2006 Fields Medals Awarded

On August 22, 2006, four Fields Medals were awarded at the opening ceremonies of the International Congress of Mathematicians (ICM) in Madrid, Spain. The medalists are Andrei Okounkov, Grigory Perelman, Terence Tao, and Wendelin Werner. [Editors Note: During the award ceremony, John Ball, president of the International Mathematical Union, announced that Perelman declined to accept the Fields Medal.]

The Fields Medals are given every four years by the International Mathematical Union (IMU). Although there is no formal age limit for recipients, the medals have traditionally been presented to mathematicians not older than forty years of age, as an encouragement for future achievement. The medal is named after the Canadian mathematician John Charles Fields (1863–1932), who organized the 1924 ICM in Toronto. At a 1931 meeting of the Committee of the International Congress, chaired by Fields, it was decided that funds left over from the Toronto ICM "should be set apart for two medals to be awarded in connection with successive International Mathematical Congresses." In outlining the rules for awarding the medals, Fields specified that the medals "should be of a character as purely international and impersonal as possible." During the 1960s, in light of the great expansion of mathematics research, the possible number of medals to be awarded was increased from two to four. Today the Fields Medal is recognized as the world's highest honor in mathematics.

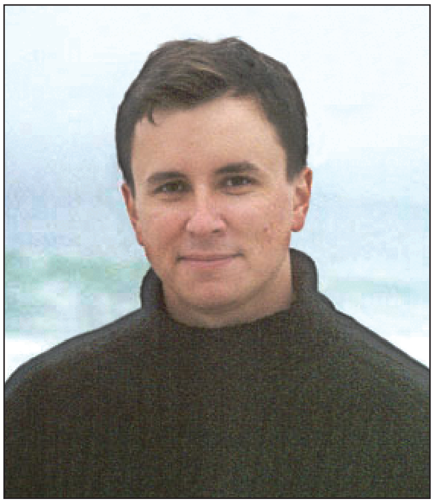
Previous recipients of the Fields Medal are: Lars V. Ahlfors and Jesse Douglas (1936); Laurent Schwartz and Atle Selberg (1950); Kunihiko Kodaira and Jean-Pierre Serre (1954); Klaus F. Roth and René Thom (1958); Lars Hörmander and John W. Milnor (1962); Michael F. Atiyah, Paul J. Cohen, Alexandre Grothendieck, and Stephen Smale (1966); Alan Baker, Heisuke Hironaka, Sergei P. Novikov, and John G. Thompson (1970); Enrico Bombieri and David B. Mumford (1974); Pierre R. Deligne, Charles L. Fefferman, Grigorii A. Margulis, and Daniel G. Quillen (1978); Alain Connes, William P. Thurston, and Shing-Tung Yau (1982); Simon K. Donaldson, Gerd Faltings, and Michael H. Freedman (1986); Vladimir Drinfeld, Vaughan F. R. Jones, Shigefumi Mori, and Edward Witten (1990); Jean Bourgain, Pierre-Louis Lions, Jean-Christoph Yoccoz, and Efim Zelmanov (1994); Richard Borcherds, William Timothy Gowers, Maxim Kontsevich, and Curtis T. McMullen (1998); Laurent Lafforgue and Vladimir Voevodsky (2002).

Andrei Okounkov

Citation: "for his contributions bridging probability, representation theory and algebraic geometry".

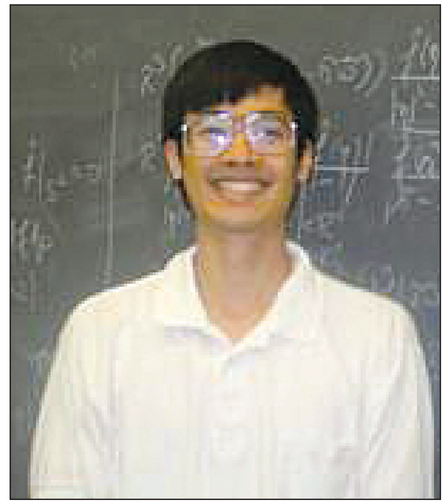
The work of Andrei Okounkov has revealed profound new connections between different areas of mathematics and has brought new insights into problems arising in physics. Although his work is difficult to classify because it touches on such a variety of areas, two clear themes are the use of notions of randomness and of classical ideas from representation theory. This combination has proven powerful in attacking problems from algebraic geometry and statistical mechanics.

One of the basic objects of study in representation theory is the "symmetric group", whose elements are permutations of objects. For example, if the objects are the letters {C, G, J, M, N, O, Q, Z}, then a permutation is an ordering of the letters, such as GOQZMNJC or JZOQCGNM. The number of



Andrei Okounkov

Grigory Perelman



Terence Tao

Wendelin Werner

possible permutations grows quickly as the number of objects grows; for 8 objects, there are already 40,320 different permutations. If we consider an abstract set of n objects, then the "symmetric group on n letters" is the collection of all the different permutations of those n objects, together with rules for combining the permutations.

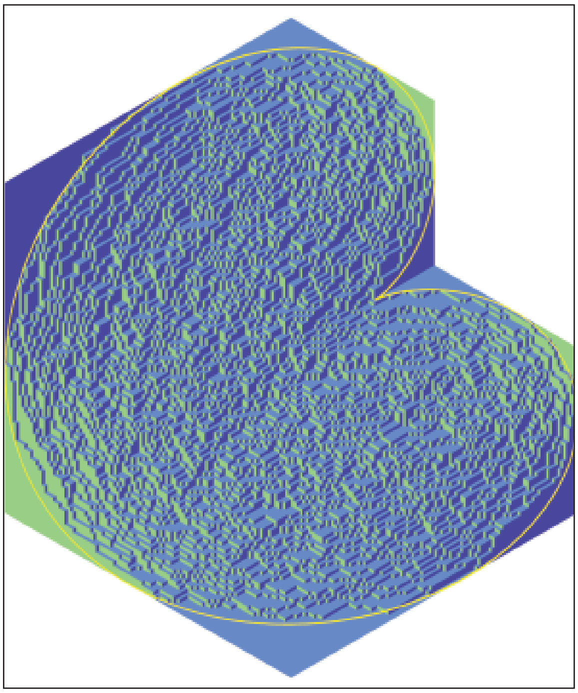
Representation theory allows one to study the symmetric group by representing it by other mathematical objects that provide insights into the group's salient features. The representation theory of the symmetric group is a well developed subfield that has important uses within mathematics itself and also in other scientific areas, such as quantum mechanics. It turns out that, for the symmetric group on n letters, the building blocks for all of its representations are indexed by the "partitions" of n. A partition of a number n is just a sequence of positive numbers that add up to n; for example $2 + 3 + 3 + 4 + 12$ is a partition of 24.

Through the language of partitions, representation theory connects to another branch of mathematics called "combinatorics", which is the study of objects that have discrete, distinct parts. Many continuous phenomena in mathematics are related by virtue of having a common discrete substructure, which then raises combinatorial questions. Continuous phenomena can also be discretized, making them amenable to the methods of combinatorics. Partitions are among the most basic combinatorial objects, and their study goes back at least to the 18th century.

Randomness enters into combinatorics when one considers very large combinatorial objects, such as the set of all partitions of a very large number. If one thinks of partitioning a number as randomly cutting it up into smaller numbers, one can ask, What is the probability of obtaining a particular partition? Questions of a similar nature arise in representation theory of large symmetric groups. Such links between probability and representation theory were considered by mathematicians in Russia during the 1970s and 1980s. The key to finding just the right tool from probability theory suited to this question derives from viewing partitions as representations of the symmetric group. A Russian who studied at Moscow State University, Andrei Okounkov absorbed this viewpoint and has deployed it with spectacular success to attack a wide range of problems.

One of his early outstanding results concerns "random matrices", which have been extensively studied in physics. A random matrix is a square array of numbers in which each number is chosen at random. Each random matrix has associated with it a set of characteristic numbers called the "eigenvalues" of the matrix. Starting in the 1950s, physicists studied the statistical properties of eigenvalues of random matrices to gain insight into the problem of the prediction and distribution of energy levels of nuclei. In recent years, random matrices have received renewed attention by mathematicians and physicists.

Okounkov has used ideas from quantum field theory to prove a surprising connection between random matrices and increasing subsequences in permutations of numbers. An increasing subsequence is just what it sounds like: For example, in a permutation of the numbers from 1 up to 8, say 71452638, two increasing subsequences are 14568 and 1238. There is a way to arrange these increasing subsequences into a hierarchy: the longest subsequence, followed by the second-longest, the third-longest, and so forth, down to the shortest.



This picture shows a random surface that can be thought of as the "melting" of a crystal. The heart-shaped curve forming the border between the melted and frozen regions is called a cardioid.

Image courtesy of Richard Kenyon and Andrei Okounkov.

Okounkov proved that, for very large n, the sequence of largest eigenvalues of an n-by-n random matrix behaves, from the probabilistic point of view, in the same way as the lengths of the longest increasing subsequences in permutations of the numbers from 1 to n. In his proof, Okounkov took a strikingly original approach by reformulating the question in a completely different context, namely, as a comparison of two different descriptions of a random surface. This work established a connection to algebraic geometry, providing a seed for some of his later work in that subject.

Random surfaces also arise in Okounkov's work in statistical mechanics. If one heats, say, a cubical crystal from a low temperature, one finds that the corners of the cube are eaten away as the crystal "melts". The geometry of this melting process can be visualized by imagining a corner to consist of a bunch of tiny blocks. The melting of the crystal corresponds to removing blocks at random. Thinking of the partitioning of the crystal into tiny blocks as analogous to partitioning integers, Okounkov brought his signature methods to bear on the analysis of the random surfaces that arise. In joint work with Richard Kenyon, Okounkov proved the surprising result that the melted part of the crystal, when projected onto two dimensions, has a very distinctive shape and is always encircled by an algebraic curve—that is, a curve that can be defined by polynomial equations. This is illustrated in the accompanying figure; here the curve is a heart-shaped curve called a cardioid. The connection with real algebraic geometry is quite unexpected.

Over the past several years, Okounkov has, together with Rahul Pandharipande and other collaborators, written a long series of papers on questions in enumerative algebraic geometry, an area with a long history that in recent years has been enriched by the exchange of ideas between mathematicians and physicists. A standard way of studying algebraic curves is to vary the coefficients in the polynomial equations that define the curves and then impose conditions—for example, that the curves pass through a specific collection of points. With too few conditions, the collection of curves remains infinite; with too many, the collection is empty. But with the right balance of conditions, one obtains a finite collection of curves. The problem of "counting curves" in this way—a longstanding problem in algebraic geometry that also arose in string theory—is the main concern of enumerative geometry. Okounkov and his collaborators have made substantial contributions to enumerative geometry, bringing in ideas from physics and deploying a wide range of tools from algebra, combinatorics, and geometry. Okounkov's ongoing research in this area represents a marvelous interplay of ideas from mathematics and physics.

Andrei Okounkov was born in 1969 in Moscow. He received his doctorate in mathematics from Moscow State University in 1995. He is a professor of mathematics at Princeton University. He has also held positions at the Russian Academy of Sciences, the Institute for Advanced Study in Princeton, the University of Chicago, and the University of California, Berkeley. His distinctions include a Sloan Research Fellowship (2000), a Packard Fellowship (2001), and the European Mathematical Society Prize (2004).

Grigory Perelman

Citation: "for his contributions to geometry and his revolutionary insights into the analytical and geometric structure of the Ricci flow".

The name of Grigory Perelman is practically a household word among the scientifically interested public. His work from 2002-2003 brought groundbreaking insights into the study of evolution equations and their singularities. Most significantly, his results provide a way of resolving two outstanding problems in topology: the Poincaré Conjecture and the Thurston Geometrization Conjecture. As of the summer of 2006 the mathematical community is still in the process of checking his work to ensure that it is entirely correct and that the conjectures have been proved. After more than three years of intense scrutiny, top experts have encountered no serious problems in the work.

For decades the Poincaré Conjecture has been considered one of the most important problems in mathematics. The conjecture received increased attention from the general public when it was named as one of the seven Millennium Prize Problems established by the Clay Mathematics Institute in 2000. The institute has pledged to award a prize of US$1 million for the solution of each problem. The work of Perelman on the Poincaré Conjecture is the first serious contender for one of these prizes.

The Poincaré Conjecture arises in topology, which studies fundamental properties of shapes that remain unchanged when the shapes are deformed—that is, stretched, warped, or molded, but not torn. A simple example of such a shape is the 2-sphere, which is the 2-dimensional surface of a ball in 3-dimensional space. Another way to visualize the 2-sphere is to take a disk lying in the 2-dimensional plane and identify the disk's boundary points to a single point; this point can be thought of as the north pole of the 2-sphere. Although globally the 2-sphere looks very different from the plane, every point on the sphere sits in a region that looks like the plane. This property of looking locally like the plane is the defining property of a 2-dimensional manifold, or 2-manifold. Another example of a 2-manifold is the "torus", which is the surface of a doughnut.

Although locally the 2-sphere and the torus look the same, globally their topologies are distinct: Without tearing a hole in the 2-sphere, there is no way to deform it into the torus. Here is another way of seeing this distinction. Consider a loop lying on the 2-sphere. No matter where it is situated on the 2-sphere, the loop can be shrunk down to a point, with the shrinking done entirely within the sphere. Now imagine a loop lying on the torus: If the loop goes around the hole, the loop cannot be shrunk to a point. If loops can be shrunk to a point in a manifold, the manifold is called "simply connected". The 2-sphere is simply connected, while the torus is not. The analogue of the Poincaré Conjecture in 2 dimensions would be the assertion that any simply connected 2-manifold of finite size can be deformed into the 2-sphere, and this assertion is correct. It is natural then to ask, What can be said about non-simply-connected 2-manifolds? It turns out that they can all be classified according to the number of holes: They are all deformations of the torus, or of the double-torus (with 2 holes), or of the triple torus (the surface of a pretzel), etc. (One actually needs two other technical assumptions in this discussion, compactness and orientability.)

Geometry offers another way of classifying 2-manifolds. When one views manifolds topologically, there is no notion of measured distance. Endowing a manifold with a metric provides a way of measuring distance between points in the manifold and leads to the geometric notion of curvature. 2-manifolds can be classified by their geometry: A 2-manifold with positive curvature can be deformed into a 2-sphere; one with zero curvature can be deformed into a torus; and one with negative curvature can be deformed into a torus with more than one hole.

The Poincaré Conjecture, which originated with the French mathematician Henri Poincaré in 1904, concerns 3-dimensional manifolds, or 3-manifolds. A basic example of a 3-manifold is the 3-sphere: In analogy with the 2-sphere, one obtains the 3-sphere by taking a ball in 3-dimensions and identifying its boundary points to a single point. (Just as 3-dimensional space is the most natural home for the 2-sphere, the most natural home for the 3-sphere is 4-dimensional space—which of course is harder to visualize.) Can every simply connected 3-manifold be deformed into the 3-sphere? The Poincaré Conjecture asserts that the answer to this question is yes.

Just as with 2-manifolds, one could also hope for a classification of 3-manifolds. In the 1970s Fields Medalist William Thurston made a new conjecture, which came to be called the Thurston Geometrization Conjecture and which gives a way to classify all 3-manifolds. The Thurston Geometrization Conjecture provides a sweeping vision of 3-manifolds and actually includes the Poincaré Conjecture as a special case. Thurston proposed that, in a way analogous to the case of 2-manifolds, 3-manifolds can be classified using geometry. But the analogy does not extend very far: 3-manifolds are much more diverse and complex than 2-manifolds.

Thurston identified and analyzed 8 geometric structures and conjectured that they provide a means for classifying 3-manifolds. His work revolutionized the study of geometry and topology. The 8 geometric structures were intensively investigated, and the Geometrization Conjecture was verified in many cases; Thurston himself proved it for a large class of manifolds. But hopes for a proof of the conjecture in full generality remained unfulfilled.

In 1982 Richard Hamilton identified a particular evolution equation, which he called the Ricci flow, as the key to resolving the Poincaré and Thurston Geometrization Conjectures. The Ricci flow is similar to the heat equation, which describes how heat flows from the hot part of an object to the cold part, eventually homogenizing the temperature to be uniform throughout the object. Hamilton's idea was to use the Ricci flow to homogenize the geometry of 3-manifolds to show that their geometry fits into Thurston's classification. Over more than twenty years, Hamilton and other geometric analysts made great progress in understanding the Ricci flow. But they were stymied in figuring out how to handle "singularities", which are regions where the geometry, instead of getting homogenized, suddenly exhibits uncontrolled changes.

That was where things stood when Perelman's work burst onto the scene. In a series of papers posted on a preprint archive starting in late 2002, Perelman established ground-breaking results about the Ricci flow and its singularities. He provided new ways of analyzing the structure of the singularities and showed how they relate to the topology of the manifolds. Perelman broke the impasse in the program that Hamilton had established and validated the vision of using the Ricci flow to prove the Poincaré and Thurston Geometrization Conjectures. Although Perelman's work appears to provide a definitive endpoint in proving the conjectures, his contributions do not stop there. The techniques Perelman introduced for handling singularities in the Ricci flow have generated great excitement in geometric analysis and are beginning to be deployed to solve other problems in that area.

Perelman's combination of deep insights and technical brilliance mark him as an outstanding mathematician. In illuminating a path towards answering two fundamental questions in 3-dimensional topology, he has had a profound impact on mathematics.

Grigory Perelman was born in 1966 in what was then the Soviet Union. He received his doctorate from St. Petersburg State University. During the 1990s he spent time in the United States, including as a Miller Fellow at the University of California, Berkeley. He was for some years a researcher in the St. Petersburg Department of the Steklov Institute of Mathematics. In 1994 he was an invited speaker at the International Congress of Mathematicians in Zurich.

Terence Tao

Citation: "for his contributions to partial differential equations, combinatorics, harmonic analysis and additive number theory".

Terence Tao is a supreme problem-solver whose spectacular work has had an impact across several mathematical areas. He combines sheer technical power, an other-worldly ingenuity for hitting upon new ideas, and a startlingly natural point of view that leaves other mathematicians wondering, "Why didn't anyone see that before?"

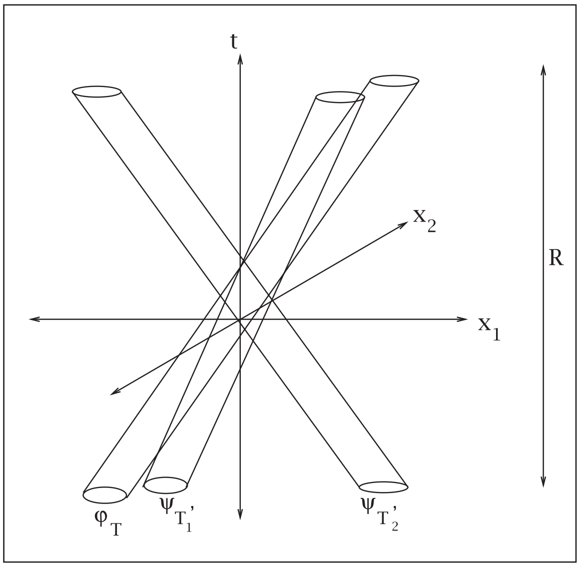
At 31 years of age, Tao has written over eighty research papers, with over thirty collaborators, and his interests range over a wide swath of mathematics, including harmonic analysis, nonlinear partial differential equations, and combinatorics. "I work in a number of areas, but I don't view them as being disconnected," he said in an interview published in the Clay Mathematics Institute Annual Report. "I tend to view mathematics as a unified subject and am particularly happy when I get the opportunity to work on a project that involves several fields at once."

Because of the wide range of his accomplishments, it is difficult to give a brief summary of Tao's oeuvre. A few highlights can give an inkling of the breadth and depth of the work of this extraordinary mathematician.

The first highlight is Tao's work with Ben Green, a dramatic new result about the fundamental building blocks of mathematics, the prime numbers. Green and Tao tackled a classical question that was probably first asked a couple of centuries ago: Does the set of prime numbers contain arithmetic progressions of any length? An "arithmetic progression" is a sequence of whole numbers that differ by a fixed amount: 3, 5, 7 is an arithmetic progression of length 3, where the numbers differ by 2; 109, 219, 329, 439, 549 is a progression of length 5, where the numbers differ by 110. A big advance in understanding arithmetic progressions came in 1974, when the Hungarian mathematician Emre Szemeredi proved that any infinite set of numbers that has "positive density" contains arithmetic progressions of any length. A set has positive density if, for a sufficiently large number n, there is always a fixed percentage of elements of $1, 2, 3, \ldots n$ in the set. Szemeredi's theorem can be seen from different points of view, and there are now at least three different proofs of it, including Szemeredi's original proof and one by 1998 Fields Medalist Timothy Gowers. The primes do not have positive density, so Szemeredi's theorem does not apply to them; in fact, the primes get sparser and sparser as the integers stretch out towards infinity. Remarkably, Green and Tao proved that, despite this sparseness, the primes do contain arithmetic progressions of any length. Any result that sheds new light on properties of prime numbers marks a significant advance. This work shows great originality and insight and provides a solution to a deep, fundamental, and difficult problem.

Another highlight of Tao's research is his work on the Kakeya Problem, which in its original form can be described in the following way. Suppose you have a needle lying flat on a plane. Imagine the different possible shapes swept out when you rotate the needle 180 degrees. One possible shape is a half-disk; with a bit more care, you can perform the rotation within a quarter-disk. The Kakeya problem asks, What is the minimum area of the shape swept out in rotating the needle 180 degrees? The surprising answer is that the area can be made as small as you like, so in some sense the minimum area is zero. The fractal dimension of the shape swept out provides a finer kind of information about the size of the shape than you obtain in measuring its area. A fundamental result about the Kakeya problem says that the fractal dimension of the shape swept out by the needle is always 2.

Imagine now that the needle is not in a flat plane, but in n-dimensional space, where n is bigger than 2. The n-dimensional Kakeya problem asks, What is the minimum volume of an n-dimensional shape in which the needle can be turned in any direction? Analogously with the 2-dimensional case, this volume can be made as small as you like. But a more crucial question is, What can be said about the fractal dimension of this n-dimensional shape? No one knows the answer to that question. The technique of the proof that, in the 2-dimensional plane the fractal dimension is always 2, does not work in higher dimensions. The n-dimensional Kakeya problem is interesting in its own right and also has fundamental connections to other problems in mathematics in, for example, Fourier analysis and nonlinear waves. Terence Tao has been a major force in recent years in investigating the Kakeya problem in n dimensions and in elucidating its connections to other problems in the field.



Tubes that are transverse can have smaller intersection, and thus larger union, than tubes that are nearly parallel. Recent progress on problems such as the Kakeya conjecture has been aided by a "bilinear" approach that excludes the latter case from consideration.
Image courtesy of Terence Tao.

Another problem Tao has worked on is understanding wave maps. This topic arises naturally in the study of Einstein's theory of general relativity, according to which gravity is a nonlinear wave. No one knows how to solve completely the equations of general relativity that describe gravity; they are simply beyond current understanding. However, the equations become far simpler if one considers a special case, in which the equations have cylindrical symmetry. One aspect of this simpler case is called the "wave maps" problem, and Tao has developed a program that would allow one to understand its solution. While this work has not reached a definitive endpoint, Tao's ideas have removed a major psychological obstacle by demonstrating that the equations are not intractable, thereby causing a resurgence of interest in this problem.

A fourth highlight of Tao's work centers on the nonlinear Schrödinger equations. One use of these equations is to describe the behavior of light in a fiber optic cable. Tao's work has brought new insights into the behavior of one particular Schrödinger equation and has produced definitive existence results for solutions. He did this work in collaboration with four other mathematicians, James Colliander, Markus Keel, Gigliola Staffilani, and Hideo Takaoka. Together they have become known as the "I-team", where "I" denotes many different things, including "interaction". The word refers to the way that light can interact with itself in a medium such as a fiber optic cable; this self-interaction is reflected in the nonlinear term in the Schrödinger equation that the team studied. The word "interaction" also refers to interactions among the team members, and indeed collaboration is a hallmark of Tao's work. "Collaboration is very important for me, as it allows me to learn about other fields, and, conversely, to share what I have learnt about my own fields with others," he said in the Clay Institute interview. "It broadens my experience, not just in a technical mathematical sense, but also in being exposed to other philosophies of research and exposition."

These highlights of Tao's work do not tell the whole story. For example, many mathematicians were startled when Tao and co-author Allen Knutson produced beautiful work on a problem known as Horn's conjecture, which arises in an area that one would expect to be very far from Tao's expertise. This is akin to a leading English-language novelist suddenly producing the definitive Russian novel. Tao's versatility, depth, and technical prowess ensure that he will remain a powerful force in mathematics in the decades to come.

Terence Tao was born in Adelaide, Australia, in 1975. He received his Ph.D. in mathematics in 1996 from Princeton University. He is a professor of mathematics at the University of California, Los Angeles. Among his distinctions are a Sloan Foundation Fellowship, a Packard Foundation Fellowship, and a Clay Mathematics Institute Prize Fellowship. He was awarded the Salem Prize (2000), AMS Bôcher Prize (2002), and the AMS Conant Prize (2005, jointly with Allen Knutson).

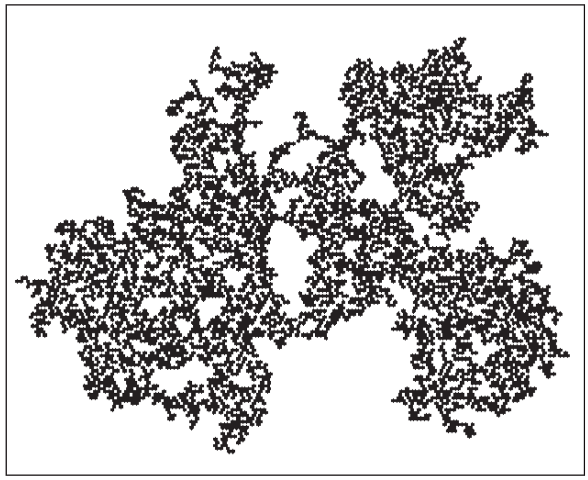
Wendelin Werner

Citation: "for his contributions to the development of stochastic Loewner evolution, the geometry of two-dimensional Brownian motion, and conformal field theory".

The work of Wendelin Werner and his collaborators represents one of the most exciting and fruitful interactions between mathematics and physics in recent times. Werner's research has developed a new conceptual framework for understanding critical phenomena arising in physical systems and has brought new geometric insights that were missing before. The theoretical ideas arising in this work, which combines probability theory and ideas from classical complex analysis, have had an important impact in both mathematics and physics and have potential connections to a wide variety of applications.

A motivation for Wendelin Werner's work is found in statistical physics, where probability theory is used to analyze the large-scale behavior of complex, many-particle systems. A standard example of such a system is that of a gas: Although it would be impossible to know the position of every molecule of air in the room you are sitting in, statistical physics tells you it is extremely unlikely that all the air molecules will end up in one corner of the room. Such systems can exhibit phase transitions that mark a sudden change in their macroscopic behavior. For example, when water is boiled, it undergoes a phase transition from being a liquid to being a gas. Another classical example of a phase transition is the spontaneous magnetization of iron, which depends on temperature. At such phase transition points, the systems can exhibit so-called critical phenomena. They can appear to be random at any scale (and in particular at the macroscopic level) and become "scale-invariant", meaning that their general behavior appears statistically the same at all scales. Such critical phenomena are remarkably complicated and are far from completely understood.

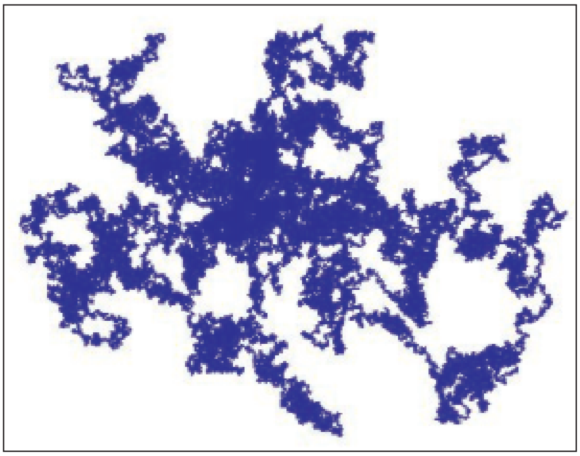
In 1982 physicist Kenneth G. Wilson received the Nobel Prize for his study of critical phenomena, which helped explain "universality": Many different physical systems behave in the same way as they get near critical points. This behavior is described by functions in which a quantity (for instance the difference between the actual temperature and the critical one) is raised to an exponent, called a "critical exponent" of the system. Physicists have conjectured that these exponents are universal in the sense that they depend only on some qualitative features of the system and not on its microscopic details. Although the systems that Wilson was interested in were mainly three- and four-dimensional, the same phenomena also arise in two-dimensional systems. During the 1980s and 1990s physicists made big strides in developing conformal field theory, which provides an approach to studying two-dimensional critical phenomena. However, this approach was difficult to understand in a rigorous mathematical way, and it provided no geometric picture of how the systems behaved. One great accomplishment of Wendelin Werner, together with his collaborators Gregory Lawler and Oded Schramm, has been to develop a new approach to critical phenomena in two dimensions that is mathematically rigorous and that provides a direct geometric picture of systems at and near their critical points.



A percolation cluster.
Image courtesy of Wendelin Werner.

Percolation is a model that captures the basic behavior of, for example, a gas percolating through a random medium. This medium could be a horizontal network of pipes where, with a certain probability, each pipe is open or blocked. Another example is the behavior of pollutants in an aquifer. One would like to answer questions such as, What does the set of polluted sites look like? Physicists and mathematicians study schematic models of percolation such as the following. First, imagine a plane tiled with hexagons. A toss of a (possibly biased) coin decides whether a hexagon is colored white or black, so that for any given hexagon the probability that it gets colored black is p and the probability that it gets colored white is then $1-p$. If we designate one point in the plane as the origin, we can ask, Which parts of the plane are connected to the origin via monochromatic black paths? This set is called the "cluster" containing the origin. If p is smaller than $1/2$, there will be fewer black hexagons than white ones, and the cluster containing the origin will be finite. Conversely, if p is larger than $1/2$, there is a positive chance that the cluster containing the origin is infinite. The system undergoes a phase transition at the critical value $p = 1/2$. This critical value corresponds to the case where one tosses a fair coin to choose the color for each hexagon. In this case, one can prove that all clusters are finite and that whatever large portion of the lattice one chooses to look at, one will find (with high probability) clusters of size comparable to that portion. The accompanying picture represents a sample of a fairly large cluster.

The percolation model has drawn the interest of theoretical physicists, who used various nonrigorous techniques to predict aspects of its critical behavior. In particular, about fifteen years ago, the physicist John Cardy used conformal field theory to predict some large-scale properties of percolation at its critical point. Werner and his collaborators Lawler and Schramm studied the continuous object that appears when one takes the large-scale limit—that is, when one allows the hexagon size to get smaller and smaller. They derived many of the properties of this object, such as, for instance, the fractal dimension of the boundaries of the clusters. Combined with Stanislav Smirnov's 2001 results on the percolation model and earlier results by Harry Kesten, this work led to a complete derivation of the critical exponents for this particular model.



The path of Brownian motion.
Image courtesy of Wendelin Werner.

Another two-dimensional model is planar Brownian motion, which can be viewed as the large-scale limit of the discrete random walk. The discrete random walk describes the trajectory of a particle that chooses at random a new direction at every unit of time. The geometry of planar Brownian paths is quite complicated. In 1982, Benoit Mandelbrot conjectured that the fractal dimension of the outer boundary of the trajectory of a Brownian path (the outer boundary of the blue set in the accompanying picture) is 4/3. Resolving this conjecture seemed out of reach of classical probabilistic techniques. Lawler, Schramm, and Werner proved this conjecture first by showing that the outer frontier of Brownian paths and the outer boundaries of the continuous percolation clusters are similar, and then by computing their common dimension using a dynamical construction of the continuous percolation clusters. Using the same strategy, they also derived the values of the closely related "intersection exponents" for Brownian motion and simple random walks that had been conjectured by physicists B. Duplantier and K.-H. Kwon (one of these intersection exponents describes the probability that the paths of two long walkers remain disjoint up to some very large time). Further work of Werner exhibited additional symmetries of these outer boundaries of Brownian loops.

Another result of Wendelin Werner and his coworkers is the proof of the "conformal invariance" of some two-dimensional models. Conformal invariance is a property similar to, but more subtle and more general than, scale invariance and lies at the roots of the definition of the continuous objects that Werner has been studying. Roughly speaking, one says that a random two-dimensional object is conformally invariant if its distortion by angle-preserving transformations (these are called conformal maps and are basic objects in complex analysis) have the same law as the object itself. The assumption that many critical two-dimensional systems are conformally invariant is one of the starting points of conformal field theory. Smirnov's above-mentioned result proved conformal invariance for percolation. Werner and his collaborators proved conformal invariance for two classical two-dimensional models, the loop-erased random walk and the closely related uniform spanning tree, and described their scaling limits. A big challenge in this area now is to prove conformal invariance results for other two-dimensional systems.

Mathematicians and physicists had developed very different approaches to understanding two-dimensional critical phenomena. The work of Wendelin Werner has helped to bridge the chasm between these approaches, enriching both fields and opening up fruitful new areas of inquiry. His spectacular work will continue to influence both mathematics and physics in the decades to come.

Born in 1968 in Germany, Wendelin Werner is of French nationality. He received his Ph.D. at the University of Paris VI in 1993. He has been professor of mathematics at the University of Paris-Sud in Orsay since 1997. From 2001 to 2006, he was also a member of the Institut Universitaire de France, and since 2005 he has been seconded part-time to the Ecole Normale Supérieure in Paris. Among his distinctions are the Rollo Davidson Prize (1998), the European Mathematical Society Prize (2000), the Fermat Prize (2001), the Jacques Herbrand Prize (2003), the Loève Prize (2005) and the Pólya Prize (2006).

—From IMU news releases