in general, and with quasi-periodic forced functions describing deviations from Cantor-like mappings (see *Cantor function*, below).

Another type of example of chaotic transition in music, is the style transformation occurring between subtle steps in the relationship between musical idiolect and ecolect, on a grammar rigid in the short term, but relatively flexible in larger cycles of transformation.[439] According to what is suggested in Chapter 4, the enormous diversity of processes of attraction and structural transformation is also evident in the context of chaotic attractors. A special case of the latter are the attractors in *real maps* as explained in the following section.

Chaos in real maps

Demidov (e2006) describes the general behaviour of iterations in the logistic map by the gradual divergence from a point $x = 0$ in the regular period for c, that represents the mode of iteration phase, and finds the tangential bifurcation point approaching the value $c = -1.7495$. For example, in ◊625 the parameter c is 1, resulting in the square of the iteration of x; instead, in ◊626$c-d$ the parameter c comes near to $-\varphi$ (~1.6180339...), close to the value Demidov indicates. Such a bifurcation results in a tendency towards chaotic behaviour within the function. Consequently, the lengths of the regular phase regions grow as they approach the bifurcation point c. According to Hanssen and Wilcox (1999) the length of the regular phase is proportional to $c\,(-c)^{-1/2}$, thus, the length of the regular phase increases twice when $c\,(-c)$ decreased four times. Demidov (e2006) also describes how regular and chaotic phases may alternate in a behaviour called *intermittency*. From the standpoint of music theory, this concept is useful in describing an analogous behaviour in transitions between harmonic regular phases and chaotic phases, and in relationships between harmonic sets and noise sets. An example of this is the transition from a source of logistic chaos, to a convergent sequence representable by rational numbers. Demidov's (e2006)

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[439] This issue is discussed in subchapter 4.8.

interactive page, that served to prepare examples ◊625 and ◊626, can also be implemented to generate a sound basis for synthesis processing after a waveform, plotting the iterations of the function on the same waveform (see ◊626).
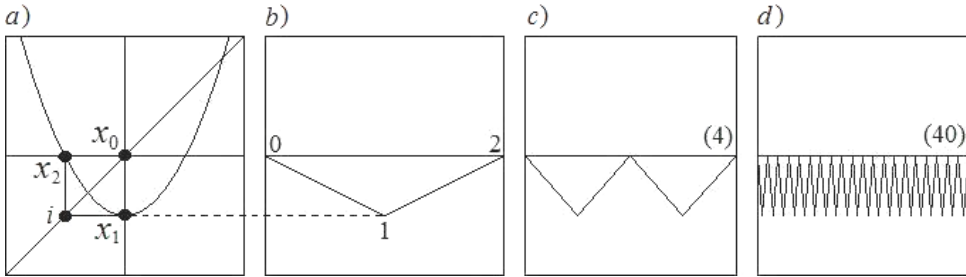
◊625. Iteration of the point x_0 in the logistic map. x_0 is the starting point and x_1 is the value obtained when mapping x_0. i is the identity (diagonal) denoting the correspondence between x_1 and x_2 (symmetrical correspondence, in this case, which does not occur for all mappings). The leftmost box (a) shows the parabola base of the map's period (in other example the parabola may have a quite different length, with different mappings). The following box (b) shows the linear graph of the function's second iteration. The next one (c) shows the fourth iteration, and the rightmost (d) shows the fortieth iteration. The general behaviour of these iterations is periodic (non-chaotic).
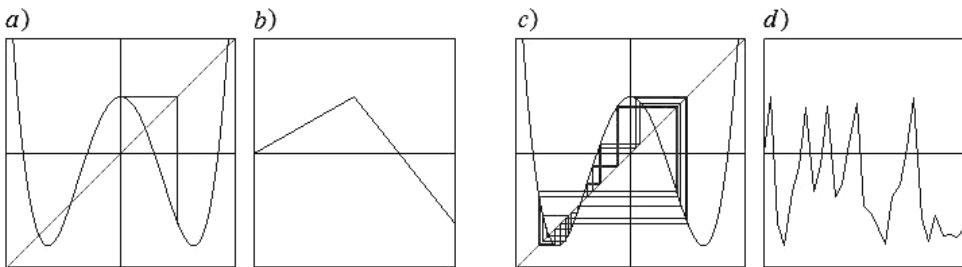
◊626. Iteration of a point in the logistic map. The parabola's period has twice the value than in example ◊625. Iteration phase in (a) is close to $-\varphi$, whilst in the former example is 1. The next box (b) shows the linear graph of the function's second iteration. In (c) the same function appears in its 28th iteration, and (d) shows its phase plot. Unlike the previous example, here the function's behaviour is chaotic (i.e. the phase plot changes unpredictably along the following iterations).

The iteration of functions in the logistic map commonly produces systems of relationships with statistical self-similarity in a similar way to the bifurcation diagram shown in ◊624. Each step of iterations in the two-dimensional space can be read as sound image, for example at coordinates x = length, y = pitch, integrating patterns of noise, timbre, and rhythms at different rates of correlation. This form of production can also be attached to a system of structural self-similarity, for example, associating micro-rhythms, mini-rhythms, meso-rhythms and macro-rhythms within a self-similar set, according to Xenakis' (1992:226) original proposal.
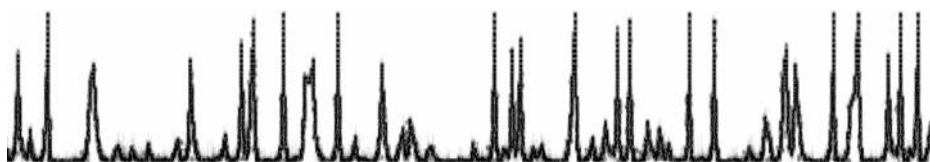
◊626 (continuation). Sequentiation of phase graph in ◊626*d* as audio signal, typically at coordinates x = length, and y = pitch. It is clear that information in this scheme can also be employed to abstract other musical parameters such as rhythm, by the repetition of duration intervals, or timbre, by compression of these intervals in a harmonic system. The overall consistency of different musical parameters based on the same statistical source can be called *sound integration*—by an 'ad hoc' analogy with parametric consistency in integral serialism.

Cantor function

In mathematics, the Cantor function is the best known case of a real function that is continuous—i.e. that is uniformly continuous, and at the same time is not absolutely continuous. In its own fashion, this feature resembles the Weierstrass function mentioned in subchapter 5.3.

There are different methods to approach the Cantor function, among which is the two-dimension graphic expression of x in base 3. This can be represented by the infinite tripartite subdivision of a straight line segment, as shown in ◊332 (see page 74), as well as by the infinite subdivision of bars in the so-called *devil's staircase* (see ◊627 on pages 355–357).

After the pioneering work of Georg Cantor (1845–1918) and Giuseppe Vitali (1875–1932), many mathematicians have explored various aspects of this function, particularly with respect to its self-similarity. Like the Koch curve or the Mandelbrot set, the devil's staircase also has drawn the attention of composers such as Charles Dodge (*Profile*, 1984) and György Ligeti (*L'escalier du diable*, 1993), because of its hierarchical self-structuring in a continuous proportional subdivision, that recalls aspects of music harmony and hierarchical distribution.

The usual definition of the standard Cantor function involves the classic 'middle-thirds' description of the standard *Cantor set*, "the most basic fractal of all" according to Devaney (1992:4). This can be visualized along the devil's staircase construction, as follows:
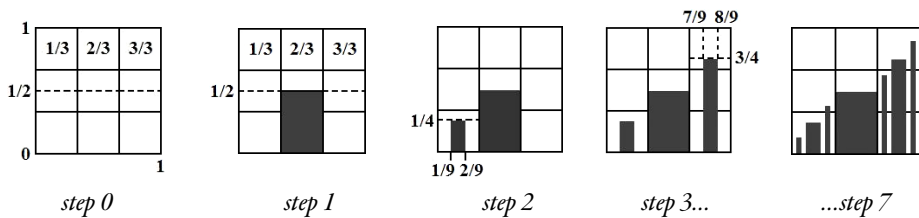
◊627a. First steps in building the devil's staircase.

The clearest way to start building this description is using a square of sides of length 1, divided into 9 equal squares. Then the whole shape is horizontally divided by 2 (see dashed line in *step 0*). In this example each minor square has the area $1/3$; thus the distance [0, 1] can be represented as [$1/3$, $2/3$, $3/3$]. In *step 1*, a solid column represents the middle third [$1/3$, $2/3$] of the base side of the square, with height $1/2$. In *step 2*, a smaller solid column—analogous to the first column—represents the middle third [$1/9$, $2/9$] of the base left-square [0, $1/3$], with height $1/4$. In *step 3*, a higher column —also analogous to the first column—represents the middle third [$7/9$, $8/9$] of the base right-square [$2/3$, $3/3$], with height $3/4$. Accordingly, successive steps should produce columns with heights $1/8$, $3/8$, $5/8$, $7/8$, and in the kth step there will be columns of heights $1/2^k$, $3/2^k$,..., $(2^k - 1)/2^k$. Analogies continuing in this manner form a 'ladder' with infinite 'steps', corresponding to the rational intervals that characterize the devil's staircase (see schematic approximations in ◊627b–d). Peitgen, Jürgens and

Saupe (2004:213–214) concludes that the Cantor set corresponds to the "density as height of the bars in each generation [of the devil's staircase]" i.e. the Cantor function is not precisely expressed by the shape in ◊627*b*, but by the density of the infinitely sharp line represented in ◊627*c*.
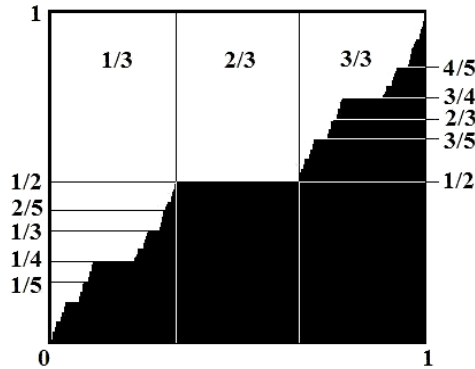
◊627*b*.

In this schematic view (◊627*b*), initial intervals $[^1/_9, {}^2/_9]$, $[^1/_3, {}^2/_3]$, $[^7/_9, {}^8/_9]$ clearly prevail, together with smaller intervals, symmetricaly shrinking in their width, and with their height approaching 0 at the left limit, and to 1 at the right limit. In the limit of the infinite construction steps—the endless iteration of the initial generative process—the whole square is symmetrically divided into two identical parts: the upper half in white and the lower half in black. Strictly speaking, the 'staircase' corresponds to the infinitely sharp line dividing the square:
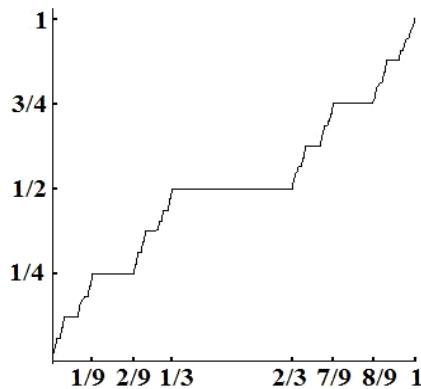
◊627*c*.
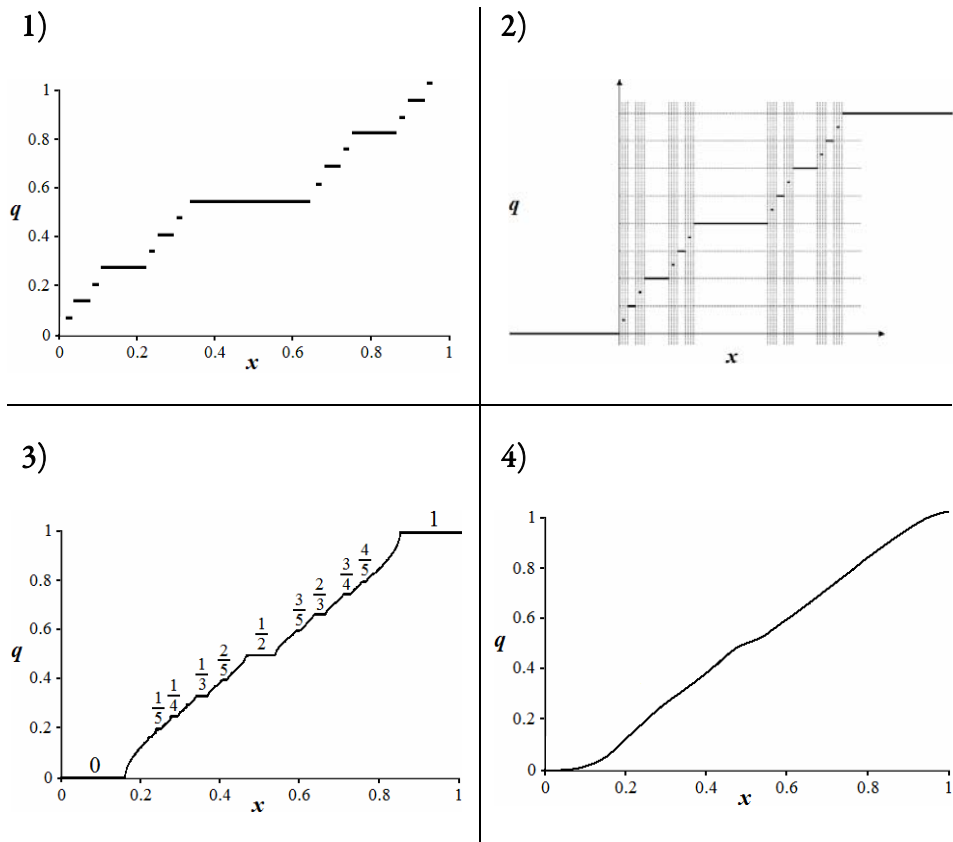
◊627d. Different construction stages of the *devil's staircase*. The ordinate q shows the inverse period or wave vector as a function of the parameter x, that represents the sequence of rational numbers between 0 and 1. The result is a 'ladder' in which simpler rational numbers correspond to longer steps. This can be associated with the notion of musical harmony, given that, as suggested by Helmholtz (1863), the simplest rational numbers are also consistent with the most prominent harmonic grades. As a whole curve, the devil's staircase is non-absolutely self-similar (since it is a finite curve), although it is commonly said that it is a 'fractal' with fractal dimension of 1.0. (Figures are adapted from the original work of Per Bak, 1982).

1) Staircase formed by the first four iterations of the function.
2) Evolving to the n iteration of the function: points on vertical subsets represent dense spaces 'emerging' as iterations continue, producing infinitely smaller intervals known in mathematics as 'Cantor dust'.
3) Staircase formed by the first 34 iterations of the function, with its main steps between rational intervals indicated as ratios.
4) Continuous mapping of $[0,1]$ in $[0,1]$ is constant everywhere except on the resulting Cantor set.

In this shape (◊627), the relative widths of the steps are hierarchically organized from bigger ratios that represent more 'visible' or locally dominant 'plateaux', to smaller and less intuitive ratios. Another interesting property of this shape is that the strucural segments of the staircase symmetrically correspond to larger and/or smaller segments, forming a relatively self-similar curve with length equal to 2, and fractal (Hausdorff/Besicovitch) dimension equal to 1. Since the staircase has a differential shrinking scale between $1/3$ for the width and $1/4$ for the height, it is said that its geometry is non-absolutely self-similar: its zoom cannot be constantly proportional; it should, for it, follow a same shrinking scale of width and height.

The approach to Cantor function from music theory has seen just first steps toward a new aesthetics. Charles Dodge's *Profile* (1984) was composed after meeting Benoît B. Mandelbrot, who acted as an advisor during the compositional process (see Dodge 1988:10). Dodge's initial goal was to create a stochastic algorithm from a collection of pre-selected pitches, following a schedule of refining-production that is intuitively comparable to the interval's refining-production process employed at the first steps of the *devil's staircase* generation. As a matter of fact, Dodge's (*op. cit.*:13–14) pitch segments written on the conventional staff, are vaguely reminiscent of this process. According to the composer himself (*ibid.*),

> The goal in writing the composition was to create a pleasing piece of music with a computer program through the recursive application of some simple rules. The plan was to use a time-filling fractal form for the structure of the piece and $1/f$ noise to choose all the musical detail of pitch, rhythm, and amplitude. The world of fractal geometry is very diverse and this musical analogy touches on but a small, simple corner of it.

This work plan reflects a basic principle of self-similar structuring in which a set of *base notes* (pitches with specific durations) is used to generate a subset of higher and shorter notes, which in turn serves to generate a third set of higher and shorter notes.[440] Dodge's emphasis on generative grammar principles, as well as his idiolectal

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[440] The functional correlation between duration and intonation was formulated theoretically by Olivier Messiaen (1908–1992) for his *Livre d'orgue* (1952), according to his concept of 'chromatic durations', associating shorter lengths to higher pitches. This correlation has a natural basis in the configuration of the bioacoustic codes (see Sotavalta 1953, 1963), pointing out that, proportionally, a 'small' organism emits a frequency higher than the common vibration of another organism, relatively larger. This principle is connected with the Weber-Fechner law, mentioned on page 212.

notion of "pleasing piece of music", reveal key aspects of his method and his intentionality for an intersemiotic translation of a self-similar set into another, analogous one, albeit with ontological and operative features completely different.

In the case of Ligeti's last part of his second book of *Études* for piano (1988–93), under the explicit title of *L'escalier du diable* (1993), Cantor function is approached with craftsmanship—and under the advice of mathematician Heinz-Otto Peitgen. Nevertheless, Ligeti's method and results also correspond, as in Dodge's case, not to a strict analogy, but to an intersemiotic translation. As Steinitz (1996a:19) notes:

> Ligeti constructs his musical staircase using his own numerical system. But it, too, exhibits recursive qualities, whilst its hemiola rhythmic cells of 2 to 3 recall the binary-ternary geometry of the devil's staircase. The piece starts softly but ominously with energetic, additive rhythms based on a 'metrical model' containing 'subgroups' of seven, nine, eleven and nine quavers. After a bar-and-a-half's false start, this metrical model is presented in full, and then repeated over and over, always subdivided into cells of 2+2+3 / 2+2+2+3 / 2+2+2+2+3 / 2+2+2+3 / 2+2+3 / 2+2+2+3 etc. The elongated 'step' of three quavers, emphasised by legato phrasing, makes a small plateau, whilst the unequal but orderly progression of the subgroups recalls the irregular staircase of the graphic image. The study also exhibits a 'pitch model' starting on the first note, so immediately out of synchronisation with the rhythmic model. As for its structure, the first note of each cell starts progressively higher, following a 12-note chromatic scale from B up to A sharp.

The intersemiotic translation that Ligeti makes of the mathematical *devil's staircase*, is confirmed by the similar operation the composer employs in the last of his *Études*, XIV, *Infinite column*. This score constitutes the intersemiotic translation of a sculpture created in 1937 by Constantin Brancusi in the town of Tirgu Jiu, Romania. The sculpture, a vertical modular structure of 29 meters high, stands in the open-air, with a steel skeleton covered with bronze plates, forming truncated pyramids placed alternately one above the other, closing base against base and vertex to vertex.[441] Ligeti translates this structure into music, taking into account the qualitative aspects of the

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[441] The content of footnote 6 (see page 2) must be highlighted here, on the paronomasia of the term 'fractal', coined by Mandelbrot (1977, 1982), with respect to the ancient Greek verb φρακτός, from φράσσω or φράζω, "to tighten the one against the other, spear against spear, shield against shield" (M. A. Bailly, 1894:609). This coincidence provides material for a history of the aesthetic conceptualization of self-similarity, as suggested, incipiently, in subchapter 6.1. It is clear that the main mechanism activating the translation, in the case Ligeti-Brancusi, implies a synecdochic intersemiosis.

column, with its $16^1/_2$ modules, each of them symbolyzed in the piano's low register, gradually ascending with upward movements in meandering form. The same freedom that characterizes Ligeti's translation of *L'escalier du diable*, reappears as part of his compositional method, creating a completely new structure.[442]

A more strict conceptualization of the mathematical devil's staircase, as a source for harmonic constructiveness, has recently suggested by Cartwright, González and Piro (2009). These authors are particularly concerned with periodically forced nonlinear oscillators, which "can exhibit an extremely rich variety of [acoustic] responses" (*op. cit.*:171). They include multi-frequency quasiperiodic and chaotic responses, and employ the devil's staircase as the "skeleton for the layout of the resonances in parameter space as Arnold tongues", somehow as the Farey tree (see section below) has been used as a source of musical harmony and (quasi-)periodicity.

An interesting topic suggested by Cartwright, González and Piro (2009), is their "nonlinear theory for the residue", structurally adapting the devil's staircase iterative approach, to the auditory system i.e. distinguishing a neo-Pythagorean notion of the harmonic hierarchies, from the actual phenomenon of human hearing. Finally, they connect their findings to their own idea of the role of the golden mean as an aesthetic and structural feature in music, associated with the self-referential character of the Cantor function—a relationship developed in the following subchapter, under the concepts of synecdoche and self-similarity.

Musicological interpretation of the Cantor function

Within the Pythagorean tradition, and particularly after Helmholtz (1863) theorizing, sound intervals are conceived as 'consonant' when they correspond to 'intuitive' or large ratios, and 'less consonant' or 'dissonant' when they become smaller or less intuitive. Congruently, hierarchical distribution in the devil's staircase has been associated with 'stability' in musical harmony (see Cartwright, González and Piro 2009). This is somehow related to the Weber-Fechner law ("the smallest discernible change in the magnitude of a stimulus is proportional to the magnitude of

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[442] The process of translation of a sculpture into music is discussed in subchapter 3.8. (see especially pages 101 and 107–108).

the stimulus"); as well as to Zipf's law or *law of least effort* in typical processes of language and music.[443] The devil's staircase is thus an extremely valuable resource for investigating structural self-similarity and synecdochic intersemiosis in music, since it analogously conjugates mathematical usefulness, aesthetic content, and fundamental principles of cognition, as explained below.

Reality, particularly because of its aesthetic components, is attainable as an elaboration of the mind, thanks to the automated relationship between primary, secondary and tertiary epistemes, in which primary categories pre-define most of the subsequent categories (see Peirce 1903a). Generalization and stereotype, as very elementary devices of cognition, depend on the principle of comparing secondary and tertiary epistemes with models or main sources.[444] This relationship is analogous in the devil's staircase, in which subsequent segments—progresively less dominant epistemes—depend on the first relationship established between main (logical, symmetric) source and its first derivation. As Kramer (1981:579) conceives for the essence of the logical thougth,

> We can [generalize] because we know the essential characteristic that each individual has if he belongs to [a] group, and that he lacks, if he does not. So it is with infinite aggregates. They may be known by their characteristics even though we cannot complete the task of counting them, one by one.

Thus, the logical relationship between 'essential characteristic' and (finite or infinite) 'aggregates' implies a form of extension, that may receive a number of cognitive functions—in the form of synecdoche, analogy or proportion, for instance. The devil's staircase pictures quite properly this relationship. Furthermore, it contributes to solve Plato's paradox: "we cannot learn anything unless we already know it" (see Chomsky 1966:11). The answer to this paradox is 'guessing', as it is already postulated by the Peircean doctrine of synechism and abduction (see Peirce 1893, 1903a–b).[445] Of course, this does not mean randomly guessing, but guessing *thirdnesses* from the evidence of primary and secondary necessarily interrelated

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[443] The Weber-Fechner law and Zipf's law are introduced on pages 212 and 216, respectively.
[444] This subject is developed in subchapters 4.2. to 4.4.
[445] An introduction to these concepts is provided in subchapters 3.2. and 3.8.

epistemes. This is the essential mechanism of the devil's staircase construction as a cognitive process.

Fractal (or pseudo-fractal) features of verbal language are suggested by Benny Shanon (1993) and Luděk Hřebíček (1994), because of language's typical relationship between word and sentence, and between sentence and its aggregates.[446] Pareyon (2007c:1304) also suggests that a 'symbolic coordination' between *implicate* and *subordinate*, in cycles of information at different rates and layers of self-organization under the same power laws, may be "responsible of self-similarity in language [and] in several scales of biological construction." Such a relationship should happen under the preponderance of few salient qualities-quantities, over a progressive dependence of—equally—gradually more abundant particularities.

Of course, the Cantor function is not the only self-organized system laying out hierarchical relationships between primary, secondary and subsequent categories, comparable to linguistic and musical structuring. As Devaney (1987:110) suggests, the Cantor function is a particular case of a devil's staircase. In mathematics, as well as in language and in music, there is a variety of staircases that can be meaningful according to a specific variety of rules affecting the aggregates.[447]

Somehow, the aim of musicological analysis is to identify the relationships between main and subsequent categories (statements and aggregates, antecedents and consequents, and so on). These relationships do not occur because music mandatorily requires establishing hierarchies, but because human perception of sound as figure, requires abbreviation and stereotyping based on the causal

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[446] This conceptualization is introduced on pages 168–169. One may note that another parallel between linguistic and musical organization, with the devil's staircase, is relative self-similarity. Speech, music and the devil's staircase can be represented as pseudo-fractal curves, since although they can have fractal dimension and apparent fractal consistency (see Pareyon 2007c), actually they have relative self-similarity. As Peitgen, Jürgens and Saupe (2004:212) point out, "the devil's staircase looks [absolutely] self-similar at first glance, but is not." Obviously, there are many differences separating this model from instability in music and in language cycles, for instance, in the stylistic loops studied in subchapters 4.7.–4.8. In practice, musical and linguistic systems are chaotic as whole systems; instead "the devil's staircase consists of commensurate states only and no chaotic states" (Bak 1982:621).

[447] See Cartwright, González and Piro (2009:172) for the specific case of a devil's staircase for the forced Van der Pol oscillator. For a more general, mathematical introduction to the dynamics of the maps in devil's staircases, see Devaney (1987:110–112).

relationship between *main* and *accessory*—something that in tonal and post-tonal music has already been theorized by authors such as Schenker (1932), Salzer (1952), Cone (1968), Lerdahl and Jackendoff (1983), Lewin (1987), Lerdahl (2001), Ockelford (2005), Vázquez (2006), and Ilomäki (2008), among many others. From this perspective, the nuances of similarity between the 'essential' and what depends 'on the essential', determine crucial aspects of identity of the former on the latter. Thus, approximate self-similarity is also nested in the bare naked perception of a great number of aesthetic elaborations. Musicology needs, therefore, to distinguish between the bias to self-similarity in music as *causal phenomenology*, and the tendency to self-similarity as *perceptual self-completeness*; besides, it is clear that in aesthetics both sorts of self-similarity are closely intertwined within a same complex system.

Circle maps and the Arnold tongues

From the physical and mathematical point of view, Per Bak's (1982) introduces a plethora of structural aspects of the devil's staircase, associating them with models of radiation, magnetism, energy distribution and vibrational systems involved in acoustic patterns and sound's behaviour affected by power laws. This is closely related to the thematic structure of Chapter 4, suggesting that specific biological relationships result from physical interactions that are power-law consistent, and therefore exhibit relative self-similarity. In this context, Bak's research is useful as a physical support for the granular theories of sound that have implicit aspects of self-similarity, at least from atomic and molecular layers.

Bak's study starts with the description of a set of 'atoms' which are interrelated as oscillatory systems. These atoms are represented by solid dots or correlated masses, connected among them by harmonic springs. In their turn, these springs behave by specific wave periods, according to a model first put forward by Poincaré (1886, 1907). Despite its extreme simplicity, this model is sufficient to support a thorough investigation on the Fourier series' periodic potential, considering quasi-periodic oscillators in modulated structures, as suggested by Bak (1982). Such a mechanical model, generalized for studying the relationships between particles and waves, can

also be associated with systems of fractons and phonons in Gabor's granular theory.[448]

Oscillators studied by Bak (1982) depend upon the differential calculus of their periodic potential cycles. The calculus is based on the circle map that translates its family of ratios into a differential equation mapping the rational numbers in $\mathbb{R}$ (i.e. the real numbers usually represented as points on an infinitely long, continuous line).

According to Devaney (1987:102), "dynamics of maps [in the circle] are somewhat different from maps of $\mathbb{R}$ since the circle is bounded." At least partially, this is why circle maps are so useful, 'collecting' huge sets of points of $\mathbb{R}$ within a bounded space. For musicological purposes, mapping a collection of rationals in the circle may be therefore practical, for example, to study the relationship of intervals within a harmonic family of ratios (e.g. from a devil's staircase).

The circle map—which basically consists of 'translating' a function of $\mathbb{R}$ within the circle—is a method originally proposed by Kolmogorov (1933) as a simplified model of mechanical rotors. Currently it has a wide variety of applications in the study of electronic circuits and electromechanical systems. These systems include the cardiac pulsation and many other endorhythms associated with cellular potentials and the spatio-temporal configuration of cognitive systems.[449]

The Cantor function in the devil's staircase, as well as other 'staircases' generated by circle maps-like boundaries in *phase mode*, are also of great interest for music theory—with analytic and compositional consequentes—, because of their systematic correlation. Consequently, the Arnold tongues, a kind of structural skeleton of the devil's staircase,[450] can be adopted in music as symbolic self-similar

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[448] For a historical introduction to this matter, see Gabor 1947 (abridged here in subchapter 4.1.). In connection with phonons and fractons, see Schroeder 1991:43; M. Clarke *et al.* 1996:212; Rocha Iturbide 1999: Ch.I; and Roads 2004:34–35. Specifically, on the mechanical model of points or 'masses' united by harmonic springs, related to acoustics and music theory, see E.R. Miranda 2002:80–99, and Dodge and Jerse 1997:283–287.

[449] See subchapters 4.1.–4.3. Congruently, subchapter 5.3. (see especially page 251) describes 'flicker noise' under this conceptualization. Such a 'spatio-temporal configuration' is also implied within the previous section, on the musicological interpretation of the Cantor function.

[450] Named after mathematician Vladimir Arnold (1937–2010), best known for formulating the Kolmogorov–Arnold–Moser theorem, crucial for the study of quasi-periodic motions under small perturbations in dynamical systems.

constructions with their typical ratios and transitions between harmony and chaos, depending on the mapping qualities.[451]
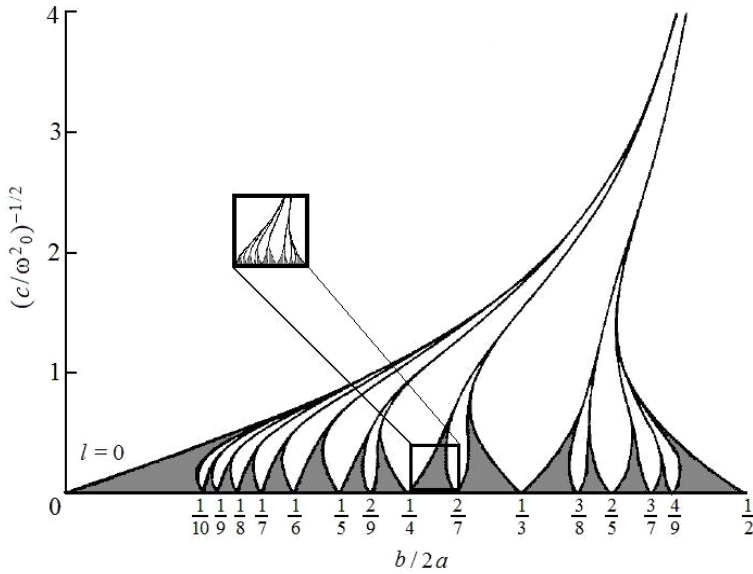
◊628. Arnold tongues in the phase diagram for the continuous model proposed by Aubry (1979), based on a differential equation. The ordinate corresponds to the intensity measure of the model's periodic potential; and the abscissa corresponds to the family of ratios obtained by the function. (For a detailed explanation see Bak 1982:612). Like in the *devil's staircase*, here the simplest ratios fill larger or more stable spaces—properly the 'tongues' or spaces between sigmoids. The inset shows the structure of an amplified segment of the system, suggesting the self-similar composition of the whole.

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[451] Considering the growing literature linking music with dynamical systems, one may say that this rich field of experimental musicology, although being in its formative period, yet promises significant results for music theory in a short future.

There are some aspects that immediately arouse musical interest in a first approximation to recurrence patterns in the circle map and its *rotation numbers*.[452] One of the best examples of this is the so-called Arnold tongues, which appear in some regions of parameters of the mapping, where the limit values are set for the recurrence frequencies—*phase locking* or *mode locking* in the language of electronic circuits. Arnold tongues, thoroughly investigated by Aubry (1979), Bak (1982), MacKay and Tresser (1985), Boyland (1986), and McGuinness and Hong (2004), among many others, are defined by Rasband (1990:130–131, 217) as "resonance zones emanating out from rational numbers in a two-dimensional parameter space of variables."

Arnold tongues (example in ◊628) emerge from *mode locking* regions (commonly symbolized by Ω), showing the rational multiples of *n* which behave chaotically towards smaller scales. Rasband (1990:218) defines mode locking as "the nonlinear interaction of a dynamical system to produce periodic behavior that persists for a range of parameters." Depending on the type of equation employed, for each mapping of the circle there is a special family of ratios corresponding to an Arnold tongues group with a specific mode locking. There is also a general tendency, shared with the devil's staircases, where simplest ratios match with larger spaces. As suggested before, this physical-mathematical behaviour is comparable to Helmholtz's (1863) harmonic theory, distinguishing stable hierarchies in terms of 'consonant' ratios, from smaller, unstable hierarchies or 'dissonant' ratios.[453]

Interestingly, although Helmholtz theory (*cit.*) is historically and culturally limited to the study of the Western concept of harmony, the boundary relationship in Arnold tongues' phase locking, seems to point out a more general, biosemiotic pattern, within a more general concept of harmony and music. For example, Vaughn (1990:116–118) suggests that in non-Western shamanic traditions of ritual vocalization, acoustic and nervous-psychological behaviour in climax situations is

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[452] According to Devaney (1987:103), "The most important invariant associated to a circle map is its *rotation number*. This number, between 0 and 1, essentially measures the average amount points are rotated by an iteration of the map."

[453] The possibility of a comparative model remains open, associating the general criteria of a phase diagram in the mapping circle, with Helmholtz's (1863) generalized results.

analogous to Arnold tongues' phase locking. Again, this relationship seems to point out the association of self-similarity in music as *causal phenomenology*, with *perceptual self-completeness*, as explained in the previous section. Pervasiveness of this relationship in a multicultural environment is further developed in subchapter 6.3., within the contex of the golden mean and the Fibonacci sequences.

Farey tree

The Farey tree[454] consists of a self-structured sequence of proportions, associated with the distribution of spaces in Arnold tongues. Milne *et al.* (2007:22) define it as follows: "A *Farey sequence* of order *n* is the set of irreducible fractions between 0 and 1 with denominators less than *n*, arranged in increasing order." The tree contains all the combinations between the numbers of the sequence. Thus, if the *order* is 4, the following sequence is generated

$$\tfrac{0}{0}, \tfrac{0}{1}, \tfrac{0}{2}, \tfrac{0}{3}, \tfrac{0}{4}, \tfrac{1}{0}, \tfrac{1}{1}, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{2}{0}, \tfrac{2}{1}, \tfrac{2}{2}, \tfrac{2}{3}, \tfrac{2}{4}, \tfrac{3}{0}, \tfrac{3}{1}, \tfrac{3}{2}, \tfrac{3}{3}, \tfrac{3}{4}, \tfrac{4}{0}, \tfrac{4}{1}, \tfrac{4}{2}, \tfrac{4}{3}, \tfrac{4}{4}.$$

The magnitude of the coefficients' quotients defines the order of appearance in the sequence, distributed from lower to higher and from left to right. Because some of these fractions have no real value, e.g. $\tfrac{0}{0}$ or $\tfrac{4}{0}$, and others are equivalent, e.g. $\tfrac{1}{1}, \tfrac{2}{2}, \tfrac{3}{3}$ and $\tfrac{4}{4}$, the sequence can be simplified as follows:

$$\tfrac{0}{1}, \tfrac{1}{1}, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{2}{1}, \tfrac{2}{3}, \tfrac{3}{1}, \tfrac{3}{2}, \tfrac{3}{4}, \tfrac{4}{1}, \tfrac{4}{2}, \tfrac{4}{3}.$$

Since the value of the fractions must be between 0 and 1, fractions above 1 must be eliminated; i.e. those whose numerator is greater than its denominator:

$$\tfrac{0}{1}, \tfrac{1}{1}, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{2}{3}, \tfrac{3}{4}.$$

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[454] Farey tree and sequence are named in honor of geologist John Farey (1766–1826), who in 1816, in a letter published in the *Philosophical Magazine*, of London, proposed a new hierarchical classification of fractions. Farey made the assumption that each new term in the expansion of the sequence corresponds to their neighbors 'mediant' or half step. However, Farey did not prove this property. The proof is attributed to A.-L. Cauchy, who read the letter of Farey and published his findings in his *Exercices d'analyse et de physique mathematique* (1840–47).

In ordering the fractions from largest to smallest and distributing them hierarchically, respect to the first and last ones, within the interval $[0,1]$, there is the distance from

$$\tfrac{1}{4} \text{ to } \tfrac{3}{4}$$

which serves to build the following hierarchical tree:

$$\tfrac{0}{1} \qquad\qquad\qquad\qquad\qquad\qquad \tfrac{1}{1}$$
$$\tfrac{1}{2}$$
$$\tfrac{1}{3} \qquad\qquad\qquad \tfrac{2}{3}$$
$$\tfrac{1}{4} \qquad\qquad\qquad\qquad \tfrac{3}{4}$$
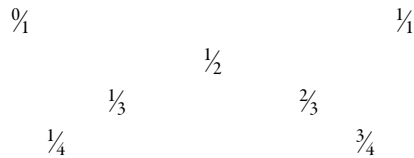
If instead of 4, the *order* is 5, then the standard Farey tree is obtained:

$$\tfrac{0}{1} \qquad\qquad\qquad\qquad\qquad\qquad \tfrac{1}{1}$$
$$\tfrac{1}{2}$$
$$\tfrac{1}{3} \qquad\qquad\qquad \tfrac{2}{3}$$
$$\tfrac{1}{4} \qquad \tfrac{2}{5} \quad \tfrac{3}{5} \qquad\qquad \tfrac{3}{4}$$
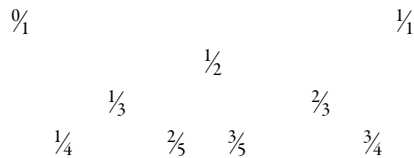
If instead of 5, the *order* is 8, the tree continues:

$$\tfrac{0}{1} \qquad\qquad\qquad\qquad\qquad\qquad \tfrac{1}{1}$$
$$\tfrac{1}{2}$$
$$\tfrac{1}{3} \qquad\qquad\qquad \tfrac{2}{3}$$
$$\tfrac{1}{4} \qquad \tfrac{2}{5} \quad \tfrac{3}{5} \qquad \tfrac{3}{4}$$
$$\tfrac{1}{5} \ \tfrac{2}{7} \qquad \tfrac{3}{8} \ \tfrac{3}{7} \ \tfrac{4}{7} \ \tfrac{5}{8} \qquad \tfrac{5}{7} \ \tfrac{4}{5}$$
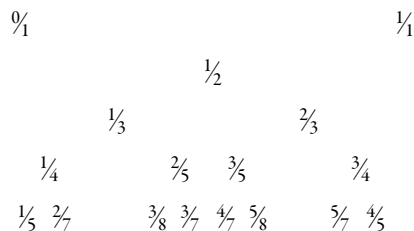
This development corresponds with a model for the analysis of rational numbers within the Cantor function, which, according to Rasband (1990:133), can also be described as a set of Arnold tongues, from a differential equation. According to Schroeder (1991:336) "Farey tree is a kind of mathematical skeleton of the Arnold tongues" (see ◊628).
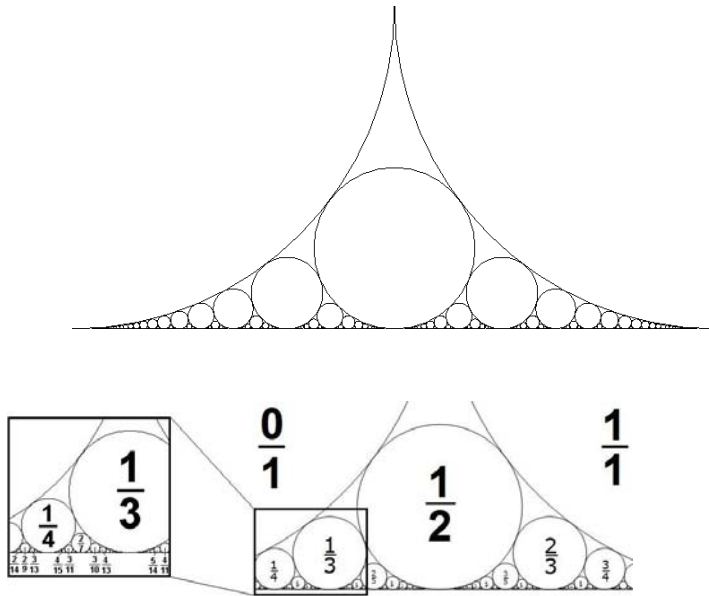
◊629. Above: Farey tree represented with Ford circles. The overall statistical self-similarity is evident, associated with the symmetry of fractal objects resulting from maps of two-folded periods.

Below: Details of the same set, showing fractional values of its major elements, corresponding to the Farey tree.

Rasch (1988) observes that the Farey tree's hierarchical distribution is akin to the notion of hierarchical harmony in music, in that major or 'more consonant' intervals correspond to simple ratios (according to Helmholtz 1863). Rasch employs the Farey tree to define 'new' sets of intervals, as an alternative to equal temperament tuning.

Carey and Clampitt (1989:187, 206) also conceive of a consistent relationship between a set of traditional music scales, under the notion of *well-formedness*:[455] "The same structure underlies the tonic-subdominant-dominant relationship, the 17-tone Arabic and 53-tone Chinese theoretical systems, and other pitch collections in non-Western music." Essentially, this assumption concerns the structural 'coherence' of scales by their proportional affinity. This notion of consistency is closely related to the *three-gap theorem*.

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[455] See independent section on pages 60–62.

The *three-gap theorem* or *Steinhaus conjecture*, formulated by mathematician Vera T. Sós (1930–), states that for any irrational number mapped in the circle, points segment the circle as arcs or intervals, at least in two different lengths and maximum at three different lengths or steps. Clearly, by this definition, the theorem relates the circle mapping, as explained on pages 363–367, with the Cantor function. Carey and Clampitt (1996) associate it also with the pitch classes structured according to well-formedness rules: "The proof of the three-gap theorem demonstrates that when the set yields three step sizes, the largest step size, call it x, is the sum of the smaller two step sizes, y and z." They conclude that the circle mapping yields meaningful information on any pitch set generated by the same interval. This includes natural numbers mappings. Continued fractions obtained by this procedure—i.e. its mapping circle—are directly related to the Farey sequence, too.

In this context, Agmon (1989, 1995) studies the "chromatic and enharmonic consequences" of the Farey tree, as well as its distributional properties contributing to structural coherence in the diatonic scale. Agmon (1996:45) defines 'coherence' or 'lack of contradiction', in terms associated with the notions of invariance and self-similarity:

> Definition: *Coherence*. Given a set of integer pairs $\{(u, v)\}$, $0 \le u \le a - 1$, $0 \le v \le b - 1$ [where a and b are steps of a tonal scale] we shall say that the set is coherent if for any pair of integer pair (u, v) and (u', v') in the given set such that $u > u'$, the relation $v' \ge v'$ holds.
> Corollary: Given a coherent set of integer pairs $\{(u, v)\}$, for any pair of integer pairs (u, v) and (u', v') in the coherent set such that $v > v'$, the relation $u \ge u'$ holds.
> Definition: *Coherent Scalar System*. We shall say that a scalar system $SS(a, b)$ is coherent if $\{(u, v)\} = I(S(a, b))$ is coherent [where I is the set of diatonic intervals].

Carey and Clampitt (1989:188–190) complement this analytical approach of the diatonic scale, highlighting its two-dimensional spatial projection as mapping circle. They employ the heptagon inscribed in the circle to represent the seven diatonic pitches consecutively arranged to yield six identical perfect fifth intervals. Then they compare this mapping with the mapping circle of other simple polygons, regular and

irregular, representing scales with different pitch collections.[456] The conclusions they obtain with this method—consistent with the characterization by Mazzola (1990), of *tonal perspectives* as (a)symmetric mappings—lead to the establishment of direct links between properties of invariance identified by pitch-class set theory,[457] with notions of consistency and structural consistency settled by the theories of self-similarity and recursiveness.

It is worth noting that the Farey tree, with its analytical and compositional traits associated with music, is also related to the notions of harmony and spatial proportion, in terms of the Fibonacci sequence and the golden mean: zigzagging down in the Farey tree, the following sequence is obtained: $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$..., whose numerators and denominators correspond to the Fibonacci sequence, and its consecutive division tends towards the golden mean. This subject develops in the next subchapter.

The systematic usage of the Farey tree's self-similar features, with its properties connected with self-referential proportional sets, as the Fibonacci sequence and the golden mean, permits a more clear definition of any deterministic intersemiotic translation process based on intrinsic relations between translation *source* and *destination*, by contrast with indeterministic intersemiotic translation, as seen in the examples given by Dodge (1984) and Ligeti (1993). Congruently, self-similar features classification according to Rasch (1988), Carey and Clampitt (1989), and Agmon (1989, 1995), provide substantial resources for understanding how both modes of intersemiotic translation, deterministic and indeterministic, interact.

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[456] Subchapter 6.4. provides a basic introduction to pitch class mappings as self-similar polygons and tessellations. The example given in ◊642*a*–*b* corresponds to a whole-tone hexagon and its self-similar mapping as projection of combined divisors.
[457] See subchapter 3.4., on invariance.

6.3. Golden mean

A straight line's segment is divided by the so-called *golden mean*, if the largest subsegment B is related to smallest subsegment C, exactly in the same way that the largest segment A is related to subsegment B:
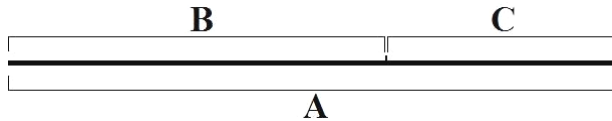
This type of relationship is known as *golden ratio*. As happens with other types of proportion, according to the Euclidean definition of ἀναλογία (see subchapter 3.3.), the golden ratio operates as a self-referential system, provided that what is related as A:B, is equally related as B:C.

If the segment A has a length equal to 1, and the largest subsegment B has a length equal to x, then

$$\frac{x}{1-x} = \frac{1}{x} \ .$$

Thus x is the solution to the quadratic equation

$$x^2 = 1 - x \ .$$

This equation has two solutions:

$$x_1 = \frac{-1 + \sqrt{5}}{2} \approx 0.618 \qquad x_2 = \frac{-1 - \sqrt{5}}{2} \approx -1.618 \ .$$

Since sub-segment B is not related to C in a subtractive, but additive form, the final result must be positive. This result (x) is conventionally denoted by the symbol φ:

$$\varphi\,(x) = \frac{1 + \sqrt{5}}{2} \ .$$

The numerical value of φ is represented by the approximation

$$\varphi = \frac{1 + \sqrt{5}}{2} \approx 1.6180339...$$

This number (1. 6180339...), historically known as *golden number*, can be estimated more exactly by recurrence of the formula $\varphi = 1 + {}^1/_\varphi$:

$$\varphi \;=\; 1 + \frac{1}{\varphi} \;=\; 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}} \; ,$$

and its reciprocal:

$$\varphi^{-1} = [0; 1, 1, 1, \ldots] = 0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$$

(Fenn 2001:22–24).

Congruently, the equation $\varphi^2 = 1 + \varphi$ corresponds to the recursion of square roots:

$$\varphi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}$$

(Walser 2001:91–93).

If this demonstrates the relationship of recurrence between golden ratio and golden number, the self-similarity of the whole system can be observed using again the proportional relationship A:B:C, as consistency of $\{a_1 : b_1 : c_1\} \in \{A : B : C\}$,

,

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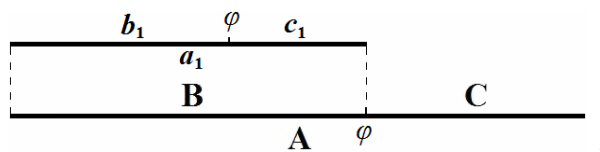
and then, of $\{a_2 : b_2 : c_2\} \in \{a_1 : b_1 : c_1\} \in \{A : B : C\}$, and so on:
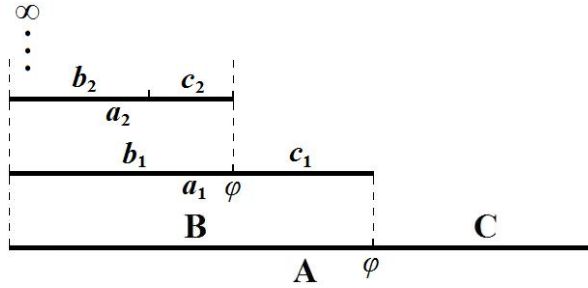
This infinite sequence of proportional relationships shows why, as suggested by Yadegari (1992:69), "self-similarity should be thought of as a portrait of a self-referential entity." The same principle can be applied to the Fibonacci sequence.

Fibonacci sequence

Godrèche and Luck (1990:3774) state that "The Fibonacci sequence is perhaps the simplest of all self-similar structures." The general definition of Fibonacci sequence is as an infinite set of consecutive integers, in which the sum of the first number with the second number equals the third number in the sequence; the sum of the second with the third equals to the fourth number of the sequence, and so on. Thus, the Fibonacci sequence can be summarized in the following recursive, self-referential relationship:

$$k_n = k_{n-1} + k_{n+2},$$

where k_n represents the nth number in the sequence, whilst k_0 is the first number of the sequence, and k_1 is the second number of the sequence. In the case of natural numbers this sequence is called Fibonacci (Fib), in which the sum of the first term with the same first term (1+1 = 2), is added to the result of the sum of the last two terms (1+2 = 3), then added to the result of the sum of the last two terms (2+3 = 5), and so on, obtaining:

Fib = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597...∞

This sequence is peculiar in that the division of any part thereof, between the previous consecutive number, approaches the *golden number* (φ), inasmuch as the divided numbers are larger. The linear projection of this sequence corresponds to the golden mean, as shown in the examples with straight line's segments, and as the recurrence of the formula $\varphi = 1 + {}^1/_\varphi$ converges to the continued fractions 1, 2, ${}^3/_2$, ${}^5/_3$, ${}^8/_5$, ${}^{13}/_8$, ... , or 1, ${}^1/_2$, ${}^2/_3$, ${}^3/_5$, ${}^5/_8$, ${}^8/_{13}$, ...

Constructive organicism

Hilbert and Cohn-Vossen (1932/1952), Mandelbrot (1967, 1977, 1982), Rasband (1990), Schroeder (1991), and Hahn (1998), among other authors, demonstrate how basic relations of symmetry are found in almost all constructive aspects of organic chemistry and biology, as well as in human aesthetic appreciation. Within these aspects, the golden ratio plays an outstanding role.[458]

Natural symmetry shows analogous ways for structural distribution of living tissues, in plants and animals, as a tendency to information consistency in biological patterns (see Livio 2002:109–119). As Campbell (1982:238) notes, "Asymmetry is not the safest strategy for evolution to adopt".[459] Consistency is achieved, thus, via patterns recursion, 'simple' in principle, and made up of few elements and few rules of association. This initial austerity explains the formative self-reference of inorganic constructive processes, with their operational consequences in organic forms. Among the most pervasive effects of this relationship, is structural self-similarity.

Some of the best known forms of golden ratio are self-similar biological structures. A common example is the Nautilus' spiral anatomy, reproduced in most textbooks on organic proportion (see Thompson 1917, Ghyka 1927, Pickover 1988, among many others), and which serves as a model for basic constructivism, drawing a

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[458] In this context, the word 'organicism' must be taken according to the first meaning given by the current edition of the *Oxford Dictionary of English* (2010): "the doctrine that everything in nature has an organic basis or is part of an organic whole." Therefore, 'organicism' is a synecdochic doctrine.

[459] Nevertheless, the productive interaction between symmetry and asymmetry, remarkable in music and in biological systems, is the main issue of subchapter 6.6.

spiral axis within a compact and eccentric sequence of squares, whose length matches with the numbers of the Fibonacci sequence:
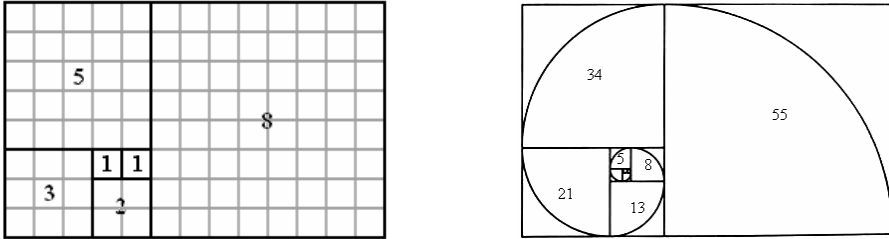
◊630. Left: initial segment 1–8 of the so-called *Fibonacci standard fractal*. Right: drawing of the logarithmic spiral, as growth axis with subsequent segment 1–55. The fractal, with infinite exact self-similarity, corresponds to the sum

$$\sum_{s=1}^{\infty} (k_s)$$

where $k_{s+1}/k_s \approx \varphi$ (k is a number of the sequence, and s is a subscript starting at 1 and going to infinity).

This example illustrates a self-similar development from a first square (with value 1), with its spiral axis logarithmically growing in a harmonious relationship evolving from its point of origin (the *atomic centre* of the first square). The sequence of squares and its associated spiral generates a fractal with exact and infinite self-similarity. Authors such as Zeising (1854), Borwick (1925), Hadow (1926), Ghyka (1927), Borissavlievitch (1958), Scholfield (1958) and Huntley (1970), consider that the most remarkable aspect of this figure—both in its constructivist and aesthetic sides— is the golden mean. However, later works by Rasband (1990), Schroeder (1991) and Walser (2001), explain that this and other proportions characterizing organic and inorganic compositions, as well as human elaborations with analogous aesthetic attributes are, more explicitly, the product of self-referential systems. In short, economy and autopoietic functionalism are the basis of the golden ratio as it is found in nature; the opposite of what theorists such as Zeising, emphasize as the effect above the generating relationship.
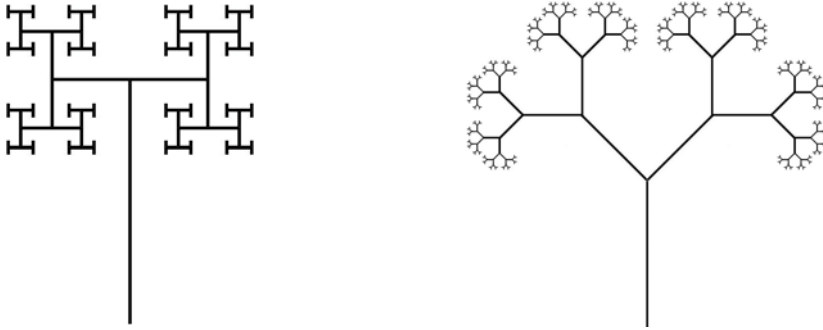
◊631. Two examples of constructive organicism, with radial symmetry in several levels, mimicking functional branchings with the Fibonacci sequence and golden ratio:

Left: Fibonacci tree made after a first straight, vertical line (the longest line, in the centre), with 377 units, forming a T with a first horizontal line of 233 units, coupled to successive T's, with lengths of 144, 89, 55, 34 and 21 units, doubling smaller segments at every constructive step.

Right: Another Fibonacci tree made with the same construction rules as above, this time including a 45-degree affine transformation for each sequence, with nine instead of seven iterations. In this form of recursive construction, the initial motif's minimum variations result in a very different final structure.

The principle of self-replacement in the Fibonacci sequence and golden ratio is easily associated with recursive tasks and organismic self-organization, due to its own trend towards self-similarity and consistency, characterizing the initial steps of the self-replacement process. For this reason, Carey and Clampitt (1996:62) believe that "Notions of self-similarity have often been invoked in organicist explanations of the evolution and unity of musical compositions." According to Mandelbrot (2002:28; with no further reference on the original source), when he refers to the music of György Ligeti, this concept is clearly identified in the composer's work:

> Ligeti [...] described fractals as 'the most complex ornaments ever, in all the arts, like the *Book of Kells* or the Alhambra. They provide exactly what I want to discover in my own music, a kind of organic development.'

One of the motivations shaping the second chapter of the present study is precisely the *transcendental analogy* mentioned therein, which attempts to capture the

structural, functional and symbolic empathy of the power laws expressed in physics and chemistry, regarding biology and biologic epiphenomena such as music and language.

Symbolism and (mis)interpretation

Golden mean, latent in a wide variety of phenomena, and permeating formative aspects of cultures, became a universal symbol of constructive relationships, partially hidden to human intellect. In this cultural development, Dench (1984:30) finds some 'advantages' of the golden mean, above other constructivist symbols:

> Golden section had several obvious advantages over a cabbalistic, gematric numerology, or any other of the esoteric choices. Firstly, it is symmetrical, within limits Fibonacci patterns map on to themselves; secondly, the golden section ratio occurs in nature bewilderingly frequently, in phyllotaxy, plant petal numbers, water movements (Debussy's beloved logarithmic spiral), etc.; thirdly, the psychological sovereignty of golden section proportions had been held from time immemorial.

In consequence, and as noted by Spitzer (1963), Huntley (1970) and Livio (2002), golden ratio's persistence in a considerable amount of aesthetic theories, may be imbued with a strong mystical and religious content.[460] Even the term 'golden' indicates an Apollonian or Christian context in that geometry, music and mathematics are intertwined with the divine. Mystical ascription, made by human enculturation of the golden mean, explains—at least partly—the radical beliefs about its alleged universalism, as Adolf Zeising (1854) attempted to justify as a positivistic notion:

> The Golden mean is a universal law [...] in which is contained the ground-principle of all formative striving for beauty and completeness in the realms of both nature and art, and which permeates, as a paramount spiritual ideal, all structures, forms and proportions,

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[460] Although Spitzer (*op. cit.*) emphasizes the association of the so-called *divine proportion* or *golden ratio*, with Christianity (by association of the golden god, Apollo, with Christ, and with the Platonic ideal of beauty), the discovery of the golden ratio's systematic usage in non-European cultures in Africa, Southeast Asia and Mesoamerica, suggests that this mythical content of Christianity is not unique, but extends to a variety of manifestations of nature, which, observed by humans, acquires magical properties. (This is a central issue in Carpentier's musicological fiction, *The Lost Steps*, 1953).

whether cosmic or individual, organic or inorganic, acoustic or optical; which finds its
fullest realization, however, in the human form.

This idea misinterprets a specific case (the golden mean) of a power law (under
which self-reference occurs in dynamical systems), deeming it as a universal law in
itself. It also reveals a willingness to place the humanity in the centre of the cosmos: a
recurring ideal in the history of religions (see Spitzer 1963:66).

Once Zeising's theory is falsified, a radical skepticism denies almost all aesthetic
qualities of the golden mean, developing new substitution hypotheses. For instance,
Webster (1950:247) believed he had discovered that "The Golden mean is not a form
of universal range, as [Adolf] Zeising (1854) thought, but limited in music almost
entirely to sonata form—though partially present in some examples of other forms."
In this fashion, the golden mean changed, from absolute universalism, to a biased
partiality: the fallacious belief that the golden mean is a phenomenon almost
exclusive to the sonata form.

In this pendular movement of speculations about the golden mean, some balance
seems to be reached when placing self-reference in the centre of autopoietic systems.
So each musical process (or form), depending on its potential for constructive
recurrence, repeats, in successive scales and with varying degrees of exactness, the
original relationships of a more general system. The golden mean would be, then, a
specific case of a wider range of relationships of self-similarity, musical or not.

Golden mean as musicological description

Literature about golden mean in music covers more than fifty years of research,
including works by Webster (1950), Norden (1964, 1972), Nørgård (1970), Lendvai
(1971, 1984), Kramer (1973, 1988), Larson (1978), Bachmann and Bachmann
(1979), Howat (1983a–b), Biles (e1998), Madden (1999/2007, 2005), Cuen (2000),
Livio (2002) and Hofman-Jablan (e2007), among many others. Most of this research
is devoted to statistical calculation and comparing models of symmetry.

Norden, Biles and Livio (*cit.*), for example, note that many musical instruments
are built following the golden ratio and Fibonacci numbers as physical and

operational features.[461] This relationship between instrumental traits and tonal music, seems to be oriented by the same human tendency that seeks for a *transcendental analogy* between physiology and mechanics (and general physics), which in the case of Western tonal tradition adopts the form of *just intonation*.

Lendvai, Nørgård, Larson, Bachmann and Bachmann, Howat, Cuen and Madden (*cit.*), pay more attention to the structural function of the golden mean in scores and recordings from a selection of music of different eras, although mainly from European tradition. They especially consider time units (seconds and minutes) and archetypal musical figures (notes, motifs, periods, phrases, sections, whole pieces), observing changes in instrumental palette and including transitions of timbre, intensity, rhythm and structural function of silences. Much less common is the study of fluctuations in local/global tempi, vertical and transversal relations of harmony, and pitch and scales generation from the golden mean and Fibonacci numbers.[462]

Larson (1978) notes that the golden mean is a structural feature in the Kyrie of the *Liber Usualis*, in Gregorian chant, setting the length of melismas and phrasing. Madden (2005:76–78) extends this approach to composers of the Renaissance and Baroque periods, up to recent times, and concludes that the golden mean is widely used in Western music.

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[461] The piano is the most cited case, with black keys distributed in the range of one or two white keys. Then the black keys are grouped into subsets of two and three. Number five appears as a set of black keys per octave, as well as within the basic relationship of harmonic triad (e.g. from C to G, then from A to E). Number eight appears as the set of white keys completing the octave, and number thirteen as a total of keys completing the chromatic octave. Then 21 appears as a set of white keys that complete the following octave—that is, completing exactly three octaves.

[462] For example, Madden (2005:xi) states that "The use of Fibonacci numbers and the golden proportion […] to establish temporal structure will be discussed in this book, while their use in tonal structure will be discussed elsewhere" (however, Madden does not suggest any source or direction).

pp   fff   ppp
   34   55   89

a)   21   68   81

(21)   (13)   (21)   (13)   (13)   (8)

b)   (21)   (12)   (22)   (13)   (9)   (4)   (7)

21   33   55   68   77   81   88

exposition's end

mutes out; timpani's pedal

climax

mutes' change

celesta's solo

coda

end
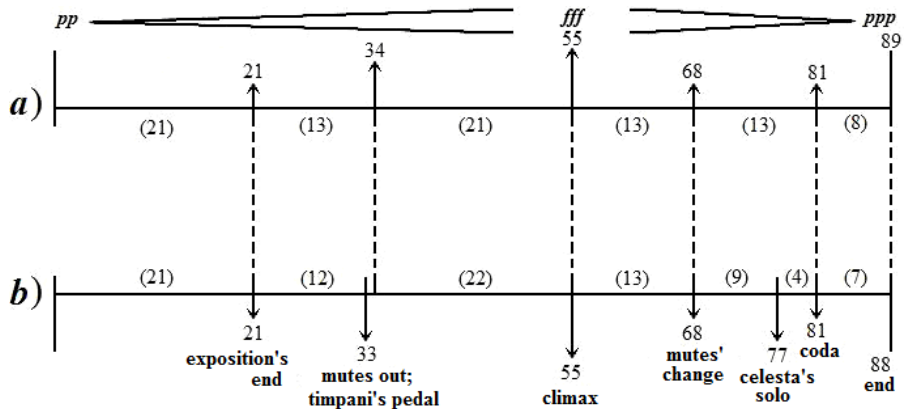
◊632. Howat (1977:287): proportional scheme of Béla Bartók's *Music for Strings, Percussion and Celesta* (1936), movement I in B♭. Golden section falls in measure 55, corresponding to climax (*fff*), with pitch A polarized with respect to the initial A. Horizontal lines represent the piece's bars: *a)* shows Lendvai's (1971) idealized proportions as Fibonacci sequence, *b)* displays the actual proportions of the piece, according to Howat (*ibid.*). Numbers in brackets denote the extension of each structural segment. Small gaps between *a* and *b* in this example are similar to lags in many biological analogies of the Fibonacci sequence.

Lendvai (1971), in his study on Bartók's music, notes that "from stylistic analysis of the music of Bartók, we conclude that the main characteristic of his chromatic technique is obedience to the laws of the golden section." Thus, in his analysis of the fugue of the *Music for Strings, Percussion and Celesta* (1936), he explains that the 89 measures of the movement are divided into two parts, one of 55 bars, and the second of 34 bars, whose division is coupled by the instrumental climax. Other sections of the fugue are also identified by terms corresponding to the Fibonacci sequence. As seen in this type of analysis, the number of elements, classification of relationships and geometric dissections are the common ways to describe a musical pattern.

Howat (1983b) asserts that the golden mean and Fibonacci numbers have also a functional significance in the works of Debussy. For example, in *Reflets dans l'eau*, an integral part of *Images* (1913) piano solo, the first reexposure of the rondo occurs after bar 34 with structural sums of 8, 13 and 21 measures. The proportional

structure of this piece presents a generalized radial symmetry, as happens in *Il canto sospeso* (1956) by Luigi Nono (1924–1990), with two golden segments at the edges.[463]

In many of the cases analyzed by this method, it is evident that the use of the golden mean and the Fibonacci sequence are rather theoretical ascriptions from analysis itself, which in any case reveal a psychological tendency in the original relationships, from which information is distributed in a self-referential scheme. As Smith Brindle (1987:47) notes "Subconsciously, man needs to work in orderly patterns or proportions, and perhaps in sound as equally in vision, these proportions are determining creative factors." Necessity of order in music reflects a preference for arrangements based on reference and similarity, and for structural negotiations between symmetry and asymmetry, leading to harmony or consistency of proportions.

A proof of this generalized trend is the evidence of golden mean in functions and structures in musical repertoires from non-Western traditions, as found in a variety of rhythmic examples from the gamelan music of Bali, Indonesia (see Canright 1990); in the traditional proportion $^{12}/_{20} \sim \varphi$ of the zanza, in the rhythm *gàdà*, from Central Africa (see Arom 1991:627); as well as in the rhythmic foots of traditional music of India, based on the ancient poetic versifying in Sanskrit. This example is documented in the history of Indian mathematics, music and poetry, taking into account methods as old as those developed by Virahanka (around 700 AD), Gopala (around 1135 AD) and Hemachandra Suri (1089–1172 AD), which explicitly mention the sequence 1, 1, 2, 3, 5, 8, 13, 21, 55... and its approximate common divisor in the form of proportion (see Singh 1985).

Whether all the examples mentioned, of traditional use of the golden mean and the Fibonacci sequence, derive from the same, hypothetical, original source in a specific culture, its presence in other cultures, with no direct relationship with the Indo-European ethno-linguistic stem, or any African ethnic group, encourages the notion that the golden mean and the Fibonacci sequence are rather ubiquitous aspects of the human cognition, related to the human appreciation of *natural* cycles and rhythms. This notion is strengthened by the analysis of the ritual songs of the

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[463] See Pareyon 2007a.

K'miai people of Baja California, Mexico (Pareyon 2002). In this case, the permutation of utterances is carefully accomplished by the Fibonacci sequence (see table ◊633). Moreover, as Martínez del Sobral (2000) suggests, the golden mean is an astonishing tendency of Mesoamerican classic architecture and design, with a massive production of objects of ritual and everyday use.

Case study: traditional songs of the K'miai people

The K'miai people are natives of the northern portion of Baja California, primarily grouped in three communities: Juntas de Neji and El Álamo, in the municipality of Tecate, and San José de la Zorra, in the municipality of Ensenada (Ochoa 1994:9). The K'miai language belongs to the Uto-Aztecan family, and speakers constitute one of the five native peoples surviving in the region, along with the Cochimee, Koo-Kah-Pah, Kiliwa and Pai-Pai; like other peoples of northern Mexico, they have a wide repertoire of music connected with native ceremonies, evidently from non-Western origin.

Currently, the K'miai people have less than a hundred members and it is estimated that their culture, including their language, poetry and music, will disappear by 2010 (see Jiménez 2005). The most important genre of K'miai music is formed by songs and ritual dance music, interpreted by a shaman, or together with a dancer, or a group of participants under the shaman's guidance. Often the shaman accompanies his songs with her/his feet tapping, hands clapping, or with a rattle, playing regular pulses. The meaning of lyrics usually integrates elements of sky, land and sea. The text, usually very short, is repeated many times; these repetitions, along with the sobriety of music and outsiders' difficulty in understanding the texts, contribute to a rather poor study and knowledge of this repertoire, beyond its original practice.

In traditional K'miai songs, the repetitions of phrases form structures that were transmitted orally by shamans, from generation to generation. For example, the ritual song *Ña yohap mäshuña, ña yohap mäshuña, ña yohap mikewe* (which in English can be translated as "At sunset, at sunset, we see the sunlight rays") is composed of two phrases that are repeated 57 times, in 18 paragraphs subdivided into two kinds of

recurring paragraphs and one unusual paragraph, plus a final figure ("ha-ha-ha" [58]), which serves as a conclusion.[464] The specific criteria in the two phrases alternation is unknown, but the self-reference of the sentences is clear, developing a sequence that reveals a constructive usage of the golden mean and the Fibonacci sequence: the song starts with the repetition of an identical element (1,1), followed by a complement (2) and a second repetition of the original element (3). This relationship completes a first paragraph, as noted in table ◊633 as 1*a* (*x, x, y, x*), where the number corresponds to each subset enumeration, ordered in this case with the elements *x, x, y, x*. The next subset is completed in 5, and the next literal repetition of the last repetition of phrases (*x, x*) starts at 8. The next analogous iteration (*x, x*) ends at 13. The last relationship (*x, x, x*) reappears in a similar way, at 21. The most prominent relationship of the set happens in line 34, marking the last paragraph change (following paragraphs are just repetitions of previous models). Finally, the last occurrence of alternate phrase (*y*) appears in 55. Rattle beats are subtly slower than the voice's prosody, in a proportion of 4:6 (see musical transcription in ◊633-continuation).

The conclusions of this section would be meaningless if there were not solid evidence suggesting that other K'miai songs have similar structures. For example, in the song *Wha mi yai matiña kuakuri* ("He cries because he goes away"), a single sentence is articulated into three distinct forms, which in turn produces 18 paragraphs (the first one is actually a complement to the latter), containing 55 lines. The first sentence is replaced by eight rattle strokes, at the end of which the singer connects four phonemes 'ɑ, as a conclusion.

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[464] The description of this song corresponds to the compact disc's contents presented by Ochoa (1994), including five ritual songs of the K'miai: *U u mi jat pa mi* (Owl and Coyote cry), *Wha mi yai matiña kuakuri* (He cries because he goes away), *Amj me yawen yangui, wi yango shimey kakap* (I want to go, I am looking for the exit), *Ña yohap mäshuña* (At sunset) and *Jay mi tiña miya home wara* (I woke up crying in the darkness) [with texts translated into Spanish by Gregorio Montes Castañeda].

1	Ña yohap mäshuña		29	Ña yohap mäshuña	
1	Ña yohap mäshuña		30	Ña yohap mäshuña	
2	Ña yohap mikewe		31	Ña yohap mikewe	
3	Ña yohap mäshuña		32	Ña yohap mäshuña	
		1a (x, x, y, x)			1c (x, x, y, x)
4	Ña yohap mikewe		33	Ña yohap mikewe	φ
5	Ña yohap mikewe		34	Ña yohap mäshuña	
		2a (y, y)			8 (y, x)
6	Ña yohap mäshuña		35	Ña yohap mäshuña	
7	Ña yohap mäshuña		36	Ña yohap mäshuña	
8	Ña yohap mikewe		37	Ña yohap mäshuña	
9	Ña yohap mikewe		38	Ña yohap mäshuña	
		3 (x, x, y, y)			7b (x, x, x, x)
10	Ña yohap mäshuña		39	Ña yohap mikewe	
11	Ña yohap mäshuña		40	Ña yohap mikewe	
		4 (x, x)			2c (y, y)
12	Ña yohap mikewe		41	Ña yohap mäshuña	
13	Ña yohap mikewe		42	Ña yohap mäshuña	
14	Ña yohap mikewe		43	Ña yohap mikewe	
15	Ña yohap mäshuña		44	Ña yohap mäshuña	
		5 (y, y, x, x)			1d (x, x, y, x)
16	Ña yohap mäshuña		45	Ña yohap mikewe	
17	Ña yohap mäshuña		46	Ña yohap mikewe	
18	Ña yohap mikewe				2d (y, y)
19	Ña yohap mäshuña		47	Ña yohap mäshuña	
		1b (x, x, y, x)	48	Ña yohap mäshuña	
20	Ña yohap mikewe		49	Ña yohap mäshuña	
21	Ña yohap mikewe		50	Ña yohap mäshuña	
22	Ña yohap mikewe				7c (x, x, x, x)
		6 (y, y, y)	51	Ña yohap mikewe	
23	Ña yohap mäshuña		52	Ña yohap mikewe	
24	Ña yohap mäshuña				2e (y, y)
25	Ña yohap mäshuña		53	Ña yohap mäshuña	
26	Ña yohap mäshuña		54	Ña yohap mäshuña	
		7a (x, x, x, x)	55	Ña yohap mikewe	
27	Ña yohap mikewe		56	Ña yohap mäshuña	
28	Ña yohap mikewe				1e (x, x, y, x)
		2b (y, y)	57	ʼaʼaʼa	

◊633. Pareyon (2002): schematic view of sentences and paragraphs of the traditional K'miai song *Ña yohap mäshuña, ña yohap mäshuña, ña yohap mikewe* ("At sunset, at sunset, we see the sunlight rays"). The two constituent phrases are symbolized by *x, y*; paragraphs are indicated by numbers and a letter, expressing the modes of alternations (e.g. 2*b* is a recurrence of 2*a*). Golden mean and Fibonacci sequence numbers (in bold characters) have a structural function in this example.

◊633. (continuation) Transcription of the K'miai song *Ña yohap mäshuña, ña yohap mäshuña, ña yohap mikewe*. The part of the rattle is played by the singer himself, traditionally a shaman. The fragment corresponds to sentences 1—11, paragraphs 1*a*—4. N.B.: accidentals preceding a musical note are valid for the following notes in the same position, within the same measure.

The possibility that the usage of the golden ratio and the Fibonacci sequence is deliberate in these examples is documented in Pareyon (2002) as a result of a relationship between the pentacimal system in K'miai numeration—usual in many other groups native to North and Central America—along with a culture of observing the lunar cycles (i.e. cycles of seven days in a year of 52 weeks). The number zero between the Kiliwa—K'miai's cousins—is called *Nyiew halah*, "Black Moon", and is represented with a closed fist (see Ochoa Zazueta, 1978). Thus, following the association of lunar cycles with the pentacimal system, numbers such as 52 or 57 (i.e. 52+5) would be more relevant than the 55 of the Fibonacci sequence. However, the emphasis of the sequence 5, 8, 13, 21, 34, 55 in K'miai songs, remains to be investigated more thoroughly.

Compositional theory after Debussy and Bartók

For some authors, the selection and musical adaptation of an abstract relationship, for example, of numeric, geometric or algebraic kind, is not part of any compositional method, but of a 'pre-compositional' method. Dench (1984:29–30) claims, thus, that Howat's (1983b) study on golden ratio in Debussy's music,

> is an investigation, not of Debussy's compositional thought, but of what has become known as his pre-compositional method. [...] What Howat is concerned with in his book is a pre-compositional system, which distinguishes it from the compositional systems of, for example, Schoenberg or Messiaen.

However, the distinction between the *pre-compositional* and *compositional* method or system, is not entirely clear. Dench, using such a distinction, seems to refer more to an explicit system, implemented by composers such as Schoenberg or Messiaen, by contrast with other, implicit (tacit) systems, as implemented by Debussy. What is relevant here is consideration of a lesser or greater degree of awareness and explicitness of Gestaltic relationships of the musical sound, in order to consistently identify a greater or lesser degree of tension between grammar and style—according to what is stated in subchapter 4.7. of this study. This relationship must be considered the same for the processes of composing and performance, rather than for systematic musicology.

Joseph Schillinger (1895–1943) is known for being one of the first theoreticians who proposed a formalized employment of golden mean and Fibonacci sequence, for determining all the parameters of a musical composition. His compositional 'system' (1946) is comparable—by constructivist goals, but not by aesthetic development—with Webern's integral serialism. Schillinger elaborates routines and mechanisms of structural symmetry and scales formation, as shown in ◊634*d*—that Nono partially used in *Il canto sospeso* (1956), as an independent development of Schoenberg's serial theory. Schillinger, pursuing the same constructivist goals that he initially tried to establish, eventually used different versions of the Fibonacci sequence, mainly the Lucas sequence or *Lucasian series* (2, 1, 3, 4, 7, 11, 18...) inverting the first relationship of sum in the sequence (2, 1 instead of 1, 2). Although Backus (1960:232) believes that Schillinger's numerical conceptualization is a "fraud" [*sic*], in the sense that it has

"no scientific or mathematical foundation", the work of Schillinger must be judged rather in the light of the 'naturalist' tradition (see subchapter 4.1.), in search of a transcendent metaphor that places musical determinism and indeterminism within a broad framework of negotiation, as suggested here, in subchapter 4.8. In this context, Schillinger also contributes to the rational and structural usage of *antiproportion*, as explained in subchapter 6.6.

Schillinger's theoretical work, developed from 1920 to 1940, fruitfully occupied a generation of scholars, particularly from the 1950s well into the 1980s. This includes Backus' article (*cit.*), entitled "Pseudo-science in music", which criticizes the lack of care with which Schillinger mixes musical ideas with acoustics and mathematics (e.g. confusing concepts such as *loudness*, of psychoacoustic kind, with *amplitude*, of a physical-mathematical character; or confusing *series* with *sequence*). Backus also notes Schillinger's over-enthusiasm for numerology, subsumed under mathematical inaccuracy. However, and despite these errors, Schillinger's work marks a first step in trying to understand interrelated aspects of physics and mathematics from modern musicology.

In addition to Nono, Stockhausen and Ligeti, other twentieth-century European composers, following a tradition laid out by Debussy and Bartók, used the golden mean and the Fibonacci sequence to shape their works. Among them, Hugo Norden (1909–1986), professor of composition at Boston University, and Per Nørgård (1932–), one of the most active Danish composers of his generation, meaningfully contributed to improving Schillinger's theory, with explicit methods for implementing these resources as mathematical transformations, which include sequence modulation from numerical remainders, and affine transformation from geometric progressions. In this way, the transition between 'pre-compositional' and 'compositional' systems, using Dench's (1984:29–30) criteria, becomes more clear.

Norden (1964, 1968, 1972), who includes these issues as central topics in his lessons on composition and analysis, also published literature on the employment of the golden mean in a musicological perspective, including examples from J.S. Bach (see comparative summary in Madden 2005), and stressing the relationship between compositional thought and the implementation of these tools, leading to a

reassessment of Schillinger's theories—accessible to musical intuition, although, as criticized by Backus (1960), often with a lack of conceptual neatness.

Nørgård, for his part, developed his idea of 'infinite series' (*Uendelighedsrækken* in Danish), to serialize pitch, harmony and rhythm in his *Second Symphony* (1970). This method takes its name from the recursive operation that generates intervals simultaneously in pitch scales and time series. The first terms of its simple form are 0, 1, −1, 2, 1, 0, −2, 3..., a variant of the Fibonacci sequence, which also produces patterns of self-similarity whilst combining its different methods of implementation.

Kevin Jones (1981, 2000) continues this development by combining probability and statistical self-generation methods, using sequences of numbers, geometric series and Fibonacci trees. More recently, Haek (2008) explains his technique for using cyclic remainders of Fibonacci sequences in modulo *m*, as a method of serial composition—considering a variety of musical parameters from Norden (1964, 1968, 1972, 1976) and Nørgård (1970, 1987).

Schemes ◊632 and ◊634 summarizes some of the most common operations of extension and time distribution; configuration of sections of a musical structure, and construction of musical scales, under the golden mean and Fibonacci numbers. However, the variety of applications and extensions of this theory is currently so wide, that Atanassov *et al.* (2001:39) believe that "the formidable quantity and quality of research in this mathematical area generate one world!" Such a 'world' is originated in a basic principle: the fact that the standard additive relationship of the Fibonacci sequence can 'translate' into families of associated relationships of numbers—as evidenced by Nørgård (1970), in a first example with 'infinite series'. Although the main objective of Atanassov *et al.* is mathematical development, many of the topics included in their treatise have immediate or potential relevance for music theory.

a)

k (2541) m (1686)

sections:	I	II	III	IV	V	VIa	VIb	VII	VIII	IX
measures:	936	400	569	636	408	360	400	750	448	488

$1 = \flat$

l (2554)

b)

α (239) β (124) γ (242)

sections:	I	II	III	IV	V	VIa	VIb	VII	VIII	IX
measures:	107	50	82	45	34	45	50	75	56	61

90
132

$-\varphi$ (bar 231) φ (bar 373)

c)

$$21 + 65 + 21 = 107$$

21	65	21
	(5 + 55 + 5)	
	(86)	

$-\varphi$ φ

φ = bar 68 (end of period IX, beginning of period X; change of tempo, instrumentation and dynamics, from *ppp* to *fff*).

d)

0+1 (semitone)   0+3   0+8
(0 = C♮)   0-2   0-5   0-13

◊634. Proportional analysis of Luigi Nono's, *Il canto sospeso* (1956):

a) General structure by metrical units (i.e. semiquavers).

b) General structure by number of measures.

c) Symmetry of the first movement (orchestral introduction) by measures: major structure (*a–b*) encompasses minor structure (*c*) under the same system of proportions.

d) Counting semitones model, based on Schillinger's (1946), with chromatic intervals as Fibonacci sequence.

(From Pareyon 2007a:71–72).

Generation of scales and intervals

The generation of intervallic systems based on Fibonacci sequences reaches a wide range of possibilities, from the simplest geometric structuring of these sequences (as suggested in ◊635), to the progression of Fibonacci *words*—variants of the standard sequence—including self-replacement of strings as explained in subchapter 6.5., and the usage of modular remainders from the sequences (see Haek 2008), such as the remainders from a *golden tessellation*.[465] As an example of a first case, let the following geometric structure (see ◊630-*left*) be the representation of a starting isometry $^1/_1$, with subsequent divisions of the length of the squares ($^1/_2$, $^1/_3$, $^1/_5$, $^1/_8$...) by a constant proportion $\sim \varphi$. Unlike series of squares gradually increasing their length, in this case the lengths decrease, allowing to control the interrelationship of elements within a limited grammar.
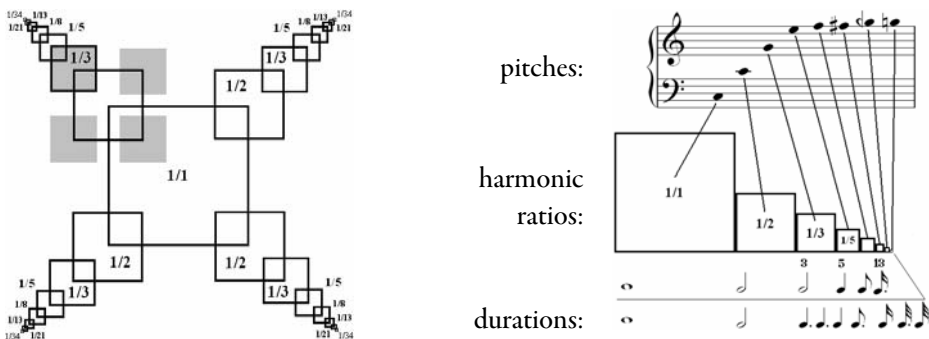
◊635. Left: Golden succession of squares as consecutive division of lengths of each side of an element (represented as $^n/_n$), according to the Fibonacci sequence. Gray boxes suggest overlapping areas in subsequent division, as segmentation of a self-similar sequence (a pattern useful for grammar self-structuring in music).

Right: Golden sequence of squares as harmonic scale, approximately represented in the staff, with a group of associated durations.

In principle, many data may be taken from these sequences, to become part of musical expressiveness. The musical usefulness of such data greatly depends on how sequences are bounded into a minimum of basic relationships, establishing a

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[465] See subchapter 6.4.

relatively simple method likely to be associated with a musical *message*. In a similar way a code elaborates information within a limited repertoire of symbols producing meaning in speech, a code of sounding symbols based on a first relation of proportion and self-similarity, can lead to a pre-musical ordering, as occurs in ◊635.

Furthermore, the arithmetic properties of the first terms of the Fibonacci sequence are also useful for musical systems of distances and intervals, and scales. In particular, number 144 has interesting qualities for self-constructivism: this number is—at least within a very large 'initial sample' of the Fibonacci sequence—the only one with a perfect square root (see table ◊636), and has close relationships with significant cardinalities 96, 48, 32, 16, 12, 8, 6, 4 and 3, typically associated with the algebra of neo-Riemannian operators (see Cohn 1997).

$12 \; (= \sqrt{144})$	Chromatic scale (and its *harmonic perspectives*).
$^{144}/_{128} = 1.125$	Size of whole tone ($^9/_8$).
$144 - 16 = 128 \; (^{128}/_{16} = 8)$	Double of a frequency divided by eight (the 'octave', in conventional terms).
$^{144}/_{16} = 18, \; ^{18}/_{12} = \; ^3/_2$	Relationship of a frequency plus its half ('perfect fifth', in conventional terms).
$^{144}/_{96} = \; ^3/_2$	'Perfect fifth'.
$^{12}\sqrt{144}$	'Fifth'.
$^{128}/_{96} = \; ^4/_3$	'Perfect fourth'.
$144 - 96 = 48$	Eighths of tone within a doubled frequency or 'octave'.
$^{96}/_8 = 12$	(96) Sixteenths of tone within a doubled frequency (8) or within a chromatic scale (12).
$^{144}/_8 = 18 \; (^{18}/_{144} \rightarrow \; ^5/_4)$	Relationship of a frequency adding the half of its half ('major third', in conventional terms).
$144 = 72 + 72$	Perspectives of tonal empathy. The ratio $^{144}/_2$ refers to 72 'consonances' and 72 'dissonances' of tonal harmony (see Mazzola 1990).
$\log 144 = 2.15836 =$ $(\log 12) \times 2 = (1.07918) \times 2$	$\log 12 \approx$ great limma $^{27}/_{25}$ (1.08), between Pythagorean double sharp $^{1125}/_{1024}$ and apotome $^{2187}/_{2048}$.

◊636. Some constructive self-referential properties of Fibonacci number 144, in relation to the chromatic scale and other harmonic intervals, from Pythagorean and just intonation.

Modular bounding

The Fibonacci sequence is characterized by starting with the sum of two integers. This has structural relevance, since sequential self-structuring comes from its initial sum, originally under the recursion, *a + b, b + b*. However, other versions of the standard sequence can start with any pair of integers *x, y* so that one can create an infinite variety of Fibonacci sequences. Consider a few examples:

standard Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...
Lucasian sequence: 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322...
Evangelist sequence: 3, 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, 343...
9φ sequence: 9, 14, 23, 37, 60, 97, 157, 254, 411...
31φ sequence: 31, 52, 83, 135, 218, 353, 571...

Consistently, other sequences starting not with the sum of two integers, but with the sum of three, can also be arranged:

0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705...

as well as other sequences starting with the sum of four integers:

0, 0, 0, 1, 1, 1, 3, 6, 11, 21, 41, 79, 152, 293, 565, 1089, 2099... .

Following the same logic, other sequences starting with the sum of *n* integers are possible. Also, for the sake of richness in the treatment of these sequences, all of them can be considered as invertible: any f_n Fibonacci sequence modulo *m* can be inverted to form an *m* (f_n modulo *m*) sequence conserving initial properties of self-reference and proportion (see mathematical proof of this proposition in Mongoven e2004:13). This property of invertibility is also important because permits forming musical series that can be (re)processed according to the four basic operations of symmetry (see subchapter 1.3.).

An obvious problem in using these numerical sets arises with the difficulty of giving musical meaning to intervals such as [*a, b*], when *a, b* are numerals with more than three digits. Given the additive properties inherent in their own build-up, after a few steps starting a Fibonacci sequence, very large numbers (i.e. hardly accessible to memory and intuition) may appear; conversely, in musical practice there is always

more preference for simple numbers—for example, to express relationships of metre, rhythm or harmony (see Helmholtz 1863, Barlow 2001). In this context, the use of remainders modulo *m* is the most productive method in the generation of operational intervals and functional scales, by virtue of shifting the criterion of bounding infinite series, into a manageable 'window' (a practicable field of intervals).

A very simple example, useful as a primary method of modular bounding or reduction, is the translation of the standard Fibonacci sequence (0, 1, 1, 2, 3, 5, 8...) into a binary modulo (0, 1):

Fibonacci sequence (bin): 0, 1, 1, 10, 11, 101, 1000, 1101, 10101...

This sequence makes it clear that the same information can easily be (deterministically) intersemiotically translated, as rhythmic pattern:

This example shows that numbers such as 13 and 21, become represented as a sequence of four articulated pulses (the last four quavers of the sample), instead of forming subsets of 13 or 21 items. There are obviously other methods of modular reduction and many other musical applications related to them. For instance, Wilcox (1992) and Mongoven (e2004) emphasize the possibility to generate Fibonacci sequences modulo *m* being adapted from pitch-class set theory—i.e. treating the modular remainders as ordered pairs and pitch classes. Haek (2008), in his turn, selects specific options to enrich serial techniques, and states that "it is indispensable (and productive and rewarding) to consider the Fibonacci numbers in light to traditional serial techniques. By doing so, I hope to (re)introduce serial research and compositional practice to the avid energy of mathematical Fibonacci research" (Haek, *op. cit.*:34; brackets in this quotation are the author's own).

Due to typical recurrences of self-referent systems, Fibonacci sequences modulo *m* intrinsically complete a certain cyclic remainder, and a numerical pattern reappears over and over again. In general, each Fibonacci sequence, as self-referential chain developing from a first set of integers, can form a sequence of integers whose remainders constitute an ordered class of remainders (Haek, *op. cit.*: 36). For example, the first twelve digits of the Fibonacci sequence modulo 8 are: [0, 1, 1, 2, 3, 5, 0, 5, 5,

2, 7, 1], whose ordered class of remainders is [0, 1, 2, 3, 5, 7]. The ordered class of remainders of a Fibonacci sequence is always a structural summary of the sequence. So, in this example, numbers 4 and 6 of set [0, ..., 8] are excluded. Insofar as the elements of the series increase, the terms of the ordered class of remainders decrease proportionally. This means that a very large collection of terms can be characterized by an ordered class of remainders, much less extensive than the number of terms in the sequence itself. Haek (*op. cit.*) employs this feature to show how the number of remainder cycles associated with each modulo, proportionally decrease each time a segment adds to the total of counted cycles. Haek, motivated by a renewal of the serial techniques, connect operations of permutation, partition, transposition, inversion and rotation of series, as a means of structural development.

Haek (2008), investigating the intrinsic properties of recurrence and self-similarity of the Fibonacci sequence modulo *m*, also discovers a family of qualities in the remainder cycles, such as the emergence of internal 'rhythms' of numbers—or numerical *words*—characterized by the recurrence of a digit; or as self-generation of numerical palindromes, synecdoches and chiasmi. According to Haek (*op. cit.*: 52), these qualities offer a limitless supply of new perspectives for a renascence of serial techniques. It also favours—one may add—an enrichment of the analytical techniques related to the basic operations of musical symmetry.

Biles–PGA scale

John A. Biles (e1998), adapts an idea of Peter G. Anderson (PGA),[466] to use the Fibonacci sequence for generating a musical scale, developing the Nørgård's concept of 'infinite series' or *Uendelighedsrækken* (see Nørgård 1970, 1987). Biles employs the so-called Fibonacci 'partition function', v_j, that computes the number of forms in

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[466] Both Biles and Anderson are researchers at the Rochester Institute of Technology, Rochester, New York.

which a non-negative integer j can be represented as the sum of distinct Fibonacci numbers. This sequence is defined as the coefficients

$$v_0, v_1, ..., v_{F_{n+2}} - 2,$$

such that

$$\sum_{j=0}^{F_{n+2}-2} v_j x^j = \prod_{k=2}^{N} \left(1 + x^{Fk}\right)$$

where F_n is a Fibonacci number, and

$$F_{n+2} - 2 = F_2 + F_3 + ... + F_n$$

is the derivation of the corresponding sequence. This version of the Fibonacci sequence also starts 1, 1, 2, followed by 1. Then each of the previous elements is added to 1, which makes 1+1, 1+1, 2+1, 1+1 = 2, 2, 3, 2. A following element 1 is inserted in the middle of the collection: 2, 2, 1, 3, 2. Taking this segment as a second segment of the sequence (the previous one was 1, 1, 2, 1), the sequence results 1, 1, 2, 1, 2, 2, 1, 3, 2. The next step applies the same production algorithm, adding 1 to the second digit, instead of inserting it (*moving by |1| intervals distributed for each new segment) and setting the previous subsegment, that for each step includes 4×n digits (8, 12, 16, etc.): 1, 2, 1, 2, 2, 1, 3, 2. For this case: 1+1, 2+1, 1+1, 2+1, 2+1, 1+1, 3+1, 2+1, makes 2, 3, 1, 3, 3, 2, 4, 2. By adding this result to the initial segment, the second part of the sequence is obtained: 1, 1, 2, 1, 2, 2, 1, 3, 2, 2, 3, 1, 3, 3, 2, 4, 2. In order for obtaining the third part, the same algorithm is invoked (this time with 12 numbers), obtaining (*applies again):

1, 1, 2, 1, 2, 2, 1, 3, 2, 2, 3, 1, 3, 3, 2, 4, 2, 3, 3, 1, 4, 3, 3, 5, 2, 4, 4, 2, 5, 3, 3, 4, 1...

whose first 140 digits correspond to this plotting:
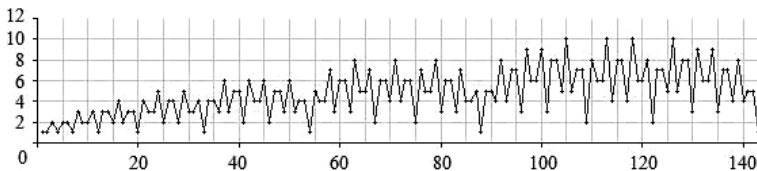
◊637. Pitch scale created by John A. Biles (e1998), based on the Fibonacci sequence. Each new segment of the scale copies the previous segment, moving one bit upwards for each step, and placing a copy in the middle of the latter segment, also moving one bit upwards for each step. The whole scale completes a self-similar structure (example from Knott e2009).

As Knott (e2009) notes, in this scale, each section begins and ends with a copy of the previous section, moving by |1| intervals distributed for each new step, and adding for each segment a copy of the last segment, moving upwards again. The ending of each aggregation is identified by structural | 1 | setting along the sequence.

Biles (e1998) theoretical proposal is accompanied by a musical motivation: "One goal of the algorithmic composer is to make mathematical self-similarity musically meaningful." Biles associates preset pitch-classes to form an initial hexatonic scale (six elements per octave, skipping the fourth grade) to develop a consecutive series of three notes for each half of the scale. The projection of this formula, the beginning of a specific musical structure—in this case an experimental work of Biles—results in a repetition of a *primary form*, product of a first modular bounding (if the sequence shown in ◊637 is applied directly to music, the result would be too simplistic, on the one hand, but also impractical, on the other, because of the increasingly open intervals). Such a bounding is basically determined by the author's choice, on instrumental decisions similar to any other of the compositional implementing processes. At the same time, subject to the evolution of the geometric progression, *primary form* is gradually extended in each cycle, increasing speed and counterpoint values (in fact, Biles simultaneously implements a variant of the scale, to develop a melodic counterpoint with the original form. In parallel, Biles creates a succession of harmonic relationships, ranging from modal ambiguity to a classical major tonality, which then gradually transforms into a texture through minor and pentatonic scales, which in turn are dispersed in a growing atonality. "Finally—Biles says—the tension is released by returning to a major tonality for the final elaboration of the form." (Biles, *ibid.*).

This summary makes evident that algorithmic tools arising from the golden mean and the Fibonacci sequence, depend upon the same universal principles of rigidity of the code and flexibility of the message. The composer, thus, acts as mediator between his own production (rigid) model, and additional structural decisions—as in this case of adaptation of a plan for contrapuntal and harmonic transitions.

Bohlen 833 scale

Although originally formulated on a combinatorial approach of pitches, the Bohlen 833 scale (Bohlen and Pierce e2009) has geometric properties related to the golden mean and the Fibonacci sequence. It contains a network of harmonic relationships with the property to match harmonic intervals cycles of 833 cents. For every ten steps in the scale there is also a coincidence with the octave respecting the base-pitch, and, at the same time, with the golden mean in the third step. By using this scale, harmonic series as multiples of 833 can be easily built, obtaining sound complexities within a palette of timbral resources.

step	proportion	cents' value	cents' step
0	1.0000	0	–
1	1.0590	99.27	99.27
2	1.1459	235.77	136.50
3	1.2361	366.91	131.14
4	1.3090	466.18	99.27
5	1.4120	597.32	131.14
6	1.5279	733.82	136.50
7	1.6180	833.09	99.27

◊638. Bohlen 833 scale (after Bohlen and Pierce e2009), combining a harmonic distribution for each tone of the scale, with the golden mean in a generalized relationship of self-similarity, both between scale units and between successive octaves.

This scale is based on the fact that the orderly distribution of intervals of 833 cents within an octave (1200 cents), is analogous to the multiplication of 0.83333×12 as total of semitones per octave. Simultaneously, the distribution of intervals having this hierarchy is also related to Zipf's distribution, so the Bohlen 833 scale is analogous to the system of melodic steps outlined in subchapter 5.4. (see especially graphs ◊548 and ◊551).

Fibonacci asymmetric trees

Whether Fibonacci trees, such as those shown in graphics ◊631, are quite common examples of radial symmetry as a whole and as constructive layers, mathematics also conceives asymmetrical Fibonacci trees. These models are commonly used in biology—mainly in taxonomy, genealogy and phyllotaxy—to explain asymmetry in generative relationships and systems of reproduction, as well as in chemistry and physics, to schematize the energy levels of an electron in a hydrogen atom (see Huntley 1970:156–163).

The scheme ◊639 presents an asymmetrical tree with ten horizontal levels. Each level corresponds to a Fibonacci number, in this case 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. At first glance, it is noticeable that the first derived systems are related to the overall aspect of the whole. A closer look also reveals that the numerical and distributional properties are linked to other, typical relationships of the Fibonacci sequence. For example, for each level, the number of black pieces corresponds to a Fibonacci number, whilst the number of white pieces corresponds to the preceding Fibonacci number.

Walser (2001:69–71) investigates the close relationship of this tree with the golden ratio, and he finds that, for the nth stratification there will be a_n black pieces and a_{n-1} white pieces. Proportion between black and white pieces tends, in (Fib$n$, ..., ∞) to $\varphi = {}^1/_\rho$, where φ is the golden number (1.6180339...) and ρ represents the difference between φ and $\sqrt{5}$.

Fibonacci asymmetric trees are evident in various aspects of music, including stylistic probability systems, systems of recursion and harmonic hierarchy, and relationships of consistency and asymmetry in rhythm and metre. A very general example of this is the probabilistic trees studied at the beginning of Chapter 5 (especially the model shown in ◊511).
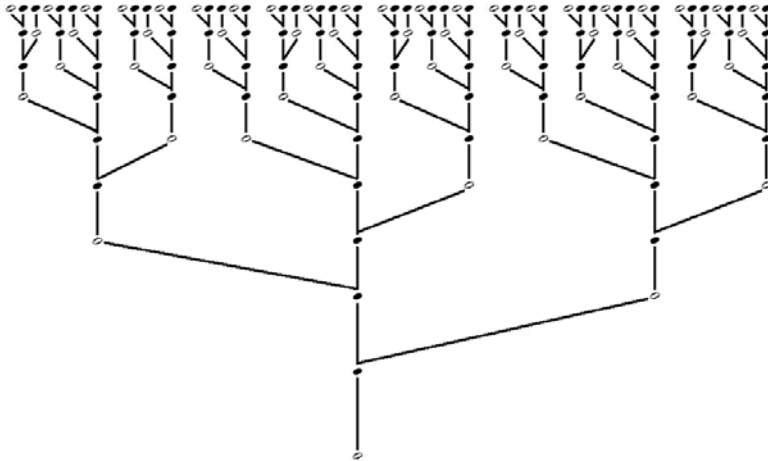
◊639. Fibonacci tree with ten horizontal levels, each of which may have one or two branches. Each level corresponds to a Fibonacci number, in this case 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. The whole tree is asymmetrical and self-similar. In this case two types of metric values were chosen (white pneumas as *long*, and black pneumas as *short*). The longer set by level has 55 elements, and extends asymmetry and proportion from previous levels. Jones (2000:5) suggests that this type of metrical organization is common to music and poetry in some of their most predictable cases (within a variety of Western traditions, at least).

Lerdahl and Jackendoff (1983:214) also use a branching model to represent a musical recursion, explaining the structural subordination to a main stem, of other secondary systems. Asymmetry in this simple model (see ◊441*d*) is analogous to the asymmetry observed in ◊639, in a more extensive schematization. Moreover, Jones (2000) suggests that self-similarity and asymmetry in these systems, are also common to metrical models in traditional versification and in vocal and instrumental music.[467] Jones (*op. cit.*:8) also refers to the constructive relationship between Fibonacci asymmetric trees and symmetric prolatio (see ◊545), suggesting that avoidance of the 'binarism'—too obvious in body symmetry and in many aspects of musical practice (see Chávez 1961:34), constitutes a 'subversive' trend, basic for a creative developing of styles and repertoires. Jones (*op. cit.*) adapts the metaphors 'safe' and 'dangerous' to, respectively, refer to such a *binarism*, and to *Fibonacci asymmetry*, formulating a

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[467] The examples given by Jones include metrical structures of limericks, as well as Scott Joplin's *Maple Leaf Rag*, for piano. The structural similarities with the Fibonacci asymmetric tree are remarkable.

hypothesis on the intuitive coordination of these two notions. The same concept of balance between symmetry and asymmetry can be attached to the link between *rigid* and *flexible*, and to the operational bonding between *tension* and *relaxation*, as the main engine of musical systems.

Absence of golden ratio

Lanza (2006:82), when criticizing Madden's book (2005) on golden ratio 'in musical form', notes that "the writer has focused more on discussing examples where the [golden] proportion is not used, or even not approximated." Madden's purpose is to show how, in a selection of examples from European classical music, there may or may not be a tendency to use the Fibonacci sequences and the golden ratio. This notion is excessively general.[468]

Tovey (1935:I, 19) suggests that "there are so many ways of taking [the] sections [of a musical piece] that I doubt whether any musical composition can avoid golden ones somewhere." Conceiving the self-referential nature of the harmonic series—a nature also evident in the golden mean and the Fibonacci sequences—it is clear that many structural aspects of music follow similar relationships, affecting musical structures and processes, without strict necessity for ubiquity of the golden mean or the Fibonacci sequences, over more general aspects of musical self-similarity.

As self-referential systems, systems of proportions are unavoidable in most of musical construction processes. In this generality, the golden ratio is one of the most noticeable systems, but—taking into account the abundance of possibilities to lay out a first generative self-reference in a musical system—cannot be the only form of structuring musical proportion. Demaree's thesis (1973) presents one of the many solid arguments that confirm this condition; Demaree reports a variety of

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[468] Reading Madden's (2005) book, one infers that the author is aware of this oversimplification; his goal is to demonstrate that, in music, there is indeed a structural tendency to employ the golden ratio and the Fibonacci sequence, albeit with numerous exceptions. The problem in Madden's statistical challenge, is that he imposes his own idea of such employment: the usage of the golden ratio and the Fibonacci sequence appears as more important than the recursions and the self-referential constructive processes of the music he analyses. This analytical method—inadequate in the light of its results—is discussed in the Introduction of the present study, as the weakness of a musical investigation from the 'engineering' perspective, only interested in structural pseudo-problems.

proportions in string quartets by Haydn, though none of them connected exactly with the golden ratio (see Demaree 1973:19; something that, partially, has already been noticed by Webster 1950:241–242). In conclusion, not all music requires nor provides a golden ratio. In Kramer's (1988:320) words:

> Not all music uses the golden ratio, and a considerable number of pieces, even those that strike us intuitively as well balanced, seem to have no consistent proportional schemes at all. What does such music tell us? That proportions are, after all, irrelevant? Surely not, for all music has proportions of some sort, whether representable by simple and consistent ratios or not. We do react to proportions that seem just right, and to proportions that seem wrong.

Therefore, for the study of the signs in music as language, self-structuring processes, resulting from functional self-referentiality, are more relevant over any preset ratio. The general possibilities for the golden mean depends on the same functional self-referentiality, in that the reference of the unit with the unit iself (i.e. a repeated element or relationship), plus the recurrence of the reference, turns into a constructive succession with potential self-similarity.

Constructivist self-referential processes in music are also diverse. They can be established from any succession of prime numbers as a reference from themselves and from the unit, to generate an analogous sequence, with or without a convergent divisor (see Barlow 1987). They can also emerge from the repetition of any geometric relationship and from the development of affine transformations (see Amiot 2003); or even from the accumulation of random values in a chaotic process in which a state x can be explained only by the immediately previous state, and not by the common source of their relationships—something that happens in *Brownian motion* or *Lévy flight* types of music, explored in Chapter 5.

6.4. Tessellations and brocades

The word 'tessellation' denotes "an arrangement of shapes closely fitted together, especially of polygons in a repeated pattern without gaps or overlapping".[469] A clear example is the Sierpiński triangle (see ◊330*a*), also a typical case of spatial self-similarity.[470] Scientific applications, however, may conceive a tessellation also as a geometrical method, including prescriptive, comparative and descriptive grammatical features.

David Lidov (1941–), composer, music theoretician and author of research on the relationships between language and music, conceives that tessellations are somehow comparable to language in its broadest sense, in terms of form, expression and sign organization (see Lidov 1999:206):

> A mosaic allows an analysis similar to language. A first articulation divides the work into figures (like morphemes). The second articulation is the tessellation with 'meaningless' stones or tiles (standing in for meaningless phonemes). It may be that just half a dozen or fewer types of tiles account for the whole work. [...] The analogy between the articulatory structures of mosaic pictures and language is not perfect but holds in some depth.

The first systematic explanation of the set of mosaics known as Archimedean tessellation, composed of regular polygons which can form up to eleven different cases, is attributed to Johannes Kepler (1571–1630)—mentioned throughout this study for his theories of proportion and harmony. Kepler's book, *Harmonice Mundi* (1619), presents one of such eleven cases. In particular, this tessellation is composed of four shapes, all of them related to the golden mean by the relationship with number 5, with √5, or by multiples of 5, forming pentagons, pentagrams, decagons and double decagons. For Kepler, this is a manifestation of harmony, which he relates—at the same time—to music as aesthetic phenomenon, to the geometric description of music, and to the spatial properties of a natural geometry (see e.g. Livio 2002:155–156; also Bailey e2005).

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[469] Source: *Oxford Dictionary of English* (2010).
[470] For a discussion on the aesthetic properties of the Sierpiński gasket, see Pickover 1990.
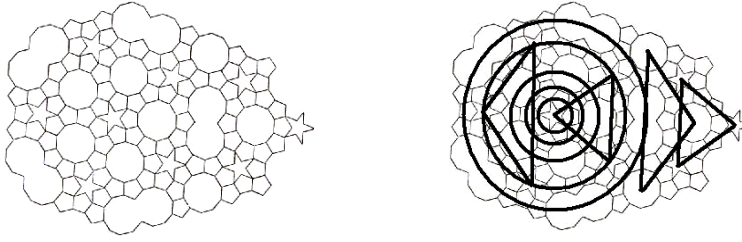
◊640. Left: Johannes Kepler's *Harmonice Mundi* (1619): tessellation with regular polygons, coupling pentagons, pentagrams, decagons and double decagons.

Right: The same figure highlighting its most obvious repetitions and internal symmetries. The five eccentric circles indicate isomorphisms through the relationship of sets of five equally spaced elements, each within the other (i.e. automorphisms as described on page 48).

Kepler's tessellation (◊640) synthesizes several of the distinctive aspects of his harmonic-astronomical theory, as noted by its numerology: This model involves arithmetic relations of 2, 3, 5, 8, 9, 10, 12, 14, 15 and 16. The first intervals recall the beginning of the Fibonacci sequence. In the *perfect intonation* scale, $^3/_2$ is equivalent to the fifth or diapente; and $^5/_3$ is equivalent to the major sixth—according to Huntley (1970:24) "there is a connection between the major sixth and the golden section". The remaining elements of the sequence (9 to 16) are related to the principles of musical harmony according to Kepler himself: $^8/_{12}$ or $^2/_3$ as the diapente reciprocal; $^9/_8$ as the whole tone interval; $^{10}/_9$ as *deep tone* or interval between grades ii–iii and v–vi. The inversion of the major seventh ($^{15}/_8$) is a minor second ($^{16}/_{15}$ or $^{15}/_{14}$), or diatonic semitone. Kepler believed that the maximum angular velocity of the Earth, measured in relationship to the Sun, varies in the proportion $^{16}/_{15}$, from E to F, between aphelion and perihelion. The ratio $^{16}/_{14}$ (1.142), close to the whole tone (1.125), also appears in Kepler's tessellation as harmony between the double decagon (with 16 sides) and the number of pentagons inscribed within its perimeter (14 figures). One may consider, in conclusion, that Kepler conceives a universe where the major relationships between the celestial bodies mirror the minor relationships hidden in musical harmony and the atomic organization of matter—a notion later developed by Emanuel Swedenborg (1688–1772).[471]

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[471] See the beginning of Chapter 4.

Another historical example of the mathematical study and the aesthetic impact of tessellations, is the decoration of the Alhambra palace in Granada, that inspired musical modernism from Debussy to Ligeti (see Lesure 1982:106; Mandelbrot 2002:28). The geometricians of Alhambra—whose mathematical work very possibly influenced Kepler—explores all the possibilities of tessellation as isometries in the plane, finding 17 ways to combine regular tiles.[472] However, it must be stressed that most of the musical implementation of these relationships consists of an indeterminate intersemiotic translation—as happens in both the cases of Debussy and Ligeti, in which the mathematical properties of the system to be translated are less relevant than the manner they are suggested in the context of a musical tradition.

Musical works developing forms and methods of tessellation analysis, synthesis and transformation, under deterministic methods, are more recent. Most of these works take into account two types of basic aspects: the fundamental relationships of symmetry in the polygons, and their affine transformation, whose products can be described as remainders of a modulo m. As explained below, these two basic aspects are closely intertwined.

In music, most of the relationships of structural repetition correspond to groups of cyclic or affine symmetry related to Z_n, where Z is a set of remainders modulo m for $n \geq 2$ (Fripertinger 2002:2). One may say, then, that basic musical symmetries regularly follow intuitive criteria such as 'up' or 'down', 'before' or 'after', and 'right' or 'left', as described in subchapter 2.3. For a geometric projection it is said that these basic symmetries are *dihedral* relationships. This is also consistent with basic self-structuring of Lindenmayer systems described in 6.5., and with the steps described in 5.4.—for example, in structuring melodic recursiveness.

The domain of Z_n is very useful for understanding the relationships of musical symmetry as tessellations bridging geometric description (graphic or algebraic) with their musical interpretation (as sound and graphic representation). Assuming that the set U, embedding the basic geometry of a tessellation, exceeds the projective space of a group of musical relationships (e.g. a metrical scheme, or a group of tonal

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[472] The mathematical proof of these 17 typologies, as well as the study of their affine transformations, appears in Montesinos (1985).

intervals), then Z_n is the dimension of the relationships of U in the modulo m of the musical relationships M. In other words, U is mapped into M, using the remainders of Z modulo m. This equates representing in a window (Z) the relationships that, for their breadth, cannot be seen in the original set (U). An analogy of this operation is the binary representation of a natural number n, in modulo 10. In this case, as in the musical bounding within Z_n, the superficial appearance of the numerical relationship changes, but the basic content of the message *remains the same*, in terms of an homeomorphism.[473] As Fripertinger (2002) notes, different objects built on Z_n are considered as equivalent if there is a symmetry motivated by music, transforming an object into another one, by cyclic or affine symmetry. This equivalence can be seen in simple examples shown in ◊644; although it can also be found in more complicated forms, as in the mosaic-fractal shown in ◊645.

The Tonnetz: harmonic-modulation theory as tessellation

In search for a mathematical conciliation between the two asymmetric parts of the octave (i.e. the third and the fifth), Leonhard Euler (1707–1783) proposed a musical lattice or *Tonnetz* (German for 'tone-network') that situates "justly tuned versions of the twelve pitch classes on a bounded 4×3 matrix whose axes are generated by acoustically pure fifths and major thirds" (Cohn 1997:7). Such a lattice, adapted and developed by Hugo Riemann (1849–1919), pictures the pitch space by its form of organizing the harmonic motion between chords and modulation between keys.

Using the Tonnetz, Riemann explored the typical relationships between pitches (elements) and their associative steps (intervals between elements) described by rows of successive perfect fifths and major thirds, functionally connected with their inversions (i.e. perfect fourths and minor thirds). In this fashion, the former spiral or continuous cycles of fifths—defended by theoreticians such as Athanasius Kircher— was embedded into a two-dimension lattice with isometric and homeomorphic

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[473] The term *homeomorphism* (ὅμοιος, similar; μορφή, form) is introduced in the context of statistical linguistics, in subchapter 4.4. It is also closely related to the concept *isomorphism* (ἰσός, same; μορφή, form), introduced in the context of musical self-similarity, at the end of subchapter 2.3., on musical symmetry.

properties, due to the general arrangement between harmonic steps. Accordingly, Riemann settled the basis for an algebra to characterize these properties, generalizing them as a homogeneous system.

After Riemann (1873, 1877), many authors revisited the Tonnetz, adding algebraic operators (see Lewin 1987, Cohn 1997, Klumpenhouwer 1998) and implementing three-dimensional vectors (see Lubin 1974, Tymoczko 2006) in order to spatially arrange the just intonation of the 12-scale rows, characterizing their tonal bonds in terms of (geometric) functional harmony with—simultaneously—'consonant' and 'dissonant' perspectives. Such a pitch space allows to conceive of a tonal-modulatory space that exists independently of individual pieces in the tonal Western tradition (an idea first put forward by Lubin, 1974). From this viewpoint, musical pieces—in their turn pieces made of pieces of pieces—emerge as partial components populating a broader, cyclical space, as suggested here in subchapters 4.4. to 4.8.

The Tonnetz has aspects in common with Kepler's tiling (see ◊640), as well as with other (pre)self-similar tilings; namely: *spatial contiguity* (i.e. tiles are distributed in the same space without any gap among them), *variety* (i.e. the tessellation admits several endomorphisms), *invariance* (i.e. the overall structure is invariant to octave transposition), *periodicity* (i.e. structural relationships reappear cyclically) and *harmonic consistency* (i.e. intervals between the elements are 'proportional'). Furthermore, the isometries in the Tonnetz' distances between elements and steps are preserved at least in four levels (i.e. in a fourth-order self-similarity). See, for instance, examples in ◊641, which, at different distances of the same Tonnetz, there are the same isomorphisms (i.e. isochords) preserving the same kind of geometric transformations. Any harmonic progression inscribed within this grid can be interpreted as a *walk* within the Tonnetz, at different step-intervals and step-orientations.[474] Here, self-similarity guarantees grammatical consistency in the sense proposed along subchapters 2.4.–2.5. and 4.4.–4.5.

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[474] Note that this form of continous tonal correlation has a direct analogy in melody. For instance, any melody within a well-defined scale can be interpreted as a *walk* within a Hilbert-like curve in three dimensions (for a brief description of the Hilbert curve in two dimensions, see pages 447–448).
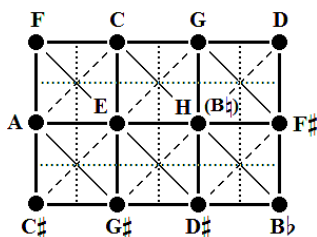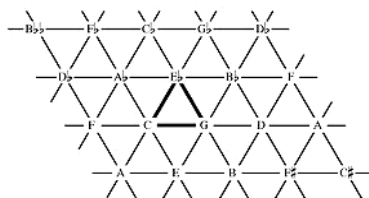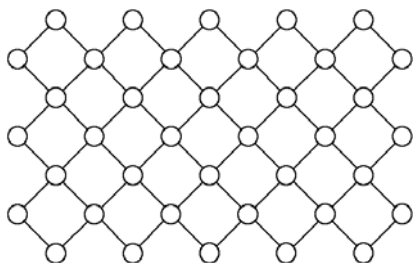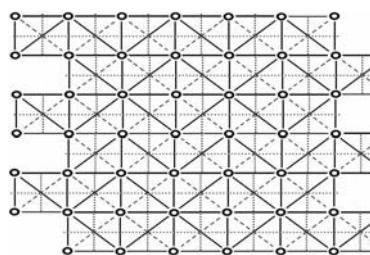
◊641. Different construction stages of the *Tonnetz*.

1) Lattice based on Euler (1739), showing tone rows associated by functional intervals. The tones are represented by dots and the periodic intervals are represented by lines: solid horizontal lines denote fifths and fourths (actually and arrangement of the circle of fifths); thin horizontal dotted lines denote whole tones between dots vertically related; thin vertical dotted lines denote minor thirds segmentation between fifths; dashed diagonals denote sixths or thirds, connected to fifths and fourths (e.g. as employed in classical trichords); and solid thin diagonals denote minor seconds or Major sevenths.

2) Typical Riemannian Tonnetz, as expansion of the previous model. Now the diagonals accentuate the third-fifth connection and the operational periodicity emphasizes the chromaticism of the whole tiling, that Riemann (1877) originally assumed as infinitely extensible on all sides (currently, after Lubin 1974, it is conventionally accepted that the Tonnetz can be represented in a cyclical three-dimensional surface: a torus).

3) Symmetrical skeleton of the Euler–Riemann lattice.

4) An abstract representation of the Euler–Riemann Tonnetz before converting into a flexible surface; a sheet that can be rolled forming a cylinder, in its turn convertible into a torus or donut-shape surface. The torus-Tonnetz presents the circle of fifths and the Major and minor thirds as continuously connected into spirals around the donut. This representation is useful for understanding the general symmetry-asymmetry and cyclical transitivity of harmonic-tonal models.

It is interesting to note that, echoing the study of Lubin (1974), on harmonic periodicity, Amiot (2008) suggests that the same logic used for bulding the Tonnetz in cycles of operational flows, can be implemented for the analysis/elaboration of self-similar melodies and self-similar 'rhythmic canons'. Actually, just like Lubin (*cit.*) proposes modelling the Tonnetz on the surface of a donut (i.e. a torus), Amiot (*cit.*) suggest using the same concept for representing self-similar canons, provided that, in general terms, music (at least its harmonic, melodic and rhythmic components considered in this case) is commonly characterized by recursive loops, representable by the geometric properties of a specific tessellation.[475]

Tonal harmony and beyond:
M.C. Escher by analogy with J.S. Bach

Before the last third of the twentieth century, few composers made explicit references to the polyphonic texture as a tessellation. An exception is Gerhart Muench (1907–1988), author of the collection of instrumental pieces titled *Tessellata Tacambarensia* (1964–1976), most of them for piano, plus one for piano with percussion, and another one for piano, percussion, and violin, intuitively developed as processes of rhythmic-harmonic tessellation. Although the constructive-discursive treatment of these pieces departs from conventions of contemporary atonal music, the explicit use of the concept 'tessellation' is rather uncommon for this historical period. In this sense, it is possible that Muench, before his exile in Mexico, was directly influenced by Hindemith's theories—especially by his book *Unterweisung im Tonsatz* (1937), where he discusses in depth the relationship between pulse, rhythm, proportion and harmony, in terms of sound tessellation and organization, explaining spatial analogies. Hindemith (e.g. 1937/1941:57) recovers fundamental concepts of Kepler, such as the relationship between spatiality and harmony:

> If we think of the series of tones grouped around the parent tone C as a planetary system, then C is in the sun, surrounded by its descendant tones as the sun is surrounded by its planets. As the distance increases, the warmth, light, and power of the sun diminish, and the tones lose their closeness of relationship. [...] In their 'melodic' function, the two

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[475] Therefore, the shape of a torus is useful for representing the cyclical, periodic arrangements of a diversity of parameters within a musical tessellation.

> successive tones of an interval are like two planets at different points in their orbits, while the formation of a chord is like a geometric figure formed by connecting various planets at a given instant.

The analogy between the 'atomic' notion of sound, continuously referred to in this study, and the 'stellar' concept of harmony, as found in musical notions of composers such as Hindemith, Messiaen, Nono, Stockhausen and Xenakis, finally corresponds to an intuition of consistency between phenomena perceived at different scales. Consistently, as Ojala (2009:397) observes,

> It seems natural that the same factors behind the process of tessellations of space and other divisions of space into category regions are also operative in attempts to process the temporal folding and unfolding of situations into intermediate and large-scale hierarchical structures [...] And detours and returns toward already more or less familiar situations do seem to account for the natural processing into hierarchical large-scale structures [...] In terms of conceptual spaces, the hierarchical structures of sound objects can be conceived as *nested* objects.

As a matter of fact, Hofstadter (1995:2–3) begins his inquiry on analogies between models of fluids, using the image of a changing tessellation—what he calls *flickering cluster* or 'fluctuating cluster' of hydrogen molecules. Although he emphasizes the structural instability of these molecules, he also states that "thanks to this unstable, dynamic, stochastic substrate, the familiar and utterly stable-seeming properties of wateriness emerge" (*ibid.*:3). This concept, which conveys the idea of disorder nesting (i.e. 'unstable substrate') within a harmonious appearance (i.e. stable properties of water), also contains the image of a *fluctuating tessellation* (oxymoron and synecdoche at the same time). Consequently, for this and other functional analogies, the spatial and temporal divisions—including the hierarchies and 'category regions' mentioned by Ojala (2009:397)—are based on the assumption of a quiescent, knowable image, attributed by Peircean abduction to the generality of the flow concept. This assumption affects equally the concept of regions and hierarchies in infinite tessellations, as happens in the descriptive-analytic treatments of aperiodic Penrose and Ammann-Beenker tilings, explained in the context of the scheme ◊643.

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When Hofstadter (1979) finds the analogy of M.C. Escher's 'spatial thought', expressed in self-similar tilings, and related to J.S. Bach's 'musical thought', he seeks to identify the cognitive processes of adjacent categories that enable the intersemiotic mapping which converts the spatial (or temporal) notion into music or image. In this context, the notion of 'infinite edges'—as found in Escher's designs—becomes comprehensible as patterns that, by abduction, can be assumed as infinite without the necessity of any explicit, causal justification. Analogously, this relationship also occurs in many conventional concepts of classical music, such as *perdendosi, morendo* or *al niente*, or such as the baroque expression *contrapunto alla mente*, refined in the musical ideas of Luigi Nono, as *contrapunto dialettico alla mente*.[476]

The musical example given by Hofstadter (1979:717) compares the notion of 'infinite loop' in Escher and Bach, a hexagonal tile representing an endless cycle of whole-tones:
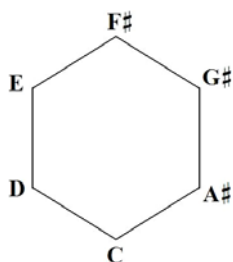
This hexagon represents the modular skeleton of Bach's *Musikalisches Opfer*, working like an infinite-loop scale generator. Obviously, for every seventh step, beginning at any of its angles {C, D, E, F#, G#, A#}, the scale ascends an octave; i.e. twice the initial frequency. However, if Shepard's cyclic system of scales is adapted to this model (see Shepard 1964), then the loop is closed, starting always with the same pitch in which the scale starts. Shepard's system consists, simply, of cycles of parallel octaves, differentiated from each other by shades of intensity: insofar as the scale ascends, its

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[476] More than a title of a specific score (i.e. *Contrapunto dialettico alla mente*, composed by Nono in 1968), this notion symbolizes a precise locus in the musical cognitive domain. In a corresponding domain, analogous conventional symbols in mathematical language are expressions such as *infinite sequence* (…), *countably infinite set* (n_∞), *infinite sum* ($\sum\limits_{n=1}^{\infty}$) or *aleph-null set* ($\aleph_0$). The aesthetic content of both, musical and mathematical symbolization, is affected and validated by the same principle of abduction.

loudness decreases proportionally, until in the last step the sound 'disappears' (*al niente*). Simultaneously, insofar as the 'original' scale ascends, the same scale (its 'shadow' or 'reflection') begins an octave lower, increasing its intensity proportionally (*dal niente*), until becoming the 'original' scale:
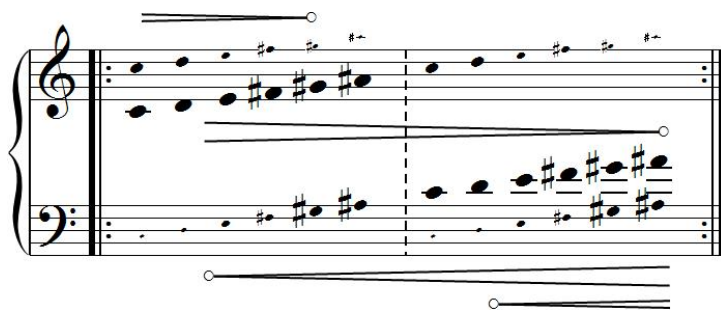
◊642*a*. 'Infinite looping' of a hexatonic scale with its own 'reflection' in different shades of loudness (based on Shepard 1964; commented and explained in Hofstadter 1979:717–719).

Assuming that the hexagon–scale {C, D, E, F#, G#, A#} is just a case of a larger set of hexagon-scales [0,5], then the whole set can be represented as a tessellation made of infinite loops. The structure of each of these cycles is comparable to the effect that Lotman (1988/1994:383) describes as *intertextual specularity*: "whatever appears to be a real object turns out to be only the deformed reflection of something that was itself a reflection [...] where the text and its frame are interwoven, so that both frame and text are framed."

The tessellation of an infinite set of hexatonic scales with absolute self-similarity cannot be expressed, however, as an ensemble of contiguous hexagons (see ◊642*b-i*). This goal requires implementing a self-similar assembly of hexagons, not exactly composed of hexagons, but of pseudo-hexagonal curves filling the plane (see ◊642*b-iii*). According to Schroeder (1991:14), although adjacent regular hexagons make a tiling, this is not a self-similar tessellation, because a hexagon surrounded by six hexagons *does not* make a bigger hexagon.

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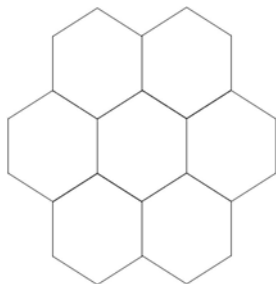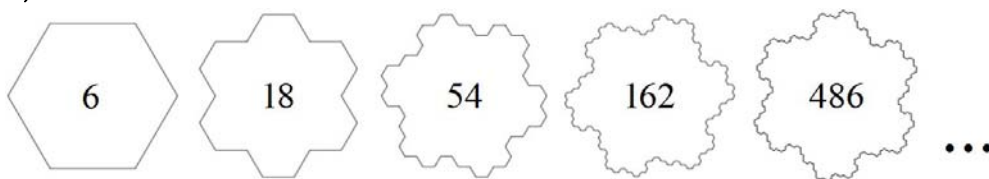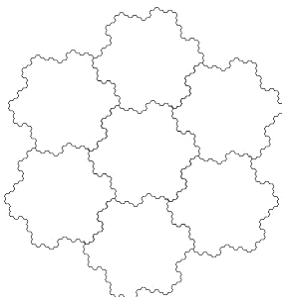
◊*642b*. Transition from a tessellation of regular hexagons, to a self-similar tessellation of pseudo-hexagons with infinite bendings (Gosper tessellation). The middle strip (*ii*) shows the steps from the regular hexagon, with levels of pre-self-similarity and relative self-similarity, going to absolute self-similarity (fractal suggested with suspension points). Numbers in the figures indicate quantity of sides, equivalent to the steps of a musical system (for example, the isometry $^1/_1$ subdivided into $^1/_6$, $^1/_{18}$, $^1/_{54}$, $^1/_{162}$ or $^1/_{486}$). The process generating the self-similar curve—and not the graphic itself—is what is interesting for music, because of similar reasons in associating music with the Cantor function (see pages 354–363). As suggested in ◊*333*, the first layers of the transition may be associated with structural and musical criteria of *motif, phrase* and *period*, or as subdivisions of a hexatonic scale with self-similar trend. Note that the following subchapter presents an introduction to the rendering of quasi-fractal curves, as a generation of musical pitch-classes, from specific examples using Hilbert and Peano curves.

This generative procedure requires a congruent segmentation of the pitch scale, at intervals corresponding to the whole self-similarity levels (transitions in ◊642b-*ii*). In this case the scaling factor corresponds to √7 at all levels. A first step in the self-similar segmentation for a hexatonic scale, produces, for example, 18 'microtones' for the whole-tone hexatonic scale. This implies that for each gap between pitches, there are three internal intervals in proportion of 3:2 for the traditional semitone. In short, while in the chromatic scale the whole tone corresponds to the segment {0, 1, 2}, for the pre-self-similarity level in ◊ 641b-*ii*, the segment {0, 1, 2, 3} is obtained, where the absolute value |0,1| decreases at a proportion of 3:2. With this procedure it is possible to get a first approximation to the Gosper tiling, shown in ◊642b-*iii*.[477]

Compatibility between mental categories

Hofstadter (1995:49–62) suggests that the primary mental operations in geometry, arithmetic and mathematics generally, are nested in music analogously such as the basic mental operations of music are nested in mathematics:

> Having played the piano for many years and composed a number of small pieces as well, I was intimately familiar with the basic building-blocks of melody in tonal music [...] Thus in the stripped-down sequence world there where clusters (and clusters of clusters, etc.) analogous to short groups of notes, full measures, and phrases made of several measures [...] Thus while I was bidding farewell to the patterned world of mathematics, I was ushering in the equally deeply patterned world of music. (Hofstadter *op. cit.*:49)

This notion supports the hypothesis put forward here, on intersemiotic translatability and synecdochic intersemiosis, as fundamental operations in music, assuming that this translatability occurs in a wide space between determinism and indeterminism. In order to reinforce this hypothesis, the discussion on atomism and musical self-similarity—already developed in subchapters 4.1. and 6.1.—, continues here in the specific field of musical tessellations.

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[477] Although this approximation is technically possible, many questions arise as to the aesthetic value of this type of operation. This 'conflict' is discussed in subchapter 5.5.

Aperiodic tessellations: finite and infinite

According to what is stated in the context of graph ◊640, Kepler seems to note that the pentagonal symmetry in his model, leads to a pattern of consistent relationships whose repetitions do not occur at regular intervals. Therefore it is clear that Kepler provides an initial insight in the investigation of aperiodic tessellations, in which tiles are repeated in a form in which their boundaries are not always the same. Following Kepler, more mathematicians found other aperiodic tessellations with symmetric properties that are also found in music, as explained below.

Around 1975 physicist Roger Penrose discovered two geometric bi-dimensional figures, which form the so-called Penrose tiling, and have very peculiar properties: (1) they can fit in the same plane without any gap or overlapping, with infinite copies of both figures, in infinite combinations; (2) none of the obtained tiles is periodic; (3) any finite region of the tessellation happens endlessly, and often within one another, which entails a relationship of deterministic self-similarity (see Ceccherini-Silberstein *et al.* 2004:102).

Grünbaum and Shephard (1986) note that the binary Fibonacci sequence is a one-dimensional (exact) analogy of the bi-dimensional Penrose tiling. Most of the recent sources that associate the golden mean and the Fibonacci sequence with tilings, also mention this correlation in the context of a powerful system of aesthetic properties, inherent to many of the examples presented in the present chapter.[478] Consistently, the musical implications of the Penrose tiling do not go unnoticed: Carey and Clampitt (1996) observe that the structural relationships of this tiling are analogies of "musical sequences", by the fact that they are constituted as self-referential harmonic systems, correlating the parts and the whole. These authors, making a rhythmic interpretation of this system, distinguish a pattern of characteristic 'accents', comparable to the asymmetrical tree with which Kevin Jones (2000) describes prosody in traditional versification, as well as in the generation of musical metre.[479]

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[478] See particularly subchapter 6.3.
[479] See example ◊639.
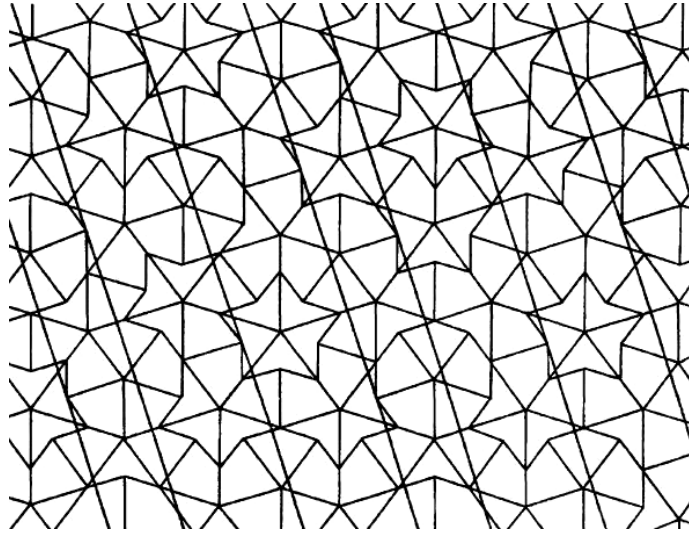
◊643. Fragment of Penrose tiling. Any region in the tiling is aperiodic, which means that it lacks any translational symmetry (i.e. its transformations are not invariant), and any of its finite regions reappears infinitely in the whole tiling. It should be noted that straight lines top-down crossing the design, are not part of the tiling, but represent the nesting of the Ammann bars. Ammann bars are formed by five sets of parallel lines, typical of Penrose tilings, and classified as *thick* and *thin*. Ammann bars are analogous to the self-similar distribution of quasicrystals (see Schroeder 1991, Caspar and Fontano 1996, Kindermann e1999), and are closely related to the Ammann-Beenken tiling. The overall relationship between *thick* and *thin* Ammann bars is proportional to ~ ϕ.

Another example of tessellation with similar characteristics is the tiling of Ammann-Beenken, generated by an aperiodic set of proto-tiles. The set is unusual in that all its arrangements are obtained with aperiodic parts (see Grünbaum and Shephard 1986), in a form of organization which has structural similarities to the Penrose tiling: given that tiles are aperiodic, they do not have translational symmetries—unlike the polyaboloes shown in ◊644*b*—and any finite region of the tiling infinitely reappears in the whole tessellation.

The Ammann-Beenker tiling has a close relationship to the silver ratio $(1 + \sqrt{2})$ and the Pell sequence,[480] which make of it an ideal candidate for implementing, along with the golden ratio, Fibonacci tilings and Penrose tilings, a deterministic context for music analysis, synthesis and transformation. It should also be noted that the square root of 2—equal to the length of the hypotenuse of a right triangle with legs of length 1—is "the irrational number determining a rational division of the octave into equal-tempered intervals" (Maconie 1997:163), but also constitutes the basis of tuning systems alternative to the just intonation system or the Pythagorean system, especially in the interval system of Julián Carrillo (1875–1965), based on the sequence $\sqrt[6]{2}$, $\sqrt[12]{2}$, $\sqrt[18]{2}$, $\sqrt[24]{2}$, $\sqrt[30]{2}$, $\sqrt[36]{2}$, $\sqrt[42]{2}$, $\sqrt[48]{2}$... (Carrillo 1956, 1957).[481]

Tessellation and atomism after D. Hilbert and J. D. Bernal

In music the concepts of *tessellation* and *atomic organization* are often complementary. This contributes to explain the structural empathy between notions of order and constructive self-similarity, analogously shared by music theory (e.g. Wilcox 1992, Fripertinger 2002, Mongoven e2004, Haek 2008) and crystallography, defined in physics as the study of interrelated systems of points, representing the structural organization of atoms and molecules (see Bravais 1848, Bernal 1926, Hilbert and Cohn-Vossen 1932/1952, Ceccherini-Silberstein *et al.* 2004).[482]

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[480] In mathematics, Pell numbers correspond to the denominators of an infinite sequence of ratios that serve as approximations to the square root of 2. The sequence starts with 0, $1/1$, $3/2$, $7/5$, $17/12$, $41/29$, $99/70$, $239/169$, $577/408$, $1393/985$, $3363/2378$,..., so that, for $n = 0$, 1,… the first eleven Pell numbers are 0, 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378, ... (Sloane e2006).

[481] Carrillo's theoretical and aesthetic program is also a landmark in the history of the musical application of the concept of 'worlds within worlds' (see pages 98, 111, 127, 457), considering the notion of scales nested within other scales, following generalized proportional laws. This is particularly evident in Carrillo's treatises *Leyes de metamorfósis musicales* [Laws of musical metamorphosis] (1927/1949) and *El infinito en las escalas y los acordes* [The infinity in the scales and chords] (1957).

[482] In a first approach to this issue, it seems clear that the 'atom' is a physical, not a mathematical concept. However, Euclid (*Elements*, Book 1, first definition) mathematically defines the 'point' (σημεῖὸν) in terms of an atom: "A point is that which has no part" (see page 325).

Hilbert and Cohn-Vossen (*cit.*:52) offer a general approach to the symmetrical principles of crystallography, in a pioneering text on the modern representation of the atomic bonds:

> The crudest picture we may form of an atom is as a point with a number of 'arms' [i.e. lines] equal to the valence of the atom. In this model we assume that the arms representing the valence bonds are arranged in space as symmetrically as possible, as long as no special reason exists for them to deviate from this symmetry. The combination of individual atoms to form a molecule is then represented by letting two arms of different atoms coincide.

Congruently, the 'crudest picture' of a crystal—an atomic assembly expressing a basic symmetry—corresponds to a set of atoms mapped in a one-dimensional space (a straight line segment), taking into account the periodic distribution of atoms in a one-dimensional crystal. Schroeder (1991:311–312) explains how such a periodic distribution matches with the *golden string* or *rabbit sequence* (defined in the next subchapter), whose segments are interrelated by the proportion $1/\varphi$. The result of this mapping also reflects the aperiodic property of the set of atoms, structured in form of an aperiodic tessellation: since the period φ corresponds to an irrational number, the intervals mapped in the line should be aperiodic.

According to Schroeder (*op. cit.*:311), in a grid made up with golden string intervals, and representing every atom of the grid by a Dirac delta function, one obtains the Fourier transform:

$$S_{nm} = \sin C \left(\frac{f}{\sqrt{5}} + m \right)$$

for frequencies

$$f_{nm} = \frac{n}{\sqrt{5}} - m \, .$$

The square root of 5, which so commonly appears in this chapter due to its relationship to the golden ratio ($\varphi \approx 1 + \sqrt{5}\,/\,2$), reappears here in the context of the symmetry of a quasicrystal. This relationship corresponds with what Roads (2004:34)

perceives as a structurally-consistent convergence, between a process of optical energy and systems of mechanical vibration, including acoustics.

Quasicrystals, discovered and described originally by Dan Shechtman in April 1982, are structures formed by elementary particles,[483] whose distribution is based on the golden mean, as three-dimensional versions of the Penrose tiling (◊643). Unlike periodic crystals, quasicrystals are structural forms that are both ordered and nonperiodic. Kindermann (e1999) emphasizes the aspect of self-organization that conducts their simultaneous order and aperiodicity, suggesting that the same power laws also govern the notions of consistency and inconsistency in a variety of acoustic and musical facts, as suggested by Gabor (1947), Rocha Iturbide (1999) and Roads (2004), from the atomistic perspective of sound.[484]

Periodic tessellations and frieze patterns

The research done by Hodges (2003) and Hofman-Jablan (e2007), on the foundations of musical geometry, present examples of structural fragments taken from different styles of music, with repetition of motifs, scales, arpeggios, tremolos and trills, in upward and downward movements. Most of these examples can be identified as *frieze patterns*, especially in monodic lines; as *polyphonic brocades* in instrumental textures; or as tessellations in harmonic relationships.[485] Such frieze patterns, brocades and tessellations meet in music a structural role analogous to the relationship between detail and totality in many self-organizing systems. For instance, the musical concept of *ornamentation*—perfected in the Baroque era and reassessed in the second half of the twentieth century by composers like Luciano Berio and Franco Donatoni—is functionally and structurally related to an analogous concept in architecture and design. For similar reasons, generalized for common ways of

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[483] This topic is introduced in subchapter 4.1.

[484] An extension of this principle, regarding negotiations of musical grammar and musical style, is introduced in subchapter 4.8.

[485] The word 'brocade' commonly refers to a fabric with fibers of one or more materials that are woven into a symmetrical pattern repeated across a strip. The term 'frieze pattern' concerns, however, the rhythmic patterns of architecture, placed as decorative strips in facades or indoors. This study uses both terms in analogy with the cyclical patterns of music, for example in the *ritornello* of a melody; in cycles of two or more contrapuntal melodies; or in an iterative bass line (a typical case of the latter is the Alberti bass).
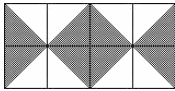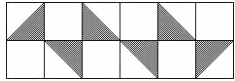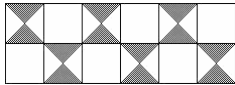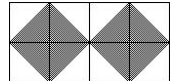
structuring in different logical–aesthetic expressions, ornamentation has a crucial role in music, especially when establishing a function of the ornamental figure, as a relationship of (pre-)self-similarity, with respect to a musical period, a phrase, or a larger musical structure.[486] It is said, then, that the *ornament*, the *fragment* to which it belongs, and the *whole* coordinating all ornaments and fragments, are all in harmonious relationship.

The spatial representation of musical relationships in the form of frieze patterns, brocades and tessellations is not necessarily arbitrary, since many forms of repetition and symmetry in metre, rhythm, harmony, timbre, melody and musical motifs can be interpreted as spatial patterns, by strict (i.e. deterministic) proportional analogies. In return, a variety of spatial patterns can be interpreted as musical relationships. To illustrate this reciprocity, examples in ◊644*a* show the deterministic intersemiotic translation between some of the simplest and most common ratios of musical metre, and their possible re-interpretation, simple as well, employing mosaics made of equal right-angled triangles. The way to assemble these tiles can have two or more varieties, such as the varieties of prosody characterizing distinct metres. These varieties can be descriptive grammars—as in this case—or, prescriptive grammars, as in the case of polyaboloes related to a pattern of rhythm, metre or timbre (harmonic spectrum), as explained below.

A polyabolo is a figure composed of *n* isosceles right triangles joined along edges of the same length (see Weisstein e2008). It is said that two or more polyaboloes are equivalent if they share the same form of their internal adjacency; for example, the following figures are equivalent to the first cases shown in ◊644*b* (leftmost figures), in triaboloes and tetraboloes, respectively:

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[486] This musical perspective has an almost direct connection with rhetoric and the poetic language. Mayoral (2004:23) characterizes the concept of ornament "as one the concepts of greater value and significance in the doctrine of rhetoric and poetry, as legacy of classical thought. [...] On this concept is based, from Aristotle and throughout the classical tradition, the concept of discourse".

time signature (*articulation*)	tessellation examples (prosodic varieties)	time signature (*articulation*)	tessellation examples (prosodic varieties)
$\frac{2}{2}$ [1] *double*		$\frac{3}{4}$ [0.75] *triple*	
$\frac{2}{4}$ [0.5] *double*		$\frac{6}{8}$ [0.75] *double*	
$\frac{4}{4}$ [1] *quadruple*		$\frac{12}{8}$ [1.5] *quadruple*	

◊644*a*. Periodic tessellations as analogies of musical metre. Examples to the right of the metrical model suggest a possibility among others, forming the prosody for each case.
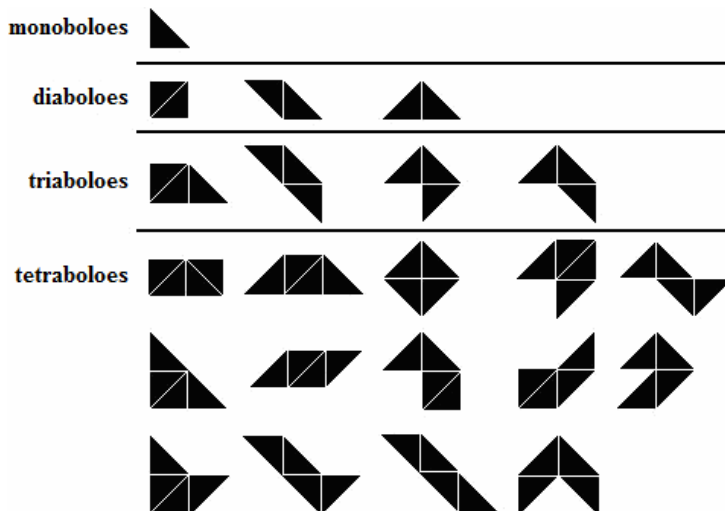
◊644*b*. Polyaboloes showing their first four groups of isometries (based on Weisstein e2008). In acoustics, some of these polyaboloes are vibration systems of plates and membranes, as described here in subchapter 4.2. (see page 141).

The number of fixed non-equivalent polyaboloes composed of *n* triangles form the sequence 1, 3, 4, 14, 30, 107, 318, 1116, 3743, 13240, 46476, 166358... . Sloane and Ploufe (1995) and Sloane (e2006) describe the same sequence as the number of different figures that can be formed with halves of *n*-squares; they then map the sequence as points in $\mathbb{R}$, by their ordinality ($1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 14, 5 \rightarrow 30... 12 \rightarrow 166358$); for each ordinal number, a pitch is mapped into its corresponding ordinate. Using this method, Sloane and Ploufe (1995) and Sloane (e2006) produce a collection of 12-points starting with narrow intervals (ordinals 1, 2, 3) that quickly run apart (ordinals 4, 5, 6, 7... 12). Finally, they produce an audible representation of the sequence, in a range logarithmically modulated from C_3 to C_6, as shown in graph ◊644c. In this way, they limit the greater intervals in the sequence, using a narrow window. Because of its heterogeneous distribution, the result resembles a twelve-tone sequence:

◊644*c*. Pitch-class mapping obtained from the numerical sequentiation of fixed non-equivalent polyaboloes, composed of halves of *n* squares (1, 3, 4, 14, 30, 107, 318, 1116, 3743, 13240, 46476, 166358). The map, in modulo 12, is distributed into the ordered pitch-space from C_3 to C_6. Example based on Sloane (e2006).

Clearly, Sloane and Ploufe's (1995) method is based on mapping the remainders modulo *m*, as pitch-classes. Essentially, this method is the same as the one used by Haek (2008), with cyclic remainders from Fibonacci sequences, also implemented to generate serial systems. However, it should be noted that the mere mapping of remainders in a limited window implies a loss of original information. For example, the number succession of fixed polyaboloes does not reflect the symmetrical qualities of polyaboloes, nor their systematic re-tessellations. In order to build a consistent

system of brocades and tessellations—in an idiomatic sense—a more balanced relationship between information and redundancy is required, simultaneously using different properties and parameters—for example by combining metrical distribution and shape of a rhythm, with the pitches obtained as a logarithmic distribution in a coordinated system. Richness, possibility and necessity of such a combination, for the sake of intersemiotic consistency between geometric representation and musical content, confirm that descriptive and prescriptive grammars are not models in radical opposition, but they can be coherently associated within the same operating system, as complementary functions.

Tessellations made of tessellations

The notions of frieze pattern and tessellation reach virtually all musical parameters, and are reflected in a variety of applications in rhythm, metre, melody, harmony, dynamics and timbre. Amiot (2003:1) emphasizes the concept of 'rhythmic canon', for example, as a form of autorepetition, and defines it as tessellation: "A rhythmic canon is a *tile* (a purely rhythmic motif) repeated in several *voices* (for instance with several different instruments) with different *offbeats*, so that two distinct notes *never fall on the same beat*." In this context, Amiot (*op. cit.*) highlights the typical properties of tessellations, as properties of musical organization: *repetition, affine transformation, reduction* and *equal-distribution*, which he uses to formulate canonical and poly-canonical rhythms.

The notion of tessellation as a system of proportions in the physical configuration of sound, particularly in timbral textures, is a common feature of the spectral analysis-synthesis of music (see Truax 1982, Waschka and Kurepa 1989, Kappraff 2000, Polotti and Evangelista 2001). In principle, the idea of mosaic made of mosaics, is compatible with the general notion of Fourier analysis (Godrèche and Luck 1990:3774–3776). In this context, certain patterns of VRA may clarify this form of analytical representation, as addressed here, in Chapter 5.[487] Subchapter 4.2., dealing with mechanical self-similarity, exposes the intimate relationship between

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[487] See pages 288–290, especially.

timbral configuration in plates and membranes, and the isospectral manifolds formed by polyaboloes.[488] The reappearance of the same form of tessellation in different complexities of the same musical system (e.g. levels of timbre, harmony, meter, rhythm or melody) obviously constitutes a relationship of self-similarity.

In counterpart, from the viewpoint of musical synthesis, self-similar tessellations made by a regular relationship between part and total, may also be useful for creating musical structures such as rhythmic sequences, pitch sequences, and—by increasing the time parameter—timbral tilings. One of the most illustrative examples of this generative procedure is the 'chair' tiling, based on continuous partitions and multiplication of squares, as shown below.

Generation of scales using 'chair' tilings

The continuous construction of a square made of converging rectangles, as shown in the following example, is an early example of scalar construction as recurrence and self-reference with embedded parts, forming the shape of a 'chair' or L:
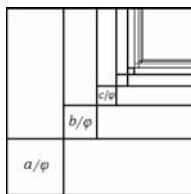
In this example the squares arranged on the diagonal of the system, a/φ, b/φ, c/φ, d/φ... form an identical sequence to that shown in ◊635-*left*, producing a musical scale analogous to ◊635-*right*. Instead, the assembly of squares by halves or doubles (as shown below as the series 1, $^1/_2$, $^1/_4$...), is analogous to the prolatio system as shown in ◊545.

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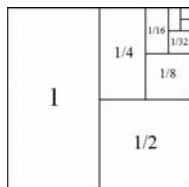
[488] See subchapter 4.2.

◊645*a*. Pre-self-similarity stage during the chair tiling construction. In this example the construction process is adapted to the generation of a musical scale: primary cells (three equal squares assembled in the shape L) are related to corresponding values, in this specific case musical tones written next to each cell. Four different cells are used to form a basic 'chair' (*α*) in a first order self-similar structure, whose values are read consecutively, following the diagonal shown in β, and always reading from left to right for the adjacent squares. Thus, the first three tones are D, G, A, according to the white cell. Obviously, the initial values of the scale may vary, as shown in the following example (below), where the numbers written next to each cell represent pitches of the chromatic scale (C = 0, C# = 1, D = 2, and so on). In consequence, a different scale is produced:
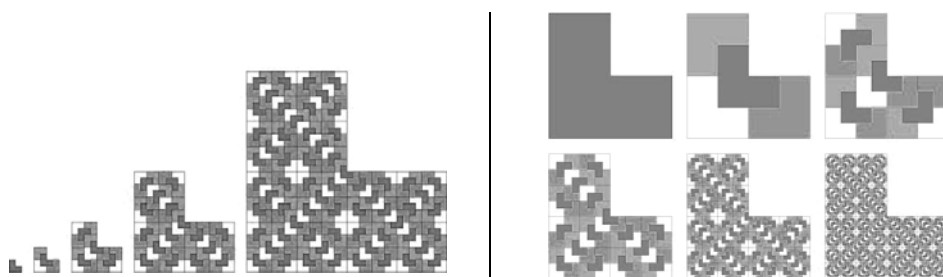
◊645*b*. Chair tiling at different constructive stages. Examples from Wright (e1997), and Blackaller, Buza and Mazzola-Paluska (e2007).

Left: tiling from an initial cell, as shown in the example above (◊645*a*). Cells are grouped in a self-similar lattice in the shape of X's juxtaposed at various scales. In this case the original chair keeps the same scale at each step of aggregation.

Right: The same process of construction shown as a scale reduction. In both cases the consecutive results can be used to generate self-similar music patterns (e.g. in the form of arpeggios, scales, melodies or timbres), as suggested in ◊645*a*.

Chair embedding around successive chair embeddings produce, however, a different type of result that may require a different form of interpretation. This is the case of the *chair tiling* (see Gähler and Klitzing 2010:165–167), a two-dimensional limit-periodic tiling that consists of one tile (the 'chair') rotating in four different basic symmetries, useful for musical purposes. A possible intersemiotic translation of the chair tiling into music should convert, for instance, its four symmetries into four different pitch orientations; its spatial articulation into rhythmic articulation; and its rotations and typical angles, into melodic motifs. A simple exercise suffices to produce musical scales through the diagonal interpretation of each primary subset of the tiling (see ◊645*b*), as a seed for algorithmic self-structuring.

Simple sieves with Fibonacci sequences

A first approach to simple sieves using Fibonacci sequences, may be a case in which a row of equal elements—e.g. regular squares—matches with a finite segment of the Fibonacci sequence (let be [*a, j*] = [1, 34]). Then, parallel to this segment, let be the same segment, but in its opposite direction (see ◊646*a*–top). By repeating this parallel-row operation, making the length [*a, j*] equal to its height, a square of squares is obtained, whose overlapping at 90° (◊646*a*) results in a brocade with relationships from the original sequence, along with other 'new' relationships.

The first result of this experiment contains repeated rows and rows with the original sequence, or other sequences with different intervals (e.g. 1, 14, 22, 27, 30, 32, 33, 34), corresponding to inner sums and subtractions of the original sequence. The diagonal linking two corners of the whole set presents the sequence 1, 2, 3, 5, 8, 13, limited by an axis (corresponding to 14), forming a palindrome. Subsets in secondary diagonals, perpendicular to the main one, also include fragments of the original sequence, along with other intervals (e.g. 1, 2, 5, 13, 14, 18, 21, 22, 24, 25; inversion: 1, 3, 4, 5, 8, 12, 13, 21, 23, 25).

The central white figure, with 48 elements, has a dislocated symmetry, in which, counting unit by unit, one finds the same type of distribution between the original relationships and their own withdrawal; for example, counting from the edges toward the centre in any direction, element 21 always appear next to the main diagonal; at

the same time, the original sequence is asymmetrically distributed and incomplete along the account. Interestingly, the pattern obtained as a whole is, at once, self-referential and self-dissimilar in various regions, something which in practice occurs in many musical examples, as described in more detail within subchapter 6.6., in the context of a theory of musical asymmetry and anti-proportion.

These results, rather than comparison to a tessellation, can be compared to a sieve, or to certain noise filters (as suggested in Chapter 5), in which the association of an original sequence with the same sequence, creates structural consistency and interference, simultaneously. This concept of sieve may be useful from several perspectives. Nono, in *Il canto sospeso* (1956), for solo voices, choir and orchestra, employs a similar sieve or matrix, in order to produce global and local symmetries and antisymmetries (see ◊660, for an analytical summary of the first movement). Under this notion of functional contrast-complement, the sieve's cells work as spaces that determine the rhythm of instructions or actions taken in music: a sort of counterpoint of algorithms in which the properties of proportion and sequentiation are only partially given by a basic self-reference—since the information inserted in each cell can vary greatly, and the sieve's cell only indicates a 'place' for implementation. Xenakis (1992:268–288) uses the concept of sieve, in a way that is familiar to this example:

> In music, the question of symmetries (spatial identities) or of periodicities (identities in time) plays a fundamental role at all levels: from sample in sound synthesis by computers, to the architecture of a piece. It is thus necessary to formulate a theory permitting the construction of symmetries which are as complex as one might want, and inversely, to retrieve from a given series of events or objects in space or time the symmetries that constitute the series. We shall call these series 'sieves'. (Xenakis, *op. cit.*:268).

Xenakis (*ibid.*:274–275) also speaks of *inversions* of sieves, convertible into modulo *m*, along with their associated partitions, reductions and *transformations*—something that Xenakis prefers to call *metabolae*—in three different possibilities: *a*) by a change of the indices of the moduli; *b*) by transformation of the logical operations (in the case given in ◊646*a*, by changing the operation of the embedded algorithms for each cell of the sieve); and *c*) by transformation of the operative units, for example, from tones to semitones, or from semitones to quarter-tones, and so on (a procedure

already suggested by Julián Carrillo, 1957). This third possibility also allows intersemiotic translation resources, such as making a range of pitches systematically correspond to a range of durations; or making a range of durations correspond to a range of intensities.

Table ◊646*a* presents three sieves produced with three different sequences of numbers: Fibonacci standard, Lucasian, and Evangelist. Musical patterns obtained have their own characteristics, correspondingly to each of the sequences. Example ◊646*b* suggests how to produce a musical system with pitches and lengths, taking a segment from the sieve as an operational preset. For the system generation, the correspondences for each filled cell (black cells in ◊646*a*) are marked with a symbol that represents a specific length and pitch. In this case the reading of cells is executed in descending diagonals, starting from the bottom leftmost cell, that corresponds to 0; the two following cells correspond to 1 and 9, and the next three, to 2, 1, 0, and so on, until completing the segment with the ultimate symbol θ, equivalent to pitch and duration C_5, with the shorter duration in the table of predetermined durations.

The same production rules employed in this example can be implemented to produce relationships of intensity, timbre, prosody and/or articulation. The production process can also feed its input by other segments from different sequences of numbers, along with the *transformations* or *metabolae* suggested by Xenakis (*cit.*). Orientation and order of the sequence's reading can be organized in distinct ways, obtaining very different outputs. Moreover, each cell of the sieve can be associated with a specific set of instructions, including, for example, values of instrumentation and formal structure. A more sophisticated method of production by 'paragraphs' of instructions (i.e. algorithms) then comes into action, as opposed to using isolated values (such as pitches or durations). In this context, the sieve, as organizer of musical expression, becomes analogous to what Rynin (1949:383) primitively called "the sieve of significance", in grammatical terms—i.e. in terms of well-formedness rules.

Conversely, decoding the 'paragraphs' of instructions for each cell in the sieve, this method—converted into an analytic device—can unveil the grammar in a musical piece composed through a systematic juxtaposition of Fibonacci sequences for all parameters. This is particularly useful in explaining and interpreting the

relationship between detail and wholeness in *Il canto sospeso* (1956), by Luigi Nono (in this specific case each cell is matched with each measure of the analyzed work). This method, combined with an initial segment of the sequence of prime numbers, that serve as a counterbalance of the well-formed structure, allows the analysis presented in table ◊660 (see pages 465–468 for full explanation).

i)
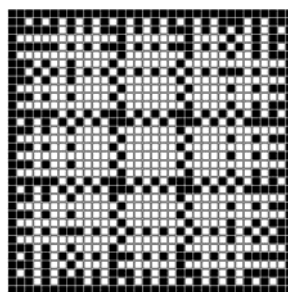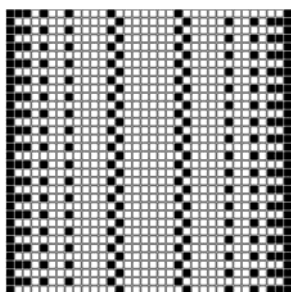
ii)
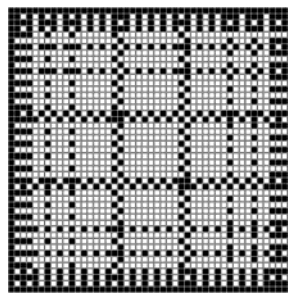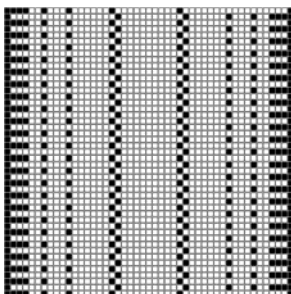
◊646*a*. See continuation on next page.

iii)
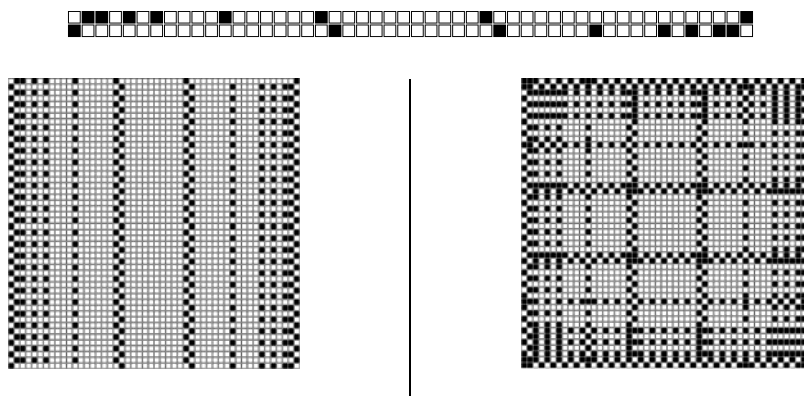
◊646*a*. Three different brocades made of rows with equal squares, elaborated from a self-referential proportional counting (~φ). The frieze pattern on the top of each example represents the counting source, with its radial inversion below. The squares of squares on the left, collect successive copies of the same pattern. The squares of squares on the right, contain a juxtaposition of the same pattern with itself, rotated to 90 degrees. From top to bottom, examples correspond to (*i*) standard Fibonacci sequence, (*ii*) Lucas sequence, (*iii*) Evangelist sequence.

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i)

(0 = Do)

ii)

◊646b. Generation of a musical structure by the intersemiotic translation of a segment of the latter example, using a sieve made from the standard Fibonacci sequence (from example ◊646a-i). In (*i*, previous page) a pitch sequence is associated with each filled cell in the *frieze* pattern; in (*ii*, previous page) the sieve's segment, elaborated by juxtaposition of the sequence source, is converted into an array of pitch (the same as assigned in *i*) and duration (table of equivalences on the right). The array runs in descending diagonals, starting at the leftmost-down cell, corresponding to 0. (*iii*) shows the sequence that results from applying the pitch-array, and (*iv*) shows the same sequence related to the duration-array, adapted to measure ⁴/₄.

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6.5. Self-replacement strings

Goethe wrote in a letter to Herder, copied in a note in his diary, dated May 17, 1787, the following statement:

> The 'Primeval Plant' (*Urpflanze*) would be the most wonderful creature in the world [...] With this model and the key to it, one could invent plants ad infinitum, even if, knowing that they do not exist, could exist not as picturesque or poetic shadows and illusions, but as the inner truth and necessity they have.[489]

This idea, assimilated into music by Anton Webern and explicitly developed by him in his text *Der Weg zur neuen Musik* (posth. 1960:53–55), summarizes the concept that, knowing the bases of certain key patterns, with adequate implementation, the creation of a primary organic structure is possible. From such a primary structure, other structures can emerge in a subsequent, derivative self-similarity. Thus, Webern suggests that musical structures are analogies of such organizational structures and, therefore, they follow similar patterns and relationships. This conceptualization also implies a synecdoche, in which any structure resulting from a replicating process is part of the 'primary structure', whilst the process itself is reflected in its derivations.

Aristid Lindenmayer (1968) implements this idea as a strict analogy, by applying a simple two-dimensional algorithm with few directional instructions, to emulate the growth and development of a *primary plant*, that, in turn, can produce other *plants*. By intersemiosis—and according to what Webern (1960) suggests—this generative concept is compatible with the ideas of musical *growth* and *development*, in empathy with generalized cases of the golden ratio and the Fibonacci sequence (explained in

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[489] "Die Urpflanze wird das wunderlichste Geschöpf von der Welt [...] Mit diesem Modell und dem Schlüssel dazu kann man alsdann noch Pflanzen ins Unendliche erfinden, die konsequent sein müßen, das heißt: die, wenn sie auch nicht existieren, doch existieren könnten und nicht etwa malerische oder dichterische Schatten und Scheine sind, sondern eine innerliche Wahrheit und Notwendigkeit haben". J. W. Goethe (incl. in *Die italienische Reise, Gedenkausgabe der Werke, Briefe und Gesprache 11*, ed. Ernst Beutler, Artemis-Verlag, Zürich, 1949; p. 413). This notion is complementary to Leibniz' aphorism §37 in his *Monadologie* (ms. 1714, original in French), published until 1840: "Chaque portion de la matière peut être conçue, comme un jardin plein de plantes, et comme un étang plein de poissons. Mais chaque rameau de la plante, chaque membre de l'animal, chaque goutte de ses humeurs est encore un tel jardin, ou un tel étang." Both concepts, *Urpflanze* in Goethe, and synecdochic plant in Leibniz, are empathically related in the context of self-replacement strings.

6.3.), and with processes described in subchapter 4.3. In short, these relationships follow a same self-organizing principle. As J.H.D. Webster (1950:248) states:

> [A]rt forms, in music as elsewhere, are similar to and spring instinctively out of natural forms of growth, in life, and in much of the inorganic world as well. Man and nature seem to be one in more ways than hitherto realized.

Self-replacement strings reflect Goethe's (1787) elementary notion, for developing a fundamental organic structure, by the systematization of organic growth, represented by a developing algorithm, as proposed by Lindenmayer (1968), Prusinkiewicz and Lindenmayer (1990), Prusinkiewicz and Hanan (1992), and Meinhardt (1998). Accordingly, under the analogy of the iterative relationship *seed-development-replication*, authors such as Prusinkiewicz (1986), Prusinkiewicz, Krithivasan and Vijayanarayana (1989), Mason and Saffle (1994), Jones (2000), Sharp (e2001), Worth and Stepney (2005), Manousakis (2006), and Lourenço, Ralha and Brandão (2009), implement various models of self-replacement strings for musical synthesis and analysis.

Self-replacement strings—and as part of them, the so called Lindenmayer systems—are essentially of intersemiotic character, as it is evident in the evolution of the concept of structural self-replication. This is relevant, since such strings operate as self-referential systems within a broader self-referential framework. So, a self-replacement string in particular may be a *uniform* system of self-similarity in *manifold* complexes of self-similarity. This statement basically concurs with what Salomaa and Rozenberg (1980:ix) recognize as an "interdisciplinary" character, at the beginning of their mathematical study on Lindenmayer (henceforth L-systems). Such a notion of 'interdiscipline' (in this case also an intersemiosis) also holds in self-replacement strings, translatable into musical systems.

Simple strings: a basic definition

Self replacement strings include a wide variety of generative grammars which, from few elements and extremely simple rules of application, can produce progressively more complex patterns for each iteration of the generating system, originally intended as a *generative seed*.

A self-replacement string consists of a recursive method for generating sequences of symbols from an *alphabet* and an initial axiom, in combination with a limited set of production rules for each symbol contained in the alphabet.[490] The application of a simple rule produces a simple string, unless the original axiom contains instructions triggering a systemic complexity from the first iterations of the system.

For the simplest case of a self-replacement string, each symbol of the alphabet is replaced by a short string; then, each string is replaced by a longer string; then, the substitution process iterates until forming a string of symbols long enough to be used as an operational string. Assuming that the initial set is *A*, and its production rule is

A. A[BBA]B[ABB] ,

then, rule R_1 indicates that, for example, symbol A must be replaced by the same string. Thus, the first substitution results in

A_1. A[BBA]B[ABB][BBA]B[ABB].

But, since the initial string already contains more than one A, all of them must also be replaced, starting from the second A in the initial string. Furthermore, a second rule R_1 may indicate that brackets cannot remain open or isolated, but can only contain non-empty sets and subsets. Thus, the second substitution results in

A_2. A[BBA[BBA]B[ABB]]B[ABB][BBA]B[ABB].

The string replacement may continue further, or accept a third rule for symbol B, as well as other rules for the subsets enclosed in brackets. A further set of rules can be introduced, for example, to replace symbols A and B by notes in a one-line staff (i.e. with two different lengths forming a rhythm), interpreting the bracketed subsets as specific dynamic values. This method produces a first, simple string of rhythmic articulation that can be combined with other strings. Clearly, with a small number of substitutions the system tends to grow rapidly. Depending on the rules implemented, the iterations may produce sequences with internal symmetries based on the reappearance of symbols A, B, [,]. The repetition of similar subsets within other similar sets, finally produces a statistically self-similar structure.

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[490] "An *alphabet* is a set of abstract symbols" (Salomaa and Rozenberg 1980:1).

The rabbit sequence

A self-replacement string that is frequently cited in the literature, and commonly used to produce musical sequences, is the *rabbit sequence*, also known as *Fibonacci word* or *golden string*. A rabbit sequence is a specific binary string with symbols 0 and 1 (or with symbols from any alphabet of two elements), whose elements are self-added using the same rule used for concatenation in the Fibonacci sequence.[491] Equally, the rule produces a sequence in which each *word* (i.e. alphabetic combination) is infinitely repeated for the previous word, adding the second last term for each recursion:

```
 1 | 1
 2 | 10
 3 | 101
 5 | 10110
 8 | 10110101                                    φ
13 | 1011010110110
21 | 101101011011010110101
34 | 1011010110110101101011011010110110
55 | 1011010110110101101011011010110110101101011011010110101
       . . .
```

◊650. Golden string or rabbit sequence, also known as Fibonacci word.

This self-referential sequence presents a growing pattern of self-similarity in which each term contains a number of digits whose distribution corresponds to the Fibonacci sequence. Also, as shown in ◊650, the synecdochic relationship of each particular length, respecting the length of the previous subsets, is equivalent to the golden ratio. Schroeder (1991:311–312) explains how this periodic distribution matches with the typical distribution of a quasicrystal, in an ordered group of atoms at aperiodic intervals, similar to the aperiodic intervals that Jones (2000) describes as models of irregular prosody in versification and musical metre[492], and which Biles (e1988) also uses to produce the pitch scale shown in example ◊637.[493]

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[491] See page 374.
[492] See page 400.
[493] See pages 395–396.

Prusinkiewicz and Hanan (1992) found that the rabbit sequence results from grammatical restrictions, considered by analogy with organic processes, with the following self-replacement characteristics, in terms of an L-system:

start: A
variables: A B
rules: $(A \to B), (B \to AB)$

which produces the sequence:

n_0: A
n_1: B
n_2: AB
n_3: BAB
n_4: ABBAB
n_5: BABABBAB
n_6: ABBABBABABBAB
n_7: BABABBABABBABBABABBAB
n_8: ABBABBABABBABBABABBABABBABBABABBAB

According to Prusinkiewicz and Hanan (*op. cit.*), this version of the rabbit sequence is directly linked to notions of production, growth and synthesis, that can be found analogously in cell interactions within plant development. Prusinkiewicz (1986) also suggests that similar processes characterize musical systems as grammatical patterns, combining fixed axioms and rules, with options of functional flexibility, by the alternation with new, simple rules (as explained in the following pages).

Thue–Morse sequence

In mathematics and information sciences, the Thue-Morse sequence is a binary sequence that begins with the words
0 1 10 1001 1001 0110 1001011001101001 ... .

Any ordered pair of symbols can be used instead of 0 and 1, since—as in the rabbit sequence—the logical structure of the Thue-Morse sequence does not depend on the symbols used to represent it. As an L-system, the sequence can be symbolized by the following instructions:

start: 0
variables: 0, 1
rules: $(0 \to 01), (1 \to 10)$

Allouche and Johnson (1995), and Kindermann (e1999), among many others, suggests to employ the Thue-Morse sequence as random numerical generator for sound synthesis. Additionally, Godrèche and Luck (1990:3788–3791) observe a variety of properties of self-similarity in the Thue-Morse sequence, including a multifractal resonance spectrum and the emergence of 'essential singularities' in its Fourier transform analysis. Adapting these properties to musical production rules —analogous to those implemented in the case of Fibonacci sequences and periodic and aperiodic tilings—can produce a great variety of rhythmic patterns, harmonic spectra, and audible textures.[494]

Musical L-systems

Within a plethora of possibilities of self-replacement strings (the previous examples are just a few among many others), the Lindenmayer systems or L-systems are the best studied formal grammars which, by their recursive nature, tend to produce self-similar patterns.[495] As happens with self-replacement strings in general, L-systems generate symbolic sequences from a restricted set of axioms and basic rules.

According to Salomaa and Rozenberg (1980:10–11), the simplest L-systems are 'context-free'. It is said that an L-system is context-free (shortened as 0L-system) if the replacement of its symbols is independent of the symbolic environment. Once these simple systems are associated with a given set of production rules, they become D0L systems (i.e. deterministic context-free L-systems). Salomaa and Rozenberg (*ibid.*) define D0L systems as a tripartite system

$$G = (\Sigma, h, \omega),$$

where Σ is an alphabet, h is an endomorphism defined on Σ, and ω, referred to as the *axiom*, is an element of Σ. The word sequence $E(G)$ generated by G consists of the words

$$h^0(\omega) = \omega, h(\omega), h^2(\omega), h^3(\omega), \ldots$$

[494] Obviously, this implies boundaries and thresholds imposed by sonological and perceptual conditions. This issue is discussed in subchapter 5.5.

[495] As mentioned earlier in this subchapter, L-systems were originally designed to model the structural growing of plants, emulating their developmental process. The initial work of Lindenmayer (1968) was continued by Prusinkiewicz, founder of the International Workshop on Functional-Structural Plant Modelling, who in 1986 also published the first formalization of L-systems as a musical grammar.

According to these authors (see Salomaa and Rozenberg *op. cit.*:ix), the fact that the rewriting processes of D0L systems allow simultaneous rewriting, is significant in terms of the modes of the strings' growth. In this regard, L-systems are dynamic (since they are built as time series), self-structuring and sequential, since only one part of the string is rewritten for every recursion of the system: "L-systems are models of *parallel* rewriting: at each step of the process all letters of the word considered have to be rewritten."

An important aspect of the linkage of L-systems with music, is the relationship between function and economy of resources that Prusinkiewicz and Lindenmayer (1990) and Meinhardt (1998) call 'algorithmic beauty', coupling conventions of aesthetics, symmetry and formal grammar.[496] In the case of music, it is clear that even for the simplest self-replacement strings, this concept of 'beauty' depends heavily on context-free and context-sensitive relationships, which determine the operability of a musical grammar: within a D0L system structural and generative freedom is such that it allows *all* kinds of translations of symbolic strings into music (as suggested in a section below, in the adaptation of self-similar curves for the generation of pitch scales and melodies). Instead, in a context-sensitive L-system, the implementation of a set of rules depends on the environment in which symbols are replaced. The difference between both types of L-systems is radical in the sense that they yield completely different results, conditioned by their recursive properties.

After Nelson's pioneering work, first put forward in 1974 with his programming system APL (*A Programming Language*; see Nelson 1992:5), other composers have used L-systems to associate self-replacement strings with specific musical elements (symbols) and production relationships (rules), usually limited to pitches with specific duration and intensity, or to sound pauses with specific duration, within a restricted set of rules and modes of combination. Since these methods imply self-organizing a musical structure by the recursion of a limited set of symbols and rules, the outcome is commonly (partially or totally) self-similar, usually with progressive variations within a self-similar whole.

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[496] An important precedent of this association is the study of D'Arcy Thompson (1917), in which the role of aesthetics is highlighted as an intuitive means of structural analysis.

In a first period of implementation, the trend of using L-systems to produce music was limited to tonal structures, using the most basic concepts of Western tonality and quite fundamental rhythmic formulae. However, since 1990, L-systems began to be employed to produce musical structures with intervals smaller than the semitone, and with increasingly intricate rhythmic figures. Prusinkiewicz, Krithivasan and Vijayanarayana (1989) also adapted specific L-systems outside the Western cultural context, to emulate traditional graphic patterns and melodic patterns of South India (Karnataka).

The study of Stelios Manousakis (2006) encompasses a wide overview on "micro-composition" techniques, related to a variety of musical parameters. Manousakis, as well as Nierhaus (2008:139–144), also distinguish between different methods of implementation, using context-free and context-sensitive L-systems, deterministic or stochastic L-systems, and parametric and non-parametric L-systems. In a parametric L-system, the rewriting of rules can be restricted to self-regulation conditions, such as changing the values associated with the symbols along the process of substitution, or involving a substitution process with a direct dependence on another system-L. The programming of L-systems can also be manipulated so that the number of symbols does not necessarily increase for each substitution; for instance, adding a boundary rule for each new string produced. Congruently, for such operational adjustments, Manousakis (2006) distinguishes three types of rules in musical L-systems: (1) production rules, (2) decomposition rules, and (3) interpretation rules.

Context-sensitive L-systems

Contextual sensitivity of L-systems allows different strains to be obtained from the same seed, depending on the elements surrounding the first applications of the rule. A short list of conditioned values $<S>$ can serve as a set of rules, where S is a symbol

that can be replaced or not, depending on the symbol that precedes it (marked with
<), or follows it (marked with >), as in this example:

axiom: F1F1F1	angle (δ):	22.5°
	direction:	90°
rules:	iterations:	30

P_1 0<0>0 → 0

P_2 0<0>1 → 1[+F1F1]

P_3 0<1>0 → 1

P_4 0<1>1 → 1

P_5 1<0>0 → 0

P_6 1<0>1 → 1F1

P_7 1<1>0 → 0

P_8 1<1>1 → 0

P_9 + → −

P_{10} − → +

The two-dimension graph produced with these instructions elaborates a self-similar 'tree'. Worth and Stepney (2005) employ this tree to illustrate a musical system that, after a process of systematic elaboration, reaches a climax at the end of its first part; a second part then begins, gradually consolidating the appearance of the whole structure—by analogy, one can find plenty of musical examples, in a variety of styles, with similar self-structuring. A plausible interpretation of this tree can have a variety of musical shapes and styles, depending on how the rules of the context-sensitive grammar are converted into rules of a specific musical system. Worth and Stepney (*ibid.*) obtain a syncopated melody which, like many tunes, "repeat the main motif, sometimes transposed." These authors believe that this kind of repetition "reflects how music is composed or improvised." More precisely, it should be noted that in elaborating a musical structure with context-sensitive L-systems, the generating string itself is relevant in the same quantity as the choice of musical elements and rules with which the string is related. Far from delivering a musical form fully completed, L-systems complete only *half* of the task considered as 'musical': the rest must be covered by the criteria and actions that are chosen to achieve the conversion of a series of algorithms into significant musical content. The dialectic between these two aspects is characterized in subchapter 5.5 as the 'negotiation between determinism and indeterminism'.

Stochastic L-systems

A string in which the initial axiom is associated with a specific rule, or to another rule, with one of the two options chosen by probability, is the seed of a stochastic L-system. It is worth recalling that a process whose behaviour is at least partially predictable, is a *stochastic* process. A process whose overall relationships are unpredictable (as in the case of white noise) is a *random* process; and a process whose relationships are well known and predictable, is a *deterministic* process. Moreover, a chaotic process can result from a deterministic process, with initial conditions that are well known, but unpredictable successive bifurcations, due to high sensitivity and an exponential growth of perturbations in initial conditions.

Many stochastic processes are associable with different forms of L-systems. In fact, the examples given above are deterministic ones, since their initial conditions are well known, i.e. their axioms, rules and first applications. Nevertheless, many other musical self-similar structures can be produced from a relatively controlled system, generating self-replacement strings within a stochastic environment.

Assuming that there is an initial axiom and rules related to two equally possible options, one of the two options can be chosen at random (e.g. throwing a coin). Each rule then has a probability of ½ , to be chosen:

P_1

0: F → ½   angle: δ= 90º

1: F-F-F+F   direction = 90º

2: FF-F-F-F+F-F+F

P_2

0: F → ½   angle: δ= 60º

1: F+F+F-F   direction = 90º

2: FF-F-F-F+F+F-F
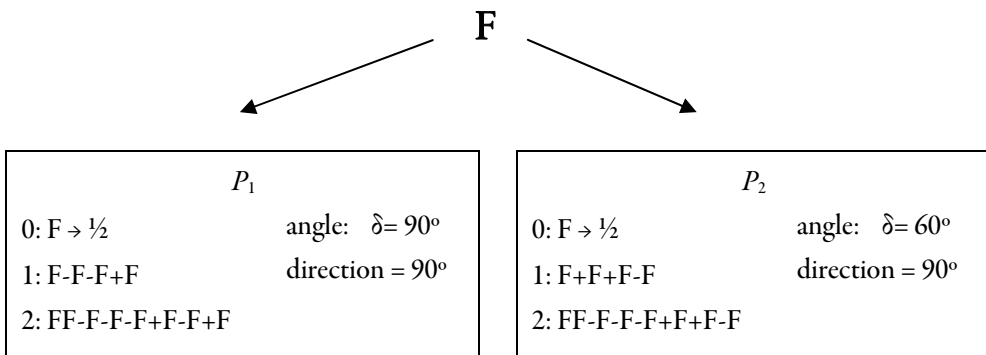
If instead of having two initial options, there are three, four or five sets of rules to be elected, then the chances are divided into $^1/_3$, $^1/_4$ or $^1/_5$, respectively, gradually opening a margin for variations produced by the stochastic system itself—especially if one

conceives that subsequent recursions can also feed back to the same stochastic system.

Worth and Stepney (2005) make an important statement regarding the iterative use of stochastic rules in L-systems applied to the generation of musical structures: they observe that applying such rules does not suffice to establish the principles of a grammatical context; these principles must be previously established. For this reason, the musical examples they offer are designed to meet in some way the Schenkerian formalization by structural levels, in order to produce results within the margin of this formalization associated with tonal music and its harmonic principles. This *filter* chosen by Worth and Stepney—and which they call *Schenkerian rendering*—is, however, equally susceptible to modifications and substitutions, in the same way that other algorithms and production rules can be modified to meet an original plan for musical synthesis or analysis.

About hidden L-systems

Webster's (1950) epigraph, borrowed from W.H. Hadow (1926), suggests that "The history of music is not different from that of organic nature." Much has been discussed about the *naturalistic* notion of music, and some authors, including Webster himself (*op. cit.*:238) and Borthwick (2000:662), claim major lacunae regarding a systematic relationship between biology and music. L-systems, along with other tools that facilitate the study of constructive self-reference, help to cover such lacunae, at least from formal, symbolic, and generativist perspectives.[497]

It is clear that many self-organizing processes observable in nature, cannot be interpreted by direct analogy with L-systems or a specific system of proportional iteration. However, as suggested by Durand *et al.* (2005), at least a substantial portion of these processes are due to overlapping or juxtaposed systems, whose rates of excitation and inhibition, along with their recursive structuring, follow particular 'versions' based on the same organizational principles—including *hidden L-systems*. By analogy, this observation is also valid in musicology, in that the analytical tools are

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[497] Other aspects, however, remain to be considered. This motivates a discussion in subchapters 4.7. and 4.8.

often appropriate for a specific case study, although it is necessary to adjust the descriptive methods employed, in order to match the shape of the analytical approach, with the rhythms and patterns that are truly meaningful in the case study. Since many organic forms already existing before analysis or emulation with L-systems are resistant to complete mirroring or reproduction by these means, computer science applied to biology constantly seeks more accurate systems of analogies—in order to better understand the rhythms and patterns of organic development. Equally, a corpus of written or recorded music cannot be automatically mirrored or reproduced by similar implementations, without first understanding which are the symbolic values and relationships to be considered relevant as self-replacement systems.

Turtle graphics

Originally, Lindenmayer's (1968) self-replacement model did not generate any automatic graphic representation. It was only in 1984 that Alvin Smith adapted L-systems for generating graphics using turtle graphics, a method of vector graphics in Logo programming language, developed by Seymour Papert in about 1969. Turtle graphics works using a relative cursor (the 'turtle') upon a Cartesian plane, and consists of four basic movement operations, represented by symbols F, f, + and −, in which means (F) 'move forward a step of length δ', (f) 'move forward a step of length d', (+) 'turn left by angle δ', and (−) 'turn right by angle δ'. In this fashion, L-systems interpreted in turtle graphics combine the simplest notions of symmetry and affine transformation, with formal grammars graphically expressed.

The simplest example provided by Prusinkiewicz and Lindenmayer (1990), has the following characteristics:

$$\omega: X \ p1: X \rightarrow F[+X][-X]FX \ p2: F \rightarrow FF$$

where ω is the initial point corresponding to the axiom, and p is the associated rule. Its generation instructions are:

start:	X
variables:	F[+X][−X]FX
rules:	FF[+F[+X][−X]FX][−F[+X][−X]FX]FFF[+X][−X]FX

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By interpreting brackets as 'beginning' ([ ) and 'ending' ( ]) of a line, and X as 'hinge' linking lines, a tree-like graphic is plotted on the computer screen—where each 'step' in the turtle graphic is interpreted as a pixel. After five iterations of the algorithm, this L-system produces a self-similar tree, such as that used by Worth and Stepney (2005) for the analogy of a musical structure with parameters pitch and length. These are similar to Fibonacci trees, such as those presented in examples ◊631 and ◊639.[498] If the Thue–Morse sequence is associated with the same algorithm, programming the following instructions,

(a) if $t(n) = 0$, move upward a step,
(b) if $t(n) = 1$, rotate counterclockwise by $\pi/3$ radians,

this sequence converges into the Koch curve (see partial representation in ◊333). This example allows visualization of the close relationship between the Thue–Morse sequence implemented as turtle graphics, and its potential capacity as a fractal engine. Moreover, it is clear that turtle graphics can be adapted to user-defined functions, defining local properties of the environment; this is particularly meaningful in terms of intersemiotically translating graphic positions and vectors, into sound sequences logically arranged by the user (and automatically developed by the algorithm). For example, Mason and Saffle (1994), Sharp (e2001), and Worth and Stepney (2005) pass turtle graphics into musical sequences, distributing time values in the abscissa, and pitch values in the ordinate. Sharp (*op. cit.*) also adds programming variations including trills and accents, as well as boundaries for the branching function, and instructions for systematically returning to a specific melodic motif.

A first example of turtle graphics conversion into a musical L-system, can be displayed using a generator (a real-time software such as Lyndyhop can be used; see Elmiger e2005), to replace a graphic network from a pixel or a point on a coordinate, with pitch-duration values in the same coordinate. This exercise makes sense assuming that points, lines and groups in the plane have a self-structuring distribution, characteristic of both, L-systems and organic-like processes in music. For practical purposes, the system can be represented in a grid on which the parts of

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[498] See pages 377 and 400, respectively.

the structure are analogously distributed. For this case, let the following be the characteristics of a specific L-system:

start:	F	angles (δ):	= 90°
variables:	F–F–F+F	direction:	= 90°
rules:	FF–F–F–F+F–F+F		

The turtle graphic corresponding to the first iteration follows here, along with its interpretation as pitch-length system in the diatonic scale with semiquavers:

◊651*a*

◊651*b*. Translation of the upper design, as monodic sequence of pitches in the diatonic scale. The conversion is done by reading the pixels from left to right and from bottom to top, articulating the filled boxes as sequential order.

This first example, although quite simple, shows that even at an early stage, the system can produce pre-self-similar cells, useful for the generation of musical motifs. The second iteration of this turtle graphic appears in ◊651*b*, followed by its interpretation, equally as a pitch-duration system in the diatonic scale, using semiquavers.

Schillinger curves as D0L systems

Schillinger (1946:I, 2) proposes interpreting a regular sequence of horizontal and vertical segments, as a 'square curve' (see below) convertible into a musical code, giving durational value to the horizontal segments, and pitch value to the vertical segments, from a predetermined starting pitch: [499]

Accordingly, this shape can be interpreted in the following way:
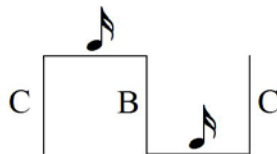
Then, using the same rules, a more complex musical system can be generated, with a more elaborated system of lines:

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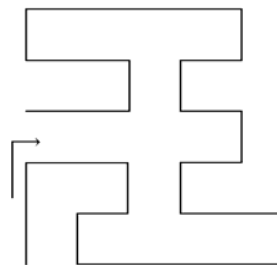
[499] Backus (1960) considers that this representation, proposed by Schillinger, is absurd. However, Schillinger's idea is taken up by Prusinkiewicz (1986), in a fruitful rediscovery of an algorithmic method for generating music. In more recent years, as seen in the works of Nelson (1992), Mason and Saffle (1994), Jones (2000), Worth and Stepney (2005), Manousakis (2006), Snyders (e2008), and Lourenço, Ralha and Brandão (2009), this method is one of the most widespread applications of musical L-systems.

In this example, as well as in the previous one, the base-pitch is C, and the durational unit is the semiquaver; the curves are read starting from the bottom-left, moving to top-right, using the diatonic scale, from which the following motif results:

Obviously, production rules can be modified using the same generative system. For example, instead of starting at C, the pattern can start at any other pitch; instead of using the diatonic scale, one can use the chromatic or another scale; instead of using the semiquaver as a unit, a different metric unit can be employed; and rather than starting to read from bottom to top and left to right, the reading movement can run from top to bottom and from right to left—for example. In short, the musical interpretation of a Schillinger curve can be very flexible, adjusting a variety of parameters. Interestingly, some 'square curves', such as Hilbert and Peano curves, can transmit to music their characteristics of iteration, self-similarity and structural consistency.

Hilbert and Peano curves

Piston (1947:13) claims that "The word curve is useful to suggest the essential quality of continuity, and to emphasize that minor decorations and indentations do not affect the main course of the melodic line." In its essence, this notion is still valid for musical L-systems and self-replacement strings in general; though, from a perspective of musical self-similarity, the "minor decorations and indentations" can also have deep and strict significance in all musical parameters.

A Hilbert curve is a fractal continuous curve—absolutely and exactly self-similarly—filling the plane. Given that the nth generation of the Hilbert curve consists of 2^{2n}, its Hausdorff dimension in the limit $n \to \infty$ is 2, which reflects the quality of the

curves covering its area (see Schroeder 1991:10). H_n is the nth approximation to the curve in its limit values, whilst the Euclidian length of H_n is

$$2^n - \frac{1}{2^n} \, .$$

This means that the curve grows exponentially with n, to infinity, whilst at the same time it is limited by a square with a finite area.

The Hilbert curve can be described by the L-system algorithm:

  alphabet: L, R (left, right)

  constants: A, +, −

  axiom: L

  production rules:

   L → +RA−LAL−AR+

   R → −LA+RAR+AL− ,

where A means 'move forward', + means 'turn 90° to left', and − means 'turn 90° to right'. The first suggestion for intersemiotically translating this system into music came from Prusinkiewicz (1986:456): "Suppose that the Hilbert curve is traversed in [a specific] direction and the consecutive horizontal line segments are interpreted as notes."
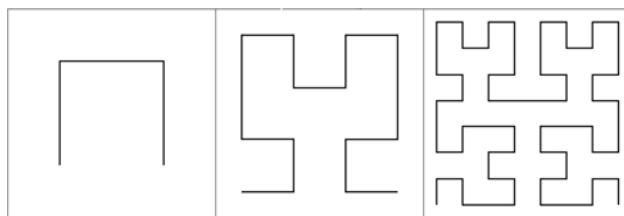
◊652. Above: Hilbert curve in its first three iterations (from left to right). Below: musical motif obtained from the second iteration, according to the rules set out for the previous example.

The Peano curve is also a self-similar construction whose limit fills the plane, with Hausdorff dimension = 2. As the Hilbert curve, the Peano curve has the geometrical property that it never passes the same point twice (a feature that cannot strictly be translated intersemiotically into music).
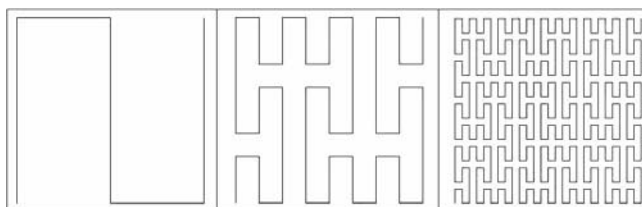
◊653. Above: Peano curve in its first three iterations (from left to right). Below: musical interpretation of the second iteration (shape in the centre), according to the rules applied to the previous two examples. As Prusinkiewicz (1986:456) notes, "Naturally, any curve consisting of horizontal and vertical segments can be interpreted in a similar way."

Some of the earliest examples of musical implementation of such curves in a compositional process are found in Gary Lee Nelson's *Summer song* (1991), for flute, and *Goss* (1993), for violin. In these musical compositions, pseudo-fractal patterns are intersemiotically translated into irregular, sounding patterns with sequential self-similarity. In addition, Mason and Saffle (1994:31–32) describe the Peano curve in terms of a musical L-system, allocated in a context of "anticipation/response models of melodic construction and analysis." These authors also provide a typology of melodic figures obtained by rotating the first iteration of the quadratic Gosper curve, structurally related to the Peano curve.

Glissandi made of glissandi

Graphics with closed regular areas, such as the examples given in ◊651, or periodic maze-like shapes, as the 'curves' shown in ◊652 and ◊652, are only a tiny fraction of the universe of self-similar structures produced as musical L-systems. Other operations create, for example, clusters of straight lines, as in the following pattern, obtained with the sixth iteration of the instructions (with nodal rewriting in L-systems type C, according to the nomenclature used by Elmiger e2005):

start:	X	angle (δ):	22.5°
variables:	X-[[F]+X]+X[+FX]-X	direction:	90°
rules:	FF ,		

which has the following graphic interpretation, embedded within a sound coordinate of pitch (from 10Hz to 10000Hz) and duration:

◊654*a*
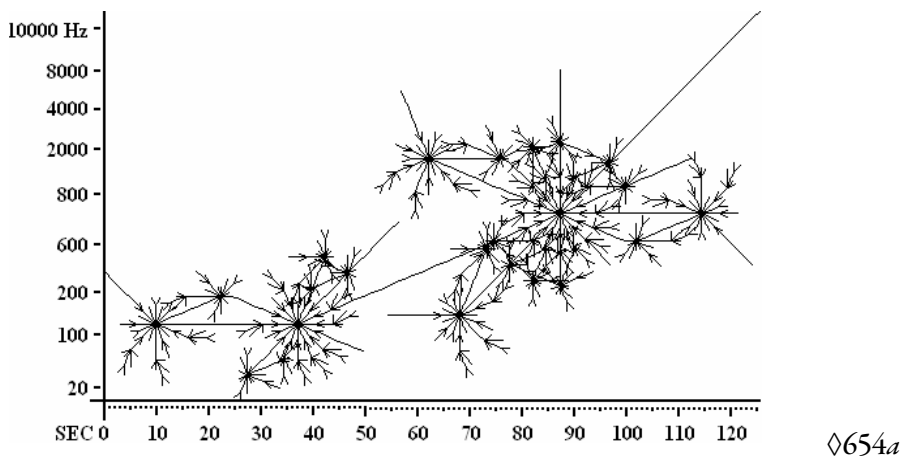
The string produced by this chart (◊652*a*) contains nearly seventy thousand symbols, which would fill more than 20 pages in this subchapter. It suffices, however, to include the above data for its generation (start, variables, rules, angle and direction) to instantly create the same graph in Lyndyhop (Elmiger e2005). The result can be situated in a sounding context, assigning pitch continuity (*tenuti*) to the horizontal lines, continuous slides (*glissandi*) to the diagonals, and pitch clusters to the verticals.[500]

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[500] The variety of pulses, articulations, intensities, and timbral possibilities in the development of a compositional plan from this figure exceeds the scope of this work. It is encouraging,

The intrinsically aesthetic qualities in this example seem to justify the process of intersemiotic translation. The arguments in favour of this operation are found in abundant literature on the perceived relationships between space, repetition and proportion For example, Bucher (1959:525)—when he analyzes the architectural theory of Borissavlievitch (1958)—notes that, whether a certain shape is considered 'beautiful' or 'harmonic' (i.e. easily grasped and assimilated as a set of proportions), "The same form repeated, however, must not be [intuitively] beautiful if it does not correspond to at least one of the [two] laws of architectural harmony." The two basic laws suggested by Borissavlievitch are (i) the law of repetition of similar figures, and (ii) the law of repetition of the same figure. These laws are operatively the same as the laws expressed in Chávez's (1961) theory of musical repetition, discussed here in subchapter 2.2., and which implies a close relation with the Gestalt criteria in musical symmetry and distribution—stating that there must be a balance between equality, similarity and transformation.

◊654*b*. Motivic detail from the previous chart. In this design, each pixel represents a basic unit in the coordinate pitch (y) – duration (x). In addition, the condensation of pixels graphically suggests loudness: the higher the concentration of information, a parallel increase in amplitude occurs.

Howard and Longair (1982) consider the feasibility of translation between basic criteria of musical harmony, by criteria similar to that in spatial constructivism. In principle, assuming that it is possible to intersemiotically translate certain musical ideas into spatial ideas, a reciprocal translation of architectural spaces into music should be possible. This speculation is solved in Xenakis' work, as intersemiotic translation of his architectural designs for the Philips Pavilion, into a structure of sounds made music in his orchestral score *Metastasis* (1954).

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however, to know that a huge number of geometries with these features await musical translation, in the terms suggested by composers such as Ligeti (for example in *L'escalier du diable* and *Désordre*) or Xenakis (in the examples presented here in ◊655 and ◊656).
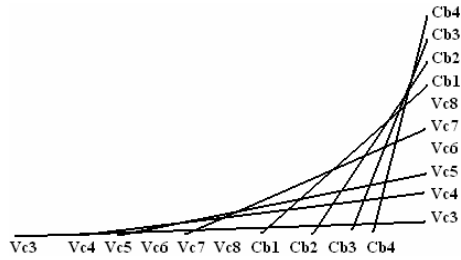
◊655. Detail from the draft of *Metastasis* (1954), for orchestra, composed by Iannis Xenakis. The design corresponds to the measure 313, with glissandi in cellos and contrabasses, based on the architectural sketch of the same author, of the Philips Pavilion proposed for the Brussels World Fair, 1958 (see Xenakis 1992:3).
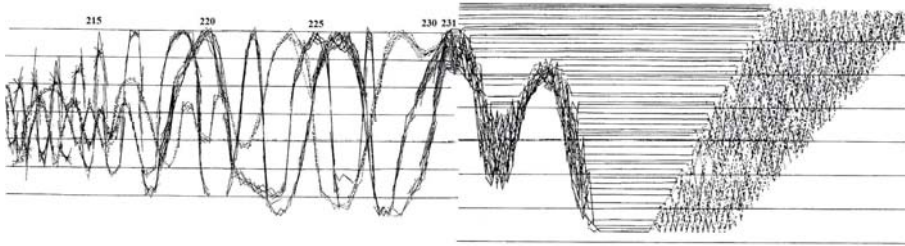
◊656. Measures 210–255 from the sketch of *Pithoprakta* (1956), composed by Iannis Xenakis, for two trombones, xylophone, Chinese box, and 46 string instruments (vni., vle., vc., cb.), each independently represented in the score. The title, in Greek, means "action by probabilities". In this work the composer performs an intersemiotic translation of the probabilistic behaviour of a set of molecules in a gas, according to the Boltzmann formula:

$$f(\text{v}) = \frac{2}{a\sqrt{\Pi}} \; e^{-v^2/a^2} \; ,$$

where *a* is the gas temperature, *v* is the molecular velocity, and $f(v)$ gives the probability that a molecule has the speed *v*. With this formula Xenakis calculates the probabilities for 58 different velocities; then, implementing Gaussian distribution, he obtains 1,148 possible velocities in a set of molecules in gaseous state, with constant temperature. The result is shown in this scheme as pizzicati-glissandi forming glissandi segments, which in turn integrate glissandi masses in a process of statistical self-similarity. In short, this intersemiotic translation is an analogy strictly proportional, since the angles of each slope are proportional ($a \propto 35$) to molecular velocities generated according to Boltzmann's kinetic theory of gases (from Xenakis, 1992:15–21).

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Critical observations

From the critical perspective of authors like Goehr (1960), Dench (1984), and Bailey (1992), self-replacement strings and self-similar tessellations explored in this and the previous subchapter, are not compositional, but pre-compositional (sub)systems. The debate arising from this notion occupies subchapters 5.5. For now, it is sufficient to consider that the creative exchange between different methods for generating patterns and structures of music provides abundant evidence throughout history, supporting the argumentation for synecdochic intersemiosis in the vortex of musical culture.

Moreover, the exchange between a logic of symmetrical relationships and self-similar patterns, within a specific musical grammar, significantly precedes musical experiments with fractals, L-systems and self-similar tessellations. According to a suggestion by Haines *et al.* (2004:340), based on a study of Giovanni Ciampini (1633–1698), Guillaume Dufay's (*ca.*1397–1474) musical thought may have been influenced by symmetry of tiles in the church of Santa Maria in Trastevere. This influence is parallel to that found in the musical works of Xenakis, inspired by the representation of a dynamical system of an unstable set of molecules (see ◊656), or of Ligeti, inspired by a mathematical model such as the Cantor function (see ◊627). This link between different models of organization of aesthetic experiences —including abstract elaborations, cannot be a whimsical relationship under a radical separation between 'compositional' and 'pre-compositional' forms. On the contrary, as suggested in the Introduction to this study, seemingly opposite notions such as 'physicalism' and tradition, or Pythagoreanism and culture, are actually making part of the same human complexity, intertwined in different ways.[501] One might conclude, following this line of thought, that the pre-compositional matter is also compositional in a substantial way.

To what extent a computational fashion or style can shape music, or how far musical tradition is able to mould computer music, is an issue that is also linked to the dynamics of the societies involved in the practical employment of these resources. Finally, it should be pointed out that computer models are also *anthropomorphic*

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[501] See section 1.1.1.

models—although it is clear that computer programming cannot encompass or replace central aspects of language as tradition. In this sense, a simple self-replacement string, even turned into melody or a chord sequence, is unable to generate idiosyncratic context that allow it to be heard as music. This requires an interaction between fixed rule and rule modifying, alongside with the intervention of a logic, not necessarily based on the operational performance of a model, but rather—and especially—according to the Gestalt qualities perceived in the model, or in a deviation from the model towards musical idiolects and ecolects; something already emphasized by authors such as Kieran (1996), Reybrouck (1997) and Ockelford (2005).[502] Also in this respect, fractional noise patterns, Markov chains, self-similar tessellations, and L-systems, provide valuable insights into understanding the meaning of music, especially for the ways in which these forms are socially adopted, adapted and transformed.

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[502] This issue is discussed in subchapters 4.7.–4.8.

6.6. Asymmetry and antiproportion

When Ossowski (1966/1978:24) states that "The possibility of a multifold figurization of Kandinsky's pictures undoubtedly adds to their charm", he also takes into account possibilities of contrast and obliteration—gradually or radically applied within an aesthetic processes. These possibilities are of paramount importance in shaping the ways in which an aesthetic complexity 'makes sense' by contrasting parts with parts, or parts with their corresponding wholes. This kind of contrast and obliteration operates in a way analogous to the complementarity between relationships of excitation and inhibition in biochemical processes (see West 1990, Prusinkiewicz and Hanan 1992, Meinhardt 1998), in which an initial *statement*—an original pattern, for example—is structurally relevant, inasmuch as its operative contradiction, limit, or reversion. The initial *statement* of a generative system 'must' be exposed to partial recomposition, in order *to be creative*; this holds true, the same in a biochemical environment than in an aesthetic analogy.

Structural repetition and proportion, both in biology in general, as in language and in music in particular, depend on the relative flexibility of a rigid structure; or —in other words—on the moderate rigidity of a flexible rule. This implies a *bounded freedom* for the sake of the message's significance, with a *minimum opennes*, desirable or necessary, to interpret the message, as balanced reciprocity between redundancy and information. This concept matches with what Umberto Eco (1968:27) identifies as the 'ordering function of the code' (*funzione ordinatrice del codice*).[503]

Jakobson (1958:358) emphasizes the role of the *poetic function* as a language generating process in the relationships between syntagma and paradigm: "The poetic function projects the principle of equivalence from the axis of selection into the axis

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[503] See pages 208–210.

of combination. [...] The selection is produced on the base of equivalence; similarity and dissimilarity; synonymity and antonymity." In empathy with this criteria, Schoenberg (see Stein 1975:165) states that "Perfect regularity (symmetry and the like) is not suited for music. Rather, coherence is achieved through contrast: antiphony, countersubject, comes, secondary theme, dominant, etc."

Howat (1977:292) emphasizes that, in music, "The measurement of proportion proceeds by the inner pulse of the music, thus absorbing fluctuations of tempo such as accelerandi and ritardandi, rather than by time as measured on a clock." Complete, mechanical rigidity, even justified by a proportional rule, is rejected in music and, in general, in a variety of aesthetic expressions. An example of this refusal is the methodical 'inconsistency' of proportion, among some of the Renaissance masters of architecture, as happens in Andrea Palladio (1508–1580), according to Howard and Longair (1982:136):

> The many anomalies and contradictions arising from the [proportional] analysis are not entirely surprising. As in so many other respects, the ideas behind [Palladio's] *Quattro libri* remain tantalizingly elusive. Despite the apparently consistent tone of the treatise, its pages betray no coherent underlying system which could have governed Palladio's principles of design. The subtlety and elusiveness of the *Quattro libri* are just two reasons for the enduring fascination of this very remarkable work.

Musical examples with analogous features are abundant—following an orientation comparable to this example—avoiding the unequivocal application of an aesthetic canon. For instance, Madden (2007:90) notes that one of Schillinger's (1946) favourite compositional techniques was to "create patterns by 'interference', running numerical sequences forward and backward against each other to produce new combinations." This way of musical development is not necessarily arbitrary; on the contrary, under a systematic employment, it resembles aspects of deterministic chaos as explained in section 3.9.4.[504] Precisely, when Xenakis (1992:275–276) speaks of "transformation of a sieve", he intends that this type of operation serves to lay out generative relationships between proportion and antiproportion.

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[504] See also subchapters 5.2.–5.4. and 6.2.

Symmetry and asymmetry in the waveforms

As suggested in subchapters 4.2.–4.3., the self-organization of matter/energy at the atomic level, keeps scalar invariance and coordination within chemical and biological layers under universal power laws.[505] The same scalar invariance has an influence in the musical practices established by culture, in coordination with physical principles. This is especially valid in terms of acoustic symmetry-asymmetry in the harmonic organization of sound, as explained below.

When a periodic wave is composed of a fundamental (f) and only odd harmonics (*f, 3f, 5f, 7f, ...*)—something common in single-reed woodwinds such as the clarinet—then the summed wave is symmetrical: whether the waveform is inverted and its phase shifted, the waveform will be exactly the same (see Fletcher and Rossing 1998:486–488). If the wave has any even harmonics (*0f, 2f, 4f, 6f, ...*), then the waveform will be asymmetrical; the top half will not be a mirror image of the bottom (see Backus 1977:111–118).

Very often, music is composed by acoustic systems combining odd and even series of harmonics (i.e. harmonic non-linearity), resulting in timbral variation and instrumental richness. Instrumental nuances in an orchestra, or changes of register within a same instrument or family of instruments and/or voices, consist of waveform changes producing a diversity of harmonic tilings, as suggested in subchapter 6.4. Thus, the interplay between symmetry and asymmetry in the mechanical and acoustic worlds, is a precursor of a more complex interplay between symmetry and asymmetry in the manifold worlds of tradition, culture, and social environment, modelling the variable shapes of music. Here appears, again, the figure of 'worlds within worlds' as foreseen by Parsegian (1968:589) and Mandelbrot (1982:2–4, 150, 209).

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[505] See section 3.9.5.

A matter of complementarity

Hofman-Jablan (e2007), adapting the mathematical theory from crystalographer Alexander Mihailovich Zamorzaev (1927–1997), introduces into music theory the concepts of asymmetry or simple antisymmetry, multiple antisymmetry, and *P*-symmetry (or permutational symmetry). Hofman-Jablan starts her investigation by mentioning the most evident contrasts within typical parameters in music (e.g. *major–minor*, *strong–weak*, *high–low*, and so on); she then explains the function of structural antisymmetry, as inhibition of sameness and repetition, providing music with an equilibrium between predictability and unpredictability.

Hofman-Jablan also refers to asymmetry or simple antisymmetry as the analogy *question–answer*, conceived in traditional counterpoint; as basic relationship *comes–dux*, in fugue; as contrast in harmonic modulation; as elementary notion in metre and prosody, by the difference between *strong* and *weak* accentuation; and as constructive relationship in melodic nuances. In conclusion, for Hofman-Jablan (*op. cit.*), "all structures based on alternation may be considered as asymmetric."

An intimate link tying logical inference and music, is the coordinated operation of the intuitions of symmetry and asymmetry. Juha Ojala (2009:138–139) suggests that this operation is achieved as synecdoche or as chiasmus (i.e. crisscrossing relations), of utmost importance in the systems of association and musical continuity:

> In symmetrical logic, a part may be equated with the whole, or a member of a set with the complete set. Ramifications of these are tremendous. In comparison to asymmetrical logic, which tends to create linear chains of ralations along the hierarchical systems of nested categories, symmetrical logic easily spawns an infinite network of relationships between elements [...] The two logics are not hermetically separated, but rather they operate together construing a continuum, facets of which are in dialogue of analysis and synthesis, in constant dynamic interplay.

This 'dialogue' of analysis and synthesis is, in itself, an example of coordination between symmetrical and asymmetrical logic. On the one hand there is the symmetry of both terms, as complementary mental processes; on the other is the asymmetry of their potentialities and results. This description, familiarized as the dichotomy between rigidity and flexibility of musical grammars, as well as with the criteria for

musical determinism and indeterminism, leads to the assumption that in general the functional similarities of music are also in 'dynamic exchange', in balance with their implied differences. Therefore, the similarities of a musical system are immersed in an ambit of flexibility, a game in which discourse, interpretation, and transformation of music have a rich intercourse.

In music, a relationship of asymmetry does not imply, necessarily, complete lack of symmetry; rather, it may signify a symmetry that for structural reasons cannot be satisfied. Then asymmetry can be interpreted as an ideal symmetry *in absentia*. This is evident in the avoidance of simple symmetries e.g. doubles or halves in rhythm, metre, harmony and melody. The systematic planning of asymmetries for structural development is a case of complex antisymmetry. Analogously, if in pursuit of a higher organization, a predictable trend of structural proportion is not met, then a case of antiproportion occurs. Symmetry, antisymmetry, proportion and antiproportion are not, however, types of relationships radically opposing each other; instead, they usually are collaborating within a common system of harmonic systems, more or less in the fashion that José Vasconcelos (1951:23, 30) suggests: "The world is made of asymmetrical compositions, contrary to dispersion in homogeneity. [...] The world is perennial dynamics, as disparities and asymmetries in concert."

Regarding the psychological functioning of the prolongation or breaking of a proportion, expected in musical time/space, a play between predictability and unexpectedness comes to the foreground. In this context, and assuming the original definition of the term *abduction*, made by Charles S. Peirce (*CP* 2.270), it is clear that antiproportion works upon the same principle:

> *Abduction* is a method of forming a general prediction without any positive assurance that it will succeed either in the special case or usually, its justification being that it is the only possible hope of regulating our future conduct rationally, and that Induction from past experience gives us strong encouragement to hope that it will be successful in the future.

This notion indicates the possibility of achieving a link of communication that is 'sufficient'—rather than 'successful'—between musical experience and predictability, or between memory and unexpectedness. In this context it is very relevant to note that the synecdoche operates as antisymmetry, creatively and efficiently placing the *well-known part* into the *whole to-be-known*. The quiddity of synecdochic

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intersemiosis in the cognitive relationships consists, therefore, of a constant game between symmetries (or *obviousnesses*) and antisymmetries (or *variations*), with which a *reality* is modelled.[506] This is an essential postulate for aesthetics, since the typical relations of constructive self-reference (e.g. the golden string, the Fibonacci sequences, or the bifurcation diagram of the logistic map) are rather systems of antisymmetry.

Decision-making procedures—including their ranges of fallibility, in which the probabilistic function alters the regular patterns of a rigid system—are part of a *code of self-vulnerability* through *local creativity* (a connection that subchapters 4.7. and 4.8. characterize as the productive relationship between idiolect and ecolect). An example of this, analogous to what happens in music, is the alteration and gradual transformation of design, in traditional brocades. Situngkir (2008:12), in examining these evolutionary changes in the batik from the Indonesian tradition of textiles, notes that

> [T]he properties of batik should be seen as a whole process from the decision on its material, the *mbatik* process including the ornamentation, and even to the appreciation on how people traditionally used batik. These properties interestingly do not emphasize batik on the ornamentation, but the batik crafting process.

A certain operational range of asymmetry in decision-making procedures are, as in this example, central to musical creativity, as Rocha Iturbide (1999) explains:

> In the late forties, the compositional technique of John Cage was to define the overall structure of the work with numbers representing the proportions; these proportions were used to rigidly define the sections of the work, but on a smaller scale, i.e. at the level of measures and beats, the composer enjoyed a certain freedom to make decisions.

This general relationship between rigid and flexible also operates in the translatability of a musical system into another one, or in the intersemiotic mapping between adjacent categories. A paradigmatic case is Nelson's (1992, 1994) interpretation of the notions of *random walk* and *dissimilarity*, in a musical negotiation between structural indeterminism and determinism. Clearly, the operating margins of asymmetry are an essential feature in the configuration, transmission, and feedback of

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[506] This concept of reality is closely related to the notion of Gestalt, as explained in subchapter 3.5.

the message in a self-referential system; for example, as shown in example ◊451, comparing the same set of code repertoire, in different interpretations; interpretations that may come from the same source, at different times.

Transformation of regular patterns

The transformation of regular patterns by antisymmetry and antipropotion is a widespread phenomenon in aesthetics, common to the structural transformations in many tessellations of M.C. Escher, and in musical patterns of J.S. Bach, studied by Hofstadter (1979) and mentioned in subchapter 6.4.

The study by D'Arcy Thompson (1917), on symmetric transformation, also presents varied evidence on how biological structures—with specific examples of plants and animals—mutate and adapt within a context, resulting in functional changes. Thompson emphasizes the aesthetic significance of these changes and formalizes structural aspects that later extend to an *organicist* perspective for understanding and interpreting (pre-)musical structures, e.g. in the cases of Mâche (1983), Josephson (1995), Head (1997), Alexjander and Deamer (1999), and Peter Gena (e1999, e2006).[507]

In an abstract form, subchapter 6.2. symbolizes the typical transformations of a simple, regular pattern, which can lead to increasing complexity. The scheme ◊620, for example, graphically represents the transformation of a regular system of points or grains of sound, into acoustic clouds or masses with a completely different aspect, with generalized transitions between symmetry–antisymmetry and proportion–antiproportion.

Antiproportion by juxtaposition

Xenakis (1992:244) notes that "there is no pattern and form recognition theory, that would enable us to translate curves synthesized by means of trigonometric functions in the perception of forms or configurations." This is closely related to what Xenakis (*op. cit.*:245) identifies as "the wrong concept of juxtaposing finite elements",

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[507] See subchapters 4.1., 4.3. and 4.6.

summarized in the fact that the quantitative manipulation of simple wave segments (i.e. sinusoidal functions) may result in the appearance of virtually any desired sound. Xenakis believes that such a massive juxtaposition jeopardizes the very quality of the constitutive elements of sound. This issue leads to the old discussion about the juxtaposition of proportions.

In his treatise on proportion, Ghyka (1927) states that it is impossible to juxtapose the same spatial relationship in various systems of proportion without obliterating each one of these proportions as they overlap. However, many composers employ this kind of juxtaposition to systematically eliminate typical proportions of constructive schemes, creating—by omission or intention—other relationships as a result of the juxtaposition in a deterministic system.[508] However, intuition and indeterminism have also a functional role in this type of equilibrium, according to the *poetic function* proposed by Jakobson (1958). Regarding the theories of Schillinger (1946), Kramer (1973:141) sees this form of 'freedom', as problematic:

> Schillinger, inspired by the 'organic' nature of the golden mean, suggests deriving melodic lines from the Fibonacci series (and also from other summation series). He freely makes octave transpositions, so that the proportional and additive properties are lost, as is the Fibonacci source of the resulting melody [...], and, if the resulting lines are in fact aesthetically pleasing, I suspect that the Fibonacci series is not the reason.

Kramer highlights the key point of the antiproportion by omission, since the argument of proportion is used as an excuse to get results justified as part of a 'free method' for musical production, while strong cultural and psychological ties remain underlying, prefiguring the audible result. Sometimes this form of antiproportion is a conscious process, as found in the works of Nono (*Il canto sospeso*, 1956), Ligeti (*Pièce électronique No. 3*, 1959–96; *Apparitions*, 1958–59) or Stockhausen (*Klavierstück IX*, 1961; *Adieu*, 1966; *Telemusik*, 1966). Toop (1999:68) claims that, in the case of

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[508] Here the role of the unconsciousness again has a prominent place in the compositional process, as in many cases in which, as suggested by Kramer (1973), Howat (1983) and Madden (2005), the golden ratio or other type of structural proportions are used. Even when most of a prescriptive grammar is properly executed, the decision-making in a range of inexactitude is common, similar to composition and music performance. This confirms a close relationship between musical determinism and indeterminism.

Ligeti, use of the golden ratio and the Fibonacci sequence is "a little more complex" than in other composers from his generation, because

> [A]s the musicologist Gianmario Borio later pointed out to him, he made so many small adjustments in the process of composition that hardly a single Fibonacci number was left. Be that as it may, ever since *Apparitions*, analysts have hastened to find golden sections and Fibonacci proportions [*sic*] in Ligeti's work. They may even be there, says Ligeti, but not by design: he used this particular kind of conscious numerical structuring once, and only once.

Toop's explanation is far from being transparent in analyzing why the case of Ligeti is "a little more complex" than others, since the same form of antiproportion by omission is also found in Debussy, Bartók, Nono, Stockhausen and Boulez, as well as in many of their followers. In any case, it is much rarer to find an *integral-proportionalism*—in empathy with Webern's radical serialism—evident in the work of a few composers such as Hugo Norden (1909–1986) and Per Nørgård (1932–). Flexibility—not generalized mathematical exactness in proportion—is the common feature in these procedures. Toop (1999:201) notes that, for the case of Ligeti, "the exactness of the analogy is of secondary interest: what the scientific model offers here is inspiration, not legitimation." However, this claim is controversial, because many aspects of different musical traditions are supported by some analogous criterion of exactness. A difference is unveiled, therefore, between these criteria and the mathematical criteria of exactness: music makes a *discretionary* use of proportions and symmetry.[509] Thus, what music finds in mathematics is not just 'inspiration', but also a certain legitimacy and rationality. Furthermore, such a relationship is reciprocal. Mathematicians, from Pythagoras to Mandelbrot, often turn to music in a search for intuitive legitimacy.[510] Nevertheless, and according to Koblyakov's conception of the 'Subject' as coordination between intellect and idiosyncrasy, "the way from music to mathematics (inversion of the traditional strategy of research) is more promising because it allows us to include the Subject's Factor into the researched phenomenon." (Koblyakov, 1995:299).

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[509] See subchapter 2.3.

[510] This particular subject is discussed by numerous sources that link mathematics to music; a summary appears in Fauvel, Flood and Wilson (2003).

A case study: Il canto sospeso (1956), *of Luigi Nono*

Thanks to a communication from Stockhausen (see Maconie 1989:50), it is known that Werner Meyer-Eppler (1913–1960), a physicist-mathematician and linguist, had direct influence on the formation of many composers of Darmstadt school's first generation. The structuralist emphasis on the golden ratio and Fibonacci sequence in this generation, is due to the influence of Meyer-Eppler and other 'naturalists' inspired on Adolf Zeising's doctrine (see Zeising 1854). Among the results obtained under this influence, there are two musical works, considered in the core of the classical repertoire of the post-war Europe: *Klavierstücke IX* (1954–61) by Stockhausen, and *Il canto sospeso* (1956) by Luigi Nono. Kramer (1973:126) notes that:

> At approximately the time that Stockhausen was composing *Klavierstück IX*, his Darmstadt colleague Luigi Nono was utilizing, in *Il canto sospeso*, the Fibonacci series in a different manner. Nono was apparently less interested in generating formal proportions than in determining individual note durations by means of the series. The second movement of this large work, which is rather celebrated in analytic literature, is 'totally' serialized, with the durations of notes generated by Fibonacci numbers 1, 2, 3, 5, 8, 13.

However, the formal construction of *Il canto sospeso* also presents aspects involving the Fibonacci sequence, in a complex way. In conceiving this work as a succession of measures—interpreting each measure as a subset characterized by a set of 'actions', capable of being registered as groups of algorithms or 'paragraphs' of instructions—it is clear that, firstly, the relevant measures in terms of succession of actions, literally match with the Fibonacci sequence.[511] On the other hand, a group of bars with contrasting actions corresponds to the initial sequence of prime numbers $\{ \wp \mid \wp \in [2, 107] \}$, where $\wp$ is a prime and 107 is the number of measures in the first movement of the examined score. This means that the 'harmonic' geometry of this composition, based on the golden ratio, intersects with a contrasting grid, in order to increase the significance of the harmonic structure (see Pareyon 2007a).

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[511] 'Literally' means that the Fibonacci sequence {1, 2, 3, 5, 8, 13, 21, 34, 55, 89} has been inserted into a grid of prime numbers {2…, 107}, in order to produce the grid of measures for Part I. The serialization of measures also includes the partial sums 5+5, 8+8, 13+13, 21+21, 34+34, as suggested in table ◊660.

This contrast of the proportion emphasizes the aesthetic values intuited in a musically organized time series. In a similar way, in drawing, design, and plastic compositions, a certain object, a certain texture or set of lines, emphasizes the perception of a particular sign that makes sense by an effect of contrast (see *Gegensatz* in Kandinsky 1926:86–87).

In short, *Il canto sospeso* is structured in a logic of counterbalances for each action, overlapping a harmonic or 'intuitive' geometry (represented by Φ in table ◊660) with a contrasting or 'anti-intuitive' plot (represented by $\wp$). Unlike the typical relationships with the golden ratio, found in many musical examples in which it appears 'naturally' (see Tatlow 2001), intervals in the prime numbers series are perceived as 'counter-rhythm'; as a form of a counterintuitive distribution, or, as Jameson (2003:vii) suggests, as "a very irregular way" of apparent distribution. Such a contrast results from a forced association of the Fibonacci sequence (matching with Φ), with intervals from a bounded segment of prime numbers ($\wp$). In the specific case of *Il canto sospeso*, this correspondence is too precise to be just an unconscious tendency (see ◊660).[512] Ultimately, the most important structural feature of this score is not the use of the Fibonacci sequence and golden ratio, but the ambiguity and unpredictability accomplished by a contrast of 'opposed' structures.[513]

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[512] In an interview held in Warsaw in October 2001, with the author of the present study, and Włodzimierz Kotoński (1925–), the Polish composer—friend of Nono and a close co-worker in academic projects with him—stated that Nono consciously used the Fibonacci sequence and the golden section in *Il canto sospeso*. When he was asked if Nono equally used the prime numbers series, Kotoński answered that he was unaware that Nono had been used them as a compositional resource.

[513] The subject of this section is restricted to antiproportion, which is the main concern of this subchapter. For a detailed discussion on the use of the Fibonacci sequence in Nono's *Il canto sospeso*, the study by Kramer (1973) is especially recommended. The article by Poné (1972) also deals with important aspects of Gestalt and self-reference in the same work, in a broader context.

bar	source	(Fib) origin	action	metre	tempo	dynamics
1	Φ	—	Part I starts. Period I, beginning of subset 1: {1, 5, 7, 9, 11} (collection B {7, 9, 11}).	$\frac{4}{8}$	♪ *ca.*92	*ppp*
2	Φ, ℘	—	Complement collection B (collection A {1, 5}).	*idem*	*rall.*	*ppp*
3	Φ, ℘	—	Timpani (1): *tr* (*mf* > *ppp*).	*idem*	*idem*	*mf*
5	Φ, ℘	—	Ending of period I, beginning of period II (beginning of subset 2, *et ss.*). Vni. con sordina + solo.	$\frac{3}{4}$	♪ *ca.*60	*ppp*
7	℘	—	Entry of ottavini; trumpet 1 (D♯) *p* < *mf*.	$\frac{4}{8}$	♪ *ca.*92	*mf*
8	Φ	—	Ending of period II, beginning of period III.	*idem*	*rall.*– *accel.*	*p*
10	Φ	5+5	Timpani (3): *tr* (*mf* > *ppp*).	*idem*	—	*mf*
11	℘	—	Ottavini 3, 4; muta in fl.; trp., trbn. < *mf*.	*idem*	*rall.*	*mf*
13	Φ, ℘	—	fl., pitch polarization (G), *mf*.	*idem*	♪ = 92	*mf*
16	Φ	8+8	Ending of period III, beginning of period IV. Vni. 1: pitch polarization (E♭), *p*.	$\frac{3}{4}$	♪ = 60	*pp*
17	℘	—	Entry of vc., cb. (divisi: pizz. & arco).	*idem*	*idem*	*pp*
19	℘	—	Ending of period IV, beginning of period V. Entry of woodwinds and brass (*mp*), strings off.	$\frac{4}{8}$	♪ = 92	*mp*
21	Φ	—	Transition between segment 'a' and 'b' of the period V. Timpani (2): *tr* (*mf* > *ppp*).	*idem*	*rall.*– *accel.*	> <
23	℘	—	Entry of flute 3 and trumpet 3.	*idem*	*idem*	*pp*
26	Φ	13+13	Conclusion of segment 'b'.	*idem*	*rall.*	*p*
29	℘	—	Ending of period V, beginning of period VI. Wind instruments off. Entry of vni. 2b and vla.	$\frac{3}{4}$	♪ *ca.*72	*mp*
31	℘	—	Entry of vc. & cb. (extended to	*idem*	*idem*	*p*

			measure 32).			
34	Φ	—	Ending of period VI, beginning of period VII.	$\frac{4}{8}$	*idem*	*p > ppp*
37	℘	—	Flute 1, register change; entry of bass cl.	*idem*	→ ♪ = 92	*p > ppp*
41	℘	—	Ending of period VII, beginning of period VIII. Entry of vni., vla.2 & vc. Wind instruments off.	$\frac{6}{8} \rightarrow \frac{3}{4}$	♪*ca.*82	*p < mf*
42	Φ	21+21	Entry of vla. 1 & cb.	$\frac{3}{4}$	*idem*	*mf*
43	℘	—	Vni. 1b + vc. E♭ completes the series for period VIII.	*idem*	*idem*	*ppp*
47	℘	—	Vni. 1a: pitch polarization fromA♭$_8$ to A♭$_9$.	*idem*	*idem*	*mf*
53	℘	—	(49–52 are transition measures). Ending of period VIII, beginning of period IX. Brass+strings (first time); timpani *tr* (*ppp < f*).	$\frac{4}{8}$	♪*ca.*92	*f >*
55	Φ	—	Ottavino (F♯); trp. 1, 3 (accents [*mf,f > ppp*]); vni. alla 8$^{va.}$ (*ppp < f > ppp*).	*idem*	*idem*	*ppp<f>ppp*
59	℘	—	Ottavino returns, tr. 1 (*mf*), trbn. (*< f*).	*idem*	*idem*	*f > ppp*
61	℘	—	Vni. 1a (*p < f*); vla. 1 (*mf > ppp*).	*idem*	*idem*	*f > ppp*
67	℘	—	Fl. 1: introduction of new rhythm (modifying of figure from measure 2) [*mf*]; ending of period IX.	*idem*	*idem*	*mf > ppp*
68	Φ	34+34	Beginning of period X. Gradual change of instrumental palette: (69) trumpets 3, 2 (*fff*); (70) fl. (*f*), hrn. 1, 2 (*fff*).	$\frac{2}{4} \rightarrow \frac{4}{8}$	♪*ca.*60 ♪*ca.*92	*ppp − fff >*
71	℘	—	Entry of trp. 1 (*p*) + trbn. 1, 2 (*p*) + timpani (*mp*).	$\frac{4}{8}$	♪*ca.*92	*fff > p*
73	℘	—	Winds ensemble *fff, f, mf*; trombone 3; pedal note (A) appears.	*idem*	*idem*	*fff > p*
79	℘	—	'Hinge' measure of period X.	*idem*	*idem*	*f − mf*

83	℘	—	Segment C of period X (ending).	*idem*	*idem*	*mf – ff*
89	Φ, ℘	—	Ending of period X, beginning of period XI. Fl. 1, 2 polarized. Fl. 3 *muta in ottavino*.	*idem*	♪$_{ca.72}$	*p < mf*
97	℘	—	'Hinge' measure of period XI. Cb. appear with accents (B_8).	*idem*	♪$_{ca.56}$	*mf < p*
101	℘	—	*Rallentando*; woodwinds end period XI.	*idem*	*rall.* ♪$_{ca.72}$	*p < mf*
103	℘	—	Ending of period XI, beginning of period XII. Viola solo.	*idem*	♪$_{ca.60}$	*mp*
107	℘	—	Pitch progression in strings. Vn. 1b: appears harmonic (*A*). End of part I.	*idem*	*idem*	*p*

◊660. Example of antiproportion: Structure by measures of *Il canto sospeso* (1956), part I, composed by Luigi Nono, for solo voices, choir and orchestra. Measures are derived from Fibonacci numbers (represented by Φ), in contrast with prime numbers (represented by ℘). The label 'source', in the second column, denotes the origin of each number as constructive reference (including relationships obtained by inversion of the original sequences, employing a method similar to that used for Fibonacci grids in example ◊646*a*). See table ◊634 (page 390), for a more general view of the whole composition.

General concept of self-dissimilarity

The typical structural dualism, e.g. similar/different, symmetric/antisymmetric or proportional/antiproporcional, can also be adapted to the systematic relationship of self-similarity/self-dissimilarity. However, this case does not necessarily imply simple oppositions; rather it deals with partial states in a certain balance between systems of relationships. Kieran (1996) recognizes this balance as the reciprocity between musical coherence and incoherence. Accordingly, self-dissimilarity can be defined as a scatter factor or entropy in a process whose ordered or quasi-ordered form, tends in the direction of discrepancy from its own structural basis. Thus, as suggested by Kieran (*op. cit.*:41) for the cases in which music establishes its significance in self-dissimilarity,

> [The] value [of] music inheres in the frustration of our attempts to engage with it, highlighting cognitive significance through perceptual incoherence. It is important to realize that this is not merely to suggest, as one might regarding Chopin's *Polonaise-Fantaisie* or Schoenberg's *Six Little Piano Pieces* op. 19 no. 3, that an analytic approach encounters difficulties in articulating how or why the piece may be coherent. Rather, it is to point out that there are musical pieces that should not and, in some cases, cannot be heard as coherent.

In this context, antiproportion does not mean no-proportion, such as self-dissimilarity does not mean no-self-similarity. Rather, these terms refer to a relationship of reciprocity that can or cannot tend toward equilibrium, in the information distribution of a system or process. Absence, contradiction and restoration of balance, but not balance itself, are the main structural references of music. This is why Schoenberg (1975:123) states that "The method by which balance is restored [is] the real idea of the composition." Therefore, the term 'self-dissimilarity' can be associated with 'relative self-similarity', in a comparable way to how Beran (2004:83) considers that the term 'information' can be associated with 'uncertainty'.[514]

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[514] A general description of this relationship is given at the beginning of Chapter 5. For an intuitive description of 'relative self-similarity', see ◊333.

The concept of self-dissimilarity is fully compatible with Xenakis' notion, about the sound 'deformation', from a periodic pattern or a set of harmonic functions (Xenakis 1992:266):

> It is a question of beginning with any form whatsoever of an elementary wave, and with each repetition, of having it undergo small deformations according to certain densities of probabilities (Gauss, Cauchy, logistic…) appropriately chosen and implemented in the form of an abstract black box. The result of these deformations is perceptible on all levels: microstructure (= timbre), ministructure (= note), mesostructure (= polyrhythm, melodic scales of intensities), macrostructure (= global evolution on the order of some tens of minutes). […Then] one would have had an effect of sounding fractals, with a sonorous effect which is impossible to predict

What is important here is Xenakis' emphasis on gradual deformation, as stochastic process. Nonetheless, structural self-dissimilarity can develop from any constructive system or process based on derivative recursion and repetition. A parallel example of this, in physics, is the successive transformation of periodic patterns into waves' diffraction. According to Schroeder (1991:44):

> A related subject [to *fractons* and *phonons*] is the diffraction of waves from fractal structures (*diffractals*). Since far-field or *Fraunhofer diffraction* is essentially a Fourier transform, the self-similarities (deterministic or statistical) of the scattering fractal must be fully reflected in the diffraction pattern of the incoming radiation, be it electromagnetic, audible, or ultrasound, electrons, neutrons, or neutrinos. […] Clearly, wave diffraction is a sensitive tool not only for classical bodies, but for fractal matter too.

Hence, structural self-dissimilarity is not—and cannot be—opposed to self-similarity. Within a variety of cases in the behaviour of waves and particles governed by the same power laws, self-dissimilarity and self-similarity reflect, instead, an association analogous to the coordinative relationship between order and chaos. For the same reason, some of the first researchers studying the structural bonds between information and entropy—among them Moles (1952, 1958) and Bucher (1959)—also find an aesthetic link between repetition and difference; between proportion and disorder. Furthermore, Bucher (*op. cit.*:525) looks forward to a first theorization of intersemiotic translation, under the notion that "developments in information theory, originally applied to the field of music, are beginning to explain some of our reactions to repeated or dissimilar architectural forms."

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Self-dissimilarity at pitch-class set theory

The way to explore the connection between self-similarity and self-dissimilarity in sets of objects and groups of musical operations—according to Lewin's (1987) general theory of transformations and intervals—, has been drawn in recent years with the implementation of the *Klumpenhouwer networks*.[515] In this context, Murphy (e2007) introduces the notion of self-dissimilarity into music analysis, within the frame of networks' recursion, and using as a case study the functions of intervals in the "Fourths" no. 131 of Bartók's *Mikrokosmos*. Murphy concludes that at least five analytical categories can be distinguished, from *exact self-similarity*—in which there is a generalized isometry between universe of networks and subsets; to *self-dissimilarity*—where the matching between network and hyper-network contrasts with exact self-similarity. These categories are compatible with other general criteria for measuring self-similar rhythms and symmetries, including fractal or absolute self-similarity, pseudofractal or quasi-perfect self-similarity, anisotropic self-similarity or self-affinity, pre-self-similarity, and self-dissimilarity (see Suppes *et al*. 1989: I, 161; Mandelbrot 2002:50, 85). However, the formalization and implementation of these degrees in musical synthesis and analysis, is still in a formative process, as seen in recent debates, particularly in pitch-class set theory (see Buchler e2007, reply in Murphy e2007, and counter-reply in Buchler e2008).

The notion of *automorphism*, extremely useful in transformational relationships of music, and defined in subchapter 2.3. (see page 48), also has its counterpart: the essential asymmetries of a musical automorphism are anti-automorphisms. Noll (2007:129) introduces this conceptualization in the context of linear isomorphisms and transformations of musical intervals "by their constitutive role for well-formed scales".[516] Such a conceptualization contributes to a possible development of the Klumpenhouwer networks theory.

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[515] The definition of this concept is given on pages 48 and 282–283.
[516] The concept of *well-formedness* is explained in subchapter 3.2. (see pages 60–62).

Conclusions

From cognitive and semiotic perspectives, self-similarity and intersemiotic mapping are two kinds of relationship integrating sense, extending from particular to general. Evidence of a transversal self-similarity—a system of intersemiotic relations crossing a wide self-structuring spectrum, from the genetic code to the configuration of messages in society—provides a deep insight into understanding the *organicism* with which, historically, scholars attempted to explain music. Such organicism is attributable, at least partially, to physical and biological self-structuring relations, analogously found in aesthetic and self-creative aspects of music. This argumentation gains more complexity when extending it to the relationship between individuals and culture, associating individuals and groups of individuals with their changing environment.

Coordination between culture and environment implies a concur of two forces: the *unification* force, and the *dispersion* force. This notion, conceived by George K. Zipf (1902–1950) as an empirical self-structuring principle for language, has been extended in the present study, as a multi-layered orientation in musical, pre-musical and meta-musical patterns constituting an interrelated whole.

The human is capable of multiplying its signs of *reality* thanks to the operative principles of similarity and difference, and—especially—thanks to generalization and stereotyping achieved by these principles. By these means the human is able to create and expand reality through a social, intersubjective invention of the world; this includes the cultural dynamics of music and its contact with an environment of re-creatable aesthetics.

Self-similarity is an inherent phenomenon of music because of the redundancy characterizing the musical styles. It is also due to the qualities of the musical resources: reflecting the physical characteristics of instruments; the interaction between grammar and pragmatics; the creative tension between ecolects and idiolects; the functioning of the organs of auditory perception; the physical vibrations in the acoustic processes; and the cyclical exchange between environment and

societies. The signs of self-similarity in music are, therefore, signs elaborating complexities in a systematic self-reference; these signs, according to Bolognesi (1983:26), embody the most primitive and intuitive structures of music, expanding as emerging pattern.

The contrast of similarities within a music system (i.e. its structural differences) responds to an effect of symbolic consolidation, defining values by comparison, or by suspension or suppression of such a comparison. Sense intermission and distortion of meaning also happen to participate in this contrast, in a similar way that the intermission of form and the distortion of symbolic functions are involved in the evolutionary processes of adaptation. In this way the conceptualization of musical repetition as genotype (see Mâche 1998:160–161) and musical recursion as phenotype becomes understandable—analogous, in a dynamical system of music, to the difference between *style inheritance* and *individual behaviour of inheritance*, closely related to the dynamics between musical ecolect and musical idiolect, respectively.

The fact that self-similarity is a pervasive feature in a wide range of musical signs, may also be associated with the cognitive tendency to adjust systems of relations partially perceived or processed, with more complete systems, taken as models in experience and memory. This relationship converts the synecdoche into a powerful, generalized mechanism of reference and recreation. The very fact that analogy and synecdoche are widespread hallmarks of human cognition, with a strong presence in music, leads to the conclusion that the interpretation of the part for the whole or the whole for the part—at different levels of structuring musical ideas—responds to a basic system, as a common platform for musical elaborations. Thus, the 'simple' intuition *this forms part of something else*, is by no means devoid of deep implications for music. Such intuition concentrates many of the operations of musical consistency and meaning.

The cognitive and creative power of synecdoche can be explained, since they are the bridge between immediateness and remoteness; between the measurable and the immeasurable: a faculty related to the mind's tendency to associate the particular with the general in the appropriation of the unknown, through the stereotype. Synecdoche, consisting of one or more operational steps, is directly involved with the

two classical modes of analogy (i.e. paradigm and proportion). It satisfies—in the same way and for the same reasons that pre-self-similarity does—the primary link between the evident and the conjecturable in a system of abduction, giving certainty to what would otherwise be lost in a plethora of unconnected points.

Interpreting self-similarity functions as *knowing-what* mappings (see Kaipainen 1994), makes evident the generality of synecdoche in music. The possibility of conjecturing self-similarity by a synecdochic stereotype, permits the confirmation of the structure itself. The sense of music emerges then as a self-referential recursion within a larger system of recursions with operational similarities and differences in each of its cycles. Consequently, the margin of *error* between origin and recursion cannot be understood as sterile or empty space, but as a field of musical recreation: an environment for the renewal between rigid grammar and flexible ecolect; coordination between musical genotype and phenotype as conceptualized by Mâche (1998), and margin for the negotiation between the forces of *dispersion* and *unification*, according to Zipf's theory (Zipf 1949).

Self-similarity is an inherent feature of the musical idiolects, due to a basic amount of individual self-reference; in a following layer the same relationship occurs with the musical ecolects defining collective styles. Moreover, although individual experience may be supplied by 'external' references, carrying them to its own domain, the individual converts them into self-referential relationships. Thus, music has an implicit self-referentiality, associable to all things the human can experience, from the impulsive and volitional, to the poetic and metaphysical; from the pre-symbolic to the synthetic-analytic. Clearly, this includes the 'pre-musical' and 'meta-musical' sources of human experience. As Fremiot (1994:253) suggests, "any material—from a gesture to a scientific phenomenon or even an abstraction—is valid encouragement for the inception of an artistic idea." As for the human capacity of bringing *the idea* to the musical language, music can originate from any idea, and consolidate its cultural and emotional meaning through cyclical practices. This involves the diversity and authenticity with which each culture recreates its own musical traditions, reifying its environmental and social experiences.

Music is a cultural self-structuring and self-referential phenomenon, open however to changes and associations of multiple semiosis, in congruence with what Deleuze and Guattari (1980:11–13) suggest for language in its broadest form. Sound—or its contingency—is the essential means of music, but it may be complemented, justified or having feedback with any kind of semiosis, making 'musical' what at its origin was 'non-musical': sonically humanizing what is humanly understandable (see Merriam 1964:145, 166; Blacking 1973:101). *Making this* what *this was not*, as an imaginary transmutation of lead into gold, equates to effectuating an intersemiotic translation. The transmutation of the *devil's staircase*–mathematical object, into the *devil's staircase*–musical process, as György Ligeti does with *L'escalier du diable* (1993), is based on this principle. Many more examples fall under this conceptualization, including a wide diversity of musical traditions beyond Western culture.

In parallel to what Bateson (1972) and Damásio (1994, 2000) suggest for language in general, emotion and intellect are efficiently correlated in music. For musicology this conclusion is not trivial, since its descriptive and analytical tools are imbued with preferential, intentional, pragmatic and idiosyncratic concerns. Music research is also fed—like the analytical protocols and like music itself—by the relatively unstable exchange between idiolect and ecolect; between style and grammar, and between correctness and preference. Whereas Bateson (1972) claims a need for narrowing gaps between human mind and human environment, and Damásio (1994, 2000) urges for changing a scientific paradigm—denying the Cartesian dualism that splits mind from body, and reason from emotion—, now it seems crucial to extend this change of paradigm into a harmonization between inner and external semiosis. This means that comprehending the form in which links are established between groups of individuals and environment—and accepting the need for ecological variety in the context of musical idiolects and ecolects—should contribute to a clearer view of the priorities and tasks of musicology.

Creation, analysis, performance, interpretation, reception, and meta-referentiality of music are manifestations of the same system of oscillations between fixation and substitution of paradigms. All of them are states of the 'struggle' between

the forces of dispersion and unification theorized by Zipf (1949), characterizing human behaviour as entropy. Also in this sense, the social and cultural aspects of music lie at a generalized synecdoche somehow mirroring the oscillatory, quasiperiodic and chaotic properties of the vibratory systems.

During the revision of the evidence used to develop the present study, incongruity could not pass unnoticed, between a qualitative register on self-similarity—related to statistical concepts of music as a redundant and hierarchical phenomenon—and the initial stage of a philosophy on self-similarity, based on the concepts of *Gestalt* (Ingarden 1962: 55, 107–108), and *rhizome*, *non-centrality* and *non-hierarchy* (Deleuze and Guattari 1980). Ingarden (1962:105) conceives a "problem" in comparing musical intersubjectivity with the "pure intentional objectivities" (*rein intentionalen Gegenständlichkeiten*), which cannot be solved since both consist of radically different epistemologies. Instead, for Deleuze and Guattari (1980:3–15) this inconsistency is solved by admitting that the *rhizome* does not contradict hierarchies such as those found in music.[517] Deleuze and Guattari rather suggest that the musical processes are created by relationships self-contained in various or many simultaneous layers, each encompassing others, in modes of arrangement between consistency and inconsistency. For this reason the present study highlights the fact that self-similarity and self-dissimilarity are coordinated in the same way that similarity and difference, or information and entropy, are coordinated as physical and aesthetic phenomena (see Vasconcelos 1951:52–53, 56–57; Campbell 1982:263–265).

Given the complementarity between information and noise (Shannon 1948; Shannon and Weaver 1949; Moles 1952, 1958), and observing the consequences that Voss and Clarke's (1975, 1978) postulates have for music, the Chapter 5 of the present study emphasizes the relevance of conceiving noise as a potential and real form of music, and music as a reciprocal nesting of noise. This reciprocity is

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[517] According to Deleuze and Guattari (1980) in the *rhizome* any element may affect or be influenced by any other element in the same environment (*op. cit.*:13). In formulating this concept, the influence of dynamical systems and fractal geometry is evident: within a fractal it is not possible to determine the rank or position of a point in respect of another; what is possible is to identify and classify its overall relations.

considered in terms of a semiotic system nested within another semiotic system: intersemiosis is contingent insofar as the germ of a semiotic set proliferates within or intersecting another set of signs. The symbolic continuum intertwining noise and music comprises a significant part of the universe of the intersemiotic relations that occur as a balance between self-similarity and self-dissimilarity.

The iterated functions of music (i.e. as musical grammars cyclically articulated within the same sound-symbolic system) elaborate dynamical maps which are more unstable in terms of outcomes, than compared to differential equations. This is why it is so difficult to satisfactorily substitute the characteristic variety and instability of music, using a mathematical model. Furthermore, the tendency to chaos in music, involves at least one period of relative self-similarity, and—on the other hand—rejects any direct comparison with exact self-similarity. This notion is closely linked to processes of assimilation, creativity, interpretation and transformation of musical repertoires: chaos, in music, means deterministic consistency within an unlimited series of indeterministic variations.

Since the ubiquity of $1/f$ noise is commonly observed in the auditory phenomena, and Zipf's distribution is verifiable in general features of music as language, such aspects are to be taken into account for a more precise definition of the *music universals*.[518] Whether self-similarity operates as a mechanism of preservation of information at a low structural and energetic cost, self-similarity can also be understood as a 'leading thread' between different forms and layers of a universal economy of language. Chapters 4 reveals how the contradiction between economy and repetition in music is apparent: music repeats what it is necessary to repeat, in order to provide the proper tension between preference and grammar, as creative coordination of the musical processes.

The alleged ubiquity of golden ratio in aesthetics and nature, highlighted by authors such as Ghyka (1927), Borissavlievitch (1958), Livio (2002) or Madden (2005), must actually be investigated in a context of self-referential systems linked to power laws. The analytical characterization of aesthetic systems should be tackled without preselecting any particular proportion (i.e. assumed as a pre-existing truth),

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[518] The concept of *music universals* is introduced in subchapter 4.6.

but rather identifying the self-referential source of such systems. Peirce's (1903a) theorization is congruent with this conclusion, in that it forecasts self-similarity as a system of conjecture and creation of reality. Peirce (*op. cit.*:160–165) anticipates, in this fashion, the concept of *cognition trees* as explained by Kaipainen (1994:94–99) in terms of "*Knowing-what* hierarchies of temporal spans" in recursive branchings.[519]

Musical self-similarity emerges in a variety of processes that cannot be expressed simply by their linearity, but by the consistency of irregularities at different layers: through the form of continuous variations within the myriad possibilities to recreate music. Musical style is a phenomenon of stochastic self-similarity and probabilistic variation, originated at the tension between grammatical determinism and preferential indeterminism. The continuity of the cycles between grammatical correctness and stylistic preference, determine—to a significant degree—the individual and collective notions of authenticity in the recreation of a musical language.

It must be stressed that the so-called Fibonacci numbers and the golden mean are not features unique to Western music; rather they are common traits of self-reference, developed in manifold expressions of individuals and societies involved in Umwelt-niche diversity. In the same way, the Peircean trichotomy,[520] as a cognitive process—such as the process for the construction of the *devil's staircase* paradigm—is not culturally exclusive, but pervasive in a wide range of logico-aesthetic elaborations.

Accordingly, the principle of 'no absolute self-reference' challenges the concept of 'pure music' defended by Ingarden (1962:48–51).[521] 'Pure music' is, like 'fractal music', fallacious. Both concepts can be conceived within a Platonic–Cartesian radicalism that extends obscurity, instead of providing a satisfactory explanation

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[519] In these 'trees', a number of possibilities extends from a cognitive level to another, finally concluding with a single possibility of *knowing something*. Kaipainen (1994:99) represents this relationship as a branch that splits several times into a following level, where a branch splits to the next level in a variety of possibilities, of which, one branch extends into another level, and so on, in a system of recursions that corresponds to a specific neural network. 'Level', in this case, "refers to [a] degree of mental abstraction" (Kaipainen *ibid.*). In general terms, Peirce (1903a/1998:162) seems to predict this conceptualization, as suggested in sections 3.8.3. and 4.4. of the present study.

[520] See section 3.8.3. (especially page 112), in connection with subchapter 5.5. (particularly on pages 354–363).

[521] See discussion on pages 314–316.

about music. This study suggests, in contrast, interpreting musical self-similarity as a universe of self-organized irregularities and analogous particularities, necessarily changing in an entropic environment—and certainly, not evolving to a specific model of music. This notion criticizes, thus, the post-modern attitude of aesthetic 'globalization' as 'unification' attempting to obliterate musical diversity.

Conceiving the strata of musical self-similarity under the notions of stereotype and abduction—and not under the generalized concept of 'fractal'—requieres better understanding of the Peircean question of the 'map within the map' (Peirce *CP*, 8.122). The consequences of this idea justify the conceptualization of musical self-similarity as a problem related to the argument of the 'house within the house', which strictly concerns to the ecology of music. The present study suggests continuity in nested sign systems revealing an axiology (i.e. a plot of links between ethics and aesthetics) by identifying the synecdochic relationship of the 'house within the house', in a complex self-similarity connecting individuals, communities, species and contexts. This notion implicates a wholeness in which music—conceived as non-exclusively human—plays a privileged role for interpreting and recreating the world, as a *world-within-the-world*.

Among the virtues of thinking in terms of a *logic of self-similarity* as a strategy for musical assessment, is the adaptability of such a logic into what Michel Foucault (1966:40) conceives as "the whole volume of the world"—according to which the fundamental relations between part and whole are not necessarily subordinated to analysis, but oriented by symmetries and asymmetries, sympathies and antipathies.[522] The association of this concept with semiotic perspectives developed in the fields of cognitive science and experimental psychology,[523] leads to the conclusion that the human dwells in a *synecdochic world* (i.e. created by a human synecdochic intersemiosis that makes language's widest range possible).

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[522] In the original text (Foucault, *loc. cit.*): "Tout le volume du monde, tous les voisinages de la convenance, tous les échos de l'émulation, tous les enchaînements de l'analogie sont supportés, maintenus et doublés par [l']espace de la sympathie et de l'antipathie".

[523] Especially within the framework outlined by Bateson (1972), Lakoff (1987), Gelman and Brenneman (1994), Frémiot (1994), Fauconnier (1997), Lakoff and Núñez (2000), and O'Halloran (2008).

Synecdoche can also characterize fundamental relations between individual and general processes in biology. For example, the development of an organism from a single fertilized egg to an individual's maturity is synecdochically comparable to the hypothetical evolution of the species, from a unicellular organism, to its present state. Many other local and general biological processes are synecdochically comparable. Moreover, many fundamental physical references are accessible by synecdoche, linking macro- with micro-phenomena. In a way, the so-called Plato's paradox, concerning 'how can we learn so much from knowing so little' (see Chomsky 1966:11), is solved by synecdochic intersemiosis, which partly 'gives' meaning to a total through the particular, and partly 'recognizes' its own performance in the power laws modelling the shape and functioning of the knowing systems. This conceptualization concurs with what Ballantine (1984:5) refers to when he states "the musical microcosm replicates the social macrocosm"—although, reciprocally, the social macrocosm somehow replicates the musical microcosm. This notion also contributes to developing ideas first put forward by Campbell (1982:219–220) and Kaipainen (1995:48, 192), in suggesting that social reality partially reflects the biological creation of reality.

The *ordering function of the code* (Eco 1968:27) and the *synecdochic function* are correlated.[524] This means that the limits of the combinatorial possibilities between grammatical rules and preferential variations, cooperate—in music as well as in a more general symbolic framework—with the limits of the combinatorial possibilities between the *known* parts, with respect to a whole *to be known*. Correlation of these two symbolic functions permits, for example, identifying figural derivation, as happens with *aposiopesis*—the figure in which a part of a musical elaboration is omitted in order to emphasize the meaning of the part *in absentia*. Among the most simple cases of this relationship is the syncopation in metre, and the deceptive or interrupted cadence, in tonal harmony. Equivalent examples in musical rhetoric, in different forms and within different traditions, are countless. In many cases, 'minor' relationships *in absentia*, within other, 'major' relationships *in absentia*, also constitute

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[524] On the *synecdochic function* see pages 44, 97, 110, introducing notions that are developed on pages 230–232, 235–237; on the *ordering function of the code*, see pages 208–210.

semiotic complexities in which the ordering function of the code and the synecdochic function are coordinated.

According to what is explained in subchapters 3.8. and 5.2., the synecdochic function is the epistemological surface of the intersemiotic continuum (IC) in which *music reality* emerges as figurativeness.[525] From this viewpoint, music is knowable and testable as an interplay between similarities and differences; between humanly audible paradigms and dissociations.[526]

Music never emerges *ex nihilo*. Moreover, any music 'translates something' in the sense that it appears by analogy and in difference with 'something similar'. Even the more abstract music reflects, in its own features, some aspects of tradition and culture, a certain ethos and a state of mind of individuals, and—congruently—a number of bodily characteristics. In this context, music is also similar to language in general and to mathematics in particular, since, despite being a cognitive domain in itself, it also cooperates in empathy—here 'empathy' implies intersemiotic translatability—with other domains that recognize the expressive uniqueness of music.

Further investigation

Aspects of experimental psychology can be developed in terms of Roman Ingarden's (1962) axiology. Gestalt relationships of music—an issue pointed out in various sections of the present study—can also be studied as experiential processes.[527] However, the link between self-similar intersemiosis, Gestalt and *mental spaces* is an issue too rich to be exhausted in a single project, as can be inferred, for example, from

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[525] Unlike pragmatics in language and in applied mathematics, in music the analogy in its forms of proportion and paradigm—not the possible materialization of the representamen or the signifier—is what prevails as orientation of sense (this is already explained in subchapter 5.5.).

[526] Again, this is related to what Foucault (1966:40) intends by the expression "tous les enchaînements de l'analogie sont supportés, maintenus et doublés par [l']espace de la sympathie et de l'antipathie".

[527] This research contains a brief introduction to Gestalt theory (pages 85–88) as well as developments on the concept of *mental spaces* (pages 16–17, 315–316). For a thorough treatment of the latter, see Lakoff and Johnson (1980), Fauconnier (1985, 1997), Lakoff (1987), Lakoff and Núñez (2000).

the structure and content of Lewis Rowell's book (1983), or from the compilation edited by Marc Leman (1997) on music and Gestalt.

The systematic implementation of resources of self-similarity, such as Fibonacci sequences modulo *m*, self-similar tessellations and textures, and Lindenmayer systems, as well as implementing pseudofractal filters and generators, already occupy a very noticeable place in literature. Obviously, these resources are not exhausted in this work, but are included in order to provide a brief overview of musical strategies and methods that are currently experiencing rapid development.

Several examples implemented in this study, covering synthesis and analysis in the context of musical composition and performance, can contribute to a renewal of music technique and theory. In particular, Arnold tongues and phase locking regions in the circle maps, as well as the Farey trees, are sources of great interest for future research exploring deterministic chaos and harmony in the context of musical self-similarity. Whether the Arnold tongues are truly the parameter space for the rational resonances of an infinite harmonic self-similarity, then a significant part of the future study of harmony will be in debt with the dynamical theory founded by Vladimir Arnold (1937–2010). Besides, linking Arnold tongues with musical inference and Gestalt figurativism is also a promising line of study, as suggested in the final sections of subchapter 6.2.

Schenkerian terminology—which has been revised and clarified by Pankhurst (2008)—and its adaptation into a system of layers (*Schichten*) as levels of functional self-similarity, is a particularly rich field awaiting further development, complemented by resources of contextualized statistical analysis.[528] Obviously, the direct adaptation of Schenker's tonal theory (1932) to a much broader perspective —i.e. beyond classical tonality, may be unjustified. In any case, an adaptation of the Schenkerian paradigm should prove compatibility with new forms of analysis of atonal music (including post-tonal music) and non-tonal repertoire (including a wide

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[528] That is, an analysis not only focused on obtaining indices, deviations or averages, but one that deals with qualitative correlations, local and overall.

variety of noise-music), by the same general criteria of functional stratification and figural similarity.[529]

Chapter 5 suggests organizing the systematic study of nestings of *music within noise* and *noise within music*, as a first step in developing an approach which would cover the function of the whole in the part, and not only the part or parts as functions of an analytic set. From a statistical perspective, the concept of invariance especially merits further development in different musical parameters, implementing tools from set and group theories.[530] Another possible extension of Chapter 5 could be the classification of musical repertoire by kinship with fractional noise patterns. This task is done incipiently in Hsü and Hsü (1991) and Madden (1999/2007:125–141), but needs a more rigorous and extensive methodology, including a much larger collection of samples—or at least a collection of samples not only confined to just one style or to the same musical tradition. In order to advance in this task, it is obvious that the spectral synthesis/analysis of a recording is not enough to produce definitive results. A comparative analysis is required between noise's tendencies or 'colours' of noise as nested music, combined with strategies of semiotic contextualization—assuming that, for instance, the notions of *noise* and *harmony* vary depending on culture. This is related to the theoretical proposals of Doležel (1969), Hřebíček (1994, 1997), and Diederich *et al.* (2003), associated with speech; Pinkerton (1956), Youngblood (1958), Knopoff and Hutchinson (1981), Harley (1995), Beran and Mazzola (1999a–b), Bigerelle and Iost (2000), and Beran (2004), related to statistical musicology; and very especially Merriam (1964), Blacking (1973), Domínguez Ruiz (2007), and Hegarty (2007), among other authors useful for a humanistic interpretation of the whole picture of these findings. Interdisciplinary research is therefore necessary for a better understanding of musical self-similarity.

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[529] For example, Murphy (e2007:8) suggests a specific type of structural analogy: "to those desiring a comparison between Schenkerian and K-net *Schichtenlehren*, I recommend embracing an analogy with 'motivic parallelisms'".

[530] Invariance has been previously studied from the point of view of pitch-class set theory, but is rarely found in a specific context of self-reference or self-similarity, which can expand the scope of this development. Some topics for this expansion are suggested by Tiits (2002), Ockelford (2005), Vázquez (2006:273–276), and Ilomäki (2008:35–53).

The relationship between structural economy and energy expense in a variety of pre-musical strata, from fundamental physical interactions and general biological principles, is an issue that also requires specialized labor by sectors for obtaining an overview of significant correlations. It is expected that a better understanding of the organic metaphor in music will make sharpen the musical relations reflecting consistency of pre-musical structures, onto a variety of cases. An example is the role played by power laws in self-structuring forms of carbon in organic chemistry, and its relationship with bioacoustics. Suggestions made by authors such as Motchenbacher and Flitchen (1973:172), and T. H. Lee (2003:345)—pointing to an association between energy release with carbon granules in a microelectronic system, following the characteristic shape of residual $1/f$ noise—could strengthen Voss' (1987, 1992, 1993) postulates on physics, biology and music. As Voss (1993:16) notes, "the connection between the ubiquitous presence of $1/f$ noise in both music and DNA sequences […] suggests that music is imitating the irregular but scale-independent correlations of many natural processes." The development of these lines of research would make more understandable the links between Larmor frequencies, $1/f$ noise, Zipf's distribution, and musical self-similarity as a phenomenon of perception and recreation in the interplay between individuals and societies, and societies and environments—what this investigation postulates under the figure of *house of the house*, as a theoretical support for ecomusicology.

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