JOINT DECISIONS AND THE ALLAIS PARADOX

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Robert Cortes

Department of Economics
Stanford University
Stanford, CA 94305
Rcc419@stanford.edu

Under the direction of
Professor Jay Bhattacharya

ABSTRACT

"Man is by nature a social animal" – Aristotle

This thesis discusses an experiment designed to determine whether the famous Allais paradox is robust to changes in the number of agents per decision. That is, if the Allais gambles are presented to two people who can collaborate, will their preferences tend to be less paradoxical? Or will they systematically violate von-Neumann Morgenstern (vNM) expected utility theory the same way individuals have been shown to do? Making use of short surveys containing Allais and common-ratio examples over small hypothetical payouts, this study compares the preferences of two types of "agents:" the control agent, a single individual, and the experimental agent, a two-person pair making shared decisions. The results indicate that allowing for collaboration generates a statistically significant reduction in the number of violations of vNM expected utility. Such findings are interpreted as evidence that people in teams tend to gravitate toward an expected utility approach because it facilitates the joint decision-making process.

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Keywords: couples, pairs, collaboration, Allais, decision-making, behavior under uncertainty, experimental economics

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INTRODUCTION

Effective application of the economics of uncertainty requires useful, descriptive models of responses to probabilistic events. To study matching, search, auctions, insurance or investment, we need a theoretical background explaining behavior toward risk. The expected utility hypothesis has widely been put toward this use. This hypothesis has a rather intuitive conceptualization: a person's valuation of risky monetary outcomes is a function of both their utility from the payouts and the probability of the payouts' occurrence. John von Neumann and Oskar Morgenstern further solidified the theoretical employment and applied use of the expected utility framework when they introduced several simple assumptions, or axioms, which, when satisfied, prove that someone is maximizing an expected (vNM) utility function. The vNM formulation has been one of the most important steps in the modeling of behavior under uncertainty.

However, empirical support of the model is sparse. Researchers have particularly scrutinized the vNM axiom of independence, which is essentially the following assumption: if lottery A is preferred to lottery B, then (aA+(1-a)C)is preferred to (aB+(1-a)C), where C is some other lottery and $0<a<1$. This assumption has not held up empirically, where it appears actual preferences are inconsistent with the expected utility hypothesis. Research findings have led to numerous generalized models of utility, both expected and non-expected, that relax assumptions of independence to encompass a much broader scope of preferences while drawing theoretical implications similar to those of expected utility hypothesis. Mark Machina (1987) said that, "like the wave versus particle aspects of light," there might never exist a unified method of considering the spectral ways we behave when facing outcomes involving chance; nonetheless, "this does not mean we cannot continue to learn by studying and modeling them separately" (pp. 149-150). And as we learn, developing newer and more accurate models, it is important that we have a good sense of when extant models can be appropriately applied. This paper discusses an experiment intended to determine whether there are circumstances in which the use of the standard expected utility model need not imply a tradeoff of descriptive validity for facility. Specifically, the experiment examines shared decision-making as a potential situation better suited for the expected utility framework.

Choices between prospects with uncertain outcomes do not always confront individuals alone. Frequently, such decisions fall on organizations of people, like households choosing between health insurance plans. Sometimes people have organized for the very purpose of collectively making such decisions, like investment companies. Consequently, the modeling of preferences over probabilistic outcomes should look beyond individual behavior to consider shared preferences, which could be free of the cognitive biases shown to obfuscate expected utility maximization in individuals. That is, a pair of individuals making probabilistic decisions as though they were a single agent may behave more like an expected utility maximizer than an actual single agent. Since most multi-agent frameworks focus on games, it is important to ascertain the relative descriptiveness of standard expected utility as it applies to *collaborative* groups.

For this reason, the simple experiment discussed here compares two-person collaborative pairs and solo individuals in terms of revealed consistency with the expected utility hypothesis. The measurement of comparison is a famous test of expected utility maximization – the Allais paradox – which simply involves the elicitation of two choices. Each choice is between two lotteries with specific levels of risk. It has been shown that particular patterns of responses violate the independence axiom of expected utility hypothesis. The experiment seeks to establish the rates of these response patterns – the violation rates – for both solo decision-makers and two-person decision-makers, intermittently called "couples," "pairs," or "teams." From a statistical comparison of the Allais-suggested rates of expected utility violation, I draw conclusions about the relative descriptiveness of the expected utility framework for the two types of agents.

LITERATURE REVIEW

The Independence Axiom

The expected utility hypothesis dates back to a 1793 treatise by Daniel Bernoulli, who proposed the theory as a response to apparently paradoxical gambles that have infinite expected values but finite, and rather small, valuations by individuals. His solution based the preferences and behavior of people making choices between uncertain monetary outcomes on utility from money. According to the hypothesis, individuals make decisions under uncertainty according to their utility from expected monetary gains, whose value equals the result of multiplying all payouts by the probability of their occurrence. A person's utility function is derived not solely from the level of monetary wealth but also from, among other things, attitudes toward risk and preferences for consumption. *Theory of Games and Economic Behavior* (1944), by John von Neumann and Oskar Morgenstern, significantly advanced expected utility theory by laying out four axioms, which, when satisfied, confirm that an individual's preferences are the result of the maximization of an expected utility function.

Consider a set of payouts **X** offering states of wealth (x_1, x_2,…, x_n); also consider a probability distribution **P** = (p_1, p_2,…,p_n) where $p_1+p_2+\ldots+p_n=1$. If you were to take part in a lottery, L, which offered you $\mathbf{X} = (x_1, x_2,\ldots,x_n)$ with probabilities $\mathbf{P} = (p_1, p_2,\ldots,p_n)$ such that $\text{prob}(x_i)=p_i$, your vNM expected utility function would look like this:

$$\mathrm{u}(\mathbf{X}, \mathbf{P}) = p_1\mathrm{v}(x_1) + p_2\mathrm{v}(x_2) + \ldots + p_n\mathrm{v}(x_n) \qquad (1)$$

where $\mathrm{v}(x)$ reflects a state-separable, state-invariant utility for x that is unique up to affine transformations of $\mathrm{v}(\cdot)$. Under the expected utility hypothesis, a person's behavior under uncertainty is the result of the maximization of $\mathrm{u}(\cdot)$, which allows the individual to distinguish between all gambles based on level of expected utility. Over a given set of outcomes, the expectation of the utility function, defined over said outcomes, represents the preference function, or preference ranking. The empirical content of expected utility theory lies in its axiomatic restrictions on the preference function. The independence axiom is implied in (1) by the property of linearity in probabilities (e.g. $\mathrm{f}(p_x\mathrm{x} + p_y\mathrm{y}) = p_x\mathrm{f}(\mathrm{x}) + p_y\mathrm{f}(\mathrm{y})$). One can see the equivalence by imagining two probability distributions $\mathbf{P}$ and $\mathbf{P}'$ defined over outcomes $\mathbf{X}$. If $\mathbf{P}$ is preferred to $\mathbf{P}'$, then $\mathrm{u}(\mathbf{X}, \mathbf{P}) > \mathrm{u}(\mathbf{X}, \mathbf{P}')$ and, by independence, some probabilistic mixture of $\mathbf{P}$ with another lottery $\mathbf{P}''$ is preferred to the same mixture of $\mathbf{P}'$ with $\mathbf{P}''$, characterized by the following preference rankings:

$$\mathrm{u}(a(\mathbf{X}, \mathbf{P}) + (1\text{-}a)(\mathbf{X}, \mathbf{P}'')) = \Sigma\mathrm{v}(x_i)(ap_i+(1\text{-}a)p_i'') = a\Sigma\mathrm{v}(x_i)p_i + (1\text{-}a)\Sigma\mathrm{v}(x_i)p_i''$$

$$= a\mathrm{u}(\mathbf{X}, \mathbf{P}) + (1\text{-}a)\mathrm{u}(\mathbf{X}, \mathbf{P}'') \qquad (2)$$

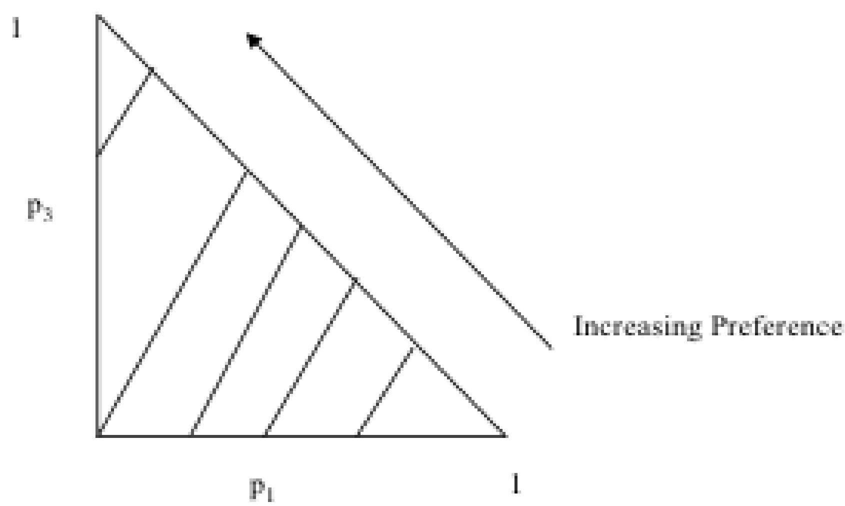
$$\mathrm{u}(a(\mathbf{X}, \mathbf{P}') + (1\text{-}a)(\mathbf{X}, \mathbf{P}'')) = \Sigma\mathrm{v}(x_i)(ap_i'+(1\text{-}a)p_i'') = a\Sigma\mathrm{v}(x_i)p_i' + (1\text{-}a)\Sigma\mathrm{v}(x_i)p_i''$$

$$= a\mathrm{u}(\mathbf{X}, \mathbf{P}') + (1\text{-}a)\mathrm{u}(\mathbf{X}, \mathbf{P}'') \qquad (3)$$

Recalling $\mathrm{u}(\mathbf{X}, \mathbf{P}) > \mathrm{u}(\mathbf{X}, \mathbf{P}')$, it is clear that (3) < (2). The property of linearity in the probabilities evidently makes it such that the preferences of expected utility maximizers have to exhibit independence. And while the other vNM axioms (completeness, continuity, transitivity) simply establish the mere *presence* of a preference ranking over all probability distributions, the independence axiom, on the other hand, imposes a specific *shape* on the preference ranking, making it the most scrutinized vNM axiom.

Before analyzing implied aspects of the violation of the independence axiom, it would be prudent to introduce a very helpful graphical representation of preferences for uncertain outcomes. Figure 1 depicts the two-dimensional probability triangle, first employed in Marschak (1950) and popularized in Machina (1982), which will be put to this purpose. Consider an outcome set (x_1, x_2, x_3) and probability distribution (p_1, p_2, p_3) where $x_1<x_2<x_3$, p_i=prob(x_i), and $p_1+p_2+p_3=1$. Taking advantage of the fact that $p_2=1-p_1-p_3$, lotteries with these elements can be expressed in terms of a probability unit triangle in the (p_1, p_3) plane:

Figure 1 – the (p_1, p_3) unit triangle with linear indifference curves



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One can see that rightward movement increases prob(x_1)=p_1, decreases prob(x_2), and maintains prob(x_3)=p_3. That is, rightward movement raises the likelihood of the smallest outcome purely at the expense of the middle outcome. Upward movement raises the likelihood of the largest outcome (p_3) purely at the expense of the middle outcome (p_2). Northwest movements raise the probability of p_2 and p_3 at the expense of p_1, thus signifying stochastically dominant[1] lotteries. In Figure 1, the parallel lines within the triangle denote indifference curves. Higher utility levels, and hence higher indifference curves, are achieved with movements to the northwest (we assume u(x) is nondecreasing in x, ensuring that stochastically dominant distributions are always preferred). The straightness of the indifference curves is rooted in the independence axiom. Levin (2006) explains: if there is indifference between distributions **P*** and **P****, then there must also be indifference between **P*** and (a**P***+(1-a)**P****) and **P****. Since, by definition, indifference between distributions places them on the same indifference curve, **P***, (a**P*** + (1-a)**P****) and **P**** will all be points on a single, linear indifference curve in the (p_1, p_2) unit triangle. The mathematical derivation of the curves points to linearity in probabilities:

$$\ddot{u} = \Sigma \mathrm{v}(x_i)p_i = \mathrm{v}(x_1)p_1 + \mathrm{v}(x_2)(1-p_1-p_3) + \mathrm{v}(x_3)p_3 = \text{constant} \quad (4)$$

$$\Rightarrow p_3 = p_1[(\mathrm{v}(x_2) - \mathrm{v}(x_1))/(\mathrm{v}(x_3) - \mathrm{v}(x_2))] \text{ (Machina 1987, p. 126)} \quad (5)$$

Since v(x) is fixed over the outcome set, the slope of (5) increases proportionately with p_2. Since this proportion does not change with different values of $\ddot{u}$, the indifference curves for every utility level in the unit triangle will be parallel.

Risk preferences are characterized by the steepness of the indifference curves, with steep indifference curves indicating risk aversion. This is because a risk averse person needs extra compensation to bear greater risk, which, in the (p_1,p_3) unit triangle, is depicted by diagonal movements to the northeast (where the probability of the middle

[1] H** is said to stochastically dominate H* if, for every positive outcome of H*, H** offers at least as high a probability of obtaining at least the same outcome, with one these probabilities being strictly larger in H**.

outcome declines and the probabilities of the large and small outcomes rise). Since utility is constant along a given indifference curve, steepness indicates that a relatively large increase in the probability of the best outcome (p_3) needs to accompany each increase in the probability of the worst outcome (p_1) to preserve the same level of utility.

The Violation of the Independence Axiom

In a 1953 essay titled, "Le comportement de l'homme rationel devant le risque: critique des postulats et axiomes de l'école Américaine," Maurice Allais presents a paradox that serves as one of the most well known counterexamples to the independence axiom of von-Neumann Morgenstern expected utility theory. The Allais paradox involves two consecutive choices. Each choice is between two gambles. The individual first makes a decision between (A) receiving $1 million with certainty versus (B) receiving $1 million with probability .89, receiving $5 million with probability .1, receiving nothing with probability .01. Next, the individual decides between (A*) receiving $1 million with probability .11 and receiving nothing with probability .89 versus (B*) receiving $5 million with a .10 probability, receiving nothing with a .90 probability. People faced with these gambles systematically select A and B* – a pattern of preferences inconsistent with a theory of expected utility maximization. Empirical evidence of this tendency can be found in Morrison (1967), Raiffa (1968), Slovic and Tversky (1974), Allais and Hagen (1979), Kahneman and Tversky (1979), MacCrimmon and Larsson (1979), Camerer (1989), Conlisk (1989). To see how these choices violate expected utility maximization, consider mathematical implications of each selection under the expected utility hypothesis:

Selecting A over B implies:

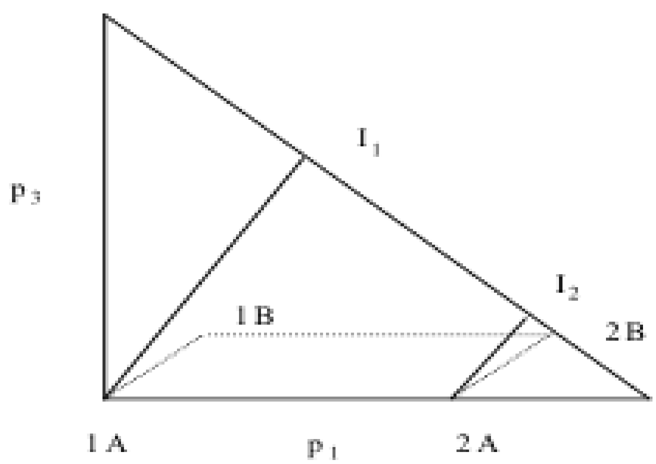
$$v(\$1M) > .1v(\$5M) + .01v(\$0) + .89v(\$1M) \tag{6}$$

while selecting B* over A* implies:

$$.11v(\$1M) + .89v(\$0) < .1v(\$5M) + .9v(\$0) \quad (7)$$
$$\Rightarrow .11v(\$1M) + .89v(\$1M) < .1v(\$5M) + .01v(\$0) + .89v(\$1M) \quad (8)$$
$$\Rightarrow v(\$1M) < .1v(\$5M) + .01v(\$0) + .89v(\$1M) \quad (9)$$

Evidently, a model of expected utility maximization dictates preferences of either AA* or BB*. That the model cannot account for the systematic Allais preferences of AB* arises from the assumption of independence required by preference functions that are linear in the probabilities. Figure 2 shows the unit triangle plotting the Allais gambles in the context of risk averse preferences. In the figure, points (1A, 1B, 2A, 2B) correspond to lotteries (A, B, A*, B*), respectively:

Figure 2 – the (p_1, p_3) unit triangle with Allais gambles and risk averse, linear indifference curves



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Graphically, it is perhaps easier to see why the expected utility hypothesis, with its parallel, straight-line indifference curves, imposes a preference pattern of AA* ((1A)(2A) in Figure 2) on risk averse individuals. One can also imagine the flattening of I_1 and I_2 to see that risk seeking preferences in a framework of expected utility maximization call for the Allais pattern BB* ((1B)(2B) in Figure 2).

The systematic Allais decision pattern of AB* has been attributed to a more general preference phenomenon dubbed the "common consequence effect," or the "certainty effect," by which people tend make the non-expected choices of A and B* among pairs of prospects that look like this:

A: $a\theta_x + (1\text{-}a)H^{**}$	OR	B: $aH + (1\text{-}a)H^{**}$
A*: $a\theta_x + (1\text{-}a)H^{*}$	OR	B*: $aH + (1\text{-}a)H^{*}$

where $0<a<1$, θ_x represents a sure gain of x, H represents a lottery offering values $(h_1,\ldots,h_n)$ such that $h_i<h_{i+1}$ and $h_1<x<h_n$, and H** stochastically dominates H*. In the Allais example, a=.11, x=\$1M, H** involves a 100% chance of winning \$1M, H* involves winning a 100% chance of winning \$0, and H involves a 10/11 chance of winning \$5M and 1/11 chance of winning \$0 (Machina 1987, p. 129). The certainty effect sheds doubt on the expected utility assumption that a specific probability change over an outcome distribution always has the same effect on the preference function. In both Allais choices, moving from B/B* to A/A* can be thought of as paying for a .01 probability movement from \$0 to \$1M with a .10 probability movement from \$5M to \$1M. People prefer this trade in the AB decision but dislike it in the A*B* decision, revealing non-linear preference effects from equivalent changes in the two probability distributions. Between A* and B*, where winning \$0 is likely, the elimination of a 1% chance of a \$0 payout is less valuable; Between A and B, where the \$0 payout is unlikely, the same .01 probability elimination is more important. People become more risk averse when such elimination creates an opportunity for a certain gain (hence the term, "certainty effect"). In terms of the (p_1, p_3) triangle diagram,

this effect makes people much more fond of the boundaries, suggesting indifference curves that get steep right before they hit an edge. So in response to the Allais gambles, it's as though people treat the two choices with *different* utility functions. The independence axiom assumes an individual's preference ranking is the expectation of a *fixed* utility function. Thus the certainty/common-consequence effect implies that our behavior under risk is fundamentally inconsistent with preference functions exhibiting linearity in probabilities. Because of this, generalized utility models, which do not assume strict independence, have been developed.

Generalized Expected Utility Models and the Relaxation of the Assumption of Independence

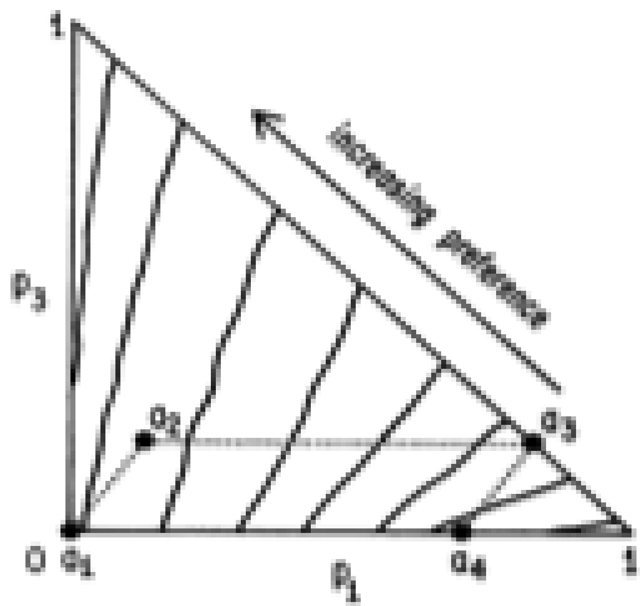
For the purposes of this paper, hypotheses will be distinguished by the indifferences curves they imply, as in Camerer (1989). Specifically, it will be useful to separate theories predicting preferences that "fan out" from those that do not.

Machina (1982) evinces the robustness of the primary implications of the expected utility model in the absence of the independence axiom. The much more modest assumption of smooth preferences still generates results consistent with expected utility maximization, owing to the fact that global non-linear preference functions locally exhibit the properties of their linear approximations. That is, a non-linear preference function $\Upsilon(\cdot)$ reflects the properties of its "local utility index" – $u_l(x_i, \mathbf{P})$ – over all outcomes at each probability distribution. Taking this into account, the author formulates Hypothesis II, or the "fanning out" hypothesis, which conjectures that local utility indices will exhibit greater concavity, or risk aversion, at stochastically dominant distributions:

$$\text{If } \mathbf{P}(x) \geq \mathbf{P}^*(x) \text{ for all values of } x \text{, then}$$
$$-u_l''(x, \mathbf{P})/u_l'(x, \mathbf{P^*}) \geq -u_l''(x, \mathbf{P})/u_l'(x, \mathbf{P^*}) \tag{10}$$

where F(·) is a cumulative distribution function and risk aversion is represented by the "Arrow-Pratt" measure: (-u′′/u′). Fanned out indifference curves in the (p_1, p_3) unit triangle are shown in Figure 3:

Figure 3 – the (p_1, p_3) unit triangle with indifference curves that fan out



Machina (1987, p. 128)

Eschewing the assumption of independent preference rankings and retaining the assumption of continuous, complete, and transitive preference rankings, Machina proves that an individual exhibiting the aforementioned certainty and common consequence effects can be described as having indifference curves that fan out like the ones above. Let us recall how the typical Allais choices imply disproportionately greater risk aversion surrounding decisions in which the relatively unfavorable event is a probability outlier. This aspect of behavior is captured by indifference curves which get steeper with movement toward p_3 = 1. In this portion of the diagram, where p_1 is lowest relative to p_2

and p_3 (i.e. where distributions are stochastically dominant), preferences are more sensitive to small changes in p_1, exactly the sort of behavior implicit in violations of independence.

Generalized utility models that are variants of expected utility also imply fanned out indifference curves. Camerer (1989) shows that fanning out characterizes the implied preference functions of: the "light hypothesis" of weighted expected utility theory (Chew and MacCrimmon, 1979; Chew, 1983) (which is actually the same as the transitive case of skew symmetric bilinear utility of Fishburn (1984)), lottery-dependent expected utility theory (Becker and Sarin, 1987), and rank-dependent expected utility theory (Quiggin, 1982).

A Subjective Expected Utility Model

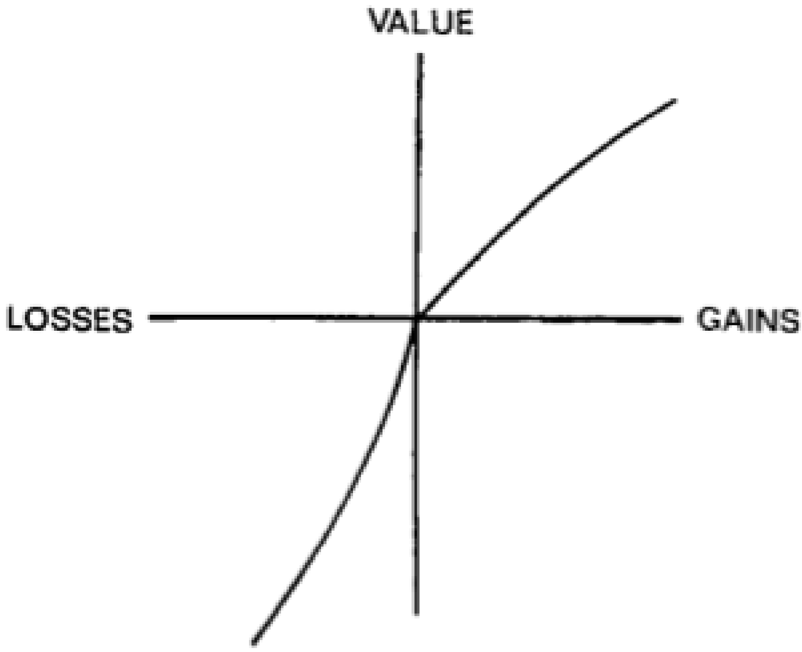
Subjective expected utility models assume that people transform given probabilities into subjective ones. In their seminal piece, "Prospect Theory: An Analysis of Decision under Risk" (1979), Daniel Kahneman and Amos Tversky present a subjective expected utility theory (prospect theory), which deals only with decisions between gambles and diverges from standard expected utility in several ways. Under prospect theory, there is no utility function for overall states of wealth. Rather, decisions are the processed through a utility-type function, itself the result of the combination of two component functions – $v(\cdot)$ and $\pi(\cdot)$ – in a manner similar to mathematical expectation:

$$u(\mathbf{X}, \mathbf{P}) = \Sigma v(x_i)\pi(p_i) \tag{11}$$

where $v(\cdot)$, the value function, has the following conjectured properties: "(i) defined on deviations from [a] reference point; (ii) generally concave for gains and commonly convex for losses; (iii) steeper for losses than for gains" (Kahneman and Tversky 1979, p. 279).

The corresponding curve, as proposed by the researchers, is represented graphically in Figure 4:

Figure 4 – conjectured value function – v(·) –*of prospect theory*

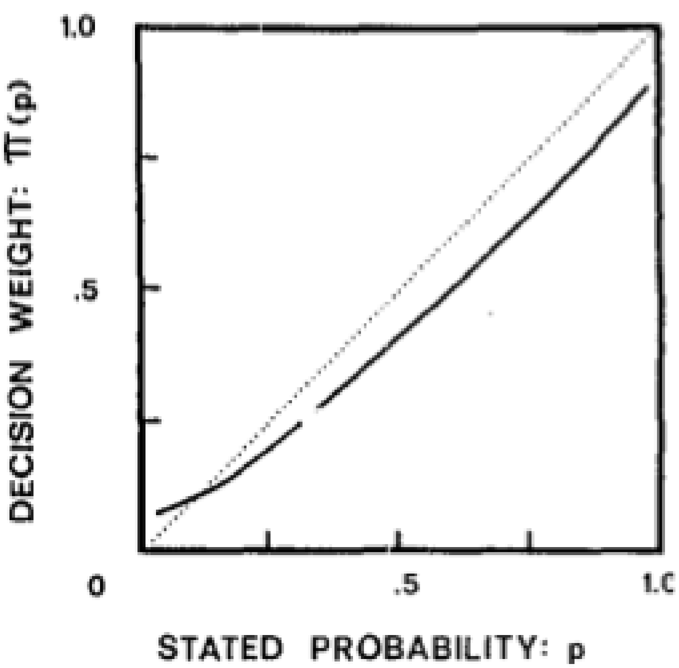


Kahneman and Tversky 1979, p. 279

This curve appears has been compared to the utility function formulated in Markowitz (1974); Markowitz also hypothesized that a decision-maker evaluates gambles from the reference point of current wealth. The works share the notion that individuals do not assess overall states of wealth, but rather *changes* in wealth. This leads to a supposition that people evaluate losses and gains fundamentally differently. The crucial difference between prospect theory and the Markowitz hypothesis, though, is the latter's preservation of the axioms of expectation: preference rankings obtained from the Markowitz utility function are linear in the probabilities and hence cannot explain the behavior revealed by Allais-type examples, unlike the value function depicted in Figure 4.

The weighting function of prospect theory, $\pi(\cdot)$, is assumed to be nonlinear (hence the term, *subjective* expected utility). Firstly, Kahneman and Tversky (1979) suggest that people overweight small probabilities. That is, $\pi(p)>p$ for small p. Another aspect of this weighting function is "subcertainty," which causes people to weight the sum of the probabilities of complementary events less than the probability of a certain event. That is, $\pi(p) + \pi(1-p) < 1$. Thirdly, the weighting function is claimed to be discontinuous at p=0 and p=1. Figure 5 is a visual representation of the resulting weighting function:

Figure 5 - proposed weighting function – $\pi(p)$ – of prospect theory



Kahneman and Tversky 1979, p. 283

Before outcomes and probabilities enter the component functions, though, the theory presumes that individuals edit prospects for the sake of simplifying the decision. For instance, people are said to discard stochastically dominated distributions, as well as outcome-probability pairs mutually contained by the gambles, before applying their evaluation to the prospects. Kahneman and Tversky note that, depending on the order of editing (which can vary with different ways of framing prospects), intransitivity and other anomalies may appear. Prospect theory therefore predicts situations in which “the decision maker does not have the opportunity to discover that his preferences could violate the decision rules that he wishes to obey” (p. 276).

Camerer (1989) graphically depicts the indifference curves suggested by prospect theory by applying its utility equation to the (p_1, p_3) probability triangle: gambles involve outcomes $0=x_1<x_2<x_3$ and familiar probabilities (p_1, p_2, p_3). The aforementioned editing phase in prospect theory implies that the terms used in the utility function depend on the value of p_1. If $p_1=0$, the individual performs an operation called segregation, looking at the gamble as a gain of x_2 with certainty and a gain of (x_3-x_2) with probability p_3, leading to a utility function that looks like this:

$$u(0 + p_2x_2 + p_3x_3) = \nu(x_2) + \pi(p_3)\nu(x_3-x_2) \qquad (12)$$

If $p_1>0$, the utility function becomes:

$$u(p_1 0 + p_2 x_2 + p_3 x_3) = \pi(p_2)\nu(x_2) + \pi(p_3)\nu(x_3) \qquad (13)$$

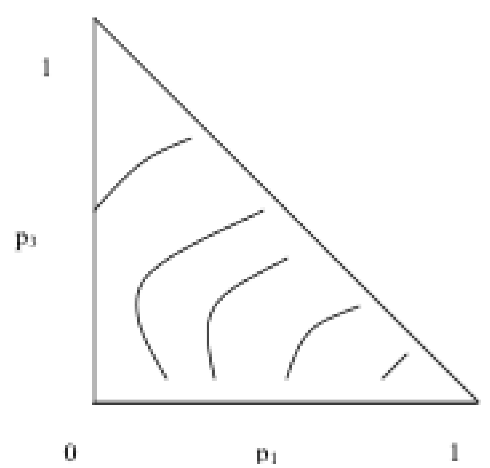
In order to find the slopes of prospect theory’s indifference curves in the (p_1, p_3) triangle, Camerer utilizes the fact that, along a given indifference curve, $du/dp_1 = 0$. He therefore differentiates (13) to get:

$$\nu(x_2)\pi'(p_2) + \nu(x_3)\pi'(p_3) = 0 \qquad (14)$$

$$\Rightarrow v(x_2)[\pi'(p_2)(-1 - (dp_3/dp_1))] + v(x_3)[\pi'(p_3)(dp_3/dp_1)] = 0 \quad (15)$$
$$\Rightarrow dp_3/dp_1 = v(x_2)\pi'(p_2)/[v(x_3)\pi'(p_3) - v(x_2)\pi'(p_2)] \quad (16)$$

Drawing upon the conjectures of Kahneman and Tversky, who suggest a convex weighting function, Camerer assumes $\pi'(p)$ is larger when p is larger. Consequently, dp_3/dp_1 increases when p_2 increases along a cross-section of p_3; dp_3/dp_1 decreases when p_3 increases along a cross-section of p_2. The result is indifference curves that get steeper with movements to the west and get flatter with movements to the northwest. Curves are steepest when closest to the bottom-left edge and flattest when closest to the top edge. Since the weighting function cannot be differentiated at 0 or 1, the indifference curves become discontinuous when they get close to the edges of the triangle. These properties can be seen in Figure 6:

Figure 6 - prospect theory indifference curves



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Other subjective utility theories, such as CE theory (Handa, 1977), subjective weighted utility theory (Karmarkar, 1978), and cumulative prospect theory (Tversky and Kahneman, 1992), imply similarly shaped indifference curves.

Toward the Joint Decisions Experiment: Empirical Studies of Generalized Utility

This study drew upon well-known empirical tests of the generalized and subjective expected utility theories (see Mosteller and Nogee, 1951; Moskowitz, 1974; Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979; Camerer, 1989; Conlisk, 1989; Carlin, 1990, 1992; Weber, 2007; Incekara-Hafalir and Stecher, 2012). However, the amount of data required for these studies to make accurate empirical distinctions between generalized utility models was leagues above the capabilities of this study. For instance, Camerer (1989) had all of 355 subjects indicate preferences over at least 13 gamble-pairs. Kahneman and Tversky (1979) discuss the results of some 14 examples (with complementary examples of reflection) given to samples of 60-100 individuals. The subjects of Weber (2007) made repeated Allais choices as part of a computerized "choice titration procedure," to be described presently.

Weber's experiment made use of the standard two Allais gamble-pairs. So each subject first decided between the gambles so far called AB. Let us recall the probability distributions of A and B: {p_1^A=0, p_2^A=1, p_3^A=0; p_1^B=.01, p_2^B=.89, p_3^B=.10}. Since this is the Allais decision in which the favorable outcome is extremely likely in both gambles, Weber dubs it the common-consequence-high ("CC-high") choice. The "CC-low" choice, therefore, involves the A*B* decision, with distributions {p_1^A=.89, p_2^A=.11, p_3^A=0; p_1^B=.90, p_2^B=0, p_3^B=.10}. The CC-high and CC-low choices were made over gains, "shifted losses," and losses. These ranges determined the size/sign of the middle and lower outcomes. Finally, the titrated matching technique was used to ascertain x_3^B for each CC level and each payout size. To see how the technique worked, consider the procedure for the CC-high choice over gains: on a computer, subjects made repeated decisions between A and B, with the aforesaid distributions and the following outcome sets picked by the researcher: {x_1^A=0,

x_2^A=\$250, x_3^A=\$0; x_1^B=0, x_2^B=\$250, x_3^B=w}. In a given round, if A(B) was selected, in the next round w would increase(decrease), until a value of w was found such that the subject repeatedly selected B when $x_3^B>w$ and A when $x_3^B<w$. According to this process, each subject had a personal w called the subject's "indifference point," since it represents the value of x_3^B at which the subject is indifferent between the risky gamble and the less risky gamble. By evaluating differences in the indifference points associated with different CCs and different payout sizes, the researcher tests the hypotheses she wanted to test: statistical analysis is performed on observed differences in Allais behavior stemming from (1) negative payouts and (2) a reframing process called "event-splitting." The results of the statistical analyses are used to compare the relative adequacy of cumulative prospect theory, prospect theory, the "transfer of attentional exchange" (TAX) model, and expected utility.

In a similar experiment found in Incekara-Hafalir and Stecher (2012), subjects chose between Allais-style gambles A and B, with probability distributions {$p_1^A=p_1^B$=.01, $p_2^A=p_2^B$=.10, $p_3^A=p_3^B$=.89} and outcomes {x_1^A=8, x_2^A=8, $x_3^A=c$; x_1^B=0, x_2^B=10, $x_3^B=c$}, where c $\in$ (0, 5, 8 10, 15 20). So, just like in Weber (2007), probabilities were fixed and the magnitude of common consequence varied. However, in this study, statistical analysis is applied to differences in Allais behavior stemming from (1) the framing of choices and (2) the "zero effect" (changes in behavior when c=0). The results of the statistical analyses are used to compare the relative adequacy of the fanning out hypothesis, disappointment aversion, rank-dependent utility, cumulative prospect theory, Prelec's probability weighting model, and expected utility.

Owing to less extensive surveys and smaller samples, the data collected in this thesis experiment is notably less abundant than in the experiments discussed above. As a result, the results discussed herein have less precision. But then again, my research question does not require the same level of meticulousness, since the intention of my experiment is not the empirical delineation of generalized utility models. Such a process requires the clear-cut mapping of gambles that separate the behavior implied by the models at hand. The usually binary (assuming indifference is prohibited) nature of decisions between lotteries makes it so that a given pattern of choices will be accounted for by multiple hypotheses. Consequently, many specific combinations of gambles to discern the specific choice patterns needed to substantiate particular models of risk preference and behavior under uncertainty. Conversely, the slight comparative advantage of my study lies in its relative simplicity, or rather, crudeness, while exploring an area somewhat neglected by models of behavior under uncertainty: two-person, collaborative decision problems. Within the context of the models discussed in previous sections, the statistical results from my data are used to demarcate a general direction for the modeling of shared decision-making under uncertainty. These ensuing sections will go over the design of the Joint Decisions experiment.

METHOD

The empirical content of this study comes from a survey-based field experiment. As one would probably imagine, the surveys had respondents choose between Allais-type gamble-pairs.

Population

A total of 181 Stanford civilians participated through the entirety of the experiment. Like the telemarketer's "cold calling" method, recruitment of these volunteers involved "cold surveying" around campus. The control group contained 65 volunteers, whose recruitment and participation were standard for most survey-based field studies; the experimental group contained 116 volunteers, who were recruited and participated in pairs of two. Consequently, a participant population of 181 was given a total of 123 surveys. The study population exhibits limited generalizability because Stanford is a rather unique geographical location. External validity is assumed up to Stanford civilians in general.

Survey Design

Each survey contained 4 gamble-pairs, equaling 2 examples (2 examples = 2 opportunities to violate independence = 4 decisions between gamble-pairs). All examples involved either a replication of the Allais distributions or a replication of the Kahneman and Tversky (1979) common-ratio distributions (the Kahneman and Tversky common-ratio examples are simplified Allais examples; they are all examples of the common-consequence effect). All gambles involved small hypothetical payouts. 94, or around 76%, of the surveys contained an Allais example and a reflected common-ratio example (see appendix A1). 29, or around 24%, of the surveys contained two common-ratio examples, varied with respect to the stated probabilities and the money outcomes (see appendix A2). The common-ratio examples are abbreviated to: "CR" for the standard common-ratio example, "R-CR" for the reflected common-ratio example, and "CR-LTM" for the common-ratio example with large-to-miniscule probabilities. The surveys varied the positions and names of the particular gamble-pairs; in this analysis, though, all less-risky lotteries are called (A, A*) and the riskier lotteries are called (B, B*), where the asterisk signals the stochastically dominated distribution. There were no *intentional* attempts to reframe prospects, however there was an unintentional alteration to the standard presentation of gamble-pairs.

General Procedure

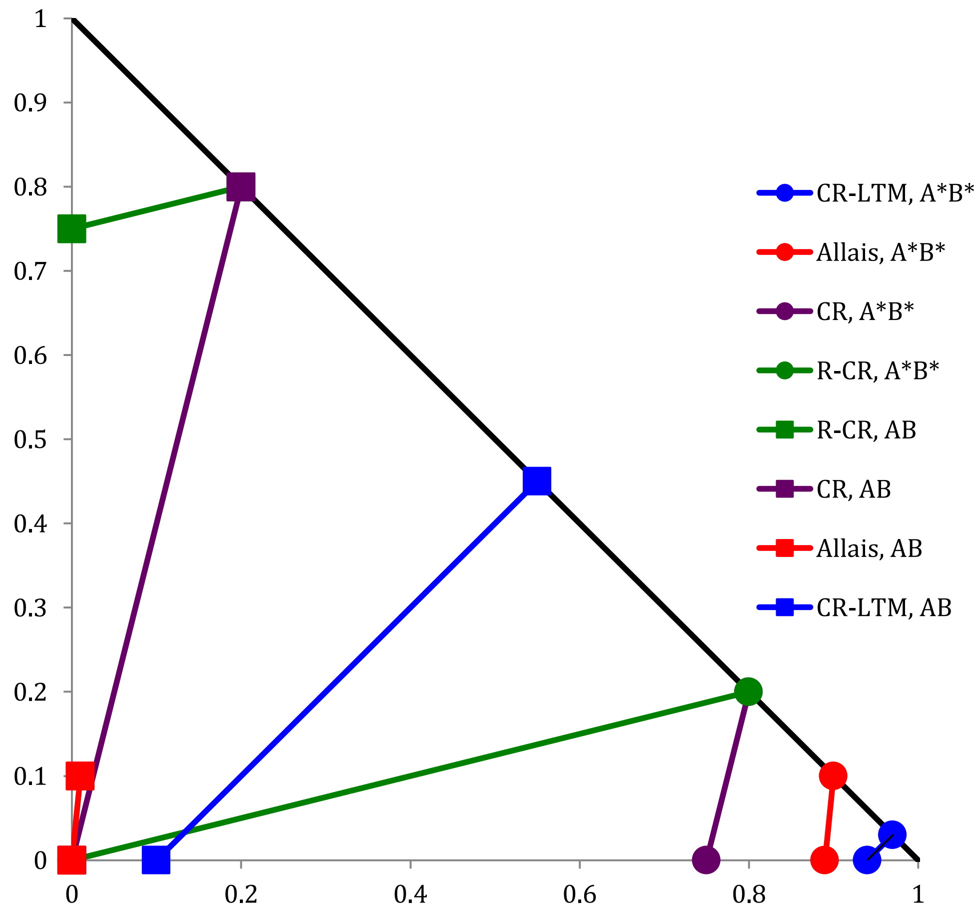
Prospective participants were randomly approached and asked if they would be willing to participate in a brief survey. (Normally a yes/no answer was given; at times prospective participants wanted to know the surveys' purpose and/or subject matter before consenting). Immediately after indicating willingness, participants were handed a pen and survey, then given succinct instructions. All surveys had the same written prompt. Verbal instructions differed slightly for the two conditions. Participants in the control condition (i.e. the solo group) were told to read the survey prompt and, before starting, to ask for clarification if the task was not totally comprehended. Participant pairs in the experimental condition (i.e. the couples group) were read the survey prompt and told to go through the survey together, providing only one preference in each part. Then, as with the control group, the participants were encouraged to ask for clarification in order to ensure total comprehension.

The data were analyzed on between-subjects, within-subjects, and within-subjects-within-surveys bases.

DISCUSSION OF METHOD

This simple field experiment sets out to determine and measure differences between single-person and two-person decision problems. The Allais and common-ratio examples have empirical histories of pointing to inconsistencies with the hypothesis of expected utility maximization. For this reason, the Joint Decisions experiment uses replications the Allais distribution and three common-ratio distributions. Figure 7 depicts these distributions as points in the *(p_1, p_3)* unit triangle:

Figure 7 – (p_1, p_3) unit triangle with distributions used in the Joint Decisions experiment



(It should be noted that the distributions of the gamble-pair, [CR-LTM, A*B*], were adjusted to the left slightly to make the points distinguishable on the (p_1, p_3) unit triangle).

These gamble-pairs were presented to the solo group and the couples group in an attempt to determine the nature of the differences between solo decision-makers and coupled decision-makers. If the couples violated independence just as frequently as solo individuals, it can be argued that, overall, a model of expected utility maximization does no better at explaining joint decisions under uncertainty. Subsequent analysis would have to compare the violation patterns of the respondent classes to see whether shared decision-making displays the same violation rate and systematic pattern (AB*). It would then appear that the same fundamental behavior underlies both single- and two-person decision problems. Conversely, if shared decisions generate a statistically significant reduction in the violation rate, support would be given to the hypothesis that subjective utility models do relatively worse at describing two-person, shared preferences for risk. More detailed analysis concerning the nature of the reduction would be needed. For instance, if systematic Allais violations decline, it should be determined whether bulk of the decline comes from changes in preferences over the stochastically dominant pair (i.e. shifts away from the certain prospect when the common consequence is large) or the dominated pair (i.e. shifts away from the risky prospect when the common consequence is small). In this way, one could assess the relative influence of the common consequence effect versus changing risk preference (i.e. fanning out). Similarly, systematic shifts in choice patterns from gains to losses could elucidate whether two-person decision makers regard gains and losses fundamentally differently, as non-expected models like prospect theory (but also expected models like Markowitz) suggest.

The naturally occurring environment in which surveys were distributed should be considered an advantage of this experiment, since within-population selection bias has been purged. Nevertheless, the advantage came with tradeoffs. The primary downside was a lack of meticulousness linked with laboratory experiments. People were unlikely to donate more than a small amount of their time, so the surveys needed to be brief, thus restricting the number of data points. Additionally, for reasons not necessarily stemming from the constraints in the field, this rather rudimentary experiment lacks other aspects of thoroughness, found in works such as Camerer (1989), Weber (2007), Incekara-Hafalir and Stecher (2012), which involved the implementation of checks for reliability, large sample sizes, and the effects of framing and incentives. One must therefore give serious consideration to the cautions and potential confounds of this study before interpreting results.

On Reliability and Targeted Violations

Whereas other experiments on generalized utility models have tested whether subjects expressed the same preferences for the same gamble, participants here were not given an explicit opportunity to do so. That is, they were never presented the chance to reverse their choices; such an option did not fit in these one-page surveys. Additionally, participants could not express indifference between gamble-pairs. So this study lacks a strong test of the reliability of responses beyond a statistical z-test of one-time decision patterns. And in light of the empirically observed "preference reversal" phenomenon, by which individuals tend assign a lower certainty equivalent[2] to their selected gamble (see Schoemaker, 1985; MacCrimmon and Wehrung 1986), it must be made clear that the Joint Decisions experiment presumes to draw conclusions regarding the discussed models' descriptive validity for *choice* under uncertainty.

Although Joint Decisions does experimentally test choices under a hypothesis of subjective expected utility, no measures were included to detect specific types of

[2] A person's certainty equivalent for a particular gamble is the money amount, *c*, at which the person would be indifferent, *ex ante*, between playing the gamble and receiving *c* with certainty

probability editing (Conlisk 1989, and Carlin 1990, for instance, use two-stage lotteries to observe whether a subjective editing phase influences obedience to expected utility maximization).

On the Understanding of the Task

After handing out of a large portion of surveys, it was brought to my attention that the gambles were abnormally aligned. In these surveys, gamble-pairs were *vertically* separated, with the second gamble below the first (see appendices A1 and A2). Thus the study unintentionally altered the structuring of examples. Zealous efforts were put toward ensuring that the participants had a clear understanding of the survey task. However, the markings of some participants indicate miscomprehension (see appendix A3). Nonetheless, such instances were quite rare and perhaps no greater than the rates of misunderstanding widely attributed to classic presentation methods. The few surveys like the one in A3, along with all others that cast doubt on the participants' understanding of the prompt, were not used in the study.

On the Understanding of EV Calculation

Harless (1988) found no difference in choice patterns when subjects were provided with expected value calculations and variances. Still, there exists a common criticism that people only violate expected utility maximization because they do not understand the concept of mathematical expectation or do not know how to calculate it. Such a criticism could be directed at this experiment, attributing couples' increases in compliance with expected utility maximization to the fact that two people are more likely than one to have knowledge of mathematical expectation (especially in a university setting). My counterargument points to qualitative data from the study, in which numerous decision-makers of both types (solo and coupled) demonstrated a thorough understanding of expected value calculation but still decided not to use it in their ordering of preferences. Most of the data of this sort comes from the two-person pairs, whose decision-making process almost always involved verbal interaction. In many cases in which one explained expected value to the other, the knowledge did not end up influencing joint preferences. When one (or even both) of the pair made it clear that “you can expect more” in B or that “.01 is so unlikely,” a simple response of “I know but I think we should have $10 for certain” was frequently potent enough to at the very least spark a more profound contemplation of how best to mutually represent a preference. Appendix 4A provides two examples in which the participants’ preferences violated expected utility maximization even though they explicitly calculated expected values or otherwise indicated the calculation of expected values. Such results oppose the argument that violators would adjust their preferences if informed of their violation. The qualitative takeaways here insinuate that many people confronted with normative appeals for the independence axiom will hold fast in favoring nonlinear preference rankings. Future studies will need to provide a more methodical account of this type of qualitative data, however.

On the Framing of Prospects

Savage (1974), Raiffa (1968, p. 80) have argued that Allais behavior would diminish with rephrasing. If respondents had a better conceptualization of the common aspects of the two decisions, they claim, the systematically observed certainty effect would disappear. However, MacCrimmon (1968), Moskowitz (1974), and Slovic and Tversky (1974) all found no change in violation rates when Allais prospects were reframed to provide a clearer representation of the common consequence. In fact, in Moskowitz (1974), the *highest* violation rate accompanied what the majority of subjects said to be the clearest framing of the decision ("decision trees"). Still, the results of MacCrimmon and Larsson (1979) show that, despite the consistent presence of violations, a decline in the rate of violations was associated with altering payouts and their corresponding probabilities away from "critical levels." And Carlin (1990) actually eliminated statistical significance of systematic Allais behavior when probabilities were expressed as numerical ranges on a numbered wheel. Carlin (1992) echoes the conclusions of the latter studies by finding that the clarity in phrasing negatively influenced the rates of typical Allais response patterns.

Other than the unintentional modification of the positions of gamble-pairs, probabilities were classically framed. Hence the Joint Decisions experiment makes no attempt to identify the existence of framing effects.

On the size and salience of payouts

Many of the experiments mentioned so far address the effects of using hypothetical, as opposed to real, payouts. This issue is a rather contentious one. Generally, economists feel that salient payouts are needed to ensure that people do not skimp on the contemplation of their preferences for financial outcomes. Other social scientists claim that people have no reason to misrepresent their preferences when payouts are not salient. Still others suggest that hypothetical payouts generate true but noisily disclosed preferences.

This experiment did not have permission to offer real payouts. And so the salience of incentives is an issue. Moreover, restrictions in both sample size and survey length precluded the ability to examine large swings in the scale of payouts, as this would have spread the data too thin. A small payout range was necessary. Ultimately a low-hypothetical payout range – $x \in (-4, 15)$ – was selected. It can be argued that this is actually better, considering the conclusions of Holt and Laury (2002), who say that "behavior is slightly more erratic under the high-hypothetical treatments" and note that, while risk aversion rises with rising real payouts, "behavior is largely unaffected when hypothetical payouts are scaled up." This is due, according to the researchers, to the fact that "subjects facing hypothetical choices cannot imagine how they would actually behave under high incentive conditions" (pp. 1653-1654).

RESULTS

Between-Subjects

Table 1 - (%) participants selecting the less risky gamble

	N	A	A*
Solo	130	58.46	23.08
Allais	46	56.52	8.70
CR	19	63.16	47.37
R-CR	46	45.65	26.09
CR-LTM	19	89.47	26.32
Couple	116	46.55	21.55
Allais	48	33.33	16.67
CR	10	60.00	0.00
R-CR	48	50.00	33.33
CR-LTM	10	80.00	10.00

The between-subjects analysis in this section deals with the study-wide, average choices for each gamble-pair. Table 1 displays the percentages of respondents choosing the less risky prospect for all examples and for each example separately. Camerer (1989) describes between-subjects analysis as "a picture of how … a hypothetical representative agent might act;" the researcher "[uses] between-subjects analyses to suggest conclusions that will be verified by within-subjects analyses" (p. 85). That is, a between-subjects approach assumes the same preferences produced all the percentages in Table 1, with random variation being the source of discrepancies. So, on this basis, expected utility cannot account for any shifts, beyond those caused by random deviation, toward/away from risk. Since constant risk preferences are implied by linear indifference curves, the classic expected utility model predicts that all of the fractions in Table 1 are the same. Thus, the expected utility model appears to be a poor fit. In almost every example, both the single-person and two-person groups modally selected the less risky A from the first gamble-pair and the riskier B* from the second gamble-pair, indicating an across-the-board shift of risk attitudes. The goodness-of-fit test compared the results in Table 1 to the expected utility requirement that the percentage of less risky choices be the same for both the AB and the A*B* parts of an example. The test yielded {$\chi^2(3, N=130) = 37.116, p > .001$} for solo respondents and {$\chi^2(3, N=116) = 18.792, p > .001$} for couples. Therefore the between-subjects results insinuate a rejection of the hypothesis of parallel, linear indifference curves for either respondent group. The population-wide movement to the riskier choice in the A*B* decision instead suggests indifference curves that are steeper toward the left edge of the (p_1, p_3) unit triangle. The between-subjects measurements intimate the presence of the common-consequence effect in both one-person and two-person decision-makers.

Nevertheless, it should be reiterated that between-subjects findings serve only to *point* toward violations, which can be formally measured by within-subjects choice patterns. Between-subjects assumes that every choice reflects an equal likelihood to violate independence. This could have led to a misrepresentation of actual variance in tastes, leading to an erroneous attribution of the discrepancies in the percentages of Table 1 to a general propensity to violate independence. Nonetheless, Table 1 still suggests that both respondent classes violated independence to a statistically significant degree.

Within-Subjects

Table 2 - (%) participants selecting each choice pattern

	N	AA*	AB*	BA*	BB*
Solo	130	11.54	46.92	11.54	30.00
Allais	46	6.52	50.00	2.17	41.30
CR	19	21.05	42.11	26.32	10.53
R-CR	46	10.87	34.78	15.22	39.13
CR-LTM	19	15.79	73.68	10.53	0.00
Couple	116	14.66	31.90	6.90	46.55
Allais	48	12.50	20.83	4.17	62.50
CR	10	0.00	60.00	0.00	40.00
R-CR	48	22.92	27.08	10.42	39.58
CR-LTM	10	0.00	80.00	10.00	10.00

Within-subjects analysis allows for the direct detection of patterns of behavior. Most importantly, within-subjects allows for tests of the *path* of axiomatic violations. Table 2 shows the within-subjects rates of each choice pattern.

Z and D statistics

The within-subjects statistical analysis conducted herein follows the procedure of Conlisk (1989): the overall violation rate (V) is defined as the percentage of participants who chose either AB* or A*B; Conlisk shows that, under the null hypothesis of equal violation rates between two groups, the groups' violation rates (V_1, V_2) and sample sizes (N_1, N_2) yield a statistic, D, whose probability distribution approximates the standard normal distribution. Page 393 provides the calculation of D:

$$D = (V_1 - V_2)[V_1(1-V_1)(N_1-1)^{-1} + V_2(1-V_2)(N_2-1)^{-1}]^{-(1/2)} \qquad (17)$$

The Conlisk D-statistic was used to determine whether statistically significant differences existed between the violation rates of solo and joint decision makers. Z-tests were performed to measure the statistical significance of particular violation patterns exhibited amongst a particular group. Violations are judged systematic if observed rates are exceedingly unlikely with a presumption of purely random violations.

For instance, table 2 shows that, on aggregate, the most frequent choice pattern among solo decision makers was AB*, chosen in 46.92% of all examples. Two-person decisions, on the other hand, most frequently involved the BB* pattern, selected in 46.55% of examples. As a result, the D-statistic testing the reduction in couple violations is calculated to be 3.13, corresponding to significance level a = $(1-\Phi(3.13)) < .001$. Thus, with respect to aggregated-example response patterns, shared decision-making caused a statistically significant reduction in violations of the independence axiom. At the same time, though, the z-statistic calculated for couples' decisions over all examples was 4.70, corresponding to significance level a = $(1-\Phi(4.70)) < .001$. And so, with respect to choice patterns accumulated over all examples, joint decision-making revealed preferences systematically inconsistent with linear-in-probabilities expected utility preferences; however, the change from solo decision-making to joint decision-making was revealed to have an undeniable downward effect on the prevalence on axiomatic violations. Similarly, an aggregation of the common-ratio examples generated z^{Solo}=3.55, z^{Pair}=4.05, and D=1.65.

Table 3 provides the z and D results for each example and respondent type (critical values are 1.285, 1.645, and 2.33 for respective confidence levels .90, .95, and .99):

Table 3 - test statistics calculated for each example

	z^{Solo}	z^{Pair}	D
Allais	5.93	2.42	2.78
CR	0.83	3.67	0.66
R-CR	1.93	1.94	1.22
CR-LTM	4.02	3.28	-0.44
All	5.93	4.70	3.13

Fanning Out, Certainty Effect and Reflection Effect

Now we can check for empirical support of the theoretical conjectures discussed in previous sections. Again, the relative deficiency of data points makes it impossible to offer clear-cut delineations between hypotheses. Joint Decisions lacks measures intended to provoke and identify such things as preference reversal, isolation effect, indirect violations of dominance, and ambiguity aversion. Nonetheless, a handful of characterizations of behavior – fanning out, certainty effect, reflection effect – are within the scope of this experiment and will be attained with a combination of between-subjects and within-subjects analyses.

Firstly, through the basis of between-subjects, we noticed a study-wide shift away from the less-risky gamble in the A*B* part of the examples. This allows us to make a tentative interpretation that greater risk seeking accompanied the decision between A* and B*. Thus, it has been hinted that indifference curves get steeper around the stochastically dominant gamble-pair. This is consistent with both fanning out and the certainty effect. The between-subjects measurements also hint at the presence of a reflection effect in single-person decision-makers: in the reflected common-ratio example, the AB decision represented the only instance in which B was the modal choice (while the other decision experienced the widespread shift toward B*). Couples, on the other hand, were split 50-50 in the AB decision of the reflected common-ratio example (while making the widespread shift toward risk in the A*B* decision).

The response patterns of solo decision-makers appear weakly consistent with an explanation of fanning out. They modally chose the fanning-consistent AB* pattern in every example except the reflected common-ratio. AB* was also the most preferred pattern for the aggregation of all the examples. However, Table 3 reveals that solo decision-makers did not systematically select the AB* pattern in the standard common-ratio example – z = 0.83. Thus, in the CR example, the fanning-inconsistent BA* pattern comprised a notable fraction of the violations. As a result, a hypothesis of fanning-out predicts almost all of the solo choice patterns.

Fanning out appears to exist more sporadically among couples. In the CR-LTM example, paired decision-makers show a very strong (80%) pattern of AB*, which is consistent with fanning out. In the CR example, 60% of the solo group violated in the same direction, although the remainder chose the fanning-inconsistent BB* pattern. Table 3 shows that couples were statistically significant in violating independence in the AB* direction for every example. However, unlike in the solo group, the AB* pattern was not modal in the accumulated, Allais or R-CR examples. In these examples, couples modally preferred the fanning-inconsistent BB* pattern. For this reason, a hypothesis of fanned out indifference curves only partially explains the preferences of joint decision-makers.

Two-person responses do not appear to exhibit a clear certainty effect either. If couples really liked certain gains (i.e. distributions on the left boundary), then the AB* choice pattern should have been more prevalent in the Allais and CR examples. Instead, AB* was at its most frequent when A moved *away* from the boundary, in the CR-LTM example. Moreover, in the remaining gamble-pairs, the choice patterns of couples indicate a relative preference for the riskier, expected value gains offered on the B/B* side.

Certainty effects appear less prevalent in the solo respondent class as well. AB* is the most frequent pattern in the Allais and standard common-ratio examples, which is a result consistent with certainty effect. However, the solo participants showed by far the most solidarity in their responses to the CR-LTM example, with only 10.53% of solo participants not picking A when facing the AB decision. Again, since AB* was the most frequent pattern when A was shifted from its position of certainty, it would appear that, for solo respondents, an explanation of certainty effects will not go as far as an explanation of fanning out.

Finally, the within-subjects measurements appear to confirm at least a weak reflection effect in both respondent classes. The modal choice patterns in the reflected common-ratio example show that, for both solo and joint decision-makers, the two most selected patterns were AB*and BB*. In both groups, therefore, the modal patterns were comprised of one EU-consistent choice and one EU-inconsistent choice. In each of these, the preferred choice pattern avoided the certain loss represented by A*.

INTERPRETATION OF RESULTS

The results of the Joint Decisions experiment are interpreted as both befuddling and encouraging factors in the modeling of choice under uncertainty. All in all, these results could easily be put toward a claim that collaboration directs people toward an expected utility framework. Although the extent to which two-person decisions currently garner an expected utility characterization remains quite small, the Joint Decisions experiment indicated that collaborative decision-making had a negative impact on violations of independence. I would argue that a *conditioning toward* expected utility maximization takes place under certain market-type scenarios in which two-person decision problems, instead of becoming games, become exercises in joint utility maximization over risky prospects.

It must be noted, though, that the common-ratio and reflected common-ratio examples had z-statistics for couples that were actually larger than those obtained from the control group. And the CR-LTM example even yielded a negative D-statistic. These examples make it clear that this respondent class was not immune to demonstrations of inconsistency with expected utility maximization. However, when results were aggregated to adjust for small sample sizes, there emerged a clear-cut, statistically significant reduction in the violation of independence. The same results applied to the famous Allais example. Furthermore, the within-subjects measurements suggested that preference patterns of couples were less vulnerable to psychological processes like the certainty effect. And while a reflection effect was observed in both solo and paired decision-makers, this does not necessarily imply incompatibility with expected utility preferences; let us recall the Markowitz hypothesis, which preserves linearity in probabilities while still accounting for evaluations based on a reference point and reflected attitudes toward losses.

Lastly, let us consider the third basis of analysis of the survey results: within-subjects-within-surveys. In Tables 4 and 5, one can observe that, in addition to lower one-time-per-example violation rates, couples exhibited significantly lower two-time-per-survey violation rates. While 33.85% of all solo decision-makers violated expected utility maximization at both opportunities, only 17.24% of couples were two-time violators. Couples had slightly larger rates repeated systematic violation, but this aspect could be tied to couples' total elimination of two-time BA* violations.

Table 4 - Two-time violation percentages for solo decision-makers (N^{SOLO}=65)

	(AB*∪BA*)–(AB*∪BA*)	AB*–AB*	BA*–BA*
%participants	33.85	24.62	3.08
%violators	100.00	72.73	9.09

Table 5 - Two-time violation rates for joint decision-makers (N^{PAIR}=58)

	(AB*∪BA*)–(AB*∪BA*)	AB*–AB*	BA*–BA*
%participants	17.24	13.79	0.00
%violators	100.00	80.00	0.00

CONCLUSION

Multi-person decision problems should have a larger presence in utility models of choice. Almost all the literature involving multiple-person, risky-decision scenarios takes a game theoretic approach, which by its nature falls into the modeling of individual behavior under uncertainty. The comparative advantage of this experiment, conversely, is the signaling of another dimension in the modeling of choice under uncertainty: collaboration effects. I believe these affects can change the nature of people's behavior when confronting risky prospects, and can in some instances engender expected utility maximization. Here, the less frequent violations of independence among joint decision-makers suggest that, to some extent, particular types of market interaction can condition people away from using a subjective utility approach. More empirical data will be needed to discern the frameworks that best characterize two-person decision problems. These studies will presumably involve more variation in the types of decisions, better incentives, larger sample sizes, and more/different types of collaborators for each decision.

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APPENDICES

A1. *Survey type #1;* $N^{Solo}=46$, $N^{Pair}=48$
In each part of this survey, there are two options (A and B). Each option represents a hypothetical lottery for small money amounts. Please circle the option you would prefer.

PART ONE

Option A:
Win $10

Option B:
Win $10 with a probability of .89
Win $15 with a probability of .10
Win $0 with a probability of .01

PART TWO

Option A:
Win $10 with a probability of .11
Win $0 with a probability of .89

Option B:
Win $15 with a probability of .10
Win $0 with a probability of .90

PART THREE

Option A:
Lose $4 with a probability of .20

Lose \$0 with a probability of .80

Option B:
Lose \$3 with a probability of .25
Lose \$0 with a probability of .75

PART FOUR

Option A:
Lose \$4 with a probability of .8
Lose \$0 with a probability of .2

Option B:
Lose \$3

A2. *Survey type #2;* $N^{Solo}=19$, $N^{Pair}=10$
In each part of this survey, there are two options (A and B). Each option represents a hypothetical lottery for small money amounts. Please circle the option you would prefer.

PART ONE

Option A:
Receive \$4 with a probability of .80
Receive \$0 with a probability of .20

Option B:
Receive \$3

PART TWO

Option A:
Receive \$4 with a probability of .20
Receive \$0 with a probability of .80

Option B:
Receive \$3 with a probability of .25
Receive \$0 with a probability of .75

PART THREE

Option A:
Receive \$3 with a probability of .90

Receive $0 with a probability of .10

Option B:
Receive $6 with a probability of .45
Receive $0 with a probability of .55

PART FOUR

Option A:
Receive $3 with a probability of .002
Receive $0 with a probability of .998

Option B:
Receive $6 with a probability of .001
Receive $0 with a probability of .999

A3. *Miscomprehension of survey prompt*

In each part of this survey, there are two options (A and B). Each option represents a hypothetical lottery for small money amounts. Please circle the option you would prefer.

PART ONE

Option A:
Win $10

Option B:
Win $10 with a probability of .89
Win $15 with a probability of .10
Win $0 with a probability of .01

PART TWO

Option A:
Win $10 with a probability of .11
Win $0 with a probability of .89

Option B:
Win $15 with a probability of .10
Win $0 with a probability of .90

PART THREE

Option A:
Lose $4 with a probability of .20
Lose $0 with a probability of .80

Option B:
Lose $3 with a probability of .25
Lose $0 with a probability of .75

PART FOUR

Option A:
Lose $4 with a probability of .8
Lose $0 with a probability of .2

Option B:
Lose $3

A4. *(i) Explicit expected value calculation with EU violation (solo)*

In each part of this survey, there are two options (A and B). Each option represents a hypothetical lottery for small money amounts. Please circle the option you would prefer.

PART ONE

Option A:
Win $10

Option B:
Win $10 with a probability of .89
Win $15 with a probability of .10
Win $0 with a probability of .01

0

PART TWO

Option A:
1.1 Win $10 with a probability of .11
Win $0 with a probability of .89

1.5 Option B:
Win $15 with a probability of .10
Win $0 with a probability of .90

PART THREE

Option A:
-0.8 Lose $4 with a probability of .20
Lose $0 with a probability of .80

Option B:
Lose $3 with a probability of .25
-0.75 Lose $0 with a probability of .75

PART FOUR

Option A:
Lose $4 with a probability of .8
Lose $0 with a probability of .2

Option B:
Lose $3

(ii) Explicit indication of expected value calculation with EU violation (couple)

In each part of this survey, there are two options (A and B). Each option represents a hypothetical lottery for small money amounts. Please circle the option you would prefer.

PART ONE

Option A:
Win $10

Option B:
Win $10 with a probability of .89
Win $15 with a probability of .10
Win $0 with a probability of .01

C

PART TWO

Option A:
Win $10 with a probability of .11
Win $0 with a probability of .89

Option B:
Win $15 with a probability of .10
Win $0 with a probability of .90

PART THREE

Option A:
Lose $4 with a probability of .20
Lose $0 with a probability of .80

Option B:
Lose $3 with a probability of .25
Lose $0 with a probability of .75

PART FOUR

Option A:
Lose $4 with a probability of .8
Lose $0 with a probability of .2

Option B:
Lose $3

we know this makes more sense but A is more fun