Algebra I Assessments

performance tasks aligned to the Texas standards

The Charles A. Dana Center
at The University of Texas at Austin

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About the development of this book

The Charles A. Dana Center has developed this standards-aligned mathematics education resource for mathematics teachers.

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This second edition updates the Texas Essential Knowledge and Skills statements and alignment charts to align with the state's 2005–06 revisions to the Mathematics TEKS. The 2009 revision of the Mathematics TEKS did not make any revisions to the Algebra I TEKS. The assessments have also been re-edited for clarity and to correct some minor errors.

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center works to raise student achievement in K–16 mathematics and science, especially for historically underserved populations. We do so by providing direct service to school districts and institutions of higher education; to local, state, and national education leaders; and to agencies, nonprofits, and professional organizations concerned with strengthening American mathematics and science education.

The Center was founded in 1991 in the College of Natural Sciences at The University of Texas at Austin. Our original purpose—which continues in our work today—was to increase the diversity of students who successfully pursue careers in science, technology, engineering, and mathematics (STEM) fields.

We carry out our work by supporting high standards and building system capacity; developing partnerships with key state and national organizations; creating and delivering professional supports for educators and education leaders; and writing and publishing education resources, including student supports.

Our staff of more than 60 researchers and education professionals has worked intensively with dozens of school systems in nearly 20 states and with 90 percent of Texas's more than 1,000 school districts. As one of the College's largest research units, the Dana Center works to further the university's mission of achieving excellence in education, research, and public service. We are committed to ensuring that the accident of where a child attends school does not limit the academic opportunities he or she can pursue.

For more information about the Dana Center and our programs and resources, see our homepage at www.utdanacenter.org. To access our resources (many of them free), please see www.utdanacenter.org/products. See our professional development site at **www.utdanacenter.org/pd** for descriptions and online sign-up for our professional development offerings.

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TEKS and TAKS Resources

The mathematics Texas Essential Knowledge and Skills (TEKS) were developed by the state of Texas to clarify what all students should know and be able to do in mathematics in kindergarten through grade 12. The mathematics TEKS also form the objectives and student expectations for the mathematics portion of the Texas Assessment of Knowledge and Skills (TAKS) for grades 3 through 10 and for the grade 11 exit test, as well as the End-of-Course Exams for Algebra I, Geometry, and Algebra II.

Texas school districts are required to provide instruction that is aligned with the mathematics TEKS, which were originally adopted by the State Board of Education in 1997 and implemented statewide in 1998. The first revision of the mathematics TEKS was adopted by the State Board of Education in 2005–06 and implemented statewide in 2006–07. This first revision of the mathematics TEKS started being assessed on the TAKS beginning in 2007–08.

The second revision of the mathematics TEKS was adopted by the State Board of Education in 2009 and is to be implemented statewide in 2009–2010. This latest revision aligned the existing mathematics TEKS to the Texas College and Career Readiness Standards (www.thecb.state.tx.us/collegereadiness/CCRS.pdf) adopted by the Texas Higher Education Coordinating Board in January 2008. In this 2009 revision of the mathematics TEKS, there were no changes to the TEKS for Algebra I.

The TEKS for mathematics, as well as for other subject areas, can be downloaded in printable format, free of charge, through the Texas Education Agency website, **www.tea.state.tx.us/teks**. The mathematics TEKS can also be downloaded free from the Mathematics TEKS Toolkit, **www.mathtekstoolkit.org**, a resource of the Charles A. Dana Center at The University of Texas at Austin. Bound versions of the mathematics, science, and English Language Arts TEKS can be ordered for a fee (to cover the costs of production) from the Dana Center product catalog at **www.utdanacenter.org/catalog** or by contacting the Dana Center at **1-866-871-9995**.

The Dana Center also provides resources, including professional development, for implementing the mathematics TEKS. Many resources can be found in the Mathematics TEKS Toolkit at www.mathtekstoolkit.org. See **www.utdanacenter.org/pd** for descriptions—and to sign up online—for our professional development offerings for mathematics educators, as well as science educators and leaders. You may also find out about professional development opportunities by calling **512-471-6190**.

For more information about the Dana Center and our programs and resources, see our homepage at **www.utdanacenter.org**.

The following products and services are also available from the Dana Center at www.utdanacenter.org/catalog:

- *Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Kindergarten–Grade 2* (first edition, 2008)
- *Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Grades 3–5* (second edition, 2008)
- *Mathematics Standards in the Classroom: Classroom Activities Aligned to the Texas Standards for Grades 6–8* (second edition, 2008)
- *Algebra II Assessments: Performance Tasks Aligned to the Texas Standards* (second edition, 2007)
- *Geometry Assessments: Performance Tasks Aligned to the Texas Standards* (second edition, 2007)
- Poster-sized Mathematics TEKS charts
- Professional development mathematics sessions for elementary, middle school, and high school teachers

CONTENTS

Introduction

The importance of mathematics assessment

The Dana Center developed *Algebra I Assessments* as a resource for teachers to use to provide ongoing assessment integrated with algebra instruction.

The National Council of Teachers of Mathematics lists as one of its six principles for school mathematics that "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students."[1]

Further, NCTM[2] identified the following six standards to guide classroom assessment:

The Mathematics Standard:	Assessment should reflect the mathematics that all students need to know and be able to do.
The Learning Standard:	Assessment should enhance mathematics learning.
The Equity Standard:	Assessment should promote equity.
The Openness Standard:	Assessment should be an open process.
The Inferences Standard:	Assessment should promote valid inferences about mathematics learning.
The Coherence Standard:	Assessment should be a coherent process.

What are the *Algebra I Assessments*?

Teachers can use these Algebra I performance tasks to provide ongoing assessment integrated with Algebra I instruction. The assessments, which reflect what all students need to know and be able to do in a high school Algebra I course, may be formative, summative, or ongoing. The assessments focus on students' understanding of concepts and their procedural knowledge, instead of on "right" or "wrong" answers.

The purpose of the assessments is to

- clarify for teachers, students, and parents what is being taught and learned in Algebra I,
- help teachers gain evidence of student insight and misconceptions to serve as a basis for instructional decisions, and
- provide problem-solving strategies to guide instruction and enhance student learning.

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[1]National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics. (Summary available at standards.nctm.org.)

[2]National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, pages 11, 13, 15, 17, 19, and 21.

What's new?

Updated performance tasks

Each of the performance tasks has been updated to reflect our new knowledge about assessing student learning.

Revised secondary mathematics TEKS

We have updated Algebra I Assessments to align with the state's 2005–06 revisions to the Mathematics TEKS and to adjust performance tasks to meet these revised TEKS where necessary. (The 2009 revision of the Mathematics TEKS did not make any revisions to the Algebra I TEKS.)

How are the assessments structured?

Teachers may use these assessments formatively or summatively, for individual students or for groups of students. Each problem:

- Includes an algebra task;
- Is aligned with the Algebra I Texas Essential Knowledge and Skills (TEKS) Student Expectations;
- Is aligned with the Texas Assessment of Knowledge and Skills (TAKS) objectives;
- Includes "scaffolding" questions that the teacher may use to help the student analyze the problem;
- Provides a sample solution*; and
- Includes extension questions to bring out additional mathematical concepts in a summative discussion of solutions to the problem.

* The sample solution is only one way that a problem may be approached. There are other approaches that may provide a correct analysis of the problem. The authors have attempted to illustrate a variety of methods in the different problem solutions. To illustrate another approach, a sample student solution is included with some problems.

Algebra I Assessments presents performance tasks and lessons from five categories that are based on key strands in the Algebra I TEKS:

1. Function Fundamentals
2. Linear Functions, Equations, and Inequalities
3. Interacting Linear Functions, Linear Systems
4. Quadratic Functions
5. Inverse Variations, Exponential Functions, and Other Functions

What is the solution guide?

The solution guide on the next page is a one-page problem-solving checklist that teachers may use to track what is necessary for a student to give a complete solution for a given problem. Because students need to know what criteria they are expected to meet in their solution, when assigning the problem, the teacher can give students copies of a solution guide customized with marks indicating which of the criteria should be considered in the problem analysis. For most problems, all the criteria will be important, but initially the teacher may want to focus on only two or three criteria.

Following the solution guide are alignments of all the problems to the TEKS and to the Grade 9 TAKS.

Algebra I Solution Guide

Mark the criteria to be considered in the solution of a particular problem, and then check the criteria that are satisfied by the solution.

Criteria to be considered	Criteria	Criteria satisfied
	Describes functional relationships	
	Defines variables appropriately using correct units	
	Interprets functional relationships correctly	
	Uses multiple representations (such as tables, graphs, symbols, verbal descriptions, concrete models) and makes connections among them	
	Demonstrates algebra concepts, processes, and skills	
	Interprets the reasonableness of answers in the context of the problem	
	Communicates a clear, detailed, and organized solution strategy	
	States a clear and accurate solution using correct units	
	Uses correct terminology and notation	
	Uses appropriate tools	

TAKS and TEKS Alignment to Assessment

Objective 1:
The student will describe functional relationships in a variety of ways.

TEKS	Assessment	Page
A.1A	Bathing the Dog	21
	Making Stuffed Animals	37
A.1B	Painted Cubes	47
	The Shuttle's Glide	65
	Stretched Spring	105
A.1C	Making Stuffed Animals	37
	The Shuttle's Glide	65
	Sound Travel	151
	The Contractor	161

TEKS	Assessment	Page
A.1D	Painted Cubes	47
	The Contractor	161
A.1E	Bathing the Dog	21
	Making Stuffed Animals	37
	The Contractor	161
	Calculating Cost	289
	Painted Cubes	47

Objective 2:
The student will demonstrate an understanding of the properties and attributes of functions.

TEKS	Assessment	Page
A.2A	Investigating Parameters	293
A.2B	Extracurricular Activities	31
	T-Shirts	85
	Motion Detector	189
A.2C	The 600-Meter Race	3
	Distance and Time	25
	Create a Situation	119
	Gas Tank	123
	The Garden	165
	Cost and Profit	183
	Motion Detector	189
	Four Cars	213

TEKS	Assessment	Page
A.2D	Stretched Spring	105
A.3A	Mosaics	7
	Paper Boxes	395
A.3B	Mosaics	7
	Paper Boxes	395
A.4A	A Ring Around the Posies	275
	Block That Kick!	279
	Grocery Carts	133
A.4B	A Ring Around the Posies	275
	Brrr!	285
	Ostrich Pens	307
A.4C	Investigating Parameters	293

Objective 3:
The student will demonstrate an understanding of linear functions.

TEKS	Assessment	Page
A.5A	Which Is Linear?	91
A.5B	CDs for the Band	61
	T-Shirts	85
A.5C	T-Shirts	85
	Which Is Linear?	91
	Finding Pairs	95
	Analysis of a Function	111
A.6A	Making Pizzas, Making Money	75
	Hull Pressure	139
	Speeding Cars	201
	Summer Money	223
	Which Plan Is Best?	243
A.6B	Hot Air Balloon	69
	Making Pizzas, Making Money	75
	Gas Tank	123
	Math-a-Thon	143
	The Submarine	169
	Speeding Cars	201
	Four Cars	213
	Summer Money	223
	Which Plan Is Best?	243

TEKS	Assessment	Page
A.6C	Hot Air Balloon	69
	The Submarine	169
	Graph It	219
	Summer Money	223
A.6D	Speeding Cars	201
	Graph It	219
	Summer Money	223
A.6E	Hot Air Balloon	69
	Summer Money	223
A.6F	Swimming Pools	17
	Hot Air Balloon	69
	Making Pizzas, Making Money	75
	Math-a-Thon	143
	The Submarine	169
	Summer Money	223
	Which Plan Is Best?	243
A.6G	Nested Rectangles	41
	Hull Pressure	139
	Speeding Cars	201

Objective 4:
The student will formulate and use linear equations and inequalities.

TEKS	Assessment	Page
A.7A	CDs for the Band	61
	Draining Pools	79
	First Aid Supplies	101
	Shopping	147
	Taxi Ride	155
	Greetings	127
A.7B	Making Pizzas, Making Money	75
	Draining Pools	79
	First Aid Supplies	101
	Taxi Ride	155
	Greetings	127
A.7C	First Aid Supplies	101
	Taxi Ride	155

TEKS	Assessment	Page
A.8A	Bears' Band Booster Club	177
	Cost and Profit	183
	The Walk	195
	Chemistry Dilemma	207
	Exercise Pens	231
	The Run	237
A.8B	Bears' Band Booster Club	177
	The Walk	195
	Chemistry Dilemma	207
	Exercise Pens	231
	The Run	237
A.8C	Bears' Band Booster Club	177
	Chemistry Dilemma	207
	Exercise Pens	231
	The Run	237
	The Walk	195

Objective 5:
The student will demonstrate an understanding of quadratic and other non-linear functions.

TEKS	Assessment	Page
A.9A	Fireworks Celebration	249
	Golfing	257
	Ostrich Pens	307
	Mathematical Domains and Ranges of Nonlinear Functions	385
	Supply and Demand	317
	What Is Reasonable?	401
A.9B	Insects in the Water	269
	Transformations of Quadratic Functions	331
A.9C	Fireworks Celebration	249
	Golfing	257
	Transformations of Quadratic Functions	331
A.9D	Fireworks Celebration	249
	Golfing	257
	Insects in the Water	269
	Block That Kick!	279
	Calculating Cost	289
	Ostrich Pens	307
	Supply and Demand	317
	Transformations of Quadratic Functions	331

TEKS	Assessment	Page
A.10A	Home Improvements	261
	How Much Paint?	265
	A Ring Around the Poises	275
	Sky Diving	313
	Supply and Demand	317
	Dog Run	325
	Window Panes	347
	Calculating Cost	289
A.10B	Home Improvements	261
	How Much Paint?	265
	Supply and Demand	317
	Dog Run	325
A.11A	Exploring Exponential Functions	359
	The Marvel of Medicine	379
	What Is Reasonable?	401
A.11B	Constructing Houses	371
	Music and Mathematics	375
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A.11C	College Tuition	353
	Exploring Exponential Functions	359
	Bright Lights	365
	The Marvel of Medicine	379

Assessment Alignment to TEKS and TAKS

Chapter 1

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
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Mosaics	7	A.3A, A.3B	A.1C, A.1D, A.5C, A.7A	2
Swimming Pools	17	A.6F	A.1A, A.1D, A.1E, A.5A, A.5C	3
Bathing the Dog	21	A.1A, A.1E	A.5A	1
Distance and Time	25	A.2C	A.1A, A.6B	2
Extracurricular Activities	31	A.2B	A.1C, A.5B, A.5C	2
Making Stuffed Animals	37	A.1A, A.1C, A.1E	A.3A	3
Nested Rectangles	41	A.6G	A.1E, A.3A, A.5A, A.5C	3
Painted Cubes	47	A.1B, A.1D, A.1E	A.1A, A.2B	2

Chapter 2

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
CDs for the Band	61	A.5B, A.7A	A.4B, A.7B, A.7C	3, 4
The Shuttle's Glide	65	A.1B, A.1C	A.1D, A.1E, A.3A, A.3B	1
Hot Air Balloon	69	A.6B, A.6C, A.6E, A.6F	A.7A, A.7B	3
Making Pizzas, Making Money	75	A.6A, A.6B, A.6F, A.7B	A.1C, A.1D	3,4
Draining Pools	79	A.7A, A.7B	A.3A, A.3B	4
T-Shirts	85	A.2B, A.5B, A.5C	A.1D	2, 3
Which Is Linear?	91	A.5A, A.5C	A.3A, A.6A, A.6B	3
Finding Pairs	95	A.5C	A.1E	3
First Aid Supplies	101	A.7A, A.7B, A.7C	A.1C, A.3A	4
Stretched Spring	105	A.1B, A.2D	A.1C, A.1E, A.7A	1, 2
Analysis of a Function	111	A.5C	A.1D, A.6E, A.2B	2, 3
Create a Situation	119	A.2C	A.1D	2
Gas Tank	123	A.2C, A.6B	A.1D, A.1E, A.5C	2, 3
Greetings	127	A.7A, A.7B	A.1C, A.3A	4
Grocery Carts	133	A.5C, A.7A	A.4A	3, 4

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
Hull Pressure	139	A.6A, A.6G	A.1B, A.1C, A.1E, A.2B	3
Math-a-Thon	143	A.6B, A.6F	A.1C, A.3A, A.5C	3
Shopping	147	A.7A	A.1D, A.3A, A.4A, A.7B	4
Sound Travel	151	A.1C	A.3A, A.5B, A.7A, A.7B	1
Taxi Ride	155	A.7A, A.7B, A.7C	A.1C, A.3A, A.3B, A.4A, A.5B, A.5C	4
The Contractor	161	A.1C, A.1D, A.1E	A.3A, A.3B, A.4A, A.5C	4
The Garden	165	A.2C	A.5B, A.5C	2
The Submarine	169	A.6B, A.6C, A.6F	A.1C, A.1D, A.1E, A.5C, A.7A, A.7B	3

Chapter 3

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
Bears' Band Booster Club	177	A.8A, A.8B, A.8C	A.1A, A.1C, A.1D, A.1E	4
Cost and Profit	183	A.2C, A.8A	A.5C, A.6A, A.6B	2, 4
Motion Detector	189	A.2B, A.2C	A.1A, A.1C, A.1D, A.1E	2
The Walk	195	A.8A, A.8B, A.8C	A.2B, A.6A, A.6B, A.6D	4
Speeding Cars	201	A.6A, A.6B, A.6D, A.6G, A.5A	A.1B, A.3A, A.3B	3
Chemistry Dilemma	207	A.8A, A.8B, A.8C	A.3B	4
Four Cars	213	A.6B, A.2C	A.5C, A.8C	2, 3
Graph It	219	A.6C, A.6D	A.3A, A.5C	3
Summer Money	223	A.6A, A.6B, A.6C, A.6D, A.6E, A.6F	A.1E, A.8C	3
Exercise Pens	231	A.8A, A.8B, A.8C	A.3A, A.3B, A.4A	4
The Run	237	A.8A, A.8B, A.8C	A.5C, A.6A, A.6B, A.6D, A.6E	4
Which Plan Is Best?	243	A.6A, A.6B, A.6F	A.5C	3

Chapter 4

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
Fireworks Celebration	249	A.9A, A.9C, A.9D	A.1D, A.10A, A.10B	5
Golfing	257	A.9A, A.9C, A.9D	A.10A, A.10B	5
Home Improvements	261	A.10A, A.10B	A.1C, A.1D, A.1E, A.4A, A.9D	5
How Much Paint?	265	A.10A, A.10B	A.4A, A.9D	5
Insects in the Water	269	A.9B, A.9D	A.4A, A.10A, A.10B	5
A Ring Around the Poises	275	A.4A, A.4B, A.10A	A.2B	2, 5
Block That Kick!	279	A.4A, A.9D	A.1C, A.10A	1, 5
Brrr!	285	A.4B, A.10A	A.9A, A.9D	1, 5
Calculating Cost	289	A.9D, A.10A, A.1E	A.1D	1, 5
Investigating Parameters	293	A.2A, A.4C, A.9B, A.9C	A.1D	5
Ostrich Pens	307	A.4B, A.9A, A.9D	A.1C, A.1D, A.1E	5
Sky Diving	313	A.10A	A.9A, A.9D	2, 5
Supply and Demand	317	A.9A, A.9D, A.10A, A.10B	A.1B	5
Dog Run	325	A.10A, A.10B	A.1E, A.3A, A.3B	5
Transformations of Quadratic Functions	331	A.9B, A.9C, A.9D	A.2A	2, 5
What Is the Best Price?	341	A.10A, A.10B	A.2A, A.2D, A.9D	5
Window Panes	347	A.10A	A.10B	5

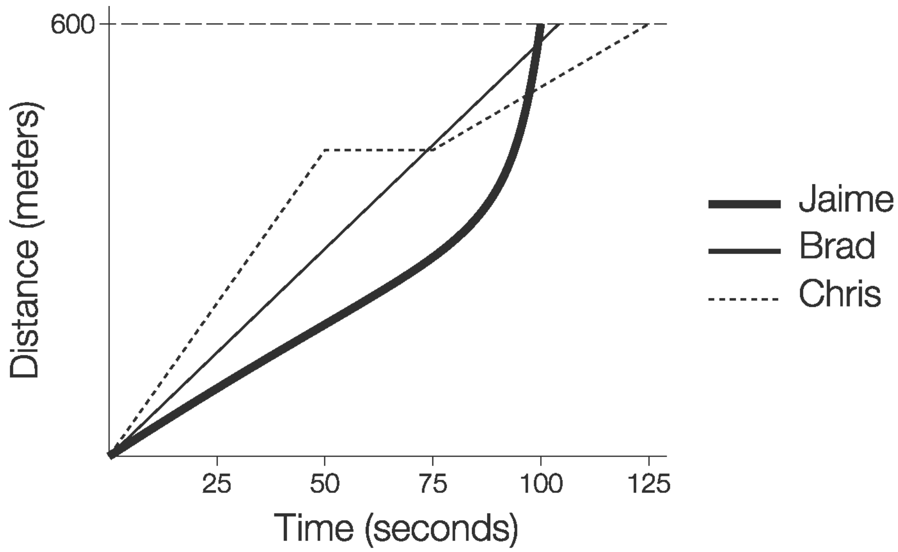
Chapter 5

Assessment	Page	Algebra TEKS Focus	Additional Algebra TEKS	TAKS Objectives
College Tuition	353	A.11C	A.1C, A.3A, A.4A	5
Exploring Exponential Functions	359	A.11A, A.11C	A.2B, A.4A	5
Bright Lights	365	A.11C		5
Constructing Houses	371	A.11B	A.2B, A.4A	5
Music and Mathematics	375	A.11B	A.2D, A.4A	5
The Marvel of Medicine	379	A.11A, A.11C	A.1D, A.2B	5
Mathematical Domains and Ranges of Nonlinear Functions	385	A.9A	A.2B, A.2C	5
Paper Boxes	395	A.3A, A.3B. A.4A, A.4B	A.1D	2
What Is Reasonable?	401	A.9A, A.11A, A.11B	A.1C	5

Chapter 1:
Function Fundamentals

The 600-Meter Race

The graph below describes what happens when three athletes—Jaime, Brad, and Chris—enter a 600-meter foot race.



Based on the graph, give a detailed interpretation of each athlete's experience of the race. Include sprints, slowdowns, and speed throughout the race as well as estimates of time and distance.

Notes

Materials:

None required

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- What is the runners' starting point on the graph? Describe its meaning.
- Can the race be broken up into time intervals? What would these intervals be?
- What does the graph show you about each runner? Who leads when? How do you know?
- How do their speeds compare? How do you know?
- Where are the runners at the end of each time interval?
- Who finishes first? Last? How do you know?
- What can you say about the runners' times?

Sample Solutions

Based on the graph, give a detailed interpretation of each athlete's experience of the race. Include sprints, slowdowns, and speed throughout the race as well as estimates of time and distance.

One approach to this problem is to break down the race by time. We know that the x-axis represents time in seconds and the y-axis distance in meters. The graph indicates that the race has four time intervals; the estimated time intervals are [0, 50], [50, 75], [75, 100], and [100, 125].

From 0 to 50 seconds:

During the first interval, Chris is in the lead, followed by Brad and then Jaime; that is, Chris has the greatest distance value for this interval, Brad has the second greatest distance value, and Jaime has the least distance value. Chris's and Brad's distances over time are modeled by straight lines for this interval, which indicate that they are running at constant rates. Chris is running faster than Brad, as shown by the greater slope of his line on the graph.

From 50 to 75 seconds:

During the second interval, Chris stops suddenly (notice the change in Chris's slope) and remains still for approximately

25 seconds. His graph at this interval is horizontal, showing no change in distance. Brad travels faster than Jaime and passes Chris at approximately 75 seconds to gain the lead. Jaime remains behind the other two runners, as shown by his distance value, which is less than Brad's and Chris's for any given time in this interval.

From 75 to 100 seconds:

Chris starts to run again; he runs at a constant rate but more slowly than he ran in the 0-to-50-second interval, as shown by his graph, which has a less steep slope than it had before. Now Brad leads, followed by Chris and then Jaime. Jaime is starting to sprint; his increase in speed can be seen in the curve that has an increasingly steep slope over this interval. At approximately 100 seconds, Brad's and Jaime's graphs intersect; Jaime catches up with Brad and overtakes him to win the race.

From 100 to 125 seconds:

Brad finishes the race just after 100 seconds. Chris completes the race at approximately 125 seconds.

Extension Questions

- What type of function could represent the distance that each athlete ran as a function of time?

  Jaime's graph might be modeled with a quadratic function after the first 75 seconds. Brad's graph appears to be linear and increasing. Chris's graph would be defined in three pieces: first, linear and increasing; then constant; and, finally, linear and increasing with less slope than in the first piece.

- For the functions described above, how do the mathematical and situation domains compare?

  The mathematical domain for each function is the set of all real numbers. The situation domains are limited to a finite time interval, starting with 0 seconds and ending with the time it takes the last athlete to complete the race.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

- Describe the mathematical and situation ranges for the problem situation. Explain how they are different.

  The mathematical range is all real numbers greater than 0. The situation range is between 0 and the total distance in the race, 600 meters. Since there is a stopping distance (600 meters), the situation range is limited.

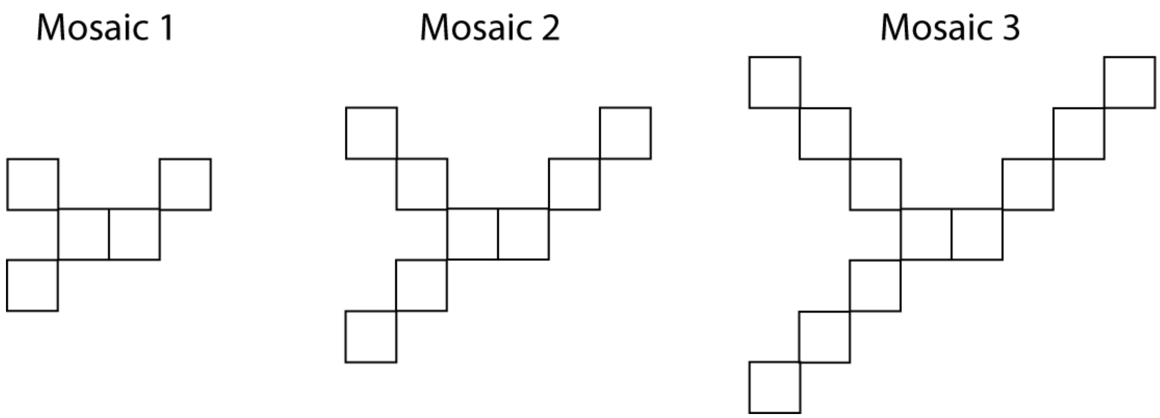
- How could you use the graph to determine the rates of speed for each runner?

  To estimate rates, we could estimate the distance traveled within a time interval and divide by an estimate of the total time in the interval to get meters per second.

- Suppose the athletes run a marathon (26.2 miles). Do you think the same functions would model their distance in relation to time? Explain. Describe the effect of this scenario on the domain and range.

  Because athletes use very different strategies when running long distances than they use when running short distances, the same functions may not apply. The domain of the situation begins with a starting time of 0 seconds; the ending time of the interval depends on how long it takes the last runner to cross the finish line. The range for the situation is between 0 and 26.2 miles, the total distance of the race.

Mosaics



Reuben learned in art class that a mosaic is made by arranging small pieces of colored material (such as glass or tile) to create a design. Reuben created a mosaic using tiles, then decided on a growing pattern and created a second and third mosaic. Reuben continued his pattern by building additional mosaics. He counted the number of tiles in each mosaic and then represented the data in multiple ways. He thinks he sees a relationship between the mosaic number and the total number of tiles in the mosaic.

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.
2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.
3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.
4. Would there be a mosaic in Reuben's set that uses exactly 57 tiles? Explain your reasoning using at least one representation.
5. In Reuben's mosaic, there are 2 tiles in the center. How would the function rule change if the center of the mosaic contained 4 tiles instead? Explain your reasoning using two different representations.

Notes

Materials:

30 tiles per student (optional)

One graphing calculator per student

Graph paper

Algebra TEKS Focus:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

Scaffolding Questions

- Build or draw the next two mosaics. How many tiles are in each?
- Make a table to record your data, including a process column. What remains constant in the relationship between the mosaic number and the number of tiles in your table? What changes?
- Make a graph of the information from the table. How does your graph illustrate what is constant and what is changing in the problem situation?
- How would you describe the relationship between the mosaic number and the number of tiles in each mosaic?
- Which quantity depends on another quantity in this problem?
- What is the rate of change for tiles with respect to the mosaic number?
- On the graph for this problem situation, should a line connect the points? Explain why or why not.
- Develop a rule to determine the number of tiles in the *n*th mosaic. What is constant and what is changing in your rule?
- What is an appropriate domain and range for the mosaics situation?
- What are the connections among the picture, verbal description, table, and graph?

Sample Solutions

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.

   Reuben's data can be represented with a verbal description, graph, table, and function rule.

   Verbal Description:

   One way to look at the mosaic is to see the 2 tiles in the center of the mosaic as the base. For each new mosaic, add 3 outside tiles.

Graph:

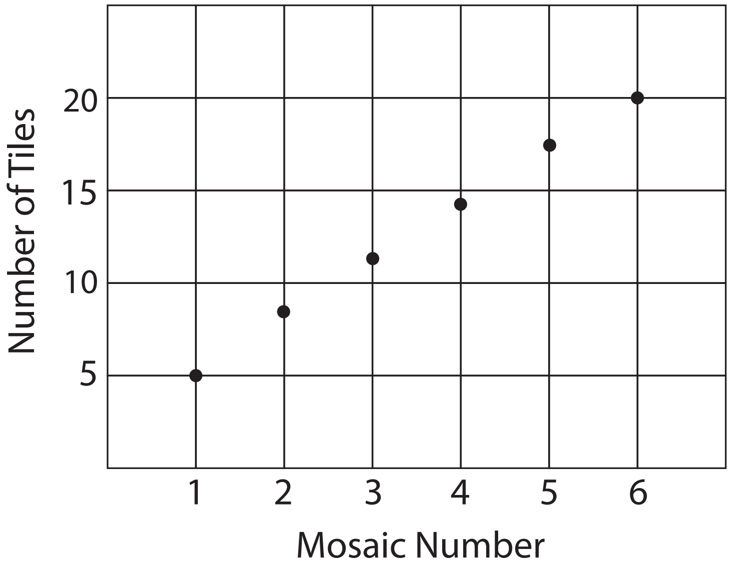


Table:

Mosaic Number	Process	Number of Tiles
1	2 + 3(1)	5
2	2 + 3(2)	8
3	2 + 3(3)	11
4	2 + 3(4)	14
5	2 + 3(5)	17
6	2 + 3(6)	20
…		
n	2 + 3(*n*)	

Function Rule:

From the table, we can find the rule for the number of tiles, T, that is 2 plus 3 times the mosaic number, n, or $T = 2 + 3n$.

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.

   The function rule is based on the picture of 2 tiles in the center and 3 tiles added for each new mosaic number. As tiles are added, the 2 tiles in the middle are constant, and the number of tiles we add each time stays the same. What changes is the mosaic number. The total number of tiles depends on the mosaic number.

   From the graph you can see that for every new mosaic, the number of tiles increases by 3. If we followed the pattern in the graph back to Mosaic 0, we would have the 2 tiles in the middle. We can also see the constant rate of change: For each new mosaic number, the number of tiles increases by the constant 3.

   We can see the relationship between the mosaic picture and the table: The 2 tiles in the center remain constant, and the 3 tiles added for each new mosaic are constant. What changes is the mosaic number. The total number of tiles depends on the mosaic number. In every representation, we see that the total number of tiles is the 2 you start with plus 3 tiles multiplied by the mosaic number.

   Alternative Solution:

   Some students may see the 5 tiles in the first mosaic as the base. The first mosaic then consists of only the base. For subsequent mosaics, students add 3 tiles each time (or 3 times 1 less than the mosaic number). If this is how students see the pattern, then the process column in the table below will change to 5 + 3(mosaic number – 1).

   - Suppose Susan's process is shown in the table below. What is her original mosaic pattern and how does it change? How does this compare to Reuben's pattern?

Mosaic Number	Process	Number of Tiles
1	5 + 3(0)	5
2	5 + 3(1)	8
3	5 + 3(2)	11

Susan's function rule would be $T = 5 + 3(n - 1)$, where n is the mosaic number. She saw the original mosaic base consisting of 5 tiles and then began adding 3 tiles starting with the second mosaic. She gets the same number of tiles for each mosaic number as Reuben, but she saw the pattern in a different way.

Note: This activity provides an opportunity to explore what is the same and what is different about the two rules. The two rules, though different, are equivalent; that is, $5 + 3(n - 1) = 2 + 3n$, but they represent two different ways of seeing the mosaic pattern. It is more important here for students to connect the way they “see” the pattern with the verbal description, rule, table, and graph. Although there is the opportunity here to illustrate the distributive property and combining like terms, these are not the objectives of the lesson. You may choose to revisit this activity later with a different focus.

3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.

   We can find how many tiles are in the tenth mosaic using the graph, table, or function rule.

Graph:

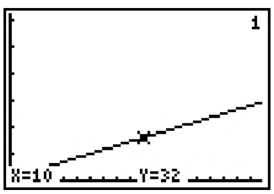
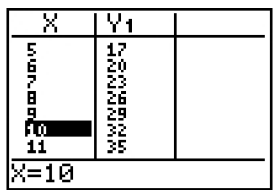


Table:



Function Rule:

$T = 2 + 3(10)$ or 32 tiles

4. Would there be a mosaic in Reuben’s set that uses exactly 57 tiles? Explain your reasoning using at least one representation.

   To determine if there is a mosaic that has 57 tiles, we need to find the mosaic number that corresponds to 57 tiles. We can look at an extended table to determine the mosaic number, or we can look at the graph.

X	Y_1
15	47
16	50
17	53
18	56
19	59
20	62
21	65

X=18

In either case, we see that since the mosaic number must be a whole number, there is not a table entry or a point on the graph corresponding to exactly 57 tiles. We can also set up an equation that arises from the function situation and solve it.

$57 = 2 + 3n$

$55 = 3n$

$n = \frac{55}{3} = 18\frac{1}{3}$

Note: You may have students who notice this possibility: If you split one tile into three equal parts and add them to each end, they could use 57 tiles.

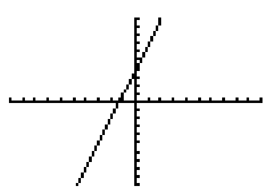
Since *n* must be a whole number, there would not be a mosaic with exactly 57 tiles.

5. In Reuben's mosaic, there are 2 tiles in the center. How would the function rule change if the center of the mosaic contained 4 tiles instead? Explain your reasoning using two different representations.

   If the center of the mosaic contained 4 tiles, the data in the table would change. Each *y*-value would be 2 greater than the *y*-value representing the number of tiles in the original mosaic. Each point in the graph would be translated up by 2 units.

X	Y_1
1	7
2	10
3	13
4	16
5	19
6	22
7	25

X=1



Generating a new rule, we see the constant would be 4. The new rule would be $T = 4 + 3n$.

Extension Questions

- If the function rule was $T = 2 + 4n$, describe the first two mosaics and the general rule.

  There would be 2 tiles in the center and 1 tile on each of the four corners for the first mosaic. The second mosaic would have 2 tiles in the center and 2 tiles at each of four corners. The general rule means that there are 2 tiles in the center and 4 tiles added for each new mosaic.

- Draw the first three mosaics for the following function rules. Make a table for each and then graph the functions on the same axes. Describe similarities and differences.

  A. $x + 3 = y$

  Picture:

  Draw Mosaic 1 with 3 tiles in the center and 1 tile on one corner, Mosaic 2 with 3 tiles in the center and 2 tiles on one corner, and Mosaic 3 with 3 tiles in the center and 3 tiles on one corner.

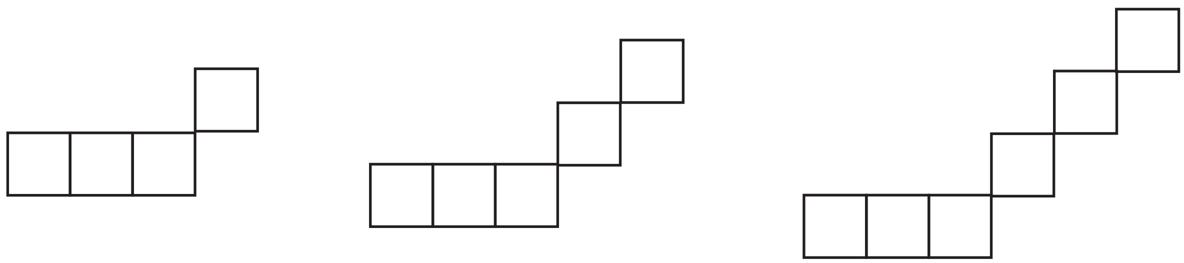


Table:

Mosaic Number	Number of Tiles
1	4
2	5
3	6

B. $4x + 3 = y$

Picture:

Draw Mosaic 1 with 3 tiles in the center and 1 tile on each of the four corners, Mosaic 2 with 3 tiles in the center and 2 tiles on each of the four corners, and Mosaic 3 with 3 tiles in the center and 3 tiles on each of the four corners.

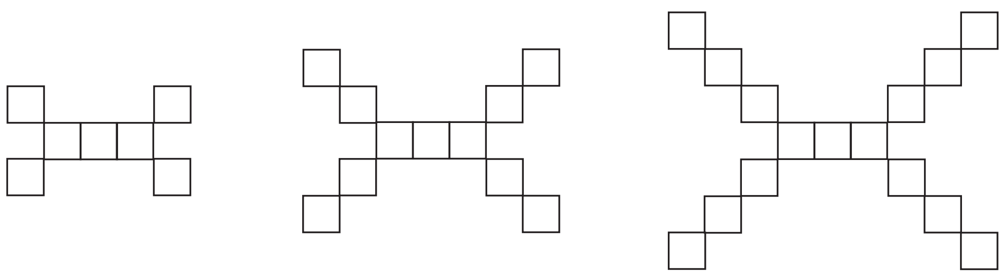


Table:

Mosaic Number	Number of Tiles
1	7
2	11
3	15

C. 4*x* + 6 = *y*

Picture:

Draw Mosaic 1 with 6 tiles in the center and 1 tile on each of the four corners, Mosaic 2 with 6 tiles in the center and 2 tiles on each of the four corners, and Mosaic 3 with 6 tiles in the center and 3 tiles on each of the four corners.

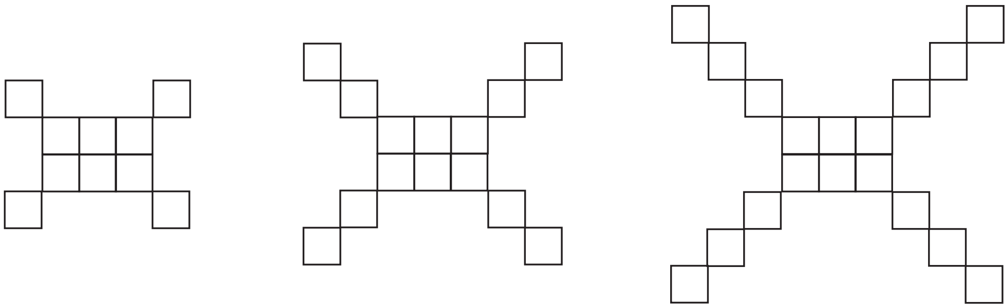
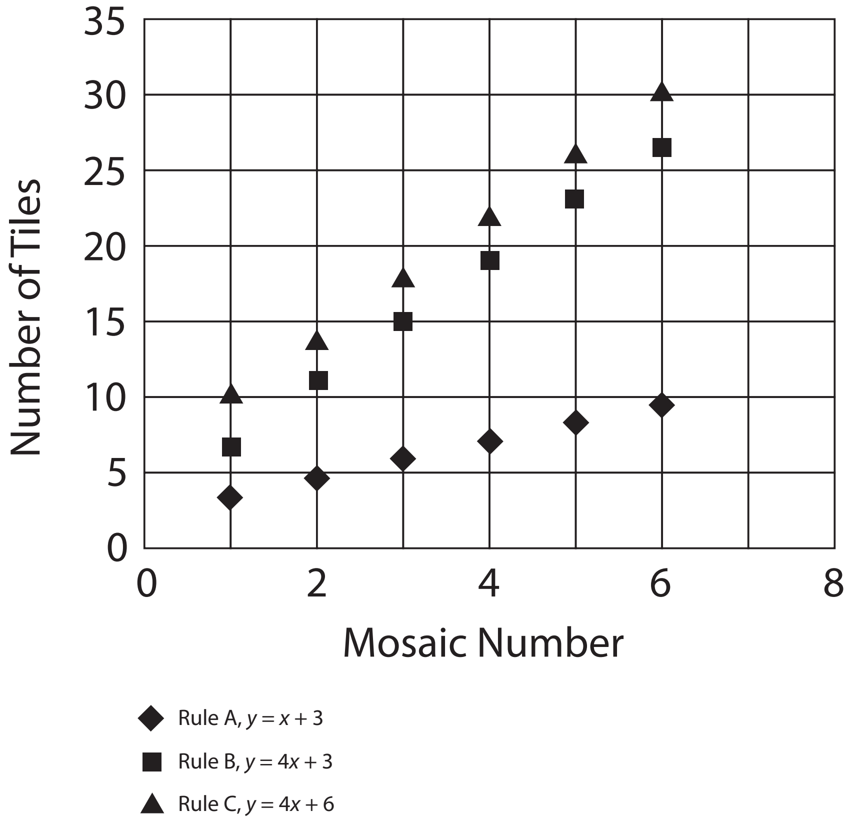


Table:

Mosaic Number	Number of Tiles
1	10
2	14
3	18

Functions graphed together

When you graph them together on the same set of axes, you can see that all of the rules are linear realtionships and that the graphs of rules B and C are parallel because they have the same slope.

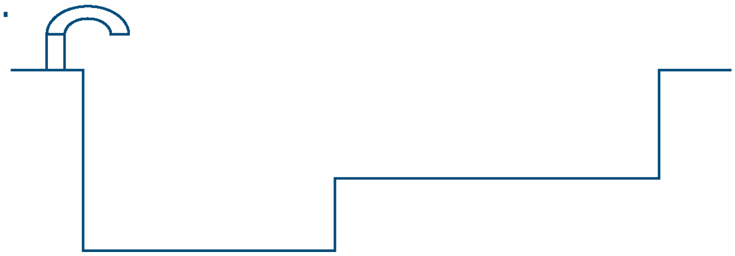

35
30
25
20
15
10
5
0
Number of Tiles
0
2
4
6
8
Mosaic Number
Rule A, y = x + 3
Rule B, y = 4x + 3
Rule C, y = 4x + 6

Swimming Pools

Two swimming pools are being filled at a constant rate. Cross sections of the pools are shown below.

A.

B.



1. For each pool, write a description of how the depth in meters of the water in the pool varies with time in minutes from the moment the empty pool begins to fill.

2. Sketch a graph to show how the depth of the water in each pool varies with time from the moment the empty pool begins to fill.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(F) interpret and predict the effects of changing slope and *y*-intercept in applied situations.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Scaffolding Questions

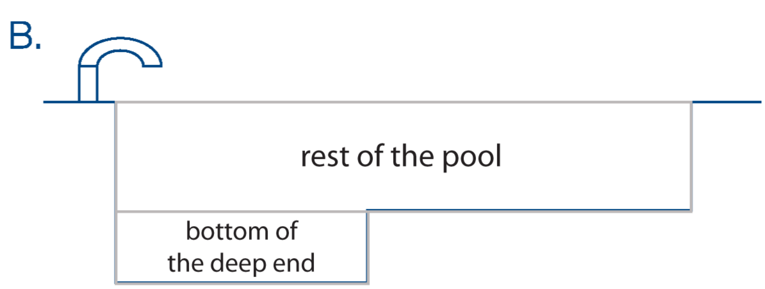
- How are the pools different from each other?
- How are the pools the same?
- Which section of Pool B will be filled first?
- What should the graph look like for Pool A? for Pool B?
- How will the graphs be different?
- How will the graphs be the same?
- What happens to the graph when the pool is filled? Can you point to the place on the graph that indicates when Pool A is filled? Pool B?

Sample Solutions

1. For each pool, write a description of how the depth in meters of the water in the pool varies with time in minutes from the moment the empty pool begins to fill.

   For Pool A, the cross section of the pool is a rectangle. If water is flowing into the pool at a constant rate, the water's depth rises at a constant rate.

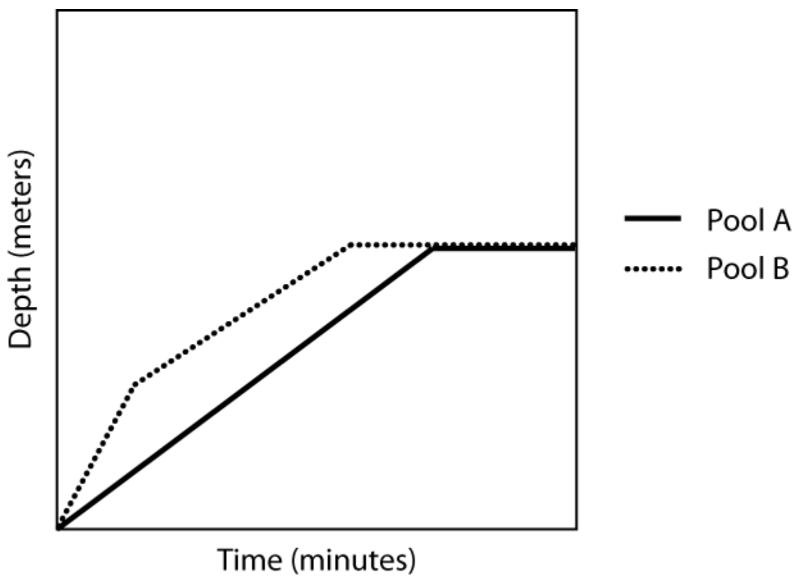
   For Pool B, we have to consider the cross section in two pieces—the bottom of the deep end (from where the shallow end stops down to the bottom of the pool) and the rest of the pool (everything except that deepest part). Both of these portions are rectangular prisms.



The water depth increases at a certain constant rate while the water fills the bottom of the deep end first. Then, as the water begins to fill the rest of the pool, the water depth increases at a slower but still constant rate until the pool is entirely full. This difference in fill rates (bottom of the deep end vs. rest of the pool) is because the deep end of the pool is a rectangular prism whose base is smaller than the base of the rectangular prism that is the whole pool.

2. Sketch a graph to show how the depth of the water in each pool varies with time from the moment the empty pool begins to fill.

Possible graph:



Extension Questions

- How are the graphs of the pools related?

  If you assume that the length and width of the pools are the same and water is flowing into the pools at the same rate, then when the deep section of Pool B is full, its line on the graph becomes parallel to the line representing Pool A.

- Describe how the graphs look once the pools are filled.

  Since Pool B holds less water, it fills up sooner than Pool A. When a pool is full, its depth is constant and its graph horizontal. If the depths of the pools are the same, the graph representing Pool B becomes horizontal before the graph of Pool A.

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

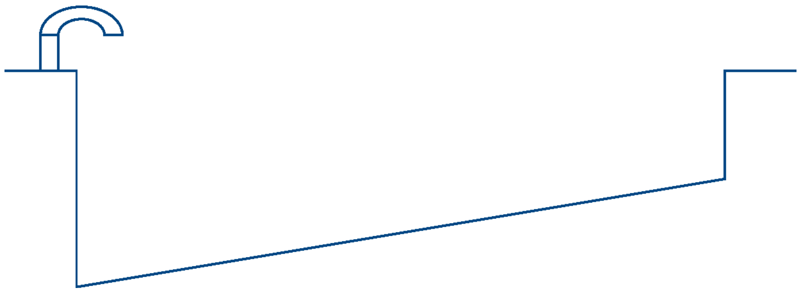
(A) determine whether or not given situations can be represented by linear functions;

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

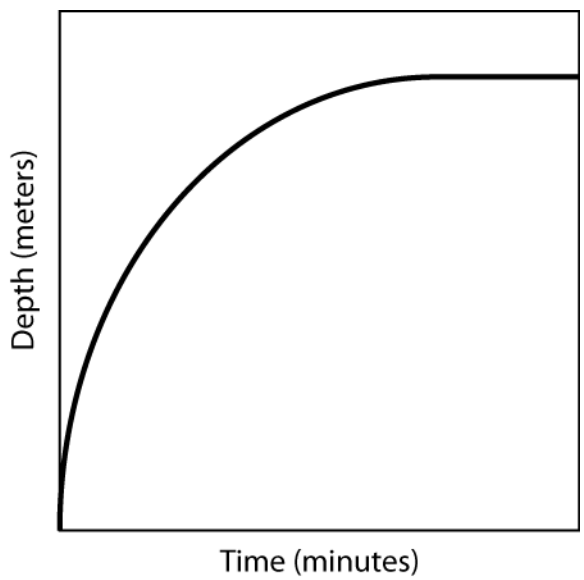
Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

- Describe a graph that represents the filling of a swimming pool whose cross section is a trapezoid.



The graph is a curve. The pool fills most quickly in the deep end, but as the surface area of the water increases, the water rises more slowly. Thus, the fill rate gradually slows until the shallowest part is covered with water. At that point, the fill rate becomes constant and is modeled by a linear graph.



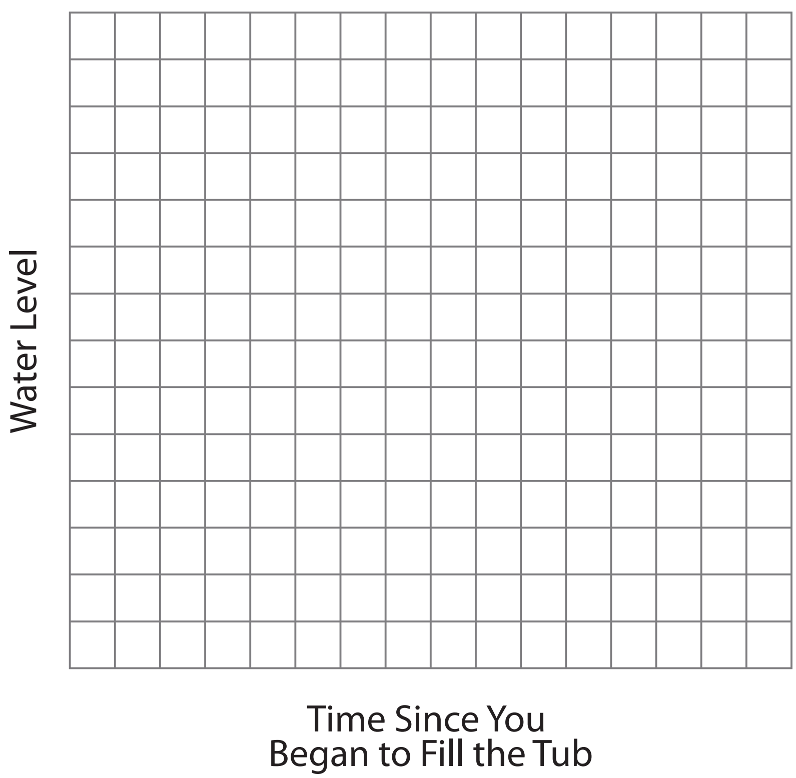
Bathing the Dog

It's time for Shadow, your German Shepherd, to have a winter bath. Shadow does not enjoy getting a bath! You fill the bathtub halfway, put Shadow in, and begin to bathe her. Shadow thrashes around, tries to escape, and gets halfway out, splashing water out the tub all the while. You pull her back into the tub and finish the bath. You get her out and then drain the tub.

Assume a lengthwise cross section of the tub is trapezoidal with the tub sides nearly vertical.

1. Describe how the water level in the tub varies before, during, and after Shadow's bath, and explain what causes the variations.

2. Sketch a graph of your description. Clearly label significant points on the graph.



Notes

Materials:

One graphing calculator per student (optional)

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

Scaffolding Questions

- How will the shape of the tub affect how the water level changes as the tub is filled?
- What is the water level when the tub starts to fill?
- What happens to the water level when you put Shadow into the tub?
- What happens to the water level when Shadow climbs halfway out of the tub?
- What happens to the water level when you get Shadow completely back into the tub?
- What happens to the water level when you finish bathing Shadow and she gets out?
- How will the shape of the tub affect how the water level changes as the tub is drained?

Sample Solutions

1. Describe how the water level in the tub varies before, during, and after Shadow's bath, and explain what causes the variations.

   First, the tub is empty. The water level rises at a nearly constant rate because the tub walls are almost vertical. Once the tub is full enough for Shadow's bath, the water level stays constant until you put Shadow in the tub.

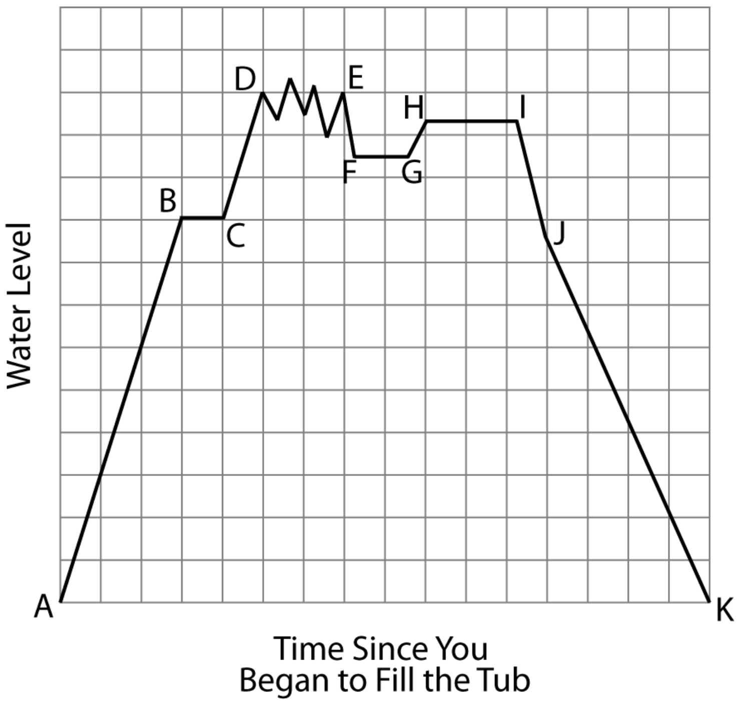
   Second, with Shadow in the tub, the water level jumps up suddenly because her body mass displaces water. Since Shadow does not enjoy being bathed, she thrashes around in the tub, varying the water depth.

   Third, Shadow climbs halfway out of the tub in a desperate attempt to escape the bath, causing the water level drop. Although she is halfway out of the tub, you continue to bathe her.

   Fourth, you get Shadow back into the tub. The water level jumps back up, but not as high, because by now some water has splashed out of the tub. You finally manage to finish bathing Shadow.

At last, Shadow's bath is done. She gets out, causing the water level to drop quickly; then you pull the plug. As the tub drains, the water level drops at a nearly constant rate.

2. Sketch a graph of your description. Clearly label significant points on the graph.



Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

Extension Questions:

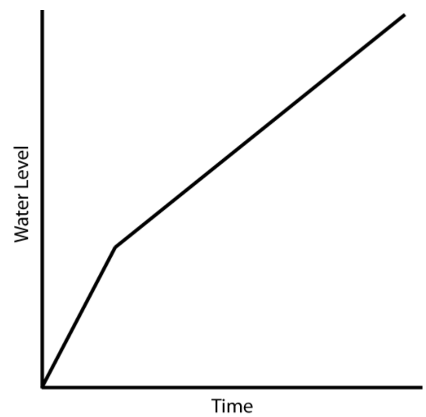
- Is it possible to write a function rule to describe how water levels in the tub change in the process of bathing Shadow? Why or why not?

  It would be extremely difficult to write a function rule for this situation. The graph would need to be divided into several time intervals, and a different function would be needed for each interval of the bath. The graph depends on a number of factors. How fast is the tub filling? Is the tub filling at a constant rate? How long will Shadow stay in the tub before she tries to climb out? How much will her thrashing and splashing affect the water in the tub? All of these factors affect the types and number of functions used to describe the graph.

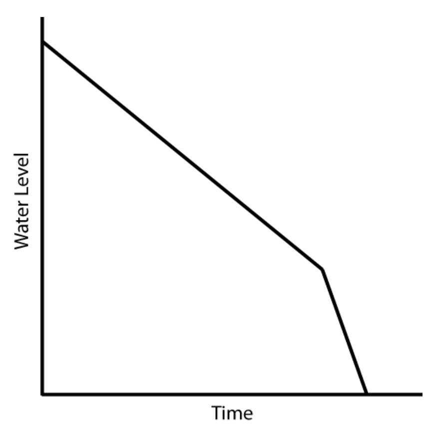
Data for one bath could be collected and a function written to fit each of the various phases. This is called a piecewise function. However, the function for each interval would change with Shadow's next bath.

- How would the graph change if the sides of the bathtub were more graduated—that is, if the sides of the tub were much wider at the top than at the bottom? Illustrate and explain your answer.

  The water level would rise quickly at first and then more slowly as the tub filled. The graph would represent an increasing function.



The water level would drop slowly at first and then more rapidly as the tub emptied. The graph would represent a decreasing function that would be concave down.

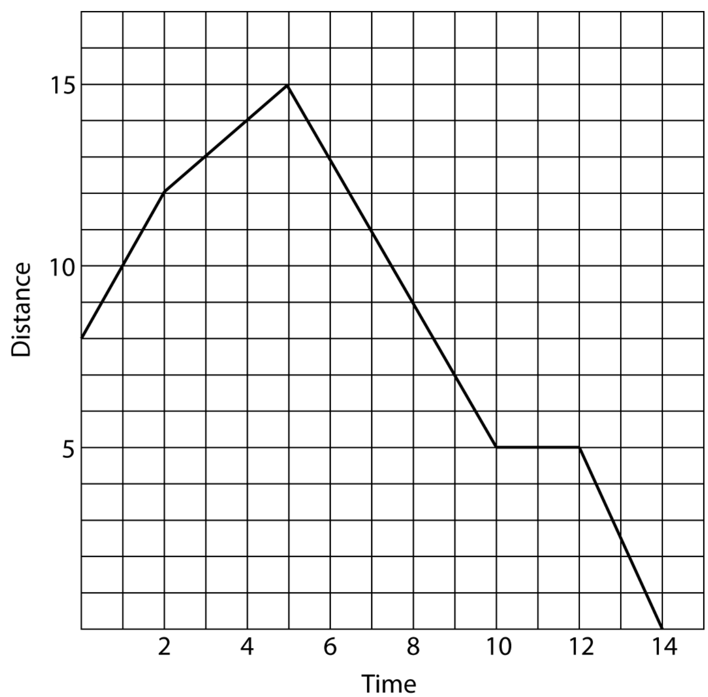


- When a dog enjoys getting a bath, the dog does not thrash around in the tub or try to escape from it. How would the graph change if Shadow enjoyed getting a bath?

  The portion of the graph that represents Shadow's time in the tub would be nearly horizontal, since there would be no thrashing or attempted escapes to change the water level.

Distance and Time

The graph below represents distance as a function of time. Create a situation that the graph might represent. Choose appropriate units for time and distance. Describe your situation in detail. Include a discussion of domain, range, dependent and independent variables, *x*- and *y*-intercepts, and slope. Write a question that you could answer from the graph.



Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or create situations that fit given graphs; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

Scaffolding Questions

- Describe some situations in which distance depends on time.
- For the situation you chose, what are reasonable units for time and distance?
- How could the graph be broken up into phases?
- What do the phases show you in terms of the function increasing, decreasing, or remaining constant? What does this mean in your situation?
- Can you determine the slope of the graph in each phase? What will this mean in your situation?
- What are the *x*- and *y*-intercepts for the graph? What do they mean in the situation you chose?

Sample Solutions

Many scenarios are possible. For example, a group of students is conducting an experiment with a motion detector, and this graph represents the motion of one walker. The student begins her walk by standing 8 meters from the motion detector and then walks further away at 2 meters per second for 2 seconds.

When she is 12 meters from the motion detector, she slows to 1 meter per second for the next 3 seconds until she reaches a distance of 15 meters away from the motion detector.

Then she walks toward the motion detector for next 5 seconds at a rate of 2 meters per second until she is 5 meters from the motion detector. Then she stops for 2 seconds, remaining 5 meters from the motion detector.

She walks toward the motion detector at a rate of 2.5 meters per second until she reaches the motion detector. In total, she travels 22 meters in 14 seconds.

In this situation, time is the independent variable and distance from the motion detector is the dependent variable. The domain is 0–14 seconds and the range is 0–15 meters. The *y*-intercept is (0, 8) and represents that at 0 seconds the walker was 8 feet from the motion detector. The *x*-intercept is (14, 0) and indicates the time (in seconds) at which the walker reached the motion detector.

The various slopes at each phase indicate the change in distance from the motion detector compared to the change in time. The slope of each phase gives the rate the walker was traveling during that time period as well as whether she was traveling toward or away from the motion detector.

One question you could ask and answer from your graph is: How far was the walker from the motion detector 8 seconds after she began walking?

Extension Questions

- How would you interpret this graph if the dependent variable were velocity instead of distance?

  In Phase A, an object is moving with constant positive velocity. In Phase B, the velocity is still steady but slower than before. In Phase C, because the distance is now decreasing at a steady rate, the velocity is a negative number. In Phase D, the velocity is zero because the object does not move. Finally, in Phase E, because the distance decreases, the velocity is negative.

- If the first phase on the graph had been from point (0, 12) to point (2, 12), how would that change your description of the graph?

  Because the distance remained constant, it is clear that the object was still for the first 2 seconds.

- Take the information from the graph and create a graph of the object's velocity as a function of time. The velocity is the speed at which the object traveled and can be found for each phase.

Time Phase	*Velocity*
0 to 2	*2*
2 to 5	*1*
5 to 10	*–2*
10 to 12	*0*
12 to 14	*–2.5*

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

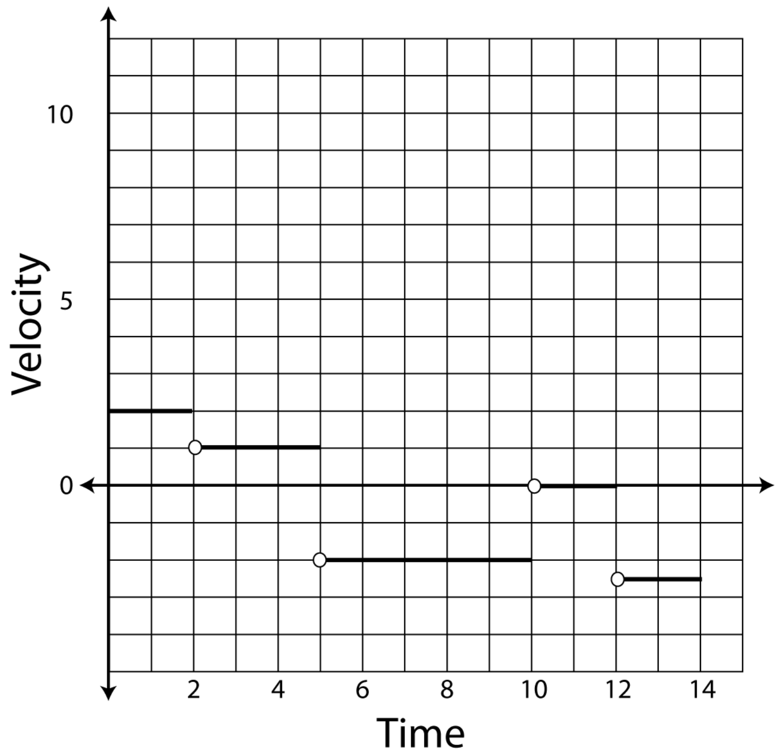
The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

The graph would be a series of horizontal line segments. There would be an open circle at one end of each line segment to indicate that a value of x may not have two function values.



- In some situations, there is a clear dependency relationship implied, so it is easy to determine what is dependent and independent. In other situations, the dependency relationship is determined by what you want to know. Think of a situation where the dependent and independent variables can be switched. What kinds of questions would each dependency relationship answer? Will switching the dependent and independent variables always lead to a functional relationship? Why or why not?

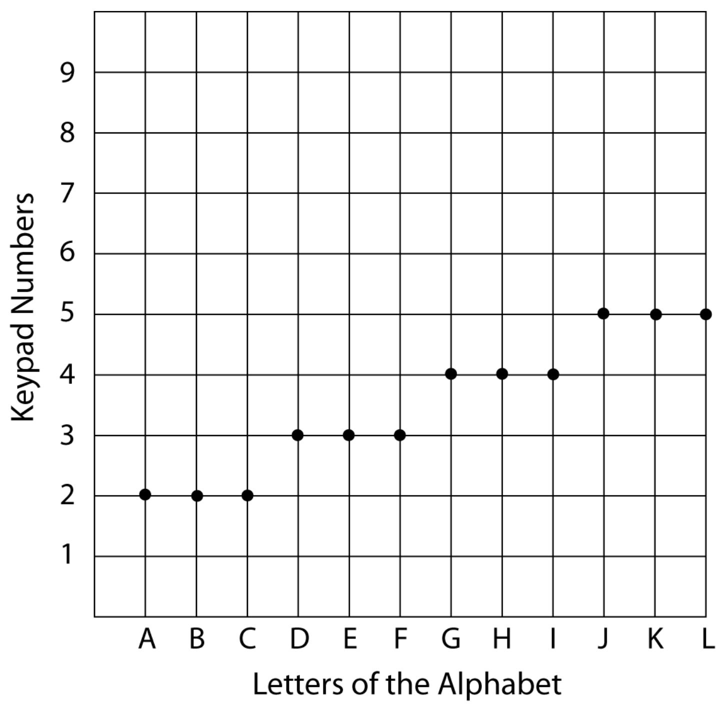
  One possible solution: If a stack of cups needs to fit into a cabinet, the number of cups that will fit depends on the height of the stack. The height of the stack is independent, and the number of cups is dependent. This relationship can answer questions such as: If you want the stack of cups to be fewer than 15 inches tall, how many cups can you stack?

  But if you have a set number of cups to stack, the height of the stack depends on the number of cups. The number of cups is independent, while height of the *stack is dependent. This relationship answers questions such as: How tall is a stack of 12 cups?*

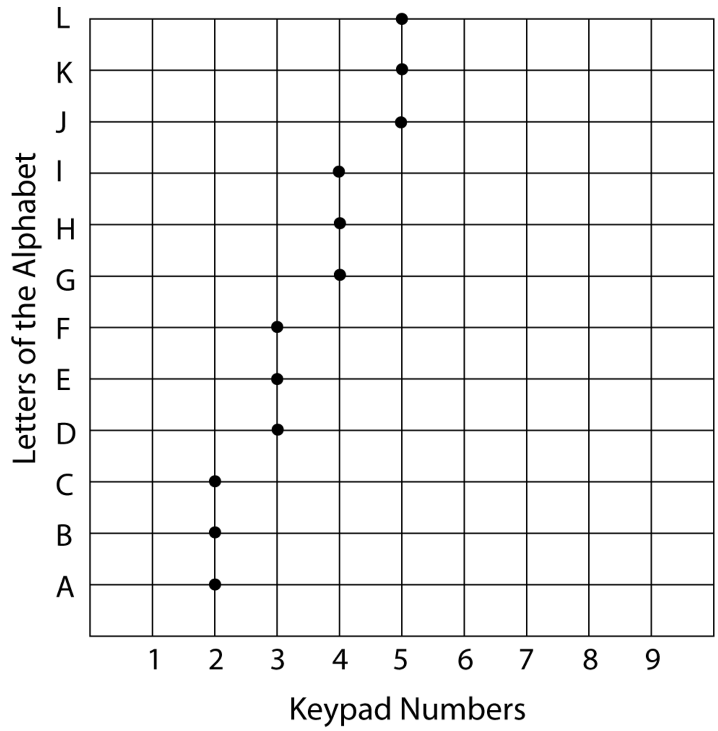
Switching the dependent and independent variables in a given situation does not always lead to a functional relationship. For example, when sending text messages on a cell phone, you press numbers on your keypad to yield letters. The number you press depends on what letter of the alphabet you need (if you need B, you press 2). The letter is the independent variable, and the number is the dependent variable since each letter corresponds to exactly one number. The inverse is not true: Because the numbers correspond to three letters each (the number 2, for example, yields A, B, or C), the dependent and independent variables cannot be switched in this situation and produce a functional relationship. The relationship between the letters and numbers on a telephone keypad is functional only when the letter represents the independent variable and the number represents the dependent variable.

When represented graphically, this distinction between functional and nonfunctional relationship may be clearer. The graphs below represent a functional and a nonfunctional relationship, respectively.

Example of a graph that represents a functional relationship:



Example of a graph that represents a nonfunctional relationship:



Note: Discussions of this type throughout the year help solidify the concepts of dependent and independent variables and set the stage for inverse functions in Algebra II.

Extracurricular Activities

Determine a function that represents each of the following situations. Describe the mathematical domain and range of the function and a reasonable domain and range for the situation. Use a table, graph, picture, or other representation to explain your choice of domain.

1. An athlete joins a fitness club that charges $3.50 per visit plus an annual fee of $150. Her total annual cost is, therefore, a function of the number of times she visits the club.

2. You buy a video game console on credit for $322.25. Your minimum monthly payment is $25.78. The first minimum monthly payment is due one month after your purchase. No interest will be charged if you make your monthly payments on time. Your account balance at any given point in time is a function of the number of months you have made the $25.78 payment.

3. A hiker arrives at an overlook at the edge of a canyon just in time to see a hot air balloon launched from the canyon floor, 100 meters below the overlook. The balloon rises at a steady rate of 10 meters per second; its maximum cruising height is 300 meters above the overlook. The balloon's vertical distance from the overlook as it rises to cruising altitude is a function of the number of seconds that have passed since it was launched.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify the mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

Scaffolding Questions

- What are the variables in this situation?
- How are those variables related?
- What is the dependent variable (range)?
- What is the independent variable (domain)?
- What does each number in this situation represent?
- In Situation 1, if the athlete made one trip to the fitness club, how would you compute the cost? Two trips? Three trips? Create a table if needed.
- In Situation 2, as the number of payments increases, what happens to the balance?
- In Situation 3, how does the height of the balloon change over time?
- In each situation, what type of function (linear, quadratic, exponential, inverse variation) best fits the data? How would you determine this?
- What restrictions does the situation place on the independent variable?
- Should you use all real numbers for the domain? Why or why not?
- What representation would best help you see the domain and range?

Sample Solutions

1. An athlete joins a fitness club that charges $3.50 per visit plus an annual fee of $150. Her total annual cost is, therefore, a function of the number of times she visits the club.

   The athlete's total annual membership cost is equal to $150 plus the product of her number of visits and $3.50. The function rule is $f(n) = 150 + 3.5n$, where n = the number of visits during the year and $f(n)$ = the total annual cost.

   Because the function is linear and not a constant value, the mathematical domain and range for this function are the set of all real numbers.

A table of values illustrates the domain and range that make sense for the situation.

Number of Visits (*n*)	Annual Cost ($)
0	150
1	153.50
2	157
3	160.50
4	164
. . .	. . .

The number of visits to the fitness center must be a whole number. Since the function value, *f(n)*, gives the initial cost of $150 plus $3.50 per visit, *f(n)* must be 150 plus whole number multiples of $3.50. The maximum number of visits, *n,* depends on how many times she visits the club in a year.

To summarize, the domain for the situation is {0, 1, 2, 3, 4, . . . , *n*}, where *n* is the maximum number of visits per year, and the range is {150, 153.5, 157, 160.5, . . ., 150 + 3.5*n*}. Both the domain and the range are finite sets, limited by the number of visits she makes in a year. The graph for the situation is discrete (i.e., a dot plot), linear, and increasing.

2. You buy a video game console on credit for $322.25. Your minimum monthly payment is $25.78. The first minimum monthly payment is due one month after your purchase. No interest will be charged if you make your monthly payments on time. Your account balance at any given point in time is a function of the number of months you have made the $25.78 payment.

   The account balance is $322.25 minus the product of $25.78 and the number of payments that you have already made. The function rule is $c(n) = 322.25 – 25.78n$, where *n* represents the number of months you pay on your account balance and *c(n)* represents your account balance. Since this is a nonconstant linear function, both the mathematical domain and range for this function are all real numbers.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Since the number of payments must be a whole number, the domain values for the problem situation must be whole numbers.

The table illustrates the domain and range that make sense for the situation.

Number of Monthly Payments (*n*)	Account Balance ($)
0	322.25
1	296.47
2	270.69
3	244.91
. . .	. . .
11	38.67
12	12.89

Your account balance zeroes out at the thirteenth month when you pay the final balance of $12.89.

To summarize, the domain for the situation is the set {0, 1, 2, 3, . . . ,13}, and the range is the set {322.25, 296.47, 270.69, . . . ,12.89, 0}. Both domain and range are finite sets. The graph of the situation consists of only 14 points plotted in the coordinate plane. The graph of the mathematical function is a line with an infinite number of points.

3. A hiker arrives at an overlook at the edge of a canyon just in time to see a hot air balloon launched from the canyon floor, 100 meters below the overlook. The balloon rises at a steady rate of 10 meters per second; its maximum cruising height is 300 meters above the overlook. The balloon's vertical distance from the overlook as it rises to cruising altitude is a function of the number of seconds that have passed since it was launched.

   The initial vertical distance between the overlook and the balloon on the desert floor at the time of the launch can be represented by –100. The vertical distance increases at a rate of 10 meters per second. The height of the balloon at any given moment during its ascent to cruising altitude is the starting vertical distance plus the product of 10 and the number of seconds that have elapsed since the launch.

   $h(t) = -100 + 10t$, where t represents time in seconds into launch, and

   $h(t)$ = height in meters of the balloon with respect to the overlook

Since this is a linear function, the mathematical domain and range for the function are all real numbers.

The domain for the situation, time in seconds into the launch, is the set of all *t* values where $0 \le t \le 40$. The range for the situation, height in meters into launch, is the set of all values *h(t)* where $-100 \le h(t) \le 300$.

The domain is time in seconds and increases continuously from 0 to 40 seconds, since the balloon starts at time 0 and takes 40 seconds to reach the cruising altitude of 300 meters. The range is height in meters and increases continuously from –100 meters to 300 meters. The graph of the situation is continuous and is a line segment instead of a set of discrete points.

Extension Questions

- What kind of function is needed to model each situation?

  Since each situation involves a constant rate of change, each situation is modeled by a linear function.

- Describe the data for each situation as discrete or continuous and justify your answers.

  In Situation 1, the data are discrete because they are based on the number of visits, not partial visits. In Situation 2, the data are discrete because they are based on the number of payments, not partial payments. In Situation 3, the data are continuous because height can be measured at any time.

- What is the parent function for these functions? How does knowing the parent function for these functions help you determine the mathematical domain and range of the function for each of these situations?

  The parent function is $y = x$. The domain and range of the parent function are the set of all real numbers. The only special case is a linear function that is constant, which would restrict the range to a single value. Multiplying x by m and adding b to the parent function to get $y = mx + b$ does not change the domain (set of x values) or the range (set of y values). It just changes the graph of the function in terms of where it crosses the y-axis and its slope.

- In Situation 2, describe how the domain and range would change if you changed the rate but not the initial value.

  If you increased the rate at which you paid the balance, the domain would shorten, since you would pay off the balance faster. If you decreased the rate—that is, if you paid less each month—the domain would lengthen, and it would take longer to pay off the debt. By the way, also pay attention to the final payment: Depending on how much you pay each month, you may overpay the final payment and end up with a credit, which would show up in your y-intercept and would change your range to include a negative value.

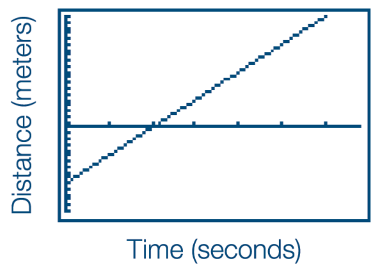
- In Situation 3, describe how the domain and range would change if the launch altitude and cruising altitude of the balloon changed. Call this the “launch to cruise distance.”

  If the “launch to cruise distance” increased, the domain would comprise a longer time interval, and the range would comprise a longer distance interval. If the “launch to cruise distance” decreased, the domain would comprise a shorter time interval, and the range would comprise a shorter distance interval.

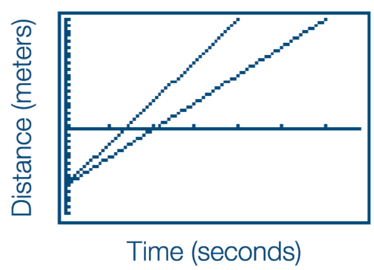
- In Situation 3, describe how the domain and range would change if the “launch to cruise distance” remained the same, but the balloon rose at a different rate.

  If the balloon rose faster, it would take less time to cover the same distance, and the domain would therefore comprise a shorter time interval. The range interval would be the same. If the balloon rose more slowly, it would take more time to cover the same distance. The domain would therefore comprise a longer time interval, but the range interval would still be the same. The graphs below illustrate these situations.

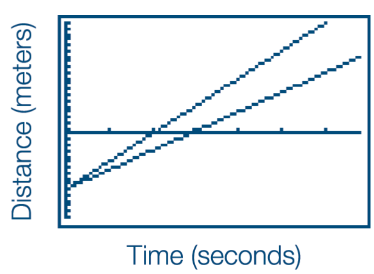
The Original Situation



Faster Rate



Slower Rate



Making Stuffed Animals

Pocket-sized stuffed animals cost $20 per box of 100 animals plus $2,300 in fixed costs to make them. Animals must be made by the box—boxes cannot be split up.

1. Write a verbal description of how you would calculate the cost of making one box of stuffed animals.
2. Describe the dependent and independent variables.
3. How much will it cost to make 200 boxes of stuffed animals? 250 boxes? 320 boxes? Justify your answers.
4. How many boxes of stuffed animals can be made with $5,000? With $50,000? Explain how you found your solutions.
5. Use a graph to predict the cost of making 425 boxes of stuffed animals.

Notes

Materials:

One graphing calculator per student

One sheet of graph paper per student

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- What are the variables?
- What is represented by the dependent variable?
- What is represented by the independent variable?
- What does the $2,300 fixed cost mean?
- What is the function rule for the cost of making any number of boxes of stuffed animals?

Sample Solutions

1. Write a verbal description of how you would calculate the cost of making one box of stuffed animals.

   The fixed cost of $2,300 plus $20 times 1 box equals $2,320 to make one box of stuffed animals.

2. Describe the dependent and independent variables.

   The dependent variable is the cost. The independent variable is the number of boxes.

3. How much will it cost to make 200 boxes of stuffed animals? 250 boxes? 320 boxes? Justify your answers.

   One approach to this problem is computing the cost of the boxes and putting the values in a table.

Number of Boxes	Process	Cost of Boxes in Dollars
0	2,300 + 20(0)	2,300
100	2,300 + 20(100)	4,300
200	2,300 + 20(200)	6,300
300	2,300 + 20(300)	8,300
400	2,300 + 20(400)	10,300
500	2,300 + 20(500)	12,300

The table shows that making 200 boxes of stuffed animals costs $6,300.

To find out how much it would cost to make 250 boxes of stuffed animals, we might consider that the supplies for making 100 boxes of animals add $2,000 to the fixed cost, so supplies for 50 boxes add $1,000. Use the table to find the cost of making 200 boxes, which is $6,300. The cost of making 250 boxes, then, is $6,300 + $1,000 = $7,300.

To find the cost of making 320 boxes: Given that supplies for 50 boxes of animals add $1,000 to the fixed costs, you can divide 1,000 by 5 to determine that supplies for 10 boxes add $200 to the fixed costs. Therefore, the cost of making 320 boxes of animals is $8,300 + $200 + $200 = $8,700.

4. How many boxes of stuffed animals can be made with $5,000? With $50,000? Explain how you found your solutions.

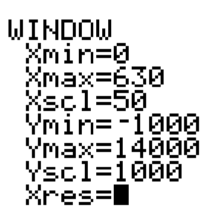
   The function is $c = 2{,}300 + 20b$, where c is the cost in dollars and b is the number of boxes. We can calculate how many boxes of animals we could make with $5,000 or $50,000 by replacing c with 5,000 or 50,000:

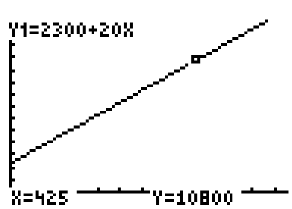
5,000	=	2,300 + 20b	50,000	=	2,300 + 20b
2,700	=	20b	47,700	=	20b
135	=	b	2,385	=	b

   We find that we can make 135 boxes of animals for $5,000 and 2,385 boxes of animals for $50,000.

5. Use a graph to predict the cost of making 425 boxes of stuffed animals.

   Using a graphing calculator, graph the line $y = 2{,}300 + 20x$. Set an appropriate viewing window. Trace along the line to find the value of y when $x = 425$.





   The cost of making 425 boxes of animals is $10,800.

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Extension Questions

- How are the graph of the problem situation and the graph of the function rule different?

  The graph of the problem situation is a set of points with x-values that are whole numbers. The number of boxes cannot be negative or a fraction since you cannot manufacture a negative number of boxes or a fraction of a box. The graph of the function rule is a line.

- How does the $2,300 in fixed costs affect the graph?

  The $2,300 represents the operational costs before any stuffed animals are actually made; in a sense, it represents the cost of 0 animals, and therefore its point on the graph line is (0, 2300). The graph does not start at (0, 0); it starts at (0, 2,300). Therefore, there is a transformation from (0, 0) to (0, 2,300). The y-intercept would be 2,300.

- What are the restrictions on the domain in this situation?

  The domain values must be whole numbers because the stuffed animals are sold by the whole box. The values begin at 2,300 and increase in increments of 20, so {2,300, 2,320, 2,340, . . . (2,300 + 20b)}.

- What makes a relationship linear?

  There is a constant increase or decrease in how one variable is related to the other variable. That is, there is a constant rate of change.

- Create an example of another linear relationship.

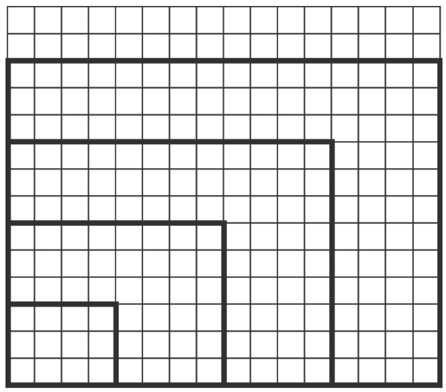
  If Jack makes $7.50 per hour, the total amount of money Jack makes depends on the number of hours he works.

- Create an example of a nonlinear relationship.

  If Jack is driving on a highway at varying speeds, the number of miles he travels depends on the time.

Nested Rectangles

A set of similar rectangles has been placed on a grid.



1. Write a function rule that shows the relationship between the length and width of the rectangles on the grid. Consider the length to be the measure of the horizontal side and the width to be the measure of the vertical side.

2. Is this relationship of length to width a proportional relationship? Explain how you know.

3. Could a rectangle with dimensions of 10 units by 8 units belong to this set? Justify your answer using two different methods.

4. Name four other rectangles that could belong to this set.

5. Describe verbally, symbolically, and graphically the relationship between the length of the rectangles and the perimeter of the rectangles.

6. Describe verbally, symbolically, and graphically the relationship between the length of the rectangles and the area of the rectangles. Compare this relationship to the relationship between length and perimeter. Is either of these relationships a direct variation?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeroes of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(G) relate direct variation to linear functions and solve problems involving proportional change.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- What are the length and width of the smallest rectangle?
- What is the relationship between these two numbers?
- What are the length and width of the second rectangle?
- What is the relationship between these two numbers?
- Does the same relationship between length and width apply for all the rectangles?
- How could you organize the information you are collecting?
- What conditions must occur for there to be a proportional relationship between length and width?
- How do you find the perimeter of the rectangles?
- In question 5, what is the dependency relationship?
- What are the variables to be considered in question 5?

Sample Solutions

1. Write a function rule that shows the relationship between the length and width of the rectangles on the grid. Consider the length to be the measure of the horizontal side and the width to be the measure of the vertical side.

   Determine the length and width of each rectangle and record the information in a table.

Length	Width
4	3
8	6
12	9
16	12

The ratio of width to length is 3:4, while the ratio of length to width is 4:3.

$\frac{w}{l} = \frac{3}{4}$, or $w = \frac{3}{4}l$

All of the rectangles satisfy this relationship.

Length	Process	Width
4	$\frac{3}{4}(4)$	3
8	$\frac{3}{4}(8)$	6
12	$\frac{3}{4}(12)$	9
16	$\frac{3}{4}(16)$	12

The relationship between length and width is that $w = \frac{3}{4}l$ or $l = \frac{4}{3}w$.

2. Is this relationship of length to width a proportional relationship? Explain how you know.

   The relationship is a proportional relationship (a direct variation) because the equation is of the form $y = kx$, where k is a constant. The ratio of length to width in any given rectangle in this set is a constant.

3. Could a rectangle with dimensions of 10 units by 8 units belong to this set? Justify your answer using two different methods.

   If we substitute 10 for the length, we get 7.5 for width.

   $\frac{3}{4}(10) = 7.5$

   The rectangle with measurements of 10 units and 8 units does not belong to this set. If the length is 10 units, the width must be 7.5 units.

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Another approach is to ask if the two ratios are equal.

$$\frac{8}{10} \neq \frac{3}{4}$$

They are not equal, so this rectangle is not similar to the given rectangles.

4. Name four other rectangles that could belong to this set.

   Possible answers are shown in the table.

Length	Process	Width
5	$\frac{3}{4}(5)$	3.75
9	$\frac{3}{4}(9)$	6.75
13	$\frac{3}{4}(13)$	9.75
14	$\frac{3}{4}(14)$	10.5

5. Describe verbally, symbolically, and graphically the relationship between the length of the rectangles and the perimeter of the rectangles.

   The perimeter is twice the length plus twice the width.

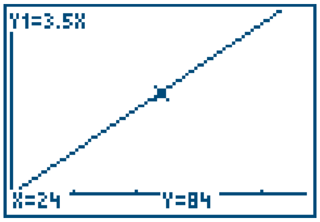
$p = 2l + 2w$ and $w = \frac{3}{4}l$

$p = 2l + 2(\frac{3}{4}l) = 2l + \frac{3}{2}l = \frac{7}{2}l$

$p = \frac{7}{2}l$

$p = 3.5l$

The perimeter is 3.5 times the length.



The graph of the function is a straight line, so the relationship is linear.

6. Describe verbally, symbolically, and graphically the relationship between the length of the rectangles and the area of the rectangles. Compare this relationship to the relationship between length and perimeter. Is either of these relationships a direct variation?

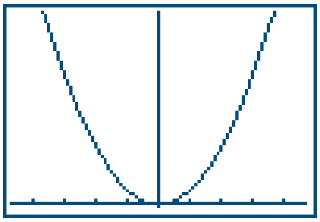
The area of the rectangle is the product of the length and the width.

$A = lw$ and $w = \frac{3}{4}l$

$A = l(\frac{3}{4}l) = \frac{3}{4}l^2$

$A = \frac{3}{4}l^2$

The area is $\frac{3}{4}$ of the length squared.



The graph is a parabola with a vertex at the origin.

The relationship between length and perimeter in the rectangle is linear, but the relationship between length and area is not linear. The length–perimeter relationship is a direct variation because it is of the form $y = kx$. Its graph is a line that passes through the origin.

Extension Questions

- What restrictions must be placed on the domain of the function $w = \frac{3}{4}l$ for this problem situation?

  The length may be any positive real number. The measurement of a side of a rectangle may not be negative or zero.

- If a rectangle similar to the original rectangles is created by doubling the length of one rectangle in this set, how would the perimeter of the new rectangle be related to the perimeter of the original rectangle?

  The perimeter of the new rectangle would be twice the perimeter of the original rectangle. If you double the length but want to keep the rectangles similar, you must also double the width.

  Students may also answer this question by drawing the rectangle, doubling the length and width segments, and seeing how the perimeter is twice that of the original rectangle.

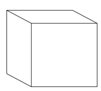
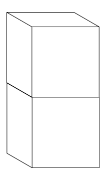
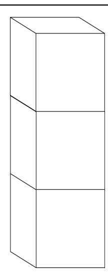
- If a new rectangle similar to the original rectangles is created by doubling the length of one rectangle in this set, how would the area of the new rectangle be related to the area of the original rectangle?

  The area of the new rectangle would be four times the area of the original rectangle. If you double the length but want to keep the rectangles similar, you must also double the width. Doubling each dimension will increase the area by a factor of 4, since the area is the product of the doubled dimensions.

  Students may also answer this question by drawing the rectangle, doubling the length and width segments, and seeing how the area is four times the original area.

Painted Cubes

Suppose you are painting a number of towers built from cubes according to the pattern below. You will paint only the square faces on the top and sides of the towers. Complete the table for figures 1 through 5. If necessary, use stackable cubes to build the figures so you can examine them.

	Figure	Process Column	Number of Painted Square Faces
1			5
2			9
3			
4			
5			

1. How many square faces would be painted for the 8th figure? Explain how you know.
2. How many square faces would be painted for the 15th figure? Explain how you know.
3. Describe in words how the number of painted square faces is related to the figure number.
4. Use your table to generate a graph, and then write an equation that represents the relationship between the figure number and the number of painted square faces.

For the following questions, show how you can use a table, equation, and graph to find answers.

5. How many painted square faces would you expect to have with a tower of 9 cubes?
6. How many cubes would you expect to have if the number of painted square faces is 53? Explain your reasoning.
7. How many cubes would you expect to have if the number of painted square faces is 66? Explain your reasoning.

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Notes

Materials:

One graphing calculator per student

Graph paper

Stackable cubes (optional)

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- How many square faces does 1 cube have?
- How many cubes does the 4th figure have?
- In the 4th figure, how many square faces does one side of the tower have?
- If we add another cube to the top of any tower, how many more square faces must be painted?
- Explain how you are going to figure out the number of square faces for any given tower.
- What are the variables in this scenario?
- Complete this sentence: “The number of ________ depends upon the number of __________.”
- How many cubes are in the 8th figure?
- How many square faces does one side of an 8-cube tower have?

Sample Solutions

1. How many square faces would be painted for the 8th figure? Explain how you know.

   Students may find the answer in one of several ways once they have determined that the 8th figure will have 8 cubes:

   - The tower has 4 sides, and each side has 8 square faces; 8 multiplied by 4 gives 32 square faces, plus the top. There are 32 + 1, or 33 painted square faces.
   - Each cube has 4 square faces to paint, and the tower has 8 cubes, so there are 32 square faces plus the top: 4 x 8 + 1 = 33.
   - Each cube has 6 square faces, but the bottom of the top cube will not be painted, and the tops and bottoms of the rest of the cubes will not be painted. This means that the top cube has 5 square faces to be painted, and the other 7 cubes have 4 square faces to be painted. So there are 1 multiplied by 5 square faces for the top cube, and 7 multiplied by 4 square faces for the rest of the cubes: (1 x 5) + (7 x 4) = 33.

2. How many square faces would be painted for the 15th figure? Explain how you know.

   Students may find the answer in one of several ways once they have determined that the 15th figure will have 15 cubes:

   - The tower has 4 sides, and each side has 15 square faces; 4 sides multiplied by 15 square faces each gives 60 square faces, plus the top. There are 60 + 1, or 61 painted square faces.
   - Each cube has 4 square faces to paint, and the tower has 15 cubes, so there are 60 square faces plus the top: (4 x 15) + 1 = 61.
   - Each cube has 6 square faces, but the bottom of the top cube will not be painted, and the tops and bottoms of the rest of the cubes will not be painted. This means that the top cube has 5 square faces to be painted, and the other 14 cubes each have 4 square faces to be painted. So there are 1 multiplied by 5 square faces for the top cube, and 14 multiplied by 4 square faces for the rest of the cubes: (1 x 5) + (14 x 4) = 61.

3. Describe in words how the number of painted square faces is related to the figure number.

   Students may find the answer in one of several ways:

   - Each figure (or tower) has 4 sides, and each side has the same number of square faces as the figure number. The total number of square faces is the product of 4 and the figure number, plus the top.
   - Each cube has 4 square faces to paint, and the number of cubes is the same as the figure number. The number of square faces is 4 multiplied by the figure number, plus 1.
   - The top cube has 5 square faces to paint, and the other cubes each have 4 square faces to paint. The number of cubes is the same as the figure number. So there are 1 multiplied by 5 square faces for the top. Take 1 less than the figure number and multiply that by 4 to get the other number of square faces to paint. The number of square faces

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

to paint is 5 plus the product of the figure number minus 1 and 4. (There are several other possible processes here; for example, see the sample solutions in question 4.)

4. Use your table to generate a graph and then write an equation that represents the relationship between the figure number and the number of painted square faces.

Tables and Equations:

Here are two possible rules and process tables:

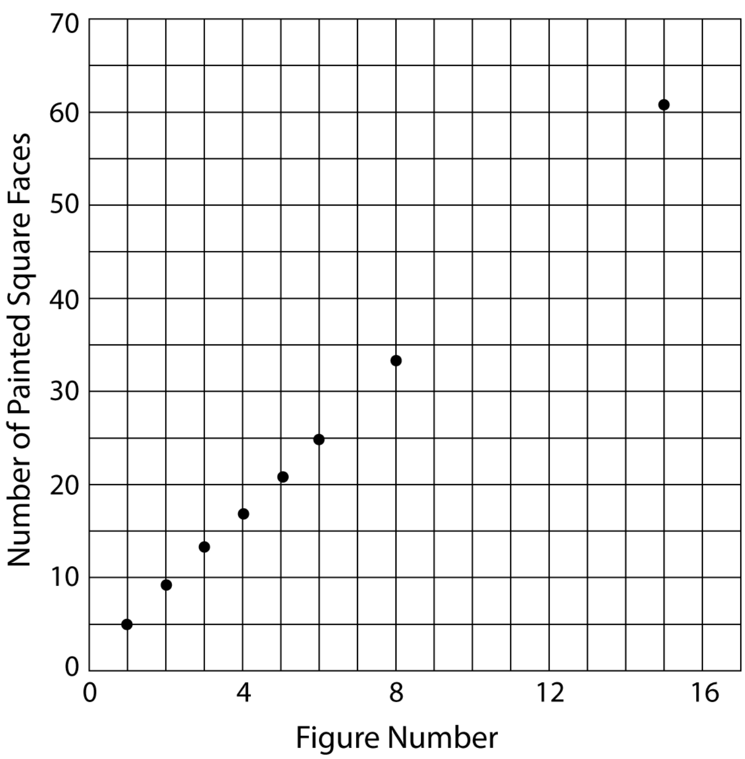
$T = 4n + 1$, where T is the total number of painted square faces and n is the figure number.

Figure Number (n)	Process	Number of Painted Square faces (T)
1	4(1) + 1	5
2	4(2) + 1	9
3	4(3) + 1	13
4	4(4) + 1	17
5	4(5) + 1	21
6	4(6) + 1	25
8	4(8) + 1	33
15	4(15) + 1	61
n	4(n) + 1	

$T = 5 + 4(n - 1)$, where T is the total number of painted square faces and n is the figure number.

Figure Number (n)	Process	Number of Painted Square faces (T)
1	5 + 4(0)	5
2	5 + 4(1)	9
3	5 + 4(2)	13
4	5 + 4(3)	17
5	5 + 4(4)	21
6	5 + 4(5)	25
8	5 + 4(7)	33
15	5 + 4(14)	61
n	$5 + 4(n - 1)$	

Graph:

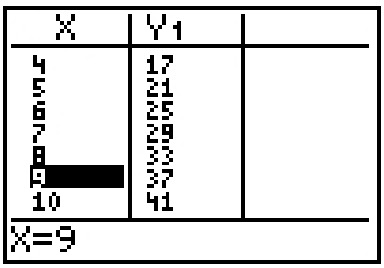


For the following questions, show how you can use a table, equation, and graph to find answers.

5. How many painted square faces would you expect to have with a tower of 9 cubes?

Table:

Possible response: 9 represents the figure number or the *x*-value in the table. Set up the table on the graphing calculator with increments of 1 and look for 9 in the first column. The number of painted square faces is 37.



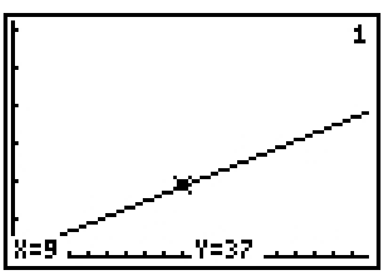
Equation:

Possible response: Substitute 9 into the function rule.

For example: 4(9) + 1 = 37, or 5 + 4(9 – 1) = 5 + 32 = 37.

Graph:

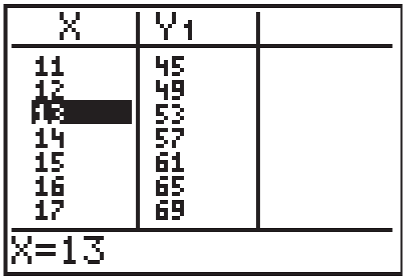
Possible response: Trace along the graph of the function rule and see that when the value of the first coordinate is 9, the second coordinate is 37. The point (9, 37) is on the line. There are 37 painted square faces on the 9th figure.



6. How many cubes would you expect to have if the number of painted square faces is 53? Explain your reasoning.

Table:

Look for 53 in the last column in the table. The value of *x* when *y* is 53 is 13. The 13th figure has 53 painted square faces.



Equation:

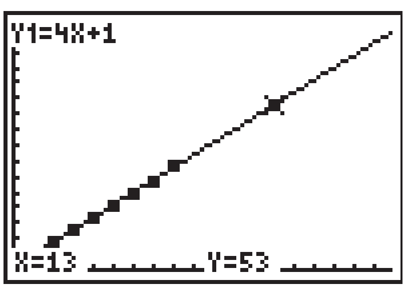
Substitute 53 for *T* in the rule and solve for *n*.

For example:

$$\begin{aligned} 4n + 1 &= 53 \\ 4n &= 52 \\ n &= 13 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 5 + 4(n - 1) &= 53 \\ 4(n - 1) &= 48 \\ n - 1 &= 12 \\ n &= 13 \end{aligned}$$

Graph:

Trace along the graph of the function rule and find where the second coordinate is about 53. The point on the graph is (13, 53). The 13th figure has 53 painted square faces.



7. How many cubes would you expect to have if the number of painted square faces is 66? Explain your reasoning.

Table:

Set up the table with increments of 1 and look for 66 in the last column in the table.

X	Y_1	
11	45	
12	49	
13	53	
14	57	
15	61	
16	65	
17	69	

X=16

The 16th figure has 65 painted square faces, and the 17th figure has 69 square faces. There is no figure with 66 square faces.

Equation:

Substitute 66 for *T* in the rule and solve for *n*.

For example:

$$\begin{aligned} 4n + 1 &= 66 \\ 4n &= 65 \\ n &= 16.25 \end{aligned}$$

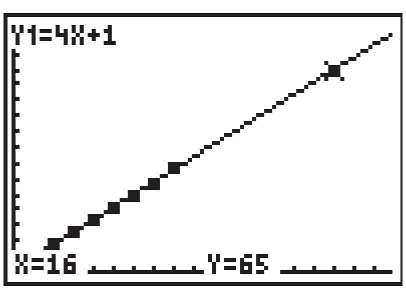
or

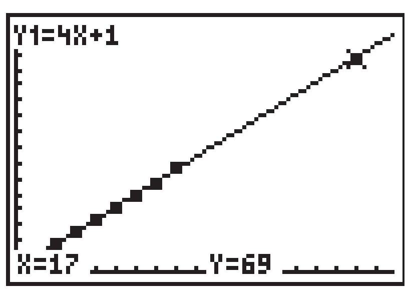
$$\begin{aligned} 5 + 4(n - 1) &= 66 \\ 4(n - 1) &= 61 \\ n - 1 &= 15.25 \\ n &= 16.25 \end{aligned}$$

No figure with complete cubes will have 66 square faces, so even though this equation can be solved, the answer does not make sense for this scenario.

Graph:

Trace along the graph of the function rule and find where the second coordinate is about 66. One point on the graph is (16, 65). The 16th figure has 65 painted square faces. The point (17, 69) is on the graph, so the 17th figure has 69 square faces. There is no figure that has 66 square faces.





Extension Questions

- Describe the domain and range for this scenario.

  The domain is whole numbers, such that $n \geq 1$, since n represents the figure number. The range is all the possible values that represent the total number of painted square faces (T). T = [5, 9, 13, . . . (1 + 4n)].

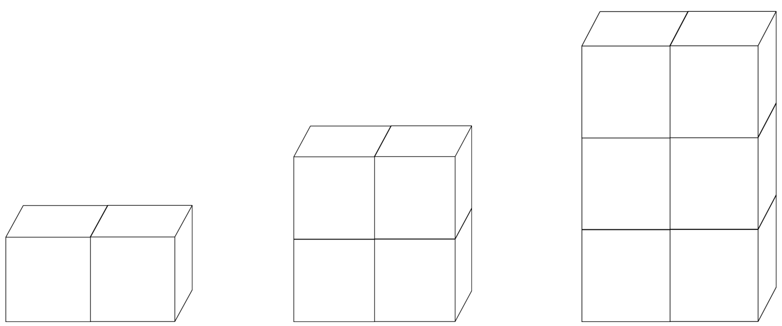
- In the graph, what does the *y*-intercept mean in the scenario?

  The y-intercept is (0, 1). It doesn't quite make sense in the scenario, since this would mean that there is 1 painted face—the top face—for the 0th tower.

- Are the data for this scenario discrete or continuous?

  The data for this scenario are discrete since it does not make sense to consider part of a tower or part of a face.

- How would your equation (or graph or table) change if there were two columns of blocks (as pictured below) instead of one?

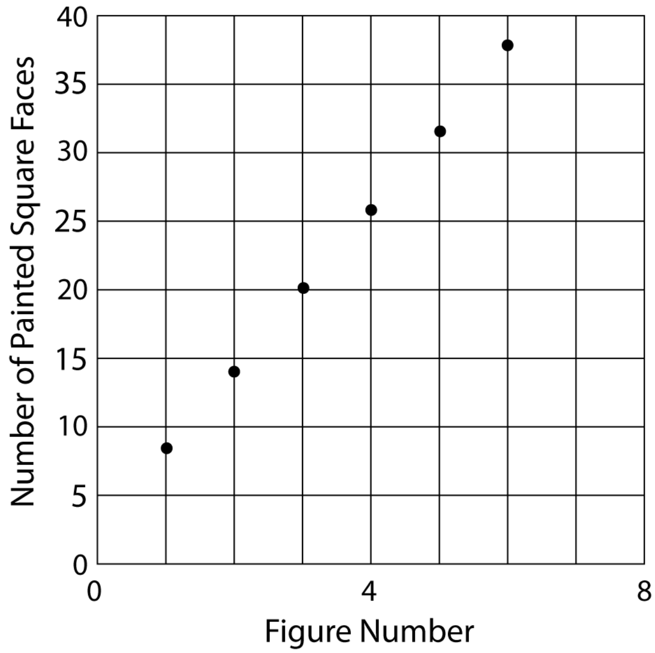


The equation for two-column towers would have a different y-intercept and a different slope since a two-column tower would have more square faces to paint.

Possible equation:

$T = 6n + 2$

The graph would have a steeper slope. The original slope was 4, but with two-column towers, the slope would be 6 since two more square faces would need to be painted for each tower.



Possible table:

The number of painted square faces would now increase by 6 for each additional figure number.

Figure Number	***Number of Painted Square Faces***
1	*8*
2	*14*
3	*20*
4	*26*
5	*32*
6	*38*
7	*44*
8	*50*

Chapter 2:

Linear Functions, Equations, and Inequalities

CDs for the Band

Bryan and his band want to record and sell CDs. The recording studio charges an initial set-up fee of $250, and each CD will cost $5.50 to burn. The studio requires bands to make a minimum purchase of $850, which includes the set-up fee and the cost of burning CDs.

1. Write a function rule relating the total cost and the number of CDs burned.

2. What are a reasonable domain and range for this problem situation?

3. Write and solve an inequality to determine the minimum number of CDs the band needs to burn to meet the minimum purchase of $850.

4. If the initial set-up fee of $250 is reduced by 50% but the cost per CD and the minimum purchase requirement do not change, will the new total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

Additional Algebra TEKS:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

Scaffolding Questions

- What is the total cost if the band purchases only one CD? Two CDs? Ten CDs?
- What are the variables? What do they represent?
- What are the constants for this situation?
- Describe in words the dependency relationship between the variables.
- What are the domain and range for this situation?
- What does the $850 represent in this situation?

Sample Solutions

1. Write a function rule relating the total cost and the number of CDs burned.

   The total cost of recording CDs is a $250 set-up fee plus the product of $5.50 and the number of CDs the band wants to purchase, so $C = 250 + 5.50n$, where C represents the total cost and n represents the number of CDs.

2. What are a reasonable domain and range for this problem situation?

   A reasonable domain for the situation is {1, 2, . . .}, since the number of CDs must be a whole number. The range values are the costs per CD at $5.50 each: {5.50, 11.00, 16.50, . . .}

3. Write and solve an inequality to determine the minimum number of CDs the band needs to burn to meet the minimum purchase of $850.

   The total cost must be greater than or equal to $850.

   Use the rule from the answer to question 1, and find the cost of various numbers of CDs.

Number of CDs	Total Cost
1	$255.50
10	$305.00
100	$800.00
110	$855.00

You know from the table that 110 CDs cost \$855.00, which is just a little over the minimum fee of \$850. Next, calculate the cost of 109 CDs using the rule; the total cost is \$250 + \$5.50(109) = \$849.50.

At \$849.50, 109 CDs cost less than \$850, so the band must burn at least 110 CDs.

Using an inequality to solve the problem:

$$\begin{aligned} 250 + 5.50n &\geq 850 \\ 5.50n &\geq 850 - 250 \\ 5.50n &\geq 600 \\ n &\geq 109.09 \end{aligned}$$

CDs must be purchased in whole number quantities. Therefore, the band can burn 109 CDs for \$849.50 but would have to pay another \$0.50 to meet the minimum purchase requirement, or they could get 110 CDs for \$855.

4. If the initial set-up fee of \$250 is reduced by 50% but the cost per CD and the minimum purchase requirement do not change, will the new total cost be less than, equal to, or more than 50% of the original total cost? Justify your answer.

   If the set-up fee is reduced by 50%, it will be 0.50(250) or \$125. The cost function becomes $C = 125 + 5.50n$.

   50% of the original cost can be calculated like this:

   $0.50(250 + 5.50n) = 0.50(250) + 0.50(5.50)n = 125 + 2.75n$

   $125 + 2.75n \leq 125 + 5.50n$

   The new cost is more than 50% of the original cost.

Extension Questions

- Suppose Bryan found another company that charges a set-up fee of \$200 and \$6.00 per CD. Would it be more economical for the band to purchase CDs from this company if it expects to spend at least \$850?

The student is expected to:

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Objective 4: The student will formulate and use linear equations and inequalities.

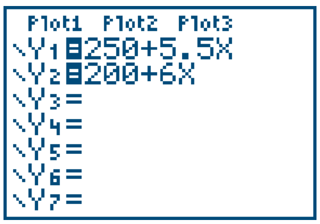
The cost function under these conditions is C = 200 + 6n.

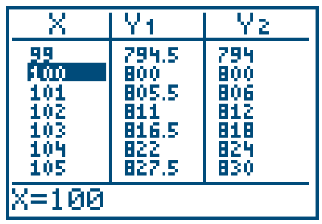
$$\begin{aligned} 200 + 6n &= 850 \\ 6n &= 850 - 200 \\ 6n &= 650 \\ n &= 108.33 \end{aligned}$$

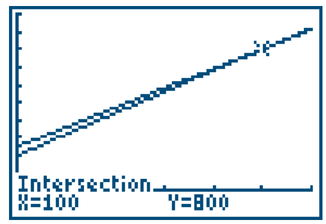
The band could purchase 108 CDs. This is not a better company to purchase from if the band plans to spend $850 since it could get 109 CDs with the first company.

- Under what circumstances would the second company be a better choice for the band to use for producing their CDs?

The tables and graphs of the two functions may be compared to determine when they are equal in cost.







The functions have the same value when x is 100. The first company's cost is greater for values of x less than 100. The second company's cost is greater for values of x more than 100. However, the first company's restriction of spending a minimum of $850 means that the first company is actually not a better deal unless the band wants to purchase more than 108 CDs.

Number of CDs	***First Company Cost Y = 250 + 5.5x***	***First Company Cost with $850 Minimum***	***Second Company Cost Y = 200 + 6x***
100	*800.00*	*850.00*	*800.00*
105	*827.50*	*850.00*	*830.00*
106	*833.00*	*850.00*	*836.00*
107	*838.50*	*850.00*	*842.00*
108	*844.00*	*850.00*	*848.00*
109	*849.50*	*850.00*	*854.00*
110	*855.00*	*855.00*	*860.00*
111	*860.50*	*860.50*	*866.00*

The Shuttle's Glide

When space shuttles return to Earth for landing, they travel in a long glide. During a return flight, an observer records the shuttle's height above Earth during the glide, beginning when the shuttle is 100 km above Earth's surface.

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

1. Using symbols and words, describe the relationship between the time in minutes and the shuttle's height above Earth's surface in kilometers.

2. Assuming that the shuttle travels at a constant speed, how far above Earth was the shuttle 2 minutes before the observer began timing? Explain your answer.

3. How long before the observer began recording data was the shuttle at 102 km above Earth's surface? How do you know?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

Scaffolding Questions

- What are the variables in this situation?
- What would you expect the height above Earth's surface to be after 5 minutes? Explain your reasoning.
- Is the relationship linear? How can you tell?
- What is the rate of change?
- What is the *y*-intercept?

Sample Solutions

1. Using symbols and words, describe the relationship between the time in minutes and the shuttle's height above Earth's surface in kilometers.

   Determine the rates of change from the table.

   10 5 5 2

time (minutes)	0	10	15	20	22
height (km)	100	97.2	95.8	94.4	93.84

   -2.8 -1.4 -1.4 -0.56

   The rate of change is –0.28 km per minute.

   The height, *h*, is the starting height plus the rate of change, –0.28, times the number of minutes, *m*. The height of the shuttle in kilometers is 100 kilometers minus the product of 0.28 kilometers per hour and the number of minutes.

   $h = 100 - 0.28m$

2. Assuming that the shuttle travels at a constant speed, how far above Earth was the shuttle 2 minutes before the observer began timing? Explain your answer.

   If the descent had begun at this constant rate at least 2 minutes before the timing began, the time would be represented by $m = -2$. So the shuttle was at 100 – 0.28(–2) or 100.56 kilometers above Earth's surface.

We could also use the table or the graph to find a solution.

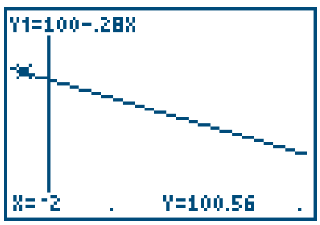
If using a graphing calculator, students can translate the equation to *y* = 100 – 0.28*x*, where *x* represents the number of minutes and *y* represents the height in kilometers. Enter the rule *y* = 100 – 0.28*x* into the graphing calculator and look at the table of values to find the value of *y* when *x* = –2.

X	Y1
-4	101.12
-3	100.84
-2	100.56
-1	100.28
0	100
1	99.72
2	99.44

X= -2

The window can also be adjusted so that the graph can be traced to find the value when *x* = –2.

WINDOW
Xmin=-3
Xmax=20.5
Xscl=5
Ymin=90
Ymax=105
Yscl=0
Xres=1



3. How long before the observer began recording data was the shuttle at 102 km above Earth's surface? How do you know?

   Students may approach this problem with a variety of strategies. For example, students may use the table in a graphing calculator and the equation *y* = 100 – 0.28*x* to determine when *y* is 102. Students will find that they need to adjust the table settings to smaller and smaller increments for the *x*-values. Shown below are possible tables that students may generate—one using increments of 1 minute, another with increments of 0.1 minutes, and the third with increments of 0.01 minutes. The calculator will round the *y*-values to the nearest hundredth, so the closest estimate that can be obtained, in answer to the question of when the

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

shuttle's altitude would be 102 kilometers, is that it would occur between –7.12 and –7.17 seconds.

X	Y_1
-9	102.52
-8	102.24
-7	101.96
-6	101.68
-5	101.4
-4	101.12
-3	100.84

X= -7

X	Y_1
-7.5	102.1
-7.4	102.07
-7.3	102.04
-7.2	102.02
-7.1	101.99
-7	101.96
-6.9	101.93

X= -7.1

X	Y_1
-7.17	102.01
-7.16	102
-7.15	102
-7.14	102
-7.13	102
-7.12	101.99
-7.11	101.99

X= -7.13

Another strategy is to solve using the equation $102 = 100 - 0.28x$, finding that $x \approx -7.14$. This means that, if the shuttle were traveling at this constant rate, then 7.14 seconds prior to when the observer began recording data, the shuttle would have been 102 km above Earth's surface.

Extension Questions

- What does the *y*-intercept represent in this situation?

  The y-intercept represents the height at time zero. In other words, the y-intercept represents the height of the shuttle when the observer first began gathering data.

- What is the *x*-intercept, and what does it mean for this problem situation?

  The x-intercept is approximately 367.14. It represents the number of minutes after the observer began recording time that the height is 0—that is, the time when the shuttle would land if it continued to descend at the same rate.

- Is this a realistic model for the descent of a shuttle?

  This is not a realistic situation. In reality, the shuttle must decrease its speed as it gets closer to landing.

Hot Air Balloon

At the West Texas Balloon Festival, a hot air balloon is sighted at an altitude of 800 feet. It appears to be descending at a steady rate of 20 feet per minute. Spectators wonder how the altitude of the balloon is changing as time passes.

1. Write a function rule (equation) to represent the relationship between the variables in this scenario.

2. How high was the balloon 5 minutes before it was sighted? Make a table of values and/or a graph that could be used to answer this question.

3. How long will it take the balloon to descend to an altitude of 20 feet? How long will it take the balloon to land?

4. At the instant the first balloon is sighted, a second balloon is also observed at an altitude of 1,200 feet and descending at a rate of 20 feet per minute. How do the descent and landing time of the second balloon compare with that of the first balloon? What does this mean graphically?

5. At the instant the first balloon is sighted, a third balloon is also observed at an altitude of 800 feet and descending at a rate of 30 feet per minute. How do the descent and landing time of the third balloon compare with that of the first balloon? What does this mean graphically?

6. At the instant the first balloon is sighted, a fourth balloon is launched from the ground. It rises at a rate of 30 feet per minute. When will the first and fourth balloons be at the same altitude? What is that altitude? What does this mean graphically?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations.

Scaffolding Questions

- What are the variables in this scenario?
- Which quantity is the dependent variable? The independent variable?
- What kind of function models the situation? How do you know?
- What decisions must you make to build a table for the function?
- What decisions must you make to graph the function?
- How can you determine the balloon's height at any given time?
- How can you determine the time it takes the balloon to reach a given height?
- How does a change in initial altitude affect the equation? The graph? The table?
- How does a change in the rate of ascent (or descent) affect the equation? The graph? The table?

Sample Solutions

1. Write a function rule (equation) to represent the relationship between the variables in this scenario.

   Possible function rules: The starting height, 800 feet, decreases at a rate of 20 feet per minute. The height (h) equals 800 minus the product of 20 and the number of minutes (m): $h = 800 – 20m$. Or, if students are using a graphing calculator, they can use y to represent height and x to represent the number of minutes: $y = 800 – 20x$.

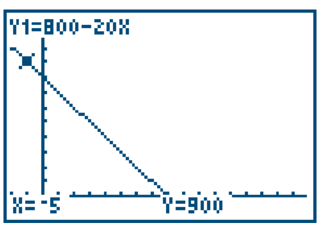
2. How high was the balloon 5 minutes before it was sighted? Make a table of values and/or a graph that could be used to answer this question.

   The value of y is 900 when m is –5. Therefore, the balloon was at 900 feet 5 minutes before it was first sighted.

Possible table:

m	$800 - 20m$	h
–5	800 – 20(–5)	900
0	800 – 20(0)	800
5	800 – 20(5)	700
10	800 – 20(10)	600
15	800 – 20(15)	500
20	800 – 20(20)	400
25	800 – 20(25)	300
30	800 – 20(30)	200
35	800 – 20(35)	100
40	800 – 20(40)	0

A graph can also be used to examine the situation:



3. How long will it take the balloon to descend to an altitude of 20 feet? How long will it take the balloon to land?

   To discover how long it will take the balloon to descend to an altitude of 20 feet, solve for m:

$$\begin{aligned} 800 - 20m &= 20 \\ -20m &= -780 \\ m &= 39 \end{aligned}$$

   It will take the balloon 39 minutes to descend to 20 feet above the ground.

   Solve $800 - 20m = 0$ for m to get $m = 40$. It will take the balloon 40 minutes to land.

   A graph or table can also be examined to determine when the height is 0.

Additional Algebra TEKS:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

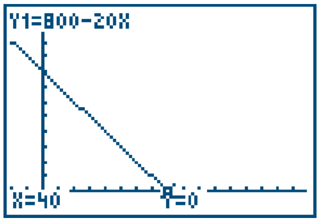
The student is expected to:

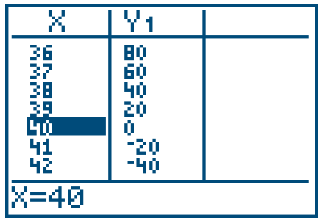
(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

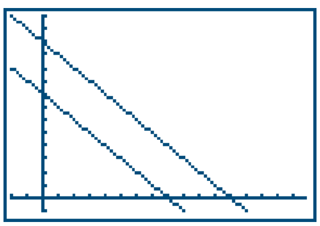




4. At the instant the first balloon is sighted, a second balloon is also observed at an altitude of 1,200 feet and descending at a rate of 20 feet per minute. How do the descent and landing time of the second balloon compare with that of the first balloon? What does this mean graphically?

   The second balloon is at a higher altitude than the first balloon but is descending at the same rate. The second balloon will take longer to land. The function rule for the second balloon is $y = 1{,}200 - 20x$. The second balloon will land in 60 minutes, or 20 minutes after the first balloon.

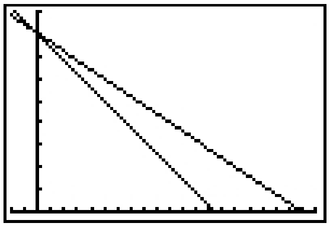
   The graph for this balloon has different *y*- and *x*-intercepts than the graph for the first balloon. The graphs for both balloons are parallel lines because they have the same slope.



5. At the instant the first balloon is sighted, a third balloon is also observed at an altitude of 800 feet and descending at a rate of 30 feet per minute. How do the descent and landing time of the third balloon compare with that of the first balloon? What does this mean graphically?

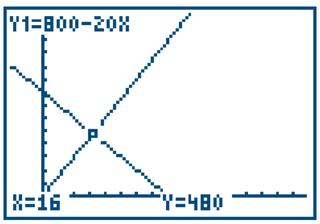
   The third balloon starts at the same height as the first but descends faster. Therefore, the third balloon will land sooner. The function rule describing this balloon is $y = 800 - 30m$. The third balloon lands in about 27 minutes, or about 13 minutes before the first balloon.

   The graph for this balloon has the same *y*-intercept as the first balloon's graph but a different *x*-intercept. The *x*-intercept for the third balloon is less than the *x*-intercept for the first balloon. The graph of the third balloon's descent is steeper than that of the first balloon's descent.



6. At the instant the first balloon is sighted, a fourth balloon is launched from the ground. It rises at a rate of 30 feet per minute. When will the first and fourth balloons be at the same altitude? What is that altitude? What does this mean graphically?

The function rule describing the fourth balloon is $y = 30x$. To see if the first and fourth balloons are ever at the same altitude, explore with tables or graphs:



X	Y1	Y2
12	560	360
13	540	390
14	520	420
15	500	450
16	480	480
17	460	510
18	440	540

X=16

We could also solve $800 - 20x = 30x$ to get $x = 16$. Sixteen minutes after the fourth balloon launches, both balloons will be at the same height, 480 feet.

Extension Questions

- If the function for the motion of a fifth balloon were $y = 700 - 20x$, how would the movement of the fifth balloon have been different from the movement of the first balloon?

  The fifth balloon would be sighted at a height of 700 feet instead of 800 feet. The rate of descent would be the same as the rate of descent of the first balloon.

- Would the fifth balloon have landed sooner or later than the first balloon? Explain how you know.

  If the fifth balloon were sighted at a lower altitude than the first balloon but descended at the same rate, it would land sooner. The x-intercept would be 700 divided by 20, which is 35 seconds.

Making Pizzas, Making Money

The CTW Pizza Company plans to produce small, square pizzas. It will cost the company $2.00 to make each pizza, and they will sell the pizzas for $5.00 each.

1. Express the profit earned as a function of the number of pizzas sold.
2. Write a verbal description of the relationship between the two variables, and then represent the relationship with a table and a graph.
3. What is the slope of the graph? What does it mean in the context of the situation?
4. Describe at least two methods for finding the number of pizzas that need to be sold to make a profit of at least $180.
5. The CTW Pizza Company found a cheaper supplier, and now it costs $0.50 less to make each pizza. Describe how this changes the function rule, graph, and table, and explain how you know.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations;

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

Scaffolding Questions

- What is the profit for one pizza? For two pizzas? Three pizzas?
- What do the variables represent in this situation?
- Can you set up a table to help you determine the relationship between the variables?
- How much profit will be made by selling 50 pizzas?
- If the CTW Pizza Company's goal is to make a profit of at least $300 a day, how many pizzas must it sell each day?
- Is the relationship between profit and number of pizzas sold proportional? How do you know?

Sample Solutions

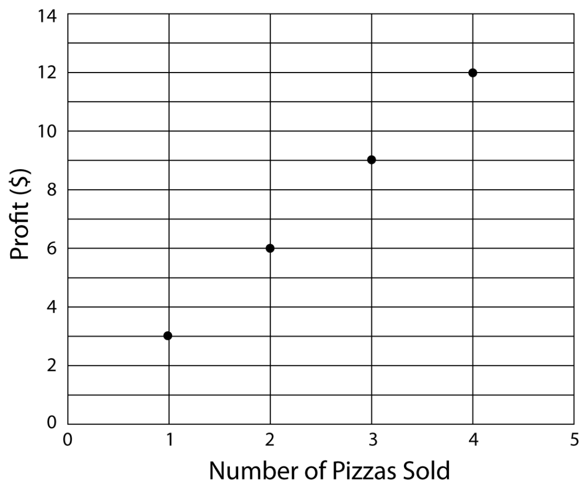
1. Express the profit earned as a function of the number of pizzas sold.

   The profit made from selling pizzas can be determined by subtracting the cost to make each pizza from the selling price. Therefore, the function rule for the profit, p, is $p = 5x - 2x$ or $p = 3x$, where x represents the number of pizzas.

2. Write a verbal description of the relationship between the two variables, and then represent the relationship with a table and a graph.

   The profit in dollars is the number of pizzas sold multiplied by $3.00. There is $3.00 profit per pizza. The more pizzas sold, the more profit made.

Number of Pizzas Sold	Profit ($)
1	3
2	6
3	9
4	12



3. What is the slope of the graph? What does it mean in the context of the situation?

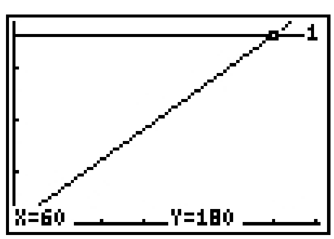
   The slope is the profit made in dollars per number of pizzas sold. The slope can be determined from the graph by looking for the rate of change: The profit goes up $3.00 for every pizza sold.

4. Describe at least two methods for finding the number of pizzas that need to be sold to make a profit of at least $180.

   Use the function $p = 3x$. We want to know when $3x$ is greater than $180, or in symbols: $180 \le 3x$. Since 3 times 60 is 180, the company must sell at least 60 pizzas to make a profit of $180 or more, or in symbols $60 \le x$. A table or graph can also be used to find y-values greater than or equal to 180. For the graph, let $Y2 = 180$, and then read from the graph where $Y1 \ge Y2$.

X	Y1	Y2
58	174	180
59	177	180
60	180	180
61	183	180
62	186	180
63	189	180
64	192	180

X=60



5. The CTW Pizza Company found a cheaper supplier, and now it costs $0.50 less to make each pizza. Describe how this changes the function rule, graph, and table, and explain how you know.

The student is expected to:

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Objective 4: The student will formulate and use linear equations and inequalities.

The new cost of making one pizza is \$1.50. The profit for each pizza is $5x - 1.50x$, or $3.50x$. The profit has increased by \$0.50 per pizza; therefore, the profit per pizza is now \$3.50. The y-intercept in the function rule $y = mx + b$ is still 0, but the slope increases by \$0.50. The slope of the graph is greater because for every pizza CTW sells it now makes \$3.50 instead of \$3.00. So the new equation for profit is $y = 3.50x$. The table also shows an increase of \$3.50 in the y-value for every increase in 1 of the x-value.

Extension Questions

- Describe how to determine the slope from your table in the answer to question 2.

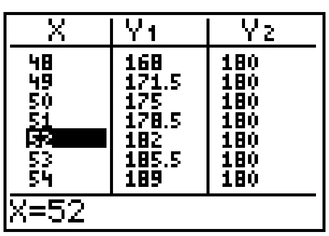
  Calculate the rates of changes by finding the difference of two y-values, divide by the difference between the corresponding x-values, and look for a constant rate of change.

- Describe how to determine the slope from the graph.

  Determine the slope from the graph by finding the ratio of the vertical change to the horizontal change between any two points on the line.

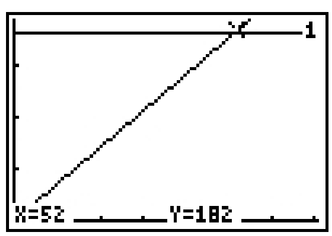
- If the CTW Pizza Company uses the cheaper supplier, how many fewer pizzas do they need to sell in order to make a profit of at least \$180?

  Using a graphing calculator with Y1 = 3.50x and Y2 = 180, we can learn that CTW needs to sell at least 52 pizzas to make a profit of at least \$180, which is 8 fewer pizzas than it needed with the first supplier.

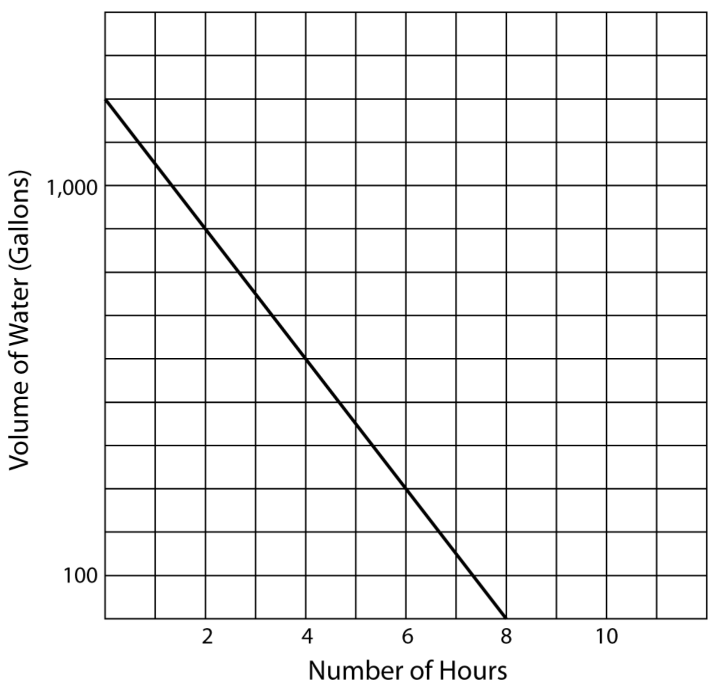


X	Y1	Y2
48	168	180
49	171.5	180
50	175	180
51	178.5	180
52	182	180
53	185.5	180
54	189	180

X=52



Draining Pools



The graph shows the relationship between the amount of water in a pool and the number of hours that have elapsed since a pump began to drain the pool.

1. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.
2. How much water is in the pool after 4 hours and 20 minutes? Describe your solution strategy.
3. How many hours after the pump began draining the pool does the pool contain 720 gallons of water? Describe two methods for determining your answer.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities;

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

Scaffolding Questions

- Define the independent and dependent variables for this problem situation.
- What type of relationship does the graph represent?
- How much water was in the pool when the pumping started? What role will this number play in the function rule? What role will this number play in the graph?
- How much water remains in the pool after 2 hours? After 4 hours? After 6 hours? Organize your responses in a table.
- At what rate is the amount of water decreasing per hour?
- Use the rate of change and the starting volume in the pool to write a function rule.
- What are the domain and range for the problem situation?

Sample Solutions

1. Describe verbally and symbolically the relationship between the amount of water in the pool and the number of hours that have elapsed since the draining began.

   The amount of water in the pool at time 0 is 1,200 gallons.

   The graph is a straight line, which means the water is draining at a constant rate. It takes 8 hours to drain the pool completely—that is, to the point that 0 gallons remain. The rate of change per hour is 1,200 gallons divided by 8 hours, or 150 gallons per hour. Because the amount of water is decreasing, the rate of change is –150 gallons per hour.

   The amount of water in the pool is the starting value plus the product of the rate of change and the number of hours. (Or, for a draining pool, it is the starting value decreased by the product of the rate of 150 gallons per hour and the number of hours the pool has been draining.)

Let w be the amount of water in the pool at time t in hours.

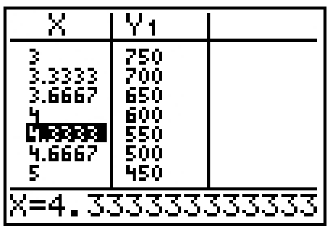
$w = 1{,}200 + (-150)t$

$w = 1{,}200 - 150t$, where t is any number from 0 to 8, inclusive

2. How much water is in the pool after 4 hours and 20 minutes? Describe your solution strategy.

The time is 4 hours and 20 minutes, or $4\frac{1}{3}$ hours.

We can obtain this answer using a graphing calculator if we set ΔTbl = 1/3 for the function $y = 1{,}200 - 150x$.



Or we can evaluate the function rule at $t = 4\frac{1}{3}$:

$w = 1{,}200 - 150(4\frac{1}{3}) = 1{,}200 - 650 = 550$

The amount of water in the pool after 4 hours and 20 minutes of draining is 550 gallons.

3. How many hours after the pump began draining the pool does the pool contain 720 gallons of water? Describe two methods for determining your answer.

When the amount of water left in the pool is 720 gallons, $y = 720$.

One way to solve the problem is by creating and solving an equation:

$$\begin{aligned} 720 &= 1{,}200 - 150t \\ t &= 3.2 \end{aligned}$$

There are 720 gallons of water in the pool after 3.2 hours, or 3 hours and 12 minutes.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Texas Assessment of Knowledge and Skills:

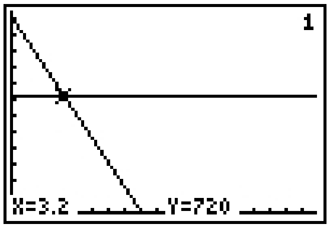
Objective 4: The student will formulate and use linear equations and inequalities.

A table or graph can also be used to determine at what point the amount of water is 720 gallons. The equation 720 = 1,200 – 150*x* asks for where the graphs of two functions intersect:

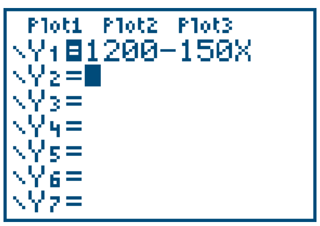
Y1 = 1,200 – 150*x*

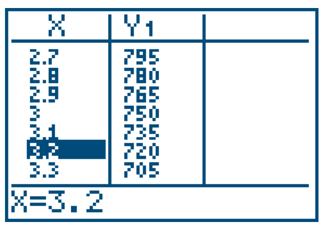
Y2 = 720

Graph and trace to find the solution.



To solve with a table, set the table minimum at 1 and increments at 0.1, and scroll down the table to find the value when *y* = 720 at *x* = 3.2. In 3.2 hours, the amount of water in the pool is 720 gallons.





Extension Questions

- What are the mathematical domain and range for the function rule you have written?

  For the general function rule not restricted by the draining pools scenario, the domain is the set of all real numbers. The range is the set of all real numbers.

- Describe the domain and range for this problem situation and explain why you selected this domain.

  The domain is the set of all real numbers from 0 to 8 inclusive, the values for the time elapsed. The domain values must be non-negative numbers and must give non-negative range values. The pool is empty after 8 hours.

  The range is the set of real numbers that show the amount of water (in gallons) in the pool: 0 to 1,200.

- How much time has elapsed if the pool is half empty?

  The original amount of water in the pool was 1,200 gallons, so when it is half empty, it contains 600 gallons. The pool ishalf empty at 4 hours. Note that this is one-half the time it takes to empty the pool.

- If half of the water drains in half the total time needed to empty the pool, predict how much of the total time has elapsed if one-third of the water has drained from the pool. Explain your reasoning.

  Draining one-third of the water takes one-third of the time it takes to drain the pool completely. There is a proportional relationship between the time and the portion of the water that has been drained.

  The time it takes to drain the pool is $\frac{1{,}200 \text{ gallons}}{150 \text{ gallons per hour}}$*, or 8 hours.*

  If one-third of the pool is drained, two-thirds of the pool volume remains.

  $\frac{2}{3}(1{,}200) = 1{,}200 - 150x$

  $150x = \frac{1}{3}(1{,}200)$

  $x = \frac{1}{3} \bullet \frac{1{,}200}{150}$

  $x = \frac{1}{3}(8)$

  Let f be the fractional portion of the pool drained. The part remaining is 1 – f.

  $(1 - f)1{,}200 = 1200 - 150x$

  $150x = 1{,}200 - (1 - f)1{,}200$

  $150x = f1{,}200$

  $x = f\left(\frac{1{,}200}{150}\right)$ *or 8f*

  Thus, if the amount of water drained is (f)(1,200), the time it takes to drain that amount of water is (f)(8), or f times the amount of time it takes to drain the pool.

- If the initial volume of the pool were 1,500 gallons, but the pool drained at the same rate, how would this affect your graph?

  The only value in the function that would change would be the y-intercept.

  $y = 1{,}500 - 150x$

  The graph would be a straight line parallel to the original line but with a y-intercept of 1,500 and an x-intercept of 10.

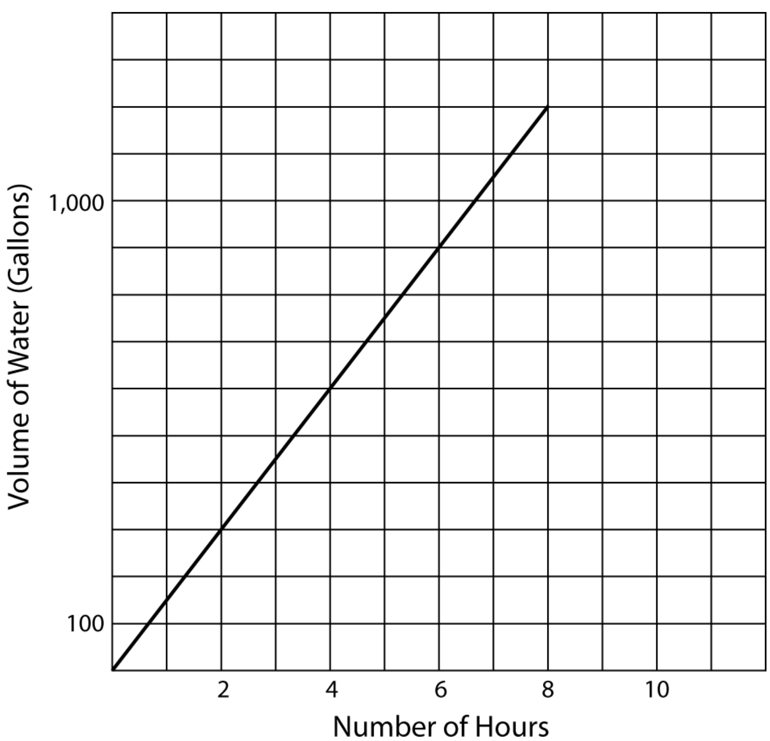
- If the pool started with the same amount of water (1,200 gallons) but emptied at 100 gallons per hour, how would the graph change?

The rate of change, or slope, would be –100. The function would be $y = 1{,}200 - 100x$.

Both the original graph and the new graph would have the same y-intercept, but since the new graph would descend at a slower rate, it would have a greater x-intercept than the original, meaning that it would take the pool longer to empty.

- Suppose an empty pool is filled at the same rate and with the same capacity of 1,200 gallons. Sketch the graph and write the function to represent this new situation.

  The function is $y = 150x$, where x varies from 0 to 8. Since the capacity of the pool is 1,200 gallons, the graph terminates at the point (8, 1,200); the graph is a line segment.



T-Shirts

Several school organizations want to sell t-shirts to raise money. Reynaldo, the treasurer for the Science Club, has found four different companies that can fill the t-shirt orders. Each function below represents the cost of placing a t-shirt order as a function of the number of t-shirts purchased.

Company A	$c = 5t$
Company B	$c = 3.25t + 55$
Company C	$c = 3t + 100$
Company D	$c = 6t - 55$

1. For each t-shirt company, write a verbal description or story that explains the company's method for calculating the cost of placing a t-shirt order. Be sure your description includes how each number in a particular function rule could be used to calculate the cost for that company.

2. Describe the differences in the methods used by Companies A and B. How are those differences represented in the function rules?

3. Make a table for each function, and then graph the four functions on the same graph.

4. Write two questions that could arise from the scenarios. Answer each one using either the graph or the table.

5. For each function, describe the difference in the domain for the function and the domain for your problem situation.

6. For each function, describe the difference in the range for the function and the range for the problem situation.

Notes

Materials:

One graphing calculator per student

Graph paper

Colored pencils (optional)

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify the mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions

- What are the variables in this scenario? Which is dependent? Independent?
- For Company A, what does the 5 represent?
- For Company B, which constant represents the cost per t-shirt?
- For Company B, what might the constant 55 represent?
- For Company C, what does the 3 represent?
- For Company C, what might the constant 100 represent?
- For Company D, which constant represents the cost per t-shirt?
- For Company D, what might the constant –55 represent?
- What are some things you might consider if you had to decide which company to use based on its pricing method?

Sample Solutions

1. For each t-shirt company, write a verbal description or story that explains the company's method for calculating the cost of placing a t-shirt order. Be sure your description includes how each number in a particular function rule could be used to calculate the cost for that company.

   Verbal descriptions or stories will vary. Examples:

   Company A. The Math Club treasurer made a deal with the manager of Company A. If the Math Club places an order, the cost will be \$5.00 per shirt.

   Company B. The Spanish Club feels Company B offers a better deal because the club will get their t-shirts for only \$3.25 each, although they do have to pay a \$55.00 set-up fee.

   Company C. The Math Club found another deal, this time with Company C. They will pay only \$3.00 per shirt with a \$100.00 set-up fee.

Company D. The president of the freshman class thinks she has the best deal: Her father's friend, who runs Company D, will sell shirts for $6.00 each and give a $55.00 discount.

2. Describe the differences in the methods used by Companies A and B. How are those differences represented in the function rules?

   Companies A and B charge different amounts per shirt. This is represented as the coefficient of *x* in each function rule. Company A charges $5.00 per shirt, and Company B charges $3.25 per shirt. Company B also charges a fee of some kind as represented by "+55" in the function rule.

3. Make a table for each function, and then graph the four functions on the same graph.

Company A
$c = 5t$

t	c
0	0
10	50
20	100
30	150
40	200
50	250
60	300

Company B
$c = 3.25t + 55$

t	c
0	55
10	87.50
20	120
30	152.50
40	185
50	217.50
60	250

Company C
$c = 3t + 100$

t	c
0	100
10	130
20	160
30	190
40	220
50	250
60	280

Company D
$c = 6t – 55$

t	c
0	–55
10	5
20	65
30	125
40	185
50	245
60	305

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

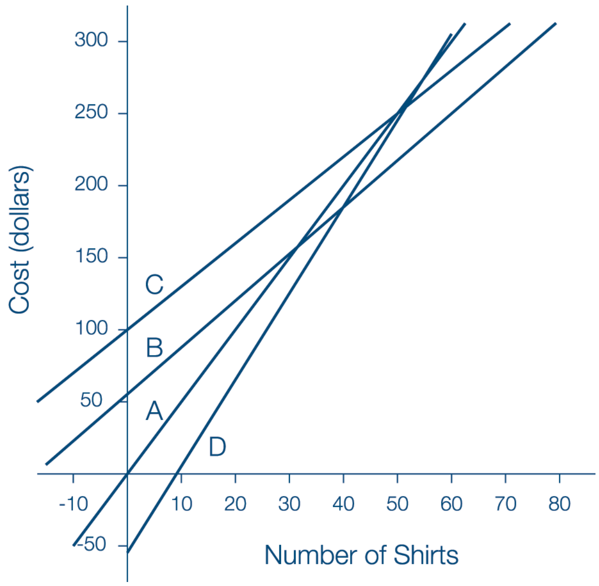
The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3: The student will demonstrate an understanding of linear functions.



4. Write two questions that could arise from the scenarios. Answer each one using either the graph or the table.

   Answers will vary. Examples of questions:

   - From which t-shirt company should a group purchase shirts if they plan to purchase 50 shirts?

     The table and graph show that the cost for 50 shirts is the least with Company B.

   - When does Company D give the better deal?

     The table and graph show that Company D is the better deal for up to 40 shirts.

   - Which company is never the least expensive?

     From the graph, we can see that Company A is never the least expensive. It also looks as if Company C is never the least expensive; however, since it charges the lowest rate per shirt, it will be the best deal if an organization wants to order at least 180 t-shirts.

5. For each function, describe the difference in the domain for the function and the domain for your problem situation.

   The domain of each function is all real numbers because each function is a linear function. For this problem situation, the domain values must be whole numbers because shirts cannot be purchased in fractions.

6. For each function, describe the difference in the range for the function and the range for the problem situation.

The range of each function is all real numbers. However, in this problem situation, the amounts are restricted to dollar values. For example, with Company B, the amounts must be \$55 plus a whole number multiple of \$3.25.

Extension Questions

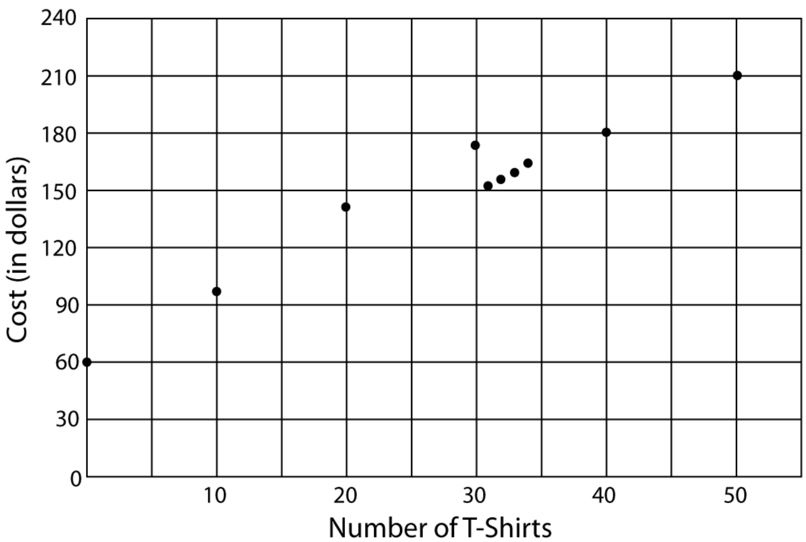
- If Company A had decided to give a discount of \$40, how would that change the function rule?

  The function rule would become $c = 5t - 40$.

- Does Company B always offer the lowest cost?

  No. Company D charges the least for up to 40 shirts. Companies B and D have the same cost for 40 shirts. Company B charges the least for between 41 and 179 shirts. Companies B and C have the same cost at 180 shirts. For more than 180 shirts, Company C has the lowest cost. These cost breakdowns can be determined by examining the graph or table, or by solving symbolically.

- A new company, Company E, charges a set-up fee of \$60 plus \$3.75 for each t-shirt; however, if a customer orders more than 30 shirts, then Company E charges \$60 plus \$3.00 per t-shirt. Make a table and graph to represent Company E's pricing plan. Discuss how this company's plan differs from the others.



Number of T-shirts	Cost (in dollars)
0	*60*
10	*97.50*
20	*135*
30	*172.50*
31	*153*
32	*156*
33	*159*
34	*162*
40	*180*
50	*210*

This plan is different because the rate changes at 31 shirts.

Note to the teacher: *The Algebra I TEKS do not require students to write function rules for this type of scenario; however, for your information, the function is a combination of two linear functions:*

$$c = 60 + 3.75t \text{ for 0 to 30 shirts, and}$$

$$c = 60 + 3.00t \text{ for 31 or more shirts}$$

A function that consists of two functions is called a ***piecewise function****, and students formally learn about these in precalculus.*

Which Is Linear?

Four function rules were used to generate the following four tables:

I

x	*y*
–1	6
0	8
1	10
2	12
3	14

II

x	*y*
0	5
3	5
6	5
9	5
12	5

III

x	*y*
–2	–5
–1	–4.5
0	–4
3	–2.5
4	–2
5	–1.5

IV

x	*y*
–1	0.5
0	0
1	0.5
2	2
3	4.5
4	8
5	12.5

1. Which table or tables represent linear relationships? Explain how you decided.
2. Make a graph of the data in each table. Describe how the graphs are related.
3. Write a function rule for each linear relationship and explain how you developed each rule.

Notes

Materials:

One graphing calculator per student

Graph paper

Algebra TEKS Focus:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

Scaffolding Questions

- As the x-values increase, what happens to the y-values?
- How are the patterns in the tables similar?
- How are the patterns in the tables different?
- What must be true about a function in order for it to be linear?
- How can you decide if a relationship is linear by looking at its table?
- What two numbers must you determine to write the linear function rule?

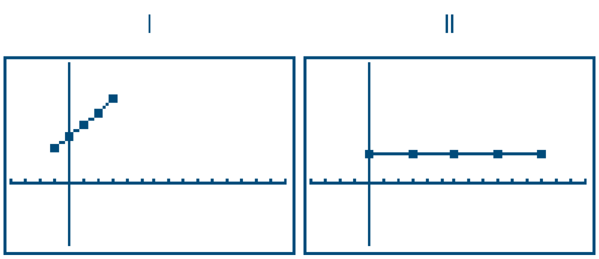
Sample Solutions

1. Which table or tables represent linear relationships? Explain how you decided.

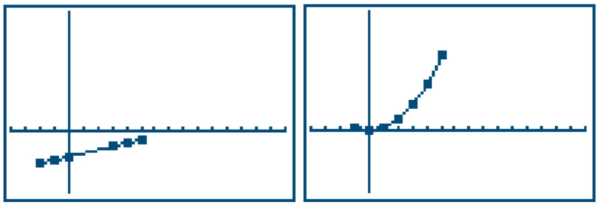
   In Table I, as x increases by 1, y increases by 2. In Table II, as x increases by 3, y stays constant. In Table III, as x increases by 1, y increases by 0.5. In Table IV, there is not a constant rate of change. Therefore, Tables I, II, and III represent linear relationships. The graphs of these sets of points form lines. As x increases by a constant number, y also increases by a constant number.

2. Make a graph of the data in each table. Describe how the graphs are related.

   The scatterplots of the data are shown below in connected mode.



III IV



Three of the graphs show a linear relationship: **I**, **II**, and **III.** The graph of Table IV is not linear. The graph of Table I has the steepest line. The graph of Table **II** has a slope of zero.

3. Write a function rule for each linear relationship and explain how you developed each rule.

   In Table **I**, the rate of change is 2 because as x increases by 1, y increases by 2**.** The point (0, 8) indicates that the line crosses the y-axis at 8, so 8 is the y-intercept.

   The function rule for Table I is $y = 8 + 2x$, or $y = 2x + 8$.

   In Table **II**, the rate of change is 0 because there is no change in y as x changes. The point (0, 5) shows where the line crosses the y-axis. The function rule for Table **II** is $y = 5$.

   In Table **III**, the rate of change is 0.5 because the ratio of the change in y to the change in x is 0.5. The point (0, –4) indicates that the line intersects the y-axis at –4. The function for Table **III** is $y = 0.5x - 4$, or $y = -4 + 0.5x$.

Extension Questions

- For each function rule you wrote for your answers to question 3, describe a real-world scenario for which that function rule might be used. Also describe the domain and range for each scenario.

  Answers will vary. Students may determine separate scenarios for each function rule, or they may suggest a scenario that includes all the function rules. Sample responses:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Last year, three sisters opened savings accounts at a nearby bank. Recently, each of them decided to check their balance weekly, and two of them will start adding money to their savings accounts each week.

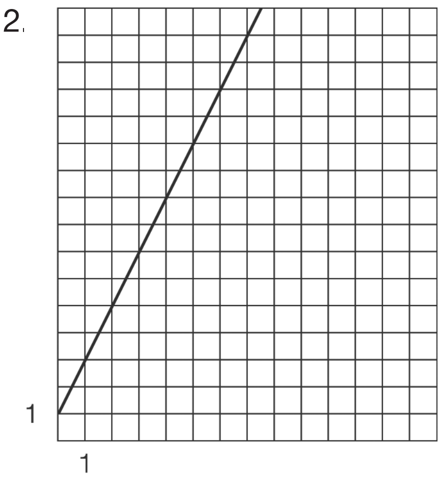
Rachel started with $8.00 in her account and will add $2.00 each week (Table I). Roxanne has $5.00 in her account and does not want to add any money each week (Table II). Rene was recently charged an annual fee, so she has –$4.00 in her account and plans to deposit $0.50 per week (Table III).

The domain for each sister's scenario is the number of weeks since they started checking their balance and is therefore whole numbers. The ranges for each sister will be the amount of money in each account. Rachel: {8.00, 10.00, 12.00, . . .}, Roxanne: {5.00}, Rene: {–4.00, –3.50, –3.00, . . .}

Finding Pairs

Six different functions are represented below. Compare and contrast the function rules, tables, graphs, and verbal descriptions. Identify which pairs represent the same functional relationship and explain how you know.

1. $y = 2x - 1$

2. 

3. The plant was growing at a rate of $1\frac{1}{2}$ inches per week.

4. The Math Club's treasurer found a place that will make the club custom t-shirts for $5.00 each, but there is a set-up fee of $50.

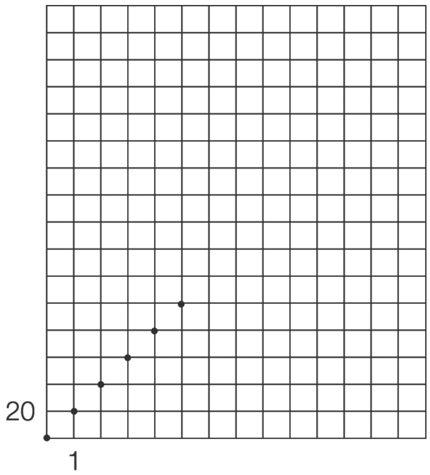
5.

x	*y*
0	50
1	55
2	60
3	65
4	70
5	75

6. $y = 2x + 1$

7. $f(x) = 1.5x$

8.



9.

x	y
–2	–5
–1	–3
0	–1
1	1
2	3
3	5

10.

x	y
–2	1.5
–1	1.5
0	1.5
1	1.5
2	1.5
3	1.5

11.

x	*y*
0	0
1	20
2	40
3	60
4	80
5	100

12. $y = 1.5$

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- What type of relationship is described by each graph, table, function rule, and/or verbal description?
- What patterns do you notice in the tables? What is causing these patterns?
- How can you determine if there is a constant rate of change in a graph? A table?
- How is a constant rate of change represented in a verbal description? An equation or function rule?
- In an equation or function rule, what do the coefficients of x represent? If another number is added or subtracted, what does it represent?
- Can you make a connection between the graph in number 2 and any of the tables? Equations? Verbal descriptions?

Sample Solutions

The following pairs represent the same functional relationship:

Table 5 and Situation 4

The \$50 set-up fee matches the entry $x = 0$, $y = 50$ in Table 5. The \$5.00 per shirt matches the increments of 5 in the y-values for Table 5.

Table 9 and Function Rule 1

The point (0, –1) from Table 9 indicates a y-intercept of –1. The increase of 2 in the y-values, for every corresponding increase of 1 in the x-values, means that the slope is 2. The function rule is $y = 2x - 1$.

Table 11 and Graph 8

The point (0, 0) indicates that the graph of the function passes through the origin. In Table 11, the slope of the line is 20, since for every unit increase in x, y increases 20 units.

Table 10 and Function Rule 12

Every *y*-value in Table 10 is 1.5, which means *y* is always equal to 1.5.

Function Rule 6 and Graph 2

The graph of the function passes through point (0, 1), which means that the *y*-intercept is 1. The coefficient of *x* is 2, and 2 is also the slope of the line in Graph 2.

Function Rule 7 and Situation 3

The coefficient of *x* is 1.5 in Function Rule 7, and that is the same as the rate of change for the plant.

Extension Questions

- How do the numbers in the function rules affect the tables?

  The coefficient, m, of x in the function y = mx + b is the slope or rate of change that can be determined from the table. The constant, b, in the function corresponds to the y-intercept, the data point (0, b).

- Make a list of patterns you notice in the tables, and explain what causes the patterns.

  In Table 5, as x increases by 1, the values of y increase by 5. This is because the constant rate of change is 5. In a function rule, 5 will show up as the coefficient of x.

  The constant rate of change for Table 9 is 2.

  The constant rate of change for Table 10 is 0. The y-values do not change as x changes. In the equation y = mx + b, the slope is zero. Therefore, y = 0x + b or y = b for every x. The constant rate of change shows that the coefficient of x is 0. This indicates that the matching function does not have a term with an x.

  The constant rate of change for Table 11 is 20.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

First Aid Supplies

Mark is the trainer for the Little Kids soccer team. He needs to buy 50 bandages and 3 ice packs for the team's first-aid kit. Mark can spend no more than \$12. Prices vary for different ice packs, but every brand of bandage costs the same: \$4.50 for 50 bandages. The sales tax is 9%.

1. Write an inequality to identify the amount Mark can spend on ice packs. Identify your variable.
2. What is the most Mark can spend for each ice pack and keep within the \$12 budget? Show how you know.
3. Suppose the booster club gives Mark another \$10 to spend on ice packs. How would this change the inequality that you wrote and your answer to question 2? Describe your solution verbally and algebraically.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

Scaffolding Questions

- Is it possible for Mark to buy the bandages and ice packs for less than $12?
- How does the tax rate affect your inequality?

Sample Solutions

1. Write an inequality to identify the amount Mark can spend on ice packs. Identify your variable.

   The cost of the bandages is $4.50. The total cost of the supplies depends on the price of the ice pack. Let x = the price of 1 ice pack.

   Since Mark needs 3 ice packs, the cost of the ice packs is 3 multiplied by the price of 1 ice pack, or $3x$. The expression for the cost of the ice packs plus the cost of the bandages is $3x + 4.50$.

   The sales tax is 9% of the cost, or $0.09(3x + 4.50)$.

   The total cost including the tax must be less than or equal to $12:

   $(3x + 4.50) + 0.09(3x + 4.50) \leq 12.00$,
   or $1.09(3x + 4.50) \leq 12$

   Other students may identify the variable as the total amount that can be spent on ice, i. An inequality for that way of thinking about this scenario might be:

   $1.09(4.50 + i) \leq 12$

2. What is the most Mark can spend for each ice pack and keep within the $12 budget? Show how you know.

   Using the first equation above, students may solve this way:

$$
\begin{aligned}
3x + 4.50 + 0.27x + 0.41 &\leq 12.00 \\
3.27x + 4.91 &\leq 12.00 \\
3.27x &\leq 7.09 \\
x &\leq 2.168195719
\end{aligned}
$$

   x represents a dollar amount and must be expressed to the nearest hundredth. However, if you round up to

$2.17, the cost of supplies is more than $12:

$$3(2.17) + 4.50 + 0.09(3(2.17) + 4.50) = 12.009$$

Mark can spend no more than $12, so he can pay up to $2.16 per ice pack.

Using the second equation above, students would get that $i \leq 6.50$ if the amount is rounded to the nearest hundredth. Dividing the total amount that can be spent on ice, Mark would have $6.50 \div 3 \approx 2.17$. However, if each bag were $2.17, that would be more than $6.50, so Mark can only spend $2.16 per pack of ice.

3. Suppose the booster club gives Mark another $10 to spend on ice packs. How would this change the inequality that you wrote and your answer to question 2? Describe your solution verbally and algebraically.

   If the booster club gave Mark an additional $10 to spend for the ice packs, the only difference in the solution would be the total amount budgeted for purchase—$22 rather than $12. The additional money would allow Mark to purchase ice packs that are more expensive or to buy more bags of ice.

   Let x = the price of an ice pack.

   The total cost may now be less than or equal to $12.00 plus $10.00.

$$\begin{aligned}
(3x + 4.50) + 0.09(3x + 4.50) &\leq 22.00 \\
3x + 4.50 + 0.27x + 0.41 &\leq 22.00 \\
3.27x + 4.91 &\leq 22.00 \\
3.27x &\leq 17.09 \\
x &\leq 5.226
\end{aligned}$$

   If this answer is rounded to $5.23 and Mark spent $5.23 per ice pack, the cost would be $27.01, so he can spend at most $5.22 per ice pack. He may also end up buying more ice packs, because more expensive packs might mean heavier packs, and the team might need lighter packs or just a greater number of packs.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

Extension Questions

- Suppose Mark found the bandages on sale at 50 bandages for $3, and he could spend no more than $15. How much could he spend per ice pack?

  Let x = the price of an ice pack.

  $$\begin{aligned} (3x + 3.00) + 0.09(3x + 3.00) &\leq 15.00 \\ 3x + 3.00 + 0.27x + 0.27 &\leq 15.00 \\ 3.27x + 3.27 &\leq 15.00 \\ 3.27x &\leq 11.73 \\ x &\leq 3.587 \end{aligned}$$

  If the bandages are on sale for $3 and Mark can spend up to $15, he could pay up to $3.58 per ice pack.

- How would a 7% tax rate influence Mark's purchase if the bandages are still on sale for $3 and he has a maximum of $15?

  Let x = the price of an ice pack.

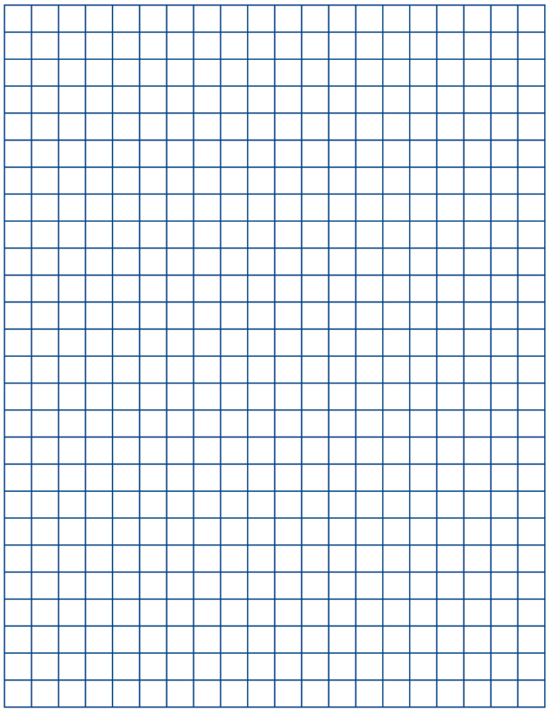
  $$\begin{aligned} (3x + 3.00) + 0.07(3x + 3.00) &\leq 15.00 \\ 3x + 3.00 + 0.21x + 0.21 &\leq 15.00 \\ 3.21x + 3.21 &\leq 15.00 \\ 3.21x &\leq 11.79 \\ x &\leq 3.673 \end{aligned}$$

Stretched Spring

The table below reports a sample of the data collected in an experiment to determine the relationship between the length of a spring and the mass of an object hanging from it. The length of the spring depends on the mass of the object.

Length versus Mass

Mass (kg)	Length (cm)
50	5.0
60	5.5
70	6.0
80	6.3
90	6.8
100	7.1
110	7.5
120	7.7
130	8.0
140	8.6
150	8.8
160	9.2
170	9.5
180	9.9
190	10.3



1. Construct a scatterplot of the data. Describe verbally and symbolically the functional relationship between the length of the spring and the mass suspended from it.

2. Predict the length of the spring when a mass of 250 kilograms is suspended from it. Describe the method you used to make your prediction.

3. Predict the mass that would stretch the spring to 15 centimeters. Explain your reasoning.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function

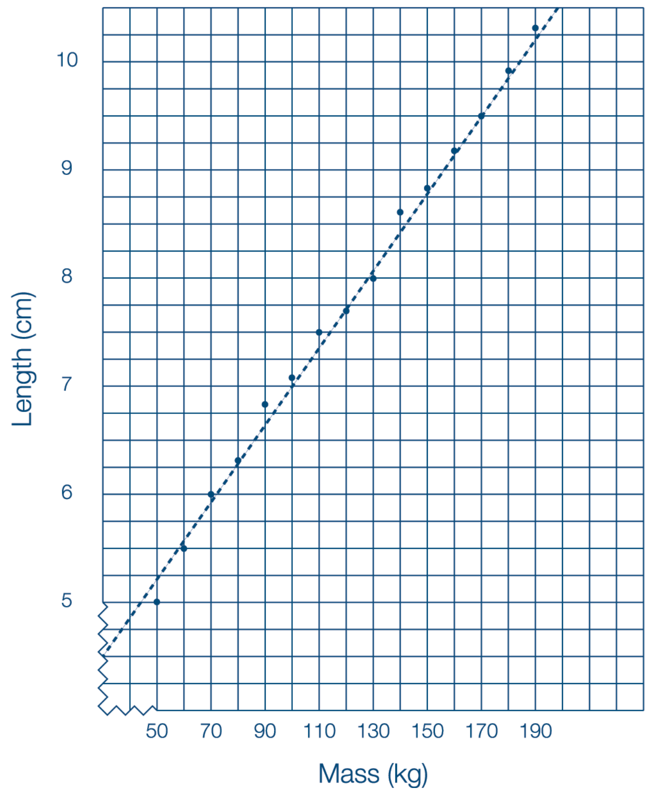
Scaffolding Questions

- How can you organize the data?
- What do you need to consider when constructing a scatterplot of the data?
- What do you need to consider when determining a reasonable interval of values and scale for each axis?
- Which function type (linear, quadratic, exponential, inverse variation) appears to best represent your scatterplot?
- What do you need to know when determining a particular function model for your scatterplot?

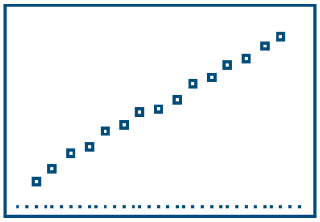
Sample Solutions

1. Construct a scatterplot of the data. Describe verbally and symbolically the functional relationship between the length of the spring and the mass suspended from it.

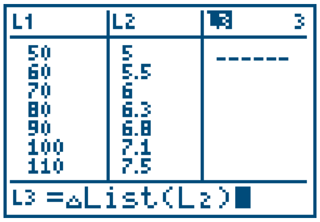
   The scatterplot is nearly linear.

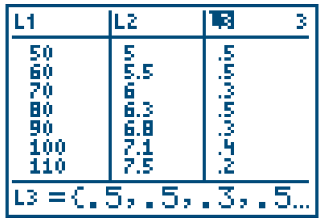


The data points may also be entered into a graphing calculator to create the scatterplot.



The consecutive difference in the length values may be computed using the list feature.





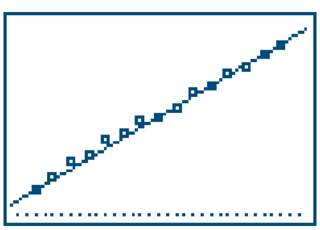
The average (mean) of these consecutive differences is approximately 0.38. The difference in the consecutive mass values is 10. The rate of change may be approximated as 0.38 divided by 10, or 0.038. Using 0.038 centimeters per kilogram as the rate of change, a trend line is of the form $y = 0.038x + b$. Use any other data point to find a possible value for b. If the point (50, 5) is used, the value is 3.1.

$5 = 0.038(50) + b$

$b = 5 - 0.038(50) = 3.1$

$y = 0.038x + 3.1$

The graph of this line is an approximate trend line for the data.



(Note: Students may discover that they can use the regression feature of the calculator to find a line of best fit. However, this calculator feature seems like a trick unless teachers help students understand the regression concept. Remember, this lies beyond the scope of Algebra I.)

represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

2. Predict the length of the spring when a mass of 250 kilograms is suspended from it. Describe the method you used to make your prediction.

   To predict the length of the spring, evaluate the function for $x = 250$.

   $y = 0.038(250) + 3.1$

   $y = 12.6$ cm

   A mass of 250 kilograms will stretch the spring to an approximate length of 12.6 centimeters.

3. Predict the mass that would stretch the spring to 15 centimeters. Explain your reasoning.

   To predict the mass that would stretch the spring to 15 centimeters, use the function rule and solve the resulting equation:

   $$\begin{aligned} 0.038x + 3.1 &= 15 \\ 0.038x &= 11.9 \\ x &= 313.16 \text{ kg} \end{aligned}$$

   A mass of about 313.16 kilograms stretches the spring to a length of 15 centimeters.

Extension Questions

- In the experiment, how did the mass suspended from the spring change and, in general, how did this affect the length to which the spring stretched?

  The mass suspended from the spring started at 50 kilograms and increased by 10 kilograms at intervals until it reached a mass of 190 kilograms. The initial amount of stretch (at 50 kilograms mass) was 5 centimeters, and the stretch increased by small amounts (0.2 to 0.5 centimeters) with each additional 10 kilograms of mass.

- With each 10-kilogram increase in mass, the spring stretched an additional 0.2 to 0.5 centimeters. What does this suggest about the functional relationship between spring length and mass?

  The relationship is approximately linear. As the mass increases in constant amounts, the additional length that the spring stretches is nearly constant. This suggests that a constant rate of change can be used to model the situation.

- How long is the spring when no mass is suspended from it?

  Use the function that models the situation, $y = 0.038x + 3.1$. When the spring has no mass attached to it, the value of x is 0, and y is 3.1 centimeters long.

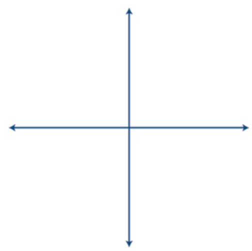
- Suppose the initial length of the spring is changed to 6.8 centimeters, and we suspend mass from the spring in increments of 20 kilograms instead of 10. How would this change the function that models this situation?

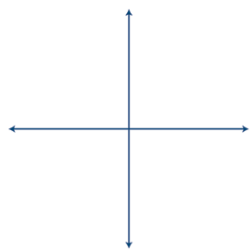
  Increasing the weight increments to 20 kilograms would not significantly affect how much the spring stretches; collecting data for different weights would not change the function that models the situation. If the spring has the same stretching ability—and the weights attached to it are within reasonable physical constraints—the rate of change would still be 0.038 centimeters per kilogram of mass. Changing the initial length of the spring to 6.8 centimeters would change the y-intercept to 6.8. The function that models the situation would be $y = 0.038x + 6.8$.

Analysis of a Function

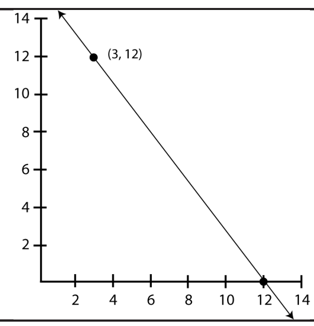
I. If given a function rule, sketch a complete graph that represents that function. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.

II. Determine the domain and range for each mathematical situation.

<table>
<tr><th></th><th>Function</th><th>Graph or Table</th><th>Domain and Range</th></tr>
<tr><td>1.</td><td>$f(x) = 5 - 2x$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>2.</td><td>$y = -2$</td><td></td><td>Domain:

Range:</td></tr>
<tr><td>3.</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>6</td><td>6</td></tr><tr><td>12</td><td>9</td></tr></table></td><td>Domain:

Range:</td></tr>
<tr><td>4.</td><td></td><td></td><td>Domain:

Range:</td></tr>
</table>

III. Describe the similarities and differences among the functions given above.

IV. Describe a practical situation that each function might represent. What restrictions on the mathematical domain and range of the function does the situation require? How does the situation affect the graph of the mathematical function?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

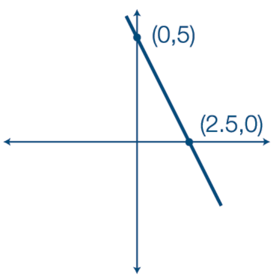
Scaffolding Questions

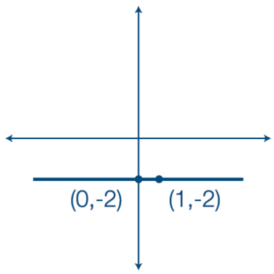
- What type of function relates the variables?
- What is the dependent variable? What is the independent variable? How do you know?
- What are the constants in the function? What do they mean?
- For the table in number 3, what restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?

Sample Solutions

I. If given a function rule, sketch a complete graph that represents that function. Show the coordinates of any intercepts. If given a graph or table, write the function representing it.

II. Determine the domain and range for each mathematical situation.

	Function	Graph or Table	Domain and Range
1.	$f(x) = 5 - 2x$		Domain: The domain is the set of all real numbers because $5 - 2x$ is defined for any value of x. Range: The range is the set of all real numbers since any number can be generated by $5 - 2x$.

2.	$y = -2$		Domain: The domain is the set of all real numbers because the function y = –2 means "*y* is equal to –2 no matter what *x* is." This is a constant function. Range: *y* is always –2, thus the range is only the number –2.
3.	The table shows a constant rate of change of $\frac{1}{2}$, so these data model a linear function. The *y*-intercept is (0, 3). The function that models this set of points is $y = \frac{1}{2}x + 3$.	*x* | *y* –2 | 2 2 | 4 6 | 6 12 | 9	Domain: The domain is the set of given *x* values {–2, 2, 6, 12}. The range is the set of *y* values {2, 4, 6, 9}. The domain and range of the function that models these data are both the set of all real numbers.

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

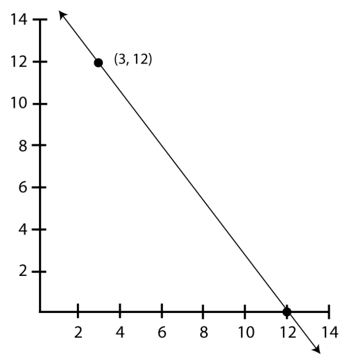
The student is expected to:

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3: The student will demonstrate an understanding of linear functions.

4.	$y = -\frac{4}{3}x + 16$		Domain: The domain is the set of all real numbers because $y = -\frac{4}{3}x + 16$ is defined for any value of *x*. Range: The range is the set of all real numbers since any number can be generated by $y = -\frac{4}{3}x + 16$.

III. Describe the similarities and differences among the functions given above.

The functions in problems 1–4 are linear, having the form $y = mx + b$. They all have as their domains the set of all real numbers because the expression for each function is never undefined. The functions in problems 1, 3, and 4 have as their ranges the set of all real numbers, since every real number can be generated by the expressions for those functions. The function in problem 2 has as its range the single number –2, since it is a constant function.

The graphs of the functions in problems 1–4 are lines. The graph of the function in problem 1 has a *y*-intercept of (0, 5) and an *x*-intercept of (2.5, 0). The line falls from left to right because the slope is negative. This is a decreasing function. The graph of the function in problem 2 has a *y*-intercept of (0, –2) and no *x*-intercept. It is a horizontal line with slope zero. This is a constant function.

The graph of the function that models the data given in problem 3 has a *y*-intercept of (0, 3) and an *x*-intercept of (–6, 0). The line rises from left to right because the slope is positive. This is an increasing function.

The graph of the function in problem 4 has a *y*-intercept of (0, 16) and an *x*-intercept of (12, 0). The line decreases from left to right because the slope is negative. This is a decreasing function.

IV. Describe a practical situation that each function might represent. What restrictions on the mathematical domain and range of the function does the situation require? How does the situation affect the graph of the mathematical function?

The function $f(x) = 5 - 2x$ could represent a toy racecar starting to race 5 feet away from the finish line and moving forward at 2 feet per second, where y is the distance in feet between the toy car and the finish line and x is the time in seconds the car has been moving. For this situation, the domain is the set of all numbers x, $0 \le x \le 2.5$, representing the time to start and complete the race. The range is the set of all numbers y, $0 \le y \le 5$, representing the range of distance traveled by the car. The graph is simply the segment from (0, 5) to (2.5, 0).

The function $y = -2$ could represent an ocean diver in the waters near a beach. The diver is floating 2 meters below the water surface. For this situation, the domain is time, x in minutes, that the diver is at this depth. For example, the domain could be the set of all numbers x, $0 \le x \le 15$, and the range is $y = -2$. The graph is a horizontal segment from (0, –2) to (15, –2).

The function $y = \frac{1}{2}x + 3$ could represent the allowance that a very young boy gets each week. The parent puts \$3 in the boy's piggy bank to start. Each week, the boy gets a 50-cent allowance and adds it to the piggy bank. The boy has been told that if he saves his allowance each week for 6 months, then he will get an increase. For this situation, the domain is the set of all values x, $x = 0, 1, 2, 3, \ldots, 24$, because there are roughly 24 weeks in the 6-month period. The range is the set of all values y, $y = 3, 3.5, 4, 4.5, \ldots, 15$, representing the amount of the boy's savings.

The graph is a discrete graph because it is simply a plot of a set of 25 points.

The fourth function could represent the remaining life of the batteries in an MP3 player. The batteries are designed to last up to 16 hours, and Jessica listens to her MP3 player 80 minutes per day as she walks to and from school. She can use the function rule to determine the number of days that the battery will last if she uses the player for 80 minutes or $\frac{4}{3}$ of an hour each day.

Extension Questions

- For problems 1–3, what is the equation of a line perpendicular to each of the given lines and having the same y-intercept?

  If a function is not a horizontal line, find the slope of the line and determine the opposite reciprocal of this slope. If the line is a horizontal line, the perpendicular line has undefined slope.

  In problem 1, the line's slope is –2; the slope of a line perpendicular to this line is $\frac{1}{2}$. The equation of the line is $y = 5 + \frac{1}{2}x$.

In problem 2, the line is horizontal with y-intercept –2. The line perpendicular to this line is a vertical line. The slope of a vertical line is undefined. The line is of the form x equals a constant. Any vertical line is perpendicular to y = –2. The equation is x = k, where k is any real number.

For problem 3, the slope of the line is $\frac{1}{2}$. The perpendicular line has slope of –2. The equation of the line is y = –2x + 3.

- Describe the domain and range of these three perpendicular lines.

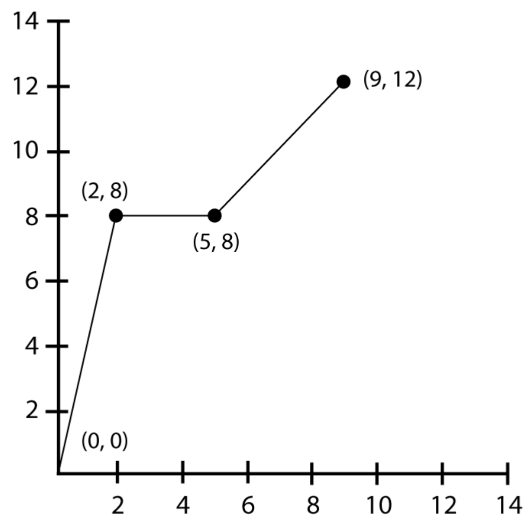
  The domain and range of the perpendicular lines in problems 1 and 3 are all real numbers.

  The domain of the line x = k is the number k. The range is all real numbers.

- Do these perpendicular lines represent functions?

  The perpendicular lines in problems 1 and 3 represent functions because for each x, there is only one y value. However, x = k does not represent a function because the x value 0 is paired with an infinite number of y values.

- Write a function rule for each phase of the graph below. Then determine the domain and range.



This function consists of three linear pieces. These could be defined as:

If 0 ≤ x ≤ 2, y = 4x.

If 2 < x ≤ 5, y = 8.

If 5 < x ≤ 9, y = x + 3.

Domain: The domain is the set of all real numbers x, 0 ≤ x ≤ 9, since this is what the graph shows.

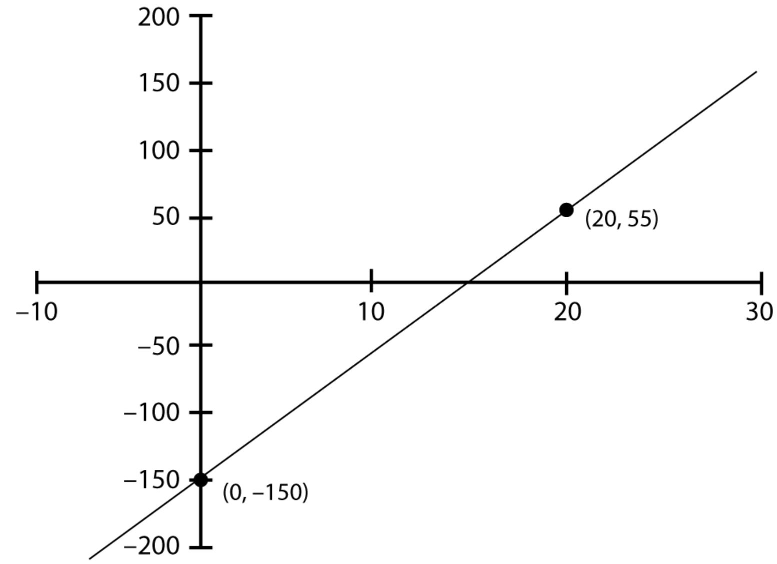
Range: The range is the set of all real numbers y, $0 \le y \le 12$, by the same reasoning.

*Note to the teacher: This is an example of a **piecewise-defined function**, which is not formally defined until precalculus. At this point, we are asking students only to determine the function rule for each phase.*

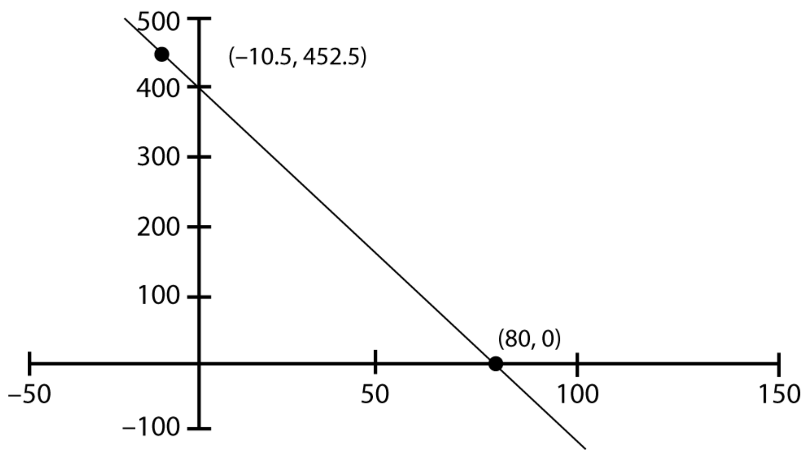
Create a Situation

Create and describe in detail a situation that each of the following graphs could represent. You may use the identified points in your situation, or you may use other points that would lie on that particular graph.

Graph A



Graph B



Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Scaffolding Questions

- What type of function do these graphs represent?
- What are the constants in the functions?
- Are the functions increasing or decreasing?
- How can you use this information to describe a situation each function might represent?

Sample Solutions

Create and describe in detail a situation that each of the following graphs could represent. You may use the identified points in your situation, or you may use other points that would lie on that particular graph.

Graph A

Using the two points given in the graph, students can determine that the slope is 10.25. The y-intercept is given. The following financial situation could be modeled by this graph and function.

> You decide to start a lawn-mowing business. You borrow $150 from your dad to buy a new mower. You charge $10.25 for each lawn you mow. The graph represents your balance after you have mowed x lawns. You will make a profit once the y-value is positive. You are in the red until you mow the 15th lawn, since your break-even point (x-intercept) lies between 14 and 15. After that you show a profit, since the y-value is positive when you have mowed 15 or more lawns.

Although it is not required, students may also write a function rule, which is $y = 10.25x - 150$. The function rule implies that the y-value is increased by 10.25 for every unit change in the x-value, and the starting amount is -150.

Graph B

Using the two points given in the graph, students can determine that the slope is -5 and the y-intercept is 400. The following situation could be modeled by this graph and function.

> This could represent the altitude of a skydiver whose parachute opens at 400 meters. The skydiver is gently drifting to a landing at a rate of 5 meters per second. The graph for this situation would lie only in the first quadrant region. The *y*-intercept, (0, 400), represents the opening of the parachute. The *x*-intercept, (80, 0), represents the number of seconds it takes the skydiver to land. The graph could be extended to include the second quadrant region by assuming that $x = 0$ is the point at which the skydiver is first sighted by someone on the ground and that she opened her parachute before that.

Although it is not required, students may also write a function rule, which is $y = 400 - 5x$. The starting value is 400, and the *y*-value is decreased by 5 units for every increase of 1 in the *x*-value.

Extension Questions

- What would happen to Graph A if the function were $y = 10.25x - 129.5$? How would this change the situation you described?

  The graph would be a line with the same slope but with a different y-intercept, (0, –129.50). In the situation described above, it could mean that you need to mow fewer yards to break even because you borrowed only $129.50 from your dad.

- What would happen to Graph B if the function were $y = 400 - 4x$? How would this change the situation you described?

  The graph would be a line with the same y-intercept. It would not be as steep, since the slope is –4 instead of –5. (The speed is the absolute value of the slope, so the original situation had a faster rate of change. The skydiver fell faster in the first situation.) The x-intercept would change from (80, 0) to (100, 0).

  In this new situation, it would mean that the skydiver drifts to her landing at 4 meters per second and lands in 100 seconds.

- What would Graphs A and B look like if you reflected the original graphs over the *x*-axis? How would doing

Texas Assessment of Knowledge and Skills:

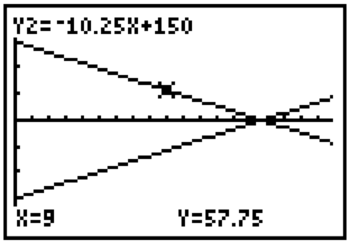
Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

this change the functions describing the graphs? How would it change the situations you chose to represent the graphs?

The function for Graph A would become y = –10.25x + 150, since reflecting over the x-axis is the same as multiplying the expression 10.25x – 150 by –1. This is a decreasing linear function. It could no longer represent a "money-earned" situation, but it could represent a "money spent out of $150" situation. For example, Jack has $150 in his savings account. He withdraws $10.25 each week. If he does not add any money to the account, y represents the amount of money in the savings account at x weeks.

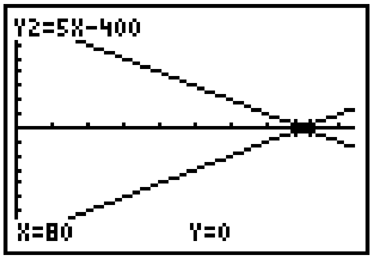
$y = 10.25x - 150$

$y = -10.25x + 150$



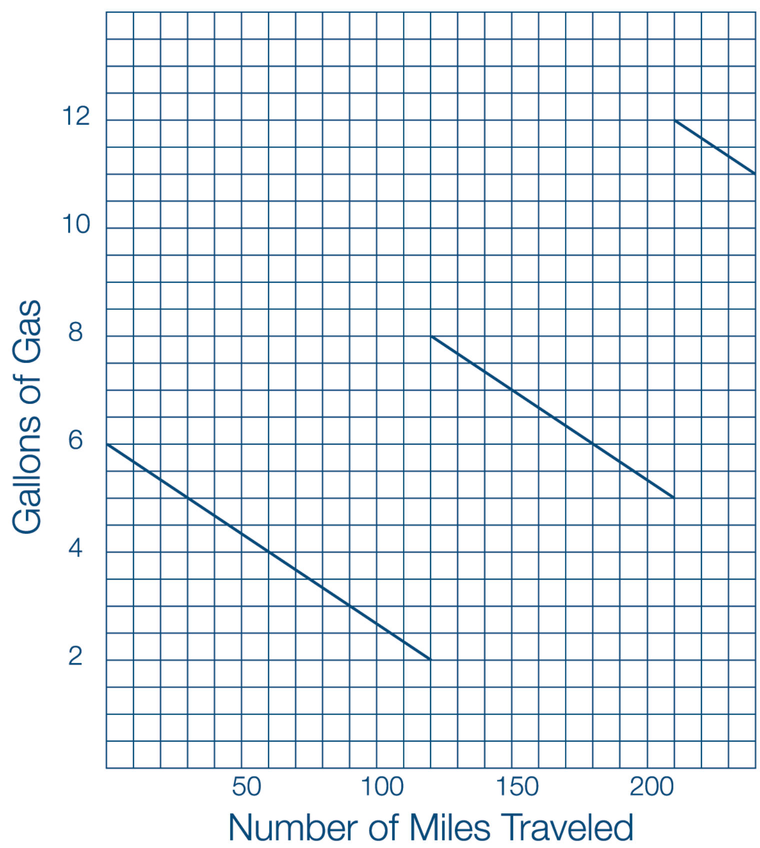
Similarly, the function for Graph B would become y = 5x – 400. The graph would have a negative y-intercept, (0, –400). This is an increasing function that starts at a negative value and could not represent the skydiver's altitude as she drifts to her landing because the altitude at time zero cannot be negative.

Instead, we must think of a situation that begins with a negative value. For example, Lance borrows $400 from his sister and pays her back at the rate of $5 per week. If he continues to pay her at this constant rate, y represents the amount of money he owes her, and x represents the number of weeks he has made payments. The x-intercept is 80; this means that after 80 weeks, Lance owes his sister 0 dollars.



Gas Tank

The following graph shows how the amount of gas in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the graph for this situation. Include in your description an interpretation of the slopes of the segments.



Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Scaffolding Questions

- How many phases do you see in the graph?
- How does the graph behave in each phase? What does this mean in the situation?
- How does the graph behave between phases? What does this mean in the situation?
- How does the amount of gas in the tank vary during the first 100 miles of the trip? During the next 120 miles? During the last 40 miles?

Sample Solutions

The graph shows how the amount of gas in a car's tank varied as a function of the number of miles traveled on a trip. Write a paragraph interpreting the graph for this situation. Include in your description an interpretation of the slopes of the segments.

The gas tank starts with 6 gallons of gas. For the first 120 miles, the gas level drops at a steady rate to 2 gallons. At 120 miles, the number of gallons jumps to 8, which suggests that the driver stopped to get gas. During the next 90 miles (from mile 120 to mile 210), the gas level drops steadily to 5 gallons. Again, at 210 miles, the number of gallons jumps suddenly, this time to 12 gallons. Then it drops steadily during the next 30 miles.

Some students may point out that the capacity of the tank is at least 12 gallons, since that is the maximum *y*-value we see. Thus, at the beginning of the trip, the tank was not full, and on the first refill, the tank was not filled to capacity.

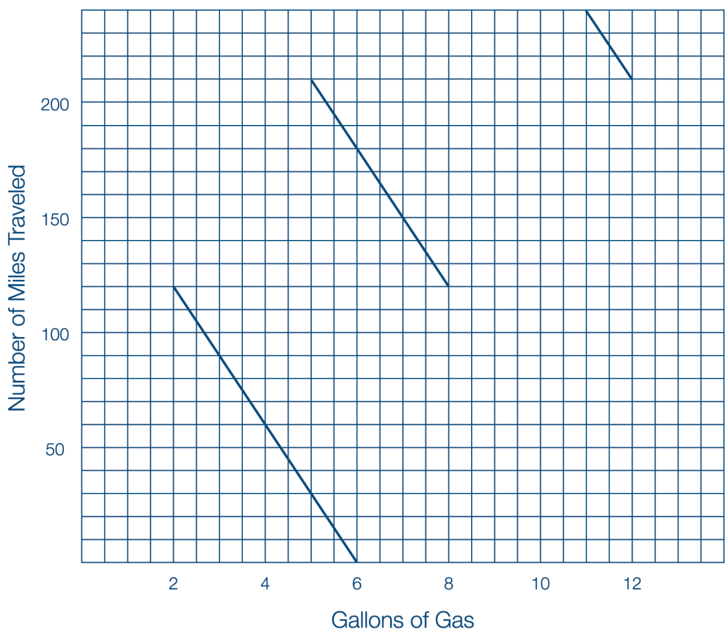
The rate of change in gas in all three phases is 1 gallon used per 30 miles (slope of $-\frac{1}{30}$), so gas consumption (gallons/mile) is occurring at a steady rate. In other words, the car uses $\frac{1}{30}$ of a gallon per mile. Some students may intuitively interpret the rate in this situation as 30 miles per gallon. While it is true that the car gets 30 miles per gallon, that rate does not match the graph and its negative slope.

Extension Questions

- Is it possible for this car to travel 400 miles on a single tank of gas?

  Answers will vary. It may be possible. The tank holds at least 12 gallons and is capable of getting 30 miles per gallon, so it could go at least 12 x 30 = 360 miles. For this car to get 400 miles on one tank, either the tank holds more than 12 gallons or the car uses less gas per gallon (possibly by driving at a slower speed).

- Create a new graph that shows the result of switching the independent and dependent variables of the original graph.



- What does the resulting rate of change (slope) in each phase now represent?

  The rate of change is miles traveled per gallon and is a decrease of 30 miles per gallon in each phase.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3: The student will demonstrate an understanding of linear functions.

Greetings

The school choir purchased customized cards from a company that charges a $100 set-up fee and $2 per box of cards. The choir members will sell the cards for $3 per box.

The function describing the choir's profit, p dollars, for selling x boxes of cards is $p = 3x - (100 + 2x)$.

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?
2. How much money will the choir make if the members sell 200 boxes? Show your strategy.
3. How many boxes must the choir sell to make a $200 profit? Explain how you found your answer.
4. How many boxes must the choir sell to make a $500 profit? Use a different strategy than the one you used in number 3.
5. How many boxes must the choir sell to break even?
6. The choir will not consider this fundraising project unless they can raise at least $1,000. Write and solve an inequality that helps them determine if they should take on this project.

Notes

Materials:

One graphing calculator per student

Algebra TEKS focus:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

Scaffolding Questions

- What does the word *profit* mean?
- What does the 3 in the expression $3x$ represent?
- What does x represent?
- What does p represent?
- Why are parentheses used in the function rule?
- How can you use the distributive property to simplify the expression?
- Which variable are you given in question 2?
- Which variable represents $200 in question 3?
- What does it mean to *break even*?
- Describe how you might use a table to answer question 3.
- Describe how you might use a graph to answer question 3.

Sample Solutions

1. What do the expressions $3x$ and $100 + 2x$ mean in this situation?

   The expression $3x$ represents the total revenue, which is the amount in dollars collected from the sale of x boxes. The $(100 + 2x)$ represents the total cost: The choir has to pay $100 plus $2 per box.

2. How much money will the choir make if the members sell 200 boxes? Show your strategy.

   If the choir sells 200 boxes, you must evaluate the function for $x = 200$.

   $p = 3x - (100 + 2x)$

   $p = 3(200) - (100 + 2(200))$

   $p = 600 - (100 + 400)$

   $p = 600 - 500$

   $p = 100$

   They would make a profit of $100.

3. How many boxes must the choir sell to make a \$200 profit? Explain how you found your answer.

   One possible solution:

   Generate a table that shows the number of boxes and the amount of profit made. Use the table to determine the number of boxes that would make a \$200 profit.

Number of Boxes	Profit in Dollars
0	–100
100	0
200	100
300	200
400	300
500	400
600	500

   The choir must sell 300 boxes to make a \$200 profit.

4. How many boxes must the choir sell to make a \$500 profit? Use a different strategy than the one you used in number 3.

   One possible solution:

   The symbolic method can be used to determine how many boxes the choir must sell to make a \$500 profit.

   Simplify the rule.

   $p = 3x - (100 + 2x)$

   $p = 3x - 100 - 2x$

   $p = x - 100$

   Substitute 500 for p.

   $500 = x - 100$

   $600 = x$

   The choir must sell 600 boxes to make \$500.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

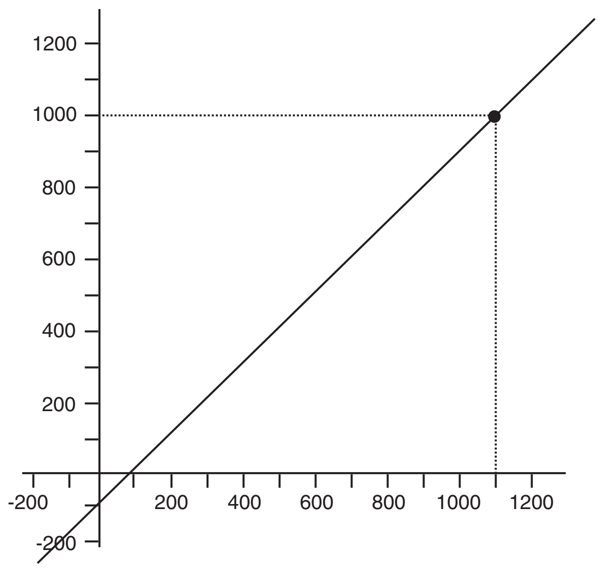
5. How many boxes must the choir sell to break even?

   To *break even* means that the cost equals the revenue, or that the profit is 0.

   One solution is to use the table from the solution for question 3 above to find where $y = 0$ (at $x = 100$). The choir must sell 100 boxes to break even.

   Another solution is to graph the function rule:

   $p = 3x - (100 + 2x)$, or $p = x - 100$



   The break-even point can be seen where the graph intersects the *x*-axis—that is, at 100 boxes. Any boxes sold after the first 100 would generate a profit (where the graph is above the *x*-axis).

6. The choir will not consider this fundraising project unless they can raise at least $1,000. Write and solve an inequality that helps them determine if they should take on this project.

   If the choir wants to make at least $1,000, then the profit must be greater than or equal to $1,000. One approach is to set up an inequality.

$$\begin{aligned} p &\geq 1{,}000 \\ x - 100 &\geq 1{,}000 \\ x &\geq 1{,}100 \end{aligned}$$

   The choir must sell at least 1,100 boxes of cards. If they feel they cannot sell at least 1,100 boxes, they should not undertake this project.

   Another approach is to examine the graph. The graphs of $y = 3x - (100 + 2x)$ and $y = 1{,}000$ intersect at the point (1,100, 1,000). That means that when the choir sells 1,100 boxes, the profit is $1,000. The graph of the profit is above the graph of y = 1,000 for values of x greater than 1,100. The choir must sell at least 1,100 boxes to make a profit of at least $1,000.

Extension Questions

- What would happen in this situation if the $100 set-up fee were waived?

  The choir would have to pay less. The profit would be represented by p = 3x – 2x, or p = x. The choir would make $1.00 per box. Now the y-intercept is zero. The rate of change is still $1.00 per box.

- For another situation, the choir's profit is represented by $p = 3x - (30 + 2.50x)$. Describe the cost and selling process for this situation.

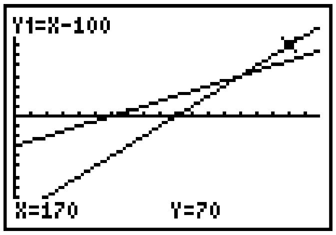
  Profit is revenue minus cost. The expression 3x means that the choir is charging $3 per box. The cost is represented by 30 + 2.50x. The choir will have to pay a set-up fee of $30 plus $2.50 per box.

- Under what conditions is the second situation better than the first?

  One strategy is to determine when the two are equal in value.

$$\begin{aligned} 3x - (30 + 2.50x) &= 3x - (100 + 2x) \\ 0.5x - 30 &= x - 100 \\ -0.5x &= -70 \\ x &= 140 \end{aligned}$$

  Another strategy is to examine the graph to determine which function has the greater value after x = 140. Enter both equations into a graphing calculator, Y1 = x – 100 and Y2 = 0.5x – 30. These graphs intersect at x = 140, meaning that for 140 boxes, cost and profit are the same.

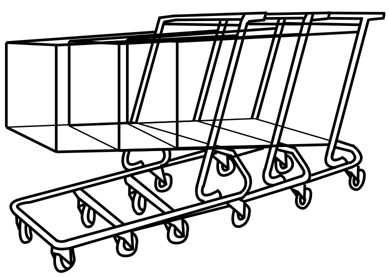


  However, when x is greater than 140, the function y = x – 100 has the greater value. For example, when x equals 170, the graph of the function Y1 = x = 100 is above the graph of Y2 = 0.5x – 30. Therefore, the profit is greater in the first situation for 140 or more boxes.

Grocery Carts

Randy must fit shopping carts into an area that has a length of 82 feet and a width of one shopping cart. He made some measurements necessary for his computations. The table shows the length of a set of shopping carts as they are nested together.

Number of (Nested) Shopping Carts	Length in Inches
1	37.5
8	116.25



Three nested shopping carts are shown.

Randy has decided to use an algebraic expression to determine the length of nested shopping carts.

1. Determine a function rule for the length, in inches, of a set of nested carts in terms of the number of nested shopping carts.
2. How do the numbers in the function rule relate to the physical shopping carts?
3. What is the length of 50 nested shopping carts?
4. How many carts fit into an area with a length of 82 feet and a width of one shopping cart? Use algebraic methods and verify your solution using a table or a graph.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

Scaffolding Questions

- What do you know about the shopping carts?
- How long is one cart?
- What is the total length of two nested carts?
- If one cart is 37.5 inches long, how could you find the additional length for each nested cart?
- Will the model for these data be linear? Why or why not?
- How can you compute the rate of change for the situation?
- Complete this new table with the missing values.

Number of (Nested) Shopping Carts	Process	Length in Inches
1	37.5	
2	37.5 +	
3	37.5 +	
4	37.5 +	
5	37.5 +	
n		

Sample Solutions

1. Determine a function rule for the length, in inches, of a set of nested carts in terms of the number of nested shopping carts.

   The length for one cart is 37.5 inches. The rate of change is 78.75 inches for 7 carts or 11.25 inches for 1 cart.

The total length is 37.5 plus 11.25 for every additional shopping cart.

$$L = 37.5 + 11.25(n - 1)$$

or

$$L = 26.25 + 11.25n$$

where n is the number of carts and L is the length of the set of carts.

The function may also be represented by a table or a graph.

2. How do the numbers in the function rule relate to the physical carts?

The 26.25 inches represents the nested length of the carts; that is, it is the length of the cart that slides into the cart in front of it each time. The 11.25 inches is the amount that hangs out for each new cart.

Number of Shopping Carts	Process	Length in Inches
1	26.25 + 11.25(1)	37.5
2	26.25 + 11.25(2)	48.75
3	26.25 + 11.25(3)	60
4	26.25 + 11.25(4)	71.25
5	26.25 + 11.25(5)	82.5
n	$26.25 + 11.25n$	

Additional Algebra TEKS:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Objective 4: The student will formulate and use linear equations and inequalities.

Graphing calculator table:

Enter Y1 = 26.25 + 11.25*x*

L1	L2	L3 1
1	37.5	------
2	48.75	
3	60	
4	71.25	
5	82.5	
6	93.75	
7	105	

L1(7)=7

Graph:

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=100
 Yscl=5
 Xres=
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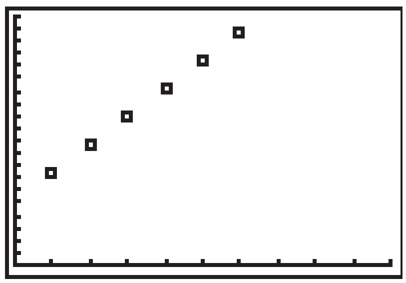


3. What is the length of 50 nested shopping carts?

When there are 50 carts, substitute 50 for *n* in the formula:

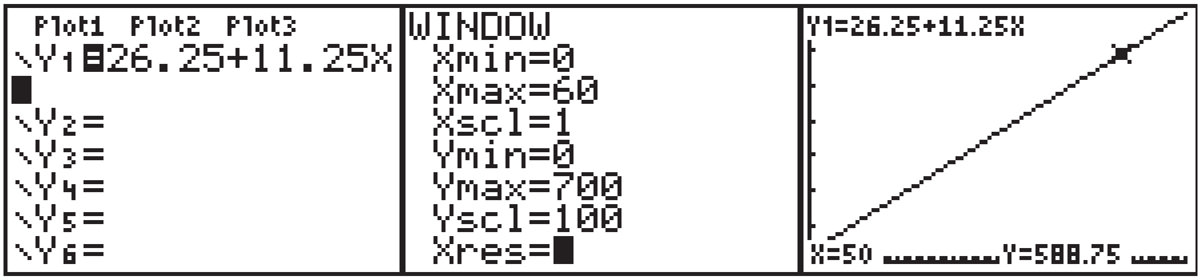
$$L = 26.25 + 11.25(50) = 588.75 \text{ inches}$$

This value can be determined using the calculator table.

X	Y1	
50	588.75	
60	701.25	
70	813.75	
80	926.25	
90	1038.8	
100	1151.3	
110	1263.8	

X=50

This value can also be determined using the calculator graph.



4. How many carts fit into an area with a length of 82 feet and a width of one shopping cart? Use algebraic methods and verify your solution using a table or a graph.

   82 feet must be converted to inches.

   $82 \bullet \frac{12}{1} = 984$ inches

   An equation may be used to determine when L is 984.

   $L = 26.25 + 11.25n$

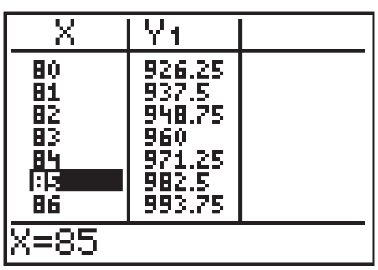
   $984 = 26.25 + 11.25n$

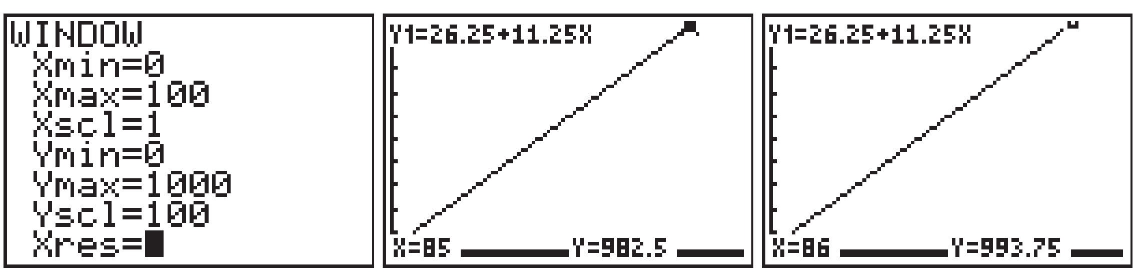
   $957.75 = 11.25n$

   $n = 85.1$, where n represents the number of carts

   There cannot be a fractional number of carts, so the number of carts that fit into 82 feet is 85 carts; 86 carts are longer than 82 feet.

   Verify this answer using the table:





Extension Questions

- If the function rule is $L = 32 + 11.25n$ for a different set of shopping carts, what is the same about the two sets of carts?

  The portion that is added for each new cart is the same because the rate, 11.25, has not changed. However, the y-intercept has changed, so the part that is nested into the rest of the carts is not the same.

- What is a reasonable domain for the function rule representing this different set of carts?

  The function L = 32 + 11.25n is a linear function. The domain of the function is all real numbers.

- What is a reasonable domain for the problem situation?

  The domain for the problem situation represents the number of carts and must be the set of positive integers. However, the domain is determined by the physical and logistical constraints of the situation, such as the available storage space and customer capacity.

- What is the rate of change for this situation.

  The rate of change is 11.25 inches for every one cart.

- How does this change affect the graph?

  The graph has a different y-intercept, but the same slope. The lines are parallel.

- If the function rule is $L = 26.25 + 14n$ for a different set of carts, what is the same about the two sets of carts?

  The portion that is added for each new cart has changed because the rate has changed (from 11.25 to 14). However, the y-intercept has not changed, so the part that is nested into the rest of the carts remains the same.

- How does this change affect the graph?

  The graph has the same y-intercept, but different slopes. The lines are not parallel, but intersect at the point (0, 26.25).

Hull Pressure

When a submarine descends into the ocean, the pressure on its hull increases in increments as given in the following table. (Pressure is measured in kilograms per square centimeter, and depth is measured in meters.)

Depth (m)	0	300	600	900	1,200	1,500
Pressure (kg/cm^2)	0	32	64	96	128	160

1. Describe verbally and symbolically a linear function that relates the depth of the submarine and the pressure on its hull.
2. How does this situation restrict the domain and range of the function?
3. How much pressure will be on the submarine's hull when it is at a depth of 1,575 meters? Describe your solution strategy.
4. If the pressure on the submarine's hull is 240 kg/cm^2, what is the depth of the submarine? How do you know?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(G) relate direct variation to linear functions and solve problems involving proportional change.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Scaffolding Questions

- How does the pressure change as the depth of the submarine increases?
- What is the initial pressure on the submarine's hull?
- What is the dependent variable in this situation?
- What is the independent variable in this situation?
- What is the rate of change in the pressure?
- How will you find the pressure for a given depth?
- How will you find the depth for a given pressure?

Sample Solutions

1. Describe verbally and symbolically a linear function that relates the depth of the submarine and the pressure on its hull.

   When the submarine is at the ocean's surface, the pressure on its hull is 0 kg/cm^2.

   For every 300 meters the submarine dives, the pressure on its hull increases by 32 kg/cm^2.

   A linear function with *y*-intercept at 0 and slope $m = \frac{32}{300} = \frac{8}{75}$ represents the situation—that is, $p = \frac{8}{75}d$, where *p* is the pressure in kg/cm^2 and *d* represents the depth in meters.

2. How does this situation restrict the domain and range of the function?

   While the mathematical domain and range for this function are both the set of all real numbers, the situation restricts the domain to the real numbers from 0 to the maximum depth the submarine can dive. The situation restricts the range to the real numbers from 0 to the maximum pressure the submarine's hull can withstand. This depends on the construction and size of the submarine.

3. How much pressure will be on the submarine's hull when it is at a depth of 1,575 meters? Describe your solution strategy.

If the submarine dives to 1,575 meters, then d = 1,575 and $p = \frac{8}{75}$.

$d = \frac{8}{75}(1{,}575) = 168$

The pressure on the submarine's hull is 168 kg/cm^2.

4. If the pressure on the submarine's hull is 240 kg/cm^2, what is the depth of the submarine? How do you know?

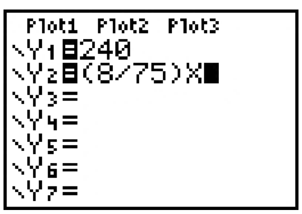
   If the pressure on the submarine's hull is 240 kg/cm^2, then p = 240 and the following equation can be solved for d:

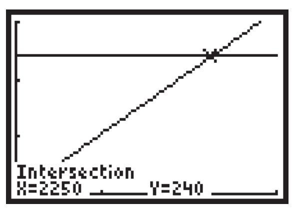
   $240 = \frac{8}{75}d$

   $d = 240 \bullet \frac{75}{8} = 2{,}250$

   The submarine's depth is 2,250 meters.

   The problem could also be solved by finding the intersection of the graphs of $y = 240$ and $y = \frac{8}{75}x$.





   The value of x when y = 240 is 2,250.

Extension Question

- Is there a proportional relationship between hull pressure and depth? Explain how you know whether the relationship is proportional.

  The graph of the function is a straight line that contains the point (0, 0). Therefore, there is a proportional relationship between hull pressure and depth.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Math-a-Thon

Catrina is participating in the school math-a-thon to raise money for the end-of-year field trip. Her mother is donating $25 to get her started. She will also receive 75 cents for every problem she answers correctly.

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.
2. Catrina's grandmother gives her an extra $20 to add to her field trip money. How does this change the previous situation's rule, graph, and table?
3. What part of the situation would you change to produce a less steep slope? A steeper slope? Explain how you know.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Scaffolding Questions

- How much money will Catrina raise in the math-a-thon if she works 20 problems correctly?
- What are the constants in this situation?
- What are the variables in this problem?
- What type of graph do you think this situation will produce?
- What role does the $25 play in the graph of this situation?
- What does adding $20 do to the graph of the situation?
- What is the rate of change for the original situation?
- What is the rate of change for the situation described in number 2?

Sample Solutions

1. What is the function rule for this situation? Explain the meaning of each constant and variable in your rule.

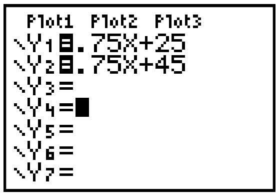
   The amount of donation is $25 plus $0.75 times the number of problems Catrina answers correctly. The function rule is $d = 0.75p + 25.00$, where d represents the total amount of donation and p represents the number of problems Catrina answers correctly.

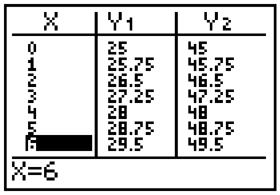
   The $0.75 is the amount Catrina earns for each problem she solves correctly. The $25 is the amount Catrina gets from her mother regardless of the number of problems she solves.

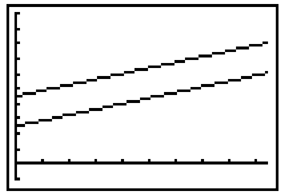
2. Catrina's grandmother gives her an extra $20 to add to her field trip money. How does this change the previous situation's rule, graph, and table?

   The $25 in the first function rule changes to $45, because Catrina now starts with $25 plus $20. The new rule is $d = 0.75p + 45$.

   The table now shows that when x is 0, y is 45 instead of 25. y still increases by 0.75 for every problem solved correctly. The two graphs are parallel lines, one starting at (0, 25) and the other starting at (0, 45).







3. What part of the situation would you change to produce a less steep slope? A steeper slope? Explain how you know.

   The amount of money Catrina receives per correct problem affects the rate of change and thus the slope of the line.

   To change the slope in this function rule, we would have to change the amount of money Catrina receives per correct problem. Anything more than \$0.75 produces a line with steeper slope, and anything less than \$0.75 produces a line with a less steep slope.

Extension Questions

- How do the domain of the function rule and the domain of the problem situation compare?

  The domain for the function that models the situation is all real numbers. However, in the problem situation, the number of problems must be a whole number. The number of problems in the competition is the maximum number Catrina could answer correctly, so the domain of the problem situation is a subset of the set of whole numbers.

- How do the graph of the function rule and the graph of the situation compare?

  The graph of the function is a straight line, but the graph of the problem situation is a set of points on a straight line in the first quadrant.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

- Write another scenario that produces a similar function rule, graph, and table.

  Johnny has a basket with 20 apples and starts picking apples at a rate of 5 apples per minute. How many apples does he have in 10 minutes?

- Jackie did not receive a starting donation. Can she still collect as much money as Catrina? Explain your answer.

  Yes, Jackie can answer more problems correctly than Catrina does, and/or she can collect more money per problem.

Shopping

Celeste is shopping for two pairs of shoes and some earrings. She can spend a maximum of $100. The shoes Celeste wants to buy cost $24.99 a pair. Earrings cost $12.99 a pair. The sales tax is 8% of the total price of Celeste's purchases.

1. Write an inequality to identify how many pairs of earrings Celeste can purchase.
2. What is the greatest number of pairs of earrings Celeste can buy? Explain your answer.
3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for $19.99?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

Scaffolding Questions

- What are you trying to find out? What do you know?
- What does the variable represent in the situation? What is changing and what stays the same?
- How will you show the tax in your inequality?

Sample Solutions

1. Write an inequality to identify how many pairs of earrings Celeste can purchase.

   Let x = the number of pairs of earrings Celeste can buy, \$24.99 = the cost of one pair of shoes, and \$12.99 = the cost of one pair of earrings.

   The cost of 2 pairs of shoes at \$24.99 each + x pairs of earrings at \$12.99 each may be represented by $2(24.99) + 12.99x$.

   The tax of 8% on the sale is represented by $0.08[2(24.99) + 12.99x]$.

   The total cost of the purchases plus the tax cannot exceed \$100. The following sample inequality describes the restriction:

   $$2(24.99) + 12.99x + 0.08\,[2(24.99) + 12.99x] \le 100.00$$

   Some students may use 1.08 for the price including tax and represent the situation using this inquality:

   $$1.08[2(24.99) + 12.99x] \le 100$$

2. What is the greatest number of pairs of earrings Celeste can buy? Explain your answer.

   Use the distributive property to simplify before solving and round to the nearest hundredth:

   $$\begin{aligned} 49.98 + 12.99x + 4.00 + 1.04x &\le 100.00 \\ 53.98 + 14.03x &\le 100.00 \\ -53.98 \qquad\qquad &\quad\; -53.98 \\ \hline 14.03x &\le 46.02 \\ x &\le 3.28 \end{aligned}$$

Celeste can buy no more than 3 pairs of earrings. She will have some money left over, but not enough for a fourth pair of earrings.

3. How many pairs of earrings could Celeste purchase if she finds the shoes on sale for $19.99?

   If Celeste finds the shoes on sale for $19.99 a pair, the inequality changes as follows:

$$2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] \le 100.00, \text{ or}$$

$$1.08[2(19.99) + 12.99x] \le 100$$

   Use the distributive property to simplify before solving, and round to the nearest hundredth:

$$\begin{aligned} 39.98 + 12.99x + 3.20 + 1.04x &\le 100.00 \\ 43.18 + 14.03x &\le 100.00 \\ -43.18 \quad\quad &\quad -43.18 \\ \hline 14.03x &\le 56.82 \\ x &\le 4.05 \end{aligned}$$

   If the shoes cost $19.99 per pair, Celeste can buy 4 pairs of earrings and still have money left over.

Extension Questions

- How would your solution change if Celeste found the shoes on sale for 25% off the original price? Show your solution algebraically.

  If Celeste found the shoes on sale for 25% off the original price of $24.99, the new shoe price could be found by starting with the idea that the shoes cost 75% of the original price. Multiplying $24.99 by 0.75 gives you the new shoe price.

  24.99(0 .75) = 18.7425; the shoes would cost $18.74 per pair.

  The new inequality would be:

$$\begin{aligned} 2(18.74) + 12.99x + 0.08[2(18.74) + 12.99x] &\le 100.00 \\ 37.48 + 12.99x + 3.00 + 1.04x &\le 100.00 \\ 40.48 + 14.03x &\le 100.00 \\ -40.48 \quad\quad &\quad -40.48 \\ \hline 14.03x &\le 59.52 \\ x &\le 4.24 \end{aligned}$$

  Celeste could purchase 4 pairs of earrings and still have money left over.

(A) use symbols to represent unknowns and variables; and

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

- How would a 9% tax rate affect how many pairs of earrings Celeste could purchase along with shoes at the original price?

  $2(24.99) + 12.99x + 0.09[2(24.99) + 12.99x] \le 100.00$

  Use the distributive property to simplify before solving, and round to the nearest hundredth:

  $$\begin{aligned} 49.98 + 12.99x + 4.50 + 1.17x &\le 100.00 \\ 54.48 + 14.16x &\le 100.00 \\ -54.48 \qquad &\quad -54.48 \\ \hline 14.16x &\le 45.52 \\ x &\le 3.21 \end{aligned}$$

  A 9% sales tax rate would cost a little more, but Celeste could still purchase 3 pairs of earrings and have money left over.

- Suppose Celeste wants to have $20 left. Describe and write your solution algebraically, assuming she finds the shoes on sale for $19.99.

  For Celeste to have $20 left, the total amount she can spend has to be reduced by $20. Rather than having $100 to spend, she now has $80. The inequality is:

  $$\begin{aligned} 2(19.99) + 12.99x + 0.08[2(19.99) + 12.99x] &\le 80.00 \\ 39.98 + 12.99x + 3.20 + 1.04x &\le 80.00 \\ 43.18 + 14.03x &\le 80.00 \\ -43.18 \qquad &\quad -43.18 \\ \hline 14.03x &\le 36.82 \\ x &\le 2.62 \end{aligned}$$

  Celeste can purchase only 2 pairs of earrings if she sets aside $20.

Sound Travel

Many fishing boats and salvage ships are equipped with sonar to measure sound waves to help them find shipwrecks and large schools of fish. Sound travels through freshwater at about 1,463 meters per second when the water temperature is 15°C. By measuring the time it takes for the sound waves to travel through the water from the boat to a school of fish, it is possible to calculate the distance from the boat to the fish.

1. Write a function rule that models the relationship between the number of seconds it takes the sound wave to return to the boat and the distance from the boat to the fish. Identify your variables.

2. Describe the graph of this function, including its domain and range. Explain how you know whether there is a direct variation between the number of seconds and the distance in meters.

3. Suppose the sound wave returned from the fish to the boat in 0.05 seconds. Estimate the distance to the fish. Justify your answer.

4. If the distance from the boat to the fish is 24,000 meters, how long does it take the sound wave to return from the fish to the boat? Explain your solution.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.1) Foundation for functions. The student understands that a function represents dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

Scaffolding Questions

- If the sound wave returns from the fish to the boat in 1 second, what is the distance from the boat to the school of fish? What if the sound wave returns in 2 seconds?
- What is the relationship between the distance from the boat and the fish at 1 second and at 2 seconds? At 2 seconds and at 3 seconds?
- What does 1,463 mean in the function?

Sample Solutions

1. Write a function rule that models the relationship between the number of seconds it takes the sound wave to return to the boat and the distance from the boat to the fish. Identify your variables.

   Let d = the distance between the boat and the school of fish in meters.

   Let t = the time in seconds it takes for the sound wave to return from the fish to the boat.

   The time it takes for the sound wave to return from the fish to the boat depends on the distance in meters between the boat and the fish. Time is the independent variable, and the distance is the dependent variable.

   The table shows the time (in seconds) that it takes for the sound wave to travel the distance (in meters) in freshwater.

Time (seconds)	Distance (meters)
0	0
1	1,463
2	2,926
3	4,389

   The function rule is linear because the rate of change is constant. The difference in the time in the table is 1 second. The difference in the distance is an increase of 1,463 kilometers for every increase of 1 second. So the function rule is $d = 1{,}463t$.

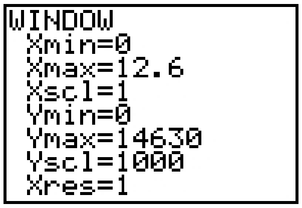
2. Describe the graph of this function, including its domain and range. Explain how you know whether there is a direct variation between the number of seconds and the distance in meters.

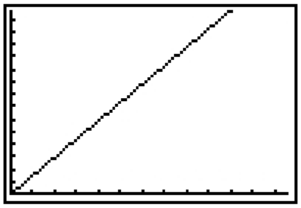
   We can tell from the table that as time increases, distance increases proportionally. We can see that there is a direct variation (proportional relationship) between the distance and the time in seconds because the graph of the function is a straight line that passes through the origin.

   The domain of the function represents the time it takes for the sound wave to return to the boat from the fish. The time in seconds must be greater than 0, or $t > 0$. The range represents the distance between the boat and the fish. The distance must also be greater than 0, or $d > 0$.

   The rate of change in the distance is an increase of 1,463 meters for every second. The slope of the equation is 1,463, representing an increase of 1,463 meters per second. The y-intercept is 0 because at 0 seconds, there is no distance to be recorded.

   The graph of the function is:





3. Suppose the sound wave returned from the fish to the boat in 0.05 seconds. Estimate the distance to the fish. Justify your answer.

   If the sound wave returned from the fish to the boat in 0.05 seconds, the distance from the boat to the fish could be found by substitution into the function as follows:

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

$d = 1{,}463t$

$d = 1{,}463(0.05)$

$d = 73.15$ meters

4. If the distance from the boat to the school of fish is 24,000 meters, how long does it take the sound wave to return from the fish to the boat? Explain your solution.

$$\begin{aligned} d &= 1{,}463t \\ 24{,}000 &= 1{,}463t \\ 16.4 &= t \end{aligned}$$

If the distance to the fish is 24,000 meters, it takes the sound wave approximately 16.4 seconds to return from the fish to the boat.

Extension Questions

- If the time is doubled, is the distance doubled? Justify your answer.

  Because there is a proportional relationship, the distance is doubled if the time is doubled.

  $$\begin{aligned} d &= 1{,}463t \\ 1{,}463(2t) &= 2(1{,}463t) = 2d \end{aligned}$$

- Describe the difference in what is asked in questions 3 and 4 in this activity.

  In question 3, we were given a value from the domain (time) and asked to evaluate the function for distance. In question 4, we were given a function value (distance) from the range and asked to find the corresponding value (time).

- Determine a function rule that expresses time as a function of distance. What type of relationship is this?

  Solve the rule for t.

  $d = 1{,}463t$

  $t = \frac{d}{1{,}463}$, *or* $t = \frac{1}{1{,}463}d$

  This function is also linear and is a proportional relationship.

Taxi Ride

The cab fees in Chicago are $1.40 for the first one-fifth of a mile and 20¢ for each additional one-tenth of a mile as shown in the table.

Miles	Cost in Dollars
$\frac{1}{5}$	1.40
$\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$	1.60
$\frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

1. How far can you travel for $10.00? Justify the reasonableness of your response.
2. Generate a table to give the cost per mile. How much will you have to pay to get to a restaurant that is 20 miles away from your hotel? Solve using two different methods.
3. If you want to include a 15% tip for the cab driver, how far can you travel for $10.00?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

(C) interpret and determine the reasonableness of solutions to linear equations and inequalities.

Scaffolding Questions

- What are the variables in this situation?
- Which is the independent variable?
- What kind of relationship is there between the two variables?
- How much would you pay for the first mile?
- How much would you pay for the second mile?
- What is the rate of increase (in dollars) per mile?
- What is the *y*-intercept?
- What is multiplied by 20¢? Could you use 40¢ instead? How?
- How many tenths are there in $\frac{1}{5}$?

Sample Solutions

1. How far can you travel for $10.00?

   Use the table to look for patterns, and from the pattern generate the rule for the situation. The pattern shows that the cost increases by 40¢ for every additional fifth of a mile.

Miles	Cost in Dollars
$\frac{1}{5}$	1.40
$\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$	1.60
$\frac{1}{5} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$	1.80
$\frac{5}{10}$	2.00
$\frac{6}{10}$	2.20
$\frac{7}{10}$	2.40
$\frac{8}{10}$	2.60
$\frac{9}{10}$	2.80
1	3.00

Determine how many tenths of a mile are left after paying $1.40 for the first fifth of a mile.

Let the number of miles be represented by m.

The number of miles left after the first fifth of a mile is

$m - \frac{1}{5}$.

In each mile, there are 10 tenths.

The number of tenths of a mile left after the first fifth of a mile is represented by $10(m - \frac{1}{5})$ or $10m - 2$.

The cost is 20¢ for every tenth of a mile or

$10(m - \frac{1}{5})(0.20)$ or $2m - 0.40$.

An expression for the total charge is $\$1.40 + 10(m - \frac{1}{5})(0.20) = \$1.40 + 2m - 0.40 = 2m + 1$.

To determine how many miles can be traveled for $10.00, substitute $10.00 for the cost and solve the inequality.

$10 \geq 1.40 + 10(m - \frac{1}{5})(0.20)$

$10 \geq 1.40 + (10m - 2)(0.20)$

$10 \geq 1.40 + 2m - 0.40$

$10 \geq 2m + 1$

$9 \geq 2m$

$m \leq 4.5$

The greatest number of miles you can travel for $10.00 is 4.5 miles.

2. Generate a table to give the cost per mile. How much will you have to pay to get to a restaurant that is 20 miles away from your hotel? Solve using two different methods.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Notes

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Generate a table to find the cost of traveling whole miles.

Miles	Cost in Dollars
1	3.00
2	5.00
3	7.00
4	9.00
5	11.00
6	13.00

The first mile costs \$3.00, but each mile thereafter costs \$2.00. Continue the table until you find the cost for 20 miles, or use the pattern to determine the rule.

Let c equal the cost in dollars of the taxi ride and m represent the number of miles.

So, $c = 2(m - 1) + 3$.

Substitute the 20 miles for the m and determine the cost.

$c = 2(20 - 1) + 3$

$c = 2(19) + 3$

$c = 38 + 3$

$c = 41$

It costs \$41.00 to travel 20 miles.

Another way to solve this problem is to use the rule for the tenth of a mile (from the answer to question 1). Substitute the 20 for the *m.*

$c = 1.40 + 10(m - \frac{1}{5})(0.20)$

$c = 1.40 + 10(20 - \frac{1}{5})(0.20)$

$c = 1.40 + 10(19.8)(0.20)$

$c = 41.00$

3. If you want to include a 15% tip for the cab driver, how far can you travel for $10.00?

   If the cost is to include a tip of 15%, the computed cost must be multiplied by 1.15 (that is, the original cost + 0.15 of the original cost).

   $10 \geq [1.40 + 10(m - \frac{1}{5})(0.20)]1.15$

   $10 \geq 1.61 + (10m - 2)(0.23)$

   $10 \geq 1.61 + 2.3m - 0.46$

   $10 \geq 2.3m + 1.15$

   $8.85 \geq 2.3m$

   $m \leq 3.847826$

   If you want to leave a 15% tip and travel for less than $10.00, the longest distance you can go is about 3.8 miles.

Extension Questions

- What are reasonable domain and range values for this situation?

  The domain values are every tenth of a mile after the first mile. The range values are $1.40 and every increment of 20¢ after $1.40.

- Does it make any difference which table you use to determine the cost of a taxi ride? Explain.

  It does not make any difference if you are going a whole number of miles, but if you are traveling partial miles, you can calculate your cost more precisely by using the original table.

- If a shuttle service charges a flat fee of $50.00 to any location from the airport, under what circumstances is it more cost effective to take a taxi?

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

This question is really asking when the taxi would cost less than $50.00.

Examine the table for the function.

X	Y_1
24.3	49.6
24.4	49.8
24.5	50
24.6	50.2
24.7	50.4
24.8	50.6
24.9	50.8

X=24.5

It would be more cost effective to take the taxi to any location that is fewer than 24.5 miles from the airport.

- If taxi rates increase, how would it affect the representation of your data?

  If the rate per tenth of a mile increased, the table entries would be greater and the slope of the graph would be steeper. If the increase also happened in the first fifth of a mile, the y-intercept would also change. If the increase happened only in the first fifth of a mile, only the y-intercept would change.

The Contractor

As a flooring contractor, Lupe sets floor tile for a living. He submits a bid for each new job. When preparing a bid, he measures the area of the floor to be tiled and then figures out how much material he will need. He charges the following prices for materials and labor:

Subflooring:	\$1.27 per square foot
Tile:	\$6.59 per square foot
Adhesive:	\$31.95 per job
Grout:	\$55.95 per job
Labor:	\$125 base price plus \$0.79 per square foot

1. Write an algebraic rule to determine the total cost of the materials and labor for a typical job in terms of the number of square feet. Explain what the numbers and symbols in the rule mean.
2. Make a table and graph to help Lupe see the amount of money he should charge for jobs with various amounts of square footage.
3. For a job tiling an area of 550 square feet, what is the amount of the bid, based on the materials listed above?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

Scaffolding Questions

- What are the constants in this problem situation?
- What are the variables?
- Make a list of all the charges Lupe must consider.
- How would he compute the cost of the tile?
- How would he compute the cost of the subflooring?
- How would he compute the labor charges?

Sample Solutions

1. Write an algebraic rule to determine the total cost of the materials and labor for a typical job in terms of the number of square feet. Explain what the numbers and symbols in the rule mean.

   The total cost is equal to the cost of the subflooring *plus* the cost of the tile *plus* the cost of the adhesive *plus* the cost of the grout *plus* the cost of the labor. Fixed materials costs are those for adhesive and grout. Costs for the subflooring and tile depend on the number of square feet to be tiled. Labor costs include both a fixed cost ($125 base price) and a cost per square foot.

   If the area in square feet is represented by the variable x and the total cost of the job is c, then the function rule is

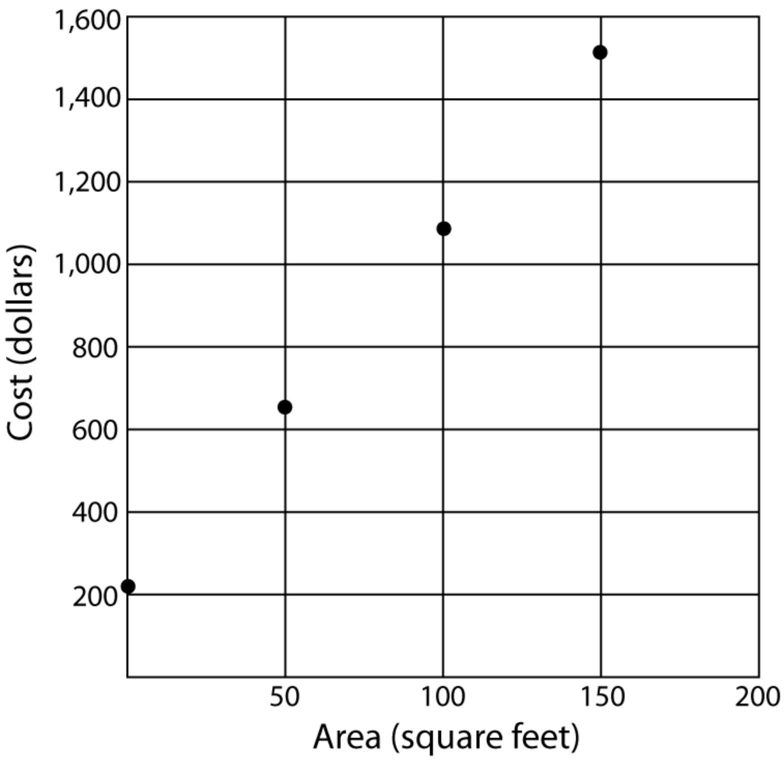
   $$c = 1.27x + 6.59x + 31.95 + 55.95 + 125 + 0.79x$$

   By combining terms, this can be rewritten as $c = 8.65x + 212.90$. $8.65 is the combined cost of the items per square foot of area. $212.90 is the total of the fixed costs that do not depend on area.

2. Make a table and graph to help Lupe see the amount of money he should charge for jobs with various amounts of square footage.

   Use the rule and put the values in a table.

Area (square feet)	**Cost (dollars)**
0	212.90
50	645.40
100	1,077.90
150	1,510.40



3. For a job tiling an area of 550 square feet, what is the amount of the bid, based on the materials listed above?

   Students may use a table to look for the *x* value of 550.

Area (square feet)	**Cost (dollars)**
500	4,537.90
550	4,970.40
600	5,402.90

   If the area is 550 square feet, the cost is $4,970.40. Students may also use an equation to find the cost: 8.65(550) + 212.90 = 4,970.40.

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

Extension Questions

- Suppose Lupe wants to make a 20% profit on each job. Write a new rule to compute how much he should charge his customers, including his profit.

  The new price can be determined by adding 20% of the original cost to the original cost. Using symbols, it is 0.20(8.65x + 212.90) + 8.65x + 212.90, or 1.20(8.65x + 212.90).

- The price of the adhesive increases to $45.95 per job. How does this affect the cost of the jobs? How does the increase show up in the rule, graph, and table?

  The adhesive cost is part of the fixed amount per job, $212.90. The fixed costs increase by the difference between the adhesive's new cost and its original cost, $45.95 – $31.95, or $14. The new fixed costs are $212.90 + $14, or $226.90. The table values all increase by this same amount. In the graph, the y-intercept is 226.90.

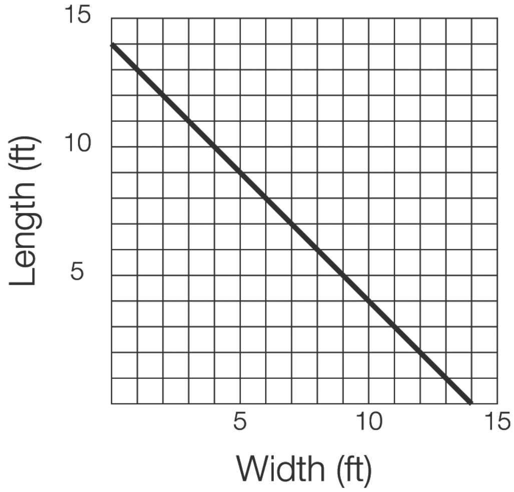
- If the charge for labor changes to $130 plus $0.85 per square foot, how does this affect the function that represents the cost of the jobs?

  The price for labor has a fixed amount, and it is also part of the charge per square foot. An increase in the fixed amount of the labor affects the y-intercept. If the fixed amount increases from $125 to $130, the y-intercept increases by 5. An increase in the charge per square foot increases the slope of the graph. The increase from $0.79 to $0.85 increases the slope of the graph by 0.06.

  The function rule changes from c = 8.65x + 212.90 to c = 8.71x + 217.90.

The Garden

Lance has a certain amount of fencing. He wants to use the fencing to enclose as much of his rectangular garden area as possible. He creates a graph to represent the relationship between the possible lengths and widths of the garden's fencing.



1. Describe verbally and symbolically the relationship between the length and the width of the garden.
2. What are reasonable domain and range values for this function? Create a table to show possible lengths and widths.
3. Explain what the graph tells you about the perimeter of the garden.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs;

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(B) determine the domain and range for linear functions in given situations; and

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions

- What type of relationship is described by the graph?
- Name some points on the graph. What does that tell you about the possible dimensions of the garden?
- Define the independent variable and the dependent variable for this problem situation.
- What would the length be if the width were 12 feet?
- What are the restrictions on the length and the width?
- How do you find the perimeter of a rectangle?

Sample Solutions

1. Describe verbally and symbolically the relationship between the length and the width of the garden.

   The graph is a line. The starting value (y-intercept) is 14. The x-intercept is 14.

   As the width of the rectangular area increases, the length decreases. Because Lance is using a fixed amount of fencing, the width and length change at a constant rate. For every 1 foot the width increases, the length must decrease 1 foot. The relationship can be described by the function $l = -1w + 14$.

   Thus, the slope of the line is –1.

   However, there could not be a garden if the width were 0 or 14 feet.

2. What are reasonable domain and range values for this function? Create a table to show possible lengths and widths.

   From the graph you can see that both the length and the width must be positive numbers less than 14. So the domain is $0 < w < 14$, and the range is $0 < l < 14$.

Sample table of possible lengths and widths:

Width (feet)	Length (feet)
0	14
2	12
4	10
8	6
10	4
14	0

The points (0, 14) and (14, 0) are not possible solutions in this problem situation. There could not be a garden if either the width or the length was 0.

3. Explain what the graph tells you about the perimeter of the garden.

Notice from the table that the length plus the width must be 14.

$$l + w = 14$$

or

$$l = -1w + 14$$

The perimeter is twice the sum of the width and the length.

$2(l + w) = 28$, or $2l + 2w = 28$

The perimeter or amount of fencing is 28 feet.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Extension Questions

- How is the domain for the function rule different from the domain for the problem situation?

  The domain for the function rule is the set of all real numbers. The domain for the problem situation is the set of all real numbers from 0 to 14.

- How would the graph have been different if the total amount of fencing had been 24 feet?

  Twice the sum of the length and width would have been 24 feet. The sum of the length and the width would have been 12 feet. The x- and y-intercepts would have both been 12.

- What would the graphs have in common if the total amount of fencing had been 24 feet?

  The slope for both graphs would have been –1.

- Describe the relationship between the area and the width of the garden.

  The area is the length times the width. The length is represented by 14 – w.

  $A = (14 - w)w$

The Submarine

A submarine is cruising 195 meters beneath the ocean's surface and begins rising toward the surface at a constant rate of 12 meters per minute.

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.

2. How long does it take the submarine to reach the ocean's surface? Justify your solution using symbols, a table, and a graph.

3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this changes the function and graph of the original situation. What is the new function, and how did you determine it?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in *m* and *b* on the graph of $y = mx + b$;

(F) interpret and predict the effects of changing slope and *y*-intercept in applied situations; and

Scaffolding Questions

- What are the constants in the problem? What quantities vary?
- What quantity is the dependent variable? The independent variable?
- Create a table and/or graph to verify your function rule for question 1.
- What kind of function models the situation?
- What is the submarine's depth when it is at surface level?
- What equation will you write and solve?
- What question will solving the equation answer?
- If the submarine must surface 5 minutes sooner, how long will it take to surface?
- What quantity in the original function rule must change?
- Will the submarine rise at the same rate? A slower rate? A faster rate?
- Try different rates in your original function rule. Use tables and/or graphs to estimate the rate at which the submarine needs to rise to reach the surface 5 minutes sooner.
- What equation can you write and solve to determine this new rate?

Sample Solutions

1. Describe verbally and symbolically a function relating the submarine's position and the amount of time it has been rising.

   The submarine's position, *D* meters below the surface, depends on the time, *t* minutes, it has been rising. The submarine starts rising from 195 meters below the surface, so its initial position is –195. It is rising toward the surface at 12 meters per minute, which is a constant rate. The function is a linear function because the rate at which the submarine is rising is constant.

The distance is the starting value plus the rate of change multiplied by the number of minutes.

This gives the function: $D = -195 + 12t$.

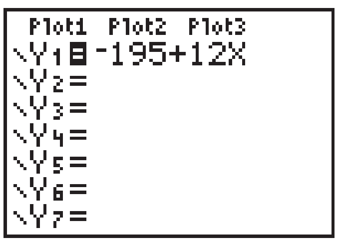
2. How long does it take the submarine to reach the ocean's surface? Justify your solution using symbols, a table, and a graph.

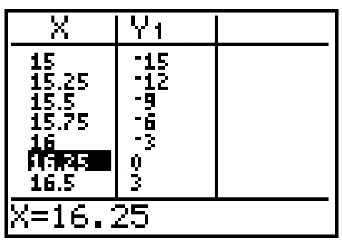
   When the submarine surfaces, its depth below the surface is 0 meters, so

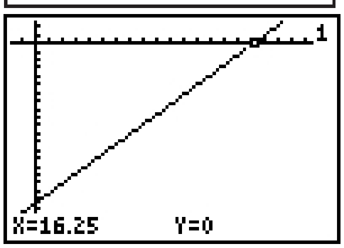
$$\begin{aligned} D &= -195 + 12t \\ 0 &= -195 + 12t \\ -12t &= -195 \\ t &= 16.25 \end{aligned}$$

   So, the submarine takes 16.25 minutes to surface if it starts at 195 meters below the surface and rises at a rate of 12 meters per minute.

   Another approach to answering the question is to use a graphing calculator.







3. Suppose the submarine started at its original depth (195 meters below sea level) and must reach the ocean's surface 5 minutes sooner than before. Describe how this changes the function and graph of the original situation. What is the new function, and how did you determine it?

   If the submarine must surface 5 minutes sooner, it must rise at a faster rate than 12 meters per minute. Before, the submarine took 16.25 minutes to surface. Now the

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Notes

(A.7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

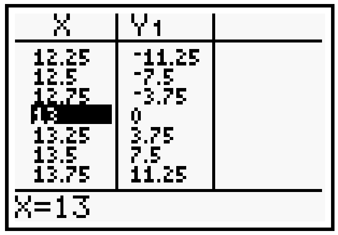
(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;

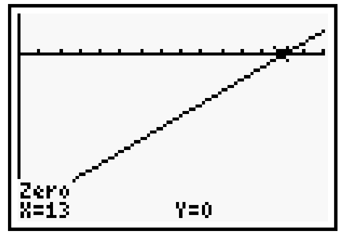
(B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and

submarine must surface in 11.25 minutes.

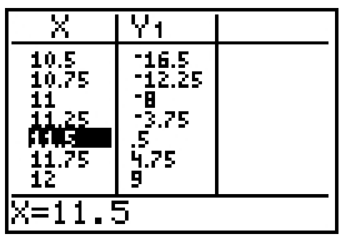
Experiment with different rates (more than 12 meters per minute) by using tables and graphs. If the submarine rises at 15 meters per minute, the equation is $D = -195 + 15t$.

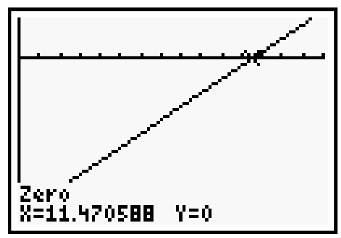
The graph and table show that it takes 13 minutes to reach the surface.



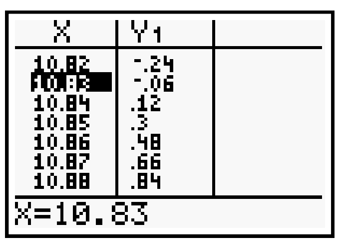


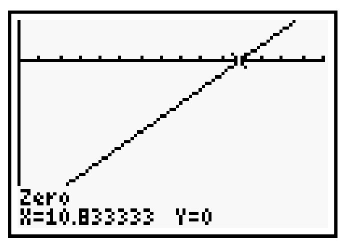
If the submarine rises at 17 meters per minute, the equation is $D = -195 + 17t$. It takes about 11.5 minutes to reach the surface.





If the submarine rises at 18 meters per minute, the equation is $D = -195 + 18t$. It takes about 10.8 minutes to reach the surface.





The submarine needs to surface at a rate between 17 and 18 meters per minute.

Another approach is to let the rate be a variable.

$D = -195 + rt$

D must be equal to 0 when $t = 11.25$.

$$\begin{aligned} 0 &= -195 + r(11.25) \\ 195 &= 11.25r \\ r &= 17.3 \end{aligned}$$

The rate is 17.3 meters per minute.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Extension Questions

- How do the domain and range of the situation compare with the mathematical domain and range of the function representing this situation? What effect does this have on how you graph the situation?

  The mathematical domain and range of the function are both all real numbers because it is a linear function. The situation restricts the domain to 0 to 16.25 minutes and the range to –195 to 0 meters. Knowing the domain and range of the situation helps determine an appropriate window for the graph on the calculator.

- What are the intercepts of the graph of the function? What information do they give about the situation?

  The y-intercept is (0, –195). The initial depth of the submarine is 195 meters below surface. The x-intercept is (16.25, 0). The submarine takes 16.25 minutes to reach the ocean's surface.

- Suppose the submarine must rise from 195 meters below the surface to the ocean's surface within 10 to 20 minutes. How does this affect the rate at which the submarine rises toward the ocean surface?

  In this case, the constant is a range in time to rise to the surface instead of the rate at which the submarine rises. So let r be the rate, in meters per minute, that the submarine rises and D the depth, in meters, of the submarine.

D = rt – 195

If d = 0 at 10 minutes,

$$\begin{aligned} 0 &= r(10) - 195 \\ 10r &= 195 \\ r &= 19.5 \end{aligned}$$

If d = 0 at 20 minutes,

$$\begin{aligned} 0 &= r(20) - 195 \\ 20r &= 195 \\ r &= 9.75 \end{aligned}$$

The rate, r meters per minute, must range from 9.75 to 19.5.

- Suppose a student modeled this situation with the function rule $y = 195 - 12t$. What do the variables represent for this rule?

  y represents the distance from the surface to the submarine. The rate of travel is –12 meters per minute because the distance from the surface to the submarine is decreasing at a rate of 12 meters per minute.

  t represents the time it takes to travel from the surface to the submarine.

Chapter 3:
Interacting Linear Functions, Linear Systems

Bears' Band Booster Club

The Bears' Band Booster Club has decided to sell calendars to raise funds for the band. The calendars cost the club $8 each, plus a $65 design fee. They decide to sell the calendars for $12 each. Investigate the situation and determine how many calendars the club must sell to make a profit.

1. Describe in words the dependency relationship between how much the calendars cost the booster club and the number of calendars sold. Translate this relationship into a function rule.

2. Describe the dependency relationship between revenue gained by selling the calendars and the number of calendars sold. Write this relationship using a function rule.

3. How many calendars must the booster club sell to make a profit? Design a system of linear equations to represent this problem situation. Solve this system of equations to answer the question.

4. If the club wants to make a profit of at least $400, how many calendars must they sell? Justify your solution in at least two ways.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

Scaffolding Questions

- How would you compute the cost of 10 calendars? 15 calendars?
- What are the constants in the situation?
- What are the variables in the situation?
- How do you compute the revenue from the sale of 10 calendars?
- What is the cost of 15 calendars? What is the revenue from the sale of 15 calendars?
- Do you make a profit when you sell 15 calendars? Explain how you know.
- How can you tell when you start making a profit? Explain your reasoning.

Sample Solutions

1. Describe in words the dependency relationship between how much the calendars cost the booster club and the number of calendars sold. Translate this relationship into a function rule.

   The total cost of the calendars depends on the number of calendars sold, so the independent variable is the number of calendars sold. The dependent variable is the total cost of the calendars. The cost is $65 plus $8 per calendar.

   The function rule is $f(x) = 65 + 8x$, where $f(x)$ is the cost in dollars and x is the number of calendars.

2. Describe the dependency relationship between revenue gained by selling the calendars and the number of calendars sold. Write this relationship using a function rule.

   The revenue generated depends on the number of calendars sold, so the independent variable is the number of calendars sold. The dependent variable is the amount of money collected (revenue). The revenue is the product of $12 and the number of calendars sold.

   The function rule is $g(x) = 12x$, where $g(x)$ is the revenue in dollars and x is the number of calendars sold.

3. How many calendars must the booster club sell to make a profit? Design a system of linear equations to represent this problem situation. Solve this system of equations to answer the question.

   To make a profit, the booster club's revenue must be greater than the cost. They break even when the revenue equals the cost.

   Cost: $y = 8x + 65$

   Revenue: $y = 12x$

   Using substitution for y in the first equation:

   $$\begin{aligned} 12x &= 8x + 65 \\ 4x &= 65 \\ x &= 16.25 \end{aligned}$$

   Since x represents the number of calendars sold, the only reasonable values in this situation are whole numbers. You cannot sell a fraction of a calendar or a negative number of calendars. Thus, 16.25 must be rounded up to 17.

   The cost becomes less than the revenue when 17 calendars are sold. Therefore, club must sell at least 17 calendars to make a profit.

4. If the club wants to make a profit of at least $400, how many calendars must they sell? Justify your solution in at least two ways.

   All of the sample solution strategies for this problem assume that students are using a calculator to create tables and graphs. They may examine the table or graph to see when the difference in the cost and revenue is at least $400.

Solution Strategy 1

One possible solution strategy is to start with a guess of 100 calendars and examine the table.

```
TABLE SETUP
 TblStart=100
 ΔTbl=10
Indpnt: Auto Ask
Depend: Auto Ask
```

X	Y1	Y2
100	865	1200
110	945	1320
120	1025	1440
130	1105	1560
140	1185	1680
150	1265	1800
160	1345	1920

X=100

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

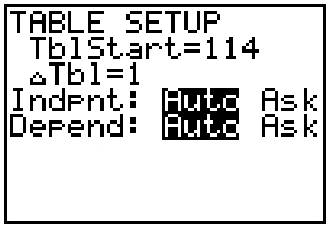
(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

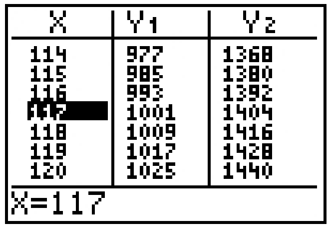
(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

In the table, the Y1 column represents the cost, and the Y2 column represents the revenue. To determine the profit, look at the difference between the columns. At 110 calendars sold, the difference between the cost and the revenue is less than \$400. At 120 calendars sold, the difference is more than \$400. Therefore, the number of calendars the club must sell to make a profit of at least \$400 is between 110 and 120. Reset the table at 110 with an increment of 1, and again determine the differences between Y1 and Y2.





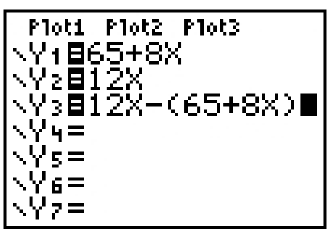
The club must sell 117 calendars to make a profit of at least \$400.

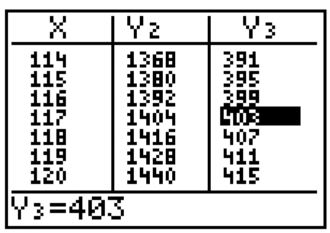
Solution Strategy 2

A second approach is to write a general rule for profit.

Profit = Revenue minus Cost

Profit = 12*x* – (65 + 8*x*)

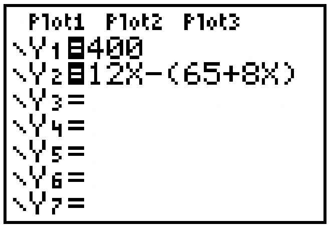


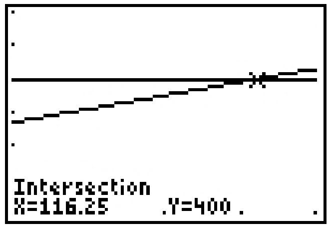


Examine the table to find the first value at which the profit is at least \$400. This occurs when *x* is 117.

Solution Strategy 3

A third approach is to graph the function $y = 12x - (65 + 8x)$ and the function $y = 400$ and find the point of intersection.



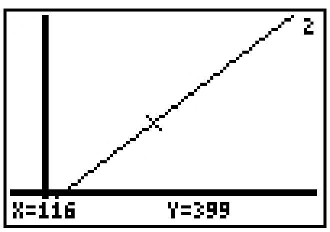


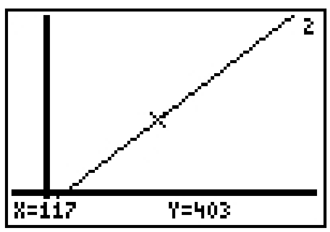
The point of intersection is 116.25, but since we cannot have a fraction of a calendar, we must round up to 117.

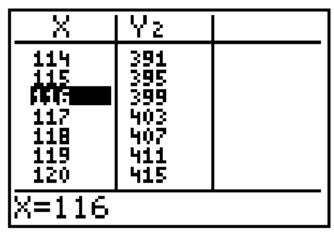
Solution Strategy 4

A fourth approach is to graph the function $y = 12x - (65 + 8x)$ and then look for the value of x that gives a y-value close to 400.

The graphs show that the profit for the sale of 116 calendars is \$399, and the profit for the sale of 118 calendars is \$407. The table shows that the profit for 117 calendars is \$403.







Extension Questions

- Using the original situation, generate a function rule for profit. Use this rule to determine how many calendars must be sold to generate a profit of at least \$600.

  To solve using an equation:

  Profit = Revenue – Cost

  $P(x) = g(x) - f(x)$

  $P(x) = 12x - (65 + 8x)$

  $P(x) = 12x - 65 - 8x$

$P(x) = 4x - 65$

To make a profit of $600, solve the inequality:

$600 \leq 4x - 65$

$665 \leq 4x$

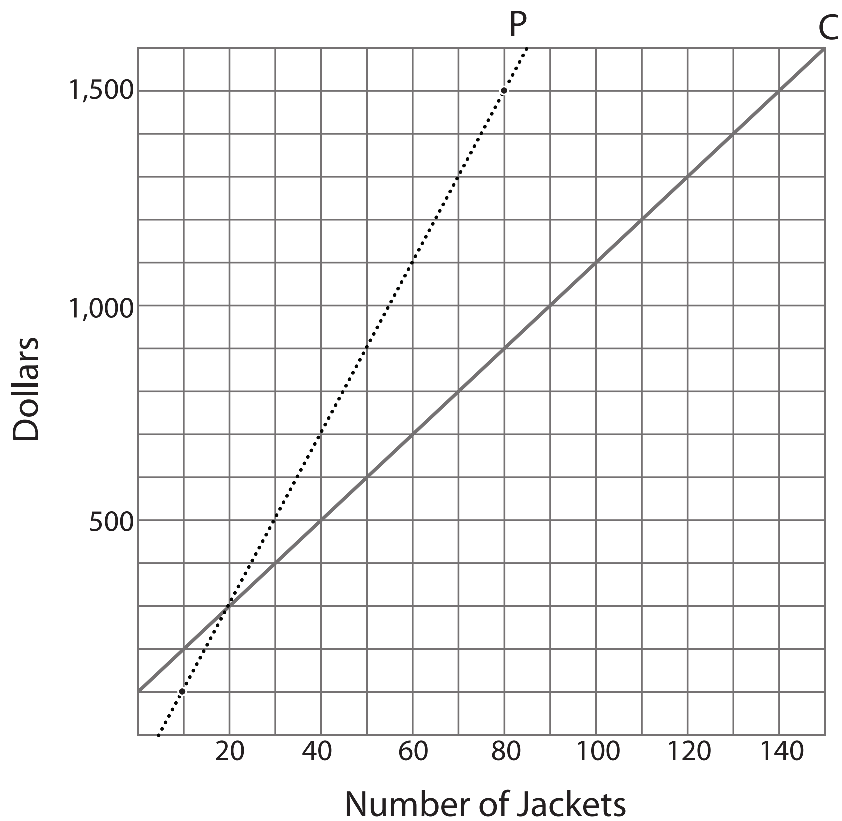
$x \geq 166.25$

The club would need to sell 167 calendars to generate a profit of at least $600.

- Consider that the situation had been represented with the function rule for cost $f(x) = 80 + 15x$ and the rule for revenue $g(x) = 20x$. Write a verbal description of the situation.

  The set-up charge was $80, and the cost per calendar was $15. They intended to sell the calendars for $20 each.

Cost and Profit



The Bartlett Booster Club is having a company make customized jackets with the school name and mascot for them to sell. Line C on the graph models the cost the company charges for making the jackets in terms of the number of jackets they make, and line P models profit in terms of the number of jackets the booster club sells.

1. Write a function rule for the cost in terms of the number of jackets the company makes for the club.
2. Describe how to find the cost of 60 jackets.
3. How many jackets were made if the total cost were $340?
4. Write a function rule to represent profit in terms of the number of jackets the club sells.
5. Explain how to determine the profit from the sale of 200 jackets.
6. If profit is revenue minus cost, what is an expression for revenue? Determine the revenue from the sale of 54 jackets?
7. What is the x-intercept of line P, and what does it mean in this problem situation?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs.

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations to solve problems;

Scaffolding Questions

- What is the y-intercept of line C? What does it mean in this situation?
- Describe the rate at which the cost of the jackets is increasing. Discuss how you found the rate.
- What is the rate of change for line P? What does it mean in this situation?
- How do you determine the y-intercept of line P? What does it mean?

Sample Solutions

Note to teachers: This assessment deals with the relationship between cost, revenue, and profit from a somewhat unique perspective. It is common to provide students with costs and revenue, as in the Bear's Band Booster Club activity. In this scenario, however, students are provided with a graph of cost and profit from which they must infer the revenue.

1. Write a function rule for the cost in terms of the number of jackets the company makes for the club.

   The initial cost, according to the graph, is $100. The rate of change is $100 for every 10 jackets, or $10 for 1 jacket. The rule for the cost is $100 plus the product of $10 and the number of jackets the company makes.

   $C(x) = 100 + 10x$, where $C(x)$ is the cost and x is the number of jackets.

2. Describe how to find the cost of 60 jackets.

   To determine the cost of making 60 jackets, evaluate the function at $x = 60$. Students can evaluate the function algebraically or determine the answer from the graph, looking at the point (60, 700).

   $C(60) = 100 + 10(60)$, or $700

   The cost of 60 jackets is $700.

3. How many jackets were made if the total cost were $340?

   To determine the number of jackets that will cost $340 to make, solve the equation.

   $340 = 100 + 10x$

   $240 = 10x$

   $x = 24$

   Making 24 jackets would cost $340.

   This solution may be hard to read from the given graph because of its scale. However, students should use the graph to check their algebraic solution for reasonableness.

4. Write a function rule to represent profit in terms of the number of jackets the club sells.

   To find the function rule that represents profit, begin by finding the rate of change. One approach that students may use is to choose two points on the profit line, for example (20, 300) and (30, 500). Students may use those points and reason that that for every 10 jackets, the profit increases by $200. This will result in a rate of $20 per jacket. One way to determine the *y*-intercept of line P is to imagine extending the graph in the negative direction from the point (20, 300). Reduce *y* by 200 for every reduction of 10 in *x* until *x* is 0.

   (20 – 10, 300 – 200) or (10, 100) is a point on the graph.

   (10 – 10, 100 – 200) or (0, –100) is a point on the graph.

   The *y*-intercept is –100.

   Another approach would be to choose any two points on the profit line, and then find the rate of change by dividing the *y*-values by the *x*-values.

   For example, (20, 300) and (30, 500).

$$\frac{500-300}{30-20} = \frac{200}{10} = \frac{20}{1}$$

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 4: The student will formulate and use linear equations and inequalities.

The rate of change is $200 for 10 jackets, or $20 per jacket.

Then substitute the slope and any point on the line into the equation $y = mx + b$ and solve for b.

$300 = \frac{20}{1}(20) + b$

$300 = 400 + b$

$-100 = b$

Therefore, $P(x) = 20x - 100$, where x is the number of jackets and $P(x)$ is the profit. This means they make a profit of $20 per jacket minus the initial start-up cost of $100. Before they sell any jackets, they start with a $100 debt.

A third way to approach the problem is to create a table of data points and use that to find a pattern. Extending the table backwards will show that when $x = 0$, the profit is $y = -100$. Use the finite differences to determine that the rate of change is $200 in profit for every 10 jackets sold, or $20 per jacket.

Therefore, $P(x) = 20x - 100$, where x is the number of jackets and $P(x)$ is the revenue.

Number of Caps Sold	Profit
10	10
20	300
30	500

Differences (Number of Caps Sold)	Differences (Profit)
10	200
10	200

5. Explain how to determine the profit from the sale of 200 jackets.

   Let $x = 200$

   $-100 + 20(200) = 3{,}900$

   The profit from the sale of 200 jackets is $3,900.

6. If profit is revenue minus cost, what is an expression for revenue? Determine the revenue from the sale of 54 jackets?

   If profit is revenue minus cost, then revenue is profit plus cost.

   $P = R - C$

   $R = P + C$

   The function rule for the profit is the difference between the rule for revenue and the rule for cost.

   $P = -100 + 20x$

   $R = (-100 + 20x) + (100 + 10x) = 30x$

   This means that the club sells each jacket for $30.

   The revenue from the sale of 54 jackets is $R = 30(54)$ or $R = \$1{,}620$.

7. What is the *x*-intercept of line P, and what does it mean in this problem situation?

   The *x*-intercept of line P is the point (5, 0), meaning that when 5 jackets are sold, the club earns a profit of $0. In this scenario, the profit function is increasing, so when 6 or more jackets are sold, the club earns a profit.

Extension Questions

- Suppose booster club was able to find someone who would sell them jackets at the same price, but with an initial cost of $80. How would the graph of the cost line be affected?

  The graph would be a line parallel to the original line, but with a y-intercept of 80. The function rule for cost would be $C = 10x + 80$.

- If the club continues to sell the jackets for $30, how is the profit affected by this new cost function? Describe the effect on the graph.

  The revenue function would still be $R = 30x$.

  Profit equals revenue minus cost.

  $P = 30x - (10x + 80) = 20x - 80$

  The previous profit function was $P = 20x - 100$. The profit would be increased by $20. The graph of the line would be raised 20 units, but it would have the same slope.

- Consider the point of intersection for cost and profit. Is that the break-even point? Why or why not?

  The break-even point is the point where cost = revenue. The point of intersection in this case represents where cost = profit, so it is not the break-even point. The true break-even point is:

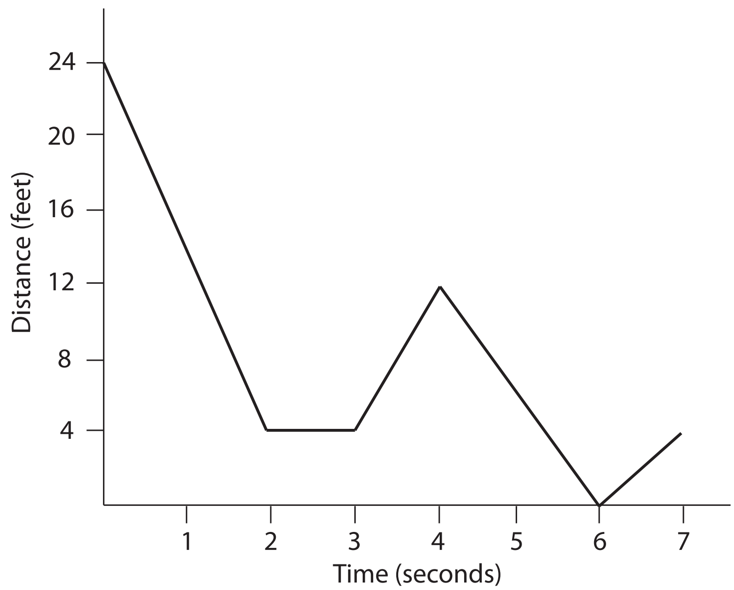
  Cost = revenue

  $$\begin{aligned} 10x + 100 &= 30x \\ 100 &= 20x \\ 5 &= x \end{aligned}$$

  So the break-even point is when 5 jackets are sold. Note that this is also the x-intercept of the profit graph (earlier interpreted in question 7 of the original problem)—the point after which the club earns a profit.

Motion Detector Problem

The graph below shows how the distance between Devon and a motion detector depends on the time that has elapsed since she began walking.



1. Give a detailed description of how Devon's position relative to the motion detector changes over the time interval of 0 to 7 seconds. Include a description of how fast she is moving for each portion of the graph.

2. When does Devon move the fastest? Explain.

3. Write a function rule for each phase you described in your solution to question 1. For each function rule, specify the domain symbolically (for example, $3 \le x \le 6$).

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(A) describe independent and dependent quantities in functional relationships;

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

Scaffolding Questions

- How far from the motion detector is Devon initially?
- How can you determine how fast Devon is moving for a given interval?
- Between 0 and 2 seconds, is she moving toward the detector or away? How fast is she moving?
- When is she standing still? For how long?
- When is Devon moving away from the motion detector?
- Does she ever reach the motion detector?

Sample Solutions

1. Give a detailed description of how Devon's position relative to the motion detector changes over the time interval of 0 to 7 seconds. Include a description of how fast she is moving for each portion of the graph.

   At the beginning, Devon is 24 feet away from the motion detector. She walks for 2 seconds and stops 4 feet from the motion detector. She is walking at a rate of 20 feet per two seconds, or 10 feet per second.

   Devon stands still 4 feet from the motion detector for 1 second.

   She walks away for 1 second at a rate of 8 feet per second. She stops 12 feet from the motion detector.

   Next, she walks for 2 seconds back toward the motion detector at a rate of 12 feet per two seconds, or 6 feet per second. She reaches the motion detector.

   Devon walks away from the motion detector for 1 second at a rate of 4 feet per second. The graph stops with Devon 4 feet away from the motion detector.

2. When does Devon move the fastest? Explain.

   Devon moves the fastest during the first 2 seconds, when she is walking toward the motion detector at 10 feet per second.

3. Write a function rule for each phase you described in your solution to question 1. For each function rule, specify the domain symbolically (for example, $3 \leq x < 6$).

For the first phase, the *y*-intercept is 24 and the rate of change is –10 feet per second. The rate is negative because the distance between Devon and the motion detector is decreasing as time increases. The function rule is $y = -10x + 24$, where $0 \leq x < 2$.

From 2 to 3 seconds, the distance stays constant. There are 4 feet between Devon and the motion detector, so the function rule is $y = 4$, where $2 \leq x < 3$.

From 3 to 4 seconds, the rate of change is 8 feet per second because the distance between Devon and the motion detector is increasing. Substituting either data point (3, 4) or (4, 12) in $y = 8x + b$ gives the *y*-intercept of the function rule.

$$4 = 8(3) + b$$
$$4 - 8(3) = b$$
$$-20 = b$$

The function rule is $y = 8x - 20$ where $3 \leq x < 4$.

From 4 to 6 seconds, the rate of change is –6 feet per second. Use (4, 12) or (6, 0) in $y = b - 6x$ to get the *y*-intercept.

$y = b - 6x$

$0 = b - 6(6)$

$0 = b - 36$

$36 = b$

The function rule is $y = -6x + 36$, where $4 \leq x < 6$.

Finally, from 6 to 7 seconds, the rate of change is 4 feet per second. Use (6, 0) or (7, 4) in $y = 4x + b$ to get the *y*-intercept $b = -24$.

The function is $y = 4x - 24$, where $6 \leq x < 7$.

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

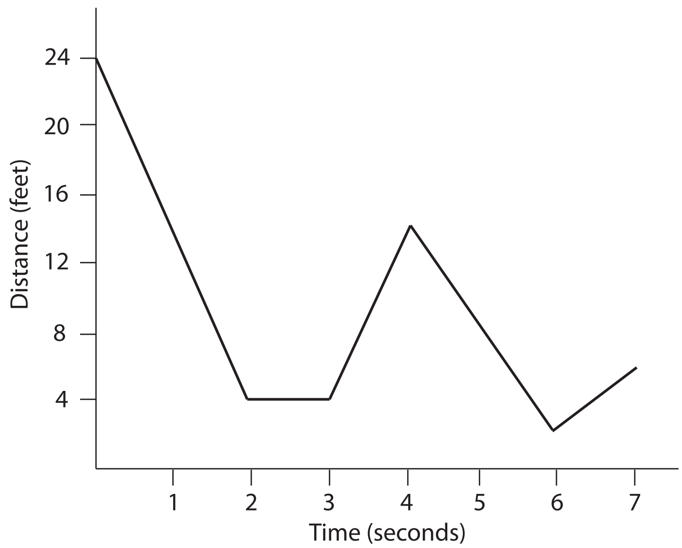
Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Extension Questions

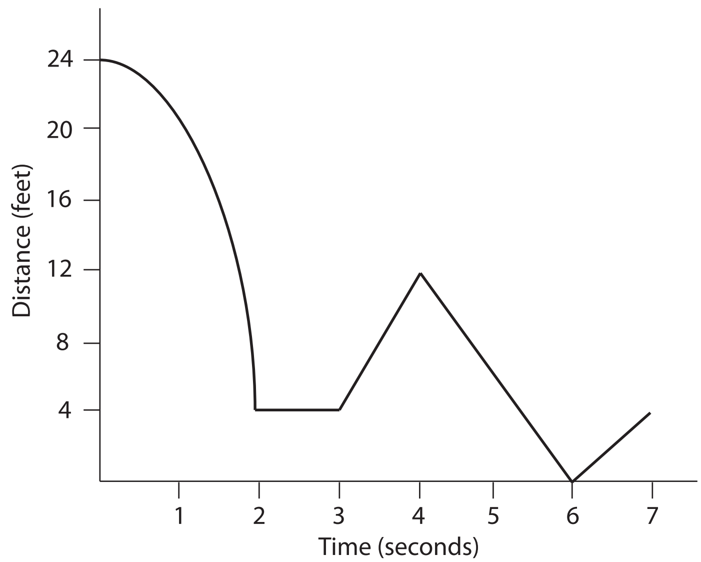
- How would the graph change for the time interval of 3 to 4 seconds if Devon walked away from the motion detector at the rate of 10 feet per second, then continued to walk at the original rates for the remainder of the time? Sketch the graph and describe how it changes from the original.

  In this scenario, Devon walks at a faster rate between 3 and 4 seconds, so the graph is steeper. At 3 seconds, she is 4 feet from the motion detector. She walks away from the motion detector for 1 second at a rate of 10 feet per second, stopping 14 feet from the motion detector. At 4 seconds, she walks back toward the motion detector at the rate of 6 feet per second, stopping 2 feet from the motion detector, then turns around and walks for 1 second at 4 feet per second. She stops 6 feet from the motion detector.



- How would the graph change for the time interval of 0 to 2 seconds if Devon walked toward the motion detector at an increasing rate? Sketch a possible graph.

  The graph would be a curve opening downward. Since the rate of change is not constant, it would not be linear. Since it is changing at an increased rate, it could be a quadratic curve.



- What would be the function rule for velocity versus time, and what would its graph look like?

  The velocity is the rate of change for each section of the graph. In each section, the slope is a constant. The values for y in each section are also constant.

  The function rules for the sections would be

  If $0 \leq x \leq 2$, $y = -10$.

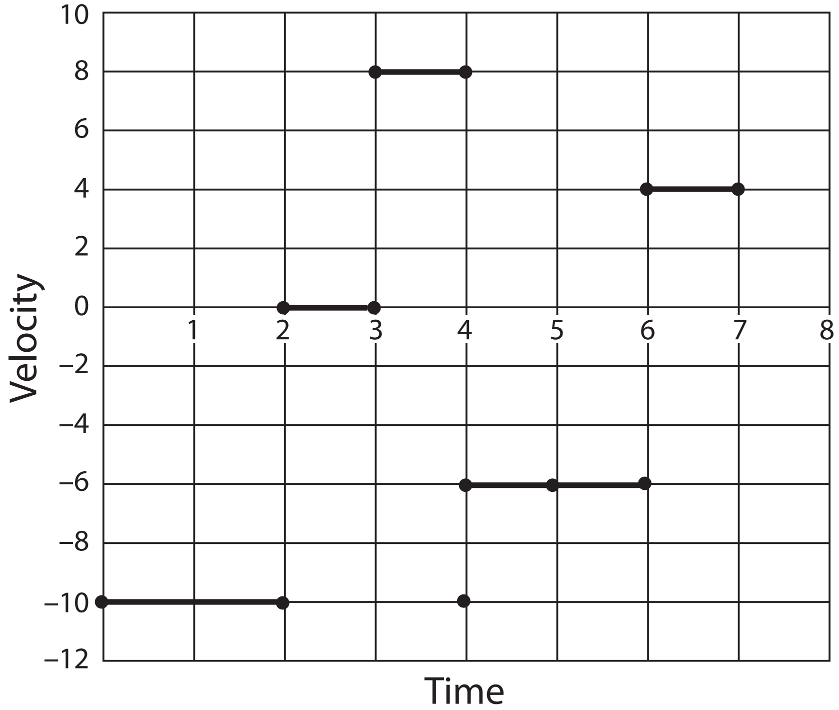
  If $2 \leq x \leq 3$, $y = 0$.

  If $3 \leq x \leq 4$, $y = 8$.

  If $4 \leq x \leq 6$, $y = -6$.

  If $6 \leq x \leq 7$, $y = 4$.

The graph would be



The Walk

Two motion detectors have been set up in a room so that two students, Pam and Abigail, may walk in parallel paths, each directly in front of a motion detector. The table below shows the data that was collected for each walk. Assume the students each walked at a constant rate and started at the same time.

Pam's Walk

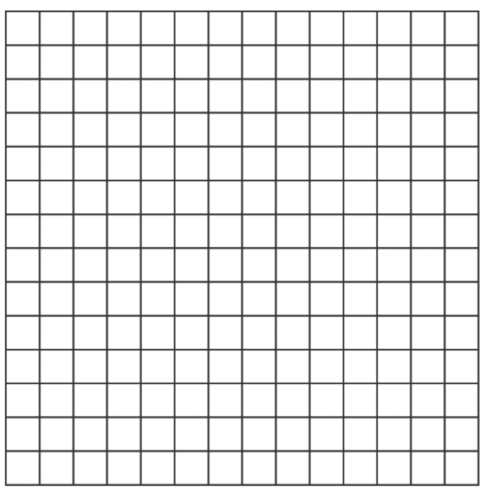
Time (seconds)	Distance (feet) from Motion Detector
1	7.9
3	5.3
6	1.4

Abigail's Walk

Time (seconds)	Distance (feet) from Motion Detector
2	3.6
4	5.2
7	7.6

1. What are reasonable domain and range values for this problem situation?

2. Create a graph to model the students' walks. Label the axes.



3. Write a function rule that models each student's distance from the motion detector in terms of the number of seconds.

4. Determine the point of intersection to the nearest tenth of the two lines.

5. Describe what the point of intersection represents in the scenario.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Scaffolding Questions

- How would you describe the shape of the graph if the students walked at a constant rate?
- How can you use the tables to determine how fast each student was walking?
- If you plot the points, what pattern do you see?
- How is Pam's walk different from Abigail's walk?

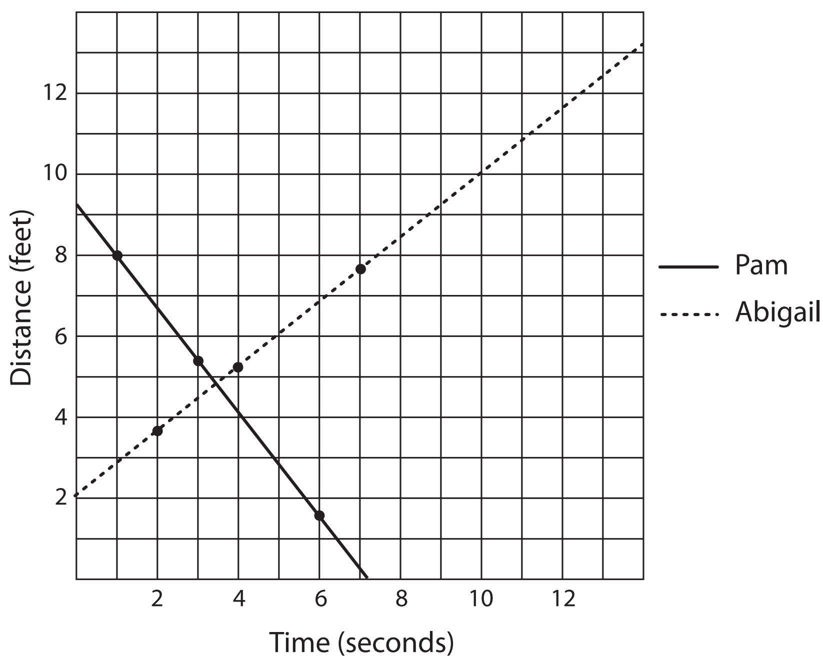
Sample Solutions

1. What are reasonable domain and range values for this problem situation?

   The domain values must be greater than or equal to 0. (In a real-life situation, there is a limit to the amount of time the students could walk in front of the motion detectors; they are limited by the room's size or the detectors' sensing range.)

   The range values represent distance and must be positive numbers. The distance or range is limited by detector's sensing range.

2. Create a graph to model the students' walks. Label the axes.



3. Write a function rule that models each student's distance from the motion detector in terms of the number of seconds.

Start by using the differences between each student's recorded distances to determine how fast each one walked.

Pam's Walk

Change	Time (seconds)	Distance (ft) from Motion Detector	Change
	1	7.9	
2	3	5.3	-2.6
3	6	1.4	-3.9

Abigail's Walk

Change	Time (seconds)	Distance (ft) from Motion Detector	Change
	2	3.6	
2	4	5.2	1.6
3	7	7.6	2.4

Pam walked at a rate of 1.3 feet per second, which is represented by –1.3. Pam's distance from the motion detector decreased, so she must have walked toward the detector.

Abigail walked at a rate of 0.8 feet per second, represented by 0.8. Abigail's distance from the motion detector increased, so she must have walked away from the detector.

The function rule for Pam's walk is the starting point plus the product of her rate and the number of seconds she walked. Use the table and the rates to determine the starting points. Add an extra row for time 0.

Pam's Walk

Change	Time (seconds)	Distance (ft) from Motion Detector	Change
	0	9.2	
1	1	7.9	-1.3
2	3	5.3	-2.6
3	6	1.4	-3.9

Abigail's Walk

Change	Time (seconds)	Distance (ft) from Motion Detector	Change
	0	2	
2	2	3.6	1.6
2	4	5.2	1.6
3	7	7.6	2.4

To determine Pam's starting point, add 1.3 feet—the distance she traveled in 1 second—to 7.9 feet—her distance from the motion detector at 1 second. This shows where Pam was at 0 seconds.

$1.3 + 7.9 = 9.2$

At 0 seconds, Pam was 9.2 feet from the motion detector. The function rule for Pam's walk is $y = 9.2 - 1.3x$.

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and *y*-intercept;

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

The function rule for Abigail's walk is her starting point plus the product of her rate and the number of seconds she walked. Her starting point is 3.6 minus the distance she traveled in 2 minutes.

$3.6 - 1.6 = 2$

At 0 seconds, Abigail was 2 feet from the motion detector. The function rule that describes Abigail's walk is $y = 2 + 0.8x$.

4. Determine the point of intersection to the nearest tenth of the two lines.

We want to know when Pam and Abigail were the same distance from their own motion detector. The graphs show that at about 3 seconds they were both about 5 feet from their own detector.

To check this solution, solve the system:

$y = 9.2 - 1.3x$

$y = 2 + 0.8x$

$$\begin{aligned} 2 + 0.8x &= 9.2 - 1.3x \\ 2.1x &= 7.2 \\ x &\approx 3.4 \end{aligned}$$

$y = 2 + 0.8(3.4) = 4.7$

The point of intersection is (3.4, 4.7).

5. Describe what the point of intersection represents in the scenario.

The point of intersection represents when Pam and Abigail were both the same distance from their own motion detector. This happened at about 3.4 seconds, when they were both about 4.7 feet from their own motion detector.

Extension Questions

- Suppose the motion detectors were set up on opposite sides of the room 10 feet apart and the girls walked on parallel paths. If the same data were used, how would your answer to question 4 be different?

If the motion detectors were 10 feet apart on opposite sides of the room, the two walkers would walk in the same direction.

The equations describing their motion relative to each person's motion detector would still be

Pam: $y = 9.2 - 1.3x$

Abigail: $y = 2 + 0.8x$

However, to determine when they would be in the same horizontal position, one must write the equations in terms of distance from one of the motion detectors. Suppose the equations are written as distance from Abigail's motion detector with respect to time.

Abigail started 2 feet from her motion detector. Pam started 9.2 feet from her motion detector. Since the distance between the motion detectors is 10 feet, Pam would be on a horizontal distance of 10 – 9.2, or 0.8 feet from Abigail's motion detector. The equation of Pam's movement relative to Abigail's motion detector is her starting point plus the product of her rate and the number of seconds.

$y = 0.8 + 1.3x$

The rate is positive because Pam's distance from Abigail's motion detector is increasing.

Abigail's rule is $y = 2 + 0.8x$.

Solving this system of equations results in a solution of

$$\begin{aligned} 0.8 + 1.3x &= 2 + 0.8x \\ 0.5x &= 1.2 \\ x &= 2.4 \end{aligned}$$

$y = 2 + 0.8(2.4)$

$y \approx 3.92$

They would both be about 3.92 feet from Abigail's motion detector 2.4 seconds after they started walking.

- Consider the previous scenario. How would the function rules change if the motion detectors were positioned 12 feet apart instead of 10 feet?

  Pam would be 12 – 9.2, or 2.8 feet from Abigail's motion detector. The equation of her movement relative to Abigail's motion detector is her starting point plus the product of her rate and the number of seconds she walked.

  $y = 2.8 + 1.3x$

  Pam's equation would be $y = 2.8 + 1.3x$.

  Abigail's rule does not change. It is still $y = 2 + 0.8x$.

Speeding Cars

On a racing route, four cars traveling at different speeds all passed through the same checkpoint at the same time. The following data were collected from the performance of the four cars based on miles driven from that checkpoint in terms of hours. Assume that the average speed of each car stayed constant during the interval in which it was collected.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

Car B

Hours	Miles
0	0
1	75
2	150
3	225
4	300

Car C

Hours	Miles
0	0
5	200
10	400
15	600
20	800

Car D

Hours	Miles
0	0
1	65
2	85
3	105
4	125

1. Using the data in the tables, determine which car was traveling the fastest. How do you know?
2. Which car was traveling the slowest? How do you know?
3. Compare and contrast the tables.
4. If possible, write a function rule to model each car's travel.
5. Create a graph to represent the distance traveled for each car. Compare and contrast the graphs.
6. Compare the domains for the functions and the domains for the problem situation.
7. Do any of the tables represent a direct variation? Explain how you know.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeroes of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and *y*-intercept;

(G) relate direct variation to linear functions and solve problems involving proportional change.

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

Scaffolding Questions

- How can you use the tables to determine the speed of each car?
- How can you tell whether a car was traveling at a constant rate?
- What are the similarities in the table values?
- What are the differences in the table values?
- How can you tell whether a table represents a linear function?
- How can you tell from a graph that a function is linear?
- What must be true if a set of points represents a direct variation?
- What happens if you do not assume that the average speed of each car stayed constant during each timing interval?

Sample Solutions

1. Using the data in the tables, determine which car was traveling the fastest. How do you know?

   One way to find the speed at which a car traveled is to divide the distance the car traveled by the amount of time it traveled. You can also use the slope formula for any consecutive interval.

Car A:

$\frac{120-0}{2-0}=60 \quad \frac{180-0}{3-0}=60 \quad \frac{300-0}{5-0}=60 \quad \frac{360-0}{6-0}=60$

Car A traveled at a constant rate of 60 miles per hour.

Car B:

$\frac{75-0}{1-0}=75 \quad \frac{150-0}{2-0}=75 \quad \frac{225-0}{3-0}=75 \quad \frac{300-0}{4-0}=75$

Car B traveled at a constant rate of 75 miles per hour.

Car C:

$\frac{200-0}{5-0}=40 \quad \frac{400-0}{10-0}=40 \quad \frac{600-0}{15-0}=40 \quad \frac{800-0}{20-0}=40$

Car C traveled at a constant rate of 40 miles per hour.

Car D:

$\frac{65-0}{1-0}=65 \quad \frac{85-0}{2-0}=42.5 \quad \frac{105-0}{3-0}=35 \quad \frac{125-0}{4-0}=31.25$

Car D did not travel at a constant rate.

If the rates are examined for the one-hour time intervals, the differences are not the same.

$\frac{65-0}{1-0}=65 \quad \frac{85-65}{2-1}=20 \quad \frac{105-85}{3-2}=20 \quad \frac{125-105}{4-3}=20$

Car D traveled 65 mph for the first hour and then at a constant rate of 20 mph for the next three hours.

Car B traveled the fastest; for every hour, it went 75 miles.

2. Which car was traveling the slowest? How do you know?

Car C traveled at the slowest constant rate; for every hour, it traveled only 40 miles. Car D, however, traveled at a slower rate after the first hour.

3. Compare and contrast the tables.

The tables are similar in several ways; all four tables start at (0, 0) and report miles and hours. All the tables show that as the hours increase, the number of miles traveled also increases. The tables are different because the time intervals are not the same in every table. In the table for Car D, there isn't a constant rate throughout the *y*-values.

Some students may note that the tables for Cars A, B, and C represent linear relationships because they indicate a constant rate of change. The table for Car D does not represent a linear relationship since the car traveled 65 miles in the first hour but only 20 miles for each additional hour.

The student is expected to:

(A) determine whether or not given situations can be represented by linear functions;

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

4. If possible, write a function rule to model each car's travel.

   The first three tables can be modeled by function rules of the form $y = mx + 0$ (where m is the slope or rate of change) because the starting value is 0.

   Car A: $y = 60x$

   Car B: $y = 75x$

   Car C: $y = 40x$

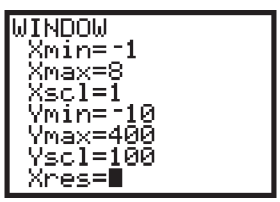
   However, the rule for Car D is different.

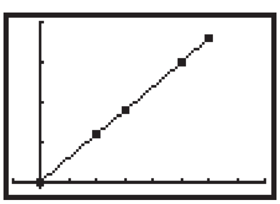
   Car D: $\begin{cases} 65x, & 0 \le x \le 1 \\ 20x + 45, & 1 < x \le 4 \end{cases}$

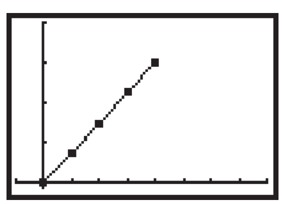
   For values of x between 0 and 1, the rate is 65 mph, but between the values of 1 and 4, the linear model $y = 20x + 45$ fits.

   Note: This is an example of a **piecewise-defined function**, which is not formally defined until Precalculus. At this point, we are suggesting only that students may determine a function rule for each phase.

5. Create a graph to represent the distance traveled for each car. Compare and contrast the graphs.



A. 

B. 

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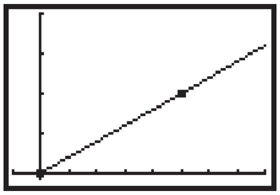

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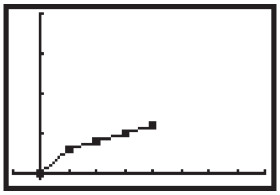


Each graph is a set of points. All graphs start at (0, 0). The lines all have a positive slope. Graph B has the greatest slope because it is the steepest and has the greatest rate of change. Graph D is modeled by a combination of two lines—a piecewise-defined function. Graphs A, B, and C represent linear relationships.

6. Compare the domains for the functions and the domains for the problem situation.

   The domains for the functions are all real numbers. The domains for the problem situation are numbers greater than or equal to 0. The graphs show connected points because the functions are continuous for the values of the number of hours. However, the upper limit on the domain values depends on how long each car traveled.

7. Do any of the tables represent a direct variation? Explain how you know.

   The tables for Cars A, B, and C represent direct variations because there is a constant rate of change and the data contain the point (0, 0). The table for Car D does not represent a direct variation because there is not a constant rate of change.

Extension Questions

- If another car traveled at twice the speed of Car C, how would the table values be affected?

  If the car is traveling at twice the speed of Car C, the rule for the distance traveled as a function of the number of hours would be $y = 80x$.

  If the x-values were the same, the y-values would have been twice the original values of Car C.

- Suppose that another car has the same table values as Car A except that 20 is added to each *y*-value. Describe how this car's motion is the same as or different from that of Car A.

Car A

Hours	Miles
0	0
2	120
3	180
5	300
6	360

New Car

Hours	Miles
0	20
2	140
3	200
5	320
6	380

The new car is traveling at the same rate as Car A—60 mph—although 20 has been added to each y-value.

$\frac{140-20}{2-0}=60$ $\frac{200-20}{3-0}=60$ $\frac{320-20}{5-0}=60$ $\frac{380-20}{6-0}=60$

One possible way to interpret the difference between the two cars' tables is that data for the new car were first collected 20 miles before the car entered the intersection.

- How would the graph of this new car's function be different from the graph of Car A's function?

  The y-intercept of this new graph would be at 20, but the graph would be parallel to the graph of Car A. The situation does not represent direct variation.

Chemistry Dilemma

Bonnie and Carmen are lab partners in a chemistry class. Their chemistry experiment calls for a 5-ounce mixture that is 65% acid and 35% distilled water. There is no pure acid in the chemistry lab, but the partners find two mixtures that are labeled as containing some acid. Mixture A contains 70% acid and 30% distilled water. Mixture B contains 20% acid and 80% distilled water.

How many ounces of each mixture should Bonnie and Carmen use to make a 5-ounce mixture that is 65% acid and 35% distilled water? Justify your solution using symbols, tables, and graphs.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

Scaffolding Questions

- How much distilled water is there in 1 ounce of Mixture A? How do you know?
- How many ounces of acid are there in 4 ounces of Mixture A?
- How many ounces of distilled water are there in 2 ounces of Mixture B?
- What are the variables in this situation?

Sample Solutions

How many ounces of each mixture should Bonnie and Carmen use to make a 5-ounce mixture that is 65% acid and 35% distilled water? Justify your solution using symbols, tables, and graphs.

Table: (shown on next page)

This is a possible student solution showing combinations of Mixture A and Mixture B and the percentages of acid and water in the new mixture.

Amount of Mixture A	Amount of Mixture B	Amount of Acid in New Mixture	Amount of Distilled Water in New Mixture	Percent of New Mixture That Is Acid	Percent of New Mixture That Is Distilled Water	Is It 65% Acid and 35% Water?
1	5 – 1 = 4	0.7(1) + 0.2(4) = 1.5	0.3(1) + 0.8(4) = 3.5	1.5 out of 5 = 30%	3.5 out of 5 = 70%	Too much water
2	5 – 2 = 3	0.7(2) + 0.2(3) = 2	0.3(2) + 0.8(3) = 3	2 out of 5 = 40%	3 out of 5 = 60%	Too much water
3	5 – 3 = 2	0.7(3) + 0.2(2) = 2.5	0.3(3) + 0.8(2) = 2.5	2.5 out of 5 = 50%	2.5 out of 5 = 50%	Too much water
4	5 – 4 = 1	0.7(4) + 0.2(1) = 3	0.3(4) + 0.8(1) = 2	3 out of 5 = 60%	2 out of 5 = 40%	Too much water
5	0	0.7(5) + 0 = 3.5	0.3(5) + 0 = 1.5	3.5 out of 5 = 70%	1.5 out of 5 = 30%	Not enough water

The student is expected to:

(B) look for patterns and represent generalizations algebraically.

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities

The amount of Mixture A that Bonnie and Carmen should use must be between 4 and 5 ounces.

4.5	0.5	0.7(4.5) + 0.2(0.5) = 3.25	0.3(4.5) + 0.8(0.5) = 1.75	3.25 out of 5 = 65%	1.5 out of 5 = 35%	The correct amounts are 4.5 ounces of Mixture A and 0.5 ounces of Mixture B.

Symbols:

Another approach to the problem, using symbols, requires that the variables be defined.

The variables are the amounts of Mixture A and Mixture B.

x = the number of ounces of Mixture A

y = the number of ounces of Mixture B

The total amount of the new mixture must be 5 ounces.

Equation 1: $x + y = 5$

The amount of acid in Mixture A plus the amount of acid in the Mixture B must be 65% of 5 ounces. The acid in Mixture A can be expressed as $0.7x$ and the acid in Mixture B as $0.2y$.

Equation 2: $0.7x + 0.2y = 0.65(5)$

Similarly, the amount of distilled water in Mixture A plus the amount of distilled water in Mixture B must be 35% of 5 ounces. The distilled water in Mixture A can be expressed as $0.3x$, and the distilled water in Mixture B as $0.8y$.

Equation 3: $0.3x + 0.8y = 0.35(5)$

To solve symbolically, use two of the equations and the substitution method:

$$x + y = 5$$

$$0.3x + 0.8y = 0.35(5)$$

$$y = 5 - x$$

$$\begin{aligned} 0.3x + 0.8\,(5 - x) &= 0.35(5) \\ 0.3x + 4 - 0.8x &= 1.75 \\ -0.5x &= -2.25 \\ x &= 4.5 \\ y &= 5 - 4.5 = 0.5 \end{aligned}$$

Bonnie and Carmen should use 4.5 ounces of Mixture A and 0.5 ounces of Mixture B to make 5 ounces of a new mixture that is 65% acid and 35% water.

Graphs:

Graphs can also be used to solve the problem. To use a graphing calculator to graph or make a table, solve the equations for *y*:

$x + y = 5$

$y = 5 - x$

$0.7x + 0.2y = 0.65(5)$

Solve for *y*.

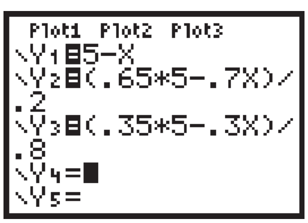
$$y = \frac{0.65(5) - 0.7x}{0.2}$$

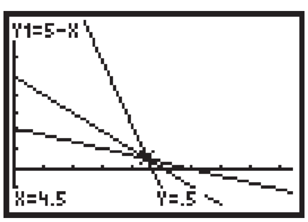
$0.3x + 0.8y = 0.35(5)$

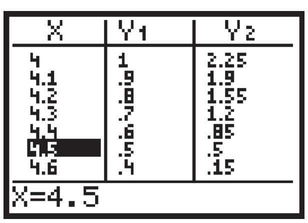
Solve for *y*.

$$y = \frac{0.35(5) - 0.3x}{0.8}$$

Graph the equations and make a table of values to find the common point, which is (4.5, 0.5). This means that Bonnie and Carmen should use 4.5 ounces of Mixture A and 0.5 ounces of Mixture B to make 5 ounces of a new mixture that is 65% acid and 35% water.







Extension Questions

- Was it necessary to have three equations to solve the problem?

  The second and third equations are complements of each other, since one gives the amount of acid needed and the other gives the amount of water needed. For instance, 70% water in one mixture means that the mixture is 30% acid. Since the amounts of water and acid must add up to 100%, an equation for both amounts is not necessary.

- Does it matter which pair of equations is used?

  The graph shows that it does not matter which pair of equations is used. If you use any pair of equations, their graphs intersect at the point (4.5, 0.5).

- What could have been determined if the total amount or 5 ounces was not given?

  The total amount could be expressed as $x + y$.

  The number 5 would be replaced by $x + y$ in the second and third equations.

  The amount of acid would be expressed as $0.7x + 0.2y = 0.65(x + y)$.

  Similarly, the amount of distilled water in Mixture A plus the amount of distilled water in Mixture B must be 65% of $x + y$ ounces.

  $$0.3x + 0.8y = 0.35(x + y)$$

  Simplify the equations:

  $$\begin{aligned} 0.7x + 0.2y &= 0.65x + 0.65y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

  $$\begin{aligned} 0.3x + 0.8y &= 0.35x + 0.35y \\ 0.05x - 0.45y &= 0 \end{aligned}$$

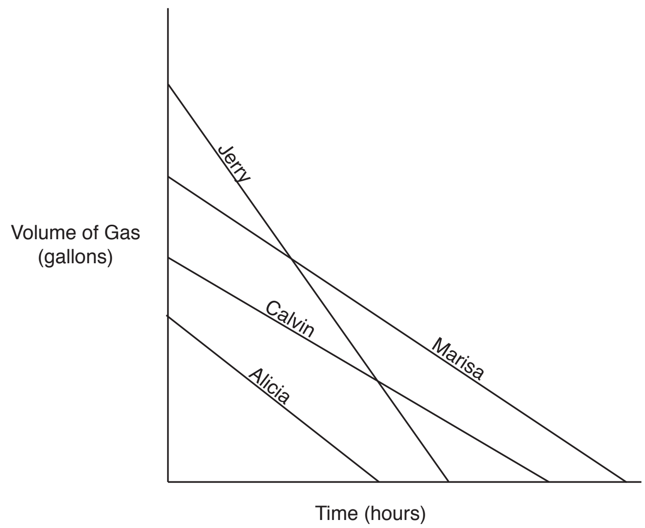
  The system could not be solved for specific values of x and y, but you would know what the ratio of x to y must be for any solution.

  $$\begin{aligned} 0.05x - 0.45y &= 0 \\ 0.05x &= 0.45y \\ \frac{x}{y} &= \frac{0.45}{0.05} = \frac{9}{1} \end{aligned}$$

  To meet the conditions of the problem, the amount of Mixture A must be 9 times the amount of Mixture B.

Four Cars

Jerry, Alicia, Calvin, and Marisa wanted to test their cars' gas mileage. Each person filled his or her car's gas tank to the maximum capacity and drove on a test track at 65 miles per hour until the car ran out of gas. The graphs given below show how the amount of gas in the cars changed over time.



1. Whose car has the largest gas tank? Explain your reasoning.
2. Whose car ran out of gas first? Explain your reasoning.
3. Whose car traveled the greatest distance? How does knowing that they all traveled at 65 miles per hour help you know who traveled the greatest distance?
4. Determine whose car gets the worst gas mileage. Describe how you used the graphs to make your decision.
5. How are Calvin's graph and Marisa's graph similar and how are they different in terms of the given situation?
6. In terms of the given situation, how does the function rule that describes Marisa's graph compare to the function rule that describes Jerry's graph?
7. In terms of the given situation, how does the function rule that describes Jerry's graph compare to the function rule that describes Alicia's graph?
8. Jerry's graph intersects Marisa's graph. What does the point of intersection represent?
9. After some automotive work, Jerry is now getting better gas mileage. How will this affect his graph?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(C) interpret situations in terms of given graphs or creates situations that fit given graphs; and

Scaffolding Questions

- Which car do you think uses the most gas?
- Which gas tank holds the most gas?
- Which gas tank holds the least gas?
- Do any of the cars get similar miles per gallon?
- Why are all the graphs in quadrant 1?
- Do these situations have positive or negative slopes?
- Which lines appear to be parallel?
- If two lines are parallel, what is the same in their equations?
- What does the slope of these graphs represent?
- What do the equations of these lines look like?
- Which person's graph has the greatest y-intercept?
- What does the y-intercept represent in this situation?
- Which person's graph has the greatest x-intercept?
- What does the x-intercept represent in this situation?

Sample Solutions

1. Whose car has the largest gas tank? Explain your reasoning.

   The given information states that each person filled his or her tank to capacity. Jerry's car has the largest tank because at 0 hours his car's gas tank had the greatest volume.

2. Whose car ran out of gas first? Explain your reasoning.

   Alicia's car ran out of gas first because it reached a volume of 0 in the shortest amount of time.

3. Whose car traveled the greatest distance? How does knowing that they all traveled at 65 miles per hour help you know who traveled the greatest distance?

   Marisa's car traveled the farthest because it took the longest time to reach a volume of 0. Each person was traveling at 65 miles per hour. Since distance traveled is rate multiplied by time, and Marisa's time was the greatest, she traveled the greatest distance.

4. Determine whose car gets the worst gas mileage. Describe how you used the graphs to make your decision.

   Jerry's car gets the worst gas mileage. We can tell because his graph has the steepest slope. This means his car's rate of change decreased at the fastest rate. The absolute value of his rate of change is the greatest. The rate of change represents the number of gallons used per hour of travel.

5. How are Calvin's graph and Marisa's graph similar and how are they different in terms of the given situation?

   It appears that the linear graphs for Calvin's and Marisa's cars' gas mileage are almost parallel, so the slopes are about the same. That is, the ratio of gallons per hour is approximately the same for both cars. Since the y-intercept of Calvin's graph is smaller than the y-intercept of Marisa's graph, Calvin's car has a smaller gas tank. Since Calvin's graph indicates a volume of 0 gallons in less time than Marisa's graph, Calvin's car ran out of gas sooner.

6. In terms of the given situation, how does the function rule that describes Marisa's graph compare to the function rule that describes Jerry's graph?

   The function rules for Marisa's and Jerry's cars are very different. Jerry's gas tank is larger than Marisa's, so the function rule (in the form $y = mx + b$) that represents his car's situation has a greater y-intercept. Because his car's graph has a steeper declining slope, his car uses gas at a faster rate; the absolute value of his car's rate of change is greater.

7. In terms of the given situation, how does the function rule that describes Jerry's graph compare to the function rule that describes Alicia's graph?

   Jerry's gas tank holds more than Alicia's gas tank, so the function rule (in the form $y = mx + b$) that represents Jerry's car's situation has a greater y-intercept, or b value. Jerry's car uses gas at a faster rate than Alicia's car; therefore, the slope, or m in the function rule, is negative, with a larger absolute value than the slope in the function rule for Alicia's car.

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

(A.8) Linear Functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 3: The student will demonstrate an understanding of linear functions.

8. Jerry's graph intersects Marisa's graph. What does the point of intersection represent?

   The point of intersection represents the time when Jerry's gas tank and Marisa's gas tank held the same amount of gas.

9. After some automotive work, Jerry is now getting better gas mileage. How will this affect his graph?

   The x-intercept on Jerry's graph would be a larger number. The slope of his graph would show a more gradual decline.

Extension Questions

- If a fifth line were added to the graph parallel to, but different from, the line for Alicia's car, what would you know about this fifth car?

  It uses gasoline at the same rate as Alicia's car, but it has a different tank capacity.

- Suppose everyone traveled at 55 miles per hour instead of 65 miles per hour. How would this affect the graphs?

  If they traveled at a slower rate, the amount of gas used per hour would decrease. The graphs would decline at a less steep slope and the x-intercepts would be greater.

- Suppose Calvin's car's graph could be represented by the rule $y = 30 - 6x$. What information do you now know about Calvin's car? When did his car run of gas? What would be a reasonable rule for Alicia's travel?

  The capacity of Calvin's car's tank is 30 gallons because the y-intercept of $y = 30 - 6x$ is 30. His car is using gasoline at a rate of 6 gallons per hour. At $0 = 30 - 6x$, x is 5, so the x-intercept is 5. This means that it takes him 5 hours to run out of gas. If he is traveling at 65 miles per hour, he will have traveled 65 • 5, or 325 miles.

  The rules for the other drivers can be estimated using the intercepts.

  The y-intercept for the graph of Alicia's car is about half of that for Jerry's car, or approximately 15 gallons.

If Jerry's car's x-intercept is 5, the x-intercept for the graph for Alicia's car is about 4.

$y = b + mx$
$y = 15 + mx$
$0 = 15 + 4m$
$m = -3.75$
$y = 15 - 3.75x$ *is a possible rule for Alicia's travel*

This function rule indicates that she is using gasoline at a rate of approximately 3.75 gallons per hour.

Graph It

1. Create a graph and write a possible function rule for each line described below. Use one set of axes to graph all three lines.

   Line A: The line has slope $-\frac{1}{2}$ and a *y*-intercept of 3.

   Line B: Any line that is parallel to Line A.

   Line C: The line has a *y*-intercept of 5 but a more steeply decreasing slope than Line A.

2. Name two points that lie on Line C.

3. What are the similarities and differences between the graphs of the lines?

4. Must any of the lines intersect? Justify your reasoning.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeroes of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(C) investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

Scaffolding Questions

- What can be determined if you know that the y-intercept is 3?
- How does knowing the slope help you create a graph?
- What part of the function rule must stay constant to produce a parallel line?

Sample Solutions

1. Create a graph, and write a possible function rule for each line described below. Use one set of axes to graph all three lines.

   Line A: The line has slope $-\frac{1}{2}$ and a y-intercept of 3.

   Line B: Any line that is parallel to Line A.

   Line C: The line has a y-intercept of 5 but a more steeply decreasing slope than Line A.

Line A:

To draw the graph, mark the y-intercept of 3 and use the slope to determine another point on the graph. The slope $-\frac{1}{2}$ means that for every change in y of –1 unit, there is a change in x of 2 units. Another point is (2, 2).

The function rule of a line can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

For this line, the rule is $y = -\frac{1}{2}x + 3$.

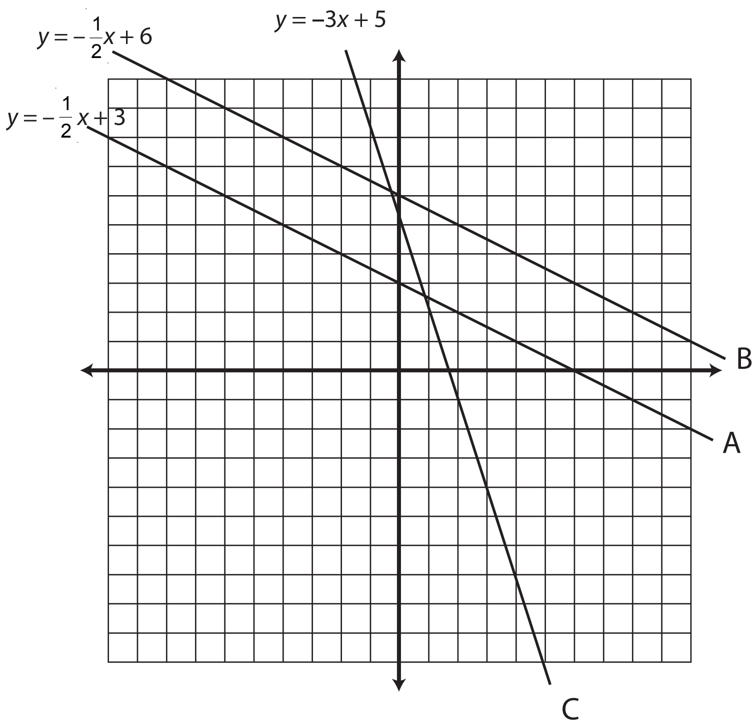
Line B:

If two lines are parallel, they have the same slope but a different y-intercept. The function rule for one possible parallel line is $y = -\frac{1}{2}x + 6$. Any line of the form $y = -\frac{1}{2}x + b$ for any real number b is correct.

Line C:

The *y*-intercept is 5, but the slope must be different. Increase the absolute value of the slope to get a steeper line. The line is still decreasing, so the slope must be negative. For example, if the slope is –3, the function rule is $y = -3x + 5$.

For the sample function rules, the graphs are as shown below. Note: Graphs may differ from those below for different function rules.



2. Name two points that lie on Line C.

   (1, 2) and (4, –7)

   Answers will vary depending on the functions students generate to satisfy the description of line C. For example, using the function rule $y = -3x + 5$, two points are (1,2) and (4, -7).

3. What are the similarities and differences between the graphs of the lines?

   The lines all have negative slopes and positive *y*-intercepts. Lines A and B have the same slope and

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

different *y*-intercepts. They are parallel lines. Line C intersects the other two lines; it has a slope of –3 (for example), and the *y*-intercept is 5.

4. Must any of the lines intersect? Justify your reasoning.

   Two lines always intersect if they have different slopes. Lines A and B, which have the same slope (and, therefore, are parallel), never intersect. Line C, which has a steeper declining slope, intersects Lines A and B.

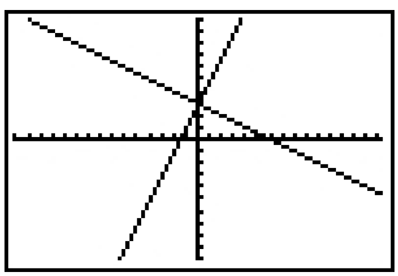
Extension Questions

- If another line is parallel to Line A and translated 6 units down, what is the function rule for the new line?

  The new line has the same slope, but its y-intercept changes to 3 – 6, or –3. The rule is $y = -\frac{1}{2}x - 3$.

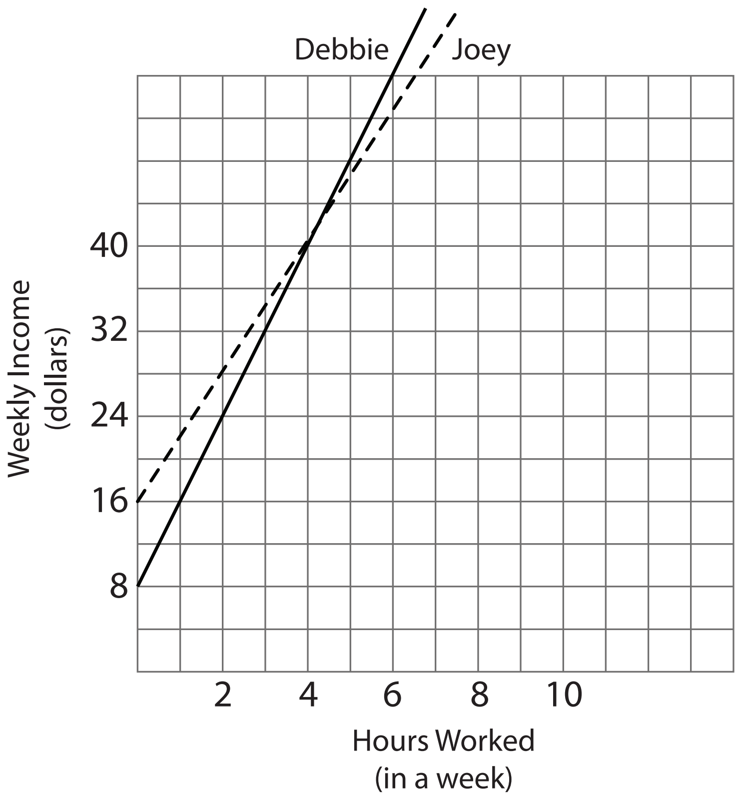
- What is the function rule of a line perpendicular to Line A with the same *y*-intercept?

  Perpendicular lines have opposite, reciprocal slopes. The slope of the new line is +2. The function rule of the new line is y = 2x + 3. Students could check this answer with a graph.



Summer Money

Debbie and Joey have decided to earn money during the summer. Each receives a weekly allowance and has also taken a job. The graphs model their weekly incomes, including allowance, as a function of the number of hours they work.



1. What information would you use to write a function rule?
2. Write a function rule that can be used to calculate the amount of money each person will earn per week in terms of the number of hours worked. Make a table of your data.
3. How will an increase in Debbie's allowance affect the graph? Entries in a table? The function rule? Use an example to justify your thinking.
4. How will an increase in Joey's hourly wages affect the graph? Entries in a table? The function rule? Use an example to justify your thinking.
5. If Debbie's weekly allowance is doubled, will her new weekly income be more or less than twice the original amount? Explain your reasoning.
6. Based on the original functions, who will have more money each week?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigate, describe, and predict the effects of changes in m and b on the graph of $y = mx + b$;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

Scaffolding Questions

- How is each person's allowance represented on the graph?
- What are you using to represent the independent and dependent variables?
- What do you need to know to determine the function rule of a line?
- How much allowance does Debbie receive? How much allowance does Joey receive?
- What does the y-intercept represent for Debbie's situation? For Joey's situation?
- What does the y-intercept mean in the context of Debbie's and Joey's incomes?
- How do you find the slope from a graph? From a table?
- What is Debbie's salary per hour? Joey's salary per hour?
- What does the slope represent for each line?
- Describe in words how much money Debbie will earn per week.

Sample Solutions

1. What information would you use to write a function rule?

   Each person has a starting amount that will be the y-intercept of the function rule, or the b in $y = mx + b$. This represents the amount of allowance that each person receives. The slope of the function rule, represented as m in $y = mx + b$, is the rate of change per hour, which in this case is the amount of money each person gets paid per hour.

2. Write a function rule that can be used to calculate the amount of money each person will earn per week in terms of the number of hours worked. Make a table of your data.

   We can tell from the graph that Debbie's starting amount (her weekly allowance) is \$8, and the rate of change from point (0, 8) to point (1, 16) is 8. So her hourly salary is $8. The function rule for this line is $y = 8x + 8$.

The graph indicates that Joey's starting amount is $16, and the rate of change from point (0, 16) to point (2, 28) is 12 for 2 hours. So his salary is $6 per hour. The function rule for this line is $y = 6x + 16$.

Debbie's Income

Hours Worked	Income in Dollars
0	8
1	16
2	24
3	32
4	40
5	48
6	56
7	64
8	72

Joey's Income

Hours Worked	Income in Dollars
0	16
1	22
2	28
3	34
4	40
5	46
6	52
7	58
8	64

3. How will an increase in Debbie's allowance affect the graph? Entries in a table? The function rule? Use an example to justify your thinking.

   If Debbie's allowance increases, the *y*-values in the original table each need to be increased by the amount of the allowance change to make the *y*-values of the new table. For example, if Debbie's allowance increases by $4.00, the corresponding income increases by $4, as shown in the second table.

(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.8) Linear Functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

Debbie's Income

Hours Worked	**Income in Dollars**
0	8
1	16
2	24
3	32
4	40
5	48
6	56
7	64
8	72

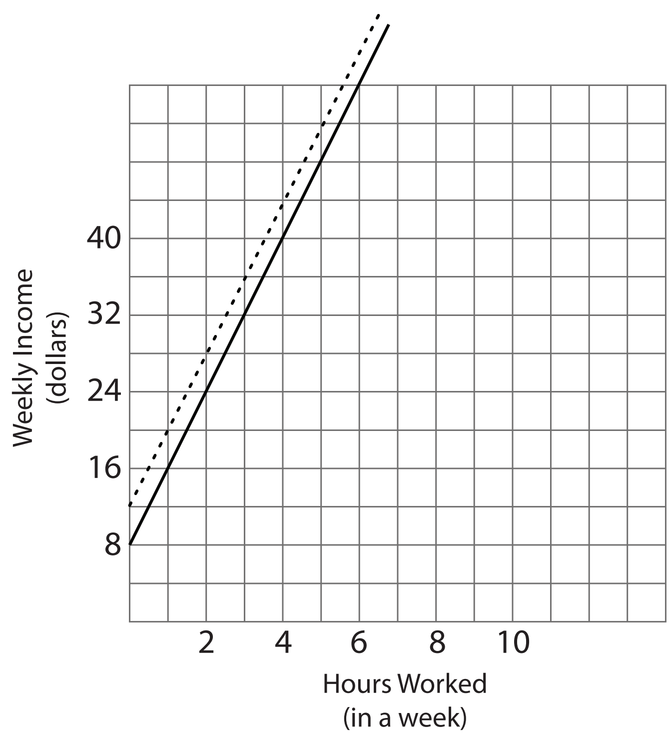
Debbie's Income with Allowance Increased by $4

Hours Worked	**Income in Dollars**
0	12
1	20
2	28
3	36
4	44
5	52
6	60
7	68
8	76

There is a difference of 4 in each y-value for the same x in the two tables. For example, the difference between the y-values for 8 hours in the two tables is 76 – 72, or 4.

In the function rule, the constant term changes. If Debbie's allowance increases by $4, the new allowance is $12, and the function rule is $y = 8x + 12$.

The graph of the new situation is a straight line parallel to the original line with a y-intercept of 12.



4. How will an increase in Joey's hourly wages affect the graph? Entries in a table? The function rule? Use an example to justify your thinking.

   If Joey's hourly rate increases by $4, the hourly rate becomes $10. His new function rule is $y = 10x + 16$.

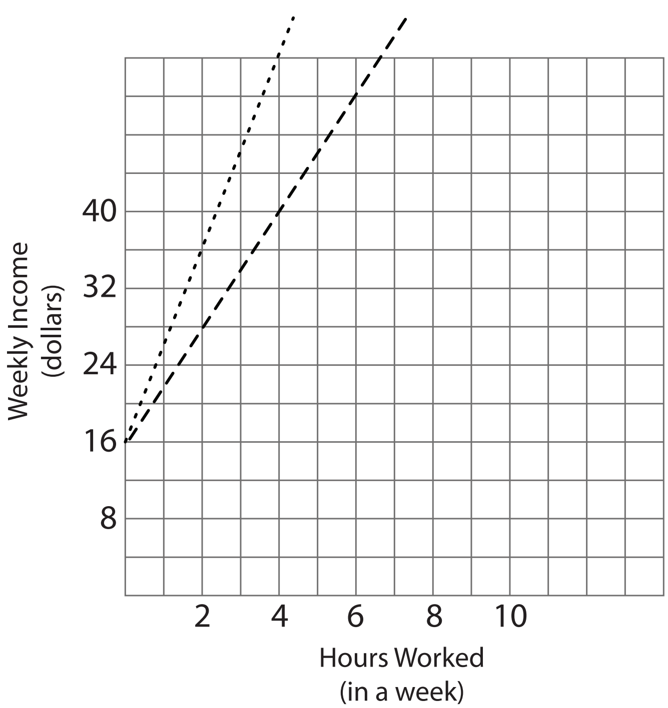
Joey's Income

Hours Worked	Income in Dollars
0	16
1	22
2	28
3	34
4	40
5	46
6	52
7	58
8	64

Joey's Income with Increase in Hourly Wage

Hours Worked	Income in Dollars
0	16
1	26
2	36
3	46
4	56
5	66
6	76
7	86
8	96

   If Joey's hourly wages increase, the constant rate of change per hour worked increases. The slope of the linear graph gets steeper, and in the function rule the coefficient of x increases. This line has the same y-intercept since his allowance did not change, but it has a different slope.



5. If Debbie's weekly allowance is doubled, will her new weekly income be more or less than twice the original amount? Explain your reasoning.

   If Debbie's weekly allowance is doubled, her function rule changes from $y = 8x + 8$ to $y = 8x + (2)8$, or $y = 8x + 16$. Her hourly wage remains the same.

   The function rule for twice her original income is $y = 16x + 16$.

   For any positive number x, $8x + 16 < 16x + 16$.

   The new income from doubling Debbie's allowance is *less* than twice her original income because doubling her allowance does not affect her hourly wage, but doubling her original income increases both her hourly wage and her allowance.

6. Based on the original functions, who will have more money each week?

   It depends on how many hours they work. If they work more than 4 hours, Debbie will earn more money. The y-values (weekly income) on Debbie's line are greater when x is greater than 4. If they work fewer than 4 hours, Joey will earn more money.

Extension Questions

- What are reasonable domain values for this function and the problem situation?

  Domain values for the situation must be numbers greater than or equal to 0. If Debbie and Joey must work only whole hours, the domain values will be whole numbers. If they can work and get paid for portions of an hour, the domain of the situation could be all real numbers greater than or equal to 0.

- A line is used to model the situation. Will all points on this graph represent the problem situation?

  If a person is usually paid for whole numbers of hours or perhaps half hours worked, not all points on the line represent the situation. Rather, the graph of the problem situation is a step function that rises in increments, not the whole line.

- Determine the point of intersection of the two lines. What does this point mean in the situation?

  The point of intersection is (4, 40), meaning that if Debbie and Joey both worked four hours in a week, they would have the same weekly income.

- How would you solve this problem using a graphing calculator?

  The point of intersection appears to be the point (4, 40). This can be verified by examining the table or graph on the calculator or by substituting into the given functions.

Symbolic

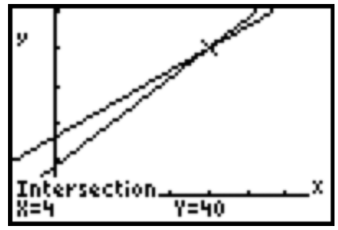
Plot1 Plot2 Plot3
\Y1 = 8X+8
\Y2 = 6X+16
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =

Table

X	Y1	Y2
2	24	28
3	32	34
4	40	40
5	48	46
6	56	52
7	64	58
8	72	64

X=2

Graph

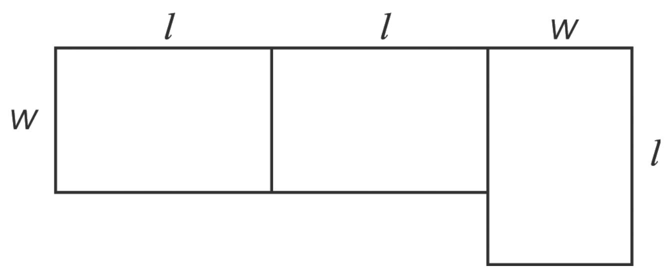


$$y = 6x + 16$$
$$40 = 6(4) + 16$$
$$y = 8x + 8$$
$$40 = 8(4) + 8$$

The point of intersection is (4, 40). This means that when Debbie and Joey work 4 hours, they both earn $40. Interpreting data from the graph, Joey earns more if they both work fewer than 4 hours; Debbie earns more if they both work more than 4 hours.

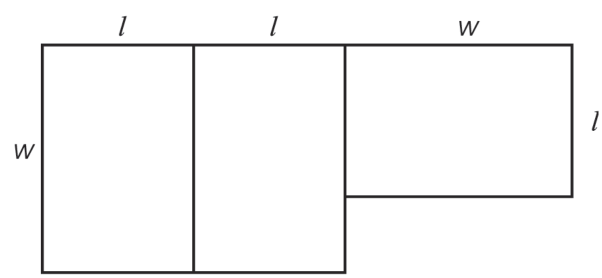
Exercise Pens

Devin is planning to build exercise pens for his three horses. Given the space available, he has decided to create three rectangular pens, as shown in the diagram. The three rectangular pens have the same size and shape. Devin has been advised that the perimeter of each pen should be 440 feet. He will use 1,200 feet of fencing for this project.



1. Use two different representations (tables, symbols, or graphs) to determine what the dimensions (in feet) of each pen should be.

2. Devin changes the orientation of the pens as shown below. What are the dimensions (in feet) of each pen? Describe how your answer relates to the problem situation.



Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Scaffolding Questions

- What are the known and unknown quantities in this situation?
- What does each known quantity represent?
- How do the perimeters of pens relate to the 1,200 feet of fencing?
- How many different relationships are described in the problem?

Sample Solutions

1. Use two different representations (tables, symbols, or graphs) to determine what the dimensions (in feet) of each pen should be.

   The unknown dimensions are the length and width of the pen. In the diagram, the width of each pen is represented by *w,* and the length of each pen is represented by *l*.

   The perimeter of each pen must be 440 feet. The perimeter is twice the length plus twice the width.

   Equation 1:

   $2l + 2w = 440$

   Divide by 2:
   $l + w = 220$

   The amount of fencing must be equal to 1,200 feet. Fencing the pens requires six lengths and four widths.

   Equation 2:

   $6l + 4w = 1{,}200$

   Divide by 2:
   $3l + 2w = 600$

   Subtract the first equation from the second equation to solve for length.

$$\begin{aligned} 3l + 2w &= 600 \\ -(2l + 2w &= 440) \\ \hline l &= 160 \end{aligned}$$

Substitute that length value into the simplified version of Equation 1 to solve for *w*.

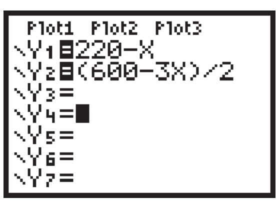
$$\begin{aligned} l + w &= 220 \\ 160 + w &= 220 \\ w &= 60 \end{aligned}$$

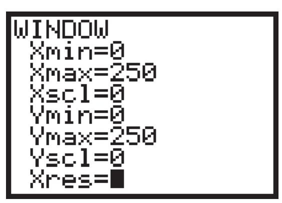
Each pen should have dimensions of 60 feet in width and 160 feet in length.

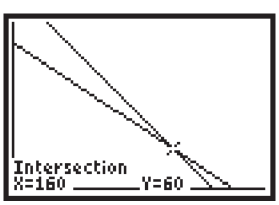
Another method for finding the pens' dimensions is to solve each equation for width, graph the functions, and find the point of intersection.

$$\begin{aligned} l + w &= 220 \\ w &= 220 - l \\ 3l + 2w &= 600 \\ w &= \frac{600 - 3l}{2} \end{aligned}$$

Enter the function rules into the graphing calculator. Let the width be the *y*-value and the length be the *x*-value. Draw the graphs on the graphing calculator and find the point of intersection.







The point of intersection is (160, 60). The *x*-value is the length of 160 feet, and the *y*-value is the width of 60 feet.

Additional Algebra TEKS:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

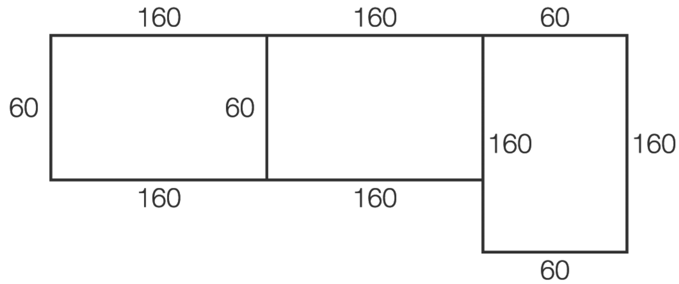
The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

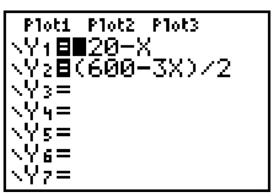
The fenced area would look like this diagram.

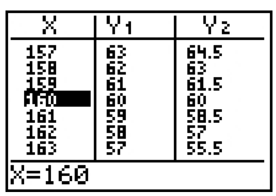


The perimeter of each pen is 2(60) + 2(160) = 440 feet.

The total amount of fencing required is 4(60) + 6(160) = 1,200 feet.

Some students may also use a table to determine the point of intersection. The equations can be entered into the graphing calculator, and then students look for the point at which the *y*-values for each function are equal, which occurs at the point (160, 60).





2. Devin changes the orientation of the pens as shown below. What are the dimensions (in feet) of each pen? Describe how your answer relates to the problem situation.

   The perimeter of each pen is still represented by the equation $2w + 2l = 440$, or $w + l = 220$.

   The amount of fencing for this figure is represented by $5w + 5l$. The total amount of fencing is 1,200 feet.

   $5w + 5l = 1200$, or $w + l = 240$

   The system of equations is

   $w + l = 220$

   $w + l = 240$

   This system has no solution. The initial restriction on the perimeter, $w + l = 220$, is contradicted by the second equation, $w + l = 240$. This means that both conditions cannot be met with one set of dimensions. In the configuration of $w + l = 240$, the perimeter of each pen will be 480 feet, which is larger than the initial restriction of 440 feet for each pen.

Extension Questions

- Two function rules were used to create lines to represent the situation. Describe the domains and ranges for the functions and the domains and ranges for the problem situation.

  The domain and range of each linear function is the set of all real numbers. However, for the problem situation, the domain and range values are restricted to first quadrant values.

  $y = 220 - x \quad 0 < x < 220 \quad 0 < y < 220$

  $y = \frac{600 - 3x}{2} \quad 0 < x < 200 \quad 0 < y < 300$

  For the two functions together, the domain is restricted to the intersection of the two domains 0 < x < 200 and the range is restricted to 0 < y < 220.

- How would your equations change for the situation in question 1 if the total amount of fencing were 800 feet?

  The first equation would not be different, but the equation for total amount of fencing would become 6l + 4w = 800, or 3l + 2w = 400.

- Solve the system from the previous extension question and explain the solution.

  One solution method is to subtract the first equation from the second equation.

  $$\begin{array}{rcl} 3l + 2w & = & 400 \\ 2l + 2w & = & 440 \\ \hline l & = & -40 \end{array}$$

  Since l represents the dimension of a rectangle, it cannot be negative. A total of 800 feet of fencing is not enough for each pen to have a perimeter of 440 feet.

- How would your equations change for the situation in question 1 if the total amount of fencing were 1,000 feet?

  The second equation becomes 6l + 4w = 1,000, or 3l + 2w = 500.

  One solution method is to subtract the first equation from the second equation.

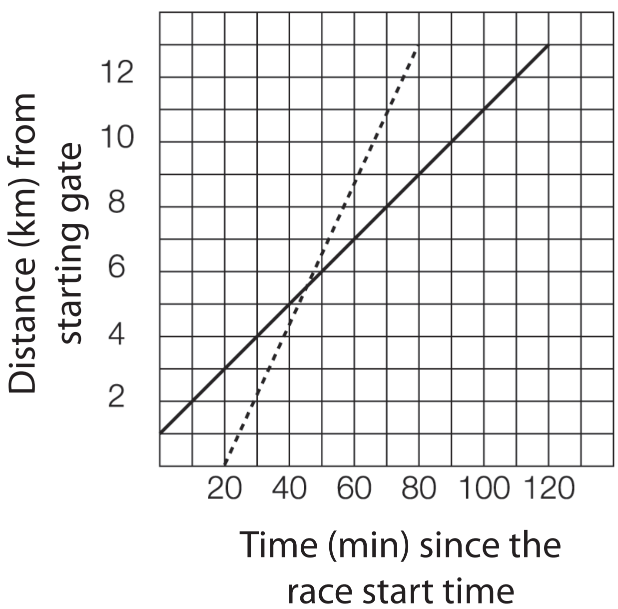
  $$\begin{array}{rcl} 3l + 2w & = & 500 \\ 2l + 2w & = & 440 \\ \hline l & = & 60 \end{array}$$

  Substitute that length value into the simplified version of the first equation to solve for w.

  $$\begin{array}{rcl} w & = & 220 - 60 \\ w & = & 160 \end{array}$$

The Run

The figure below represents the average speeds for two participants in a race—Eloise and Ty. The solid line segment represents Eloise's average speed, and the dashed line segment represents Ty's average speed. The graphs do not imply that each participant maintained a constant speed, since speed varies over time according to terrain, fatigue, or other factors.



1. Study the graphs and write a description of each participant's progress during the race.
2. Write a function rule that models each participant's progress.
3. For each participant, describe the meaning of the slope and intercepts represented in the graph.
4. What is the point of intersection of the two line segments? What does it represent?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.8) Linear functions. The student formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student is expected to:

(A) analyze situations and formulate systems of linear equations in two unknowns to solve problems;

(B) solve systems of linear equations using concrete models, graphs, tables, and algebraic methods; and

(C) interpret and determine the reasonableness of solutions to systems of linear equations.

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Scaffolding Questions

- How do you determine the rate at which Eloise is traveling?
- What is Ty's average speed?
- What is the length of the race? How do you know?
- Which participant took the least amount of time to complete the race?

Sample Solutions

1. Study the graphs and write a description of each participant's progress during the race.

   Possible description:

   Eloise started when the time was 0 and was positioned 1 kilometer from the starting gate. That first point on the line segment is (0, 1). Since her average speed is represented as a line segment, choose any two points on the line segment to determine the rate. For example, the points (0, 1) and (10, 2) can be used to find the rate. The rate of change from the point (0, 1) to the point (10, 2) is 1 kilometer in 10 minutes, or $\frac{1}{10}$ of a kilometer per minute.

   Ty left 20 minutes after Eloise. The first point on Ty's line segment is (20, 0). One reason for this could be that he arrived late at the race site or experienced some equipment problems such as difficulty with his shoes. Another point on that line is (80, 13). The rate of change from (20, 0) to (80, 13) is 13 kilometers in 60 minutes. The rate is about 4.61 minutes per kilometer, or $\frac{13}{60}$ kilometers per minute.

2. Write a function rule that models each participant's progress.

   The function rule for Eloise is the starting value (1) plus the product of the rate ($\frac{1}{10}$) and the number of minutes (x).

   $y = 1 + \frac{1}{10}x$

   Each line segment stops when y is 13; therefore, the race is 13 kilometers long. The y-value for Eloise is 13 when x is 120 minutes. Eloise started the race at the 1-kilometer mark, so she traveled 12 kilometers in 120 minutes.

Ty left the starting gate at 20 minutes after the race start time and traveled at $\frac{13}{60}$ kilometers per minute. Using the point-slope equation of a line, the function rule for Ty is

$y = \frac{13}{60}(x - 20) + 0$, or

$y = \frac{13}{60}x - 4\frac{1}{3}$

3. For each participant, describe the meaning of the slope and intercepts represented in the graph.

   The slopes of the lines represent the average speed at which each person traveled.

   Ty traveled 13 kilometers per 60 minutes or $\frac{13}{60}$ of a kilometer in 1 minute. Eloise traveled at 1 kilometer per 10 minutes or $\frac{1}{10}$ of a kilometer in 1 minute.

   Eloise's y-intercept indicates that she was 1 kilometer from the starting gate at 0 minutes. Her x-intercept can be inferred from the graph to be (–10, 0), which does not make sense for this situation since time a time of –10 isn't possible. Ty's y-intercept can be inferred to be (0, –4), which also does not make sense since a distance of –4 is not possible. His x-intercept is (20, 0), which means that at 20 minutes after the race start time, Ty was at the starting gate.

4. What is the point of intersection of the two line segments? What does it represent?

   The graphs of $y = \frac{1}{10}x + 1$ (Eloise) and $y = \frac{13}{60}(x - 20) + 0$ (Ty) intersect at about (45, 5.5).

   To determine the exact values, solve the equation:

   $\frac{1}{10}x + 1 = \frac{13}{60}(x - 20) + 0$

   Multiply both sides of the equation by 60.

   $6x + 60 = 13(x - 20)$

   $6x + 60 = 13x - 260$

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(D) graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representations;

Texas Assessment of Knowledge and Skills:

Objective 4: The student will formulate and use linear equations and inequalities.

$-7x = -320$

$x = \frac{-320}{-7}$

$x = 45\frac{5}{7}$

To find the *y*-value, substitute into the original equation for Eloise:

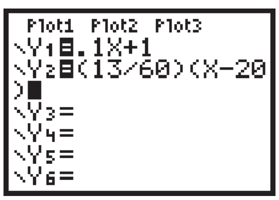
$y = \frac{1}{10}x + 1$

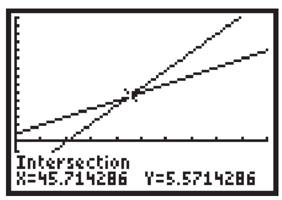
$y = \frac{1}{10}\left(\frac{320}{7}\right) + 1$

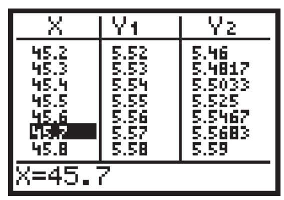
$y = 5\frac{4}{7}$

The point of intersection represents the point at which Eloise and Ty were the same distance from the starting line at the same time. The two participants met $5\frac{4}{7}$ kilometers from the starting line $45\frac{5}{7}$ minutes after the race start time. At this point, Ty overtook Eloise and stayed ahead for the rest of the race.

A graphing calculator may also be used to determine the point of intersection.



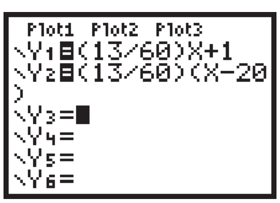


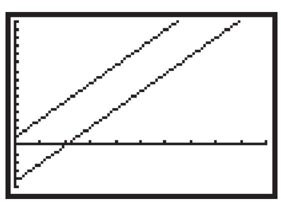


Extension Questions

- Describe ways in which the Eloise could have won the race.

  If she had traveled at the same rate as Ty, he would never catch up because she started first.





  Any rate for Eloise that would determine a line that intersects with Ty's line after 13 kilometers would allow her to win. (This is a good calculator exploration of slope at the Algebra I level.)

  Or students could reason that Eloise would have to run faster than $\frac{1}{10}$ of a *kilometer per minute and finish the race in less than 80 minutes. To find a possible rate, substitute the x and y values of (80, 13) in the function rule to find the slope:*

$13 = a(80) + 1$

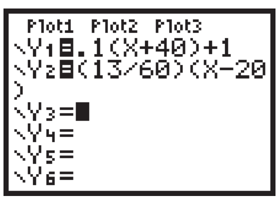
$a = \frac{12}{80}$

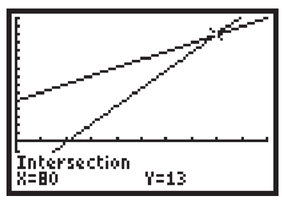
$a = \frac{3}{20}$

Eloise would have to run faster than $\frac{3}{20}$ of a kilometer per minute to win the race.

- If both participants traveled at the same rate, what factors or variables could be changed so that Eloise wins the race?

The given graph shows that Eloise finished 40 minutes after Ty. If her graph is moved 40 units to the left, (x + 40) replaces x and both participants finish at the same time. That is, Eloise needs a head start of an additional 40 minutes to tie with Ty.





Thus, to win the race, Eloise's rule would have to be changed to reflect an amount greater than 40. For example, replace x with x + 41.

$y = \frac{1}{10}(x + 41) + 1$

$y = \frac{1}{10}x + 4.1 + 1$

$y = \frac{1}{10}x + 5.1$

She would need to start 5.1 kilometers from the starting line to win.

Which Plan Is Best?

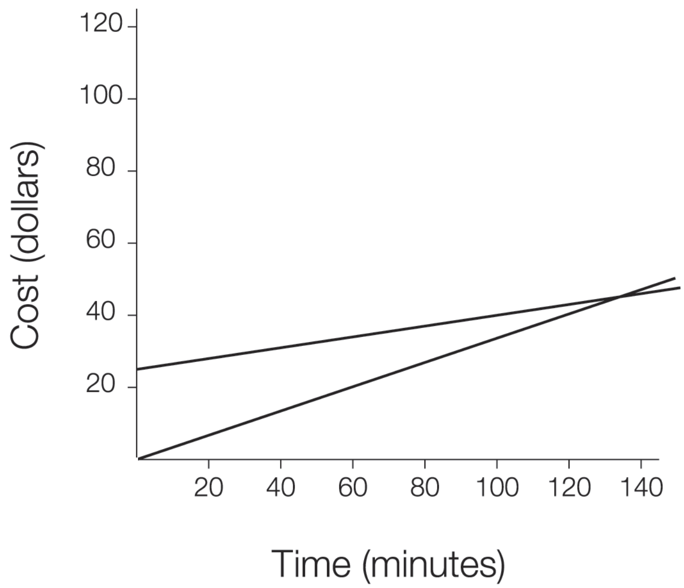
Students were given two cell phone plans to compare.

Plan 1: $C = \$0.35m$

Plan 2: $C = \$0.15m + \$25$

C represents the monthly cost in dollars, and m represents the time in minutes.

One group of students graphed the two plans.



1. Explain the differences between the two phone plans.
2. Explain the meaning of the slope for each plan.
3. Explain the meaning of the *y*-intercept for each plan.
4. Which plan offers the better deal? Explain your thinking.
5. If the second plan charged 20 cents per minute, what would be different about its graph?
6. If the first plan were changed so that the base fee was $10, how would its graph change?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.6) Linear functions. The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeroes of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations;

(B) interpret the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(F) interpret and predict the effects of changing slope and y-intercept in applied situations; and

Scaffolding Questions

- How can you determine the slope for each cell phone plan?
- How is the cost per minute represented in each plan?
- Does either plan have a base or starting fee? How do you know?
- If you talk for 40 minutes, which plan costs more? How do you know?
- What does the point of intersection of the two lines mean in the context of this problem?

Sample Solutions

1. Explain the differences between the two phone plans.

   Plan 1 charges $0.35 per minute, while Plan 2 charges only $0.15 per minute. Plan 2 also has a base fee of $25, and Plan 1 does not have a base fee.

2. Explain the meaning of the slope for each plan.

   In this situation, the slope of each plan's line represents the rate of change per minute, or the cost per minute in each plan. Plan 1's slope is 0.35. Plan 2's slope is 0.15.

3. Explain the meaning of the *y*-intercept for each plan.

   In this situation, the *y*-intercept represents each plan's cost at 0 minutes. Plan 1's *y*-intercept is 0 because there is no charge for the plan itself. Plan 2's *y*-intercept is 25 because there is a $25 base fee connected with this plan.

4. Which plan offers the better deal? Explain your thinking.

   The better deal depends on the number of minutes you plan to use.

   Examine the table for the two plans:

X	Y_1	Y_2
123	43.05	43.45
124	43.4	43.6
125	43.75	43.75
126	44.1	43.9
127	44.45	44.05
128	44.8	44.2
129	45.15	44.35

X=123

If you plan to use fewer than 125 minutes, you should go with Plan 1. If you plan to use more than 125 minutes, the best plan would be Plan 2.

5. If the second plan charged 20 cents per minute, what would be different about its graph?

   The slope of the graph would change. The new graph would have a steeper slope.

6. If the first plan were changed so that the base fee was $10, how would its graph change?

   The *y*-intercept of the original Plan 1 is 0. If a base fee of $10 were added, the *y*-values for each point would increase by 10 units. The *y*-intercept would be 10.

Extension Questions

- What ways can Plan 1's method of charging change so that it is always a better deal than Plan 2?

  In general, if Plan 1 had a slope value less than 0.15 and a y-intercept less than or equal to 25, it would always be a better deal than Plan 2.

  Examples:

  If Plan 1 charged the same base rate as Plan 2 but decreased the slope, Plan 1's fee would always be less. For example, Plan 1 could charge 14 cents per minute with a base fee of $24.99.

  $C = 0.14m + 24.99$

  Another option would be for Plan 1 to charge the same rate per minute as Plan 2 (15 cents) but decrease the base fee of $25. Plan 1's price would be less for any number of minutes. For example, Plan 1 could charge 15 cents per minute with a base fee of $20.

  $C = 0.15m + 20$

- Suppose the cell phone companies that are offering these plans merge. Together they come up with a new plan. The new plan has no base fee, charges the customer $0.25 per minute, and provides the first 40 minutes free. The customer doesn't start paying until

Additional Algebra TEKS:

(A.5) Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations.

The student is expected to:

(C) use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Texas Assessment of Knowledge and Skills:

Objective 3: The student will demonstrate an understanding of linear functions.

he or she has used the phone for more than 40 minutes. What is the function rule for this new plan?

First, make a table.

Minutes	***Cost***
40	*$0.00*
41	*$0.25*
42	*$0.50*
43	*$0.75*
44	*$1.00*

To figure out a function rule, we need to account for the 40 free minutes. The charge after 40 minutes is 25 cents per minute, or $2.50 for 10 minutes. Extend the table to determine the y-intercept (example below). There are negative costs for less than 40 minutes. These negative costs represent the money the customer is not paying or the money the customer is saving. Continuing to backtrack in the table, we learn that at 0 minutes, the cost is –$10.00.

Minutes	***Cost***
0	*–$10.00*
10	*–$7.50*
20	*–$5.00*
30	*–$2.50*
40	*$0.00*
41	*$0.25*
42	*$0.50*
43	*$0.75*
44	*$1.00*

The y-intercept is –10. Our function rule is $C = 0.25x - 10$, where x is the number of minutes and is greater than or equal to 40.

Chapter 4:
Quadratic Functions

Fireworks Celebration

At a fireworks celebration, a bottle rocket is launched upward from the ground with an initial velocity of 160 feet per second. Spectators watch and wonder how high the bottle rocket will go before it begins to descend back toward the ground.

The formula for vertical motion of an object is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is –32 feet per square second, v is the initial velocity, and s is the initial height. Time, t, is measured in seconds, and height, h, is measured in feet.

1. What function describes the height, h, of the bottle rocket t seconds into launch?
2. Sketch a graph of the position of the bottle rocket as a function of time into launch, and give a verbal description of the graph. Include a reasonable domain and range.
3. How high is the bottle rocket 3 seconds into launch? When is it at this height again?
4. For the audience's safety, the bottle rocket, as it descends, should be set to explode at least 250 feet above the ground. The operator has a choice of fuses to put into the bottle rocket. Fuse A will detonate the bottle rocket between 3 and 5 seconds into launch, Fuse B between 4 and 6 seconds into launch, and Fuse C between 6 and 8 seconds into launch. Which fuse should be used? Why?
5. Suppose the bottle rocket is launched from the top of a 200-foot-tall building. How does this change the position function for the bottle rocket? How does the graph of the new position function compare with the graph of the first position function? What does the new graph tell you about the situation?
6. Suppose you are the fireworks operator and you want to launch a bottle rocket from the ground and have it stay in the air 3 seconds longer than in the original scenario. How can you accomplish this? What effect does this change have on the bottle rocket's maximum height?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

(D) analyze graphs of quadratic functions and draw conclusions.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Scaffolding Questions

- What are the initial velocity and height of the bottle rocket?
- How can you rewrite the function so that it is easier to determine an appropriate window for graphing?
- What decisions must be made to determine an appropriate window for the graph of the function $h(t) = -16t^2 + 160t$?
- How can you use your graph to answer questions about time and height?
- As the situation changes, which values for vertical motion in the formula change? Which values stay the same?
- What is a reasonable domain for this situation? How do you know?
- What is a reasonable range? How do you know?

Sample Solutions

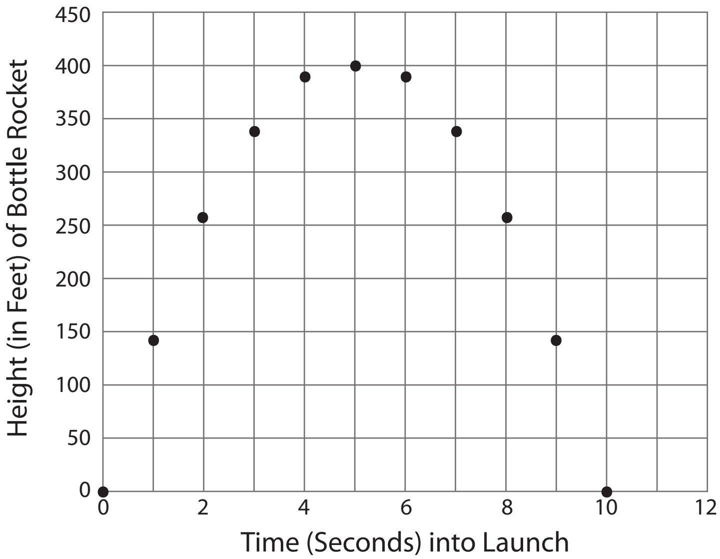
1. What function describes the height, h, of the bottle rocket t seconds into launch?

   The vertical motion formula is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is –32 feet per square second. The initial velocity, v, is 160 feet per second, and the initial height, s, is 0 because the bottle rocket is launched from the ground.

   $h(t) = -16t^2 + 160t$

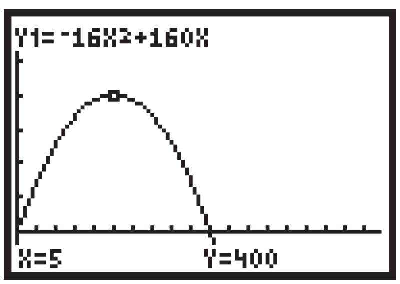
2. Sketch a graph of the position of the bottle rocket as a function of time into launch, and give a verbal description of the graph. Include a reasonable domain and range.

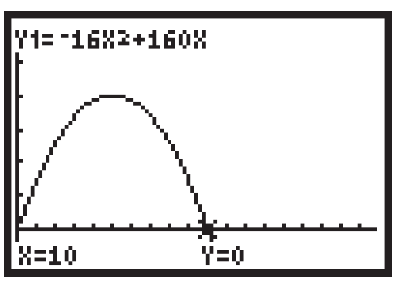
   The graph below shows the bottle rocket rising to reach its maximum height of 400 feet at 5 seconds and then falling to hit the ground at 10 seconds; therefore, a reasonable domain is 0 to 10 seconds, and a reasonable range is 0 to 400 feet.



Students may also choose to graph the equation in the calculator and examine the graph there. The graph is a parabola opening down, with the vertex halfway between 0 and 10 at $t = 5$.

$h(5) = -16(5)^2 + 160(5) = 400$





3. How high is the bottle rocket 3 seconds into launch? When is it at this height again?

By tracing the graph, we see that the bottle rocket reaches a height of 336 feet when $t = 3$ seconds and again when $t = 7$ seconds.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

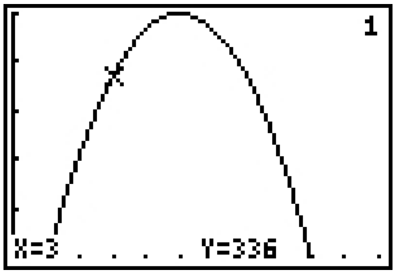
The student is expected to:

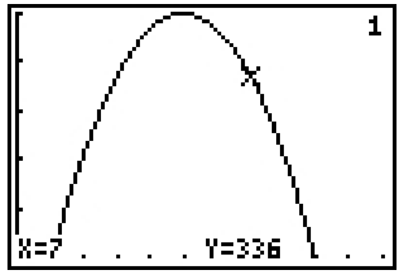
(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Texas Assessment of Knowledge and Skills:

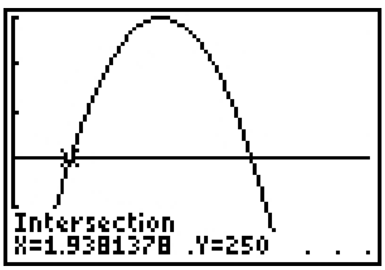
Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

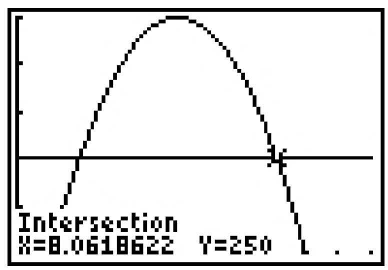




4. For the audience's safety, the bottle rocket, as it descends, should be set to explode at least 250 feet above the ground. The operator has a choice of fuses to put into the bottle rocket. Fuse A will detonate the bottle rocket between 3 and 5 seconds into launch, Fuse B between 4 and 6 seconds into launch, and Fuse C between 6 and 8 seconds into launch. Which fuse should be used? Why?

   By drawing the line $y = 250$ and finding its intersection points with the graph of the parabola, we see that the bottle rocket ascends to 250 feet 2 seconds into launch and descends back to 250 feet 8 seconds into launch.





This value can also be found by solving the equation:

$$\begin{aligned}
250 &= -16t^2 + 160t \\
0 &= -16t^2 + 160t - 250 \\
0 &= 16t^2 - 160t + 250 \\
t &= \frac{160 \pm \sqrt{(-160)^2 - 4(16)(250)}}{2(16)} \\
t &= \frac{160 \pm \sqrt{9600}}{2(16)}
\end{aligned}$$

$t = 8.062$ or $t = 1.938$

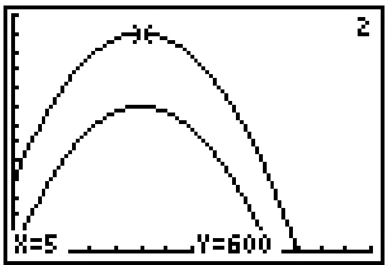
It does not make sense to set the rocketbottle rocket to detonate during its ascent. Therefore, it should be set to detonate after reaching its maximum height, at about 5 seconds. The operator should use Fuse C, since it would ensure that the rocketbottle rocket detonates after it reaches its maximum height but before (or just as) it begins to descend.

5. Suppose the bottle rocket is launched from the top of a 200-foot-tall building. How does this change the position function for the bottle rocket? How does the graph of the new position function compare with the graph of the first position function? What does the new graph tell you about the situation?

   Since the initial height of the bottle rocket is now 200 feet instead of 0 feet (that is, at ground level) and the initial velocity is still the same, the new position function is

   $h(t) = -16t^2 + 160t + 200$

   The new graph is a parabola and opens downward as in the original graph, but it is translated up 200 units. The new graph and the original graph are shown below.

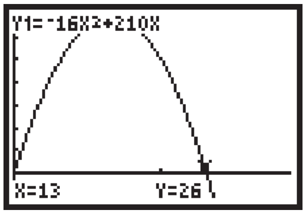


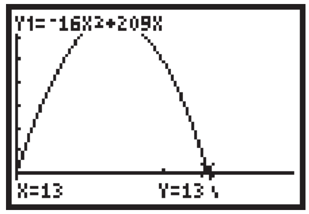
   The graphs have the same axis of symmetry, $x = 5$, but the vertex of the new graph is (5, 600). This makes sense since the original vertex has been translated up 200 units. By the same reasoning, the new y-intercept is (0, 200). The x-intercepts for the new graph cannot be found as easily. Tracing or using the "zero" function of the calculator shows that one x-intercept is between 11 and 12, and the other is between –1 and –2.

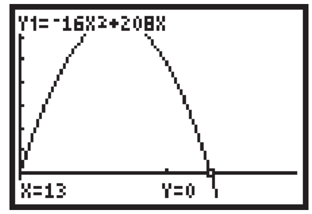
   The part of the graph that makes sense for the situation is the portion with domain values of 0 to approximately 11. The graph shows that the bottle rocket starts at a height of 200 feet, reaches a maximum height of 600 feet in 5 seconds, and then lands after a little more than 11 seconds.

6. Suppose you are the fireworks operator and you want to launch a bottle rocket from the ground and have it stay in the air 3 seconds longer than in the original scenario. How can you accomplish this? What effect does this change have on the bottle rocket's maximum height?

   In the original function, the gravitational constant, –16 feet per square second, does not change. Therefore, students can experiment with the initial velocity of the bottle rocket. By experimenting with different initial velocities and graphing, they can see that the initial velocity needs to increase to about 200 feet per second and that the bottle rocket reaches its maximum height of about 676 feet in about 6.5 seconds.







Another method uses the roots of the function. We see that the original time interval of 0 to 10 seconds shows up when we look at the original position function in factored form:

$h(t) = -16t(t - 10)$

Therefore, change the function to

$h(t) = -16t(t - 13)$

$h(t) = -16t^2 + 208t$

Now the *x*-intercepts are (0, 0) and (13, 0), meaning that the bottle rocket is in the air for 13 seconds. The bottle rocket reaches its maximum height in 6.5 seconds, and evaluating the new function shows that the maximum height is 676 feet. Increasing the bottle rocket's velocity keeps it in the air longer and shoots it to a greater height.

Extension Questions

- What do you know about the shape of the graph of the function that helps you determine the domain and range? How will you use this information?

  The graph is a parabola opening downward. One x-intercept is 0. Therefore, the vertex of the parabola is above the x-axis. Solving $0 = -16t^2 + 160t$ by factoring and using the Zero Product Property gives the other x-intercept. This means that the appropriate x-values for the calculator window should be a little less than 0 and a little more than the second x-intercept.

  The x-coordinate of the vertex is halfway between the x-intercepts. We can evaluate the function at that x-coordinate to get the maximum height. Then we know the range in y-values for the calculator window are from a little less than 0 to a little more than the maximum height.

- What parameter(s) in the function cannot change? What does this tell you must change in order to launch the bottle rocket from the ground and have it stay in the air longer?

  Since the gravitational constant and initial height are constant, we must vary the initial velocity. By experimenting with different values, we can see that the initial velocity should be increased.

- Is it possible for a bottle rocket to launch from the ground, travel through the air, and land on the ground in the same amount of time as the first bottle rocket but go higher than the first bottle rocket? Explain.

  Since the bottle rocket launches from the ground, its initial height is 0 and the function has the form

  $h(t) = -\frac{1}{2}gt^2 + vt + 0$

  $h(t) = t\left(-\frac{1}{2}gt + v\right)$

  The t-intercepts are those values that give a height of 0.

  $h(t) = t\left(-\frac{1}{2}gt + v\right)$

  $t = 0 \text{ or } -\frac{1}{2}gt + v = 0$

  $t = 0 \text{ or } t = \frac{2v}{8}$

  Changing the initial velocity, v, changes the positive t-intercept of the graph so that the bottle rocket is in the air a different amount of time. The constant, g, is the gravitational constant, –32 feet per square second, and cannot change if the bottle rocket is being launched on Earth. The bottle rocket cannot be in the air for the same amount of time as the first bottle rocket and go higher.

Golfing

The height, h (in feet), of a golf ball depends on the time, t (in seconds), it has been in the air. Sarah hits a shot off the tee that has a height modeled by the velocity function $f(h) = -16t^2 + 80t$.

1. Sketch a graph and create a table of values to represent this function. How long is the golf ball in the air?

2. What is the maximum height of the ball? How long after Sarah hits the ball does it reach the maximum height?

3. What is the height of the ball at 3.5 seconds? Is there another time when the ball is at this same height?

4. At approximately what time is the ball 65 feet in the air? Explain.

5. Suppose the same golfer, Sarah, hit a second ball from a tee that was elevated 20 feet above the fairway. What effect does this have on the values in your table? Write a function that describes the new path of the ball. Sketch the new relationship between height and time on your original graph. Compare and contrast the graphs.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

(D) analyze graphs of quadratic functions and draw conclusions.

Additional Algebra TEKS:

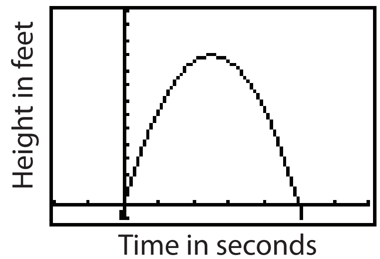
(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

Scaffolding Questions

- Describe the height of the ball over time.
- What are some possible *x*- and *y*-values for this scenario? Record these values in a table.
- Describe what your graph will look like.
- What values are reasonable for the domain and range in this situation?

Sample Solutions

1. Sketch a graph and create a table of values to represent this function. How long is the golf ball in the air?



The graph of this function is a parabola that opens downward. It has a domain (representing the time in seconds) of $0 \le x \le 5$. The range (representing the height of the ball) is $0 \le y \le 100$.

X	Y1
0	0
.5	36
1	64
1.5	84
2	96
2.5	100
3	96

X=0

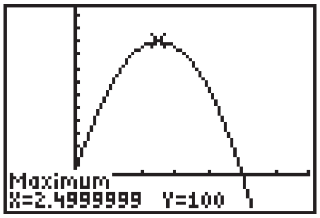
X	Y1
3.5	84
4	64
4.5	36
5	0
5.5	-44
6	-96
6.5	-156

X=6.5

According to the table, the golf ball is in the air a total of 5 seconds.

2. What is the maximum height of the ball? How long after Sarah hits the ball does it reach the maximum height?

The table suggests that the golf ball reaches a maximum height (*y*) of 100 feet when the time (*x*) is 2.5 seconds. Another method of estimating the maximum height is to use the Calculate function on the graphing calculator. Setting a lower and upper bound on the curve yields a maximum height value of 100 feet and a corresponding time value that rounds to 2.5 seconds. (The Trace feature can also be used to estimate the maximum height.)

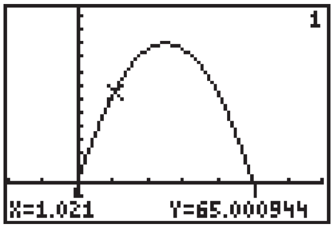


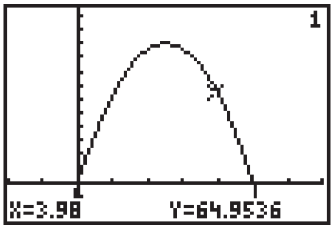
3. What is the height of the ball at 3.5 seconds? Is there another time when the ball is at this same height?

   The height of the ball at 3.5 seconds can be found in the table in the answer to question 1, when $x = 3.5$. At 3.5 seconds, the ball is at 84 feet during its descent. The ball is also at 84 feet during its ascent, at 1.5 seconds.

4. At approximately what time is the ball 65 feet in the air? Explain.

   The golf ball is at 65 feet at two different times. Because the vertex of the parabola is greater than 65 and the vertex is the maximum value on the graph, the *y*-value of 65 appears twice. The table above does not show an exact *y*-value of 65 because the values are rounded. The value of 64 is found in the table at about 1 second and 4 seconds. To find a more exact answer, you can use the graphing calculator. The calculator's results show very small differences in time compared to the values in the table.





The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Texas Assessment of Knowledge and Skills:

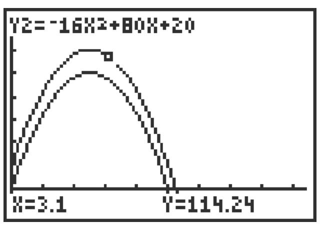
Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

5. Suppose the same golfer, Sarah, hit a second ball from a tee that was elevated 20 feet above the fairway. What effect does this have on the values in your table? Write a function that describes the new path of the ball. Sketch the new relationship between height and time on your original graph. Compare and contrast the graphs.

   If the same golfer hit a second ball from a tee that was elevated 20 feet above the fairway, the values for the time remain the same in the table. The height values for the elevated golf shot are larger because the shot started 20 feet higher.

   The new function is $f(h) = -16t^2 + 80t + 20$. The graph of the new function at time 0 seconds begins at 20 feet (0, 20). This is 20 feet higher than in the original graph. The maximum for the new function is 20 feet higher than the maximum for the original function. The maximum can be found by tracing the function or looking at the table. For the first situation, the ball hits the ground at 5 seconds. For the second situation, the ball hits the ground at about 5.27 seconds.

   The graph shows the original function and the new function.



Extension Questions

- If the initial velocity were 60 feet/second, how would the velocity function be written?

  Replacing the 80 in the original function with 60 indicates that the initial velocity is 60 feet per second. The function would be $f(h) = -16t^2 + 60t$.

- How would the graph of your function with an initial velocity of 60 feet/second compare to your original graph?

  In the new function, the ball reaches a maximum height of approximately 56.25 feet after about 1.9 seconds. The original function has a maximum at 100 feet after about 2.5 seconds.

- How does a greater initial velocity appear to affect the flight of the ball?

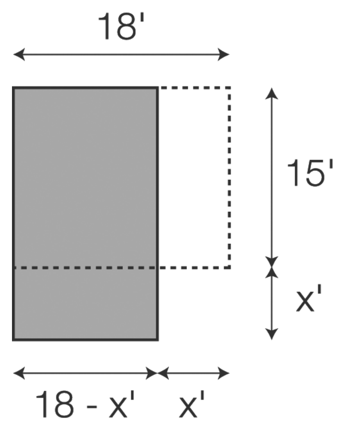
  From the example, it appears that the greater the initial velocity, the higher the maximum height of the ball and the longer the period of time the ball is in flight.

- How do the *x*-intercepts of the two graphs compare?

  If the velocity is increased, the x-intercept for the graph of the ball with greater velocity increases—that is, the ball takes longer to land on the ground.

Home Improvements

Ken's existing garden is 18 feet long and 15 feet wide. He wants to reduce the length and increase the width by the same amount, according to the diagram below.



1. Write a function that models the area of the new garden plot.
2. What value of *x* produces a new area of 266 square feet? Justify your solution.
3. What value of *x* produces a new area of 280 square feet? Justify your solution.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables,

Scaffolding Questions

- How can you find the area of any rectangle?
- What are the dimensions of the new garden plot? Explain.
- What methods can you use to solve your equation?
- How do the given values of the area relate to your algebraic area representation?

Sample Solutions

1. Write a function that models the area of the new garden plot.

   The new garden plot is a rectangle with dimensions of $(15 + x)$ by $(18 - x)$. The formula for finding the area of a rectangle is $A = l \bullet w$.

   The function that models the area is:

   $A = (15 + x)(18 - x)$

   $A = 270 + 3x - x^2$

2. What value of x produces a new area of 266 square feet? Justify your solution.

   To determine values of x that produce an area of 266 square feet, set the area function equal to 266 as follows.

   $270 + 3x - x^2 = 266$

   $0 = x^2 - 3x - 4$

   $0 = (x - 4)(x + 1)$

   $x = 4, -1$

   Both values produce an area of 266 square feet when substituted back into the original equation.

$$(15 + x)(18 - x) = 266$$

$$(15 + 4)(18 - 4) = 266 \quad \text{and} \quad (15 + -1)(18 - (-1)) = 266$$

$$(19)(14) = 266 \qquad (14)(19) = 266$$

However, the value of x cannot be negative because a measurement cannot be negative. The problem states that Ken wants to reduce the length and increase the width, so only the value of 4 makes sense for this situation. The dimension change that produces an area of 266 square feet is an increase of 4 feet in width and a decrease of 4 feet in length.

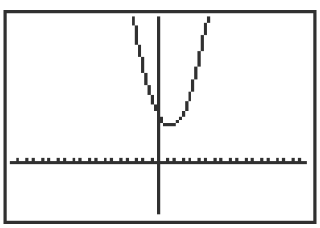
3. What value of x produces a new area of 280 square feet? Justify your solution.

   To find the value of x that produces a new area of 280 square feet, substitute the new area for the variable A in the function above.

   $270 + 3x - x^2 = 280$

   $0 = x^2 - 3x + 10$

   The graph of the function $y = x^2 - 3x + 10$ never crosses the x-axis, so there are no roots.

   This means that there are no values for x that produce a garden area of exactly 280 square feet.

   Another way to solve this is to check the discriminant. The value of $b^2 - 4ac$ is less than zero:

   $b^2 - 4ac = (-3)^2 - 4(1)(10) = 9 - 40 = -31$

   This shows that there are no solutions for this equation.

graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(D) analyze graphs of quadratic functions and draw conclusions.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

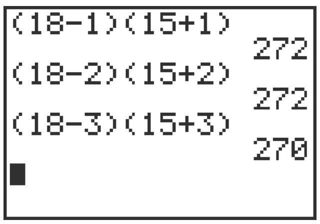
Extension Questions

- What values of *x* give Ken a garden with the dimensions of the original garden?

  Ken's original garden was 18 feet by 15 feet. If the length and width are reduced and increased by the same amount (x), the value of x has to be 3. If you reduce the 18-foot side by 3, it will result in a side of 15 feet. If you increase the 15-foot side by 3, it will produce a side of 18 feet. Therefore, the dimensions of the new garden will match that of the original garden.

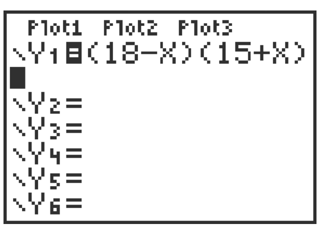
- What values of *x* produce a garden with the maximum area?

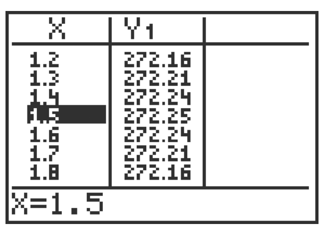
  Using the area formula, the length and width can be decreased and increased by the same amount to find the maximum area. Using the calculator shows that values of 1 foot and 2 feet produce the same, and the greatest, area.

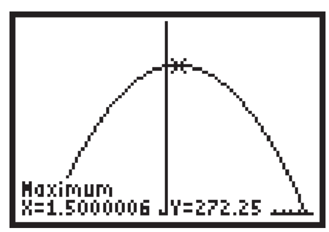


These values produce dimensions of 16 feet by 17 feet. These whole-number values come the closest to producing a garden plot that is a square figure, which produces the maximum area. The area of the garden could be maximized more if decimal values were used. By increasing and decreasing the length by 1.5 feet, the area becomes 272.25 square feet.

The table and graph show that this maximum area occurs when the increase and decrease are both 1.5 feet.

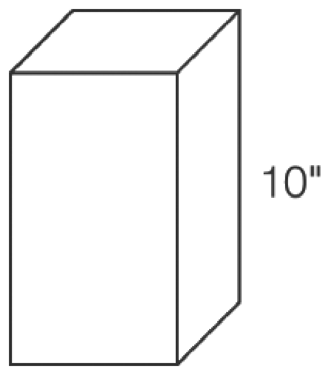






How Much Paint?

Ana has a can of paint that will cover 3,800 square inches. She wants to build a small wooden box with a square base and a height of 10 inches. The paint will be used to finish the box.



1. Write a function to represent the total surface area of the box.
2. What equation will allow you to determine the dimensions of a box whose surface area is 3,800 square inches? Show how to solve the equation you wrote symbolically.
3. Describe how to solve the equation using a graph.
4. What is the measure of the base of the largest box Ana can build? Explain your answer.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph the function.

Additional Algebra TEKS:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

Scaffolding Questions

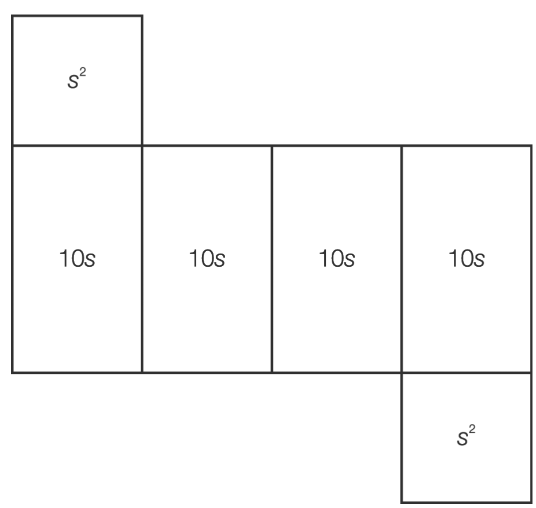
- How can you algebraically represent the area of each face of the box?
- How can you use the area of each face to determine the total area of the box?
- Describe how to use a graph to determine the number of roots.
- What are some different methods of solving for side length?

Sample Solutions

1. Write a function to represent the total surface area of the box.

   Let s = the measure of a side of the square base.

   Then s^2 = the area of the square base. There are 2 square bases, so the area is represented by $2s^2$.



The area of one rectangular side of the box is $10s$, using the formula of (base)(height). There are 4 sides on the box, representing an area of $4(10s)$ or $40s$. The total area of the box equals $2s^2 + 40s$.

The function is $A(s) = 2s^2 + 40s$.

2. What equation will allow you to determine the dimensions of a box whose surface area is 3,800 square inches? Show how to solve the equation you wrote symbolically.

   The total surface area of the box equals $2s^2 + 40s$. To find the dimensions of a box with a surface area of 3,800, solve the equation $2s^2 + 40s = 3{,}800$.

   Transform the equation into standard form:

   $2s^2 + 40s - 3{,}800 = 0$

   Use the quadratic formula with $a = 2$, $b = 40$, and $c = -3{,}800$.

   $$s = \frac{-40 \pm \sqrt{(40)^2 - 4(2)(-3{,}800)}}{2(2)}$$

   $s = 34.72$ inches or $s = -54.72$ inches

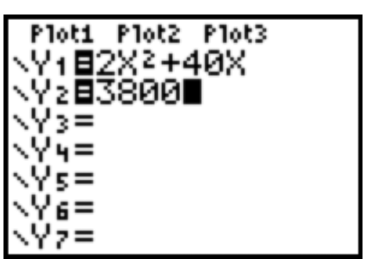
   There are two solutions, but only one makes sense for this situation. Since length cannot be negative, the measure of the side of the square base should be about 34.72 inches.

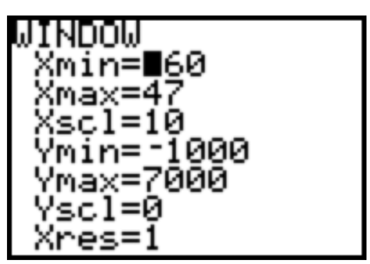
3. Describe how to solve the equation using a graph.

   Graph the function for the surface area, $y = 2x^2 + 40x$.

   Graph the line $y = 3{,}800$.

   Note: See Extension Questions for another possible method.





The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

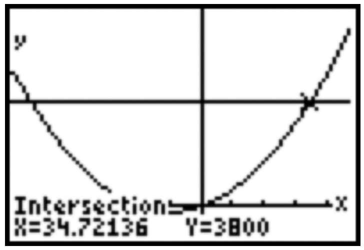
(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(D) analyze graphs of quadratic functions and draw conclusions.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.



The graph shows the intersection points at approximately (–54.72, 3,800) and (34.72, 3,800). The *x*-value of the intersection points represents the possible lengths for the base edge on the box.

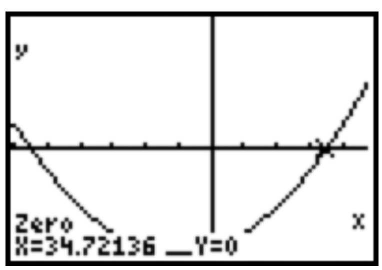
4. What is the measure of the base of the largest box Ana can build? Explain your answer.

   Since lengths cannot be negative, only *x*-values between 0 and 34.72 can be considered. The largest possible side length within the paint coverage limit is 34.72 inches.

Extension Questions

- Describe another graph that can be used to determine the measure of the base of a box with a total surface area is 3,800 square inches.

  The function $y = 2x^2 + 40x - 3{,}800$ can be graphed. Determine for which values of x the function is 0.



- What is the meaning of the function $y = 2x^2 + 40x - 3{,}800$ for the problem situation?

  Since $2x^2 + 40x$ represents the surface area, the function represents the amount of the surface area less 3,800 square feet. When the function value is 0, the surface area is equal to 3,800 square feet. If the function value is > 0, then Ana needs more than one can of paint.

- How does the original function change if the height of the box is 15 inches?

  The function is $A(s) = 2s^2 + 4(15)s$.

Insects in the Water

A biologist was interested in the number of insect larvae present in water samples of various temperatures. He collected the following data:

Temperature (C°)	0	10	20	30	40	50
Population	20	620	920	920	620	20

1. Make a scatterplot of the data. Given the function $y = ax^2 + bx + c$, where b is 75, experiment with the values of a and c to fit a quadratic function to your plot.
2. Write a verbal description of the relationship between the larvae population and the temperature of the water samples. What do the x- and y-intercepts mean? At what water temperature is the larvae population greatest?
3. The water sample is considered to be mildly contaminated but does not need to be treated if the larvae population is 300 or less. At what temperatures is the larvae population 300 or less? Explain.
4. Suppose that testing shows virtually no larvae present at 0°C, and the model for this situation is the function $y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(B) investigate, describe, and predict the effects of changes in *a* on the graph of $y = ax^2 + c$;

(D) analyze graphs of quadratic functions and draw conclusions.

Additional Algebra TEKS:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

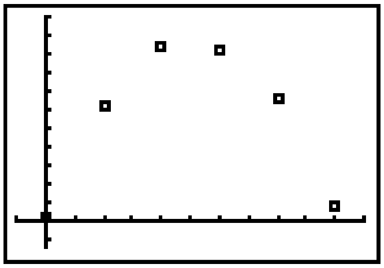
Scaffolding Questions

- How do the data in the table help you determine a reasonable window for your plot?
- What does the table indicate is a reasonable value for *c*? What does the shape of the graph tell you about the value of *a*?
- What is the function that most closely models this scatterplot?
- What happens when you experiment with values of *a* between –1 and –2?
- How can you use the graph of $y = 300$ to answer question 3?

Sample Solutions

1. Make a scatterplot of the data. Given the function $y = ax^2 + bx + c$, where *b* is 75, experiment with the values of *a* and *c* to fit a quadratic function to your plot.

   Scatterplot of larvae population vs. water temperature:

   x-axis = Temperature

   y-axis = Population

   In looking for values of *c*, students may reason that, since $y = 20$ when $x = 0$, the value for *c* in $y = ax^2 + 75x + c$ is 20. Since the scatterplot shows the larvae population increasing and decreasing, *a* must be negative.

   Looking for values of *a*, graphing $y = -1x^2 + 75x + 20$ gives a parabola that opens wider than the (original) plot appears. The following table shows values for this function. According to the values below, the larvae population is growing too fast compared to the original table of values.

Temperature (C°)	0	10	20	30	40	50
Population	20	670	1120	1370	1420	1270

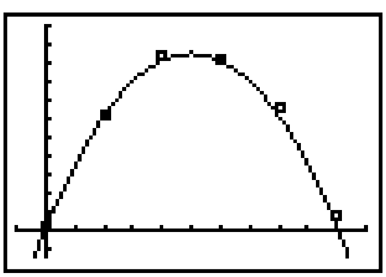
Graphing $y = -2x^2 + 75x + 20$ gives a parabola that opens narrower than the (original) plot appears. The following table shows values for this function. According to the values below, the larvae population is growing too slowly compared to the original table of values.

Temperature (C°)	0	10	20	30	40	50
Population	20	570	720	470	–180	–1230

Trying a value close to $a = -1.5$, we find that a good-fitting quadratic is $y = -1.5x^2 + 75x + 20$. The values in the table below (which are the values for this function) are the same values as those in the original table.

Temperature (C°)	0	10	20	30	40	50
Population	20	620	920	920	620	20

Using $a = -1.5$, then, is the most appropriate quadratic.



x-axis = Temperature

y-axis = Population

2. Write a verbal description of the relationship between the larvae population and the temperature of the water samples. What do the x- and y-intercepts mean? At what water temperature is the larvae population greatest?

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

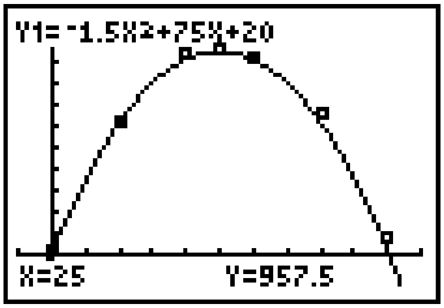
(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x–intercepts) of the graph of the function.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

The *y*-intercept of the graph shows that at 0°C, there are 20 insect larvae in the water sample. As the temperature increases to 25°C, the population increases to about 957. This is shown by finding the coordinates of the vertex, (25, 957.5), using the graph. Then the population decreases to 0 at about 50°C (the *x*-intercept to the right of the origin).

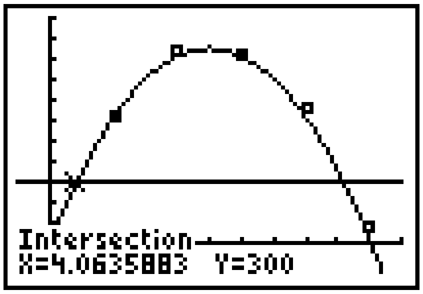


x-axis = Temperature

y-axis = Population

3. The water sample is considered to be mildly contaminated but does not need to be treated if the larvae population is 300 or less. At what temperatures is the larvae population 300 or less? Explain.

By graphing $y = 300$ along with the population graph and finding the points of intersection, we can determine the temperatures when the population is no more than 300.



x-axis = Temperature

y-axis = Population

The graph shows that for temperatures up to 4°C, the population of insect larvae is no more than 300. Since this point is 21 units to the left of the axis of symmetry, $x = 25$, the other intersection point is 21 units to the right of the axis of symmetry, and, therefore, is (46, 300). When the temperature is between 46°C and 50°C, the insect larvae population will again be no more than 300.

4. Suppose that testing shows virtually no larvae present at 0°C, and the model for this situation is the function
$y = -1.5x(x - 50)$. How does this function compare with the original function? How well does it appear to fit the data?

   Since the first model ($y = -1.5x^2 + 75x + 20$) is in polynomial form and the second model (given in the question) is in factored form, rewrite the second model in polynomial form:

$$y = -1.5x(x - 50)$$

$$= -1.5x^2 + 75x$$

   Finding the x-intercepts and vertex of the graph either with a calculator or analytically, we find that the second model has x-intercepts (0, 0) and (50, 0) and vertex (25, 937.5). It is a translation of the first model, $y = -1.5x^2 + 75x + 20$, shifted down 20 units. The first model accounted for the 20 larvae at 0°C. Compared to the first model, the second model underestimates the number of insect larvae present at any temperature by 20 larvae.

Extension Questions

- What trends in the table and the graph tell you what is happening with the insect larvae in the water samples?

  The table shows that as the temperature increases to 20°C, the larvae population increases to 920, and then the population decreases to 20 at 50°C. The graph provides a more accurate picture. Since it is a parabola opening downward, the maximum number of larvae occurs at the vertex, which is when the temperature of the water is 25°C. The graph shows the population decreasing to 20 at 50°C and no larvae present at a fraction of a degree hotter than 50°C.

- How can you graphically investigate when the insect larvae population is no more than 300?

  "No more than 300" means "less than or equal to 300." By graphing the line $y = 300$, we can find the portion of the larvae population graph that lies below the line graph, including the intersection points. We can do this using calculator features such as Trace or Intersect.

- Suppose $y = -1.5x(x - 50) + 20$ is considered a usable model for predicting the number of insect larvae present in water samples, and a second round of experiments shows that the population at each of the previous temperatures in the table doubles. How will this affect the scatterplot and the function that models this new scatterplot?

  The new table will be

Temperature (C°)	0	10	20	30	40	50
Population	40	1240	1840	1840	1240	40

  The original scatterplot will be stretched vertically by a factor of 2 since the larvae population doubles. All of the y-values from the original function are multiplied by 2.

  The new function will be $y = 2\,[-1.5x(x - 50) + 20]$ or $y = -3x(x - 50) + 40$.

- How does the new function in polynomial form compare with the original one?

  The coefficients and the constant in the new function are twice those of the original function.

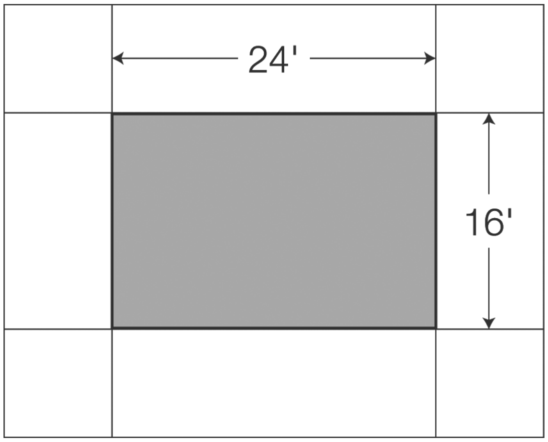
  The original is $y = -1.5x(x - 50) + 20 = -1.5x^2 + 75x + 20$.

  The new function is $y = -3x(x - 50) + 40 = -3x^2 + 150x + 40$.

A Ring Around the Posies

The Lush Landscaping Company is involved in a project for the city. They are expanding the garden in front of the city hall by planting a border of flowers around it. The current dimensions of the garden are 24 feet long by 16 feet wide. The border will have the same width around the entire garden. The flowers planted in the border will fill an area of 276 square feet.

Find the width of the border surrounding the garden using symbolic and/or algebraic methods. Explain the meaning of each number and symbol that you use.



Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

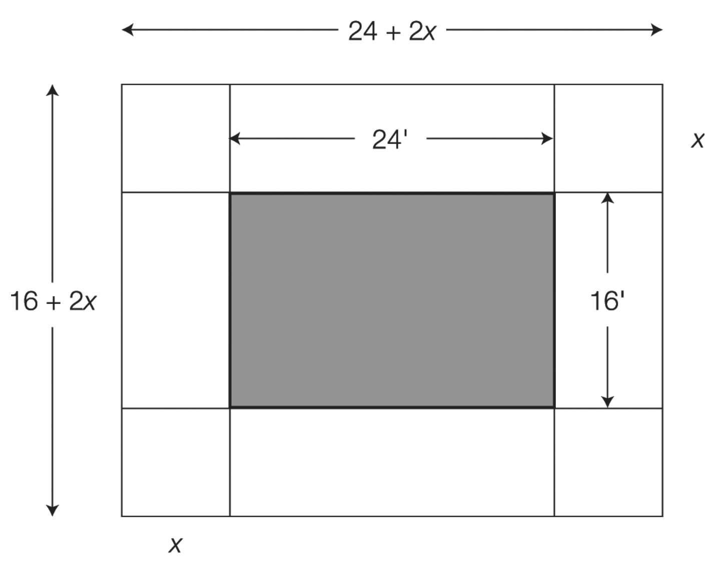
Scaffolding Questions

- How can you find the area of the existing garden?
- What are the variables in this situation?
- What are a possible length and width of the larger garden that includes the new border?
- What are a second possible length and width of the larger garden?
- How can you represent the dimensions of the larger garden algebraically?
- How can you find the area of the border?
- How does the area of 276 square feet relate to this situation and your equation?
- How does the area of the original garden relate to the area of the larger garden that includes the new border?

Sample Solutions

Find the width of the border surrounding the garden using symbolic and/or algebraic methods. Explain the meaning of each number and symbol that you use.

Let x = the width of the new border.



The area of the original garden, with dimensions of 24 feet by 16 feet, is calculated using the formula for the area of a rectangle, which is length • width, or $A = l \bullet w$.

$A = 24 \bullet 16$

$A = 384 \text{ ft}^2$

The area of the new garden with the border can be calculated with the same area formula.

$A = (24 + 2x)(16 + 2x)$

$A = 384 + 48x + 32x + 4x^2$

$A = 384 + 80x + 4x^2$

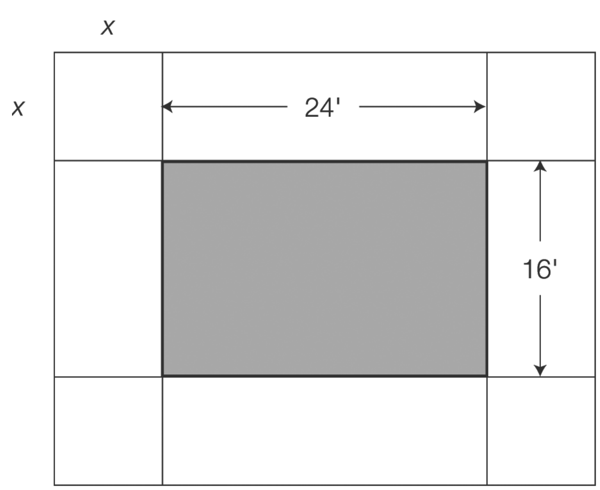
The flowers will be planted in the border that surrounds the 24-foot by 16-foot rectangle. To find that area, subtract the area of the small inside rectangle from the large rectangle.

Border area = large garden area – small original area

Border area = $(384 + 80x + 4x^2) - (384)$

Border area = $80x + 4x^2$

Another way to look at the border is to think of it as the sum of the rectangles that make up the border.



There are two rectangles at the top and bottom with measurements 24 and x. The two rectangles on the sides have measurements 16 and x. There are four corner squares with side length x.

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

The border area is

$A = 2(24x) + 2(16x) + 4x^2$

$A = 48x + 32x + 4x^2$

$A = 80x + 4x^2$

The problem stated that the flowers planted in the border would fill an area of 276 ft^2. Substitute this value for the border area.

$276 = 80x + 4x^2$

Factor to solve this equation.

$0 = 4x^2 + 80x - 276$

$0 = 4(x^2 + 20x - 69)$

$0 = 4(x + 23)(x - 3)$

$0 = x + 23$ and $0 = x - 3$

$x = -23, 3$

The solutions for x are –23 and 3. Since x represents the width of the border, the value cannot be negative. The only solution that makes sense is 3. The width of the border is 3 feet.

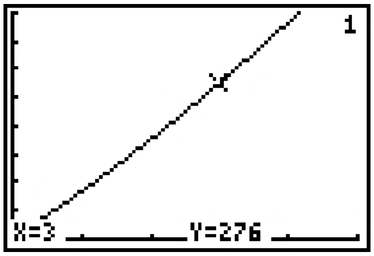
Extension Questions

- Find the border area as a function of border width x.

  The border area is the same as illustrated in the sample solution, $A = 80x + 4x^2$.

- Graph this function, and illustrate the solution to the original problem on the graph.

  The solution shown on the graph indicates that with a border width of 3 feet, the area of the border is 276 square feet.



Block That Kick!

A kicker kicks a football upward from the ground at an initial velocity of 63 feet per second. The height of the football stadium is 70 feet. The height an object reaches with respect to time is modeled by the following equation:

$$h = \frac{1}{2}gt^2 + vt + s$$

In the equation, g is –32 ft/sec^2, v is the initial velocity, s is the initial height, and t is time in seconds.

1. Write a function that models this situation as related to the number of seconds since kickoff.

2. Sketch and describe the graph of this function, including intercepts and maximum height.

3. At what times is the football the same height as the stadium? Explain your answer.

4. Suppose the initial velocity of the kicked football is 68 feet per second. At what times is the football the same height as the top of the stadium? Justify your answer.

5. Now consider that the kicker is trying to kick an extra point. A linebacker on the opposing team has a maximum reach of 10 feet, which includes his height, full extension of his arms, and his vertical jump. If he blocks the kick, at what time (in seconds after the kick) does this occur?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(D) analyze graphs of quadratic functions and draw conclusions.

Scaffolding Questions

- How would you describe the relationship between height and time?
- What are reasonable values for the domain and range?
- What is the value of *v* in the function for the first scenario?
- What is the initial height of the football?
- What do you expect the graph to look like?
- How can you tell from the graph how many solutions there will be? Explain your answer.

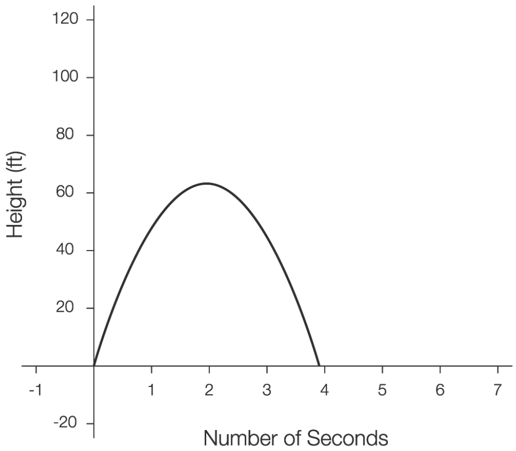
Sample Solutions

1. Write a function that models this situation as related to the number of seconds since kickoff.

   The football's initial velocity is 63 feet per second, and its initial height is 0 since it was kicked from ground level. The height of the football *t* seconds after it was kicked is modeled by the function $f(t) = -16t^2 + 63t$.

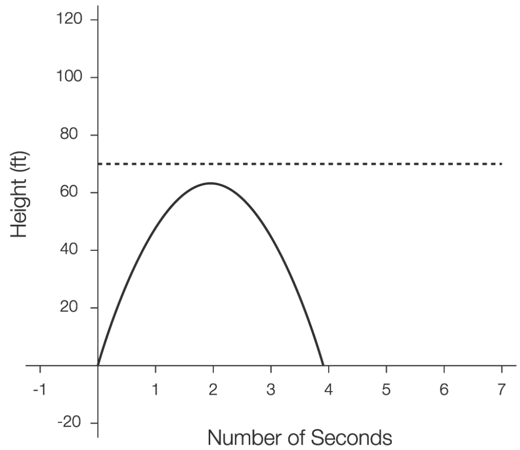
2. Sketch and describe the graph of this function, including intercepts and maximum height.

   The graph is a parabola that opens downward. The *x*-intercepts are 0 and 3.94, meaning that the ball is on the ground initially and then again almost 4 seconds later. The maximum height of the ball is approximately 62 feet. The *y*-intercept is 0 since at 0 seconds the ball is on the ground initially.



3. At what times is the football the same height as the stadium? Explain your answer.

   The stadium is 70 feel tall, so the question is asking when the football is 70 feet in the air. The graph below shows that there is no time when the ball is at the height of 70 feet.



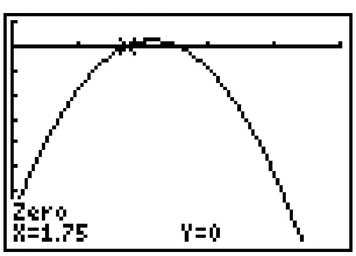
4. Suppose the initial velocity of the kicked football is 68 feet per second. At what times is the football the same height as the top of the stadium? Justify your answer.

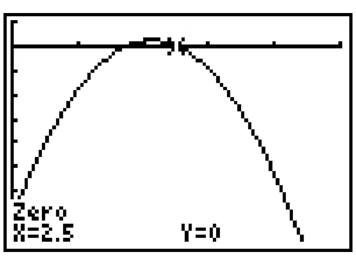
   If the initial velocity of the football is increased to 68 feet per second, the function becomes $f(t) = -16t^2 + 68t$. The equation becomes $70 = -16t^2 + 68t$.

   $0 = -16t^2 + 68t - 70$

   The related quadratic function is $f(t) = -16t^2 + 68t - 70$.

   The function has two roots, because the graph crosses the x-axis twice.





   The roots of the function are at approximately 1.75 and 2.5. This means that the football is at 70 feet on the

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

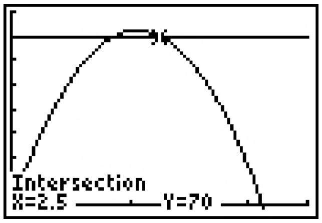
Texas Assessment of Knowledge and Skills:

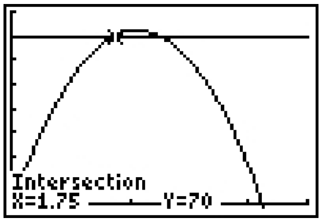
Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

ascent, after 1.75 seconds, and again on the descent, at 2.5 seconds. Students may reason about this scenario with or without considering whether or not the stadium has a roof. If it has a roof, then once the ball hits the roof, its velocity instantly changes, and the only time that makes sense is 1.75 seconds.

Another approach that students can take to this question is to examine the intersection of the functions $f(t) = -16t^2 + 68t$ and $y = 70$.





5. Now consider that the kicker is trying to kick an extra point. A linebacker on the opposing team has a maximum reach of 10 feet, which includes his height, full extension of his arms, and his vertical jump. If he blocks the kick, at what time (in seconds after the kick) does this occur?

   Students can approach this problem by setting up and solving the equation,

$$10 = -16t^2 + 63t$$

$$0 = -16t^2 + 63t - 10$$

$$t = \frac{-63 \pm \sqrt{63^2 - 4(-16)(-10)}}{2(-16)}$$

$$t = \frac{-63 \pm \sqrt{3969 - 640}}{-32}$$

$$t = \frac{-63 \pm \sqrt{3329}}{-32}$$

$$t \approx 3.77 \text{ or } 0.166$$

The times at which the ball is at a height of 10 feet are 3.77 and 0.166 seconds. Since the linebacker wants to block the kick, this must occur at 0.166 seconds.

Extension Questions

- If the ball were kicked at an initial velocity of 63 feet per second, what would its initial height have to be for it to reach a height of 70 feet?

  The maximum value of the parabola in the answer to question 2 is about 62 feet. If the ball were kicked from 8 feet off the ground, the graph of the function would be raised vertically 8 feet and would reach the height of 70 feet.

- Suppose the football is kicked from the ground and reaches the same maximum height as the stadium. Predict the initial velocity needed to achieve a maximum height of 70 feet to the nearest tenth if the ball is kicked from ground level.

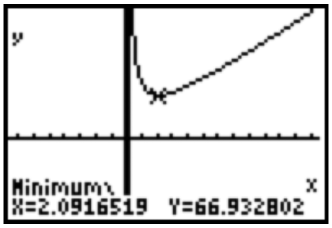
  One approach students can take in answering this question is to notice that they need to solve for v, given that h is 70. Also, students can use what they know from questions 2 and 5—that the velocity they are looking for must be between 63 and 68 feet per second, and that the number of seconds for the maximum height should be around 2 seconds.

$$h = \frac{1}{2}gt^2 + vt + s$$

Recalling that g is –32 ft/sec², v is the initial velocity, and s is the initial height, the new equation will be:

$$\begin{aligned} 70 &= \frac{1}{2}(-32)t^2 + vt + 0 \\ 70 &= -16t^2 + vt \\ 70 + 16t^2 &= vt \\ \frac{70+16t^2}{t} &= v \end{aligned}$$

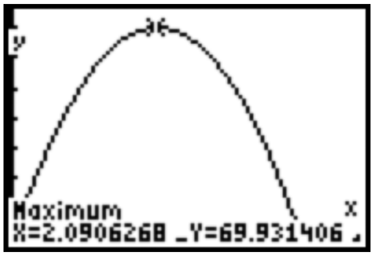
Graphing that function, x represents time and y represents velocity. Time cannot be negative, so it only makes sense to examine positive time values. Tracing along the function, there is a local minimum value of 66.9 feet per second at 2.09 seconds.



Testing this velocity into the original equation and then graphing the new function shows that the maximum height is very near 70 feet.

$$h = \frac{1}{2}gt^2 + vt + s$$

$$h = -16t^2 + 66.9t$$



Brrr!

Wind chill is the term used to describe how wind makes the air temperature feel colder. Wind carries away the warm air around your body; the greater the wind speed, the colder you feel.

The wind chill, c, at a given temperature in Fahrenheit (F) is modeled by a quadratic function of the wind speed in miles per hour, s.

For example, at 40ºF, the function $c = 0.018s^2 - 1.58s + 3.48$ models the wind chill with wind speeds from 0 to 45 miles per hour.

1. Graph the function and describe how the function models the situation.

2. Use the quadratic formula to find the wind speed for a wind chill of –10ºF.

Notes

Materials:

One graphing calculator per student

The function on the student activity page was obtained using data from the wind chill index at www.weather.gov/om/windchill and finding the quadratic regression equation with a graphing calculator.

Algebra TEKS Focus:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

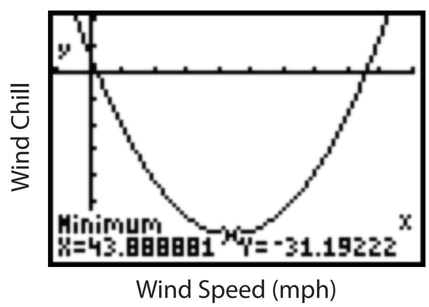
Scaffolding Questions

- What is the relationship between wind chill and wind speed?
- Use the function rule to determine the wind chill for a wind speed of 20 mph. What does your answer mean for the situation?
- How would you describe the graph of this relationship?
- How can you determine the roots of the function and how they relate to the given situation?

Sample Solutions

1. Graph the function and describe how the function models the situation.

   The graph of this function is a parabola that opens upward. The graph shows that the greater the wind speed (up to about 44 mph), the more the temperature drops. After about 44 mph, the wind chill values begin to increase, which doesn't really make sense.



2. Use the quadratic formula to find the wind speed for a wind chill of –10°F.

   Wind chill, c, is –10.

   $-10 = 0.018s^2 - 1.58s + 3.48$

   Add 10 to each side of the equation to put it in standard form.

   $0 = 0.018s^2 - 1.58s + 13.48$

Use the quadratic formula to solve: $a = 0.018$, $b = -1.58$, $c = 13.48$. Values used are rounded to the nearest tenth.

$$s = \frac{-(-1.58) \pm \sqrt{(-1.58)^2 - 4(0.018)(13.48)}}{2(0.018)}$$

$$s = \frac{1.58 \pm \sqrt{2.5 - 0.97}}{0.036}$$

$$s = \frac{1.58 \pm 1.24}{0.036}$$

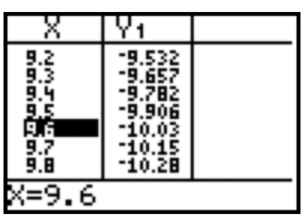
$s =$ 78.33, 9.44

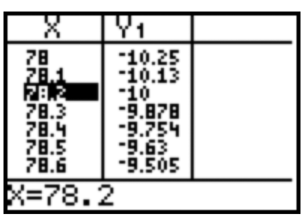
Only a wind speed of 9.44 miles per hour makes sense for this situation, since the value of s must be between 0 and 45 miles per hour.

Extension Questions

- Describe how to solve the second problem using a table.

  Examine the table of the function. Set the increments small enough to find a value close to –10. The values are 9.6 and 78.2. The value of 9.6 miles per hour is the reasonable value.





- Describe how to solve the second problem using a graph of a quadratic function.

  The function is $c = 0.018s^2 - 1.58s + 13.48$.

  The function can be graphed in a graphing calculator. Use the Trace feature to examine the graph for y-values near –10. This method yields a less precise answer.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

Additional Algebra TEKS:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

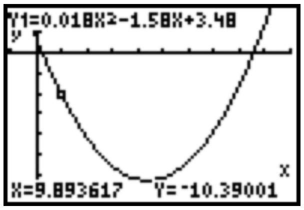
(A) determine the domain and range for quadratic functions in given situations;

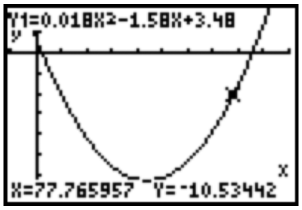
(D) analyze graphs of quadratic functions and draw conclusions.

Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.





The approximate wind speeds are 9.89 mph and 77.77 mph. The wind speed of 9.89 makes the most sense.

- Why is the given model not reasonable for wind speeds above 45 miles per hour?

  After about 44 mph, the function doesn't really make sense because the graph shows that after 44 mph, as the wind speed continues to increase, the temperature drops less. Wind speeds of up to 30 mph are fairly typical in winter. Wind speeds greater than 45 mph are considered extreme weather, and different models are used in such situations.

Calculating Cost

The marketing team for Capital Computer Company is working to find the most profitable selling price for the company's new laptop computer. After much research, the team determines that the function $N = -100p^2 + 300000p$ represents the expected relationship between N, the net profit in dollars, and p, the retail price of the laptop computer.

Use the given function to determine what price the Capital Computer Company should charge for the new laptop computer.

- Use symbolic and graphical solution methods to prove that the laptop price you suggest is the best choice for the company.
- Write a brief paragraph explaining your recommendation to the company.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

Scaffolding Questions

- What does the term *net profit* mean?
- What different methods can you use to determine a selling price?
- What does the variable *p* represent?
- What does it mean when the net profit equals 0?
- What does the graph tell you about the net profit as the retail price increases?

Sample Solutions

Use the given function to determine what price the Capital Computer Company should charge for the new laptop computer.

- Use symbolic and graphical solution methods to prove that the laptop price you suggest is the best choice for the company.
- Write a brief paragraph explaining your recommendation to the company.

Using symbolic solution methods, students can solve the equation $-100p^2 + 300000p = 0$ by factoring:

$-100p(p - 3000) = 0$

Examining each factor separately:

$-100p$	=	0	and	$p - 3000$	=	0
p	=	0		p	=	3000

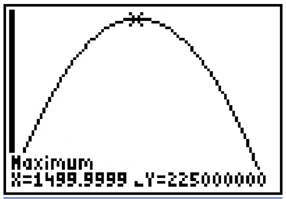
These solutions indicate that the net profit will equal 0 if the price of the laptop is \$0 or \$3,000. These values also indicate the reasonable domain for this situation and can be used to find a reasonable range. Students may realize that this function rule indicates a quadratic relationship and that when graphed it will show a parabola opening downward. Using the middle value in the domain of $x = 1{,}500$ and substituting into the expression,

$-100p^2 + 300000p$

$-100(1500)^2 + 300000(1500) = \$225{,}000{,}000$

This indicates that the maximum net profit is achieved at a laptop price of $1,500.

This reasoning can be verified using graphical solution methods. Again, students may realize that because the highest exponent of the equation is 2, the graph will be a parabola. The negative sign of the coefficient indicates that the graph will open downward. The graph of the function is as follows:



The graph shows that as the price of the laptop increases from $0 to $1,500, the net profit increases from $0 to $225 million. As the price continues to increase up to $3,000, the net profit decreases back to $0.

The marketing team should propose $1,500 as the price for the laptop because the maximum net profit occurs at that price.

Extension Questions

- If the coefficient of p is changed to 200,000, how does the change affect the solutions?

  The answer changes to 0 and 2,000.

  $-100p\,(p - 2000) = 0$

  $$\begin{aligned} -100p &= 0 \quad \text{and} \quad & p - 2000 &= 0 \\ p &= 0 & p &= 2000 \end{aligned}$$

- Without graphing, tell how you expect this change to affect the graph.

  The graph's vertex will have an x-value halfway between 0 and 2,000. The maximum net profit will occur at p = $1,000.

The student is expected to:

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 1: The student will describe functional relationships in a variety of ways.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Investigating Parameters

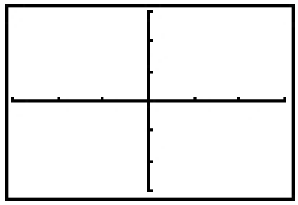
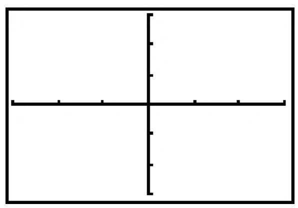
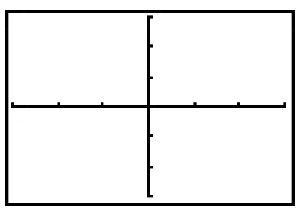
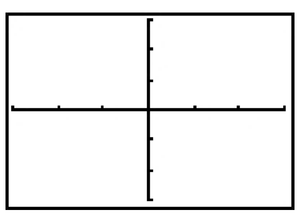
1. For each function in Problem Sets A–D, do the following:

   - Predict how each function rule will transform the parent function, $y = x^2$.
   - Complete the table and sketch a graph. In the Graph column, show the parent graph as a thin line and the transformed graph as a bold, thick line.
   - Compare each of your sketches with the graph of the parent function $y = x^2$ and write a verbal description of the transformation.

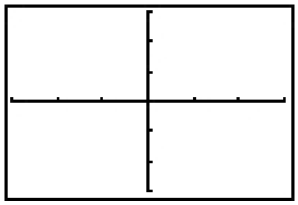
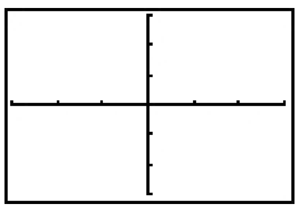
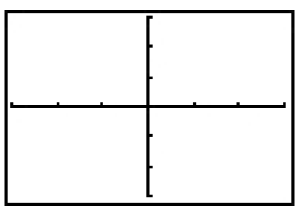
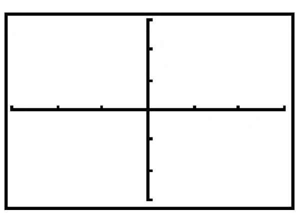
2. Each of the following describes transformations on the graph of $y = x^2$. Explain how the vertex, axis of symmetry, intercepts, and function rule are changed by the transformation.

   - Vertically stretch by a factor of 4 and translate up by 2 units.
   - Vertically compress by a factor of $\frac{1}{2}$ and translate down by 2 units.
   - Vertically stretch by a factor of 2 and reflect over the x-axis.

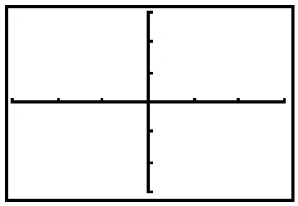
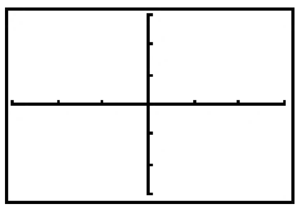
Problem Set A

	Function	Graph	Table	Verbal Description of Transformation
A1.	$f(x) = 2x^2$		x | y −2 −1 0 1 2	
A2.	$g(x) = (2x)^2$		x | y −2 −1 0 1 2	
A3.	$h(x) = \frac{1}{2}x^2$		x | y −2 −1 0 1 2	
A4.	$m(x) = (0.5x)^2$		x | y −2 −1 0 1 2	

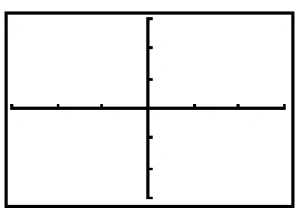
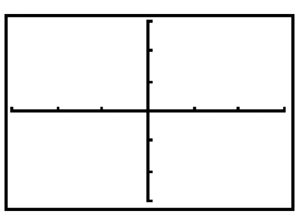
Problem Set B

	Function	Graph	Table	Verbal Description of Transformation
B1.	$f(x) = -x^2$		x | y −2 −1 0 1 2	
B2.	$g(x) = -3x^2$		x | y −2 −1 0 1 2	
B3.	$h(x) = (-3x)^2$		x | y −2 −1 0 1 2	
B4.	$m(x) = -(3x)^2$		x | y −2 −1 0 1 2	

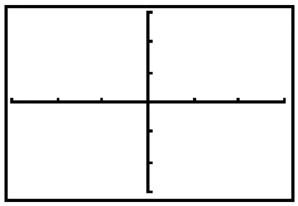
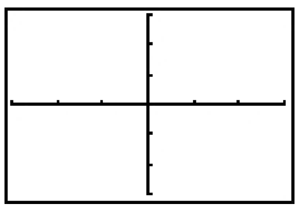
Problem Set C

<table>
<tr><th></th><th>Function</th><th>Graph</th><th>Table</th><th>Verbal Description of Transformation</th></tr>
<tr><td>C1.</td><td>$f(x) = x^2 - 1$</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td></td></tr><tr><td>−1</td><td></td></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr></table></td><td></td></tr>
<tr><td>C2.</td><td>$g(x) = x^2 + 2$</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td></td></tr><tr><td>−1</td><td></td></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr></table></td><td></td></tr>
</table>

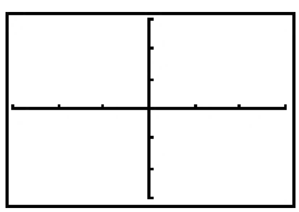
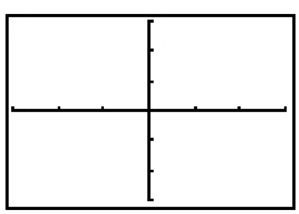
Extension Questions

<table>
<tr><th></th><th>Function</th><th>Graph</th><th>Table</th><th>Verbal Description of Transformation</th></tr>
<tr><td>C3.</td><td>$h(x) = (x - 1)^2$</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td></td></tr><tr><td>−1</td><td></td></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr></table></td><td></td></tr>
<tr><td>C4.</td><td>$m(x) = (x + 2)^2$</td><td></td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td></td></tr><tr><td>−1</td><td></td></tr><tr><td>0</td><td></td></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr></table></td><td></td></tr>
</table>

Problem Set D

	Function	Graph	Table	Verbal Description of Transformation
D1.	$f(x) = 2x^2 + 1$		x y −2 −1 0 1 2	
D2.	$g(x) = -3x^2 + 12$		x y −2 −1 0 1 2	

Extension Questions

	Function	Graph	Table	Verbal Description of Transformation
D3.	$h(x) = 0.5(x - 2)^2$		x y −2 −1 0 1 2	
D4.	$m(x) = -(x + 2)^2$		x y −2 −1 0 1 2	

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(C) connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

Scaffolding Questions

- What does the graph of the parent function $y = x^2$ look like? What characterizes this graph?

For Problem Set A

- How are the functions different from the parent function?
- How does a table of values help you see the difference between the graphs of the new function and the parent function?
- What does it mean to change a quantity by a scale factor?
- How is the point (1, 1) in the original function affected by the scale factor?

For Problem Set B

- How are the functions different from the parent function?
- How are they different from the functions in Set A?
- What effect does the negative coefficient have on the graph?

For Problem Set C

- How are the functions different from the parent function?
- How does comparing a table of values for the parent function with these four functions help you see what is happening to the graph of the parent function?

For Problem Set D

- For this problem set, you are working with combinations of transformations on the parent function. For each function, how would you describe the sequence of transformations performed on the parent function that result in the new function?

Sample Solutions

In each set, to compare the parent function with each of the four transformed functions, we can use graphs and tables to show us the effect each transformation has on the graph of the parent function.

Note: We recommend that you assign this assessment to be completed *without* a graphing calculator. We include the screen shots in the sample answers to clarify the look of the graphs. Student graphs will be hand drawn.

The student is expected to:

(B) investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

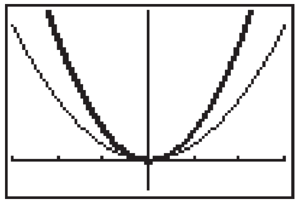
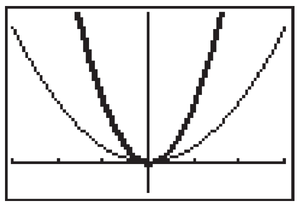
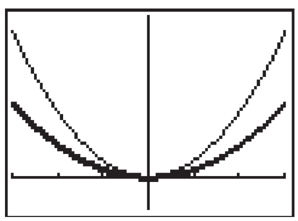
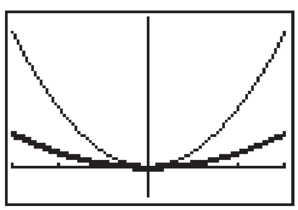
(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Texas Assessment of Knowledge and Skills:

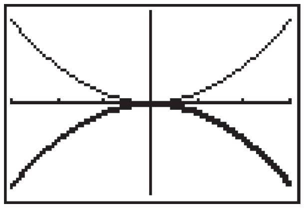
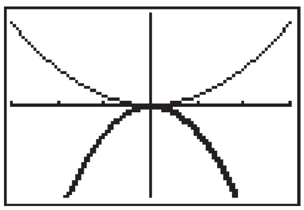
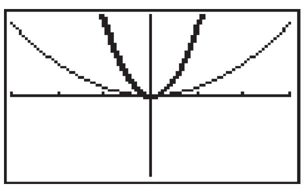
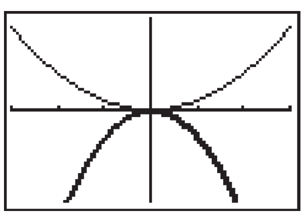
Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

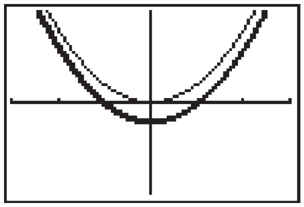
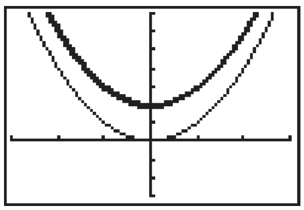
Problem Set A

	Function	Graph	Table	Verbal Description of Transformation
A1.	$f(x) = 2x^2$		x | y −2 | 8 −1 | 2 0 | 0 1 | 2 2 | 8	Vertically stretch $y = x^2$ by a factor of 2. The vertex remains at (0, 0).
A2.	$g(x) = (2x)^2$		x | y −2 | 16 −1 | 4 0 | 0 1 | 4 2 | 16	Vertically stretch $y = x^2$ by a factor of 4, $(2)^2$. The vertex remains at (0, 0).
A3.	$h(x) = \frac{1}{2}x^2$		x | y −2 | 2 −1 | 0.5 0 | 0 1 | 0.5 2 | 2	Vertically compress $y = x^2$ by a factor of 0.5. The vertex remains at (0, 0).
A4.	$m(x) = (0.5x)^2$		x | y −2 | 1 −1 | 0.25 0 | 0 1 | 0.25 2 | 1	Horizontally compress x by a factor of 0.25, $(0.5)^2$. The vertex remains at (0, 0).

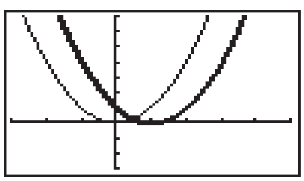
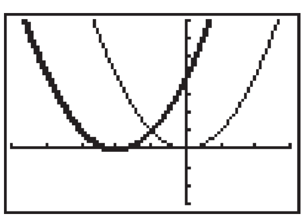
Problem Set B

	Function	Graph	Table	Verbal Description of Transformation
B1.	$f(x) = -x^2$		x | y −2 | −4 −1 | −1 0 | 0 1 | −1 2 | −4	Reflect graph of parent function over x-axis. The vertex remains at (0, 0).
B2.	$g(x) = -3x^2$		x | y −2 | −12 −1 | −3 0 | 0 1 | −3 2 | −12	Vertically stretch parent graph by a factor of 3. Reflect over x-axis. The vertex remains at (0, 0).
B3.	$h(x) = (-3x)^2$		x | y −2 | 36 −1 | 9 0 | 0 1 | 9 2 | 36	Vertically stretch $y = x^2$ by 9, $(-3)^2$. The vertex remains at (0, 0).
B4.	$m(x) = -(3x)^2$		x | y −2 | −36 −1 | −9 0 | 0 1 | −9 2 | −36	Vertically stretch $y = x^2$ by a factor of 9, $(3)^2$. Reflect over x-axis. The vertex remains at (0, 0).

Problem Set C

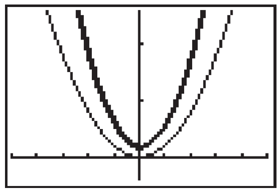
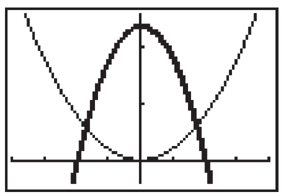
	Function	Graph	Table	Verbal Description of Transformation
C1.	$f(x) = x^2 - 1$		x | y –2 | 3 –1 | 0 0 | –1 1 | 0 2 | 3	Translate vertically down 1 unit. Vertex moves to (0, –1). The *y*-intercept is (0, –1). The *x*-intercepts are the points (–1, 0), (1, 0).
C2.	$g(x) = x^2 + 2$		x | y –2 | 6 –1 | 3 0 | 2 1 | 3 2 | 6	Translate vertically up 2 units. Vertex moves to (0, 2). The *y*-intercept is (0, 2). There are no *x*-intercepts.

Extension Questions

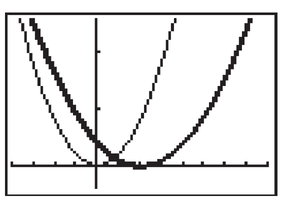
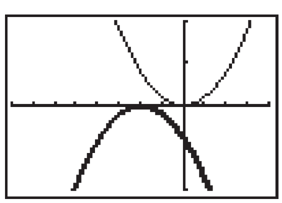
	Function	Graph	Table	Verbal Description of Transformation
C3.	$h(x) = (x - 1)^2$		x | y –2 | 9 –1 | 4 0 | 1 1 | 0 2 | 1	Translate right 1 unit. Vertex moves to (1, 0). The *y*-intercept is (0, 1). The *x*-intercept is (1, 0).
C4.	$m(x) = (x + 2)^2$		x | y –2 | 0 –1 | 1 0 | 4 1 | 9 2 | 16	Translate left 2 units. Vertex moves to (–2, 0). The *y*-intercept is (0, 4). The *x*-intercept is (–2, 0).

Problem Set D

Complete the table to compare the graphs of the given functions with the graph of the parent function $y = x^2$. Describe how changing the parameters *a* and *c* affects the shape of the graph of the parent function, its intercepts, and its orientation.

	Function	Graph	Table	Verbal Description of Transformation
D1.	$F(x) = 2x^2 + 1$		x | y −2 | 9 −1 | 3 0 | 1 1 | 3 2 | 9	Vertically stretch parent graph by a factor of 2. Then translate up 1 unit. The vertex and *y*-intercept become (0, 1). There are no *x*-intercepts.
D2.	$G(x) = -3x^2 + 12$		x | y −2 | 0 −1 | 9 0 | 12 1 | 9 2 | 0	Vertically stretch parent graph by a factor of 3. Reflect over *x*-axis. Translate up 12 units. The vertex and *y*-intercept become (0, 12). The *x*-intercepts are (2, 0) and (–2, 0).

Extension Questions

	Function	Graph	Table	Verbal Description of Transformation
D3.	$h(x) = 0.5(x - 2)^2$		x | y −2 | 8 −1 | 4.5 0 | 2 1 | 0.5 2 | 0	Translate parent graph right 2 units. Vertically compress by a factor of 0.5. The vertex becomes (2, 0). The *y*-intercept is (0, 2).
D4.	$m(x) = -(x + 2)^2$		x | y −2 | 0 −1 | −1 0 | −4 1 | −9 2 | −16	Translate parent graph left 2 units. Reflect over the *x*-axis. The vertex becomes (–2, 0). The *y*-intercept is (0, –4).

2. Each of the following describes transformations on the graph of $y = x^2$. Explain how the vertex, axis of symmetry, intercepts, and function rule are changed by the transformation.

- Vertically stretch by a factor of 4 and translate up by 2 units.

  If the graph of $y = x^2$ is vertically stretched by a factor of 4 and translated up 2 units, the new function is $y = 4x^2 + 2$. The original vertex (0, 0) translates to (0, 2). The axis of symmetry is still the y-axis. The y-intercept is the vertex. There are no x-intercepts since the equation $4x^2 + 2 = 0$ has no real solution.

- Vertically compress by a factor of $\frac{1}{2}$ and translate down by 2 units.

  If the graph of $y = x^2$ is vertically compressed by a factor of $\frac{1}{2}$ and translated down 2 units, the new function is $y = \frac{1}{2}x^2 - 2$. The original vertex (0, 0) translates to (0, –2). The axis of symmetry is still the y-axis. The y-intercept is the vertex. Solving the equation $\frac{1}{2}x^2 - 2 = 0$ implies $x = \pm 2$, so the x-intercepts are (–2, 0) and (2, 0).

- Vertically stretch by a factor of 2 and reflect over the x-axis.

  If the graph $y = x^2$ is vertically stretched by a factor of 2 and reflected over the x-axis, the new function is $y = -2x^2$. The vertex is still (0, 0). The axis of symmetry is still the y-axis. The y-intercept and x-intercepts are both 0.

Extension Questions

- What kinds of transformations on the graph of $y = x^2$ can be performed so that the resulting graph continues to be that of a function?

  Since a function is a relation between x and y that generates exactly one output value, y, for each input value, x, the only transformations on $y = x^2$ we can consider are dilations, translations, and reflections over the axes.

- What parameter causes a dilation on the graph of $y = x^2$? What is another way of saying *dilation*? What else does this parameter tell you?

  The parameter a in $y = ax^2$ causes a dilation. This dilation is a vertical stretch or a vertical compression on the graph of $y = x^2$. A "stretch" is another way of saying "dilation." The parameter a also causes a reflection over the x-axis if a is negative. As the magnitude of $|a|$ increases, the graph of $y = ax^2$ becomes steeper.

- What parameter causes a translation and what kind of translation?

  The parameter c in $y = ax^2 + c$ causes a vertical translation. The parent graph is translated c units up if c is positive and –c units down if c is negative.

- What does the order of operations sequence tell you about the sequence of transformations performed on the parent graph to generate the graph of $y = ax^2 + c$?

  If $a > 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Then translate vertically c units. If $a < 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Next, reflect the graph over the x-axis, and then translate vertically c units.

- What is the difference between the transformations $y = ax^2$ and $y = (ax)^2$?

  The first transformation is a vertical stretch or compression of the graph of $y = x^2$. The second transformation is a horizontal stretch or compression of x before it is squared. This transformation can also be described as a vertical stretch or compression, but the dilation factor is a^2, since $(ax)^2 = a^2x^2$.

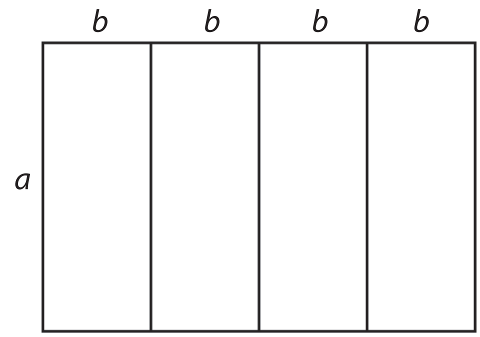
- What is the difference between the transformations $y = x^2 + c$ and $y = (x + c)^2$?

  The first transformation is a vertical translation c units up if c is positive and –c units down if c is negative. The second transformation is a horizontal translation c units left if c is positive and –c units right if c is negative.

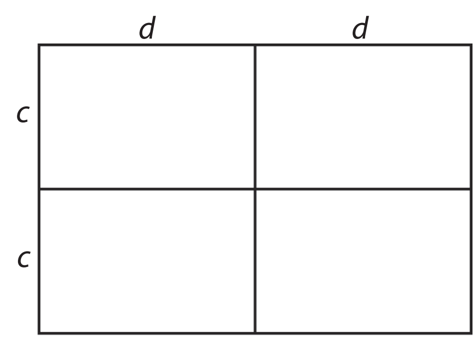
Ostrich Pens

A rancher who raises ostriches has 60 yards of fencing to make four equally sized rectangular pens for his flock. He is considering two possible pen arrangements, as shown in the diagrams below.

Option 1: Arrange the smaller pens in a line with adjacent pens sharing one common fence.



Option 2: Arrange the four smaller pens so that they share exactly two common fences.



1. For each option, write the function for the total area in one variable. Use A_1 to represent the area under Option 1 in terms of either *a* or *b*, and use A_2 to represent the area under Option 2 in terms of either *c* or *d*.

2. Create a graph of each function. Compare the domains and ranges for the problem situation under each option.

3. For each option, determine the dimensions that create the greatest total area for the pens. How do the total areas of the two options compare? How do the areas of the individual pens in the two options compare?

4. If the total amount of fencing for the pens were doubled, how would this change your responses to questions 1, 2, and 3?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(D) analyze graphs of quadratic functions and draw conclusions.

Scaffolding Questions

- What are the known and unknown quantities in this situation?
- What does each known quantity represent?
- What are the dimensions of the sides of the pens under Option 1 and Option 2?
- If we have two variables, how can we express the area under Option 1 in one variable?
- What representation(s) will best help you describe the domain and range for each option? How do the situations restrict the domains and ranges?
- How can you determine the maximum area by looking at the graph?

Sample Solutions

1. For each option, write the function for the total area in one variable. Use A_1 to represent the area under Option 1 in terms of either *a* or *b*, and use A_2 to represent the area under Option 2 in terms of either *c* or *d*.

   The total area, *A*, is given by the product of the length and width. The total amount of fencing, 60 yards, will be used to fence the perimeters of the pens. The 60 yards can, therefore, be used to relate the length and width variables.

   For Option 1,

   $$5a + 8b = 60$$

   $$b = \frac{60-5a}{8}$$

   For Option 2,

   $$6c + 6d = 60$$

   $$d = \frac{60-6c}{6} = \frac{6(10-c)}{6} = 10 - c$$

Area equals width multiplied by length. If we solve for length, the area functions for the first and second sets of pens, respectively, are

$A_1 = a \bullet 4b$

$A_1 = a \bullet 4\left(\frac{60 - 5a}{8}\right)$

$A_1 = \frac{4}{8}a(60 - 5a)$

$A_1 = 0.5a(60 - 5a)$

$A_2 = 2c \bullet 2d$

$A_2 = 2c \bullet 2(10 - c)$

$A_2 = 4c\,(10 - c)$

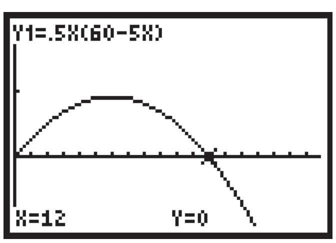
Students can also approach this problem by solving first for the other variable.

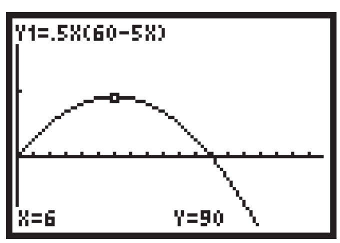
2. Create a graph of each function. Compare the domains and ranges for the problem situation under each option.

The domains and ranges for the two area functions are easily seen by building a table or by graphing. This question asks for a graph.

For Option 1, rewrite the area function using the variables *y* and *x*.

$y = 0.5x(60 - 5x)$, where *y* is the area and *x* is one of the dimensions (replacing *a* in the original context).





The domain for Option 1 is the set of all values, $0 < x < 12$. These are the only values that make sense in the situation, since the area must be a positive value.

The range is the set of all values, $0 < A_1 \leq 90$, because the area must be positive and 90 is the largest value on the graph.

Similarly, the domain and range for Option 2 can be

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

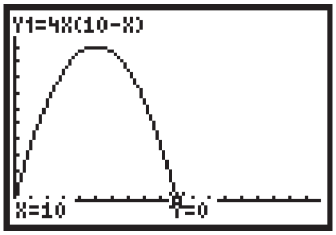
(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

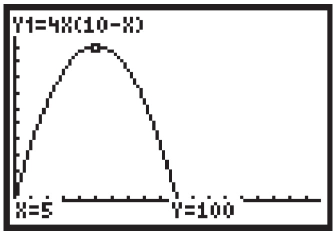
Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

determined by examining the graph of $y = 4x(10 - x)$.





The domain is the set of all values, $0 < x < 10$.

The range is the set of all values, $0 < A_2 \leq 100$, because the area must be positive and 100 is the largest value on the graph.

3. For each option, determine the dimensions that create the greatest total area for the pens. How do the total areas of the two options compare? How do the areas of the individual pens in the two options compare?

For Option 1, the greatest possible area occurs at the vertex of the parabola, which is the shape of the graph of each area function. When x is 6, the greatest area for Option 1 is 90 square units. (x represented the variable, *a,* in the answer to question 2.)

$5a + 8b = 60$, and from above we know that $a = 6$, so

$$b = \frac{60 - 5a}{8} = \frac{60 - 5(6)}{8} = 3.75$$

So the dimensions of each small pen under Option 1 are 3.75 yards multiplied by 6 yards, and the area of each pen is 3.75(6) = 22.5 square yards. The area of the larger arrangement of pens under Option 1 can be represented in terms of *a* and *b* as $A_1 = a \bullet 4b$, so the dimensions of this arrangement are 6 yards and 4(3.75) = 15 yards.

For Option 2, the maximum area is 100 square units at $x = 5$. (x represented the variable, *c,* in the answer to question 2.)

The dimensions of the pen are $2c$ multiplied by $2d$.

$6c + 6d = 60$, and from above we know that $c = 5$, so

$d = 10 - c = 10 - 5 = 5$

So the dimensions of each small pen under Option 2 are 5 yards multiplied by 5 yards, and the area of each pen is 5(5) = 25 square yards. The area of the larger arrangement of pens under Option 2 can be represented in terms of *c* and *d* as $A_2 = 2c \bullet 2d$, so the dimensions of this arrangement are 10 yards by 10 yards. A square arrangement under Option 2 yields the maximum area.

4. If the total amount of fencing for the pens were doubled, how would this change your responses to questions 1, 2, and 3?

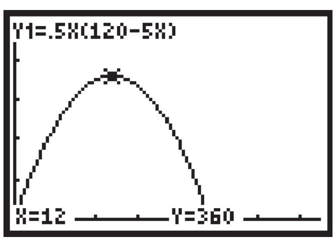
   If the total amount of fencing were doubled, the area functions would change and the domains and ranges would increase. This can be investigated easily using the table of values for the functions or the graphs.

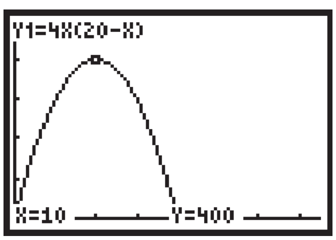
   Doubling the fencing changes the 60 feet of fencing to 120 feet. The first area function changes.

$$\begin{aligned} 5a + 8b &= 120 \\ b &= \frac{120 - 5a}{8} \\ A_1 &= 4b \bullet a = 4ab \\ A_1 &= 4a\left(\frac{120 - 5a}{8}\right) \\ A_1 &= 0.5a(120 - 5a) \end{aligned}$$

$$\begin{aligned} 6c + 6d &= 120 \\ d &= \frac{120 - 6c}{6} = \frac{6(20 - c)}{6} = 20 - c \\ A_2 &= 2c(2d) \\ A_2 &= 2c \bullet 2(20 - c) \\ A_2 &= 4c(20 - c) \end{aligned}$$

Graphing these two functions and looking for maximum *y*-values shows that the parabola vertices are (12, 360) and (10, 400).





The square arrangement of pens still gives the greater area. Doubling the fencing doubles the dimensions of the pens and increases their areas by a factor of 4.

Extension Questions

- What quantities vary in this situation, and how do these quantities affect the total area, *A*?

  The dimensions of the pens vary, and the arrangement of the pens varies. This means that initially the area, A, is a function of the two variables. The fixed perimeter lets us relate the two variables, and this relation depends on the arrangement of the pens. With this relation, we can express one variable in terms of the other, and then the total area, A, as a function of one variable.

- What type of function is *A(x),* and how do you know this function has a maximum value? How does this help you determine the dimensions of the pens that give the greatest possible area?

  The function A(x) is a quadratic function with a negative quadratic coefficient. Therefore, its graph is a parabola opening downward. The vertex is the highest point on the graph, and its coordinates tell you the value of x that gives the maximum area and what that maximum area is. Once x is known, using that value in the perimeter equation gives the corresponding value of y.

- How do you know which arrangement of pens gives the greater maximum area? Which arrangement is this, and is it realistic? What other factors may need to be considered?

  By comparing the parabolas that are the graphs of the area functions, we can determine which has the higher vertex and therefore the greater area. The graph of the function for the second arrangement has the higher vertex, indicating a total maximum area of 100 square yards. This is realistic, but it may not be practical. The pens are square, and that may not be the best shape for this animal. The pens built with the amount of fencing available also may not be large enough. It depends on the space ostriches need for exercise.

Sky Diving

An airplane is flying at an altitude of 4,000 meters. A skydiver jumps from the airplane and descends straight toward the ground.

1. Use the vertical motion formula $h = \frac{1}{2}(-9.8)t^2 + vt + s$ to write the skydiver's height above ground during free fall as a function of the time since she jumped. h = new height, t = time in seconds, v = initial velocity, and s = starting height. The initial velocity of a free fall is 0. Graph your function, and identify its zeros. Relate the zeros to the problem situation.

2. If the skydiver has fallen approximately 100 meters, how many seconds have passed? Explain how to use your graph to estimate the solution. Show how to find the number of seconds algebraically.

3. If the skydiver deploys her chute at 1,000 meters, how much time has passed since she jumped out of the plane?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

Additional Algebra TEKS:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(D) analyze graphs of quadratic functions and draw conclusions.

Scaffolding Questions

- Can you determine a reasonable domain and range for this situation?
- Which values are not reasonable for the domain and/or range? Explain.
- How can you use the function rule or graph to identify the zeros for this problem situation?

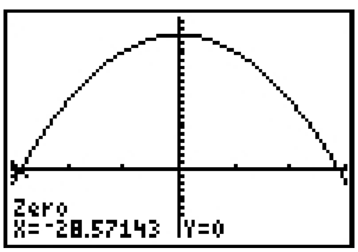
Sample Solutions

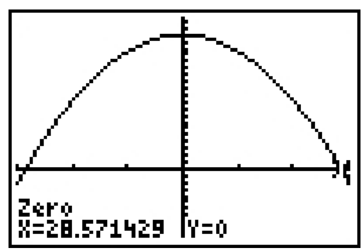
1. Use the vertical motion formula $h = \frac{1}{2}(-9.8)t^2 + vt + s$ to write the skydiver's height above ground during free fall as a function of the time since she jumped. h = new height, t = time in seconds, v = initial velocity, and s = starting height. The initial velocity of a free fall is 0. Graph your function, and identify its zeros. Relate the zeros to the problem situation.

   Substitute values into the vertical motion formula $h = \frac{1}{2}(-9.8)t^2 + vt + s$.

   The initial velocity for this situation is zero, therefore $v = 0$. The skydiver's starting height is the altitude of 4,000 meters, so $s = 4{,}000$. The function is $h = \frac{1}{2}(-9.8)t^2 + 4{,}000$, or $h = -4.9t^2 + 4{,}000$.

   The graph of the function is a parabola that opens downward.





   The zeros of the function are at approximately –28.57 and 28.57. Only the nonnegative values make sense in the situation because these values represent time. The number of seconds cannot be negative. In the problem situation, the zeros represent the number of seconds until the skydiver hits the ground if she does not open her chute, but this would be a tragedy. Opening her chute would change her velocity and would require a new function rule to model the rest of her descent.

2. If the skydiver has fallen approximately 100 meters, how many seconds have passed? Explain how to use your graph to estimate the solution. Show how to find the number of seconds algebraically.

   The graph can be used to estimate the amount of seconds that have passed when the skydiver has fallen approximately 100 meters. Using the trace function on the calculator, you can find that the value of 3,900 meters for the height yields a value of about 4.5 seconds for the time.

   Another way to determine the number of seconds is to solve algebraically. Substitute the value 900 for the height into the original function and solve for *t*.

   $h = -4.9t^2 + 4{,}000$

   $3900 = -4.9t^2 + 4{,}000$

   $0 = -4.9t^2 + 100$

   Using the quadratic formula to solve.

   $a = -4.9 \quad b = 0 \quad c = 100$

   $$t = \frac{0 \pm \sqrt{0^2 - 4(-4.9)(100)}}{2(-4.9)}$$
   $$t = \frac{\pm\sqrt{-4(-4.9)(100)}}{2(-4.9)}$$
   $$t = \frac{\pm\sqrt{1{,}960}}{-9.8}$$
   $$t = \pm 4.5$$

   The solutions are approximately –4.5 and 4.5. The number of seconds cannot be negative, so only the positive value is reasonable.

3. If the skydiver deploys her chute at 1,000 meters, how much time has passed since she jumped out of the plane?

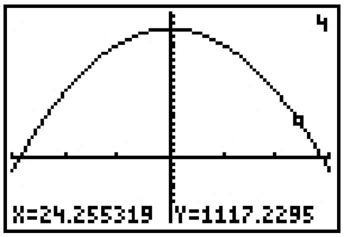
   To find the elapsed time if the skydiver deploys her chute at least 1,000 meters from the ground, students

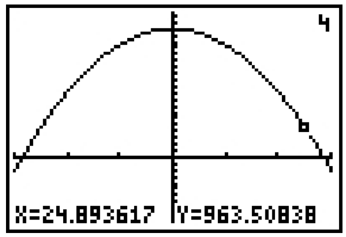
Texas Assessment of Knowledge and Skills:

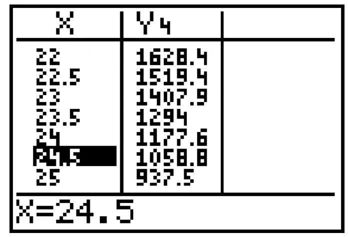
Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

can use the table or graph feature of their graphing calculator. Using the trace feature with the graph, students can see that the skydiver reaches an altitude of 1,000 meters at about 24.5 seconds. This is confirmed using the table. In reality, the skydiver should deploy her chute at or before the 24 seconds to ensure a safe landing.







Extension Questions

- Imagine that skydiving on the moon is possible. The moon's gravity is $\frac{1}{6}$ that of the Earth. What would the vertical motion formula be if she were skydiving from 4,000 meters above the moon?

  Our formula for Earth is

  $H = -4.9t^2 + 1{,}000$

  For the moon it would be

  $H = \frac{1}{6}(-4.9)t^2 + 1{,}000$
  $H = -0.817t^2 + 1{,}000$

Supply and Demand

Each year, the senior class sponsors a Teen Idol contest. Last year, they charged \$3 per ticket and sold 2,500 tickets. Based on a survey of the student body, the senior class leaders know that for every 25¢ the ticket price increases, they will sell 125 fewer tickets. The senior class president must help her fellow officers decide how much to charge per ticket for this year's Teen Idol contest.

1. Suppose the senior class must collect at least \$7,800. What range of ticket prices would allow them to do this?
2. Write a function for the amount of money, M dollars, that would be collected in terms of x, the number of 25¢ price increases.
3. Sketch a reasonable graph of the function and write a verbal description of what the graph tells you about the situation.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(D) analyze graphs of quadratic functions and draw conclusions.

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

Scaffolding Questions

- What decisions must the senior class make to determine the amount of money they can collect with the fundraiser?
- How much money was raised last year? Explain how you know.
- What processes and operations are you using to determine the number of tickets that will be sold for a given ticket price or number of price increases?
- What processes and operations are you using to determine the amount of money collected for a given ticket price or number of price increases?
- What does the vertex of the parabola in the graph of the situation tell you?

Sample Solutions

1. Suppose the senior class must collect at least $7,800. What range of ticket prices would allow them to do this?

   Students can approach this question in several ways. Below are an example using tables and an example using a graph.

   Tables:

   Students can construct a table, initially considering the relationship between the ticket price and the number of tickets sold:

Ticket Price	Predicted Ticket Sales
3.00	2,500
3.25	2,375
3.50	2,250

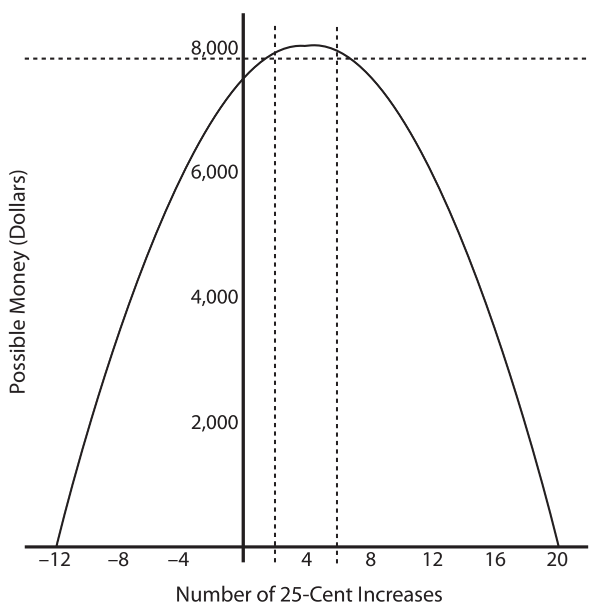
   Students then can use that information to determine the amount to be collected, noting that the price must increase for the senior class to meet the $7,800 goal. But they should also note that eventually the drop-off in the number of ticket sales due to the price increase makes it impossible to collect enough money.

Ticket Price	Predicted Ticket Sales	Possible Money Collected, *M* Dollars
3.00	2,500	7,500.00
3.25	2,375	7,718.75
3.50	2,250	7,875.00
3.75	2,125	7,968.75
4.00	2,000	8,000.00
4.25	1,875	7,968.75
4.50	1,750	7,875.00
4.75	1,625	7,718.75

By studying the pattern in the amounts collected, students can see that ticket prices between $3.50 and $4.50 will allow the senior class to meet its goal.

Graph:

Students can also construct a graph to see the range of price increases the class must consider to collect at least $7,800. Add the graph of $y = 7{,}800$ to the money-collected graph and determine the *x*-coordinates of the points of intersection of the two graphs:



(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(B) gather and record data and use data sets to determine functional relationships between quantities;

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

The number of 25¢ price increases must fall between 2 and 6. This means the class should keep the ticket price between 3.00 + 0.25(2) = \$3.50 and 3.00 + 0.25(6) = \$4.50 inclusive, and they need to sell between 2,500 – 125(2) = 2,250 and 2,500 – 125(6) = 1,750 tickets.

2. Write a function for the amount of money, *M* dollars, that would be collected in terms of *x*, the number of 25¢ price increases.

   Students can use a table to study the relationships in this scenario and determine a function rule.

Number of 25¢ Price Increases	Ticket Price	Process	Predicted Ticket Sales	Process	Possible Money Collected, *M* Dollars
0	3.00	2500 – 125(0)	2,500	(2500 – 125(0)) (3.00 + 0.25(0))	7,500.00
1	3.25	2500 – 125(1)	2,375	(2500 – 125(1)) (3.00 + 0.25(1))	7,718.75
2	3.50	2500 – 125(2)	2,250	(2500 – 125(2)) (3.00 + 0.25(2))	7,875.00
3	3.75	2500 – 125(3)	2,125	(2500 – 125(3)) (3.00 + 0.25(3))	7,968.75
4	4.00	2500 – 125(4)	2,000	(2500 – 125(4)) (3.00 + 0.25(4))	8,000.00
5	4.25	2500 – 125(5)	1,875	(2500 – 125(5)) (3.00 + 0.25(5))	7,968.75
6	4.50	2500 – 125(6)	1,750	(2500 – 125(6)) (3.00 + 0.25(6))	7,875.00
7	4.75	2500 – 125(7)	1,625	(2500 – 125(7)) (3.00 + 0.25(7))	7,718.75
x		2500 – 125*x*		(2500 – 125*x*) (3.00 + 0.25*x*)	*M* = (2500 –125(*x*)) (3.00 + 0.25(*x*))

Since *x* is the number of 25¢ price increases and each price increase results in 125 fewer tickets being sold, the number of tickets sold is given by the expression 2,500 – 125*x* and the price per ticket is given by the expression 3.00 + 0.25*x*.

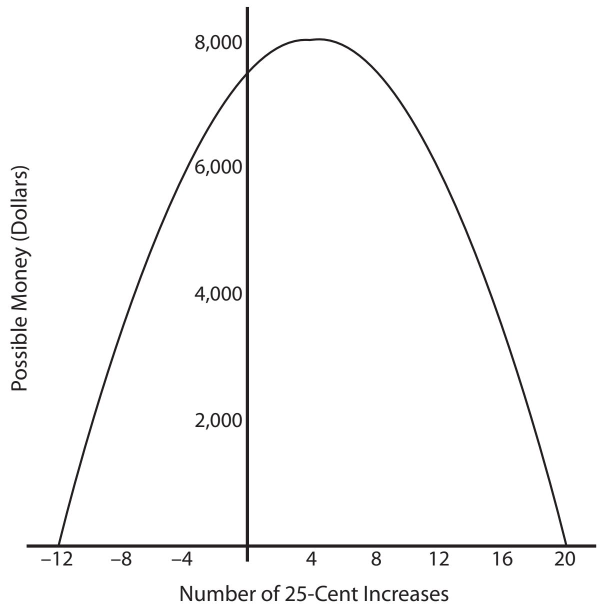
The amount of money collected equals the product of the price per ticket and the number of tickets sold, so

$M = (3.00 + 0.25x)(2{,}500 - 125x)$

$M = 7{,}500 + 250x - 31.25x^2$

3. Sketch a reasonable graph of the function and write a verbal description of what the graph tells you about the situation.

A reasonable graph is pictured below. The graph indicates that for the first few price increases, the money raised increases, but after the sixth price increase (ticket prices greater than $4.50), the number of students willing to pay those prices decreases enough to cause the total collected to drop below $7,800.



Students can also discuss domain, range, and intercepts. The domain for *M* is easily seen when *M* is in factored form and helps in determining *x*-intercepts. Clearly, $x \geq 0$, since *x* represents the number of price increases, which must be positive. Also, $x \leq 20$, because otherwise $2{,}500 - 125x$ represents a negative number of tickets.

To determine the range for *M*, first locate both intercepts, –12 and 20; then find the *x*-coordinate of the vertex by determining the midpoint of the intercepts, $x = 4$. Evaluate the function for $x = 4$.

$M = 7{,}500 + 250(4) - 31.25(4)^2$

$= 8{,}000$

The range values for the function must be less than or equal to 8,000.

For the problem situation, the range values must also be greater than or equal to 0.

The intercepts that make sense for this situation are (0, 7500) and (20, 0). The number of 25¢ price increases can range from none ($x = 0$) to 20 ($x = 20$).

The ticket prices are represented by $3.00 + 0.25x$. The ticket prices can range from $3.00 + 0.25(0) = \$3$ to $3.00 + 0.25(20) = \$8$. The maximum amount of money, \$8,000, is collected when $x = 4$. The graph shows that the amount of money collected grows if the number of 25¢ price increases is between 0 and 4. The senior class collects the most money when the ticket price is $3.00 + 0.25(4) = \$4$, and they sell 2,500 – 125(4) or 2,000 tickets.

Extension Questions

- The vice president of the senior class conducts another student survey. Based on his survey, he predicts that if the class charged \$2.50 per ticket, they could expect to sell 2,700 tickets, and each 25¢ increase in price per ticket would result in 75 fewer tickets being purchased. If he is correct, how does the money collected under this plan compare with the money collected under the original plan?

  In this situation, the number of tickets is changed to 2,700, and the price per ticket is changed from \$3.00 to \$2.50.

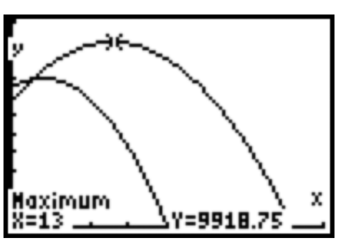
  The number of tickets sold is given by the expression 2700 – 75x, and the price per ticket is given by the expression 2.50 + 0.25x, where the first factor is the price per ticket per 25¢ increase and the second factor is the number of tickets sold per price increase.

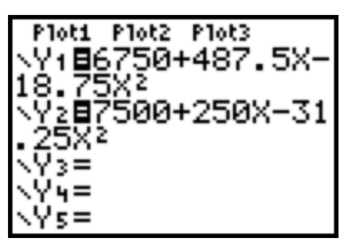
  The money-collected function for this second situation is

  $M_2 = (2.50 + 0.25x)\ (2{,}700 - 75x)$

  $\quad = 6{,}750 + 487.5x - 18.75x^2$

  The following graph compares the two situations:





  For the second situation, the maximum amount of money collected is when the number of 25¢ price increases is 13. At 13, the ticket price is 2.50 + 0.25(13) =

$5.75, so a price of $5.75 has the potential to raise the most money according to this model.

The vertex of the parabola for the second situation is higher and further to the right than that of the first situation, which shows that the second situation results in more tickets sold and more money made.

The y-intercept for the first graph is (0, 7500), while the y-intercept for the second graph is (0, 6750). This means that with no increase in ticket price, the money collected for the first situation is $7,500, and the money collected for the second situation is $6,750. By finding the intersection of the two graphs, we know that the first situation is more lucrative up to an increase of 50¢ per ticket. With an increase of 75¢ or more, the second situation makes more money for the class.

The wider spread of the graph of the second situation also shows that more students are willing to accept a greater number of 25¢ price increases.

Dog Run

Sam owns a kennel and needs to build a rectangular dog run for a new litter of puppies. He has 22 meters of chain-link fence to enclose all four sides of the run.

1. Construct a table of values (with at least five entries) relating the area, *A*, of the run in square meters to its length, *l*, in meters.
2. Write a function rule relating *A* and *l*, and construct a graph.
3. What side length maximizes the area of the pen? Should Sam build the pen to maximize the area? Why or why not?
4. How does your function change if the perimeter of the dog run is 24 meters? 26 meters? 28 meters?
5. Describe how you can find the maximum area of the pen and corresponding side length for any given perimeter.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratics equations, the zeros of their related functions, and the horizontal intercepts (*x*-intercepts) of the graph of the function.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(E) interpret and make decisions, predictions, and critical judgments from functional relationships.

Scaffolding Questions

- What lengths make sense in this situation?
- If the length of one side of the dog run were 1 meter, what would the width of the dog run be?
- What is the relationship between length, width, and amount of available fencing?
- For each possible length, what are the corresponding areas?
- What function type describes the area in terms of the dog run's length?
- What will the graph of your function look like?
- How do your table and/or graph help you find the maximum area and corresponding length for the dog run?

Sample Solutions

1. Construct a table of values (with at least five entries) relating the area, A, of the run in square meters to its length, l, in meters.

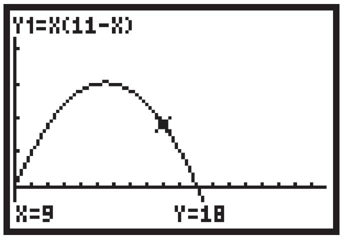
   Since the perimeter of the dog run is 22 meters, building a width and length of the dog run takes 11 meters. The width of the pen is 11 minus the length. If the length is represented by l, the width is represented by $11 - l$.

Length in Meters	Width in Meters	Area Process	Area in Square Meters
1	10	1(10)	10
3	8	3(8)	24
5	6	5(6)	30
7	4	7(4)	28
9	2	9(2)	18
l	$11 - l$	$l(11 - l)$	

2. Write a function rule relating *A* and *l*, and construct a graph.

   The function is $A = l\,(11 - l)$, where $0 < l < 11$.

   The following is a graph of the situation:



3. What side length maximizes the area of the pen? Should Sam build the pen to maximize the area? Why or why not?

   The maximum area occurs when the length equals the *x*-coordinate that is halfway between the *x*-intercepts, 0 and 11. Therefore, the length that gives the maximum area is $l = 5.5$ meters, and the maximum area is $A = 5.5(11 - 5.5)$, or 30.25 square meters.

   The shape of the dog run that maximizes the area is a square with sides of 5.5 meters. Usually, a dog run is longer than it is wide so that the dog has plenty of room to run. Sam may decide not to build a square run. He should research the best length-to-width ratio for a dog run and use that information to make his decision.

4. How does your function change if the perimeter of the dog run is 24 meters? 26 meters? 28 meters?

   The length and width of the dog run must add up to one-half of the total amount of fencing. The table below gives the area, *A*, as a function of length, *l*, for varying perimeters:

Perimeter	Area as a Function of Length
22	$A = l\,(11 - l)$
24	$A = l\,(12 - l)$
26	$A = l\,(13 - l)$
28	$A = l\,(14 - l)$

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

In general, for any given perimeter, *P*, the area as a function of length is

$$A = l\left(\frac{P}{2} - l\right)$$

5. Describe how you can find the maximum area of the pen and corresponding side length for any given perimeter.

To find the maximum area for any given perimeter, you can trace along the graph or look at the table. The maximum area occurs when the pen's length equals its width. A square dog run creates the maximum area. The length and the width are each one-fourth of the total perimeter.

$$w = l = \frac{P}{4}$$

$$A = w \bullet l = \frac{P}{4} \bullet \frac{P}{4} = \frac{P^2}{16}$$

Extension Questions

- Suppose Sam investigates and determines that the best dog run has a length-to-width ratio of 2:1. If Sam still has 22 meters of fencing, what is the area of this dog run?

  If the length-to-width ratio is 2:1, then

  $l = 2w$ *and*

  $l + w = 11$

  $2w + w = 11$

  $3w = 11$

  $$w = \frac{11}{3} = 3\frac{2}{3}$$

  $$l = 2\left(3\frac{2}{3}\right) = 7\frac{1}{3}$$

  The dimensions of the dog run are $3\frac{2}{3}$ *meters and* $7\frac{1}{3}$ *meters. The area is* $26\frac{8}{9}$ *square meters.*

- If a given pen's perimeter is doubled, how does the new pen's maximum area relate to the original pen's maximum area ?

  Let P represent the original perimeter. The area for the original perimeter is given by

  the rule $A = l\left(\frac{P}{2} - l\right)$.

When the perimeter is doubled, the area becomes $A = l\left(\frac{2P}{2} - l\right) = l(P - l)$.

For the original perimeter, the maximum area occurs when $l = \frac{P}{4}$. *The area is* $\frac{P}{4} \bullet \frac{P}{4}$, *or* $\frac{P^2}{16}$. *For the new perimeter, the maximum area occurs at* $\frac{P}{2}$, *which is twice the value for the original perimeter. The maximum area is* $\frac{P}{2} \bullet \frac{P}{2}$ *or* $\frac{P^2}{4}$, *which is four times the maximum area for the original perimeter.*

Transformations of Quadratic Functions

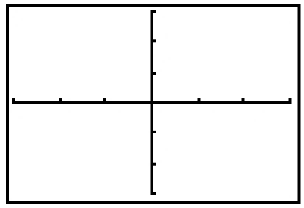
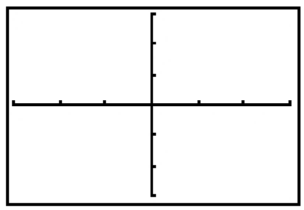
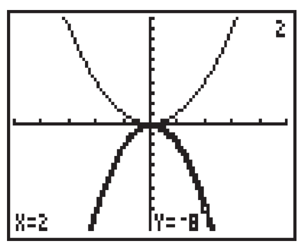
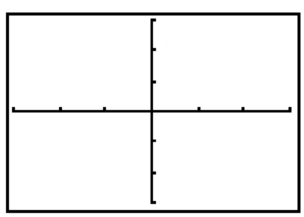
In this activity, your task is to investigate, describe, and predict the effects of changes in the parameters a and c on the graph of $y = ax^2 + c$ in comparison to the graph of the parent function $y = x^2$.

1. For each problem set, one representation of the parameter change is given, and you are to complete the others.

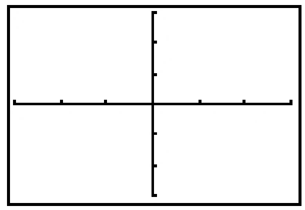
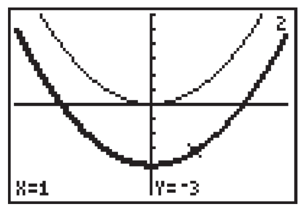
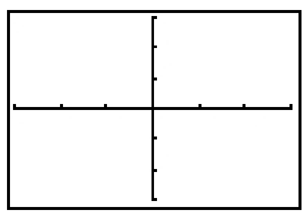
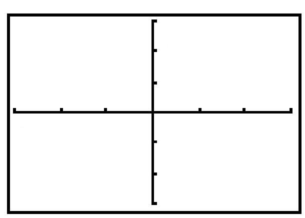
   - In the Function column, write the function rule in $y = ax^2 + c$ form.
   - In the Verbal Description column, describe the transformation in words and include the images of (0, 0) and (1, 1) under the transformation. (See A1 for an example.)
   - Graph the parent function and the transformed function on the same grid. Draw the parent function as a thin line and the transformed function as a thick line The graph should show the vertex and x- and y-intercepts.
   - The table should include the vertex and images of (±1, 1) and (±2, 4). (See A4 for an example.)

2. Write a summary of the effects of the changes in the parameters a and c on the graph of $y = ax^2 + c$. Include a description of the effects of the transformations on the vertex, axis of symmetry, and intercepts.

Problem Set A

	Function	Verbal Description of Transformation(s)	Graph	Table
A1.	$y = 3x^2$	Vertically stretch $y = x^2$ by a factor of 3. $(0, 0) \rightarrow (0, 0)$ $(1, 1) \rightarrow (1, 3)$		$y = x^2$: x: −2, −1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x | y (blank)
A2.		Vertically compress $y = x^2$ by a factor of $\frac{1}{2}$. $(0, 0) \rightarrow (0, 0)$ $(1, 1) \rightarrow (1, 0.5)$		$y = x^2$: x: −2, −1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x: −2, −1; y: 2, 0.5
A3.				$y = x^2$: x: −2, −1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x | y (blank)
A4.				$y = x^2$: x: −2, −1, 0, 1, 2; y: 4, 1, 0, 1, 4 $y =$: x: −2, −1, 0, 1, 2; y: 16, 4, 0, 4, 16

Problem Set B

	Function	Verbal Description of Transformation(s)	Graph	Table
B1.	$y = x^2 + 1$			$y = x^2$: x, y: −2, 4; −1, 1; 0, 0; 1, 1; 2, 4 $y =$: x, y:
B2.				$y = x^2$: x, y: −2, 4; −1, 1; 0, 0; 1, 1; 2, 4 $y =$: x, y:
B3.		Vertically stretch $y = x^2$ by a factor of 2 and translate down 2 units. $(0, 0) \rightarrow (0, 2)$ $(1, 1) \rightarrow (1, 0)$		$y = x^2$: x, y: −2, 4; −1, 1; 0, 0; 1, 1; 2, 4 $y =$: x, y:
B4.				$y = x^2$: x, y: −2, 4; −1, 1; 0, 0; 1, 1; 2, 4 $y =$: x, y: −2, 1; −1, 4; 0, 5; 1, 4; 2, 1

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(B) investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$;

(C) investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$; and

(D) analyze graphs of quadratic functions and draw conclusions.

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

Scaffolding Questions

- What points help you graph the parent function? What are its vertex, axis of symmetry, and intercepts?
- What does the graph show you about the effect of *a* on the shape and orientation of the graph of $y = ax^2 + c$?
- Describe how the value of *y* varies in your tables as the value of *a* varies.
- What does the graph show you about the effect of *c* on the position of the graph of $y = ax^2 + c$?
- Describe how the value of *c* affects the value of *y* in your tables.
- What is the order of operations in the expression $ax^2 + c$?

Texas Assessment of Knowledge and Skills:

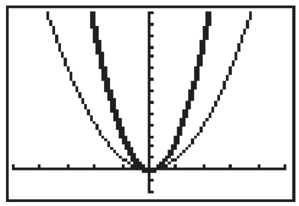
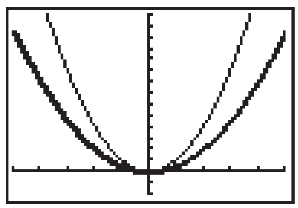
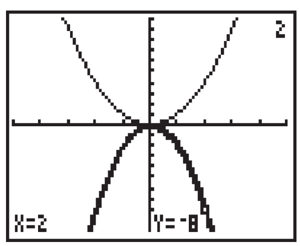
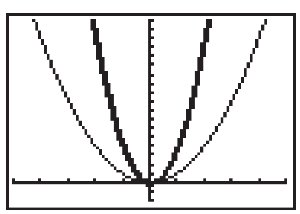
Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

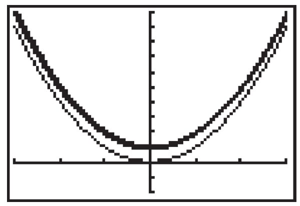
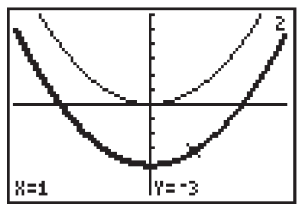
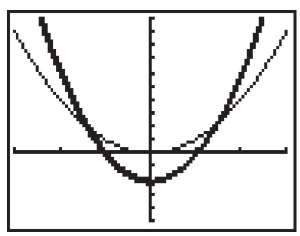
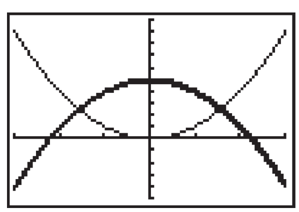
Sample solutions are on the following page.

Sample Solutions

Problem Set A

	Function	Verbal Description of Transformation(s)	Graph	Table
A1.	$y = 3x^2$	Vertically stretch $y = x^2$ by a factor of 3. $(0, 0) \to (0, 0)$ $(1, 1) \to (1, 3)$		x | y −2 | 12 −1 | 3 0 | 0 1 | 3 2 | 12
A2.	$y = \frac{1}{2}x^2$	Vertically compress $y = x^2$ by a factor of $\frac{1}{2}$. $(0, 0) \to (0, 0)$ $(1, 1) \to (1, 0.5)$		x | y −2 | 2 −1 | 0.5 0 | 0 1 | 0.5 2 | 2
A3.	$y = -2x^2$	Vertically stretch $y = x^2$ by a factor of 2 and reflect over the x-axis. $(0, 0) \to (0, 0)$ $(1, 1) \to (1, -2)$		x | y −2 | −8 −1 | −2 0 | 0 1 | −2 2 | −8
A4.	$y = 4x^2$	Vertically stretch $y = x^2$ by a factor of 4. $(0, 0) \to (0, 0)$ $(1, 1) \to (1, 4)$		x | y −2 | 16 −1 | 4 0 | 0 1 | 4 2 | 16

Problem Set B

	Function	Verbal Description of Transformation(s)	Graph	Table
B1.	$y = x^2 + 1$	Translate $y = x^2$ up 1 unit. $(0, 0) \rightarrow (0, 1)$ $(1, 1) \rightarrow (1, 2)$		x: −2, −1, 0, 1, 2 y: 5, 2, 1, 2, 5
B2.	$y = x^2 - 4$	Translate $y = x^2$ down 4 units. $(0, 0) \rightarrow (0, 4)$ $(1, 1) \rightarrow (1, 3)$		x: −2, −1, 0, 1, 2 y: 0, −3, −4, −3, 0
B3.	$y = 2x^2 - 2$	Vertically stretch $y = x^2$ by a factor or 2 and translate down 2 units. $(0, 0) \rightarrow (0, 2)$ $(1, 1) \rightarrow (1, 0)$		x: −2, −1, 0, 1, 2 y: 6, 0, −2, 0, 6
B4.	$y = 5 - x^2$	Reflect $y = x^2$ over x-axis and translate up 5 units. $(0, 0) \rightarrow (0, 5)$ $(1, 1) \rightarrow (1, 4)$		x: −2, −1, 0, 1, 2 y: 1, 4, 5, 4, 1

2. Write a summary of the effects of the changes in the parameters a and c on the graph of $y = ax^2 + c$. Include a description of the effects of the transformations on the vertex, axis of symmetry, and intercepts.

   The effect of a on the graph of $y = ax^2 + c$ depends on the signed value of a and the magnitude of a. If $a > 0$, the graph of the transformed function still opens up. If $a < 0$, the graph of $y = x^2$ is reflected over the x-axis, and the transformed function opens downward. If the magnitude of a is greater than 1, the graph of $y = x^2$ is vertically stretched by a factor of the magnitude of *a*. If the magnitude of *a* is less than 1, the graph of $y = x^2$ is vertically compressed by a factor of the magnitude of *a*; in particular, $(1, 1) \to (1, |a|)$. The vertex and the axis of symmetry are preserved under this transformation.

The effect of *c* on the graph of $y = x^2 + c$ is to translate the graph of $y = x^2$ vertically *c* units. If $c > 0$, the graph of $y = x^2$ is translated up *c* units. If $c < 0$, the graph of $y = x^2$ is translated down $|c|$ units. The *x*-coordinate of the vertex is still 0, and the axis of symmetry is still $x = 0$. But the vertex has been translated to the point (0, *c*). This is the new *y*-intercept. The new *x*-intercepts are $(\pm\sqrt{c}, 0)$, since to get the *x*-intercepts we solve $x^2 + c = 0$.

Extension Questions

- What kinds of transformations on the graph of $y = x^2$ can be performed so that the resulting graph continues to be that of a function?

  Since a function is a relation between x and y that generates exactly one output value, y, for each input value, x, the only transformations on $y = x^2$ we can consider are dilations, translations, and reflections over the axes.

- Which parameter causes a dilation on the graph of $y = x^2$? What is another way of saying *dilation*? What else does this parameter tell you?

  The parameter, a, in $y = ax^2$ causes a dilation. If $|a| > 1$, the dilation is a vertical stretch. A "stretch" is another way of saying "dilation." If $|a| < 1$, then there is a vertical compression on the graph of $y = x^2$. The parameter, a, also causes a reflection over the x-axis if a is negative.

- Which parameter causes a translation and what kind of translation?

  The parameter, c, in $y = x^2 + c$ causes a vertical translation. The parent graph is translated c units up if c is positive and $|c|$ units down if c is negative.

- What does the order of operations sequence tell you about the sequence of transformations performed on the parent graph to generate the graph of $y = ax^2 + c$?

  If $a > 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Then translate vertically $|c|$ units. If $a < 0$, vertically stretch or compress the graph of $y = x^2$ by a factor that equals the magnitude of a. Next, reflect the graph over the x-axis, and then translate vertically $|c|$ units.

- If $a > 0$, what is the difference between the transformations $y = ax^2$ and $y = (ax)^2$?

  The first transformation is a vertical stretch or compression of the graph of $y = x^2$. The second transformation is a horizontal stretch or compression on x before squaring. This transformation can also be described as a vertical stretch or compression, but the dilation factor is a^2, since $(ax)^2 = a^2x^2$.

- What is the difference between the transformations $y = x^2 + c$ and $y = (x + c)^2$?

  The first transformation is a vertical translation c units up if c is positive and –c units down if c is negative. The second transformation is a horizontal translation c units left if c is positive and –c units right if c is negative.

What Is the Best Price?

Laura makes earrings to sell at craft fairs. Given all of her expenses, she decided that the cheapest price she can sell a pair of earrings for is $15. She experimented with different selling prices at several fairs and recorded the data in a table.

Selling Price ($)	Number Sold
15	118
16	115
17.50	110
19	102
20	99
21.50	93
22	91
24	79
25	75
27.50	62
28.50	56
30	51
35	27

Laura thinks that the number of pairs of earrings sold depends on the selling price. The revenue—the amount of money she receives from the sales—depends on the selling price of the earrings and the number of pairs sold.

1. Use a graphing calculator to create a scatterplot of the data. Determine a model for the number of pairs of earrings sold as a function of the selling price.
2. If Laura sets the selling price at $32, how many pairs of earrings could she expect to sell?
3. Revenue is the amount of money received from sales. For example, if you sell 118 items for $15 each, the revenue is $1,770. Make a table comparing the selling price and the revenue. Create a scatterplot of the points (revenue, selling price).
4. Use the function rule you found for the number of items sold to find a function for the revenue in terms of the selling price.
5. Evaluate the revenue function at a selling price of $32.
6. Explain what you think the selling price should be to generate the greatest revenue. Justify your reasoning using algebraic representations, tables, or graphs.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(A) identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) parent functions;

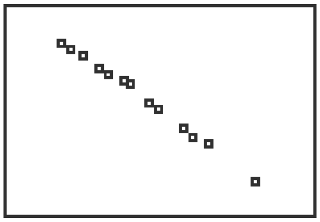
Scaffolding Questions

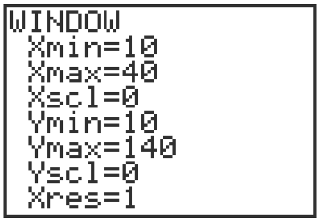
- What are the variables in this situation? Which one is the dependent variable?
- Examine the table and determine the rate of change. What is the average rate of change in the scatterplot?
- How can you determine the y-intercept for the linear model?
- What is the function rule that shows the relationship between the number of pairs of earrings sold and the selling price?
- What do you need to know to determine the revenue?
- What is the function rule for the revenue?

Sample Solutions

1. Use a graphing calculator to create a scatterplot of the data. Determine a model for the number of pairs of earrings sold as a function of the selling price.

   The points determined by the table were plotted with the number sold depending on the selling price. The graph appears to be linear.





   The finite differences can be computed to determine the rate of change.

Difference	Selling Price ($)	Number Sold	Difference
	15	118	
1			-3
	16	115	
1.5			-5
	17.50	110	
1.5			-8
	19	102	
1			-3
	20	99	
1.5			-6
	21.50	93	
0.5			-2
	22	91	
2			-12
	24	79	
1			-4
	25	75	
2.5			-13
	27.50	62	
1			-6
	28.50	56	
1.5			-5
	30	51	
5			-24
	35	27	

The rate of change can be found by finding the ratios of the differences.

$\frac{-3}{1} = -3$ $\quad \frac{-5}{1.5} = -3.33$ $\quad \frac{-8}{1.5} = -5.33$

$\frac{-3}{1} = -3$ $\quad \frac{-6}{1.5} = -4$ $\quad \frac{-2}{0.5} = -4$

$\frac{-12}{2} = -6$ $\quad \frac{-4}{1} = -4$ $\quad \frac{-13}{2.5} = -5.2$

$\frac{-6}{1} = -6$ $\quad \frac{-5}{1.5} = -3.33$ $\quad \frac{-24}{5} = -4.8$

The ratios are –3, –3.33, –5.33, –3, –4, –4, –6 , –4, –5.2, –6, –3.33, and –4.8.

Summing these ratios and then dividing the total by 12 gives the average of this set of numbers: –4.33. This number can be used as the rate of change of the linear function that models the set of data.

The function rule is of the form $y = -4.33x + b$.

Use one of the given points, (20, 99), and solve for b.

$99 = -4.33(20) + b$

$b = 185.6$

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(D) analyze graphs of quadratic functions and draw conclusions.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

The number sold, n, as a function of the selling price, p, is

$n = -4.33p + 185.6$

Note that using a different data point would give a different y-intercept and thus a different rule. This is an approximate value. Students may find other approximations.

It is also possible to use the regression line from a graphing calculator.

2. If Laura sets the selling price at $32, how many pairs of earrings could she expect to sell?

   If Laura sets a selling price of $32, the function must be evaluated for $p = 32$.

   Using the function rule that was determined by averaging of ratios, you get:

   $n = -4.33(32) + 185.6$

   $n = 47.04$

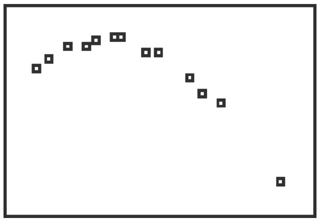
   She could expect to sell about 47 items.

3. Revenue is the amount of money received from sales. For example, if you sell 118 items for $15 each, the revenue is $1,770. Make a table comparing the selling price and the revenue. Create a scatterplot of the points (revenue, selling price).

   To find the revenue, multiply the selling price by the number of pairs of earrings sold.

Selling Price ($)	Number Sold	Revenue ($)
15	118	1,770
16	115	1,840
17.50	110	1,925
19	102	1,938
20	99	1,980
21.50	93	1,999.5
22	91	2,002
24	79	1,896
25	75	1,875
27.50	62	1,705
28.50	56	1,596
30	51	1,530
35	27	945

The scatterplot of selling price and revenue shows what appears to be a quadratic relationship.



4. Use the function rule you found for the number of items sold to find a function for the revenue in terms of the selling price.

   To develop the symbolic representation, remember that revenue equals the product of the number sold and the selling price.

   $R = np$

   And using the possible student answer to question 1, $n = -4.33\text{p} + 185.6$.

   $R = (-4.33p + 185.6)p$

   $R = -4.33p^2 + 185.6p$

5. Evaluate the revenue function at a selling price of $32.

   The value of the function at $32 is

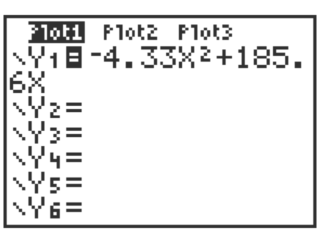
   $R = -4.33p^2 + 185.6p$

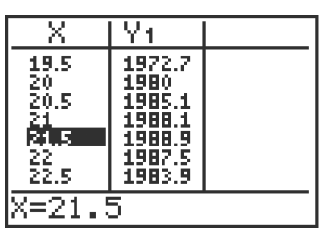
   $R = -4.33(32)^2 + 185.6(32)$

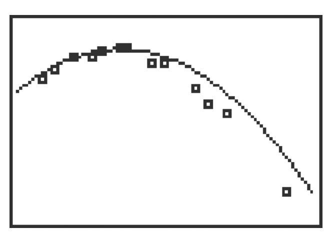
   So the revenue for a selling price of $32 is $1,505.28.

6. Explain what you think the selling price should be to generate the greatest revenue. Justify your reasoning using algebraic representations, tables, or graphs.

   A graphing calculator can be used to determine the selling price that gives the highest revenue.







   The table gives a maximum revenue of $1,988.90 at the selling price of $21.50.

Extension Questions

- Describe the domain of the linear function used to model the situation. Compare the domain of the function to the domain for the problem situation.

  The domain of this linear function is all real numbers, but the domain of this problem situation requires that p be a selling price in dollars and cents. Thus, it must be a positive rational number with at most two decimal places. Further restrictions given in the problem require that p be greater than or equal to 15. The x-intercept is between 42 and 43. For any integer greater than 42, the value of n will be negative. Since the number sold cannot be negative, $15 \leq x \leq 42$.

- What is the meaning of the slope in the linear equation?

  As the selling price increases by $1.00, the number sold decreases by an average of 4.33 earring pairs.

- How does the domain of the revenue function compare to the domain of the linear model?

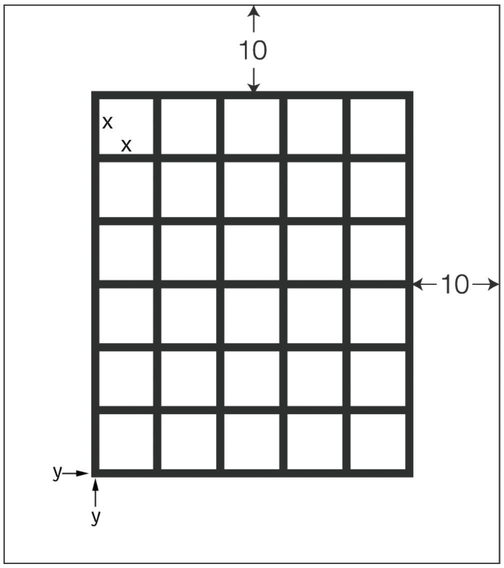
  There are no restrictions for the function rules on x, but for the problem situation, the same restrictions apply to x: $15 \leq x \leq 42$.

- What other conditions can be considered in this situation?

  The cost of producing the goods affects how many pairs of earrings Laura makes. The amount of time required to make the earrings also affects her production.

Window Panes

The window shown below is made up of square panes of glass, the wooden strips that surround each pane, and the border that frames all the panes. The individual panes have dimensions of *x* inches by *x* inches. The wooden strips surrounding each pane are *y* inches thick. The width of the border that frames all the panes is 10 inches.



1. Write expressions in terms of *x* and *y* for the dimensions of the entire window (including the border).

2. Write an expression in terms of *x* and *y* for the perimeter of the entire window. Simplify the expression.

3. Write an expression in terms of *x* and *y* for the area of the entire window. Simplify the expression.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

The student is expected to:

(A) solve quadratic equations using concrete models, tables, graphs, and algebraic methods; and

Additional TEKS Focus:

(A.10) Quadratic and other nonlinear functions. The student understands there is more than one way to solve a quadratic equation and solves them using appropriate methods.

(B) make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Scaffolding Questions

- Describe the width of the entire window.
- Describe the length of the entire window.
- Describe the shape of the window.
- How can you determine the perimeter of the entire window?
- How can you determine the area of the entire window?

Sample Solutions

1. Write expressions in terms of x and y for the dimensions of the entire window (including the border).

   The window's horizontal side has 5 panes plus 6 wooden strips and 2 borders.

   The width is represented by $5x + 6y + 20$.

   The height of the window is 6 panes plus 7 wooden strips and 2 borders. The height is $6x + 7y + 20$.

   The dimensions of the window are $(5x + 6y + 20)$ by $(6x + 7y + 20)$.

2. Write an expression in terms of x and y for the perimeter of the entire window. Simplify the expression.

   The perimeter is found by using the formula $P =$ 2(length) + 2(width).

   $P = 2(5x + 6y + 20) + 2(6x + 7y + 20)$

   $P = 10x + 12y + 40 + 12x + 14y + 40$

   $P = 22x + 26y + 80$

3. Write an expression in terms of x and y for the area of the entire window. Simplify the expression.

   The area of the window is found by multiplying its length by its width.

   $A = (5x + 6y + 20)(6x + 7y + 20)$

   $A = 30x^2 + 35xy + 100x + 36xy + 42y^2 + 120y + 120x + 140y + 400$

   $A = 30x^2 + 71xy + 220x + 42y^2 + 260y + 400$

Extension Question

- Suppose you want the window to be 98 inches wide by 113 inches long and have the same number of panes in the same formation as the window pictured previously. Write and solve a system of equations to find the widths of the panes (x) and the widths of the wooden strips surrounding the panes (y).

  The expression for the width is 5x + 6y + 20. The width must equal 98 inches.

  $$5x + 6y + 20 = 98$$

  The expression for the length is 6x +7y + 20. The length must equal 113 inches.

  $$6x + 7y + 20 = 113$$

  Subtract 20 from each side of each equation.

  $$5x + 6y = 78$$

  $$6x + 7y = 93$$

  Multiply the first equation by 6 and the second by –5.

$6(5x + 6y = 78)$	$30x + 36y$	$=$	468
$-5(6x + 7y = 93)$	$-30x - 35y$	$=$	465
	y	$=$	3

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Substitute the y value back into one of the original equations to find x:

$$
\begin{aligned}
5x + 6y &= 78 \\
5x + 6(3) &= 78 \\
5x + 18 &= 78 \\
5x &= 60 \\
x &= 12
\end{aligned}
$$

The width each pane (x) in 12 inches, and the width of each wooden strip (y) is 3 inches.

Chapter 5:
Inverse Variations, Exponential Functions, and Other Functions

College Tuition

In 1980, the average annual cost for tuition and fees at two-year colleges was $350. Since then, the cost of tuition has increased an average of 9% annually.

1. Make a table and develop a function rule that models the annual growth in tuition costs since 1980. Identify the variables, and describe the dependency relationship.
2. Determine the average annual cost of tuition for 2001. Justify your answer using tables and graphs.
3. Predict the cost of tuition for the year you will graduate from high school.
4. When did the average cost double the 1980 cost?
5. When did the average cost reach $1,000?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

Scaffolding Questions

- What are the variables in this situation?
- How would you represent the annual growth factor as a decimal?
- What is the starting value and what does it represent?
- What patterns do you notice in the table?
- How do tuition amounts change with each additional year?

Sample Solutions

1. Make a table and develop a function rule that models the annual growth in tuition costs since 1980. Identify the variables, and describe the dependency relationship.

   The cost of tuition depends on the number of years since 1980. Each year, the tuition is the previous year's tuition plus 9% of that amount. This can be thought of as 100% + 9%. This is equivalent to multiplying by 1 + 0.09, or 1.09. To find the cost after 1 year, multiply 1.09 by the starting amount ($350). To find the cost after 2 years, multiply 1.09 twice and then multiply that result by the starting amount ($350).

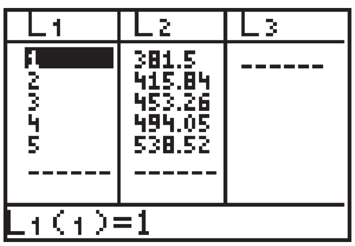
Number of Years Since 1980	Pattern	Tuition Cost
0	350	350
1	$350 \bullet 1.09^1$	381.50
2	$350 \bullet 1.09^2$	415.84
3	$350 \bullet 1.09^3$	453.26
4	$350 \bullet 1.09^4$	494.05
5	$350 \bullet 1.09^5$	538.52
n	$350 \bullet 1.09^n$	$350 \bullet 1.09^n$

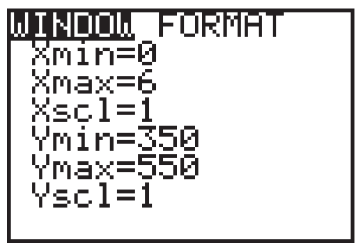
   The function created by this situation is $T = 350 \bullet 1.09^n$, where n represents the number of years since 1980 and T represents the cost of tuition. The minimum for the range is $350.

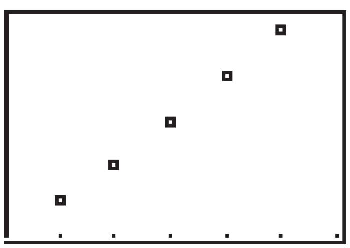
There is no constant rate of change in the table. Therefore, the relationship is nonlinear.

2. Determine the average annual cost of tuition for 2001. Justify your answer using tables and graphs.

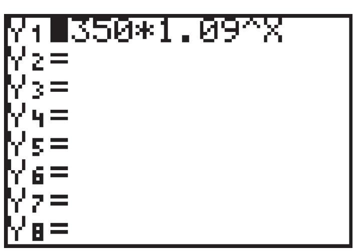
The situation can be represented graphically and tabularly using a graphing calculator. The data can be plotted using the statistic feature on the calculator. Let *y* represent tuition cost and *x* represent the number of years.







The graph is an exponential curve showing growth. The starting amount is $350, and the growth factor is 1.09. The growth factor shows 100% of the initial cost plus 9% of the cost.



Use the table feature on the calculator to explore cost increases since 1980.

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

X	Y_1
18	1651
19	1799.6
20	1961.5
21	2138.1
22	2330.5
23	2540.3
24	2768.9

X=21

The function can also be used to find the cost in the year 2001. The value of n is 2001 – 1980, which is 21. The cost is $\$350 \bullet 1.09^{21} = \$2,138.08$.

3. Predict the cost of tuition for the year you will graduate from high school.

   Answers will vary.

4. When did the average cost double the 1980 cost?

   To determine when the average cost was double the average cost in 1980, find the table value that is at least 2 times 350.

X	Y_1
6	586.99
7	639.81
8	697.4
9	760.16
10	828.58
11	903.15
12	984.43

X=8

   The cost doubled between 1988 and 1989, or between 8 and 9 years after 1980.

5. When did the average cost reach \$1,000?

   Look on the table for y values that are at least 1,000.

X	Y_1
9	760.16
10	828.58
11	903.15
12	984.43
13	1073
14	1169.6
15	1274.9

X=13

   The cost of tuition reached \$1,000 between 12 and 13 years after 1980, or between 1992 and 1993.

Extension Questions

- Describe the domain and range for this function.

  The domain is all whole numbers greater than 1. The domain represents the number of years since 1980. The years can only be represented in whole numbers because the increase is calculated on a yearly basis. The range represents the cost of the tuition. For this problem, the year 1980 is a starting point, and the tuition that year was $350. The cost will continue to increase at a rate of 9% per year for as long as the school is operating. The range for this problem is $y \geq 350$.

- How does the graph change if the annual cost increase is 12%? How does this affect costs?

  The graph would be "skinnier," indicating a steeper rise per year. The cost of tuition would increase at a faster rate.

- Predict the annual cost of tuition in the year 2020 at the 9% growth rate.

  $y = 350 \bullet 1.09^{40}$ *equals approximately $10,993.30*.

Exploring Exponential Functions

A rectangular sheet of notebook paper is folded in half. The fold divides the paper into 2 rectangles. The folded paper is then folded in half again. When it is opened, there are 4 rectangles formed by the folds.

1. Take a sheet of notebook paper and repeat this process. Record the results in a table similar to the one shown below. Continue folding until you cannot make another fold. Is this a linear situation? Why or why not?

Number of Folds	Number of Rectangles	Process
0	1	
1	2	
2	4	
3		
4		
5		

2. Identify the variables for this relationship and describe the domain and range for this situation. Describe how the values of y change as the values of x increase.

3. Express symbolically the relationship between the variables.

4. If your paper had 128 rectangles, how many folds would you have made? Explain your answer.

5. Describe the graph of your data.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Scaffolding Questions

- What is the relationship between the original number of rectangles and the number of rectangles after 1 fold?
- What is the relationship between the number of rectangles after 1 fold and the number of rectangles after 3 folds?
- What do you think the relationship will be between the number of rectangles after 1 fold and the number of rectangles after 5 folds?
- What pattern do you notice in the number of rectangles as the number of folds increases?
- How do exponents relate to the values in your table?

Sample Solutions

1. Take a sheet of notebook paper and repeat this process. Record the results in a table similar to the one shown below. Continue folding until you cannot make another fold. Is this a linear situation? Why or why not?

   There is no constant change in the number of rectangles. Therefore, the situation is not linear. The values for the rectangles are all multiples of 2. The number of factors is the same as the number of folds, and a pattern with exponents emerges. Successive ratios are constant.

Folds	Rectangles	Process	Ratio
0	1		
1	2	$= 2 = 2^1$	$2/1 = 2$
2	4	$= 2*2 = 2^2$	$4/2 = 2$
3	8	$= 2*2*2 = 2^3$	$8/4 = 2$
4	16	$= 2*2*2*2 = 2^4$	$16/8 = 2$
5	32	$= 2*2*2*2*2 = 2^5$	$32/16 = 2$

2. Identify the variables for this relationship and describe the domain and range for this situation. Describe how the values of y change as the values of x increase.

   The x-values represent the number of folds, and the y-values represent the number of rectangles. The domain represents the number of folds, so $x = 0, 1, 2, 3, 4, 5 \ldots.$ There will be a limit to the number of folds. The range represents the number of rectangles formed by the folds. The range is $y = 1, 2, 4, 8, 16, 32 \ldots$

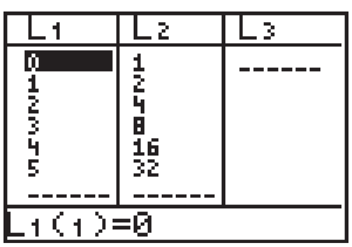
   As the value of x increases by 1, the value of y increases by a factor of 2.

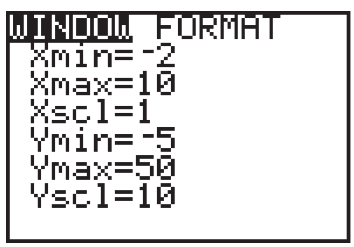
3. Express symbolically the relationship between the variables.

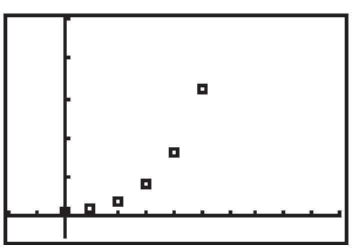
   The pattern involves repeated multiplication by 2. The pattern can be represented as powers of 2: $y = 2^x$.

4. If your paper had 128 rectangles, how many folds would you have made? Explain your answer.

   This question can be answered using a graphing calculator. Let the y-values represent the number of rectangles and the x-values represent the number of folds.







   The function rule and the table can help find the number of folds (x) when the number of rectangles (y) is 128. It takes 7 folds to get 128 rectangles.

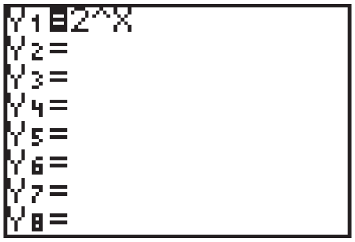
(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

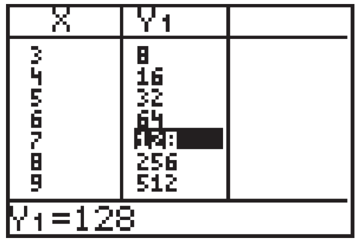
The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.





5. Describe the graph of your data.

   The graph is nonlinear because the rate of change is not constant. The rate of change of successive terms is a constant ratio of 2. The graph is in the first quadrant. The y-intercept is (0, 1). [In an exponential function, the y-intercept is always (0, 1) unless the function has been translated. $y = n^0$ always equals 1, because any value raised to the 0 power equals one.] The graph curves upward at a rapid rate. As the value of x increases, the value of y increases exponentially.

Extension Questions

- Describe how the situation would be different if the paper had been folded into thirds each time instead of halves.

  The multiplier would be 3. The equation would be $y = 3^x$.

- Suppose that you begin with a square sheet of paper that measures 2 feet on a side. What is the relationship between the number of folds and the area of the rectangle after each fold?

  The area of the sheet of paper is 4 square feet. Each time the paper is folded, the area is multiplied by $\frac{1}{2}$.

Fold Number	Area of Rectangle in Square Feet
0	4
1	$4(\frac{1}{2})$
2	$4(\frac{1}{2})(\frac{1}{2})$
3	$4(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$
x	$4(\frac{1}{2})^x$

The area, A, would be a function of the number of folds.

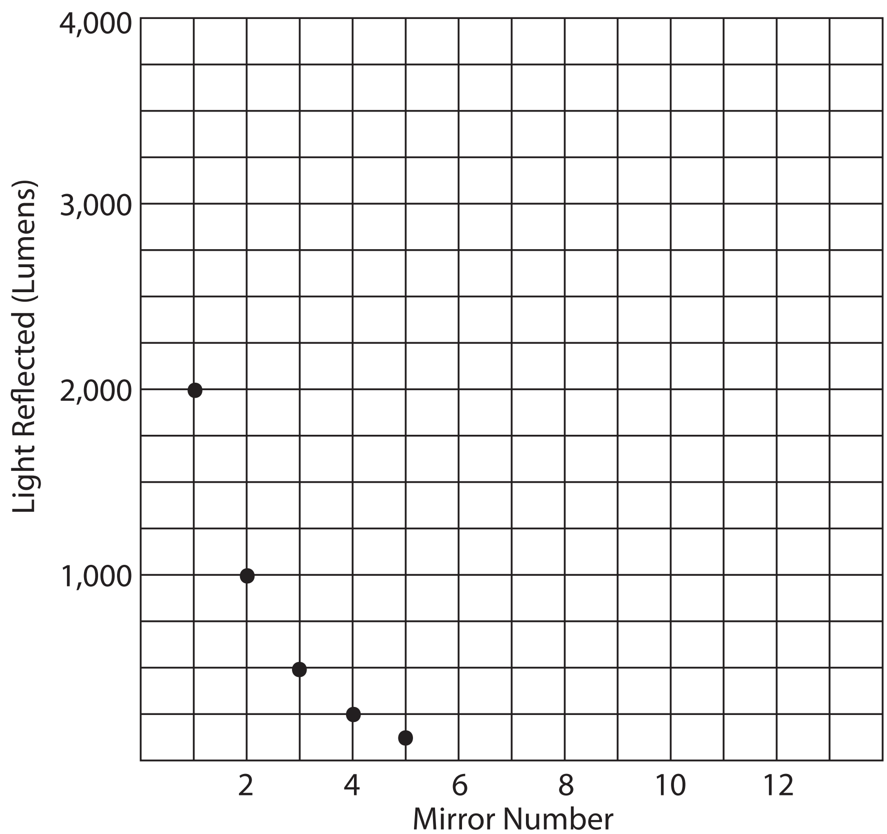
$A = 4(\frac{1}{2})^x$

- How is the graph of this function, $A = 4(\frac{1}{2})^x$, different from the graph of the original function, $A = 2^x$?

  The graph of the function $A = 4(\frac{1}{2})^x$ decreases as x gets larger. (This is generally referred to as a decay curve.) The starting value of the function is 4. The function $A = 2^x$ increases as x increases. (This is generally referred to as a growth curve.) It has a starting value of 1.

Bright Lights

The brightness of light is measured in lumens. A light of 4,000 lumens is shined on a mirror. Then the reflected light is shined on another mirror, and the reflected light from that mirror is shined on a third mirror, and so on. The resulting number of lumens of reflected light is recorded on this graph. The brightness of the light decreases in the same way after each reflection.



1. Describe the relationship between the number of mirrors and the lumens. Give your description in words and symbolically. Identify the variables.
2. If this type of reflection continues, what would be the lumens reflected from the 6th and 7th mirrors? Explain.
3. If this type of reflection continues, which mirror would you expect to have a measurement of 50 lumens? Explain.
4. Which mirror might have a measurement of 3.9 lumens? Explain.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Scaffolding Questions

- What kind of relationship is modeled in this function? Explain your reasoning.
- What is the relationship between the original measurement in lumens and the 1st measurement?
- What is the relationship between the 1st and 4th measurements?
- Where does the function begin on the *y*-axis?

Sample Solutions

1. Describe the relationship between the number of mirrors and the lumens. Give your description in words and symbolically. Identify the variables.

   The amount of light reflected each time is $\frac{1}{2}$ of the previous mirror's measurement in lumens.

   The 1st mirror's lumen measurement is $4{,}000(\frac{1}{2})$.

   The 2nd mirror's lumen measurement is $4{,}000(\frac{1}{2})(\frac{1}{2})$, or $4{,}000(\frac{1}{2})^2$.

   The 3rd mirror's lumen measurement is $4{,}000(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$, or $4{,}000(\frac{1}{2})^3$.

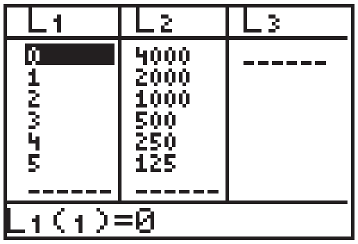
   The *n*th mirror's lumen measurement is $4{,}000(\frac{1}{2})^n$.

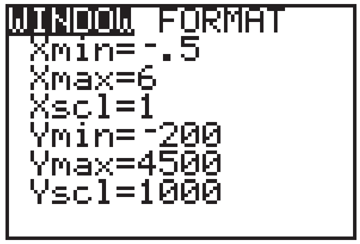
   The function that models this relationship is $L = 4{,}000(\frac{1}{2})^n$, where L is lumens, 4,000 is the initial amount of light in lumens, $(\frac{1}{2})$ is the factor by which the light decreases from mirror to mirror, and n is the mirror number.

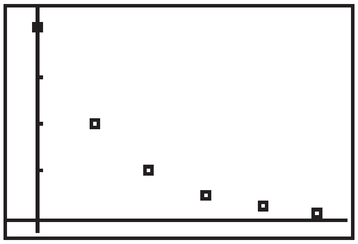
Mirror Number	0	1	2	3	4	5
Number of Lumens	4000	2000	1000	500	250	125

Successive ratios $\frac{2000}{4000} = \frac{1000}{2000} = \frac{500}{1000} = \frac{250}{500} = \frac{125}{250} = \frac{1}{2}$

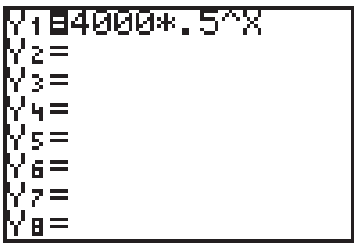
The values for the domain (the *x*-values) are based on the number of mirrors in the table. The *y*-values in the table represent the number of lumens. The ordered pairs plot a curve.

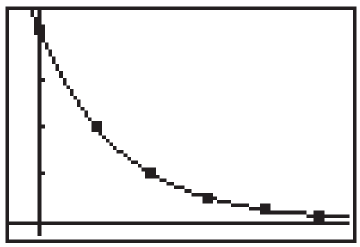






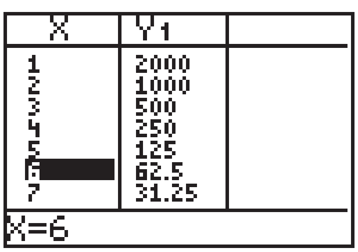
The function $y = 4,000(0.5)^x$ models the situation. The curve goes through all of the graphed points.

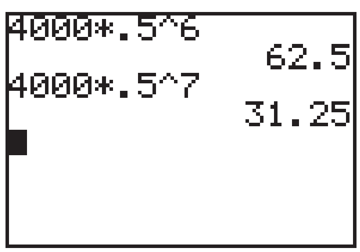




2. If this type of reflection continues, what would be the lumens reflected from the 6th and 7th mirrors? Explain.

   The table feature on the calculator verifies the lumens for the 6th and 7th mirrors. The lumen value can also be verified by substituting the values into the function.





   The 6th mirror reflects $4,000(\frac{1}{2})^6$, or 62.5 lumens.

   The 7th mirror reflects $4,000(\frac{1}{2})^7$, or 31.25 lumens.

3. If this type of reflection continues, which mirror would you expect to have a measurement of 50 lumens? Explain.

The table feature used in the previous answer also tells us if any mirror reflects exactly 50 lumens. No mirror reflects exactly 50 lumens.

Although the graph of the equation is a curve, not all of the points on the curve make sense for this situation. We can consider only whole numbers. Since 50 is between the values 31.25 and 62.5—or mirrors 6 and 7—there is no whole number that gives a function value of 50.

4. Which mirror might have a measurement of 3.9 lumens? Explain.

   The table can be used to determine that the 10th mirror reflects approximately 3.9 lumens.

X	Y_1	
6	62.5	
7	31.25	
8	15.625	
9	7.8125	
10	3.9063	
11	1.9531	
12	.97656	

X=10

Extension Questions

- Describe the domain in this situation.

  The domain as graphed is {0,1,2,3 . . ., n}, where n is the number of mirrors.

- Compare the domains of the problem situation and the function rule that models the situation.

  The domain of the function rule can be all real numbers. The domain of the problem situation is whole numbers.

- Describe the range in this situation.

  The range is $\{4{,}000,\ 2{,}000,\ .\ .\ .,\ 4{,}000 \bullet (\frac{1}{2})^n\}$

- Compare the ranges of the problem situation and the function rule that models the situation.

  The range of the function rule is $y > 0$. *The range of the problem situation is* $4{,}000 \geq y > 0$.

- Describe the rate of change.

  There is not a constant rate of change in this problem. The rates of change are:

  2,000 lumens per mirror after the 1st mirror

  1,000 lumens per mirror between the 2nd and 1st mirrors

  500 lumens per mirror between the 2nd and 3rd mirrors

250 lumens per mirror between the 3rd and 4th mirrors

125 lumens per mirror between the 4th and 5th mirrors

Because the rate of change is not constant, the equation is not linear. There is a constant ratio of successive terms: $\frac{1}{2}$ *.*

- If the initial measurement had been 3,500 lumens, how would the function have been written?

  $L = 3{,}500(\frac{1}{2})^n$

- If the rate of decay changes, how does that affect the function?

  The base of the exponential function changes by the rate of decay.

- If you continue the reflection process, when will the amount of light reflected be 0 or less than 0?

  Theoretically, the value would never reach 0 or below. It would continue to get closer and closer to 0.

Constructing Houses

A volunteer crew constructs houses for low-income families. It always takes 200 individual workdays to complete one house. For example, if it takes 1 person 200 days to build the house, then it has taken 1 times 200 workdays; if it takes 2 people 100 days to build the house, then it has taken 2 times 100 workdays; if a crew of 20 people can complete a house in 10 days, it has taken 20 times 10 workdays, and so on.

1. Use a table to determine how long it takes 40 people to complete the house. Justify your answer.
2. Express the number of workdays as a function of the crew size. Define the variables and explain how you created your function. What type of relationship is formed in the situation?
3. What are appropriate values for the domain and range of this problem situation?
4. Write a verbal description of the effect of the crew size on the number of construction days.
5. About how long would it take a crew of 32 to complete a house? How do you know?
6. If a crew completed a house in 12.5 days, how big was the crew? Explain.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Scaffolding Questions

- What relationship do you observe between the crew size and the construction days?
- What are the variables in this scenario?
- How do the values of *y* change as the values of *x* increase?
- How would you create a table to help you determine the function rule for this situation?

Sample Solutions

1. Use a table to determine how long it takes 40 people to complete the house. Justify your answer.

Crew Size (*x*)	Construction Days (*y*)	Individual Workdays (*t*)
2	100	200
4	50	200
8	25	200
10	20	200
20	10	200
40	5	200

The number of construction days, *y*, can be found by dividing the total number of days by the crew size, *x*.

2. Express the number of workdays as a function of the crew size. Define the variables and explain how you created your function. What type of relationship is formed in the situation?

The domain for this scenario is the possible crew size. Realistically, as the crew size increases, eventually there are too many people to have enough space to work effectively. For example, if 200 people were working on the house all at once, the function rule would tell us that the house could be completed in 1 day, but that is unlikely because 200 people all trying to work together in or around one house would too crowded.

A reasonable domain would be the natural numbers greater than or equal to 1 and less than or equal to 20. Students may suggest a higher or lower upper limit, and that would be fine as long as they recognize the limits of the scenario.

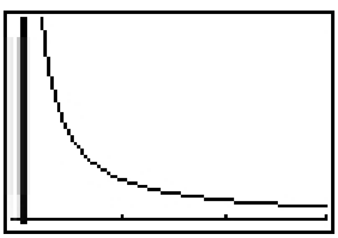
The range for this scenario is the number of individual workdays. Given the limitations discussed above, a reasonable range would be the set of real numbers greater than or equal to 10 and less than or equal to 200.

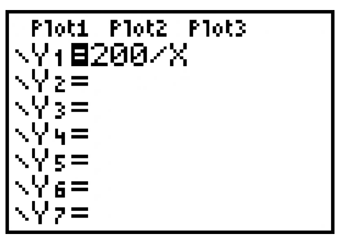
3. What are appropriate values for the domain and range of this problem situation?

   The domain of this function is the set of natural numbers from 1 to 200, and the range is the set of real numbers from 200 to 1.

4. Write a verbal description of the effect of the crew size on the number of construction days.

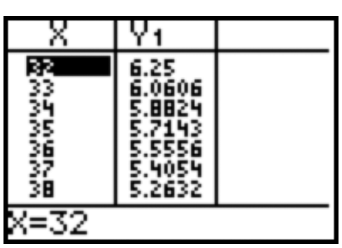
   As the crew size increases, the number of construction days decreases. The product of the quantities remains constant (200 total individual workdays) and forms an inverse variation. (This constant product is called the *constant of variation*.)





5. About how long would it take a crew of 32 to complete a house? How do you know?

   The table feature on the calculator allows exploration of the number of days it takes to build the house with various size crews.



X	Y1
32	6.25
33	6.0606
34	5.8824
35	5.7143
36	5.5556
37	5.4054
38	5.2632

X=32

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations; and

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

It takes 32 crew members about 6.25 days to build a house.

These values can also be verified on the home screen of the calculator by substituting into the function for each situation.

6. If a crew completed a house in 12.5 days, how big was the crew? Explain.

   If the number of days, *y*, is 12.5, the equation can be used to solve for *x*, the number of crew members.

   $x(12.5) = 200$

   $x = \frac{200}{12.5} = 16$

   It takes 16 crew members to complete the work in 12.5 days.

Extension Questions

- What are the similarities between an exponential function and an inverse variation?

  Both functions are similar in shape and do not ever touch the x-axis. There is an asymptote at the x-axis. These characteristics occur only if the functions have not been translated.

- What are the differences between an exponential function and an inverse variation?

  The exponential function must pass through the y-axis at the point (0, 1) unless the function has been translated. The inverse variation also has an asymptote at the y-axis, unless the function has been translated.

- How do you determine the domain and range for an inverse variation?

  The domain cannot be 0, because if one of the variables, x or y, is 0, the product, xy, is 0 and that results in an undefined fraction. The product of the independent and dependent variables must equal a given value k. Therefore, the domain and range are the same in an inverse variation, only the type of numbers might be different. For example, the domain can be the natural numbers from 1 to 10 and the range can be all real numbers from 1 to 10.

Music and Mathematics

Stringed instruments such as violins and guitars produce different pitches on a musical scale depending on the length of the string and the frequency at which the string vibrates. The product of the string length and the vibration frequency of the string is a constant value. When the length of the string is under equal tension, the frequency of its vibration varies inversely with its length.

1. Complete the table to find the string lengths for a C-major scale. Round your answers to the nearest whole number.

Pitch	C	D	E	F	G	A	B	C
Frequency (cycles/sec)	523	587	659	698	784	880	988	1,046
String Length (mm)	420	______	______	______	______	______	______	______

2. Find a function that models this variation.

3. Describe how the values of the frequency change in relation to the string length.

4. Make a scatterplot of your data. Describe the graph.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(D) collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

Scaffolding Questions

- What is true about the product of the frequency and string length for the first pitch, C?
- What does it mean to say that the frequency and string length vary inversely?
- What is true about the product of the frequency and string length for the second pitch, D?
- How can you determine the value of the string length for the second pitch, D?

Sample Solutions

1. Complete the table to find the string lengths for a C-major scale. Round your answers to the nearest whole number.

   The problem states that the frequency of a vibrating string varies inversely with the string's length. Two quantities vary inversely if their product remains constant. In this situation, the product of the frequency and the string length must remain constant. The product of the frequency for pitch C (523) and the string length for that pitch (420) is 219,660. This product can be used to work backward and obtain the string length for different vibration frequencies. Divide 219,660 by each given frequency to complete the table.

Pitch	C	D	E	F	G	A	B	C
Frequency (cycles/sec)	523	587	659	698	784	880	988	1,046
String Length (mm)	420	374	333	315	280	250	222	210

2. Find a function that models this variation.

   Because the situation is described as inverse variation, the product of the quantities remains constant at 219,660. This pattern was used to complete the table, and it is used to write the function.

   $xy = 219{,}660$

   $y = \dfrac{219{,}660}{x}$

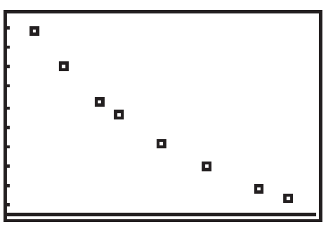
where x represents frequency and y represents string length.

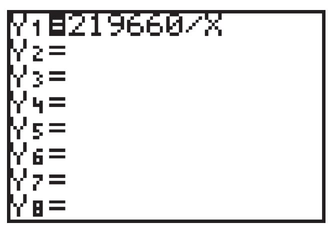
3. Describe how the values of the frequency change in relation to the string length.

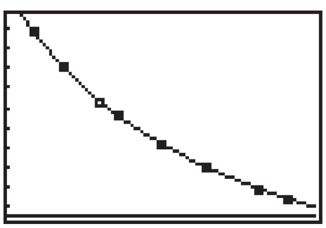
   As frequency increases, string length decreases.

4. Make a scatterplot of your data. Describe the graph.

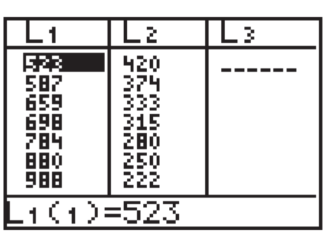
   Let the x-values represent frequency and the y-values represent string length. The graph does not touch the x- or y-axis, and it has a curve in it. Because the graph does not cross the y-axis, it is not an exponential decay curve. The scatterplot is not linear, because the slope or rate of change is not constant. (Use the trace function on the calculator to confirm that the inverse function does not ever cross the x- or y-axis.)

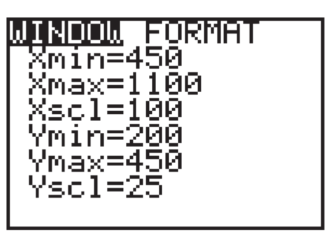






Students can use the statistic feature on the graphing calculator to create the scatterplot. They can see L3 as the product of L1 and L2 (shown below) and confirm that the products of each corresponding frequency and string length are always very near 219,660.





(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

L1	L2	L3 3
523	420	219660
587	374	219538
659	333	219447
698	315	219870
784	280	219520
880	250	220000
988	222	219336

L3(1)=219660

Extension Questions

- Describe the domain and range for the function rule.

  The domain cannot be 0, because if one of the variables, x or y, is 0, the product, xy, is 0 and that results in an undefined fraction. The product of the independent and dependent variables must equal a given value k. Therefore, the domain and range are the same in an inverse variation, only the type of numbers might be different. For example, the domain can be the natural numbers from 1 to 10 and the range can be all real numbers from 1 to 10.

- Describe the domain and range for the problem situation.

  The frequencies for the given pitches are constant as shown in the table in the answer to question 1. They can be extended in the positive direction. They cannot be negative numbers. String length can approach 0 but cannot equal 0.

- Compare the function $y = \frac{219{,}660}{x}$ with the function $y = 219{,}660x$.

  The function $y = \frac{219{,}660}{x}$ *represents an inverse variation. As the value of y increases, the value of x decreases. The domain and range are all numbers except 0. The graph of the function is not a line.*

  The function y = 219,660x represents a direct variation. As the value of y increases, the value of x increases. The domain and range of the function are all real numbers. The graph is a straight line.

- What happens if the string length changes?

  As the string length approaches 0, the frequency becomes greater. As the string length becomes greater, the frequency approaches 0.

The Marvel of Medicine

A doctor prescribes 400 milligrams of medicine to treat an infection. Each hour following the initial dose, 85% of the concentration remains in the body from the preceding hour.

1. Complete the table showing the amount of medicine remaining after each hour.

Number of Hours	Process	Number of Milligrams Remaining in the Body
0	400	400
1	400 (0.85)	340
2	400 (0.85)(0.85)	
3	400 (0.85)(0.85)(0.85)	
4		
5		
x		

2. Using symbols and words, describe the functional relationship in this situation. Discuss the domain and range of both the function rule and the problem situation.

3. Determine the amount of medicine left in the body after 10 hours. Justify your answer in two ways.

4. When does the amount of medicine still in the body reach 60 milligrams? Explain how you know.

5. Suppose that the level of medicine in the patient's body must maintain a level greater than 100 milligrams. How often does the patient need to take the medicine?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;

(C) analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Scaffolding Questions

- How much is the initial dosage?
- What percentage of the medicine is left in the body after 1 hour? Express this percentage as a decimal.
- How is the amount of medicine in the body at 2 hours related to the amount at 1 hour?
- What are the variables in this situation?
- How do the variables change in relation to each other?
- How would a graph help you understand the problem situation?

Sample Solutions

1. Complete the table showing the amount of medicine remaining after each hour.

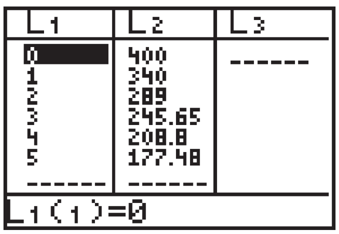
   Repeated multiplication by 0.85 was used to complete the table.

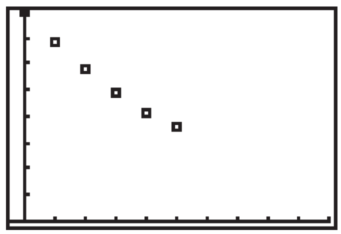
Number of Hours	Process	Number of Milligrams Remaining in the Body
0	400	400
1	400(0.85)	340
2	400(0.85)(0.85)	289
3	400(0.85)(0.85)(0.85)	245.65
4	400(0.85)(0.85)(0.85)(0.85) = $400(0.85)^4$	208.8
5	400(0.85)(0.85)(0.85)(0.85)(0.85) = $400(0.85)^5$	177.48
x	$400\ (0.85)^x$	

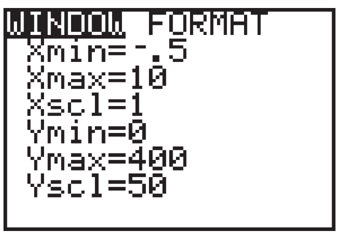
2. Using symbols and words, describe the functional relationship in this situation. Discuss the domain and range of both the function rule and the problem situation.

A function that models this situation is $y = 400(0.85)^x$. The x-values represent the number of hours the medicine is in the patient's system. The y-values represent the amount of medicine (in milligrams) that remains in the patient's system. Each hour, 15% of the medicine filters through the body, leaving 85% of the medicine in the body to fight the infection.

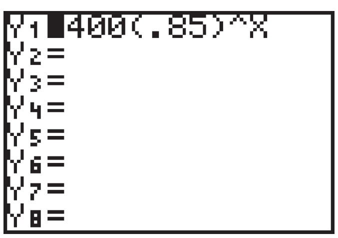
A scatterplot can be used to analyze the data.

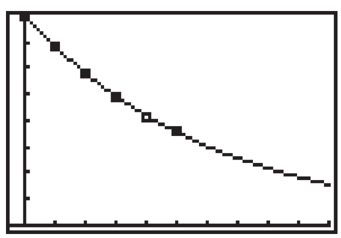






This model applies until another dose of medicine is administered.





The domain of the function is the set of all real numbers, but the domain of the problem situation is the set of nonnegative numbers because x represents time in this situation. The range of the function is the set of all real numbers greater than 0, but the range for the problem situation is the set of all numbers less than or equal to 400 but greater than 0.

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

3. Determine the amount of medicine left in the body after 10 hours. Justify your answer in two ways.

   After 10 hours, there will be 78.75 milligrams left in the body.

   The calculator can be used in two ways to explore the amount of medicine left in the patient's system after different numbers of hours. Students could input the function into *y* = and look at the table, or they could use the calculator to evaluate the algebraic expression at 10 hours.

X	Y1
7	128.23
8	109
9	92.647
10	78.75
11	66.937
12	56.897
13	48.362

X=10

```
400(.85)^10
     78.74976174
```

4. When does the amount of medicine still in the body reach 60 milligrams? Explain how you know.

   Use a table to find when the dose of medicine is at 60 milligrams. The patient will have 60 milligrams left in his system between 11 and 12 hours after the initial dosage.

X	Y1
9	92.647
10	78.75
11	66.937
12	56.897
13	48.362
14	41.108
15	34.942

X=11

5. Suppose that the level of medicine in the patient's body must maintain a level greater than 100 milligrams. How often does the patient need to take the medicine?

X	Y1
6	150.86
7	128.23
8	109
9	92.647
10	78.75
11	66.937
12	56.897

X=8

   The patient needs to take the medicine every 8 hours.

Extension Questions

- If the rule had been $y = 500(0.85)^x$ instead of $y = 400(0.85)^x$, how would this situation be different from the given situation?

  The initial dose of the medicine changes from 400 to 500 milligrams.

- What would the equation be if the concentration of medicine decreased by 30% each hour?

  If the concentration was reduced by 30% each hour, then 70% of the original dose would be left in the patient's body after 1 hour. If the original dose was 400 milligrams, the equation would be $y = 400(0.70)^x$*.*

- If the patient took a second 400-milligram dose at the twelfth hour, how much medicine would the patient have in his system at the fifteenth hour?

  The amount of medicine in the patient's system at the fifteenth hour is approximately 280.59 milligrams.

X	Y1	
10	78.75	
11	66.937	
12	56.897	
13	48.362	
14	41.108	
15	34.942	
16	29.7	

X=12

  From the table, the amount in the patient's system at the twelfth hour is 56.897 milligrams. If 400 milligrams are added, the amount in the patient's system is 456.897 milligrams. If you want to know the amount of medicine left at the fifteenth hour, use the expression $456.897(0.85)^3$*, because three hours have passed from the twelfth to the fifteenth hour.*

Mathematical Domains and Ranges of Nonlinear Functions

1. For the following problems:

   - Sketch a complete graph for the given function. Show the coordinates of any intercepts.
   - Describe the domain and range for each mathematical situation.

	Function	Graph	Process Area (optional)	Domain and Range
A.	$f(x) = \frac{1}{2}x^2$			Domain: Range:
B.	$y = x^2 + 3$			Domain: Range:
C.	$y = -3x^2$			Domain: Range:
D.	$y = x(5 - x)$			Domain: Range:

	Function	Graph or Table	Process Area (optional)	Domain and Range
E.	$h(x) = 3^x$			Domain: Range:
F.	$m(x) = \left(\frac{1}{3}\right)^x$			Domain: Range:
G.	$g(x) = \frac{4}{x}$			Domain: Range:

2. Write a summary comparing the domains, ranges, and graphs of the functions.

3. Describe a practical situation that the functions in problems D, E, and F might represent. What restrictions will the situation place on the mathematical domain and range of the function? How will the situation affect the graph of the mathematical function?

Notes

Materials:

One graphing calculator per student (optional)

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

Additional Algebra TEKS:

(A.2) Foundations for functions. The student uses the properties and attributes of functions.

The student is expected to:

(B) identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;

(C) interpret situations in terms of given graphs or create situations that fit given graphs; and

Scaffolding Questions

- What kind of function is this (linear, exponential, etc?)
- What is the dependent variable?
- What is the independent variable?
- What are the constants in the function? What do they mean?
- What restrictions does the function place on the independent variable?
- What is a reasonable domain for the function?
- What is a reasonable range for the function?
- If the graph of a function approaches but does not cross an *x*- or *y*-axis, what might that indicate about either the domain or the range of that function?

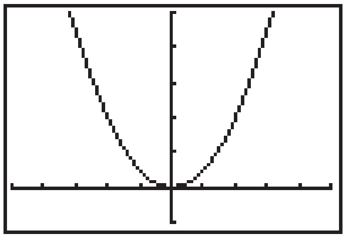
Sample Solutions

Note: We recommend that you assign this assessment to be completed *without* a graphing calculator. We include the screen shots in the sample answers to clarify the look of the graphs. Student graphs will be hand drawn. Students can use the process column on the chart to help them reason about the fuctions (with scratch work, a table, initial ideas, etc.)

1. For the following problems:

   - Sketch a complete graph for the given function. Show the coordinates of any intercepts.
   - Describe the domain and range for each mathematical situation.

   A. $f(x) = \frac{1}{2}x^2$



The y-intercept and the x-intercept for this function are at the origin (0, 0).

The domain for this function is the set of all numbers greater than or equal to 0, since any value for x can be squared and multiplied by $\frac{1}{2}$.

The range for this function is the set of all numbers greater than or equal to 0, which is the result of squaring any number and multiplying by $\frac{1}{2}$.

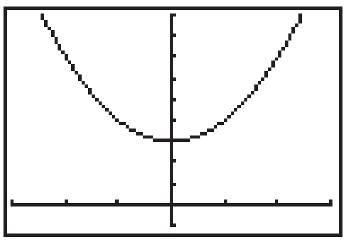
$x^2 \geq 0$

$\frac{1}{2}x^2 \geq 0$

Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of the quadratic and other nonlinear functions.

B. $y = x^2 + 3$



The y-intercept for this function is (0, 3).

There is no x-intercept because the graph does not intersect the x-axis.

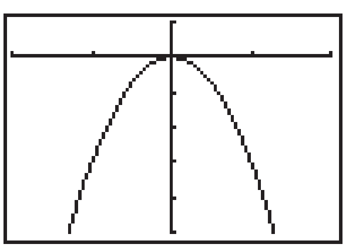
The domain for this function is the set of all real numbers, since any value for x can be squared and increased by 3.

The range for this function is the set of all numbers greater than or equal to 3.

$x^2 \geq 0$

$x^2 + 3 \geq 3 \geq 0$

C. $y = -3x^2$



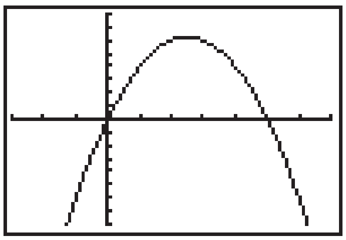
The y-intercept and the x-intercept for this function are at the origin (0, 0).

The domain for this function is the set of all real numbers, since any value for *x* can be squared and multiplied by –3. The range for this function is the set of all numbers less than or equal to 0, which is the result of squaring any number and multiplying by –3.

$x^2 \geq 0$

$-3x^2 \leq 0$

D. $y = x(5 - x)$

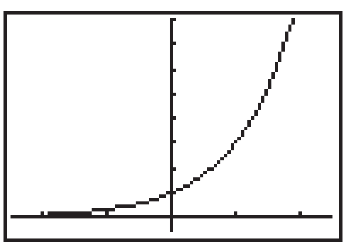


The *y*-intercept for this function is (0, 0).

The function crosses the *x*-axis at two points so that the *x*-intercepts are (0, 0) and (5, 0). The domain for this function is the set of all real numbers, since the expression $x(5 - x)$ is always defined for any value of *x*.

To determine the range, trace along the graph to find the largest *y* value. It occurs at the point (2.5, 6.25). The range for this function is the set of all numbers less than or equal to 6.25. This maximum *y*-value occurs at $x = 2.5$, which is the average of the two *x*-intercept values, 0 and 5.

E. $h(x) = 3^x$

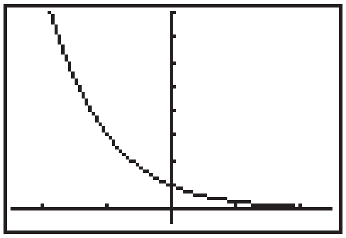


The *y*-intercept for this function is (0, 1) because the value of the function at 0 is 1. There are no *x*-intercepts, since no power of 3 equals 0. The trace function on the calculator helps show that the graph does not cross the *x*-axis.

The domain for this function is the set of all real numbers, since any real number can be used as an exponent on 3.

The range for this function is the set of all positive real numbers, since powers of 3 are always positive. Negative powers of 3 give *y*-values between 0 and 1, and positive powers of 3 give values greater than 1.

F. $m(x) = \left(\frac{1}{3}\right)^x$



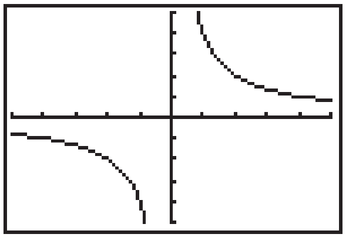
This function is similar to the function in problem E, only this graph is reflected across the *y*-axis. The *y*-intercept for this function is still (0, 1). There are no *x*-intercepts, since no power of $\frac{1}{3}$ equals zero.

The domain for this function is the set of all real numbers, since any real number can be used as an exponent.

The range for this function is the set of all positive real numbers, since powers of positive number are always positive.

Negative powers of a positive number produce values between 0 and 1, and positive powers of a positive number produce values greater than 1.

G. $g(x)=\frac{4}{x}$



This function has no y-intercept, since division by 0 is undefined. There is no x-intercept, since $0=\frac{4}{x}$ does not have a solution. Both of these facts are shown on the graph, as the graph does not cross either axis.

The domain for this function is the set of all real numbers except 0. The range for this function is the set of all real numbers except 0.

2. Write a summary comparing the domains, ranges, and graphs of the functions.

The first four functions are quadratic functions. They all have the same domain, the set of all real numbers. Their ranges vary because the expression ax^2 is positive if $a > 0$ and negative if $a < 0$. The expression $x^2 + c$ is greater than or equal to c for any c.

The functions in problems E and F are exponential functions. They have the same domain and range.

The function in problem G is an inverse variation function with both domain and range being the set of all real numbers except 0.

3. Describe a practical situation that the functions in problems D, E, and F might represent. What restrictions will the situation place on the mathematical domain and range of the function? How will the situation affect the graph of the mathematical function?

A practical situation that $y = x(5 - x)$ could represent is the area of a rectangle.

In this situation, the domain would be all numbers between 0 and 5 because the numbers less than 0 or greater than 5 result in a negative product, and the area cannot be negative. The range would be the numbers between 0 and

6.25. (6.25 would be included in this range.) The graph would be that part of the parabola above the *x*-axis.

A practical situation that $h(x) = 3^x$ could represent is the number of rectangles formed by folding a rectangular sheet of paper repeatedly into thirds. The domain would be the number of folds made, which must be the set of whole numbers {0,1,2,3,4...}. The range would be the number of rectangles formed by the folds. The number of rectangles is a whole number power of 3 {1,3,9,27,...}. The graph would be only those points on the original graph with nonnegative integer coordinates.

For a practical situation that might represent $m(x) = \left(\frac{1}{3}\right)^x$ consider the area of the original sheet of paper (in the example above) as one square unit; the function $m(x) = \left(\frac{1}{3}\right)^x$ would represent the area of each rectangle resulting from the folds. The domain is the set of the possible number of folds {0,1,2,3,4...}, and the range is $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right\}$. The graph would be the points on the original graph where *x* equals a nonnegative integer.

Extension Questions

- Compare the range of $f(x) = \frac{1}{2}x^2$ to the range of $f(x) = \frac{1}{2}x^2 + 3$.

  The range of the first function is the set of all numbers greater than or equal to 0. If 3 is added to all values of the function, the range will be the set of all numbers greater than or equal to 3.

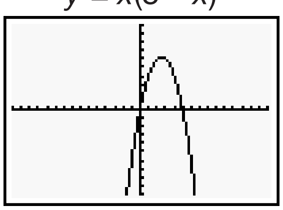
- Compare the range of $f(x) = \frac{1}{2}(x-3)^2$ to the range of $f(x) = \frac{1}{2}x^2$.

  The function $f(x) = \frac{1}{2}(x-3)^2$ will be similar to $f(x) = \frac{1}{2}x^2$, only shifted 3 units to the left. The vertex will change, but the range values will still be all the numbers greater than or equal to 0.

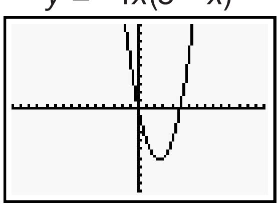
- If the function $y = x(5 - x)$ is multiplied by –1, how will the domain and range be affected?

  The domain of $y = x(5 - x)$ and $y = -1x(5 - x)$ will both be the set of all real numbers because the products are defined for any x. However, the ranges will be different. The range of the first function is $y \le 6.25$. The range of the second function will be $y \ge -6.25$ because all values of the function are multiplied by –1.

$y = x(5 - x)$

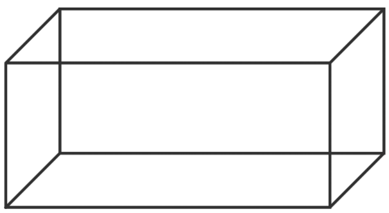


$y = -1x(5 - x)$



Paper Boxes

Marcy used a sheet of 8-inch by 11-inch paper to make an open box. She cut squares measuring x by x out of each of the paper's corners and folded up the sides. The diagram below shows the finished box. Simplify your expressions and justify your answers to each of the questions.



1. Draw a diagram of the sheet of paper showing the fold lines needed to make the box. Label your diagram with the dimensions of the cut-out pieces and the lengths of the fold lines. Use your diagram to find the dimensions of the open box.
2. Marcy decides to put a ribbon around the bottom edge of the box. She needs to determine the perimeter of the base. Write a polynomial to represent the perimeter of the base of the box, simplify the expression, and explain how you determined the answer.
3. Write a polynomial to represent the area of the base of the box. Explain how you found the area of the base.
4. Suppose the length of the box is increased by 3 units. How does this affect the area of the base?
5. Write a polynomial expression function to represent the volume of the box. Justify your answer.
6. Suppose all of the box's dimensions are doubled. What is the effect on the area of the base? On the volume of the box?

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.3) Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and

(B) look for patterns and represent generalizations algebraically.

(A.4) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

The student is expected to:

(A) find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations;

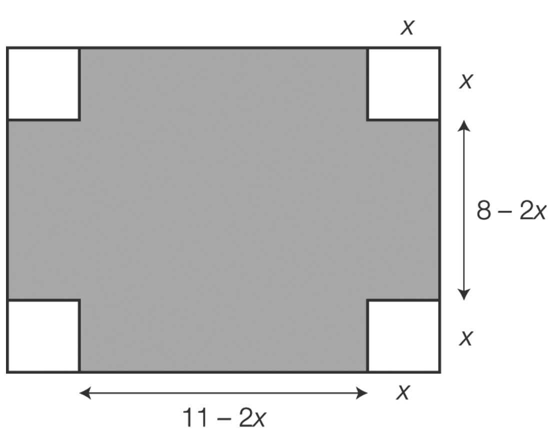
Scaffolding Questions

- What are the dimensions of the base?
- What is the height of the box represented by?
- How can you find the perimeter of the base?
- What would you do to find the area of the base?
- What is the formula for the volume of a rectangular prism?

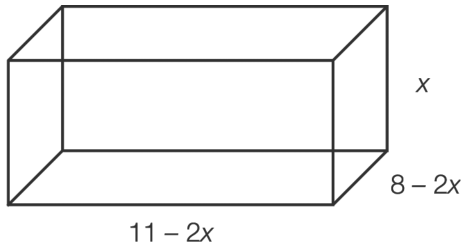
Sample Solutions

1. Draw a diagram of the sheet of paper showing the fold lines needed to make the box. Label your diagram with the dimensions of the cut-out pieces and the lengths of the fold lines. Use your diagram to find the dimensions of the open box.

   The sheet of paper is 8 inches by 11 inches. The length of the side of the square that is cut from each corner may vary. Let the length of the side of the square in inches be represented by *x*. The length of each side of the sheet of paper is decreased by 2*x*. The length of the fold lines is 8 – 2*x* and 11 – 2*x*.

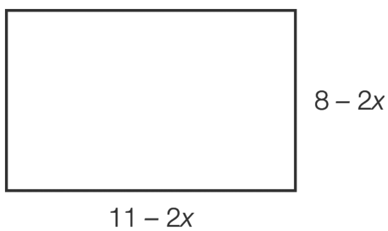


   The height of the open box is the same as the length of the side of one of the cut-out squares.



2. Marcy decides to put a ribbon around the bottom edge of the box. She needs to determine the perimeter of the base. Write a polynomial to represent the perimeter of the base of the box, simplify the expression, and explain how you determined the answer.

   The perimeter is the distance around a figure. The base of the box is a rectangle having dimensions of $11 - 2x$ and $8 - 2x$. Opposite sides of a rectangle are congruent, so there are two sides with length $11 - 2x$ and two sides with length $8 - 2x$.



   Using the formula for finding the perimeter [P = 2(length) + 2(width)], the perimeter can be calculated as follows:

   $P = 2(11 - 2x) + 2(8 - 2x)$

   $P = 22 - 4x + 16 - 4x$

   $P = 38 - 8x$

3. Write a polynomial to represent the area of the base of the box. Explain how you found the area of the base.

   The base of the box is a rectangle. The area can be found using the formula for the area of a rectangle (A = length times width).

   $A = (11 - 2x)(8 - 2x)$

   $A = 88 - 22x - 16x + 4x^2$

(B) use the commutative, associative, and distributive properties to simplify algebraic expressions; and

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

Texas Assessment of Knowledge and Skills:

Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.

$A = 88 - 38x + 4x^2$

$A = 4x^2 - 38x + 88$

4. Suppose the length of the box is increased by 3 units. How does this affect the area of the base?

   If the length of the base increases by 3 units, the box has a width of $8 - 2x$ and a length of $14 - 2x$. Using the formula A = (length)(width), the area of the base with an increased length of 3 units creates an area that was increased by 3 times the width. This new area can be represented by

   $A = (14 - 2x)(8 - 2x)$

   $A = 112 - 28x - 16x + 4x^2$

   $A = 112 - 44x + 4x^2$

   $A = 4x^2 - 44x + 112$

5. Write a polynomial expression function to represent the volume of the box. Justify your answer.

   Volume can be found by using the formula V = (length)(width)(height).

   The value of the length times the width was calculated when the area of the base was determined. The area was $4x^2 - 38x + 88$. The height of the box is represented by x. The volume of the original box is

   V = (length)(width) (height)

   $V = (4x^2 - 38x + 88)(x)$

   $V = 4x^3 - 38x^2 + 88x$

6. Suppose all of the box's dimensions are doubled. What is the effect on the area of the base? On the volume of the box?

   If the length and width are doubled, then the area of the base is $(2l)(2w) = 4(lw)$, 4 times (or 2^2) the old area. If the length, height, and width are doubled, then the volume becomes $(2l)(2w)(2h) = 8(lwh)$, 8 times (or 2^3) the original volume.

Extension Questions

- Using one sheet of 8-inch by 11-inch paper, which box holds more: a box with a height of 2 or a box with a height of 3? Justify your solution.

  Substitute the values of 2 and 3 into the volume formula to find the larger volume. If $x = 2$, the volume of the box is

  $V = (\text{length})(\text{width})\ (\text{height})$

  $V = (4x^2 - 38x + 88)(x)$

  $V = 4x^3 - 38x^2 + 88x$

  $V = 4(2)^3 - 38(2)^2 + 88(2)$

  $V = 56\text{ in.}^3$

  If $x = 3$, the volume of the box is

  $V = (\text{length})(\text{width})\ (\text{height})$

  $V = (4x^2 - 38x + 88)(x)$

  $V = 4x^3 - 38x^2 + 88x$

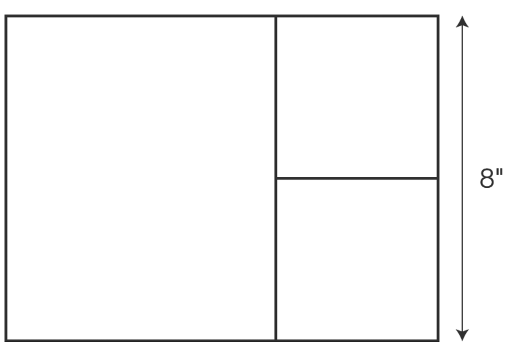
  $V = 4(3)^3 - 38(3)^2 + 88(3)$

  $V = 30\text{ in.}^3$

  Therefore, the smaller x-value of 2 yields the larger volume.

- What is the largest value of *x* that fits on an 8-inch by 11-inch paper? The smallest?

  The lengths of the sides of the piece of paper are 8 inches and 11 inches. Two squares are cut out of each side of the paper to form the box. If on the 8-inch side of the paper the cut-out squares are 4 inches long, the dimensions of that side of the box are 0. Therefore, the value for the side of the square, represented by x, must be greater than 0 and less than 4 inches.



What Is Reasonable?

Determine the reasonableness of the following situations. Use the questions to guide your work.

Part A

A rectangular garden plot will be enclosed with 40 meters of fencing. The area of the garden is a function of the dimensions of the rectangle.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.
2. Describe the mathematical domain and range of the function.
3. Describe a reasonable domain and range for the situation.

Part B

You fold a square piece of paper in half. Then you fold the resulting paper in half again, and so on. The number of rectangles formed by the folds is a function of the number of folds you make.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.
2. Describe the mathematical domain and range of the function.
3. Describe a reasonable domain and range for the situation.

Part C

The time it takes to proofread a certain book varies inversely with the number of people assigned to the proofreading task. Suppose 5 people proofread the book in 30 hours.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.
2. Describe the mathematical domain and range of the function.
3. Describe a reasonable domain and range for the situation.

Notes

Materials:

One graphing calculator per student

Algebra TEKS Focus:

(A.9) Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.

The student is expected to:

(A) determine the domain and range for quadratic functions in given situations;

(A.11) Quadratic and other nonlinear functions. The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

The student is expected to:

(A) use patterns to generate the laws of exponents and apply them in problem-solving situations;

(B) analyze data and represent situations involving inverse variation using concrete models, tables, graphs, or algebraic methods; and

Scaffolding Questions

- What is the constant in Part A?
- What are the variables in each situation?
- What type of function (linear, quadratic, exponential, inverse variation) relates to the variables?
- Should you use all real numbers for the domain? Why or why not?
- What representation would best help you see the domain and range?

Sample Solutions

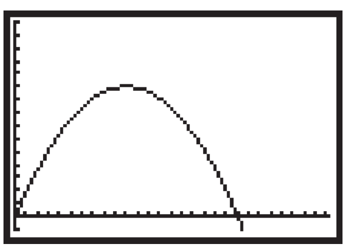
Part A

A rectangular garden plot will be enclosed with 40 meters of fencing. The area of the garden is a function of the dimensions of the rectangle.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.

   With 40 meters of fencing for the perimeter of the garden, there are 20 meters of fencing for the semi-perimeter (halfway around the rectangle). The dimensions of the garden can therefore be represented by x and $20 - x$, and the area of the garden is the product of the length and width. The area can be expressed as a function of the width.

   $A(x) = x(20 - x)$



2. Describe the mathematical domain and range of the function.

   The mathematical domain for this function is all real numbers, since no value for x makes the expression

$x(20 - x)$ undefined. The mathematical range for the function is all real numbers less than or equal to 100.

The graph shows that x can be any number (negative, 0, or positive) and that the range includes all values for y less than or equal to 100.

X	Y_1	
0	0	
2	36	
4	64	
6	84	
8	96	
10	100	
12	96	

X=0

The range for the function is the set of all numbers less than or equal to 100. For the problem situation, the area must be positive and less than or equal to the maximum possible area, 100 square meters.

3. Describe a reasonable domain and range for the situation.

   The domain of the situation is the set of all numbers between 0 and 20, because the x-value must be less than 20 and greater than 0 to produce a positive area. For the garden to exist, the length of a side, x, must be greater than 0 but less than half of 40 meters, the amount of fencing available for the whole garden. The range can be determined by examining a table or a graph. The greatest value of y is 100.

Part B

You fold a square piece of paper in half. Then you fold the resulting paper in half again, and so on. The number of rectangles formed by the folds is a function of the number of folds you make.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.

   Each time the paper is folded the number of rectangles is doubled, or multiplied by 2.

Additional Algebra TEKS:

(A.1) Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(C) describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

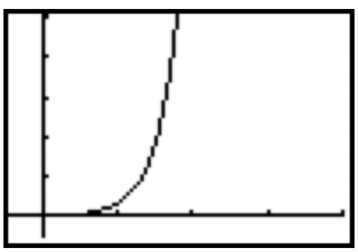
Texas Assessment of Knowledge and Skills:

Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.

Number of Folds	Number of Rectangles
0	1
1	2
2	4
3	8

The number of rectangles is a power of 2.

Number of Folds	Number of Rectangles
0	2^0
1	2^1
2	2^2
3	2^3
4	2^4



The function is $r = 2^n$, where n = the number of folds made and r = the number of nonoverlapping rectangles formed.

2. Describe the mathematical domain and range of the function.

   The mathematical domain for this function is the set of all real numbers since no value of n makes the function undefined. The range is the set of all positive real numbers since no power of 2 gives 0 or a negative value.

3. Describe a reasonable domain and range for the situation.

   The domain of the situation is $\{0, 1, 2, 3, \ldots, n\}$, where n is the maximum number of folds you can make. The number of folds possible depends on the dimensions of the piece of paper and its thickness. The resulting range of the situation is $\{1, 2, 4, 8, \ldots, 2^n\}$.

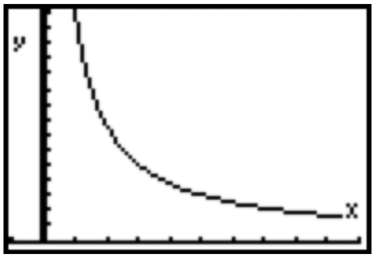
Part C

The time it takes to proofread a certain book varies inversely with the number of people assigned to the proofreading task. Suppose 5 people proofread the book in 30 hours.

1. Describe the function in words. Represent the function with an algebraic rule, graph, and table.

   If two quantities vary inversely, their product is a constant. In this case, the product is 5 times 30, or 150.

Number of People Proofreading	Number of Hours to Proofread
1	150
2	75
3	50
4	37.5
5	30
.	.
.	.
149	1.01
150	1



$nh = 150$ or $h = \frac{150}{n}$, where n is the number of people proofreading the book and h is the number of hours it takes to proofread the book.

2. Describe the mathematical domain and range of the function.

   The mathematical domain and range are both the set of all real numbers except 0. If $n = 0$, then h is undefined, and $h = \frac{150}{n}$ will never equal 0. The quotient of 150 and a number will never be 0.

3. Describe a reasonable domain and range for the situation.

   The domain and range for the situation are best described in a table. The number of people proofreading the book must be positive integer values.

Extension Questions

- In Part A, how do the domain and range change if you change the amount of fencing used to enclose the garden?

  If you decrease the amount of fencing, both the domain and the range decrease since both the dimensions and area of the garden decrease. If you increase the amount of fencing, both the domain and the range increase since the dimensions and area of the garden increase.

- In Part B, determine the function that would relate the area of each of the rectangles formed in the folding process to the number of folds. Describe the domain and range of this function. Compare the area function with the Number of Rectangles function.

  The initial area is 12^2, or 144 square inches. Each fold produces a new rectangle that is half as large as the previous rectangle. The function is $A = 144(\frac{1}{2})^n$.

  The domain is the set {0, 1, 2, . . ., n}, where n is the maximum number of folds you can make.

  The range is $\{144, 72, 36, 18, \ldots, 144(\frac{1}{2})^n\}$.

  The area function is a decreasing exponential function, while the Number of Rectangles function is an increasing exponential function. Both functions have finite domains and ranges.

- In Part C, suppose 6 people complete the proofreading task in 30 hours and that the time to complete the task must be measured in whole hours. How does this change your function and the domain and range for the situation?

  The function becomes $h = \frac{180}{n}$, since the task now requires 6 multiplied by 30, or 180 people-hours to complete. The domain and range now consist of factor pairs of 180, since we are measuring by both whole people and whole hours. There are 18 factor pairs for the new situation.

Number of people	1	2	3	4	5	6	9	10	12	15	18	20	30	36	45	60	90	180
Number of hours	180	90	60	45	36	30	20	18	15	12	10	9	6	5	4	3	2	1

- In general, how does changing the number of proofreaders in Part C affect the time required to complete the proofreading task?

  As the number of people increases, the time to complete the task decreases, and this occurs at a nonlinear rate. This can be seen by building a table where the number of people, x, increases by a constant amount and then comparing the corresponding change in the time, y, required to complete the task.