Accretion, black holes, AGN and all that…..

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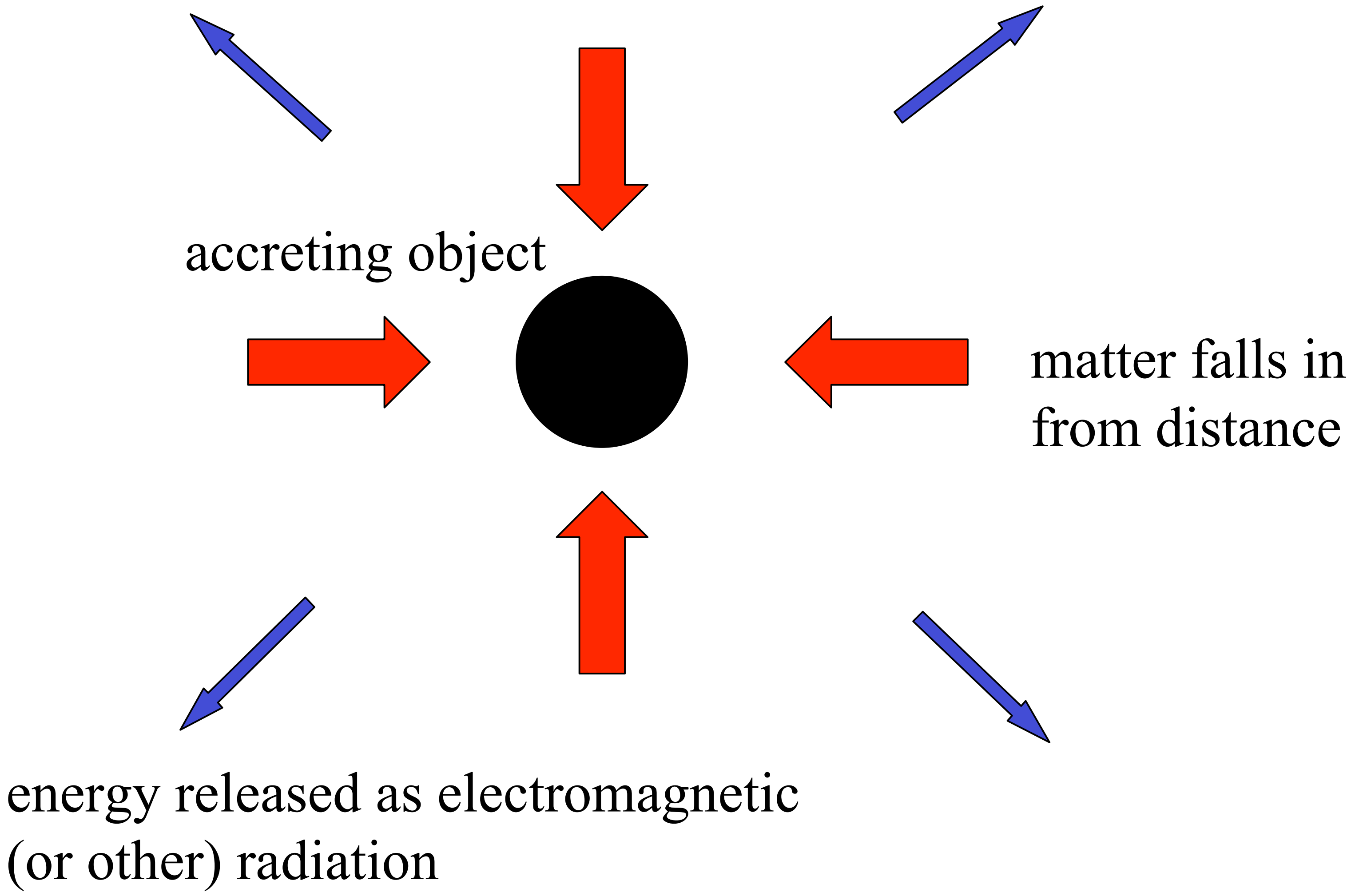
accretion = release of gravitational energy from infalling matter



If accretor has mass M and radius R, gravitational energy release/mass is

$$\Delta E_{acc} = \frac{GM}{R}$$

this *accretion yield* increases with *compactness* M/R: for a given M the yield is greatest for the smallest accretor radius R

e.g. for accretion on to a neutron star $(M = M_{sun}, R = 10km)$

$$\Delta E_{acc} = 10^{20} erg / gm$$

compare with nuclear fusion yield (mainly H→ He)

$$\Delta E_{nuc} = 0.007c^2 = 6\times10^{18} erg / gm$$

Accretion on to a black hole releases significant fraction of rest—mass energy:

$$R \approx 2GM / c^2 \Rightarrow \Delta E_{acc} \approx c^2 / 2$$

(in reality use GR to compute binding energy/mass:
typical accretion yield is roughly 10% of rest mass)

This is the most efficient known way of using mass to get energy:

accretion on to a black hole must power the most luminous phenomena in the universe

$$L_{acc} = \frac{GM}{R}\dot{M} = \eta c^2 \dot{M}$$

Quasars: $L \approx 10^{46} erg/s$ requires $\dot{M} = 1 M_{sun}/yr$

X—ray binaries: $L \approx 10^{39} erg/s$ $10^{-7} M_{sun}/yr$

Gamma—ray bursters: $L \approx 10^{52} erg/s$ $0.1 M_{sun}/\sec$

NB a gamma—ray burst is (briefly!) as bright as the rest of the universe

Accretion produces radiation: radiation makes pressure – can this inhibit further accretion?

Radiation pressure acts on electrons; but electrons and ions (protons) cannot separate because of Coulomb force. Radiation pressure force on an electron is

$$F_{rad} = \frac{L\sigma_T}{4\pi c r^2}$$

(in spherical symmetry).
Gravitational force on electron—proton pair is

$$F_{grav} = \frac{GM(m_p + m_e)}{r^2}$$

$$(m_p >> m_e)$$

thus accretion is inhibited once $F_{rad} \geq F_{grav}$, i.e. once

$$L \geq L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T} = 10^{38} \frac{M}{M_{sun}} erg/s$$

Eddington limit: similar if no spherical symmetry: luminosity requires
minimum mass

$$M > 10^8 M_{sun}$$

bright quasars must have

$$M > 10 M_{sun}$$

brightest X—ray binaries

In practice Eddington limit can be broken by factors ~ few, at most.

Eddington implies limit on *growth rate of mass*: since

$$\dot{M} = \frac{L_{acc}}{\eta c^2} < \frac{4\pi G M m_p}{\eta c \sigma_T}$$

we must have

$$M \leq M_0 e^{t/\tau}$$

where

$$\tau = \frac{\eta c \sigma_T}{4\pi G m_p} \approx 5\times 10^7\ yr$$

is the *Salpeter timescale*

Emitted spectrum of an accreting object

Accretion turns gravitational energy into electromagnetic radiation. Two extreme possibilities:

(a) all energy thermalized, radiation emerges as a blackbody. Characteristic temperature T_b , where

$$T_b = \left(\frac{L_{acc}}{4\pi R^2 \sigma} \right)^{1/4}$$

i.e. significant fraction of the accretor surface radiates the accretion luminosity. For a neutron star near the Eddington limit

$$L \approx 10^{38} erg / s, R = 10km \Rightarrow T_b \approx 10^7 K$$

(b) Gravitational energy of each accreted electron-proton pair turned directly into heat at (shock) temperature T_s . Thus

$$3kT_s = \frac{GMm_p}{R}$$

For a neutron star $T_s \approx 5 \times 10^{11} K$

Hence typical photon energies must lie between

$$kT_b = 1keV \leq h\nu \leq kT_s \approx 50MeV$$

i.e. we expect accreting neutron stars to be X—ray or gamma—ray sources: similarly stellar—mass black holes

Good fit to gross properties of X—ray binaries

For supermassive black holes we have

$$M \approx 10^8 M_{sun}, R = 2GM / c^2 \approx 3\times10^{13} cm$$

so

$$T_b \approx 10^7 (M_{sun}/M)^{1/4} K \approx 10^5 K$$

and T_s is unchanged. So we expect supermassive BH to be *ultraviolet, X—ray and possibly gamma—ray emitters.*

Good fit to gross properties of quasars

Modelling accreting sources

To model an accreting source we need to

(a) choose nature of compact object – black hole, neutron star, … to agree with observed radiation components

(b) choose minimum mass M of compact object to agree with luminosity via Eddington limit

Then we have two problems:

(1) we must arrange accretion rate $\dot{M}$ ***to provide observed luminosity, (the feeding problem) and***

(2) we must arrange to grow or otherwise create an accretor of the right mass M ***within the available time (the growth problem)***

examine both problems in the following

compare accreting binaries and active galactic nuclei (AGN)

for binaries
feeding: binary companion star
growth: accretor results from stellar evolution

for AGN
feeding: galaxy mergers?
growth: accretion on to `seed’ black hole?

both problems better understood for binaries, so ideas
and theory developed here first.

Modelling X—ray binaries

Normal galaxies like Milky Way contain several 100 X—ray point sources with luminosities up to $10^{39}\, erg\,/\,s$

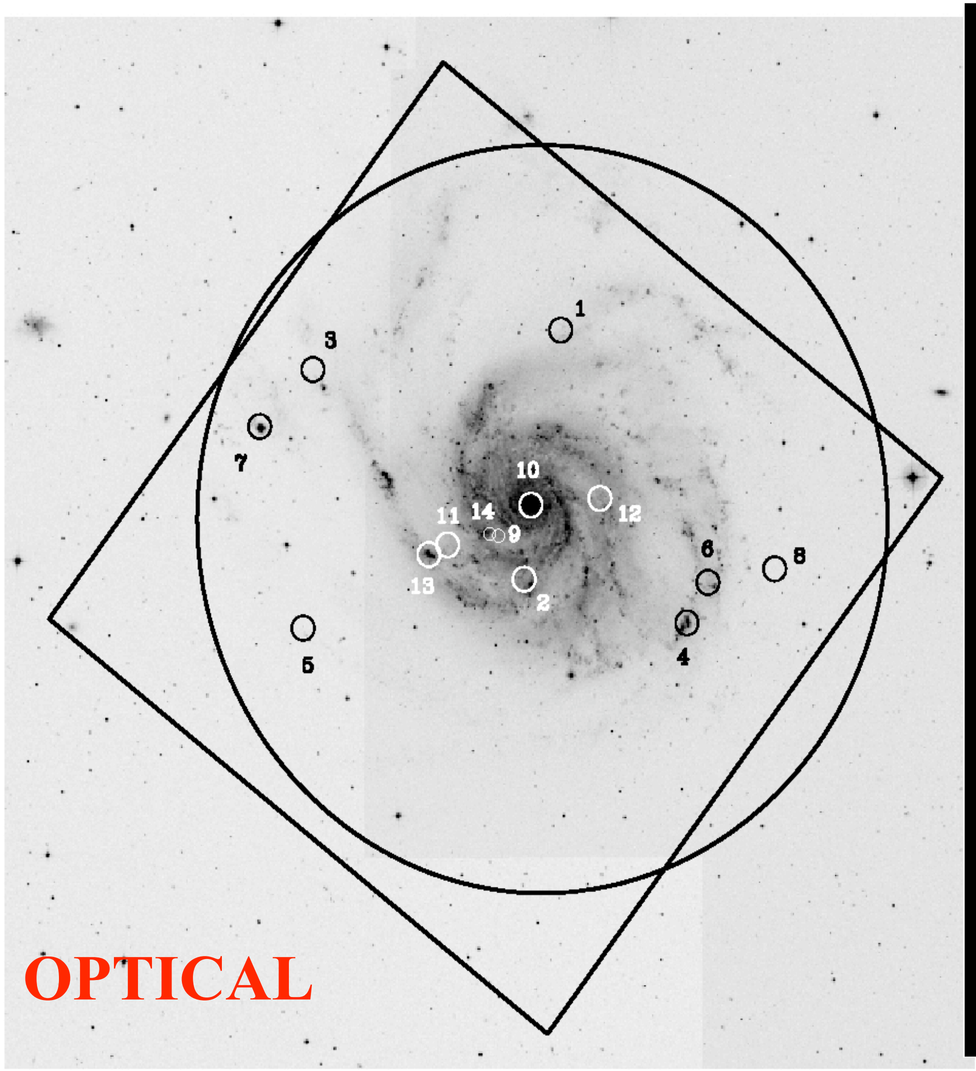
Typical spectral components ~ 1 keV and 10 – 100 keV

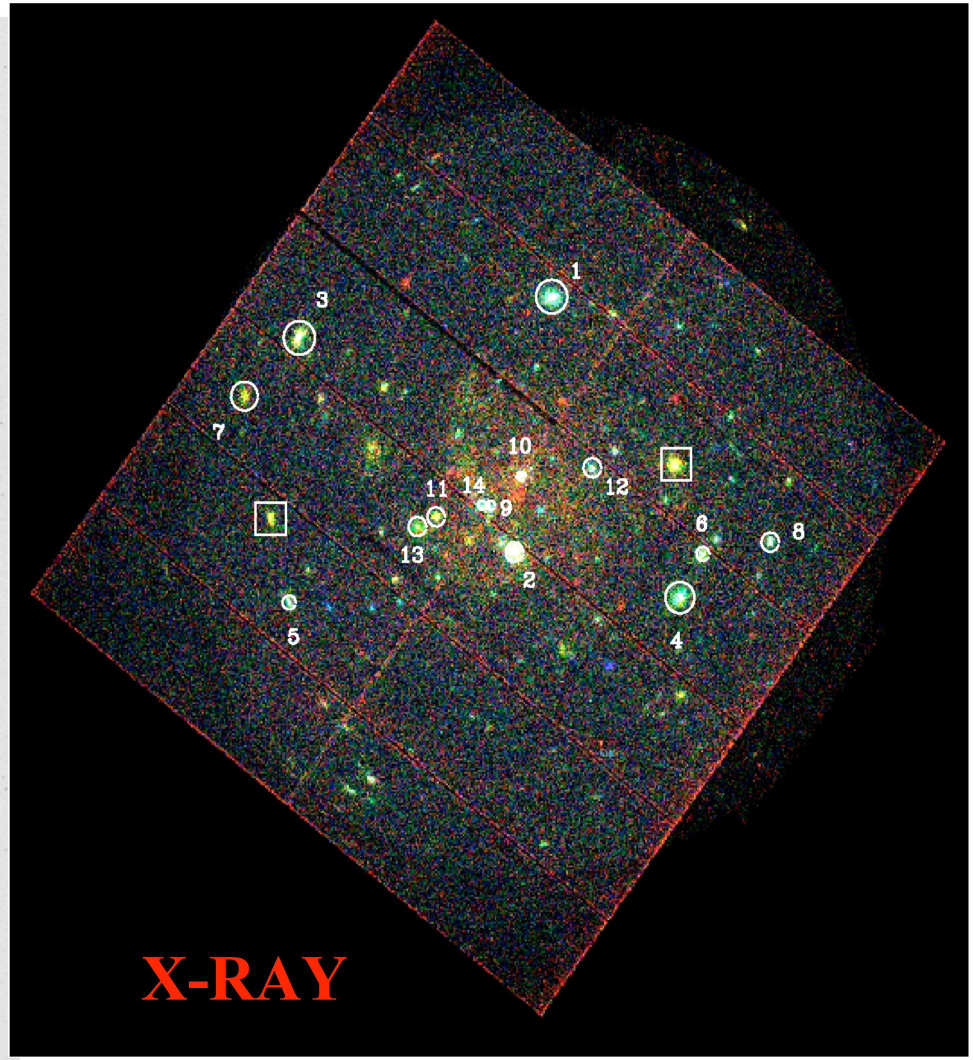
Previous arguments suggest *accreting neutron stars and black holes*
Brightest must be *black holes*.

Optical identifications: some systems are coincident with luminous hot stars: *high mass X—ray binaries* (HMXB).

But many do not have such companions: *low mass X—ray binaries* (LMXB).

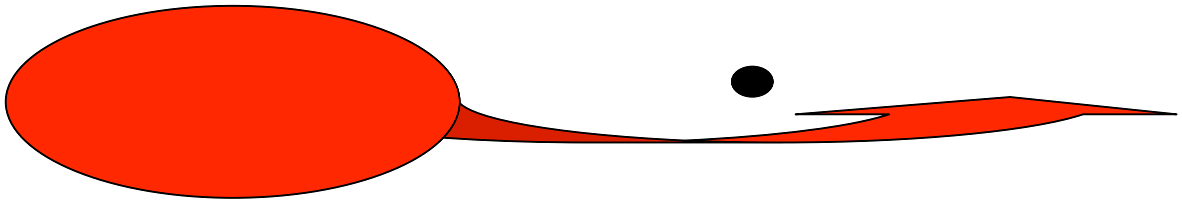
Accreting Black Holes in a Nearby Galaxy (M101)





Accretion disc formation

Transferred mass does not hit accretor in general, but must *orbit* it



— initial orbit is a rosette, but self—intersections → dissipation → energy loss, but no angular momentum loss

Kepler orbit with lowest energy for fixed a.m. is circle.

Thus orbit *circularizes* with radius such that specific a.m. j is the same as at L_1

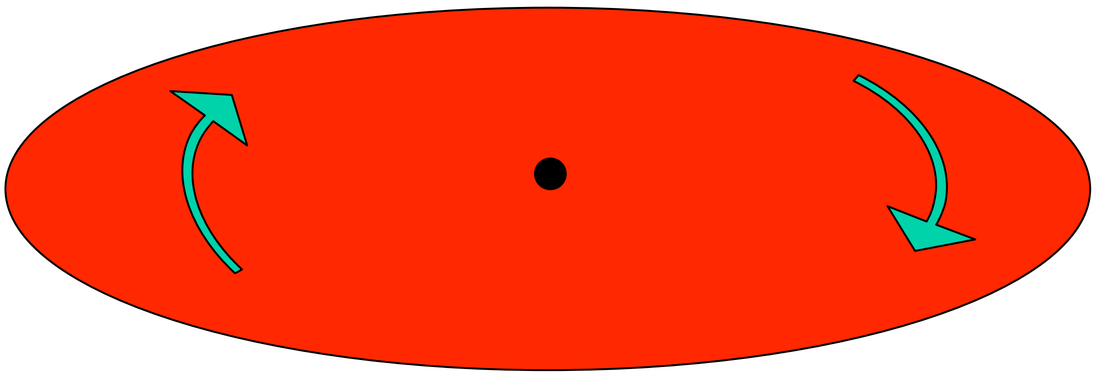
Thus in general matter orbits accretor. What happens?

Accretion requires angular momentum loss – see later: specific a.m. at accretor (last orbit) is smaller than initial by factor

$$(R / r_{circ})^{1/2} \geq 100$$

Energy loss through dissipation is quicker than angular momentum loss; matter spirals in through a sequence of circular Kepler orbits.

This is an *accretion disc*. At outer edge a.m. removed by tides from companion star



Accretion discs are *universal*:

matter usually has far too much a.m. to accrete directly – matter velocity not `aimed' precisely at the accretor!

in a galaxy, interstellar gas at radius R from central black hole has specific a.m. $\sim (GMR)^{1/2}$, where M is enclosed galaxy mass; *far* higher than can accrete to the hole, which is

$$\sim (GM_{bh}R_{bh})^{1/2} \sim (GM_{bh} \cdot GM_{bh}/c^2)^{1/2} = GM_{bh}/c$$

angular momentum increases in dynamical importance as matter gets close to accretor: matter may be captured gravitationally at large radius with `low' a.m. (e.g. from interstellar medium) but still has far too much a.m. to accrete

Capture rate is an upper limit to the accretion rate

- expect theory of accretion discs developed for XRBs to apply equally to supermassive black—hole accretors in AGN as well

- *virtually all phenomena present in both cases*

Thin Accretion Discs

Assume disc is closely confined to the orbital plane with semithickness H, and surface density

$$\Sigma = \int_{-\infty}^{\infty} \rho dz \approx 2H\langle\rho\rangle$$

in cylindrical polars (R, ϕ, z). Assume also that

$$v_\phi = v_K = (GM/R)^{1/2}$$

These two assumptions are consistent: both require that *pressure forces are negligible*

Accretion requires angular momentum transport outwards. Mechanism is usually called `*viscosity*', but usual `molecular' process is too weak. Need torque *G(R)* between neighboring annuli

Discuss further later – but functional form must be

$$G(R) = 2\pi R \nu \Sigma R^2 \Omega'$$

with $$\Omega' = d\Omega / dR$$

reason: *G(R)* must vanish for rigid rotator $(\Omega' = 0)$

Coefficient $\nu \sim \lambda u$, where λ = typical lengthscale and u = typical velocity.

Net torque on disc ring between $R, R+\Delta R$ is

$$G(R+\Delta R) - G(R) = \frac{\partial G}{\partial R}\Delta R$$

Torque does work at rate

$$\Omega \frac{\partial G}{\partial R}\Delta R = \left[\frac{\partial}{\partial R}(G\Omega) - G\Omega'\right]\Delta R$$

but term

$$\frac{\partial}{\partial R}(G\Omega)\Delta\Omega$$

is transport of rotational energy – (a divergence, depending only on boundary conditions).

Remaining term represents dissipation: per unit area (two disc faces!) this is

$$D(R) = \frac{G\Omega'}{4\pi R} = \frac{1}{2}\nu\Sigma(R\Omega')^2$$

Note that this is positive, vanishing only for rigid rotation. For Keplerian rotation

$$\Omega = (GM / R^3)^{1/2}$$

and thus

$$D(R) = \frac{9}{8}\nu\Sigma\frac{GM}{R^3}$$

Assume now that disc matter has a small radial velocity v_R.

Then mass conservation requires

$$R\frac{\partial\Sigma}{\partial t}+\frac{\partial}{\partial R}(R\Sigma v_R)=0$$

Angular momentum conservation is similar, but we must take the `viscous' torque into account. The result is

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega)+\frac{\partial}{\partial R}(R\Sigma v_R R^2\Omega)=\frac{1}{2\pi}\frac{\partial G}{\partial R}$$

We can eliminate the radial velocity v_R, and using the Kepler assumption for Ω we get

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left\{ R^{1/2}\frac{\partial}{\partial R}\left[\nu \Sigma R^{1/2} \right] \right\}$$

Diffusion equation for surface density: mass diffuses in, angular momentum out.

Diffusion timescale is *viscous timescale*

$$t_{visc} \sim R^2/\nu$$

For accretion disc equations etc see

Frank et al.(2002) Accretion Power in Astrophysics, 3rd Ed.

For general astrophysical fluid dynamics see

Pringle & King (2007) Astrophysical Flows

Steady thin discs

Setting $\partial / \partial t = 0$ we integrate the mass conservation equation as

$$R\Sigma v_R = const$$

Clearly constant related to (steady) accretion rate through disc as

$$\dot{M} = 2\pi R\Sigma(-v_R)$$

Angular momentum equation gives

$$R\Sigma v_R R^2\Omega = \frac{G}{2\pi} + \frac{C}{2\pi}$$

where $G(R)$ is the viscous torque and C a constant.
Equation for $G(R)$ gives

$$\nu\Sigma\Omega' = \Sigma(-v_R)\Omega + \frac{C}{2\pi R^3}$$

Constant C related to rate at which a.m. flows into accretor.
If this rotates with angular velocity << Kepler, there
is a point close to the inner edge R_* of the disc where

$$\Omega' = 0 \qquad \text{or equivalently} \qquad G(R_*) = 0$$

(sometimes called `no—stress' boundary condition). Then

$$C = -\dot{M}(GMR_*)^{1/2}$$

Putting this in the equation for Ω' and using the Kepler form of angular velocity we get

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left(1-\left(\frac{R_*}{R}\right)^{1/2}\right)$$

Using the form of D(R) we find the surface dissipation rate

$$D(R) = \frac{3G\dot{M}M}{8\pi R^3}\left(1-\left(\frac{R_*}{R}\right)^{1/2}\right)$$

Now if disc optically thick and radiates roughly as a blackbody,

$$D(R) = \sigma T_b{}^4$$

so *effective temperature* T_b given by

$$T_b{}^4 = \frac{3GM\dot{M}}{8\pi\sigma R^3}\left(1 - \left(\frac{R_*}{R}\right)^{1/2}\right)$$

Note that T_b is *independent of viscosity*!

T_b is effectively observable, particularly in eclipsing binaries: this confirms simple theory.

Condition for a thin disc (H<<R)

Disc is almost hydrostatic in z-direction, so

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left(\frac{GM}{(R^2+z^2)^{1/2}}\right)$$

But if the disc is thin, $z << R$, so this is

$$\frac{1}{\rho}\frac{\partial P}{\partial z} = -\left(\frac{GMz}{R^3}\right)$$

With $\partial P / \partial z \sim P / H, z \sim H$

and $P \sim \rho c_s^2$, where c_s is the sound speed, we find

$$H \sim c_s \left(\frac{R}{GM} \right)^{1/2} R \sim \frac{c_s}{v_K} R$$

Hence *for a thin disc we require that the local Kepler velocity should be highly supersonic*

Since $c_s \propto T^{1/2}$ *this requires that the disc can* ***cool.***

If this holds we can also show that *the azimuthal velocity is close to Kepler*

Thus for discs,

thin $\Longleftrightarrow$ Keplerian $\Longleftrightarrow$ efficiently cooled

*Either **all three** of these properties hold, or **none** do!*

Thin discs?

Thin disc conditions hold in many observed cases.

If not, disc is *thick, non—Keplerian, and does not cool efficiently*.
Pressure is important: disc ~ rapidly rotating `star'.

Progress in calculating structure slow: e.g. flow timescales far shorter at inner edge than further out.

One possibility: matter flows inwards without radiating, and can accrete to a black hole `invisibly' (ADAF = advection dominated accretion flow). Most rotation laws → dynamical instability (PP).

Numerical calculations suggest indeed that most of matter flows out (ADIOS = adiabatic inflow—outflow solution)

Viscosity

Early parametrization: $\nu \sim \lambda u$ with typical length and velocity scales λ, u . Now argue that

$$\lambda < H, u < c_s$$

First relation obvious, second because supersonic random motions would shock. Thus set

$$\nu = \alpha c_s H$$

and argue that $\alpha < 1$. *But* no reason to suppose $\alpha = const$

`Alpha—prescription' useful because disc structure only depends on low powers of α . But *no predictive power*

Physical angular momentum transport

A disc has

$$\frac{\partial}{\partial R}(R^2\Omega) > 0, \qquad \text{but} \qquad \frac{\partial \Omega}{\partial R} < 0$$

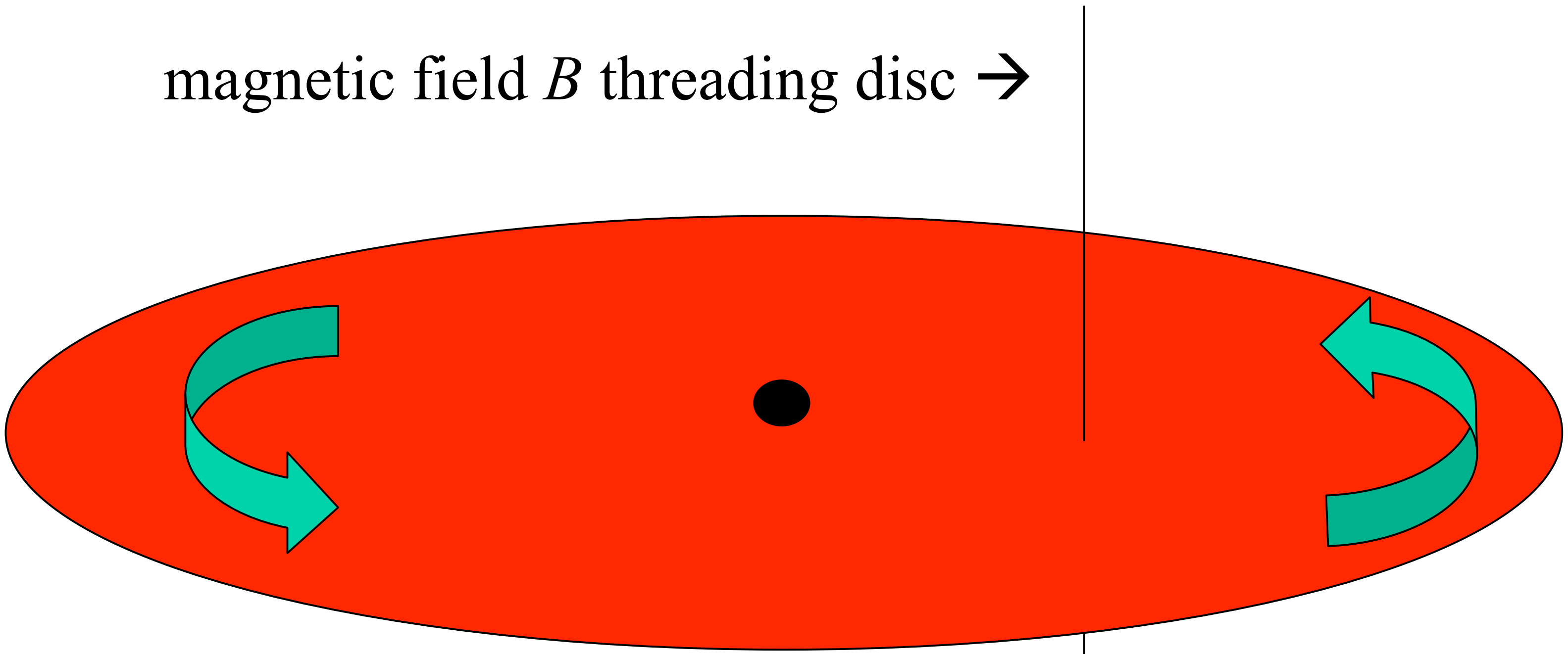
accretion requires a mechanism to transport a.m. outwards, but first relation → *stability* against axisymmetric perturbations (Rayleigh criterion).

Most potential mechanisms sensitive to a.m. gradient, so transport a.m. *inwards*!

need a mechanism sensitive to Ω or v_K

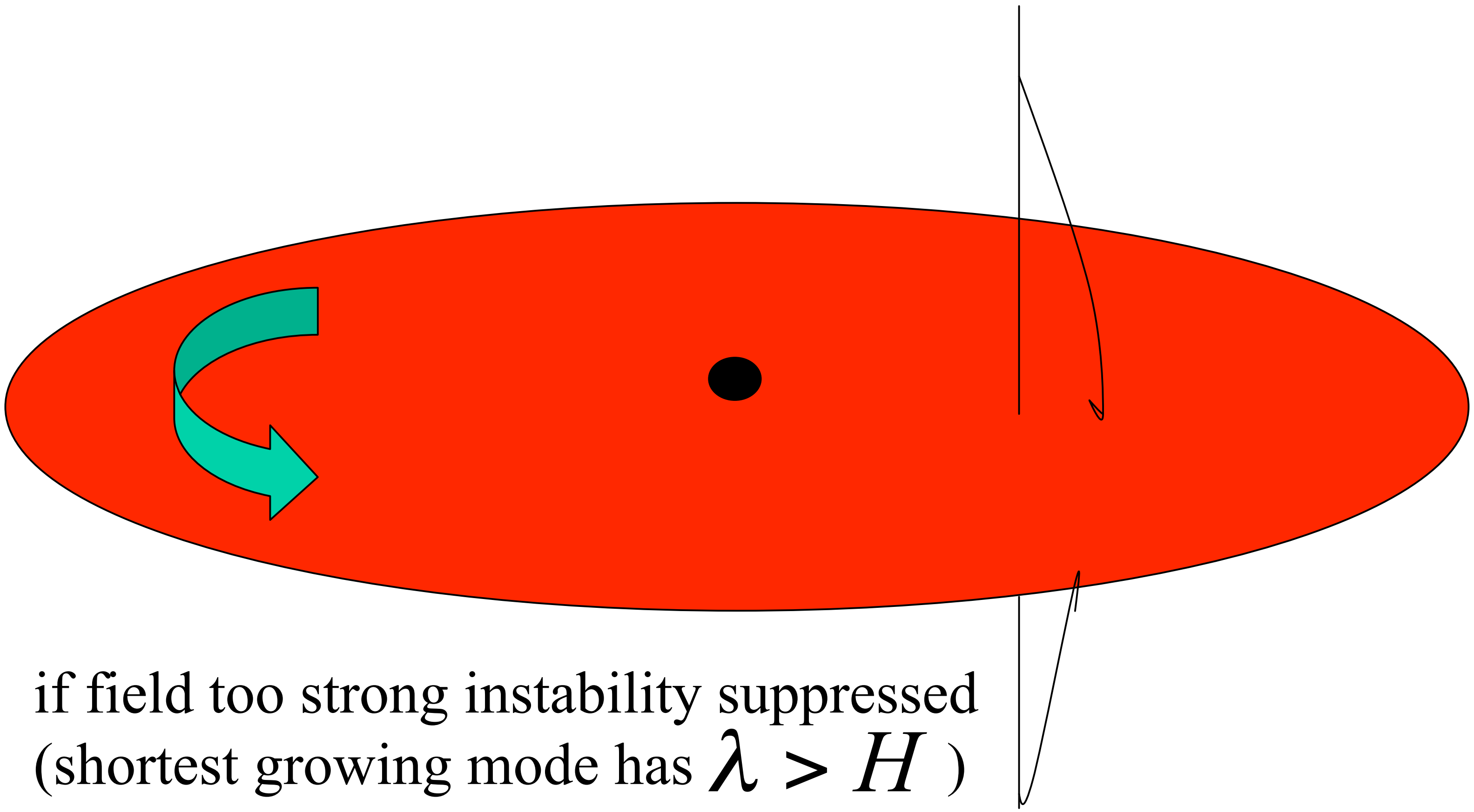
Balbus—Hawley (magnetorotational, MRI) instability

magnetic field *B* threading disc →



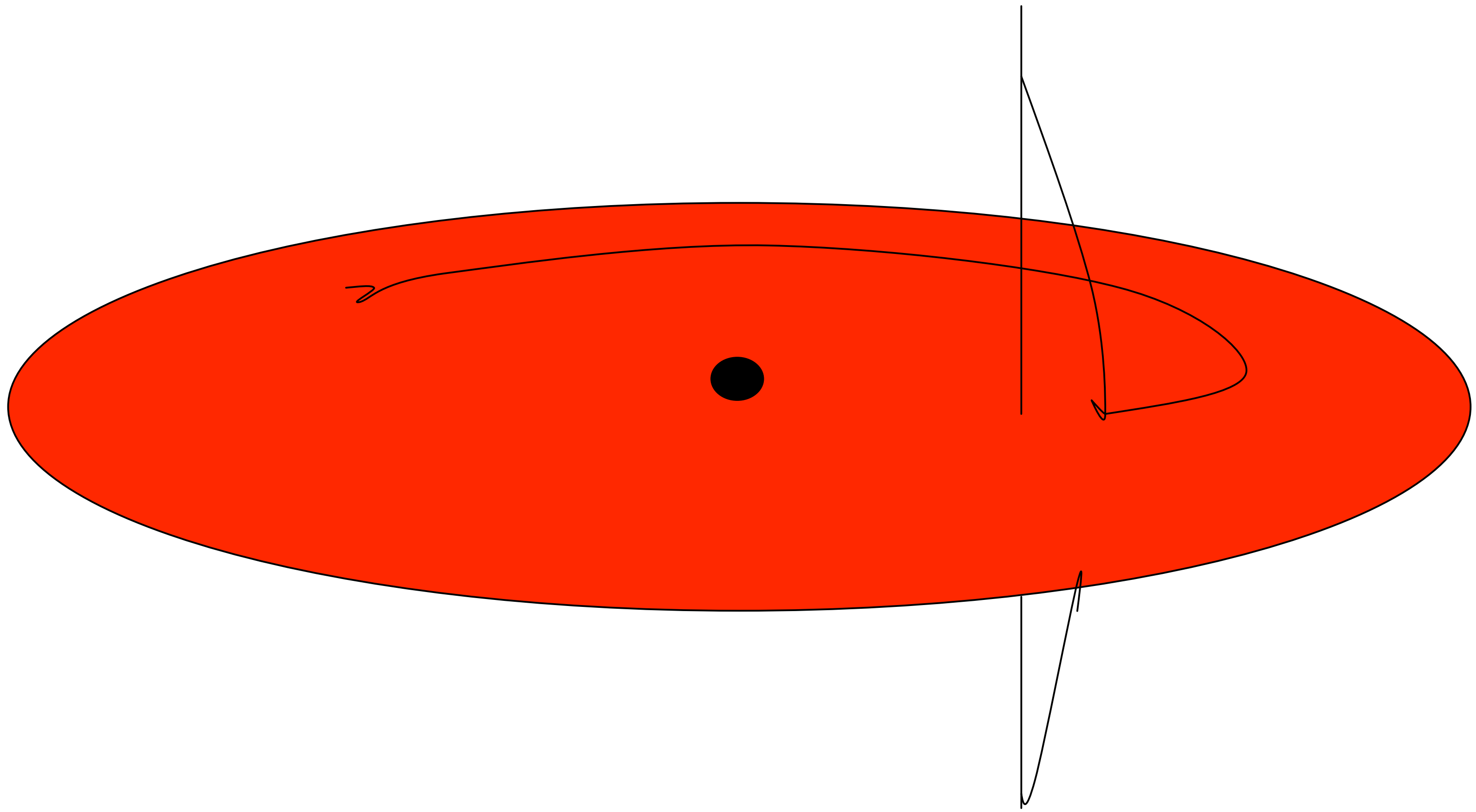
magnetic tension tries to *straighten* line
imbalance between gravity and rotation *bends* line

Vertical fieldline perturbed outwards, rotates faster than surroundings, so centrifugal force > gravity → *kink increases.* Line connects fast-moving (inner) matter with slower (outer) matter, and speeds latter up: ***outward a.m. transport***



if field too strong instability suppressed (shortest growing mode has $\lambda > H$)

distorted fieldline stretched azimuthally by differential rotation, *strength grows*

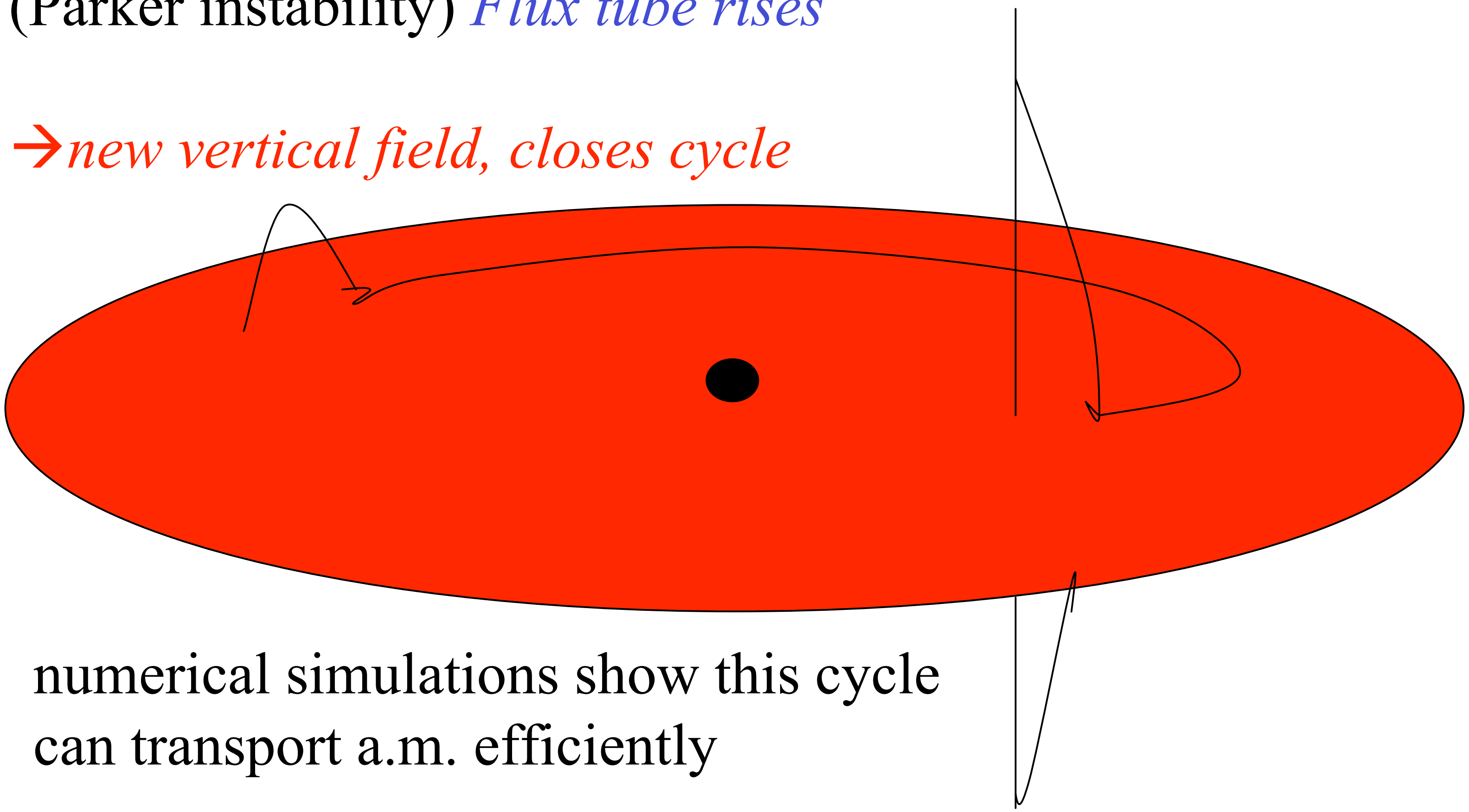


pressure balance between flux tube and surroundings requires

$$\frac{B^2}{8\pi} + P_{gas,in} = P_{gas,out}$$

so gas pressure and hence density lower inside tube → *buoyant* (Parker instability) *Flux tube rises*

→*new vertical field, closes cycle*

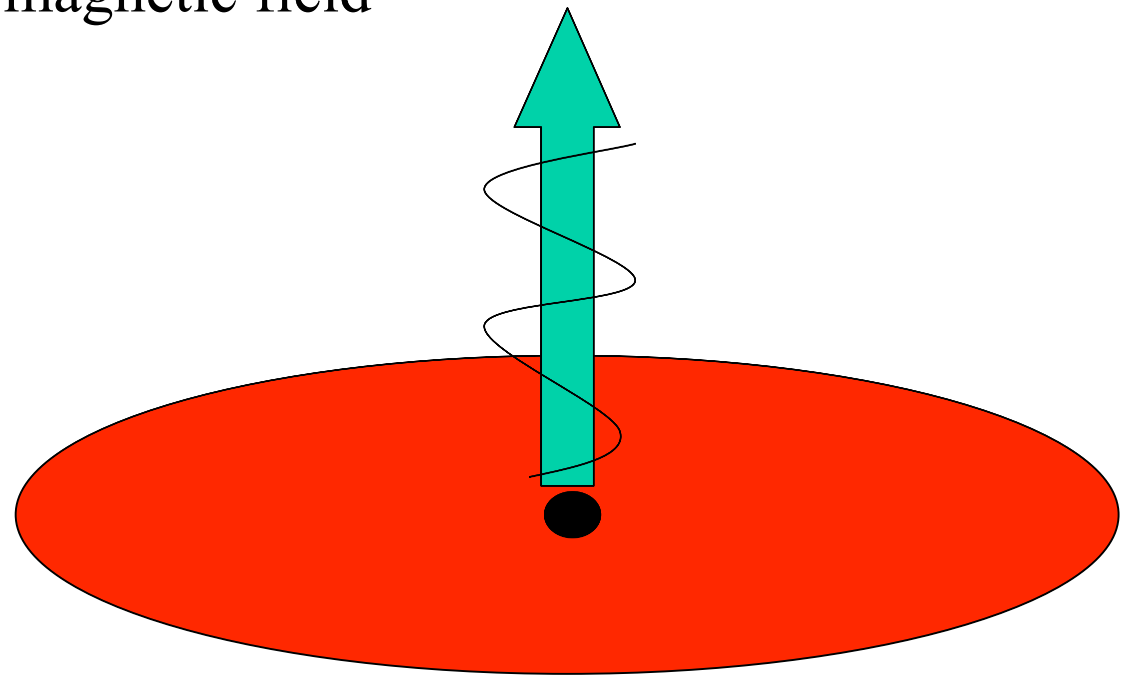


numerical simulations show this cycle can transport a.m. efficiently

Jets

One observed form of outflow: jets with ~ escape velocity from point of ejection, ~ *c* for black holes

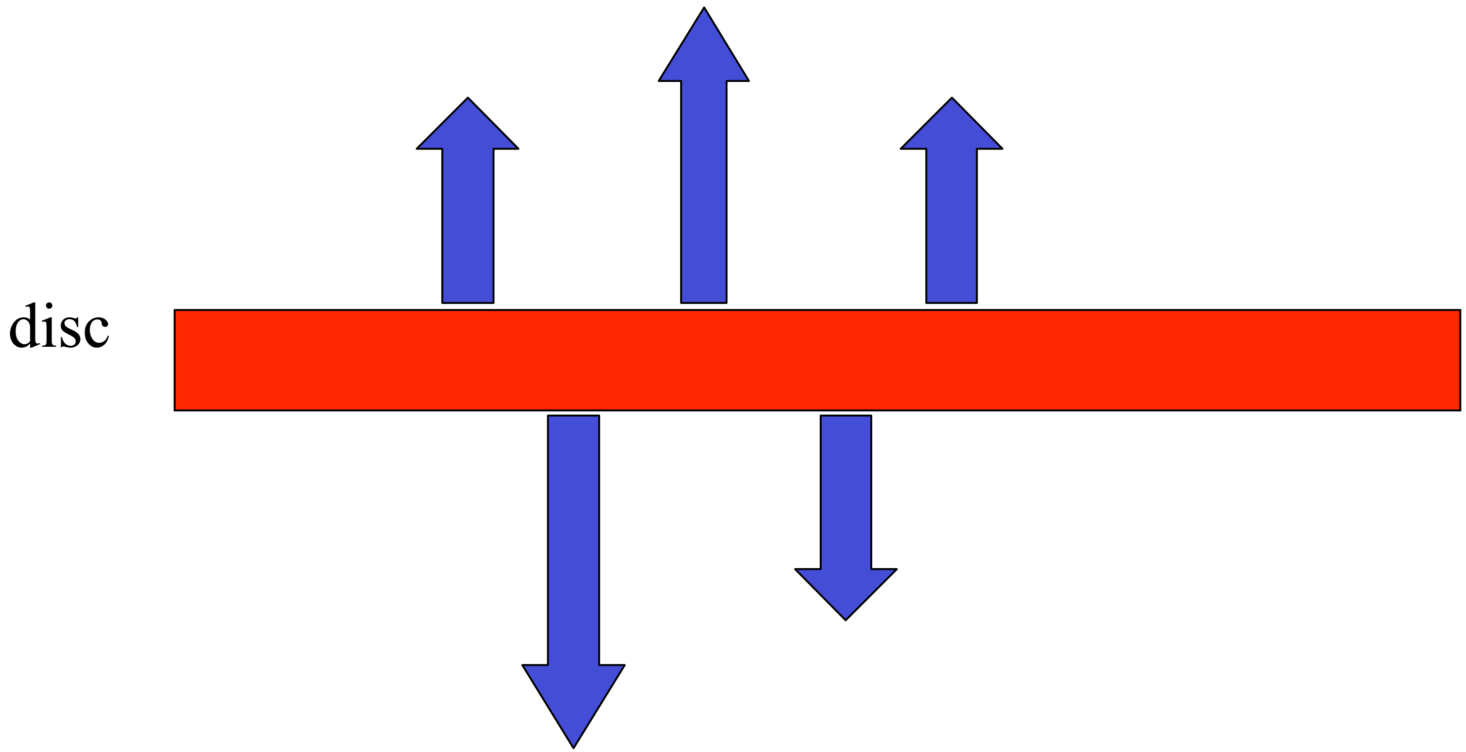
Launching and collimation not understood – probably requires toroidal magnetic field



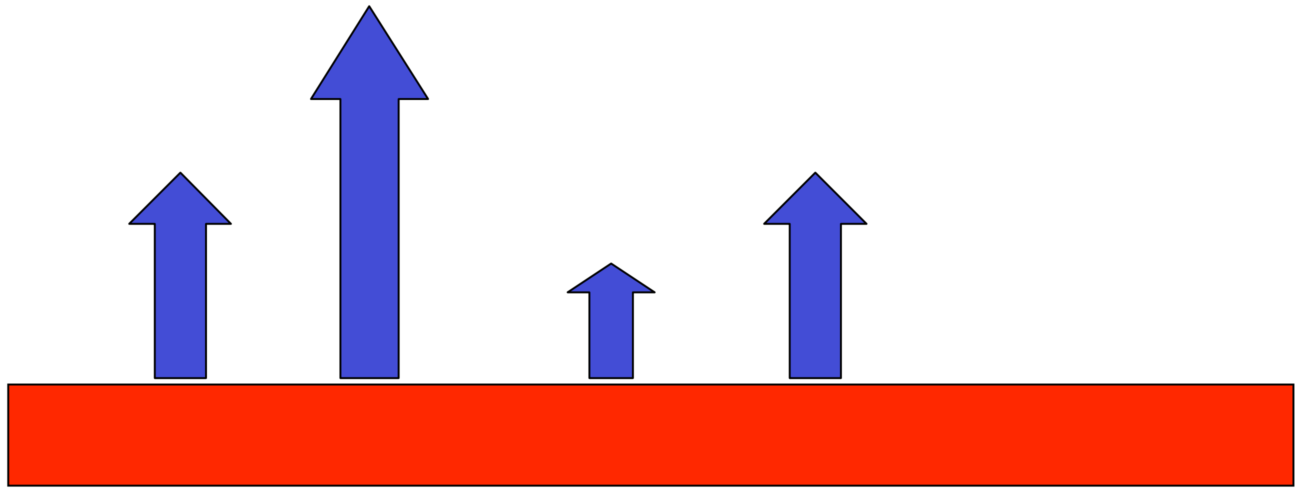
Disc may have *two* states:

1. infall energy goes into radiation (standard)

2. infall energy goes into winding up internal disc field – thus



generally vertical field directions uncorrelated in neighboring disc annuli (dynamo random); BUT



occasionally all fields line up → matter swept inwards, strengthens field → energy all goes into field → jet ???

(see King, Pringle, West, Livio, 2004)

jets seen (at times) in almost all accreting systems: AGN, LMXBs etc

Disc timescales

Have met dynamical timescale

$$t_{dyn} = R / v_K = (R^3 / GM)^{1/2}$$

and viscous timescale

$$t_{visc} = R^2 / \nu$$

We define also the thermal timescale

$$t_{th} = \Sigma c_s^2 / D(R) = \frac{R^3 c_s^2}{GM\nu} = \frac{c_s^2}{v_K^2} \frac{R^2}{\nu} = \frac{H^2}{R^2} t_{visc}$$

so

$$t_{dyn} < t_{th} < t_{visc}$$

Disc stability

Suppose a thin disc has steady—state surface density profile Σ_0

Investigate stability by setting $\Sigma = \Sigma_0 + \Delta\Sigma$
With $\mu = \nu\Sigma$ so that $\Delta\mu = (\partial\mu / \partial\Sigma)\Delta\Sigma$
diffusion equation gives (Exercise)

$$\frac{\partial}{\partial t}(\Delta\mu) = \frac{\partial\mu}{\partial\Sigma}\frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(R^{1/2}\Delta\mu)\right]$$

Thus diffusion (stability) if $\partial\mu / \partial\Sigma > 0$,
but
anti—diffusion (instability) if $\partial\mu / \partial\Sigma < 0$ — mass flows towards denser regions, disc breaks up into rings , on viscous timescale.

origin of instability:

$$\mu = \nu\Sigma \propto \dot{M}$$

so

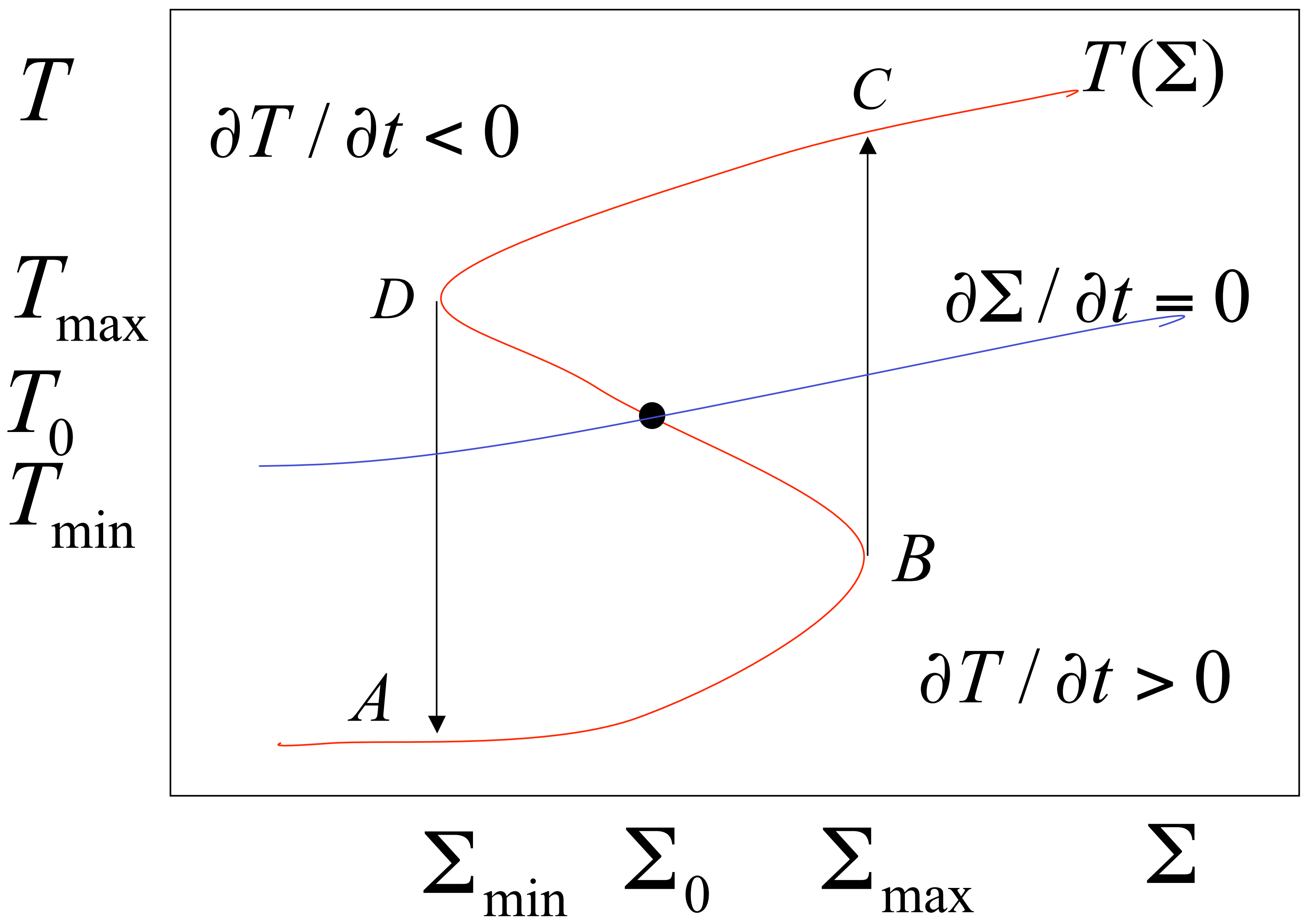
$$\partial\mu / \partial\Sigma < 0 \Rightarrow \partial \dot{M} / \partial\Sigma < 0$$

i.e. local accretion rate *increases* in response to a *decrease* in Σ (and vice versa), so local density drops (or rises).

To see condition for onset of instability, recall

$$\mu = \nu\Sigma \propto \dot{M} \propto T_b^4$$

and $T_b \propto$ internal temperature T. Thus stability/instability decided by sign of $\partial T / \partial \Sigma$ along the equilibrium curve $T(\Sigma)$ i.e. $\partial T / \partial t = 0$



Equilibrium

$$\partial T / \partial t = \partial \Sigma / \partial t = 0$$

here lies on unstable branch $\partial T / \partial \Sigma < 0$

System is forced to hunt around limit cycle ABCD, unable to reach equilibrium.

evolution A→B on long viscous timescale
evolution B→C on very short thermal timescale
evolution C→D on moderate viscous timescale
evolution C→A on very short thermal timescale

Thus get *long low states* alternating with *shorter high states*, with *rapid upwards and downward transitions* between them – **dwarf nova light curves.**

origin of wiggles in equilibrium $T(\Sigma)$ *curve* is hydrogen ionization threshold at $T \sim 10^4 K$

If all of disc is hotter than this, disc remains stably in the high state – no outbursts.

Thus *unstable discs must have low accretion rates*:

$$T_b{}^4 = \frac{3GM\dot{M}}{8\pi\sigma R_{out}{}^3} < 10^{16} K^4$$

where R_{out} is outer disc radius

X—ray irradiation by central source: disc is *concave or warped*

thus $T_b = T_{irr} \propto R^{-1/2}$ not $T_{visc} \propto R^{-3/4}$ *so dominates at large R* (where most disc mass is)

ionization/stability properties controlled by CENTRAL $\dot{M}$

Thus an outburst of an irradiated disc cannot be ended by a cooling wave, but only when central accretion rate drops below a critical value such that

$$T_{irr}(R_{out}) = T_{ion} \approx 6500K$$

→ mass of central disc drops significantly → *long*!

K & Ritter (1998): in outburst disc is roughly steady—state, with

$$\Sigma \approx \frac{\dot{M}_c}{3\pi\nu}$$

$\dot{M}_c$ the central accretion rate. Mass of hot disc is

$$M_h = 2\pi \int_0^{R_h} \Sigma R dR \approx \dot{M}_c \frac{R_h^2}{3\nu}$$

Now hot zone mass can change only through central accretion, so

$$\dot{M}_h = -\dot{M}_c$$

thus

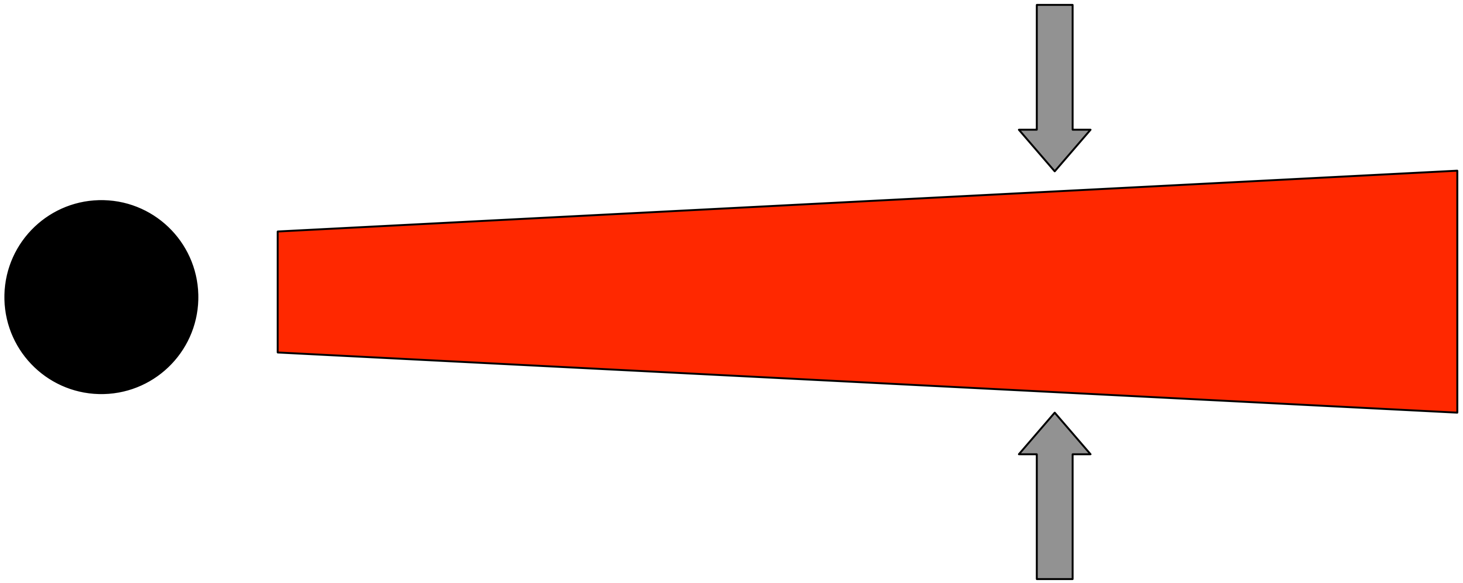
$$-\dot{M}_h = \frac{3\nu}{R_h^2} M_h$$

i.e.

$$M_h = M_0 e^{-3\nu t / R_h^2}$$

so central accretion rate, X—rays, drop exponentially for small discs

observation indeed shows that outbursts in small, fully irradiated discs are exponential (`soft X—ray transients') *size of AGN disc* set by *self—gravity*



vertical component of gravity from central mass is $\sim GMH / R^3$

cf that from self—gravity of disc $\sim G\rho H^3 / H^2 \sim G\rho H$

Thus self—gravity takes over where $\rho \sim M / R^3$, or

$$M_{disc} \sim R^2 H\rho \sim \frac{H}{R} M$$

disc breaks up into stars outside this

accretion to central object

central object gains a.m. and *spins up* at rate

$$\sim \dot{M}(GMR_{in})^{1/2}$$

reaches maximum spin rate ($a \sim M$ for black hole) after accreting $\sim M$ if starts from low spin. `Centrifugal' processes limit spin. For BH, photon emission limits $a/M < 1$

in AGN, BH gains mass significantly – does it spin up?

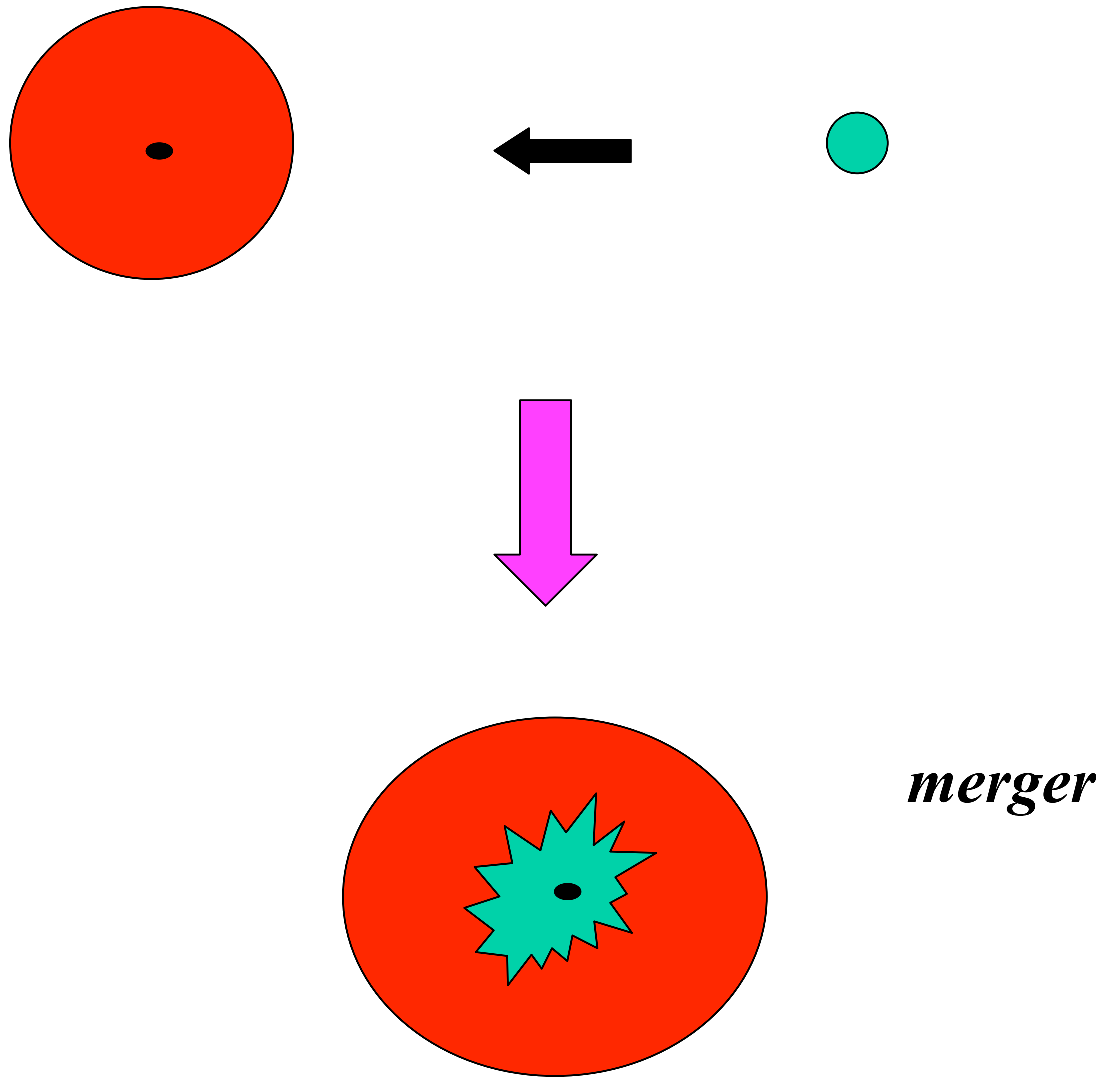
active galactic nuclei

supermassive BH $(10^6 - 10^9 M_{sun})$ in the centre of every galaxy

how did this huge mass grow?

cosmological picture:

big galaxy swallows small one



galaxy mergers

two things happen:

1. *black holes coalesce*: motion of each is slowed by inertia of gravitational `wake' – *dynamical friction*. Sink to bottom of potential and orbit each other. GR emission → *coalescence*

2. *accretion*: disturbed potential → gas near nuclei destabilized, a.m. loss → accretion: *merged black hole grows*: radiation → *AGN*

black hole coalescence

Hawking's theorem: black hole *event horizon area*

$$A = \frac{8\pi G^2}{c^4}[M^2 + (M^4 - c^2 J^2 / G^2)^{1/2}]$$

or

$$\boxed{A \propto M^2[1 + (1 - a_*^2)^{1/2}]}$$

where $J =$ a.m., $a_* = cJ / GM^2$, *can never decrease* thus can give up angular momentum and still increase area, i.e. *release rotational energy* – e.g. as gravitational radiation

then *mass M decreases!* – minimum is $M/\sqrt{2}$ (irreducible mass) – start from $a_* = 1$ and spin down to $a_* = 0$ keeping *A* fixed

coalescence can be both *prograde* and *retrograde* wrt spin of merged hole, i.e. orbital opposite to spin a.m.

Hughes & Blandford (2003): long—term effect of coalescences is *spindown* since last stable circular orbit has larger a.m. in retrograde case.

black hole accretion

Soltan (1982): total rest—mass energy of all SMBH

consistent with radiation energy of Universe

if masses grew by luminous accretion (efficiency ~10 %)

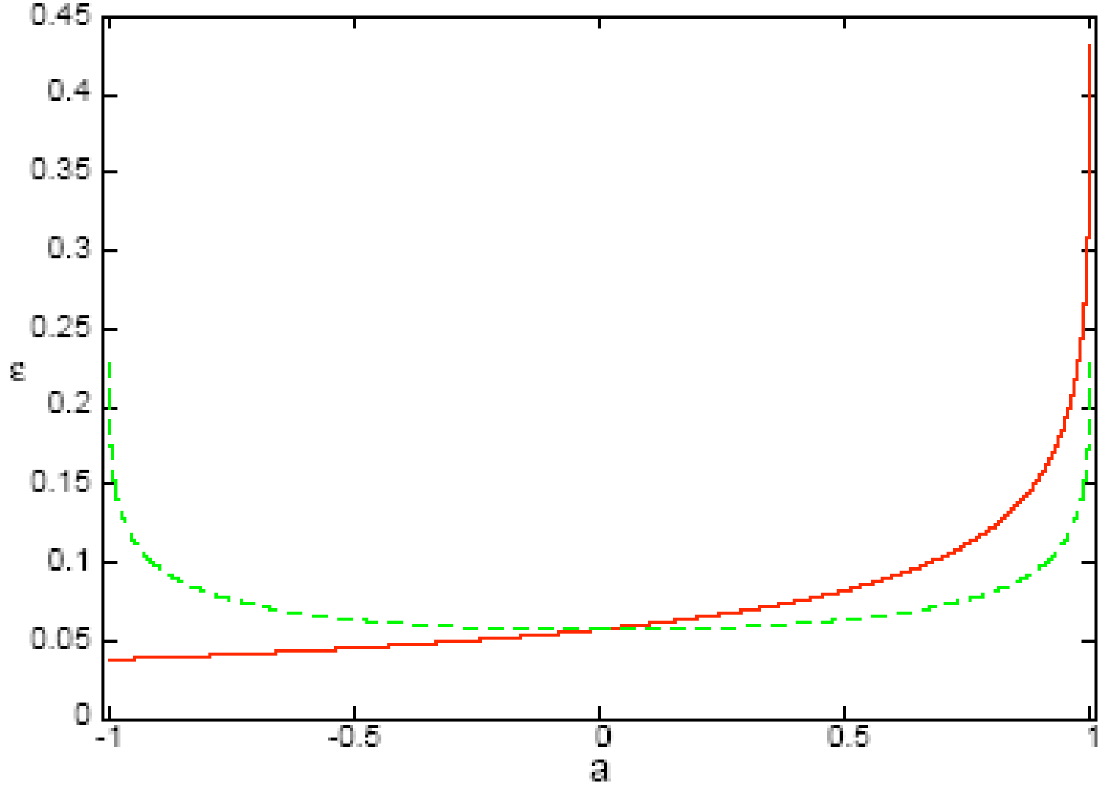
thus ADAFs etc unimportant in growing most massive black holes *merger picture of AGN: consequences for accretion*

• mergers do not know about *black hole mass* M, so accretion may be super—Eddington

• mergers do not know about *hole spin* a, so accretion may be retrograde

*

efficiency versus spin parameter



• ***super—Eddington accretion***:

must have been common as most SMBH grew (z ~2), so

outflows

what do we know about accretion at super (hyper)—Eddington rates?

Hyper-Eddington Accretion: ***SS433***

- 13.1—day binary with huge mass transfer rate (~ 3000 Eddington)

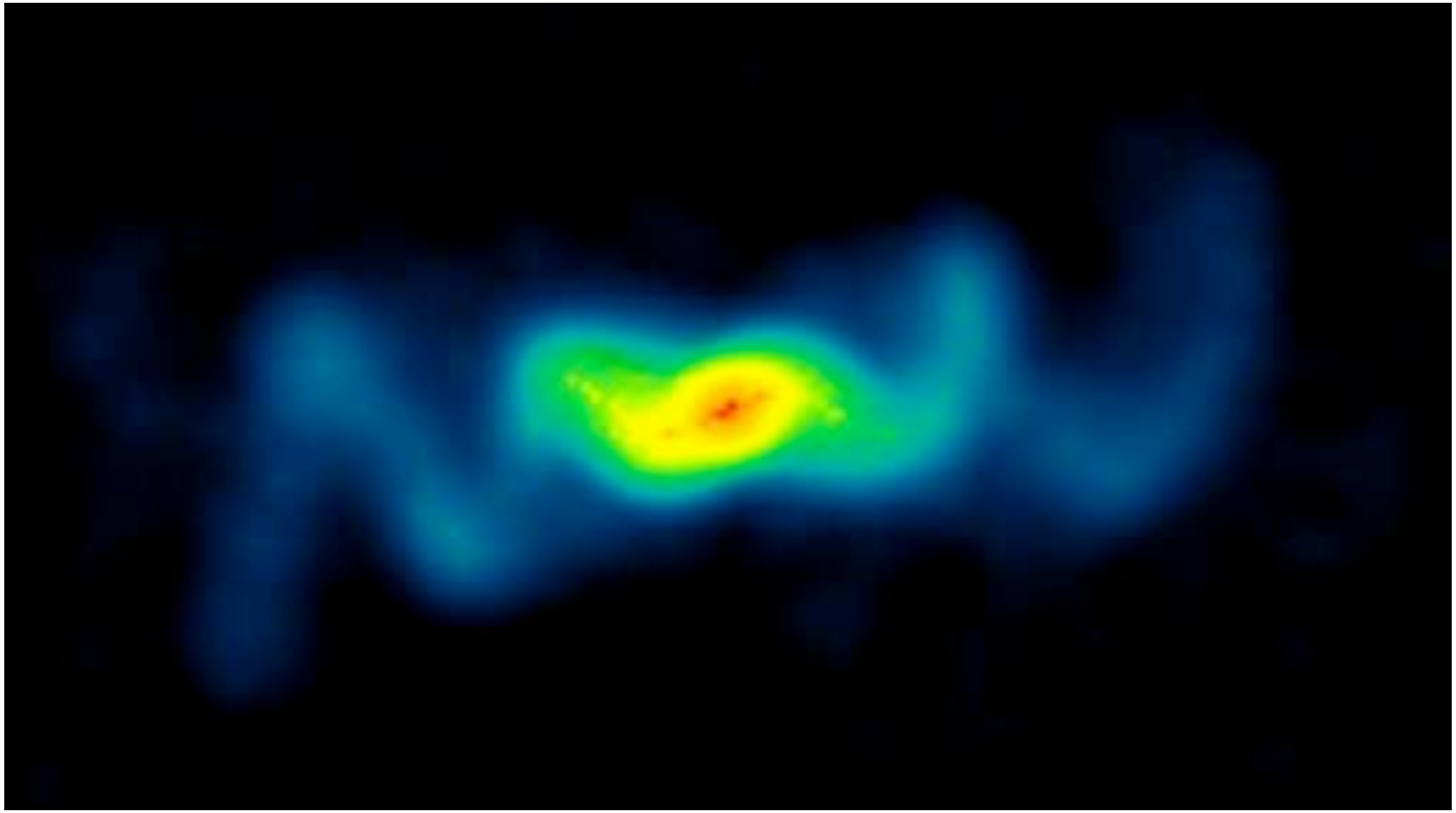
- pair of jets (v = 0.26c) precessing with 162—day period, at angle $\theta = 20^o$ to binary axis

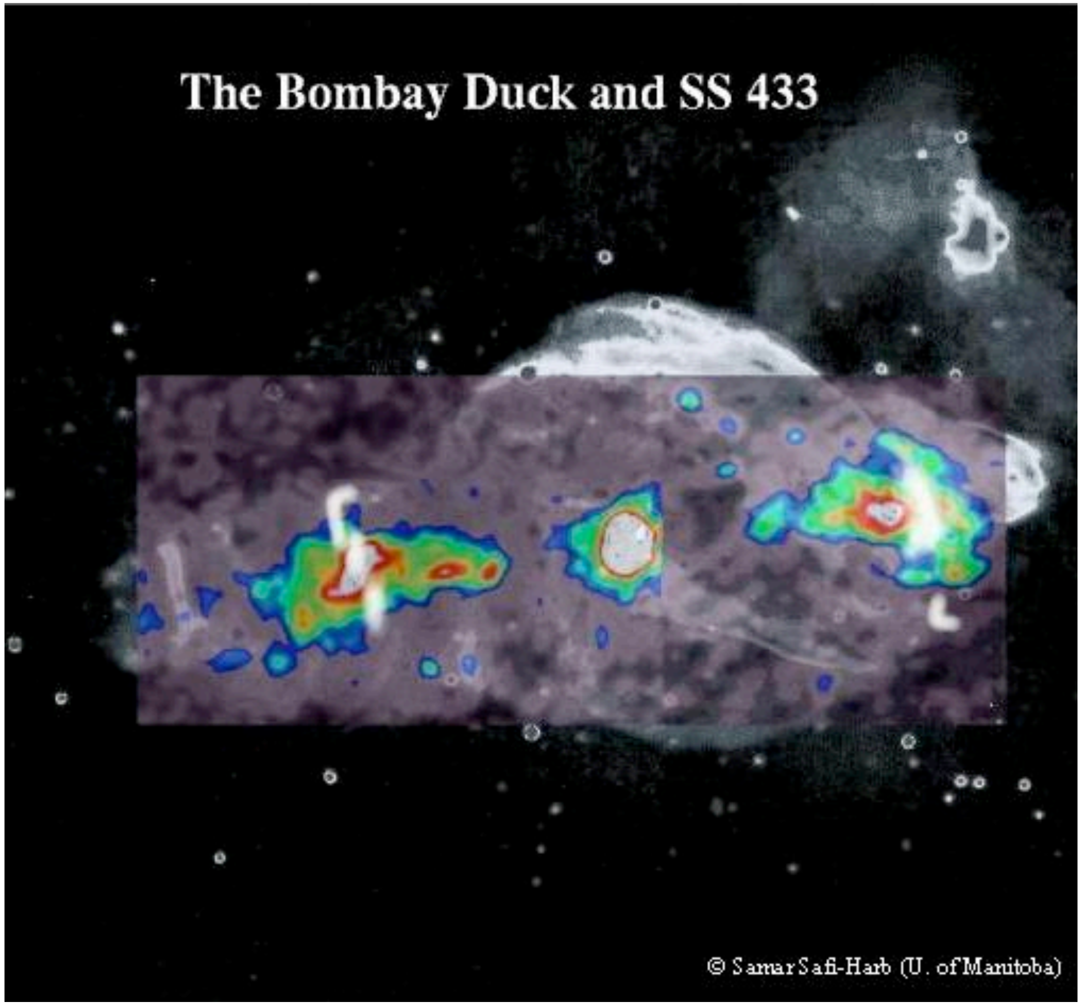
- seen in H alpha, radio, X—rays

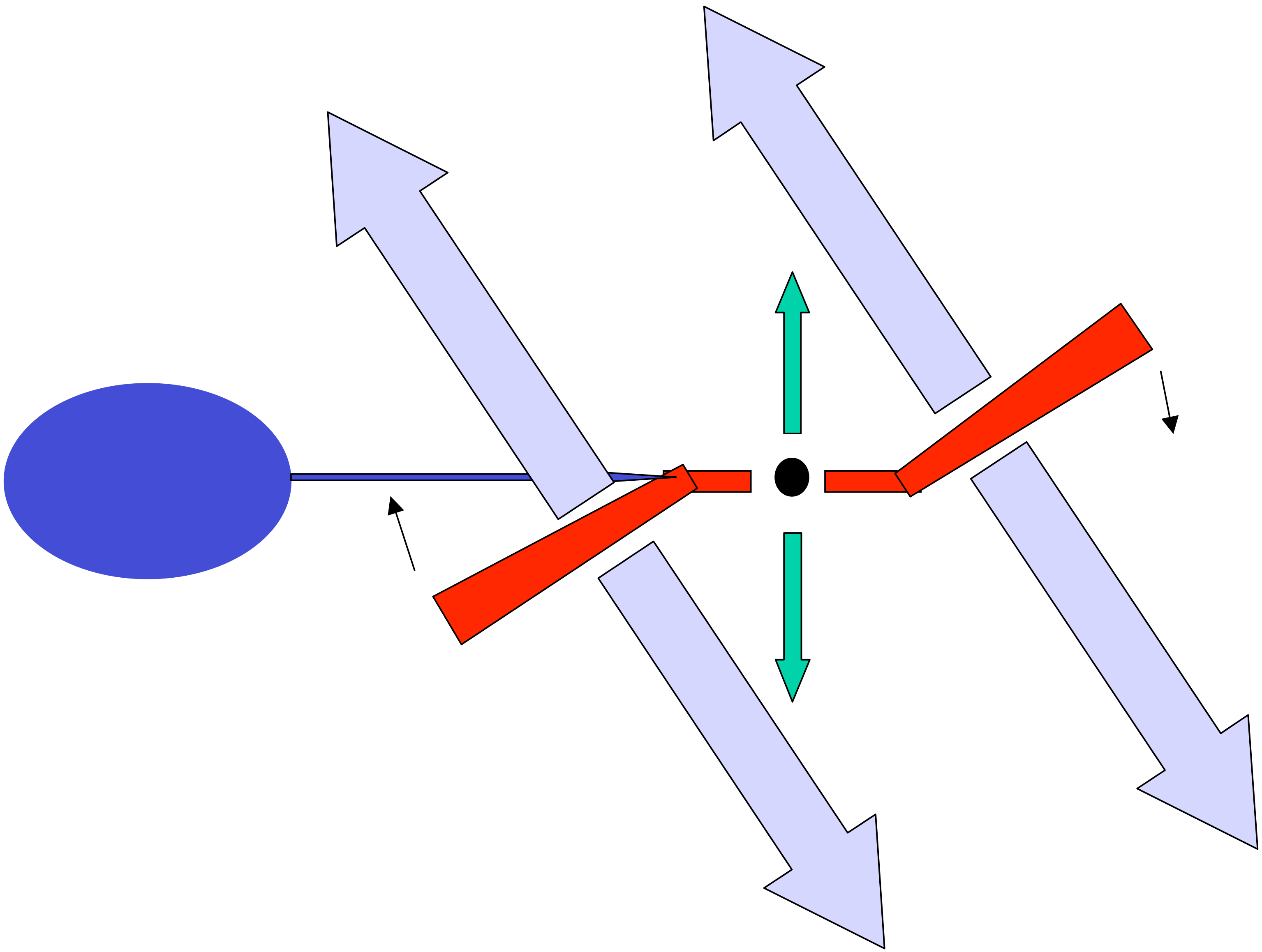
- kinetic luminosity of jets ~ 10^{39} erg/s, but radiative luminosity less, e.g. $L_x \approx 10^{36}$ erg/s

- huge outflow (`stationary H alpha') at 2000 km/s — this is where hyper—Eddington mass flow goes

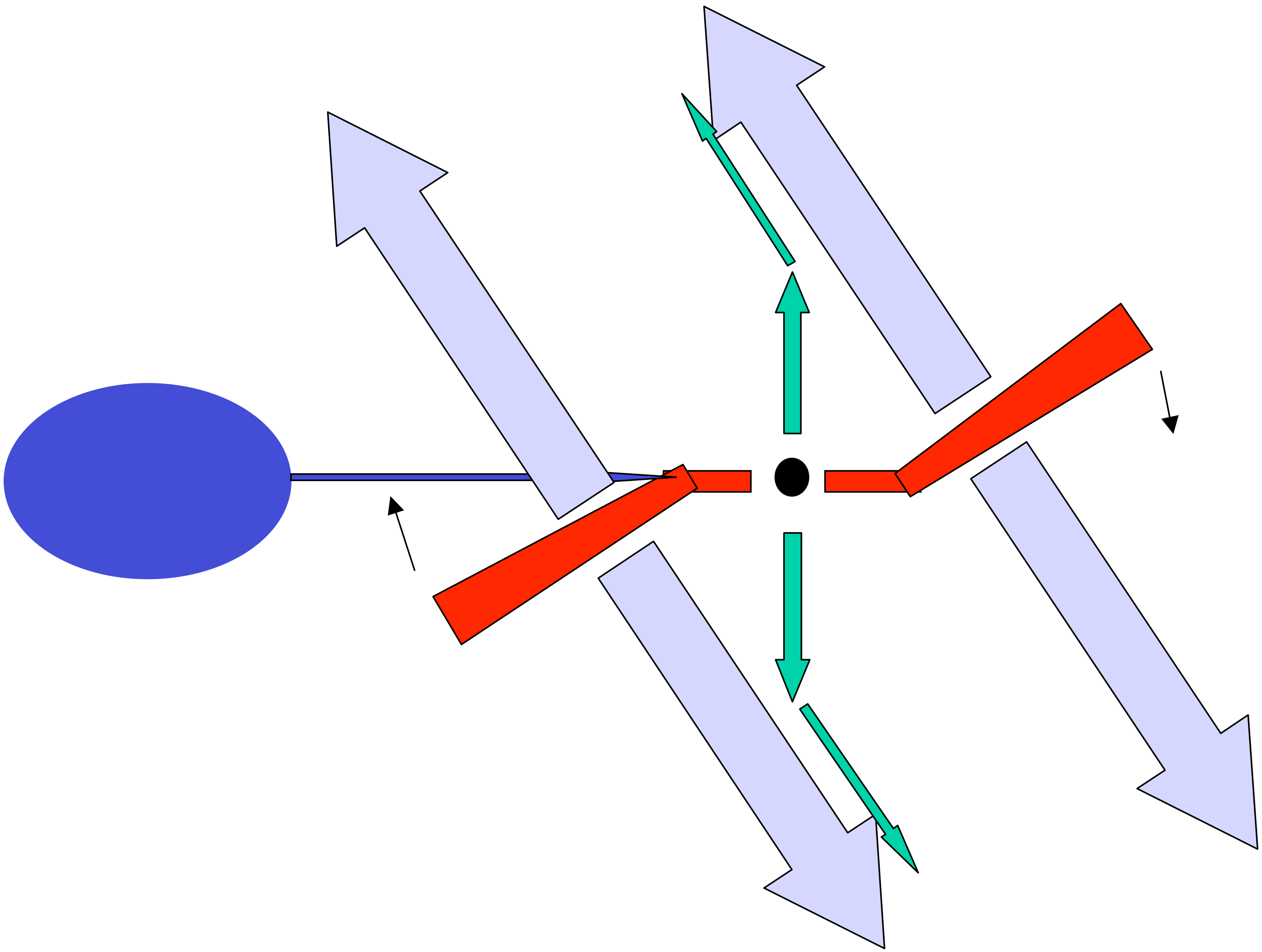
- this outflow inflates surrounding nebula (W50) and precessing jets make `ears'




The Bombay Duck and SS 433
© Samar Safi-Harb (U. of Manitoba)



outflow *can* be sensitive to outer disc plane if from large enough R



outflow bends jets parallel to axis of outer disc, since far more momentum

Where is the outflow launched?

Shakura & Sunyaev (1973): `spherization radius'

$$R_{sp} = \frac{27\dot{M}_{out}}{4\dot{M}_{Edd}} R_s, \quad R_s = \text{ Schwarzschild radius}$$

Outflow velocity is v ~ 2000 km/s, suggesting

$$R_{sp} \cong \frac{2GM}{v^2} \cong \frac{c^2}{v^2} R_s \cong 7 \times 10^{10}\, cm$$

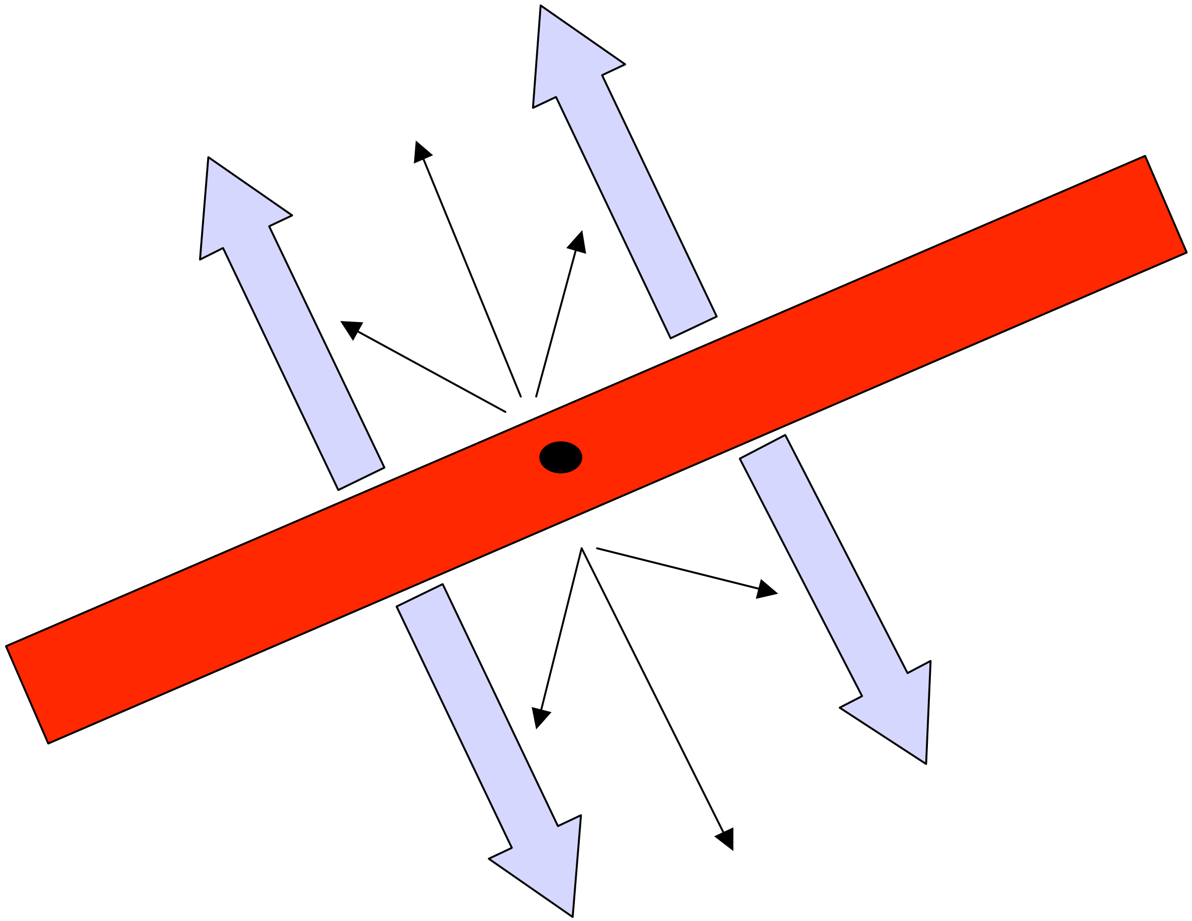
for 10 Msun black hole

within R_{sp} accretion rate must drop as ~R, to keep each radius below Eddington rate. This leads (cf Shakura & Sunyaev, 1973) to

$$L_{acc} \approx \int_{R_{in}}^{R_{sp}} \frac{3GM\,\dot{M}_{Edd}R}{8\pi R^3} 2\pi R dR \cong L_{Edd} \times \ln(R_{sp} / R_{in})$$

Now $R_{sp} \approx 10^4 R_{in}$, so logarithm is ~ 10.

Thus a 10 Msun black hole can emit 10^{40} erg/s



Moreover ‘walls’ of outflow are very optically thick (tau ~ 80) so all luminosity escapes in narrow cone

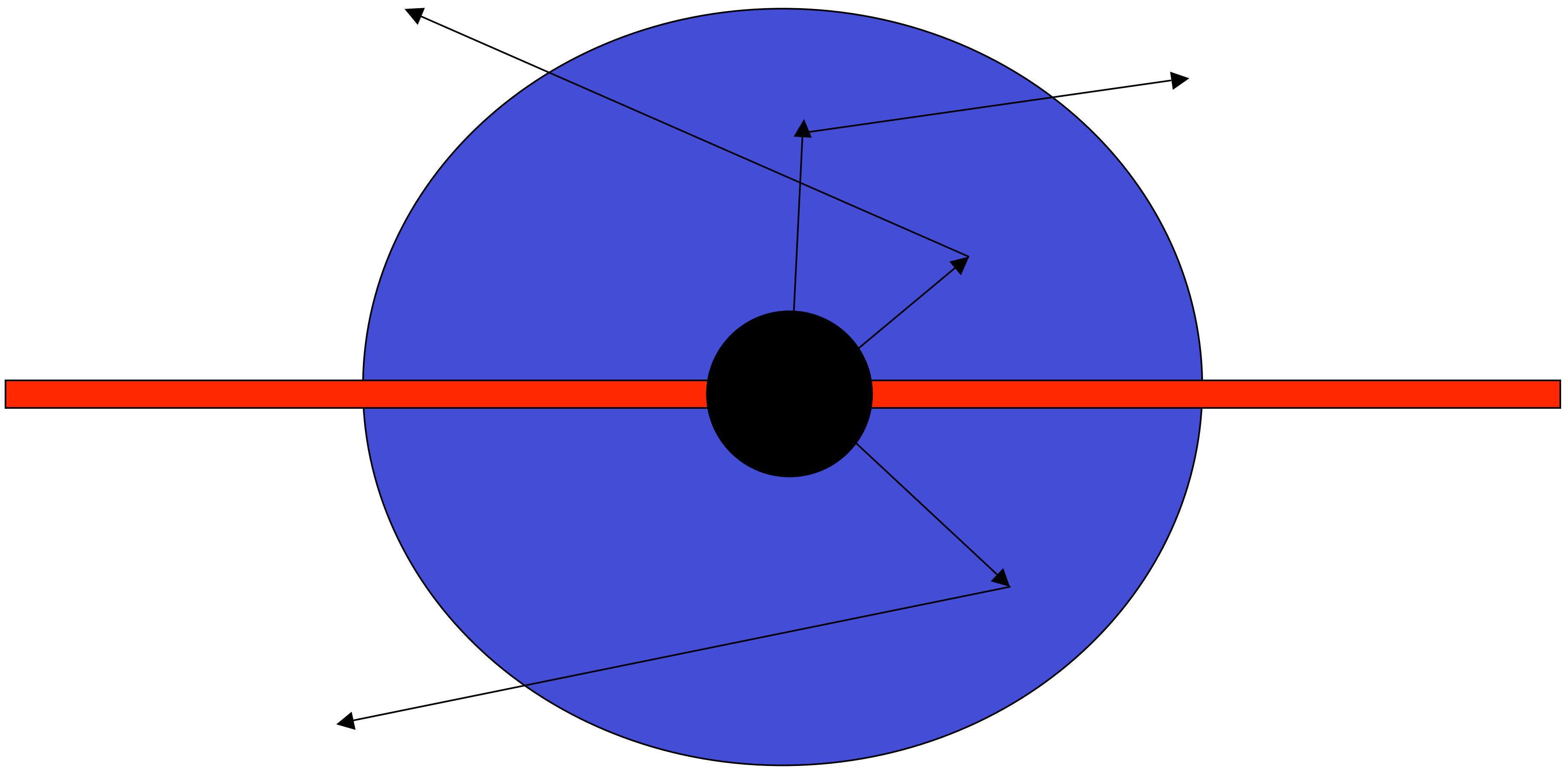
An observer viewing the system down this axis would infer an isotropic luminosity

$$L \approx 10^{40} b^{-1} \text{ erg/s}$$

where b is the collimation factor.

Ultraluminous X—ray sources (ULXs) may be (non—precessing) systems like this: even with only b = 10% collimation they can reach the luminosities of observed ULXs.

outflow is optically thick to scattering: radiation field L » L_{Edd} transfers » *all* its momentum to it



- *response to super—Eddington accretion*: expel excess accretion as an outflow with ***thrust*** given purely by L_{Edd} , i.e.

$$\dot{M}_{out} v \approx \frac{L_{Edd}}{c}$$

- *outflows with Eddington thrust must have been common as SMBH grew*

- NB mechanical energy flux $\frac{1}{2}\dot{M}_{out} v^2 \approx \frac{L_{Edd} v}{c}$ requires knowledge of v or $\dot{M}_{out}$

- *effect on host galaxy large*: must absorb most of the outflow momentum and energy – galaxies not `optically thin’ to matter – unlike radiation

- e.g. could have accreted at » 1M− yr^{-1} for » 5£10^7 yr

- *mechanical energy* deposited in this time » 10^{60} erg

- cf *binding energy* » 10^{59} erg of galactic bulge with *M » 10^{11} M−* and velocity dispersion σ » 300 km s^{-1}

- examine effect of super—Eddington accretion on growing SMBH (K 2003)

• model protogalaxy as an isothermal sphere of dark matter: gas
density is

$$\rho(R) = \frac{f_g \sigma^2}{2\pi G r^2}$$

with $f_g = \Omega_{\text{baryon}}/\Omega_{\text{matter}}$ ' 0.16

• so gas mass inside radius R is

$$M(R) = 4\pi \int_0^R \rho r^2 dr = \frac{2 f_g \sigma^2 R}{G}$$

- dynamics depend on whether gas cools (`momentum—driven') or not (`energy—driven')

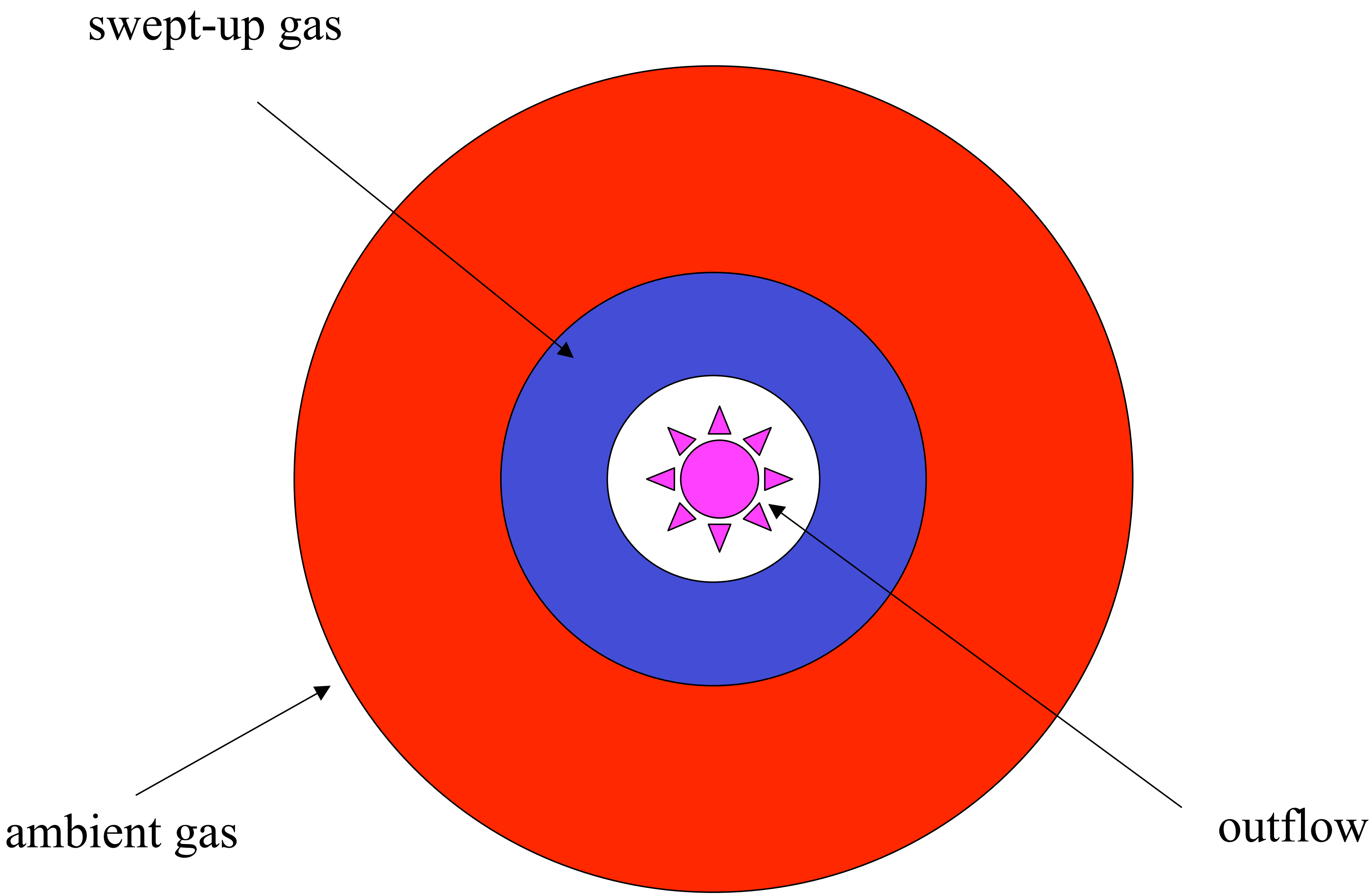
- Compton cooling is efficient out to radius R_c such that

$$M(R_c) \approx 2 \times 10^{11} \sigma^3_{200} M_8^{1/2} M_\odot$$

where $\sigma_{200} = \sigma/200\ km\ s^{-1}$, $M_8 = M/10^8 M_\odot$

- flow is momentum—driven (i.e. gas pressure is unimportant) out to $R = R_c$

for $R > R_c$ flow speeds up because of pressure driving


swept-up gas
ambient gas
outflow

ram pressure of outflow drives expansion of swept-up shell:

$$\frac{d}{dt}[M(R)\dot{R}] = 4\pi R^2 \rho v^2 = \dot{M}_{out} v - \frac{GM^2(R)}{R^2}$$

$$= \frac{L_{Edd}}{c} - 4 f_g \frac{\sigma^4}{G} = const$$

(using $M(R) = 2f_g\sigma^2 R/G$ etc)

thus

$$R^2 = \left[\frac{GL_{Edd}}{2 f_g \sigma^2 c} - 2\sigma^2\right] t^2 + 2R_0 v_0 t + R_0^2$$

for small L_{Edd} (i.e. small *M*), *R* reaches a maximum

$$R_{\max}^2 = \frac{R_0^2 v_0^2}{2\sigma^2 - GL_{Edd}/2f_g\sigma^2 c} + R_0^2$$

in a dynamical time $\sim R_{\max}/\sigma$

R cannot grow beyond $R_{\max}$ until *M* grows: expelled matter is trapped inside bubble

M and *R* grow on Salpeter timescale $\sim 5\times10^7\, yr$

gas in shell recycled – star formation, chemical enrichment

- ***starbursts accompany black—hole growth***
- AGN accrete gas of *high metallicity*

ultimately shell too large to cool: drives off gas outside

- velocity large: *superwind*
- remaining gas makes bulge stars — *black—hole bulge mass relation*
- no fuel for BH after this, so *M* fixed: *M—sigma relation*

thus *M* grows until

$$M = \frac{f_g \kappa}{\pi G^2} \sigma^4$$

or

$$M = 2 \times 10^8 \sigma_{200}^4 M_\Theta$$

for a dispersion of 200 km/s

Note: predicted relation

$$M = \frac{f_g \kappa}{\pi G^2} \sigma^4$$

has no free parameter!

• M—sigma is very close to observed relation (Ferrarese & Merritt, 2000; Gebhardt et al., 2000; Tremaine et al, 2002)

• only mass inside cooling radius ends as bulge stars, giving

$$\boxed{M \sim 7\times10^{-4} M_8^{-1/4} M_{bul}}$$

actually

$$M \sim (m_e / m_p)^2 (c/\sigma) M_{bul}$$

• in good agreement with observation

• argument via Soltan assumes standard accretion efficiency

• but ***mergers*** imply accretion flows initially *counter*aligned in half of all cases, i.e. low accretion efficiency, initial spindown

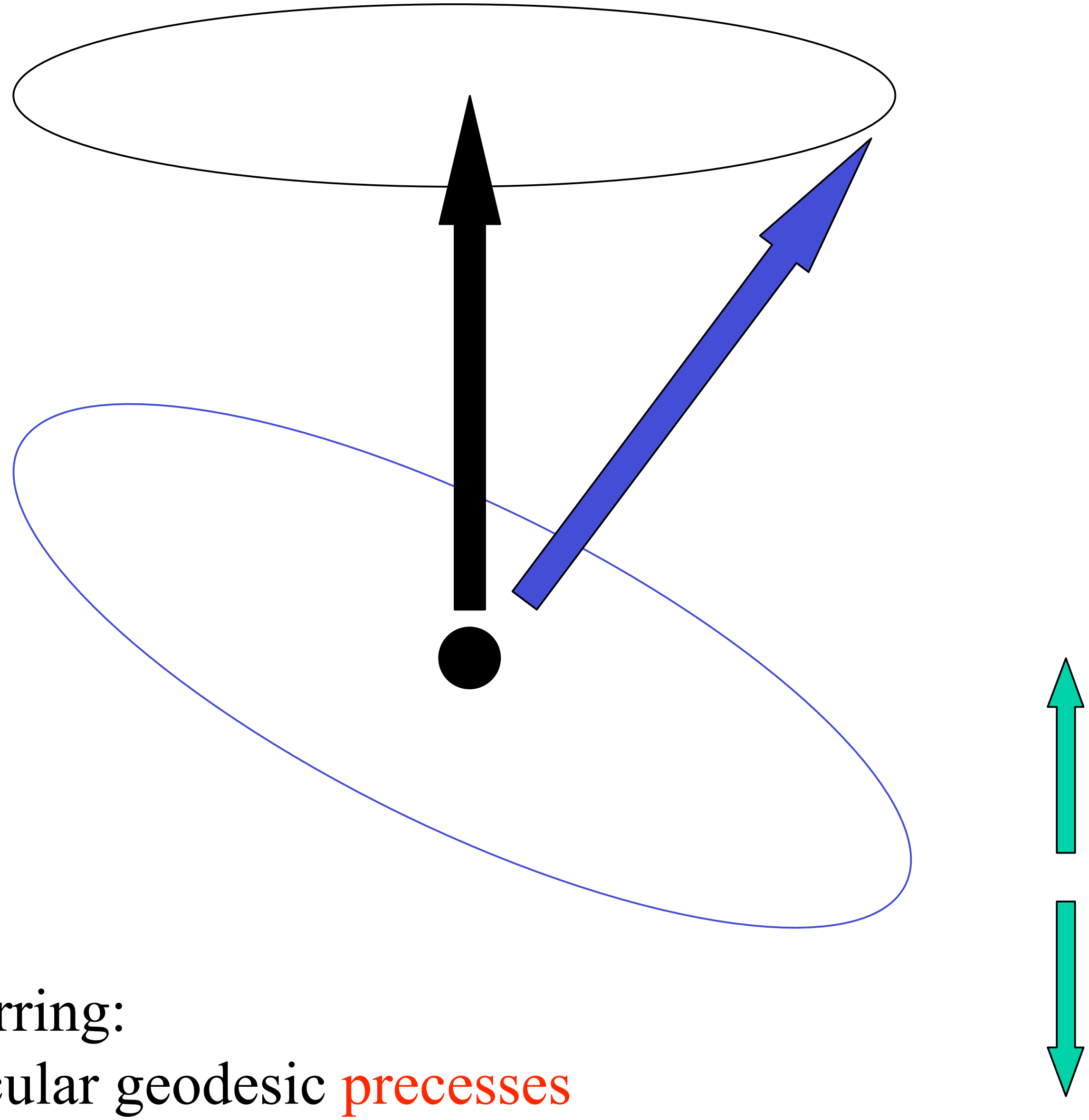
- how does SMBH react? i.e. what are torques on hole?

- two main types:
  1. accretion – spinup or spindown – requires hole to accrete ~ its own mass to change *a/M* significantly — *slow*
  2. Lense—Thirring from misaligned disc viscous timescale — *fast* in inner disc

- standard argument: *alignment* via Lense—Thirring occurs *rapidly*, hole spins up to keep a ~ M, accretion efficiency is *high*

- but L—T *also* vanishes for *counteralignment*

- alignment or not? (King, Lubow, Ogilvie & Pringle 05)



Lense—Thirring:
plane of circular geodesic precesses
about black hole spin axis: dissipation causes alignment or counteralignment

Torque on hole is pure precession, so *orthogonal to spin*.

Thus general equation for spin evolution is

$$\frac{\mathrm{d}\mathbf{J}_h}{\mathrm{d}t} = -K_1[\mathbf{J}_h \wedge \mathbf{J}_d] - K_2[\mathbf{J}_h \wedge (\mathbf{J}_h \wedge \mathbf{J}_d)].$$

Here $K_1, K_2 > 0$ depend on disc properties. First term specifies precession, second alignment.

Clearly magnitude J_h is constant, and vector sum $\mathbf{J}_\mathrm{t}$ of $\mathbf{J}_\mathrm{h}$, $\mathbf{J}_\mathrm{d}$ is constant. Thus $\mathbf{J}_\mathrm{t}$ stays fixed, while tip of $\mathbf{J}_\mathrm{h}$ moves on a sphere during alignment.

Using these, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{J}_h \cdot \mathbf{J}_t) = \mathbf{J}_t \cdot \frac{\mathrm{d}\mathbf{J}_h}{\mathrm{d}t} = \mathbf{J}_\mathrm{d} \cdot \frac{\mathrm{d}\mathbf{J}_h}{\mathrm{d}t}.$$

thus

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{J}_h \cdot \mathbf{J}_t) = K_2[J_d^2 J_h^2 - (\mathbf{J}_d \cdot \mathbf{J}_h)^2] \equiv A > 0.$$

But J_h, J_t are constant, so angle θ_h between them obeys

$$\frac{\mathrm{d}}{\mathrm{d}t}(\cos\theta_h) > 0$$

— hole spin *always* aligns with *total* angular momentum

Can further show that J_d^2 always *decreases* during this process – dissipation

Thus viewed in frame precessing with $\mathbf{J}_h$, $\mathbf{J}_d$,

$\mathbf{J}_t$ stays fixed: $\mathbf{J}_h$ aligns with it while keeping its length constant

J_d^2 decreases monotonically because of dissipation

Since

$$J_t^2 = J_h^2 + J_d^2 - 2J_h J_d \cos(\pi - \theta)$$

there are two cases, depending on whether

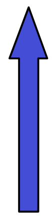
$$\cos\theta < -\frac{J_d}{2J_h}$$

or not. If this condition fails, $J_t > J_h$ and alignment follows in the usual way – older treatments implicitly assume

$$J_d >> J_h$$

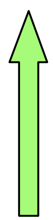
so predicted alignment

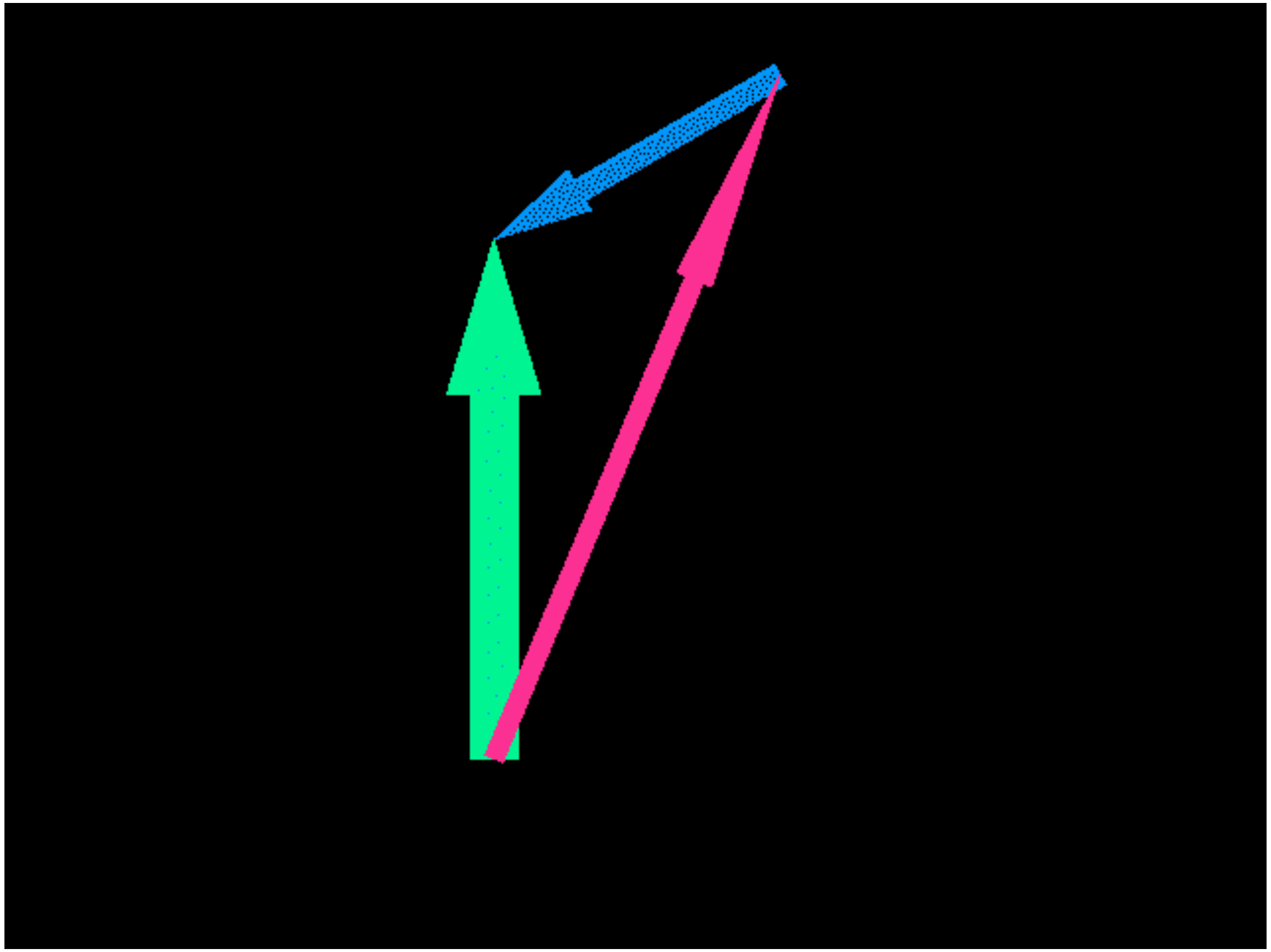
$$\mathbf{J}_{\mathrm{h}} =$$



$$\mathbf{J}_{\mathrm{d}} =$$

$$\mathbf{J}_{\mathrm{t}} = \mathbf{J}_{\mathrm{h}} + \mathbf{J}_{\mathrm{d}} =$$



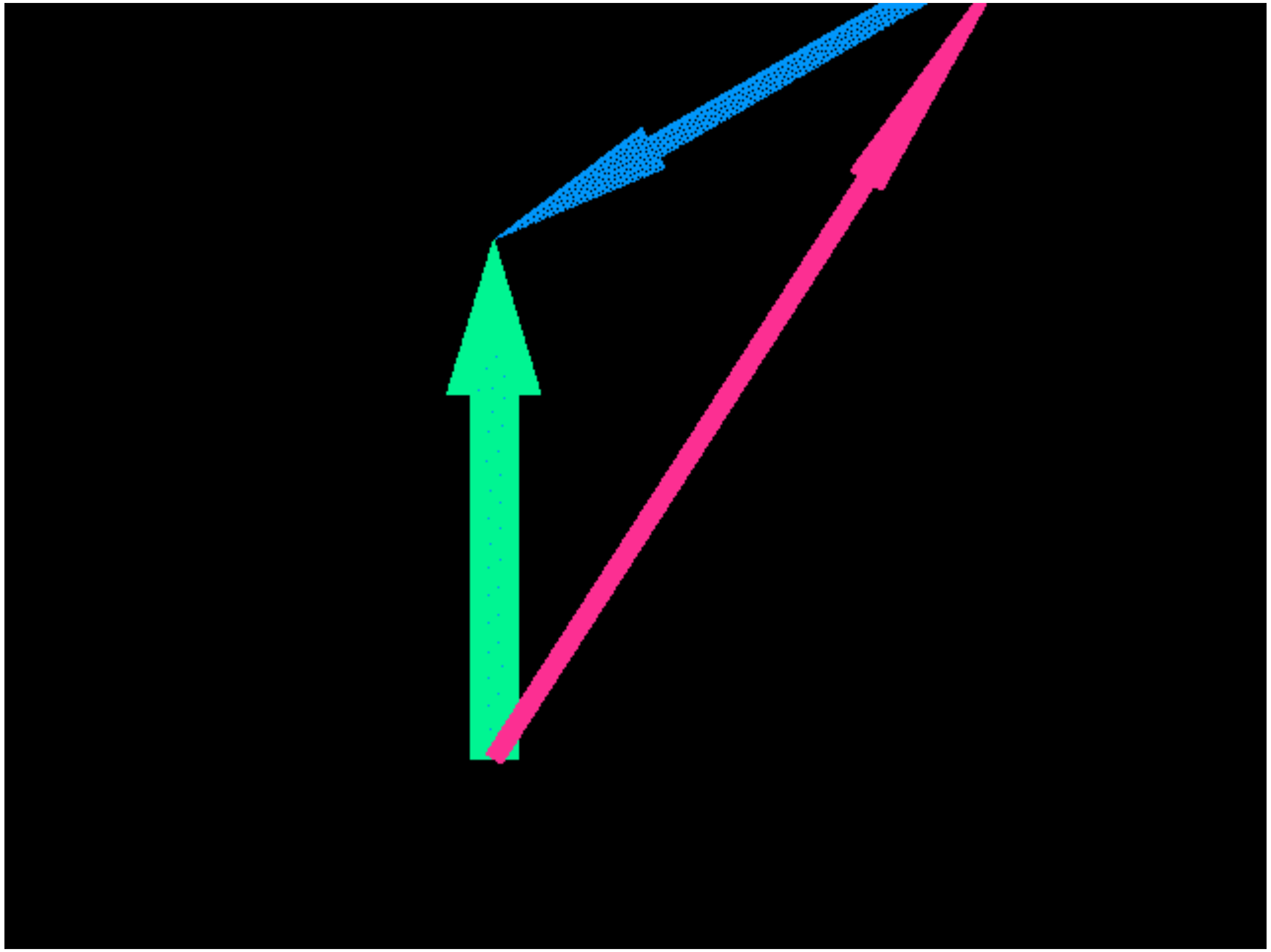


but if $\quad \cos\theta < -\dfrac{J_d}{2J_h} \quad$ *does* hold,

which requires $\theta > \pi/2$ and $J_d < 2J_h$,

then $J_t < J_h$, and

counteralignment occurs

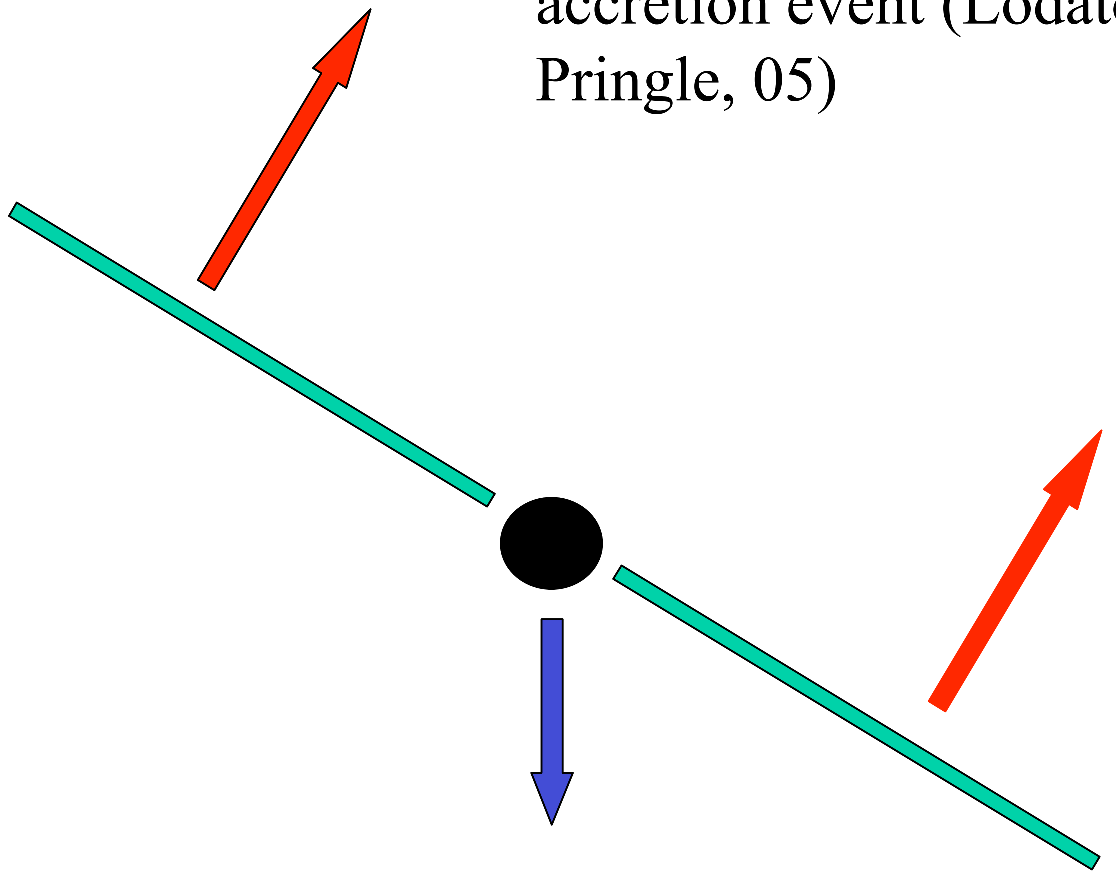


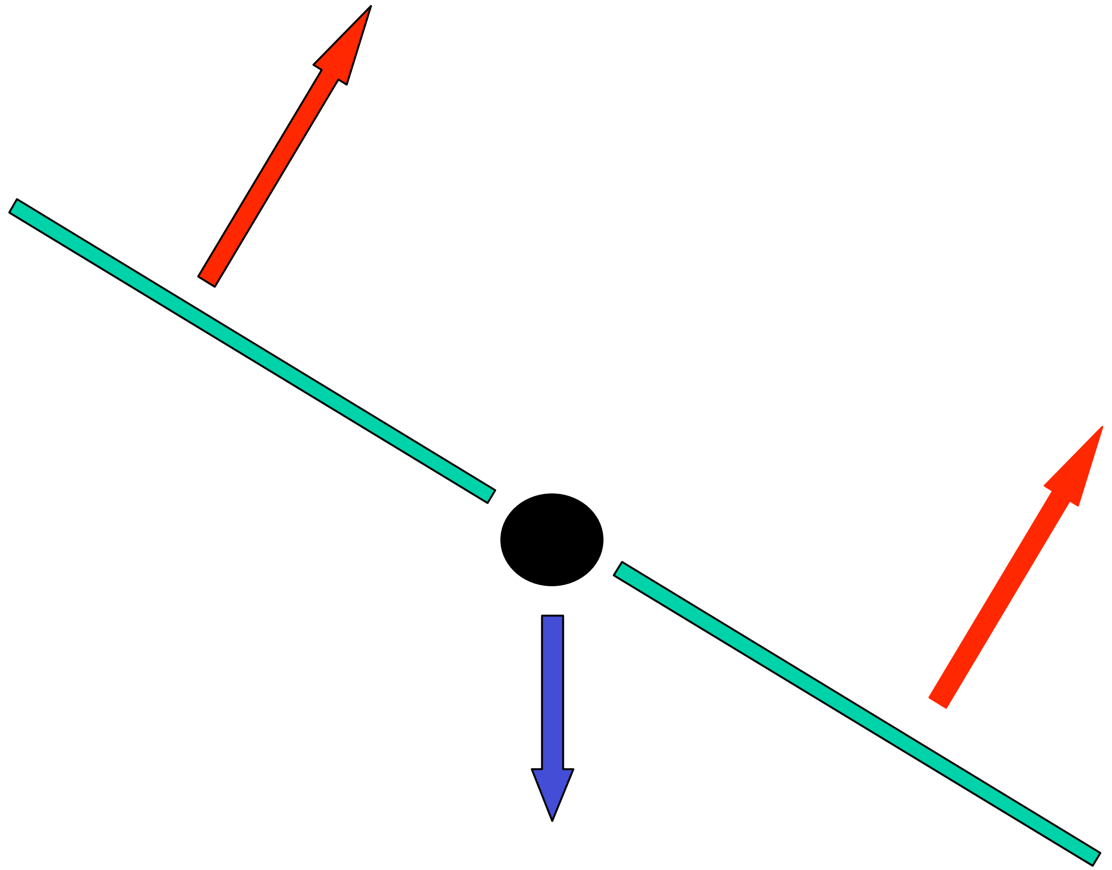
- small counterrotating discs anti—align

- large ones align

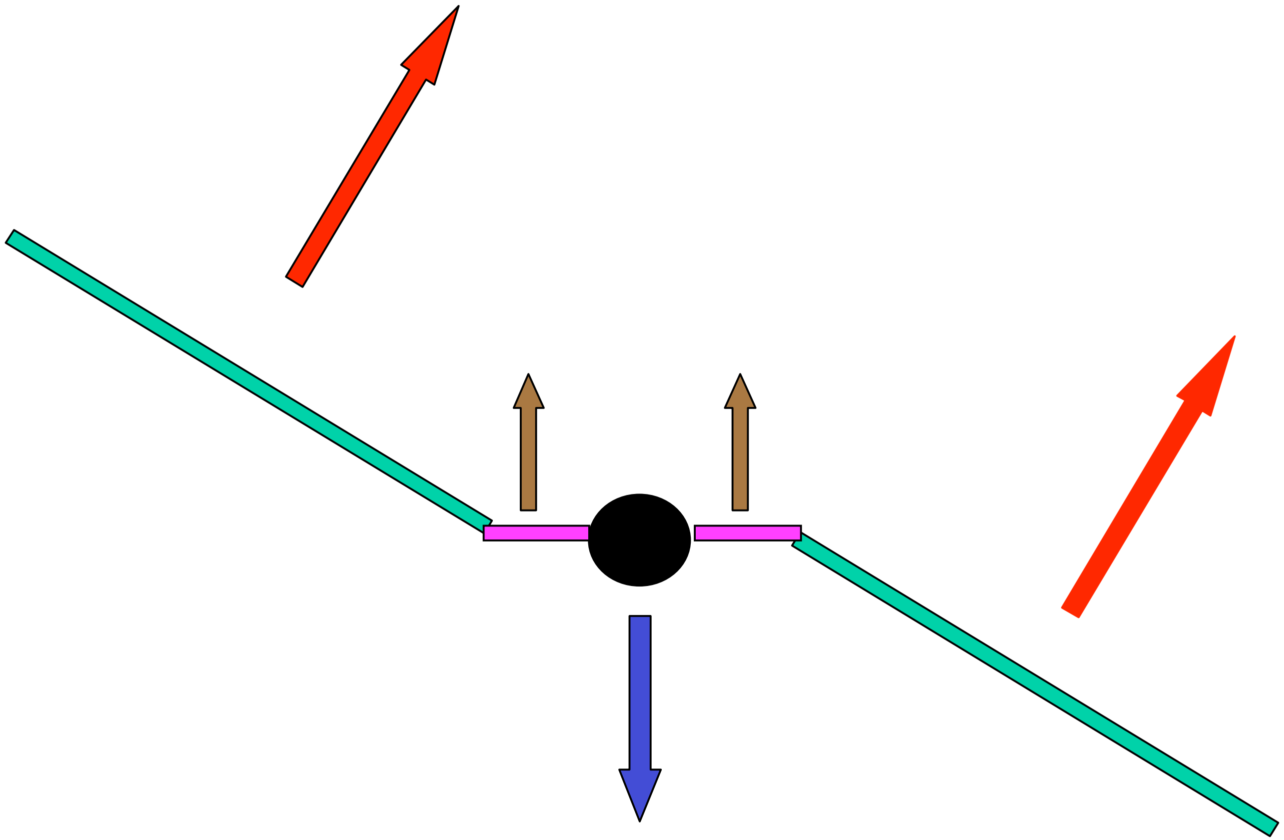
- what happens in general?

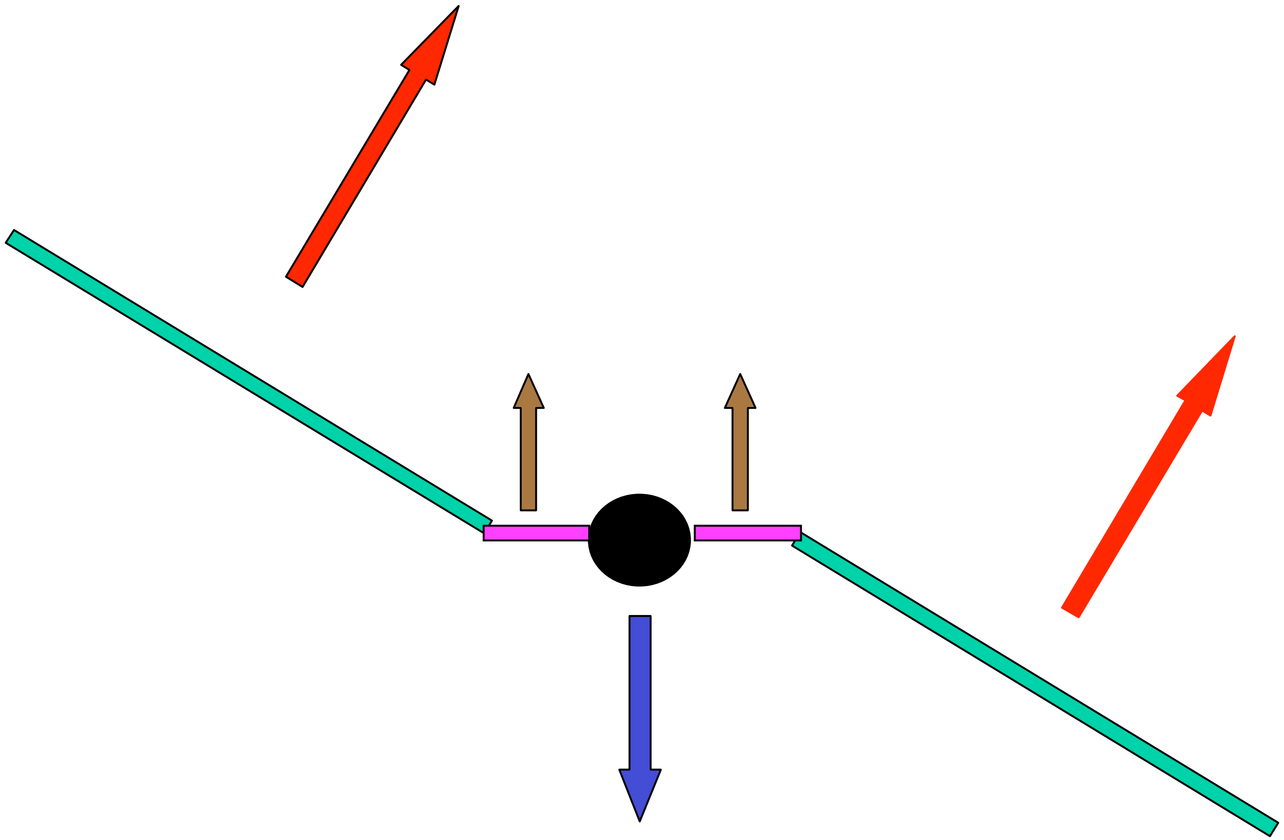
consider an initially counteraligned accretion event (Lodato & Pringle, 05)



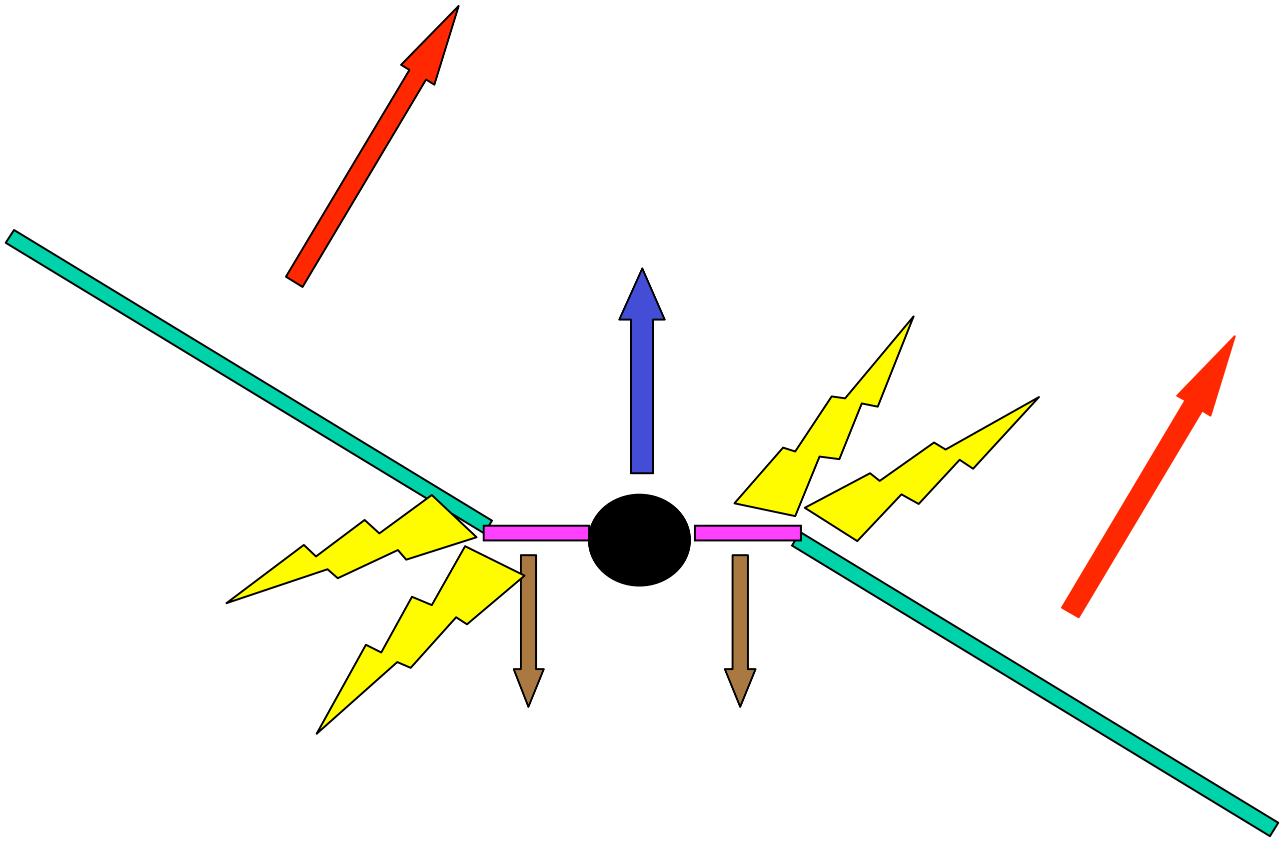


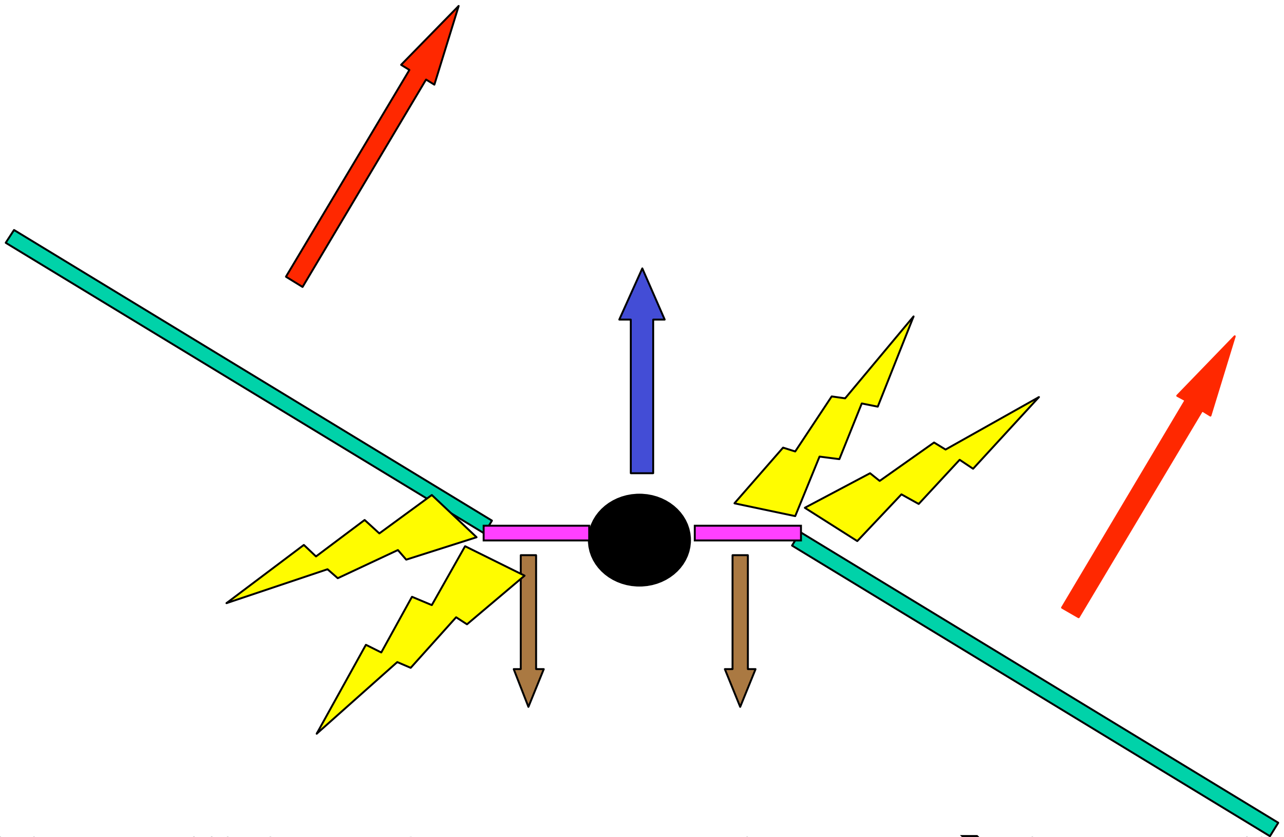
L—T effect first acts on *inner disc*: less a.m. than hole, so central disc *counteraligns*, connected to outer disc by warp: timescale $< 10^8$yr





but outer disc has more a.m. than hole, so forces it to *align*, ***taking counteraligned inner disc with it***





resulting collision of counterrotating gas ➔ intense dissipation ➔ high central accretion rate

accretion efficiency initially low (retrograde): *a/M* may be lower too

- *merger origin of AGN ➔ super—Eddington accretion ➔ outflows*

- these can explain

1. M—sigma
2. starbursts simultaneous with BH growth
3. BH—bulge mass correlation
4. matter accreting in AGN has high metallicity

*

black hole growth

can we grow masses $M > 5\times10^9 M_{sun}$

at redshifts z = 6 (Barth et al., 2003; Willott et al., 2003)?

must grow these masses in $\leq 10^9$ yr after Big Bang

and $\dot{M}$ is limited by Eddington, i.e.

$$\dot{M} = (1-\varepsilon)\,\dot{M}_{acc}$$

with

$$\varepsilon c^2 \dot{M}_{acc} \leq L_{Edd} = \frac{4\pi G M c}{\kappa}$$

these combine to give

$$\dot{M} < \frac{1-\varepsilon}{\varepsilon}\frac{M}{t_{Edd}}$$

with

$$t_{Edd} = \frac{\kappa c}{4\pi G} = 4.5\times 10^{8} \quad \mathrm{yr}$$

thus

$$\frac{M}{M_0} < \exp\left[\frac{1}{\varepsilon_{\min}} - 1\right]\frac{t}{t_{Edd}}$$

final mass exponentially sensitive to 1/efficiency thus $\varepsilon = 0.43$ (maximal a = 1) restricts growth to only

$$M / M_0 < 20 :$$

growing from $10 M_{sun}$ to $M > 5 \times 10^9 M_{sun}$ by z = 6 requires

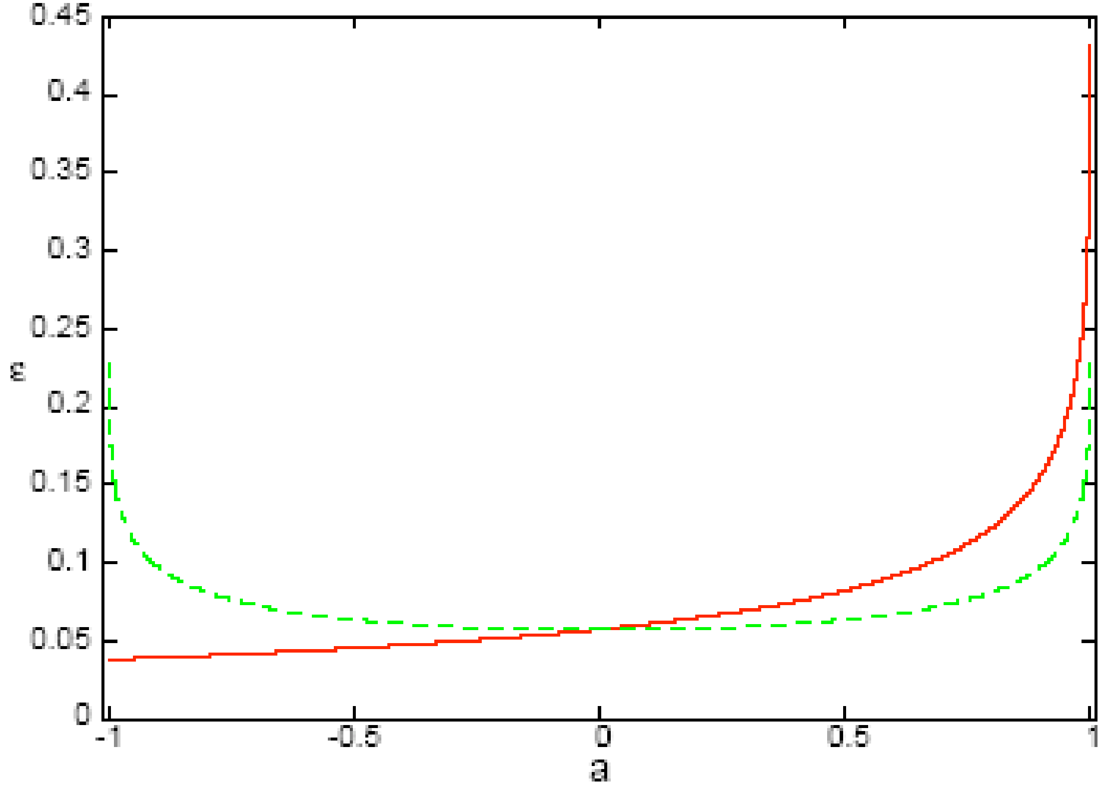
$$\varepsilon_{\min} < 0.08$$

i.e. $a < 0.5$ — even lower a is needed if BH does not accrete continuously at Eddington rate

rapid black—hole growth requires low spin

is this possible?

efficiency versus spin parameter



argument

- how fast can BH mass grow during mergers?
  can we account for masses $M \approx 5 \times 10^9 M_{sun}$ inferred at z = 6?

BH spin evolution during mergers?

nature of accretion flow during mergers?

clues from nearby AGN, Sgr A*

black hole spin

mergers imply accretion flows initially *counter*aligned in half of all cases—how does the BH spin evolve?

interaction of spin and mass evolution

usual argument (Volonteri & Rees, 2003; Volonteri et al, 2005):

LT effect ➔ rapid co—alignment of hole spin with disc,
so accretion of rest of disc always produces high spin, high ε
—difficult to grow large BH masses at high redshift

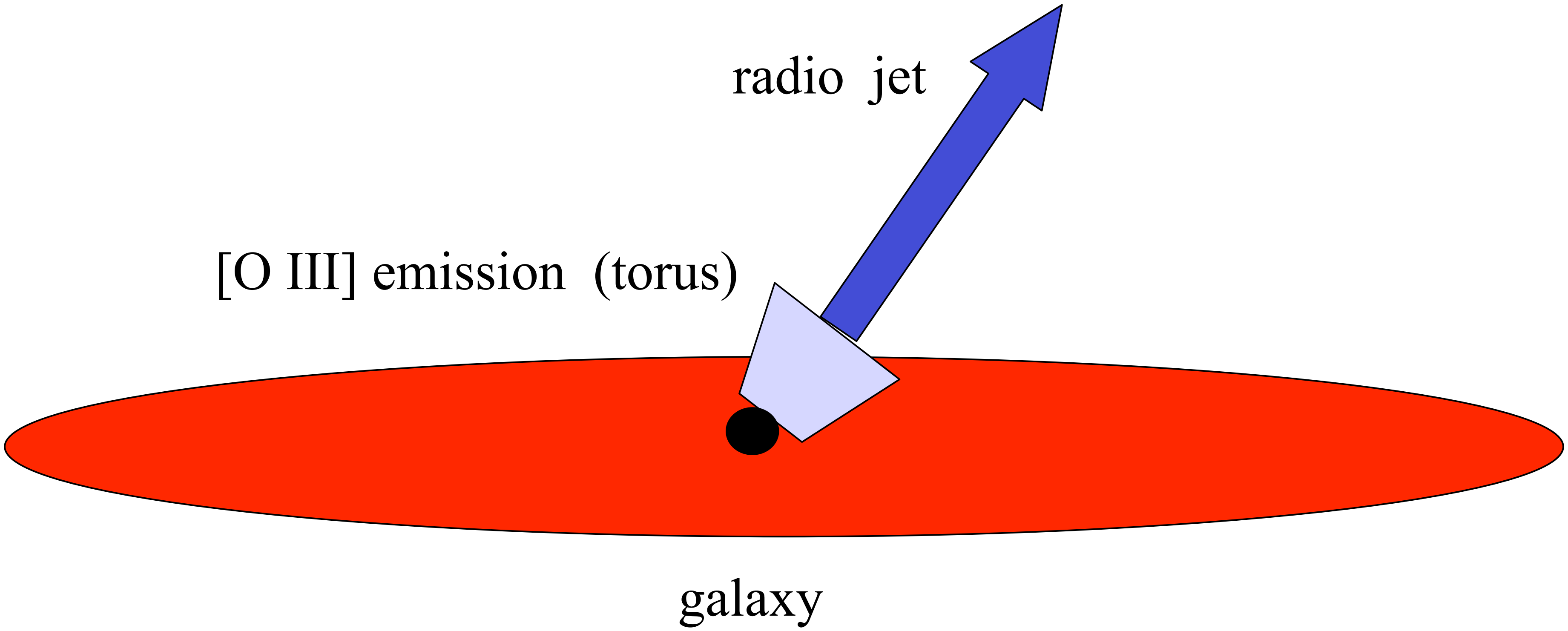
but KLOP result changes this: if $J_d < 2J_h$, counter—alignment can occur ➔ possible *spindown*

• spindown is more effective than spinup since ISCO is larger for retrograde accretion

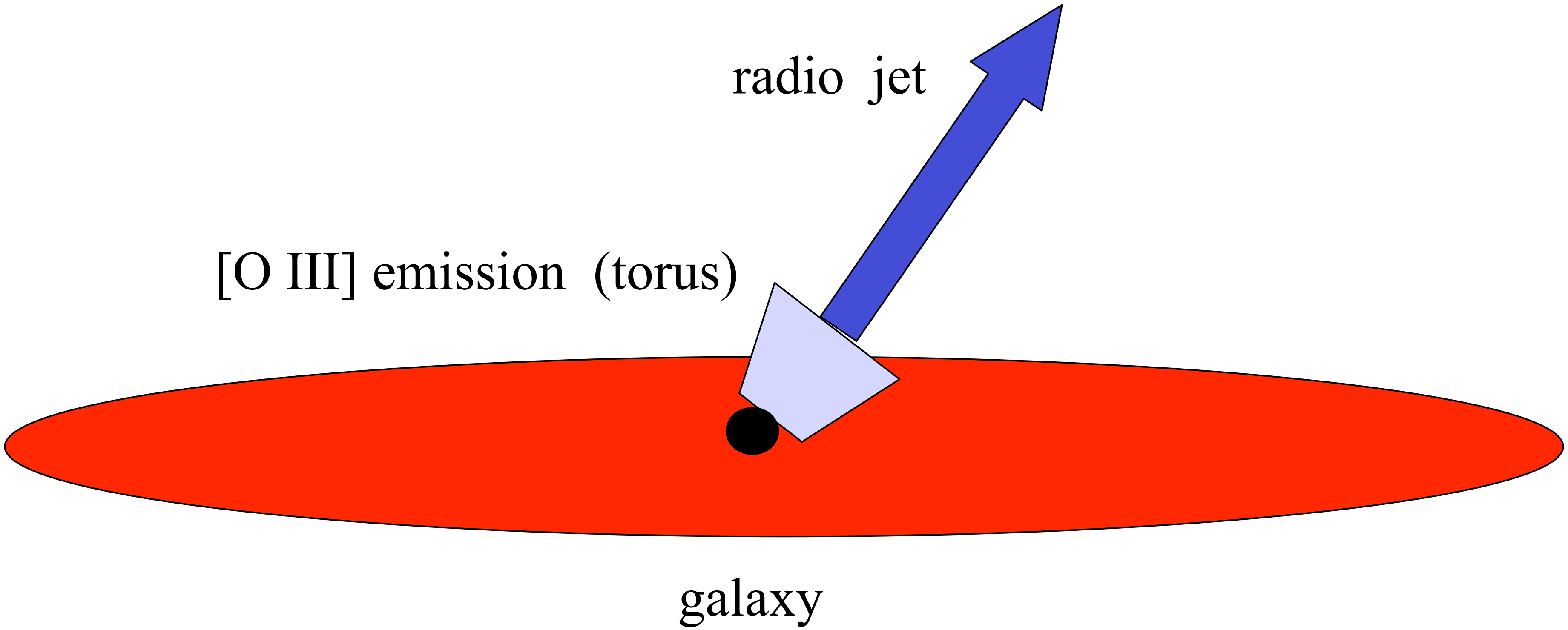
• low spin and thus rapid BH growth is possible provided accretion is *chaotic* (K & Pringle 2006) – does this happen?

what happens in nearby AGN?

orientations



jet and torus directions correlate with each other, but are *uncorrelated* with galaxy major axis at low redshift (Kinney et al., 2000; Nagar & Wilson, 1999; Schmitt et al, 2003) *orientations*



jet and torus directions correlate with each other, but are *uncorrelated* with galaxy major axis at low redshift (Kinney et al., 2000; Nagar & Wilson, 1999; Schmitt et al, 2003)

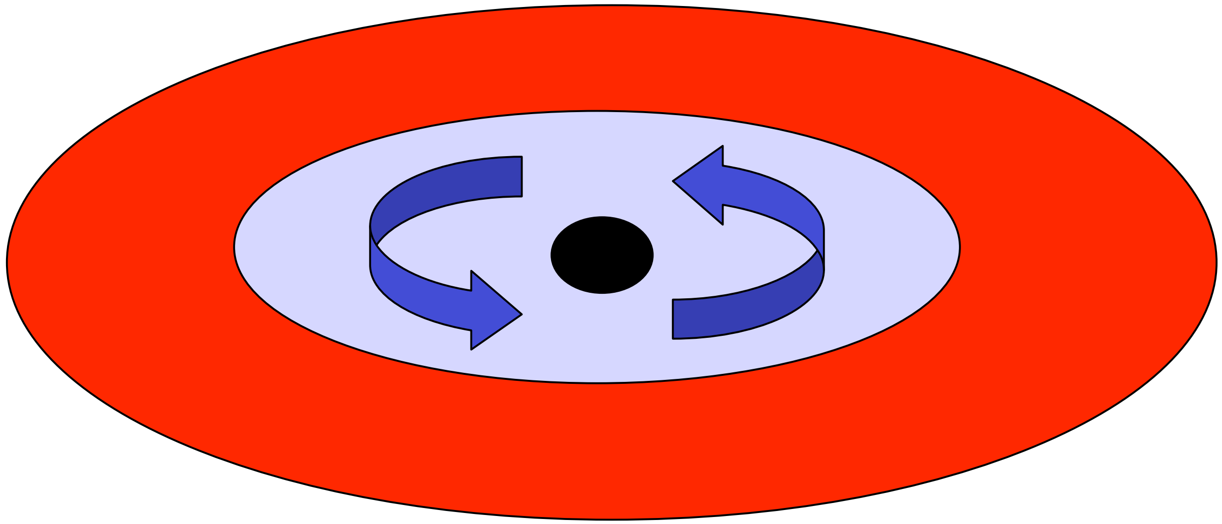
➔ central disc flow has angular momentum *unrelated* to host

• merger must deposit gas close to BH (~1 pc) if it is to accrete—
viscous time ~ Hubble at this distance:
requires accurate `shots' (cf Kendall, Magorrian & Pringle, 2003)

• without a randomizing mechanism, gas must come from *outside*
galaxy — cf cosmological simulations of structure formation

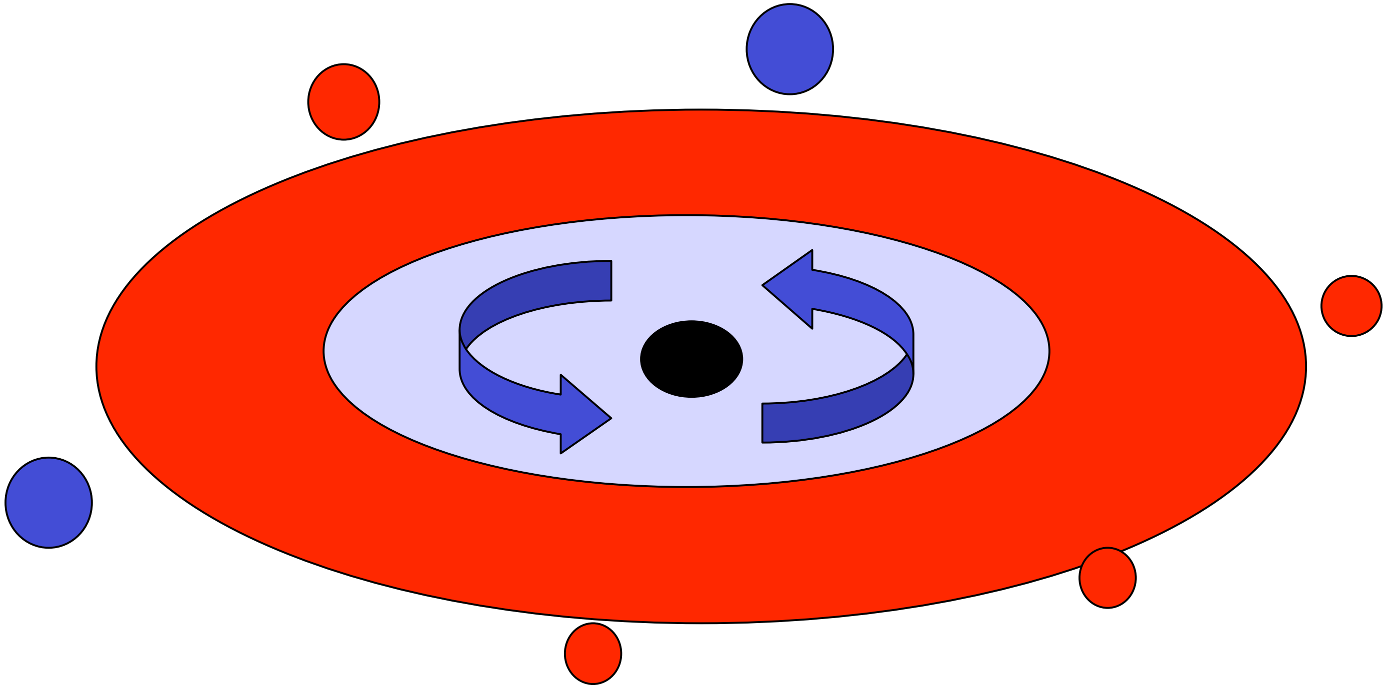
requirements are met if feeding is via

small—scale, randomly—oriented accretion events what happens in an event?



accreting matter forms ring and spreads into a disc
(few orbital times)

disc is self—gravitating outside some radius R_{sg} , accretes viscously within (K & Pringle, 2007a)



in this case we expect that

most of the gas outside R_{sg} *forms stars* *disc properties*

Shakura—Sunyaev eqns ➔ $\frac{H}{R} \approx 2\times10^{-3} \approx$ constant

➔self—gravity radius R_{sg} reached at disc mass

$$M_d \sim \frac{H}{R}M \approx 2\times10^{-3}M \approx 2\times10^{4}M_7\left(\frac{L}{L_E}\right)M_{sun}$$

disc properties (2)

R_{sg} defined by

$$M_d \sim \frac{H}{R} M = \int_0^{R_{sg}} 2\pi \Sigma R dR = \int_0^{R_{sg}} \frac{2\dot{M}R}{3\nu} dR \approx \frac{2}{3} \frac{R}{H} \frac{\dot{M}}{\alpha c_s} R_{sg}$$

$(\nu = \alpha c_s H)$, so

$$\boxed{R_{sg} \approx \frac{3}{2} \left(\frac{H}{R} \right)^2 \frac{M}{\dot{M}} \alpha c_s \sim 0.01 \frac{L}{L_E} pc}$$

otherwise independent of BH mass

disc properties (3)

disc evolution timescale is

$$\tau_{sg} \sim \frac{H}{R}\frac{M}{\dot{M}} \sim 10^5 \frac{L_E}{L} yr$$

detailed equations show that $\Sigma \propto \dot{M}^{3/5} R^{-3/5}$;

since $\dot{M} \propto \nu\Sigma$ we must have $\nu \propto \Sigma^{2/3} R$ for such discs:

then similarity solutions (Pringle, 1991) ➔ $L \sim t^{-19/16}$

at late times

thus luminosity evolution of individual events should follow

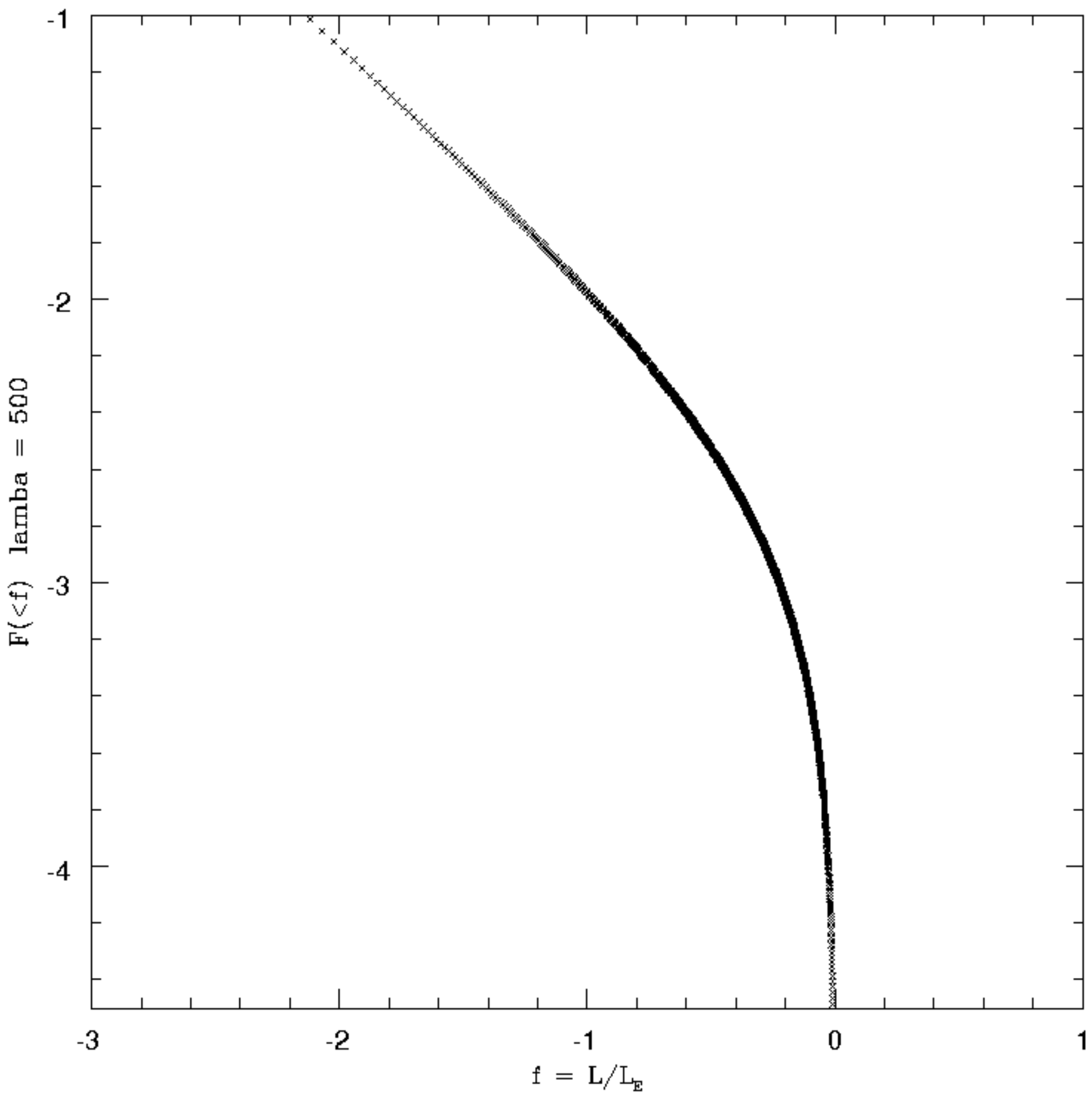
$$L = L_0[1 + (t/\tau_{sg})]^{-19/16}$$

for independent fuelling events starting at Eddington luminosity
this gives an AGN luminosity function

$$F(>L) = \frac{(L/L_E)^{-16/19} - 1}{(L_{end}/L_E)^{-16/19} - 1}$$

for typical BH masses $\sim 10^7 M_{sun}$ (cf Heckman et al, 2004):

typically ~100 events of duration $\sim 10^5\, yr$ recurring every $\sim 10^8\, yr$



-1
-2
-3
-4
F(<f) lamba = 500
-3
-2
-1
0
1
f = L/L_E

compare self—gravity radius $R_{sg} \sim 0.01 pc$

with inner edge ~ 0.03 pc of ring of young stars around Sgr A*

expect ring to be slightly larger as disc within R_{sg} must pass angular momentum outwards to stars

inner edge of current ring consistent with an event with $L \sim L_E$

at an epoch given by age of these stars

- suggest feeding of nearby AGN via small—scale accretion events uncorrelated with large—scale galaxy structure

- chaotic nature ➔ low BH spin ➔ radio jets aligned with obscuring torus, not with large—scale galaxy structure *accretion events in major mergers?*

cosmological simulations ➔ galaxy gains a large mass

$$\Delta M_{gal} \sim M_{gal}$$

in a major merger, while BH acquires only a mass

$$\Delta M_{merge} \sim M_{BH} \sim 10^{-3} M_{merge}$$

i.e. only a tiny part of the merging mass accretes on to the hole

moreover this fraction must have almost *zero angular momentum wrt the hole*

accretion is close to the Eddington limit, and star formation is vigorous ➔ feedback ➔ *chaotic flow near BH*

suggest that *flow is **episodic**, via a sequence of randomly—oriented accretion discs*, whose masses are limited by self—gravity, i.e.

*perhaps self—gravity limit on disc size hold at **all** redshifts*

(K, Pringle & Hofmann, 2007)

then $J_d < 2J_h$, so retrograde disc accretion tends to limit a

but as J_h decreases towards $J_d / 2$, probability

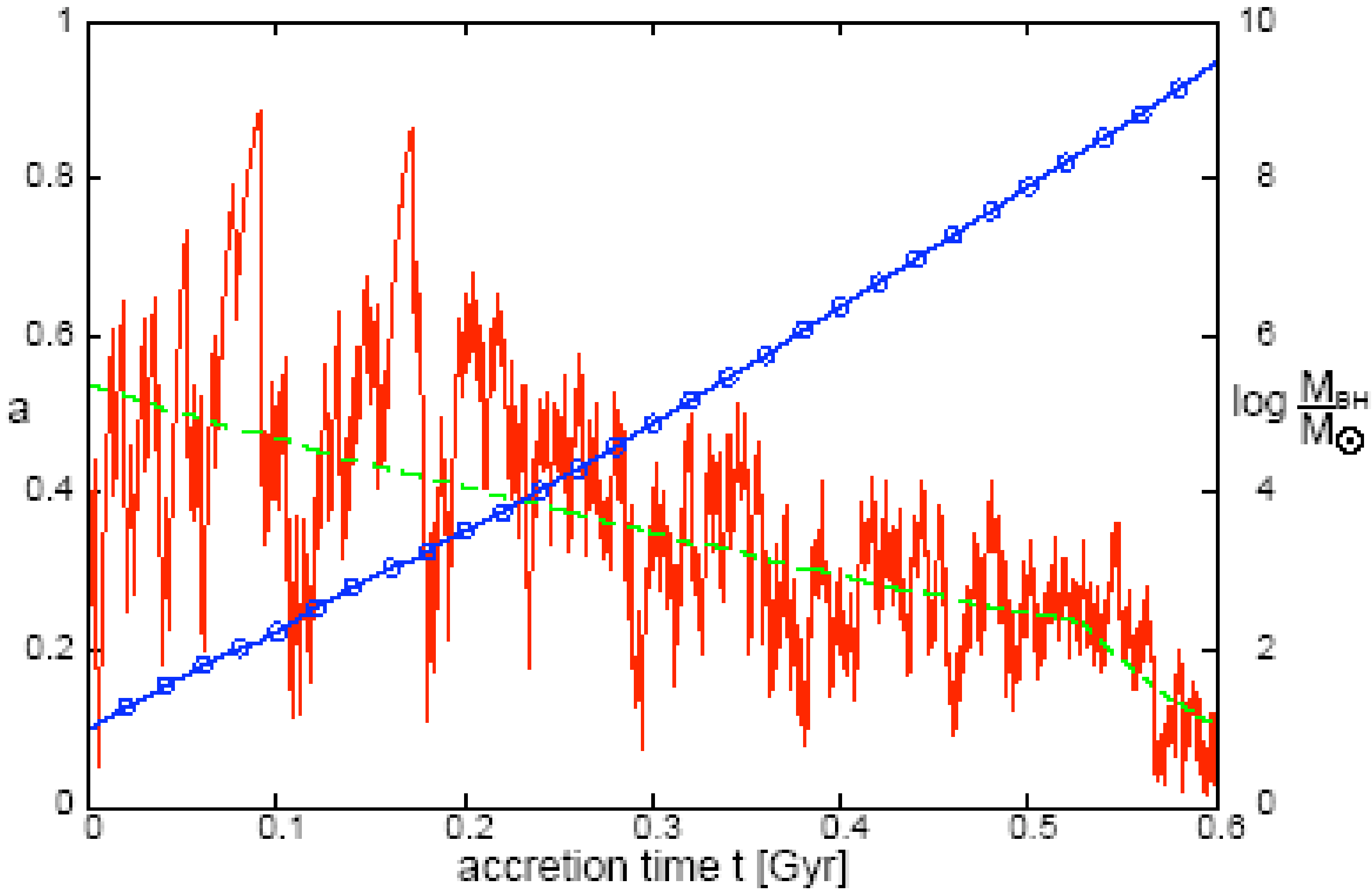
$$p_{counter} = \frac{1}{2}\left[1 - \frac{J_d}{2J_h}\right]$$

of counteraligning torque → 0, so spinup again: spin equilibrium is such that

$$p_{counter} \frac{da}{dM}_{spindown} = p_{co} \frac{da}{dM}_{spinup}$$

which gives nonzero a

Evolution of BH mass and spin



repeated random accretion events keep BH spin

low

$$(a < 0.2 - 0.3)$$

- large a must be unusual in SMBH:
  rare cases from prograde coalescence of SMBH of similar mass:
  probably in giant ellipticals

- growth to supermassive values even from stellar masses is possible at high redshift

- coalescences have no long—term effect on a: this converges to the mean very rapidly

- for *a* near its mean value ~ 0.2 – 0.3 we have

$$J_d / J_h \sim 0.2$$

thus *direction of hole spin (jets!) does not change much in an accretion episode* – cf `double—double' radio sources; but randomizes after a few episodes

- coalescence of two holes with mean *a* produces low GR recoil velocities < 200 km/s – *coalesced hole not ejected from host* *conclusions*

- super—Eddington accretion probably establishes M—sigma and M—M relations through momentum feedback

- SMBH spin remains fairly low unless accretion very well ordered, so

  SMBH can grow from stellar BH masses at high z
  coalesced holes are not ejected from host
  successive accretion episodes can produce jets in similar directions

- observations of nearby AGN, Sgr A*, may support this picture