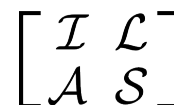


IMAGE



Serving the International Linear Algebra Community
Issue Number 54, pp. 1–44, Spring 2015

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ISSN 1553-8991. Produced by the International Linear Algebra Society (ILAS). Two issues are published each year, on June 1 and December 1. The editors reserve the right to select and edit material submitted. All issues may be viewed at <http://www.ilasic.org/IMAGE>. *IMAGE* is typeset using L^AT_EX. Photographs for this issue were obtained from websites referenced in articles, university department websites, or from reporters directly, unless indicated otherwise here: photo [p. 31] from SIAM at <http://fellows.siam.org/index.php?sort=year&value=2015>.

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FEATURE INTERVIEWS

“Venturing Into Something New”

Ravindra Bapat Interviewed by **A.K. Lal¹,**
Manjunath Prasad², and **Sukanta Pati³**



Ravindra Bapat

Q - When and how did you get interested in mathematics?

Bapat - During my school days, I liked mathematics as a subject but I thought it was something very natural, that everybody would get it easily, and I found it somewhat predictable. Perhaps it had something to do with the school syllabus not being very challenging. In that sense, I did not have to make any special effort to study the subject and never really made an attempt to sincerely study until a few days before the examination. This continued throughout the initial high school classes; maybe in the last one or two years I put in some effort. I liked geometry and had the sense that I liked mathematics but there wasn't any strong inclination towards mathematics. Later on, after going to college, I found mathematics interesting but found pure mathematics a little too abstract. So, for my bachelor's, I didn't choose pure mathematics as the main subject but rather chose statistics because I liked mathematical statistics. I realize now that I do not have the feeling for data that is required to do statistics. But I continued with statistics as a subject even for my master's and the combination of statistics and mathematics was something that I found interesting.

While starting research towards my Ph.D., again initially I was just trying to do something in game theory and a linear complementarity problem because of the interest of my advisor; but then, within the first one or two semesters, I started liking linear algebra. I was attracted by very simple statements which connected together and formed somewhat of a broad theory; for example, I was fascinated by the various characterizations of positive definite matrices. That certainly was the point when I got interested in matrix theory as a subject rather than mathematics as a whole. And then I started to study nonnegative matrices. At that time the book of Berman and Plemmons on nonnegative matrices had come out and we started doing some work on the van der Waerden conjecture which was a major open problem at that time. My thesis was about some attempt towards the conjecture. Permanents of nonnegative matrices continued to be my main interest for several years.

Q - As per our information, after your undergraduate degree, you joined a famous management institute in Mumbai. At what point did you decide to leave it and why?

Bapat - There were two instances in my academic career when I strayed away from the path of academics or mathematics as a subject. The first instance was when I finished the first two years of college and appeared for the entrance examination of the Indian Institute of Technology, which continues even now to be an attraction for students in India, because of either parental pressure or pressure from the community and friends. I did succeed in the examination in the sense that I was called for the interview and was sure to get a seat. However, while studying for the examination, I realized that I did not want to go for engineering and decided not to appear for the interview. It was a very difficult decision at that time, because admission to the institute is considered very prestigious. Thus I did not go for engineering but instead went for the bachelor of science.

The second instance came when I finished the bachelor of science and there was an option to join the Bajaj Institute of Management, a very prestigious management institute where the admission was considered very difficult, but I could get admission on the basis of my bachelor's score. I, in fact, joined the institute and completed one semester and during that semester I realized this is not what I wanted to do, because completing the course would have meant immediately going for employment in some commercial place or industry, and I did not want to do that so early. So, after one semester, I decided to leave the management institute and then it was too late to join the regular master's programme at Bombay University. But I found a way out as the Indian Statistical Institute had an examination which is called the Statisticians' Diploma where one studies and writes the exam, and it is equivalent to the first year of a master in science. I decided

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to write that examination. I studied for that exam for about two or three months and completed all the ten papers and thus became eligible for admission into the second year of the master of statistics at the Indian Statistical Institute in Delhi.

Q - Did you choose your advisor or did your advisor choose you when you were a student at the Indian Statistical Institute?

Bapat - In a way it was mutual. The Delhi campus of the Indian Statistical Institute started in 1975–76 and our class was the first batch. It was a very exciting place. Professor C. R. Rao was in the institute, S. K. Mitra was there, very famous statisticians like Godambe and Khatri used to visit and there were very famous economists like T. N. Srinivasan, Kirit Parikh and so on and we took courses from some of them. I remember that professor Godambe gave some seminars, maybe three or four, on sampling theory, at the end of which we were told that it was actually a course and were asked to write an exam. So things were quite informal. We also took a course from Prof K. R. Parthasarathy. Professor T. E. S. Raghavan was on sabbatical from the University of Illinois at Chicago during that period and I took two courses from him in Statistical Decision Theory and Game Theory. I liked his approach as well as the area and hinted to him that I would be interested in following him to the University of Illinois at Chicago for graduate studies. I did not apply to any other university, which was unusual, as places such as Berkeley and Stanford used to be, and still are, the top preferences of students going to the USA. At the same time, Raghavan was also looking for a prospective student and backed me completely. He even wrote a personal check of 20 dollars which was the application fee, as getting foreign exchange involved a lot of formalities at that time.

Q - Which is (are) the result(s) of yours that you like most? And how do you see its (their) development?

Bapat - I will mention a few. The first is a result with Sunder that the eigenvalues of the Schur product of a Hermitian matrix A and a correlation matrix (a positive definite matrix with ones on the diagonal) are majorized by the eigenvalues of A . It neatly generalizes Schur's classical result that the eigenvalues of a Hermitian matrix majorize its diagonal elements. We came to this result while attempting a conjecture in the book by Marshall and Olkin, which was also proved in the same paper. Another result I would mention is that if A is a symmetric, entrywise positive matrix with exactly one positive eigenvalue, then the entrywise reciprocal of A is positive definite. This result was developed while proving a conjecture due to Karlin and Rinott about characterizing a multinomial distribution with maximum entropy. I should also include the formula for the determinant of the q -distance matrix of a tree, obtained with Lal and Pati. Given a tree with n vertices, the q -distance between distinct vertices i and j at distance t , is defined as $d_q(i, j) = 1 + q + q^2 + \dots + q^{t-1}$. The q -distance matrix D_q is the $n \times n$ matrix with (i, j) -element $d_q(i, j)$ if $i \neq j$, and with zeros on the diagonal. Then the determinant of D_q is given by $(-1)^{n-1}(n-1)(1+q)^{n-2}$. When $q = 1$, we recover a well-known formula of Graham and Pollak. The result is quite unexpected and provides a link with the Ihara zeta function, although we learnt about this connection much later. Finally, I will mention a recent result with Sivasubramanian that if D is the distance matrix of a tree with at least 3 vertices, then the number of negative eigenvalues of the Hadamard square $D \circ D$ equals the number of pendant vertices in the tree. It is a result with no explanation or intuition behind it. Some such recent results were invariably obtained by MATLAB experiments and guesswork.

Q - Are there any turning points in your academic career?

Bapat - The turning points are essentially the times when I visited other places and got introduced to a new area. After my Ph.D. I worked for a year at Northern Illinois University and returned to India in 1983. I joined the Department of Statistics at the University of Bombay. The first turning point was when I joined the Indian Statistical Institute in Delhi after two years. I met V. S. Sunder and we worked on majorization inequalities as well as permanents. Sunder moved to the Bangalore campus of the Institute and I visited him in 1989. There I got in touch with Bhaskara Rao and his student Manjunatha Prasad when I became interested in generalized inverses, an interest that continued for a long time. Another turning point was when I visited Miki Neumann for a year at the University of Connecticut. I worked with Neumann and Kirkland and that is when I picked up my interest in matrices and graphs, which continues even now. I should also mention my short visits during which I worked with Pauline van den Driessche, Dale Olesky, Stephane Gaubert, Marianne Akian, Devadatta Kulkarni and S. K. Jain when I benefited by venturing into something new.

Q - How come you wrote a mathematics book in Marathi?

Bapat - I grew up in a middle class, culturally strong Marathi neighborhood of Mumbai and had primary education in Marathi. I am not a big reader of English fiction or general literature but I have kept up with literature in Marathi to

some extent. When I want to write about nontechnical subjects or even explain technical material in a nontechnical way, I find it easier and enjoyable to do so in Marathi. I believe that Indians have been robbed of the pleasure of learning deep technical subjects in their mother tongue due to excessive dependence on English. So when I decided to write a book about mathematics with a recreational flavor, Marathi was the natural choice. It was a very pleasant experience and the book was well-received. Due to a government initiative, appropriate words have been coined in Indian languages for technical terms and are waiting to be used. One such word I like is “dyut-siddhanta” for game theory. “Siddhanta” means theory and “dyut” is the grand game of dice depicted in the *Mahabharata*.

Q - If you look at the present scenario of graduate education in India, then in most of the places the size of the class is quite large with the added problem of heterogeneity. In such a situation, what should be the ideal content of a 40-lecture introductory course on linear algebra?

Bapat - I will not be able to answer this question satisfactorily. But I will share some thoughts in general. I have served on several syllabus committees. Usually the discussion ends up in completely diverse views with a lot of ill feelings and with no signs of consensus. The amount of basic mathematics that we would like our students to know is just too vast. We would be deceiving ourselves if we claim that a few courses in a three year undergraduate or a two year graduate program can equip the students with the necessary amount of mathematics needed. I do not have a solution. One thought is to go back to the earlier days of studying books rather than subjects. We did not study probability, rather we studied Feller Volume I. That way you are sure which topics are covered and which aren't. We may compile a list of books that students must study. The list can be revised once in five years. So to partially answer your question, find a suitable introductory book on linear algebra and make the students master it to the best of their abilities. The syllabus will consist of only the name of the book, indicating the chapters/sections to be covered.

Q - You were one of the first researchers in India to work in spectral graph theory. Did you ever expect to see such a huge gathering of Indian researchers joining this topic?

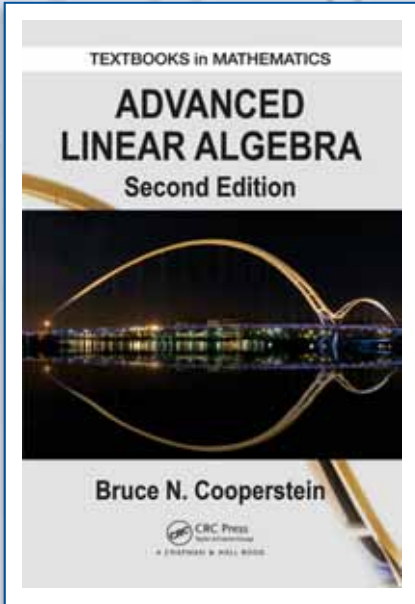
Bapat - My first interest has always been linear algebra and matrix theory. I was drawn to graph theory due to the interesting properties of the matrices associated with a graph. So I do not really work in “spectral graph theory” but rather “linear algebraic graph theory.” But this is a minor point. Broadly speaking, the area may still be called spectral graph theory. Traditionally, graph theory and linear algebra are taught at the master's level in many universities in India. So students already have the basic tools to work in spectral graph theory. Many researchers who initially worked in pure graph theory later took up the linear algebra connection since it leads to interesting problems. This may be a reason for the resurgence in the area. My students Lal and Pati got jobs in the Indian Institutes of Technology where they could attract students in the area. So I am happy if I contributed a little bit to this development. But I still do not see a substantial growth. One indicator would be the number of researchers with at least two papers in spectral graph theory listed in the Mathematical Reviews in the past five years. That number is still small.

Q - What are three important pieces of advice that you would like to give to young researchers in India?

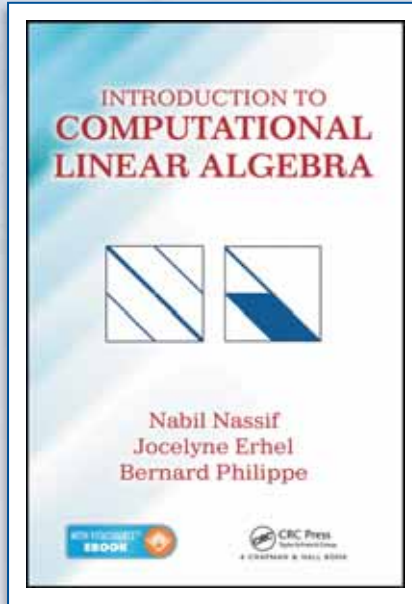
Bapat - There used to be a time when I was enthusiastic about persuading young students to take up a career, research or related, in mathematics. After looking at the current situation I am no longer so. A student with some mathematical abilities is much better off entering the field of engineering or finance. So my first advice (though I don't like the word so much) would be to take up research in mathematics only if you are so passionate about it that you cannot possibly do anything else. The other two pieces of advice are the usual ones: Try to look for and formulate problems that are interesting and that you can possibly solve, and develop a set of techniques of your own, normally after reading the work of masters in the area, that you can effectively use.



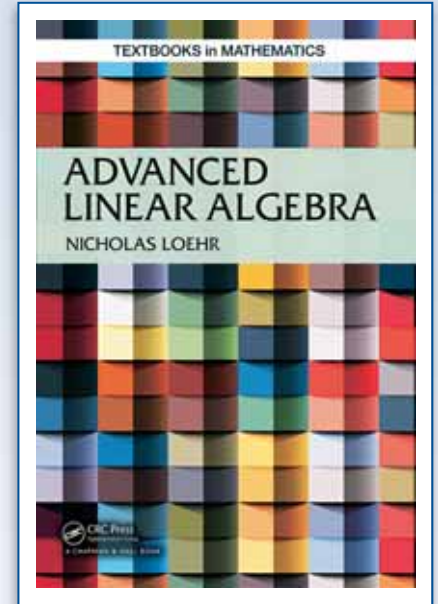
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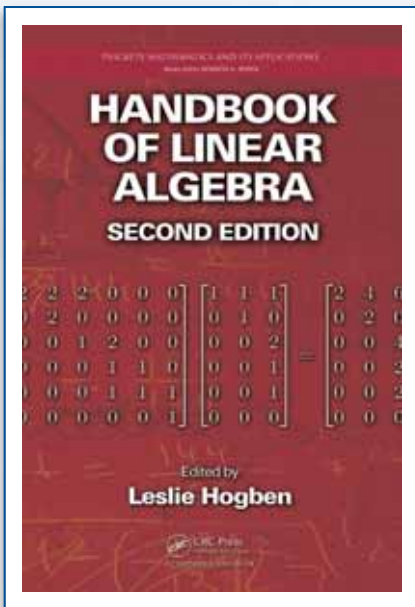
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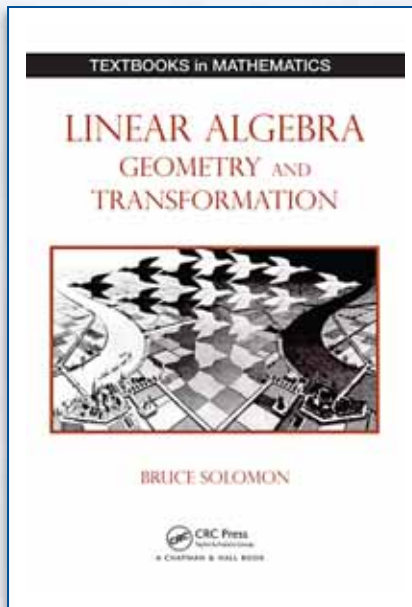
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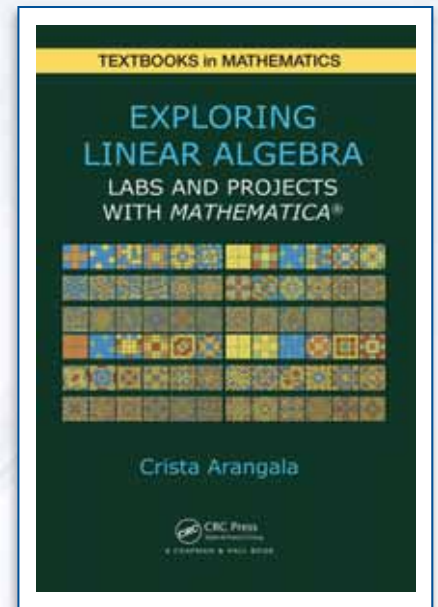
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“Try to be Really Good in Two Fields”

Miroslav Fiedler Interviewed by Jeff Stuart¹

Professor Miroslav Fiedler is well-known for his extensive work in linear algebra, especially the role of special families of matrices in numerical analysis, the interplay of matrix theory and graph theory, and in particular, M -matrices and P -matrices. In recognition of his many contributions to linear algebra, in 1993 he was one of three recipients of the first Hans Schneider Prize. Dr. Fiedler has been a leader of the Czech mathematics community for more than three decades, and has earned multiple national awards for his mathematical contributions and his leadership.



Miroslav Fiedler

Q - How did you come to mathematics in general, and to linear algebra in particular?

Fiedler - In secondary school in Prague, my favorite subjects were mathematics and physics. When, in the final course, I won the mathematical problem competition in the journal *Rozhledy matematicko-přírodovědecké*, I decided that after graduation in 1945, I would enter Charles University in Prague to study mathematics and physics.

For me, the most influential professors were Bohumil Bydžovský (1880) and Eduard Čech (1896). Bydžovský, who had attended lectures of Eduard Weyr and had authored a book on determinants and matrices, lectured on classical algebraic geometry. Čech, well-known for his results in topology and geometry, and who, in my opinion, was one of the last people to understand mathematics as a whole, held lectures on the fundamentals of geometry.

Although it was not easy to avoid political turbulence at Charles University after the communists took over power in Czechoslovakia in 1948, I successfully finished my studies by receiving the title RNDr in 1950. Just a few days after graduation, by a coincidence which happens once in life, I heard that Professor Čech was organizing a program of higher mathematical studies for a group of about a dozen participants. So I applied and was accepted.

The goal of the program was to stimulate research in those modern and applicable parts of mathematics that were not taught before the war. (I should mention that during the war all Czech universities were closed.)

Already at the secondary school, I had admired the richness of properties in the geometry of a triangle. It seemed to me later that much of that could be true for higher-dimensional simplices. This, of course, needed, as the main tool, matrices. I already had some experience with the subject, but, among other obligatory lectures, one was advanced linear algebra. We read Malcev's excellent book in Russian. (I also learned the Russian alphabet from that book.) I have liked the topic ever since.

Q - What were some of the most important events, ideas or colleagues that propelled the development of your career?

Fiedler - My scientific career got a new impulse in Professor Čech's group. We received a stipend (my first earned money) and, in addition to listening to first class lectures (even on computer programming), we met once a week with Eduard Čech, who presented some mathematical problems. There were really excellent people. Let me mention just a few names: Ivo Babuška, Jaroslav Hájek, Jaroslav Kurzweil, Jan Maří, Vlastimil Pták, and Miloš Zlámal. To encourage group bonding, we had an annual retreat week at a mountain resort.

In 1952, the Czechoslovak Academy of Sciences was founded, and Eduard Čech became the first director of the Institute of Mathematics. A good part of the group became the employees of the Institute. Both Mila (Vlastimil) Pták and I were located in a small room together, Mila writing a dissertation on functional analysis, and I on simplex geometry using matrices. We discussed problems of iterative methods for solving systems of linear equations. Norms are a significant tool for measuring the rate of convergence. We succeeded in comparing the rates in the case of a special kind of matrices.

It was not easy to do research in Czechoslovakia in the 1950s. We had very limited access to foreign literature, and then only Russian translations to acquaint us with western mathematical books. We were allowed to publish only in Czech or Russian, and, starting around 1955, also in German.

My first visit abroad was to the Riemann-Tagung in Berlin in 1955. In 1956, I attended the Fourth Congress of Austrian

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Mathematicians in Vienna where I spoke about the location of eigenvalues using norms. Olga Taussky-Todd and her husband John Todd were present and contacted me. Sometime later, John Todd extended an invitation for me to visit Caltech. For personal reasons (my first wife became seriously ill, she died in 1963), I postponed the visit until 1964.

Behind the iron curtain, reality was sometimes quite ridiculous. At a research institution, one had to report any contact with a foreigner. Once in Moscow, I wished to have a short chat with a famous linear algebraist I had never met before. To avoid bureaucracy, he preferred to meet and have a chat on a chair in a park.

After 1960, the political situation in Czechoslovakia was slightly better and the authorities allowed a group from the Institute (including Mila and me) to attend the 1962 ICM in Stockholm. There we met Richard Varga, Alston Householder and others. Mila discussed with Richard the similarity of our results and the priority questions, concluding independence. Alston was very interested in our results on generalized norms, and ever since, he extended invitations to both of us to attend the Gatlinburg meetings. A short time later, I even became a member of the Gatlinburg steering committee.

Just before 1960, I was asked by Štefan Schwarz, a prominent Slovak mathematician, to be an external reviewer for the doctoral dissertation of Professor Anton Kotzig. Schwarz said it was from graph theory, and that I need read only one book, König's, to learn everything I needed. Ever since, I have used graphs in my research wherever it was appropriate.

Q - In the 1950s and 1960s, matrix theory was widely considered a useful tool but not a serious subject for research, and yet you and Professor Pták did significant work on M -matrices that helped reawaken the field. How did that come about?

Fiedler - We were inspired by the work of the prematurely deceased Russian mathematician Koteljanskij. We even spoke about matrices of class K to honor him. We considered our seminal paper as a survey paper, and were surprised by the response. In fact, the notion of P -matrices began there.

Q - You spent much of your career at the Czechoslovak Academy of Sciences. What roles and responsibilities did you have there?

Fiedler - Altogether, I spent more than 50 years there. (Since Czechoslovakia split apart, it has been called the Academy of the Czech Republic.) Until 1989, it was, on the one hand, an asylum for politically questionable mathematicians, and on the other hand, a place on which the highest political authorities focused their attention. Most of the time, I was head of a small department that concentrated on numerical analysis, logic and graph theory. We had productivity plans to fulfill, and I considered an important part of my task to protect people in the department from political pressure of the authorities.

Q - Among the many papers that you have written, are there particular results of which you are fondest?

Fiedler - Although I had the most success with M -matrices, algebraic connectivity and, recently, companion matrices, I like most the following easily described result. Color the edges of an n -simplex red, blue, or white according to whether the opposite dihedral angle in the simplex is acute, obtuse, or right. Characterize all possible colorings. The answer is that such a coloring is possible if and only if the set of red edges connects all vertices of the simplex.

Q - What areas of matrix theory do you currently find most interesting?

Fiedler - Totally positive matrices.

Q - Any additional thoughts that you wish to share?

Fiedler - In addition to general advice such as don't be lazy and preserve your physical health, I think that young people should try to be really good in two fields, such as mathematics and biology, or in (at least) two parts of mathematics, one theoretical and one applicable. One should have imagination, even crazy ideas, but also technical skill. In mathematics, one usually tries to generalize, but this should not mean simply thinning the hypotheses of a previous result. When presenting a mathematical result, one should give a proof and not just try to convince the audience about its correctness by using too many words. In the old times, an ideal mathematical paper was such that no word was superfluous. Nowadays, one is expected to use what is probably a better approach for the reader – explaining the idea of the proof before giving the proof itself. Mila claimed that good mathematical results are usually also aesthetic. Finally, one should support mathematical olympiads and similar competitions for high school students; successful participants will better resist the seduction of easier earnings elsewhere.

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Athanasios S. Fokas
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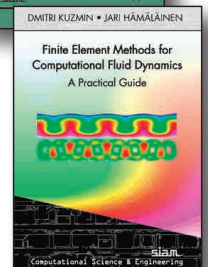
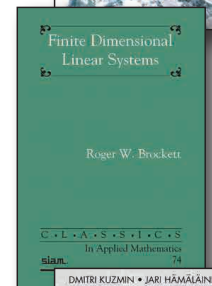
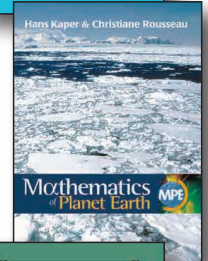
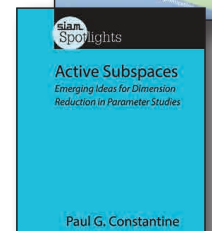
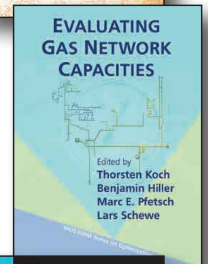
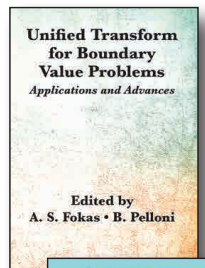
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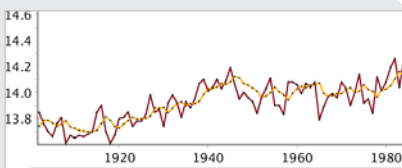
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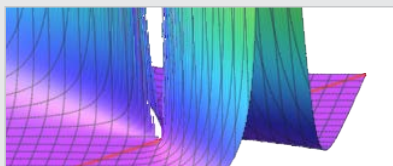
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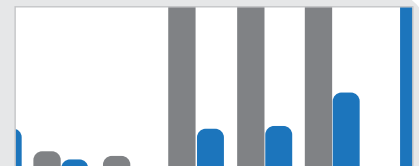
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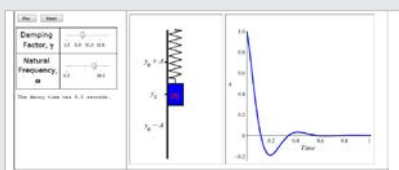
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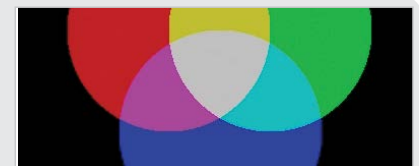
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The Person Behind ILASIC & ILAS-Net: Who is Sarah Carnochan Naqvi?

Sarah Carnochan Naqvi Interviewed by Shaun Fallat¹

S.F. - Sarah, many thanks for agreeing to this interview. Let me begin by introducing you to our membership by asking: “Who is Sarah Carnochan Naqvi?”

S.C.N. - I am the Laboratory Instructor in the Department of Mathematics and Statistics here at the University of Regina in charge of running and maintaining the Mathematics & Statistics Computer lab. I also develop weekly labs for undergraduate course offerings in both mathematics and statistics. More recently, I have become very interested in online course and curriculum development and I am currently developing such materials for our 3rd year course in Linear Programming. Outside of the university, I am a mother of three and a very proud grandmother to three wonderful grandchildren. As part of fulfilling the service component of my position as Lab Instructor, I am also the ILASIC Manager, which includes maintaining the ILAS website and administering the messages sent via ILAS-NET. I am also an Assistant Managing Editor for *ELA*.



*Sarah Carnochan Naqvi
and Shaun Fallat*

S.F. - What exactly are your duties of ILAS? Do you enjoy the work you do for the society?

S.C.N. - First and foremost, I am the ILAS Information Centre (ILASIC) manager. As manager, I oversee maintaining the ILAS website and development of the website content moving forward. I also manage – receive and distribute – messages for ILAS-NET. Finally, I have been an Assistant Managing Editor for the past seven years. Personally, I really enjoy the work that I do for ILAS. I have always been fond of linear algebra and being a part of the society in this capacity keeps me informed of the current trends, and so on, in linear algebra. Of course, there are always ups and downs with every job. . . sometimes I get the feeling that when people submit a message to ILAS-NET they expect to have it distributed right away. However, as I have many other duties and family commitments, I am not always able to do this so quickly. But overall, I appreciate interacting with people in this manner. I also find it very gratifying that I have some creative freedom associated with this position. For example, securing our domain name, outsourcing the website to a professional provider, and web design have all been rather fulfilling personally. I just hope that most ILAS members appreciate the work I do, and see that I fill an important role for the society.

S.F. - How long have you been associated with ILAS?

S.C.N. - I joined ILAS as a student member in 1998 and have continued to be an ILAS member. I attended the ILAS conference in Barcelona in 1998 and in Regina in 2005 and I took over for you as ILASIC manager in 2007.

S.F. - How did you become interested in linear algebra in the first place?

S.C.N. - As an undergraduate student, I was advised by Judi McDonald to consider studying mathematics. Through her guidance I ended up pursuing an honours degree in Mathematics and Computer Science and eventually a master’s degree under her supervision. Judi was so wonderful to work with. I owe much of my success as a university student and subsequently my position as Lab Instructor to her unwavering support and encouragement. My master’s thesis was on ‘Eventually Nonnegative Matrices’ and I continue to stay involved in this subject through these positions within ILAS and *ELA*.

S.F. - Describe the significance of the work you do for ILAS.

S.C.N. - I do believe that having a functional website, etc., is vital for ILAS and its membership, so it is work that I believe must be done. I know sometimes there is frustration with the technology (e.g., ILAS-NET crashing) aspect, but like all of us working for ILAS (or *ELA* or *IMAGE*), we are volunteers, and our day jobs and families are our main priority. Personally, I do my best, but sometimes there are problems in the middle of the night that I just cannot tend to immediately.

S.F. - How might an ILAS member know you?

S.C.N. - I suppose some people might recognize my name as it appears on the ILAS-NET messages that I forward. I do get the feeling that I am invisible and partly anonymous, which is fine from my point of view, as I like it that way.

¹Department of Mathematics and Statistics, University of Regina, Regina, SK, Canada; Shaun.Fallat@uregina.ca

S.F. - What do you see in the future for ILASIC, etc.?

S.C.N. - That is a good question. When I took over for you, one of the things I wanted to accomplish was to separate ILASIC from the University of Regina, securing a domain name for ILAS. I have done this, which in turn means that my job can change hands very easily. In the past, all of the data, etc., had to be moved from the Technion to Regina, and we had to rely on IT services here at the university. In some sense, we are much more self-sufficient as far as the website is concerned. As social media changes and adapts, we may find that our technology platform may have to be transformed accordingly. But for now, I will proceed as the ILAS Executive directs. I hope that I can continue to manage the ILAS website and look forward to working with the Executive and other volunteers associated with ILAS.

S.F. - Many thanks for taking the time out of your busy schedule to chat with me. On behalf of ILAS and its members, I want to personally thank you for the outstanding job that you do for this society. I know firsthand how much work is required to be the ILASIC manager and am so glad to see where you have taken the ILAS website. You should be very proud!

Announcement: 2016 Hans Schneider Prize in Linear Algebra Committee

The 2016 Hans Schneider Prize in Linear Algebra Committee has been appointed by ILAS President Peter Šemrl. It consists of Richard Brualdi (chair), Shmuel Friedland, Ilse Ipsen, Thomas Laffey, Peter Šemrl (ILAS president, ex-officio member), and Xingzhi Zhan. The purpose of the committee is to solicit nominations, and to make a recommendation to the ILAS Executive Board, for this Prize to be awarded at the ILAS conference in Leuven, Belgium, July 11-15, 2016.

Previous recipients of this prize and the date of the meeting in which the prize was given are:

1993 - Miroslav Fiedler, Shmuel Friedland, Israel Gohberg
 1996 - Mike Boyle, David Handelman, Robert C. Thompson
 2005 - Richard Varga
 1999 - Ludwig Elsner
 2006 - Richard Brualdi
 2002 - Tsuyoshi Ando
 2010 - Cleve Moler and Beresford Parlett
 2004 - Peter Lancaster
 2013 - Thomas Laffey

Nominations of distinguished individuals judged worthy of consideration for the Prize are now being invited from members of ILAS and the linear algebra community in general. In nominating an individual, the nominator should include: (1) a brief biographical sketch and (2) a statement explaining why the nominee is considered worthy of the prize, including references to publications or other contributions of the nominee which are considered most significant in making this assessment.

Nominations are open until December 1, 2015 and should be sent, preferably by email, to the Chair, Richard A. Brualdi (Department of Mathematics, University of Wisconsin, 480 Lincoln Drive, Madison, WI 53706, brualdi@math.wisc.edu). Guidelines for the Hans Schneider prize are available at <http://www.ilasic.org/misc/hsguidelines.html>.

Send News for *IMAGE* Issue 55

IMAGE Issue 55 is due to appear online on December 1, 2015. Send your news for this issue to the appropriate editor by October 1, 2015. *IMAGE* seeks to publish all news of interest to the linear algebra community. Photos are always welcome, as well as suggestions for improving the newsletter. Send your items directly to the appropriate person:

- problems and solutions to Bojan Kuzma (bojan.kuzma@upr.si)
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- history of linear algebra to Naomi Shaked-Monderer (nomi@tx.technion.ac.il)
- book reviews to Douglas Farenick (Doug.Farenick@uregina.ca)
- linear algebra education news to David Strong (David.Strong@pepperdine.edu)
- announcements and reports of conferences, workshops and journals to Minerva Catral (catralm@xavier.edu)
- interviews of senior linear algebraists to Carlos Fonseca (carlos@sci.kuniv.edu.kw)
- advertisements to Amy Wehe (awehe@fitchburgstate.edu).

Send all other correspondence to Kevin N. Vander Meulen (kvanderm@redeemer.ca).

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ARTICLES

Gems of Linear Algebra in Combinatorics

Yufei Zhao

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yufeiz@mit.edu.

In this short article, I present four gems on applications of linear algebra to combinatorics. Every one is quite classical, and none is original. In fact, all of them appear in the wonderful little book by Jiří Matoušek titled *Thirty-three miniatures* [3]. Sadly, Matoušek passed away recently, in March 2015, at the age of 51. I hope this article will bring more readers to enjoy his book, which is a treasure trove where one can find many more gems just like these.

Theorem 1 (Oddtown). *In Oddtown, there are n citizens and m clubs. Every club has an odd number of members, and every pair of clubs share an even number of members in common. Then $m \leq n$.*

It is indeed possible to form n clubs following these rules, since every citizen can be the lone member of his own club.

Proof. Associate the i -th club with the incidence vector $v_i \in \{0, 1\}^n$, so that the k -th coordinate of v_i is 1 if citizen $\#k$ is a member of the club, and 0 otherwise. Then $v_i \cdot v_j$ is the number of members in common between the i -th and the j -th club.

Working over the finite field \mathbb{F}_2 , we claim that the vectors $v_1, \dots, v_m \in \mathbb{F}_2^n$ are linearly independent. Indeed, the hypothesis of the theorem gives that $v_i \cdot v_i = 1$ for all i , and $v_i \cdot v_j = 0$ if $i \neq j$. If

$$\lambda_1 v_1 + \dots + \lambda_m v_m = 0$$

with $\lambda_1, \dots, \lambda_m \in \mathbb{F}_2$, by taking the dot product with v_i on both sides, we obtain that

$$\lambda_i = v_i \cdot (\lambda_1 v_1 + \dots + \lambda_m v_m) = 0$$

for each i . Therefore $v_1, \dots, v_m \in \mathbb{F}_2^n$ are linear independent, and therefore $m \leq n$. \square

Theorem 2 (Fisher's inequality). *We have distinct nonempty subsets $S_1, \dots, S_m \subseteq \{1, \dots, n\}$, and some t such that $|S_i \cap S_j| = t$ for all $i \neq j$. Then $m \leq n$.*

Proof. As before, consider incidence vectors $v_i = 1_{S_i} \in \{0, 1\}^n$, but now viewed as vectors in \mathbb{R}^n . We have $v_i \cdot v_j = t$ for all $i \neq j$.

If $|C_i| = t$ for some i , then $|C_i \cap C_j| = t$ implies that $C_j \supseteq C_i$ for all $j \neq i$, and $C_j \setminus C_i$ are disjoint for different j , from which one easily deduces that $m \leq n$. So let us assume from now on that $|C_i| > t$ for all i .

We claim (as before) that the vectors v_1, \dots, v_m are linearly independent (now in \mathbb{R}^n). Indeed, if $\lambda_1 v_1 + \dots + \lambda_m v_m = 0$, then

$$\begin{aligned} 0 = \|\lambda_1 v_1 + \dots + \lambda_m v_m\|^2 &= \sum_{i=1}^m \lambda_i^2 \|v_i\|^2 + 2 \sum_{i < j} \lambda_i \lambda_j v_i \cdot v_j \\ &= \sum_{i=1}^m \lambda_i^2 |C_i| + 2 \sum_{i < j} \lambda_i \lambda_j t = \sum_{i=1}^m \lambda_i^2 (|C_i| - t) + (\lambda_1 + \dots + \lambda_m)^2 t. \end{aligned}$$

All the terms on the RHS are nonnegative, and hence they must all be zero. Since $|C_i| > t$ for all i , we must have $\lambda_i = 0$ for all i . Thus $v_1, \dots, v_m \in \mathbb{R}^n$ are linearly independent. Hence $m \leq n$. \square

The two previous theorems demonstrate the power of the elementary fact that a vector space of dimension n cannot have more than n linearly independent vectors. The next two examples use related facts about ranks of matrices.

Theorem 3 (Graham–Pollak [1]). *If the edge set of the complete graph K_n can be decomposed into a disjoint union of m complete bipartite graphs, then $m \geq n - 1$.*

Note that it is always possible to decompose K_n into $n - 1$ complete bipartite graphs, for example by peeling off one vertex at a time:



Proof. Label the vertices of K_n by $\{1, \dots, n\}$. Let the k -th subgraph in the decomposition be a complete subgraph between X_k and Y_k . Let A_k be the $n \times n$ matrix whose entries are given by

$$(A_k)_{ij} = \begin{cases} 1 & \text{if } i \in X_k \text{ and } j \in Y_k, \\ 0 & \text{otherwise.} \end{cases}$$

Let $A = A_1 + \dots + A_m$. Then $\text{rank } A \leq m$ since A is a sum of m rank 1 matrices. We will show that $\text{rank } A \geq n - 1$.

Let I and J denote the $n \times n$ identity matrix and all-ones matrix, respectively. The decomposition hypothesis is equivalent to

$$A + A^T = J - I.$$

If $\text{rank } A \leq n - 2$, then there is some $0 \neq x \in \mathbb{R}^n$ such that $Ax = 0$ and $1^t x = 0$ (i.e., $Jx = 0$), so that

$$0 = x^t(A + A^t)x = x^t(J - I)x = -x^t x < 0,$$

which is a contradiction. Thus $n - 1 \leq \text{rank } A \leq m$. □

Theorem 4 ([2]). *There do not exist four points in \mathbb{R}^2 whose pairwise distances are all odd integers.*

Proof. Suppose these four points do exist. We can translate them so that one of them is at the origin. Let the four points be $0, a, b, c \in \mathbb{R}^2$. Then $|a|, |b|, |c|, |a - b|, |c - a|, |b - c|$ are all odd integers, so their squares are all $1 \pmod{8}$. It follows that

$$2a \cdot b = |a|^2 + |b|^2 - |a - b|^2 \equiv 1 \pmod{8}.$$

Let V be the 2×3 matrix whose columns are a, b, c . Consider the Gram matrix

$$B = V^t V = \begin{pmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{pmatrix}.$$

We have

$$2B \equiv \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \pmod{8}.$$

Thus $\det(2B) \equiv 4 \pmod{8}$, and hence $\det(B) \neq 0$. However, this is impossible, since $\text{rank } B = \text{rank}(V^t V) \leq \text{rank } V \leq 2$ as V is a 2×3 matrix. □

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Quantity of Contributions of the Middle East to Linear Algebra

Mohammad Sal Moslehian,
Department of Pure Mathematics, Ferdowsi University of Mashhad,
P. O. Box 1159, Mashhad 91775, Iran, moslehian@um.ac.ir

Country	Population	Total papers	SIMAX	LAA	LAMA	NLAA	ELA	per million pop.	JOT
Israel	8,146,300	179	24	113	11	7	24	21.97	10
Iran	77,777,500	148	1	107	19	2	19	1.90	7
Turkey	76,667,864	46	5	26	7	4	4	0.60	6
Jordan	6,637,160	26	0	14	10	0	2	3.92	0
Saudi Arabia	30,770,375	16	0	11	5	0	0	0.52	1
Kuwait	4,044,500	9	3	4	0	2	0	2.22	0
Oman	4,055,418	6	0	5	0	1	0	1.48	1
Lebanon	4,822,000	5	0	3	2	0	0	1.04	0
Egypt	87,230,800	5	1	3	0	1	0	0.06	0
United Arab Emirates	9,446,000	4	1	3	0	0	0	0.42	2
Qatar	2,077,357	1	0	0	1	0	0	0.48	0
Yemen	25,956,000	1	0	1	0	0	0	0.04	0
Cyprus	858,000	0	0	0	0	0	0	0	2
Iraq	36,004,552	0	0	0	0	0	0	0	0
Syria	21,987,000	0	0	0	0	0	0	0	0
Bahrain	1,348,000	0	0	0	0	0	0	0	0

A region consisting of countries mostly in southwest Asia is known as the Middle East. While most of its member countries have suffered in recent decades from some internal or external wars or conflicts, there are some active mathematicians in these countries working hard in the world of Linear Algebra.

The table shows how many papers were published in the years 2004-2013 by mathematicians with affiliations in the Middle East in the leading journals in linear algebra, i.e., *SIAM Journal on Matrix Analysis and Applications* (SIMAX), *Linear Algebra and its Applications* (LAA), *Linear and Multilinear Algebra* (LAMA), *Numerical Linear Algebra with Applications* (NLAA) and *Electronic Journal of Linear Algebra* (ELA). The first and the second columns give the list of Middle East countries and their population based on Wikipedia data. The third column presents the total numbers of papers published from 2004 to 2013 in SIMAX, LAA, LAMA, NLAA and ELA. (The next five columns show the number of publications in each of these journals.) The data were obtained from Scopus.

As we can see, Israel and Iran top the list in number of total publications, significantly ahead of other countries. Since the number of publications is often correlated with the population of countries (in fact, mathematicians), we present the number of paper publications of each country per one million population in the eighth column. As one observes, the situation of Jordan is rather unique – being fourth in the total number of publications, and second in the number of publications per one million people. It is worth noting that among 26 papers of Jordanian mathematicians, 15 papers were published by one mathematician. The situation is rather similar in some other countries, e.g., one may notice that all 5 papers of Lebanon were written by one person. Of course, this is a common facet of third world (developing) countries, that any progress in such countries is often based on activities of a small number of its individuals and not on the existence of any scientific traditions or systematic program.

In Operator Theory, which can be regarded as a sort of infinite-dimensional linear algebra, one can see the same model of the situation of these countries as we examine the *Journal of Operator Theory* (JOT) and insert the numbers of paper publications of the Middle East countries in this journal in the period 2004-2013. Even the numbers of mathematics journals of high level in these countries obey a similar pattern. If we consider the prestigious list “The Reference List of Journals” of MathSciNet we may observe that only three Middle East countries have journals on this list: Iran, Israel and Turkey.

The situation of Middle East countries brings to mind the saying “everything suits everything else” of Mozaffar ad-Din Shah Qajar, a king of Iran about 120 years ago, when people asked him about the poor economics, unbalanced social welfare, weak culture, and political problems throughout the country. In other words, the scientific contribution of these countries is similar to the other components of their life.

LINEAR ALGEBRA EDUCATION

Introduction

David Strong, *IMAGE* Education Editor

Each January we organize a session on “Innovative and Effective Ways to Teach Linear Algebra” at the annual Joint Meetings of the American Mathematical Society and the Mathematical Association of America. This past January, in San Antonio, Texas, we heard from nineteen speakers. Each year there are a few presentations that stand out to me as especially interesting and useful. From this past January’s set of speakers, I particularly enjoyed the presentation of Jeremy Case, department chair and professor of mathematics at Taylor University. (A more complete list of presentations is available at <http://faculty.pepperdine.edu/dstrong/LinearAlgebra/index.html>.) In his presentation, Dr. Case discussed how he uses journal articles when he teaches Linear Algebra. I asked Dr. Case to put his presentation into a short article, which you will find below. I hope you find his thoughts helpful. Undoubtedly, some of our *IMAGE* readers do something similar in their own teaching. With this in mind, we would like to include a follow-up article to the article below, in which we discuss how other *IMAGE* readers have used journal articles in teaching Linear Algebra. To this end, please email me at David.Strong@pepperdine.edu with information (including links) about articles you have used and how you have used them in your own teaching of Linear Algebra, both undergraduate and graduate courses. We hope to include this follow-up article in the next issue of *IMAGE*, so please email me by September 15 of this year.

My Favorite *MAA* Articles for Teaching Linear Algebra

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Introduction.

When I was an idealistic young instructor, I had this great idea of assigning journal articles for my students to read. For my sophomore level Linear Algebra course, it was a failure. Students read the article but not for understanding. Students were frustrated, overwhelmed, and viewed the reading as a waste of their time by detracting from “what was going to be on the test.”

I still use journal articles, but I use them more judiciously. I list here my favorite Mathematical Association of America (*MAA*) articles which help shape my course. These are not ranked like a top 10 list but are ordered to aid the exposition of how one might use journal articles in framing the course, designing course activities, and assigning them as reading. Incorporating journal articles is not as easy as I first imagined, and I share a few strategies to address some of the hurdles and to integrate journal articles into a course successfully.

Framing the Course.

1. ***Down with determinants!*** Sheldon Axler [2]. When I first started teaching, it was shortly after the Linear Algebra Curriculum Study Group released their recommendations to reform the teaching of Linear Algebra. The 1993 Special Issue on Linear Algebra in the *College Mathematics Journal* [8] led to the 1997 *Resources for Teaching Linear Algebra* [6], a compilation of *MAA* articles which continue to inform my approach. Sheldon Axler’s provocative article profoundly affected me since I could not believe determinants were superfluous. While I do not take his approach in totality nor use his textbook [3], his aim towards understanding the structure of linear operators on finite dimensional vector spaces challenged the objectives of my course. This article helped me answer the questions: What is my student population, and what is the purpose of my course?
2. ***Pitfalls in Computation, or why a Math Book isn’t Enough*** George E. Forsythe [9]. One of the first topics in a Linear Algebra course is Gaussian elimination. This classic article retains its relevance with regard to numerical error and Gaussian elimination.

Course Activities.

3. ***Arithmetic Matrices and the Amazing Nine-Card Monte*** Dean Clark and Dilip K. Datta [7]. Everyone who teaches Linear Algebra should have a card trick up his or her sleeve. To explain the trick, this article uses the vector spaces of arithmetic matrices. On a day before spring break when class attendance is down, I become a carnival showman who tries to take the students’ money. Students report this is the most memorable experience of the class. Now if they could only remember the normal equations. . .

4. ***Applications of Linear Algebra in Calculus*** Jack W. Rogers, Jr. [17]. Not all Introductory Linear Algebra courses have multivariate calculus as a prerequisite, but this article demonstrates the benefits of bases, linear operators and basis coordinates in applications to calculus. The article illustrates how notation can condense information in new ways while reinforcing ideas in calculus and linear algebra. I assign a MATLAB project written by Tom Polaski of Winthrop University based on this article for use with a particular textbook [12].
5. ***Visualization of Matrix Singular Value Decomposition*** Cliff Long [16]. Tom Polaski also developed a project using MATLAB, *Mathematica*, or Maple to demonstrate image compression using the Singular Value Decomposition. Having students use their own pictures (sometimes of me) provides an extra incentive. Since the SVD has a lot of moving parts, I have found that using this project as a preview of the SVD is more beneficial than after a class presentation. The article and the project elevate student curiosity and motivation to understand all of those parts.

Reading Journal Articles.

Completing a course does not end the discussion of the material because there is always more to learn. In order to emphasize this point, I have students read journal articles. After all, reading journal articles is supposed to enhance analytic abilities, promote confidence in reading the literature, provide insight into the research process, and facilitate the transition to graduate school [10, 15].

There were several reasons why my first journal reading assignments were failures. I expected complete understanding emanating from their joy of learning but instead got frustration. There are three strategies that I now employ to address these complaints: providing context, providing choice, and employing annotated reading.

First, I inform the students of why I am having them read the articles. I want them to get a taste of Linear Algebra's usefulness rather than a complete understanding of the article. I tell them how the article and its notation relate to the material we are studying. In short, I provide context. I now assign about four articles requiring a short but not exhaustive response. Here is a sample of the articles.

6. ***Turning Lights Out with Linear Algebra*** Marlow Anderson and Todd Feil [1]. Several websites have the game Lights Out illustrating the puzzle where the goal is to turn off all of the lights. When one presses a button, the pushed button and its adjacent neighbors change their on/off state. After demonstrating the game on a website and providing an overview of the article's purpose, I ask students to read Anderson and Feil's article and to answer six short questions. The questions include how particular vectors relate to winning the game. I like that the article incorporates the null space and column space with mod 2 arithmetic to solve the puzzle.

Another strategy is to provide choice. A colleague in the English department told me that a choice of topics provides students with a sense of control over their own learning which increases motivation. Furthermore, students can select an article pertaining to their interest of study. As an example of one assignment, I provide four choices. The math education majors analyze student errors on a national exam. The following three articles are possibilities for the others.

7. ***The Hat Problem and Hamming Codes*** Mira Bernstein [4] and ***A Dozen Hat Problems*** Ezra Brown and James Tanton [5]. I try to find results which are intriguing and unforeseen. The problem involves guessing the color of the hat on your head by seeing the other player's hat colors but without communicating with one another. Surprisingly, a group may improve their success probability above 50% by employing mathematics involved with error-correcting codes. My purpose for this article is not for students to understand completely the strategy but to develop a sense of how linear algebra contributes to such an unexpected result.
8. ***Singular Vectors, Subtle Secrets*** David James, Michael Lachance, and Joan Remski [11] and ***Spectral analysis of the supreme court*** Brian L. Lawson, Michael E. Orrison, and David T. Uminsky [14]. How many Supreme Court judges do we really need? These articles provide several examples of the power of spectral analysis and the SVD, including the area of politics. I also have students list the Supreme Court judges and label them as conservative or liberal since presumably over half of the population cannot even name one of the nine justices.
9. ***The Linear Algebra behind the Search Engines.*** Amy Langville [13]. This Loci/JOMA article goes through the field of information retrieval and the basic components of search engines. The module shows the tremendous power of matrices and vector spaces and is particularly agreeable to computer science majors.

Even if a student does not read a particular article, a classmate has. I will ask a student during class for an extemporaneous report on their reading in class and whether they liked the result. The objective is to convince students of the applicability of linear algebra in a variety of contexts.

Besides context and choice, I also employ annotated reading where students mark up an article with questions and comments. Depending on its implementation, annotated reading demonstrates that students can identify the main ideas, can trace the development of the ideas, and can provide their own thoughts and reactions. Students can ask questions on the side to demonstrate they are wrestling with the content. Since students have access to and can print out journal articles, the barriers to annotated reading are minimal compared to when I started teaching.

10. *The Growing Importance of Linear Algebra in Undergraduate Mathematics*. Alan Tucker [18]. This Tucker article is my favorite one, and I wish other courses would have such a theme article tracing the course's history and its importance. I assign this article towards the end of the course. Students are to highlight all of the terms we used in class in yellow and all of the terms we did not use in red. They should provide comments in the margin. They are then to reflect on the connections of this class to other disciplines and courses. I ask, "What is so important about Linear Algebra?" I receive information as to their thinking and understanding. They receive a review of the course and its main topics.

Why MAA articles? My main reason is their accessibility to my students. In one sense of accessibility, these articles are easily found on JSTOR using our school's library website. The MAA has a generous use clause allowing institutions with an educational license to use the articles as long as they are for educational purposes. The other sense of accessibility is that the MAA articles are readable and a bridge to more research-oriented articles. Students can develop confidence in their reading to learn new information on their own.

Linear Algebra has many wonderful applications to a wide variety of disciplines. These applications and ideas cannot be contained in a single textbook. In the words of Alan Tucker [18], "Linear Algebra is what a math course ought to be!" Journal articles have the potential to excite students to make the same statement.

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Additional MAA articles. Here are additional articles that I consider. Almost all were published before 2011 because of the delay between current journals and the archived journals made available through JSTOR.

A. Framing the Course, Numerical Issues, and Determinants.

1. Carlson, David, Charles R. Johnson, David C. Lay, and A. Duane Porter. The Linear Algebra Curriculum Study Group recommendations for the first course in linear algebra. *College Mathematics Journal* (1993) 41–46.
2. Carlson, David. Teaching Linear Algebra: Must the Fog Always Roll In? *College Mathematics Journal* 24:1 (1993) 29–40.
3. Corless, Robert M. Six, lies, and calculators. *American Mathematical Monthly* (1993) 344–350.
4. Nievergelt, Yves. Numerical linear algebra on the HP-28 or how to lie with supercalculators. *American Mathematical Monthly* (1991) 539–544.
5. Day, J., and Dan Kalman. Teaching linear algebra: Issues and resources. *The College Mathematics Journal* 32, no. 3 (2001): 162–168. [Provides further resources as a follow-up to the 1993 theme issues in the *College Mathematics Journal*.]
6. Hannah, John. A geometric approach to determinants. *American Mathematical Monthly* (1996): 401–409.
7. Hooley, Donald E. Using the College Mathematics Journal Topic Index in Undergraduate Courses. *College Mathematics Journal* 28:2 (1997) 106–9. [This article includes related ideas to this paper.]
8. Nievergelt, Yves. How (not) to solve quadratic equations. *College Mathematics Journal* (2003) 90–104. [Shows pathological examples of rounding errors related to quadratic equations.]
9. McWorter, William A., and Leroy F. Meyers. Computing eigenvalues and eigenvectors without determinants. *Mathematics Magazine* (1998) 24–33. [Another “Down with Determinants” article.]
10. Scheinerman, Edward R. When close enough is close enough. *American Mathematical Monthly* (2000) 489–499. [2001 Ford winner.]

B. Course Activities, Magic Tricks, and the SVD.

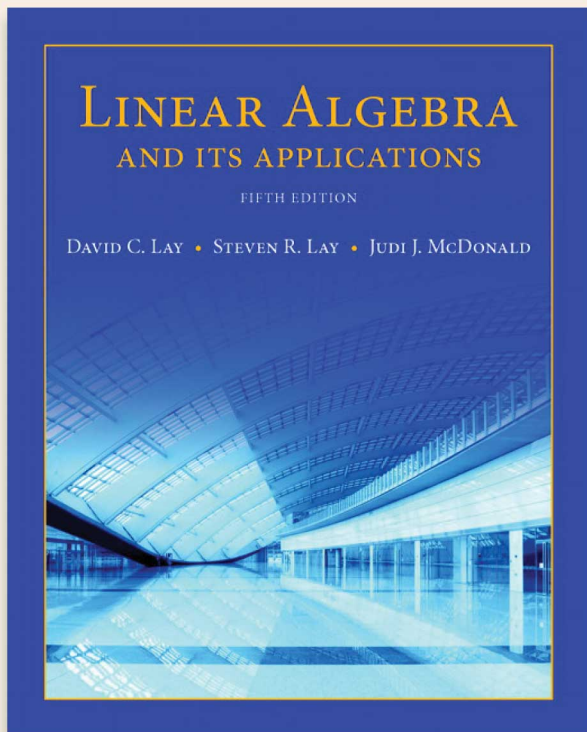
1. Carlson, David, Charles R. Johnson, David Lay, and A. Duane Porter. Gems of exposition in elementary linear algebra. *College Mathematics Journal* (1992) 299–303.
2. Diaconis, Persi, and Jason Fulman. Carries, shuffling, and an amazing matrix. *American Mathematical Monthly* 116:9 (2009) 788–803.
3. Kedzierawski, Andrzej, and Olympia Nicodemi. Image Reconstruction in Linear Algebra. *College Mathematics Journal* (2001) 128–134.
4. Long, Andrew E., and Clifford A. Long. Surface approximation and interpolation via matrix SVD. *College Mathematics Journal* 32:1 (2001) 20–25. [Includes an image compression of a bust of Abraham Lincoln.]
5. Martin-Sanchez, Oscar, and Cristobal Pareja-Flores. Two reflected analyses of lights out. *Mathematics Magazine* (2001) 295–304. [Another article on Lights Out.]
6. Missigman, Jennie, and Richard Weida. An easy solution to mini lights out. *Mathematics Magazine* (2001) 57–59. [An additional article on Lights Out.]
7. Mulcahy, Colm, and John Rossi. A Fresh Approach to the Singular Value Decomposition. *College Mathematics Journal* (1998) 199–207.
8. Sprows, David. A Tricky Linear Algebra Example. *The College Mathematics Journal* 39:1 (2008) 54–56. [Another magic trick.]

9. Strang, Gilbert. The fundamental theorem of linear algebra. *American Mathematical Monthly* (1993) 848–855. [The classic four fundamental subspaces.]
10. Thoo, J. B. A picture is worth a thousand words. *College Mathematics Journal* (1998) 408–411. [Another chart of the four fundamental subspaces.]

C. Reading Journal Articles. These are articles I have considered assigning or have assigned to read.

1. Abbott, Stephen D., and Matt Richey. Take a Walk on the Boardwalk. *College Mathematics Journal* (1997) 162–171. [The use of Markov chains in examining the game Monopoly.]
2. Aceto, Lidia, and Donato Trigiane. The matrices of Pascal and other greats. *American Mathematical Monthly* (2001) 232–245.
3. Brualdi, Richard A., and Jennifer JQ Massey. Some Applications of Elementary Linear Algebra in Combinatorics. *College Mathematics Journal* (1993) 10–19.
4. Cahill, Nathan, John D’Errico, Darren Narayan, and Jack Narayan. Fibonacci determinants. *College Mathematics Journal* (2002) 221–225.
5. Call, Gregory S., and Daniel J. Velleman. Pascal’s matrices. *American Mathematical Monthly* (1993) 372–376.
6. Clifford, John H., and Michael Lachance. Quartic coincidences and the singular value decomposition. *Mathematics Magazine* 86:5 (2013) 340–349.
7. Cowen, Carl C. Teaching and testing mathematics reading. *American Mathematical Monthly* (1991) 50–53. [Uses linear algebra as an example to help students with reading mathematics.]
8. de Medeiros, Airton von Sohsten. Elementary Linear Algebra and the Division Algorithm. *College Mathematics Journal* (2002) 51–53. [Links linear algebra to the rings of polynomials.]
9. Goldstein, Jerome A., and Mel Levy. Linear algebra and quantum chemistry. *American Mathematical Monthly* 98:10 (1991) 710–718.
10. Hirst, Holly P., and Wade T. Macey. Bounding the roots of polynomials. *College Mathematics Journal* (1997) 292–295.
11. Holte, John M. Carries, combinatorics, and an amazing matrix. *American Mathematical Monthly* (1997) 138–149.
12. Ipsen, Ilse C.F., and Carl D. Meyer. The angle between complementary subspaces. *American Mathematical Monthly* (1995) 904–911.
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15. Kominers, Scott Duke. Finding Matrices that Satisfy Functional Equations. *College Mathematics Journal* (2009) 289–292.
16. Loehr, Nicholas. A Direct Proof That Row Rank Equals Column Rank. *College Mathematics Journal* 38:4 (2007) 300–303.
17. Rosentrater, C. Ray. The Sample Correlation Coefficient from a Linear Algebra Perspective. *College Mathematics Journal* (2006) 47–50.
18. Shader, Bryan L., and Chanyoung Lee Shader. Scheduling conflict-free parties for a dating service. *American Mathematical Monthly* (1997) 99–106.
19. Spivey, Michael Z. Fibonacci identities via the determinant sum property. *College Mathematics Journal* 37:4 (2006) 286.
20. Swart, Barbara. Linear Algebra in the Financial World. *College Mathematics Journal* (2001) 208–210.
21. Walsh, James A. Fractals in linear algebra. *College Mathematics Journal* (1996) 298–304.

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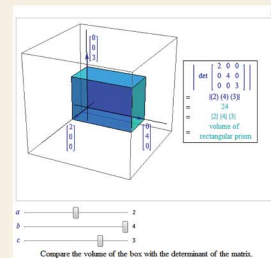
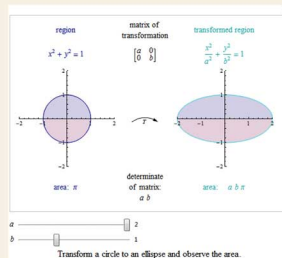


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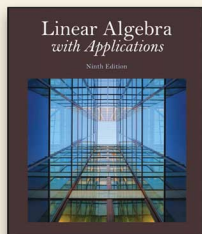


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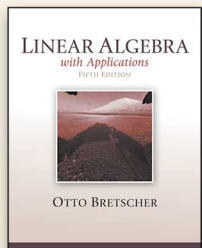
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CONFERENCE REPORTS

2014 International Conference on Tensors and Matrices and their Applications Suzhou, China, December 17–19, 2014

Report by Changqing Xu

The 2014 Conference on Tensors and Matrices and their Applications (TMA2014) was held at Suzhou University of Science and Technology (USTS), Suzhou, China, December 17-19, 2014. TMA2014 was intended to allow researchers from both the tensor theory and matrix communities to discuss novel methods and applications as well as theoretical advances. TMA2014 was endorsed by the International Linear Algebra Society (ILAS), and jointly sponsored by the China Natural Science Foundation, the Hong Kong Polytechnic University, Fudan University, Jiangsu Normal University (JSNU), and the Suzhou University of Science and Technology (USTS). It also got sponsorship from the Mathematical Center of Ministry of Education of China and the Suzhou Government. Changqing Xu (USTS), Yimin Wei and Zhengke Miao (JSNU) co-chaired TMA2014.

The preparation of TMA2014 began in March 2013 when Prof. Liqun Qi (The Hong Kong Polytechnic University) was hosting Changqing Xu (USTS) and Yimin Wei (Fudan University). They devoted most of their precious time during Xu and Wei's 2013 HK visit to discuss TMA2014. The central topics of their frequent discussions during that time were mainly about the interplay of the matrix and the multi-way matrix whose nickname is "tensor." The discussion prompted the idea of the Suzhou conference. TMA2014 turned out to be the largest and most fruitful meeting ever held on tensors and matrices, and attracted many people to present their latest results related to matrices and tensors.

The influence of TMA2014 can be seen most strongly in fruitful works published recently by the SIAM or IEEE journals, *LAA*, and others, reflecting the interweaving of tensors and matrices, covering structured tensors, tensor decompositions, tensor spectra, tensor approximations, tensor optimizations, etc. Participants from a Shanghai workshop before TMA2014 (hosted by Shanghai University, Dec. 14–16, 2014) had an unforgettably joyful journey, with open discussion sessions in the shuttle by participants excitedly talking about activity in matrix theory and tensor analysis. Many graduate students also contributed ideas to the topic.

Suzhou is an ancient city with more than 2500 years of history. It is less than 25 minutes from Hongqiao, Shanghai by high speed express train. Suzhou is noted for its world-famous garden arts, as well as Lake Taihu which skirts the city.

Just as Prof. Liqun Qi described it, TMA2014 was a large conference whose influence will be seen at least for the next 10 years. As a matter of fact, we have already seen some evidence of this conjecture.



Group Photo – TMA2014

International Conference on Linear Algebra & Its Applications Department of Statistics, Manipal University, India, December 18–20, 2014

Report by K. Manjunatha Prasad

The theme of the conference was focused on (i) Matrix Methods in Statistics, (ii) Combinatorial Matrix Theory and (iii) Classical Matrix Theory covering different aspects of Linear Algebra. This conference was in sequence to the conference CMTGIM 2012 held in Manipal during January 2012. The main goal of the ICLAA 2014 conference was to bring experts,

researchers, and students together and for those to present recent developments in this dynamic and important field. The conference also aimed to stimulate research and support interaction between scientists by creating an environment in which participants could exchange ideas. The conference was supported by the National Board for Higher Mathematics, the Department of Science and Technology, the Council of Science and Industrial Research and the Indian National Science Academy.

The scientific activities of ICLAA 2014 were dedicated to eminent matrix theorist Prof. Ravindra B. Bapat on the occasion of his 60th birthday. Many of his contemporaries, collaborators and students came together to serve the objectives of the conference on the occasion, in honor of him. The list of invited speakers consisted of R. Balakrishnan, Abraham Berman, Rajarama Bhat, Rajendra Bhatia, Arup Bose, Steve Haslett, Surender Kumar Jain, Andre Leroy, Steve Kirkland, S. H. Kulkarni, Arbind Kumar Lal, S. K. Neogy, Sukanta Pati, Simo Puntanen, T. E. S. Raghavan, Bhaba Kumar Sarma, Ajith Iqbal Singh, K. C. Sivakumar, Sivaramakrishnan Sivasubramanian, Murali K. Srinivasan, Wasin So, and V. S. Sunder. For the detailed list of participants and speakers, abstracts of the talks, photos, and videos please visit <http://conference.manipal.edu/ICLAA2014/>.



Felicitating of Prof. R.B. Bapat

The book entitled *Linear Algebra with Applications – A Volume in Honour of Prof Ravindra B. Bapat* has been released on the occasion. The book (<http://www.mup.manipal.edu/BookDetails.aspx?BookId=61&Type=B>) is published by Manipal University Press, Manipal India, and consists of selected papers of Bapat. The editors of this book are Steve Kirkland, K. Manjunatha Prasad, Sukanta Pati, and Simo Puntanen.

A special issue of the *Electronic Journal of Linear Algebra* will be dedicated to this conference; the last date for submission of articles for this issue was April 15, 2015.



Participants of ICLAA-2014

OBITUARY

Leiba Rodman (June 9, 1949 – March 2, 2015)



Leiba Rodman

Professor Leiba Rodman passed away on March 2, 2015 after a battle with cancer. Dr. Rodman has been noted for his work in linear algebra and matrix theory, matrix and operator-valued functions, Krein-space operator theory, linear preserver problems, and, most recently, quaternionic linear algebra. His recently published book, *Topics in Quaternion Linear Algebra*, was published by Princeton University Press; a review appears in this issue of *IMAGE*. Altogether he published eight books and over 330 research papers. Professor Rodman was Editor-in-Chief of *Operators and Matrices*, served as a Senior Editor for *Linear Algebra and its Applications* and was an Associate Editor of two other journals. A special issue of *Linear Algebra and its Applications* dedicated to his 65th birthday will appear shortly.

Dr. Rodman earned his Ph.D. from Tel-Aviv University, and served as a faculty member at Tel-Aviv University and at Arizona State University before joining the College of William & Mary in 1987. In 2009, Professor Rodman was one of the first recipients of the Plumeri Award at the College of William & Mary, an award recognizing exemplary achievements in teaching, research and service. Colleagues have made special note of his generous demeanor and diligent service. He is survived by his wife, Ella; two sons, Daniel and Benjamin; and two daughters, Ruth and Naomi.

Further information and personal reflections can be found at <http://rodman.blogs.wm.edu/>.

BOOK REVIEWS

Topics in Quaternion Linear Algebra, by Leiba Rodman

Princeton University Press, 2014, ISBN 978-0-6911-6185-3, 384 pages.

Reviewed by Haixia Chang, Shanghai Finance University, hcychang@163.com and
Fuzhen Zhang, Nova Southeastern University, zhang@nova.edu

There are many books on (real) quaternions and applications of quaternions. This book, *Topics in Quaternion Linear Algebra* by Rodman, however, is the first (and only) one devoted to linear algebra and matrix analysis with the underlying “number field” the quaternions. It not only gathers classical and fundamental properties of quaternions but also covers recent results in the area of quaternion linear algebra and matrix theory. The book contains discussions of almost all aspects of linear algebra and matrices under the setting of quaternions. There is no doubt that the book will become a standard reference for students and professionals who have an interest in quaternions and matrices.

Quaternions are known to have been invented by William Rowan Hamilton in 1843 as an extension of the complex numbers. (Carl Friedrich Gauss might have known these mathematical objects, as revealed in his notes as early as 1819. See <http://en.wikipedia.org/wiki/Quaternion>.) A feature of quaternions is noncommutative multiplication; that is, if a and b are quaternions, then $ab \neq ba$ in general. For undergraduate students in mathematics, quaternions may be the very first “strange” “numbers” they have ever encountered. Well, they are in fact the first discovered noncommutative division algebra, or skew field. In mathematics, “quaternion algebras sit prominently at the intersection of many mathematical subjects. They capture essential features of noncommutative ring theory, number theory, K-theory, group theory, geometric topology, Lie theory, functions of a complex variable, spectral theory of Riemannian manifolds, arithmetic geometry, representation theory, the Langlands program – and the list goes on. Quaternion algebras are especially fruitful to study because they often reflect some of the general aspects of these subjects, while at the same time they remain amenable to concrete argumentation” (John Voight, *The arithmetic of quaternion algebras*). Or in short, as Leiba put it: “Quaternions were a starting point in many important developments in modern algebra: octonions, division algebra, and Clifford algebra, to name a few.” (In the Notes on page 27.) Over the years, quaternions have been extensively studied and have found applications in many fields. Now they are not only part of contemporary mathematics, but also widely used in computer graphics, control theory, signal processing, altitude control, physics, and mechanics (mainly for representing rotations and orientations of objects in three-dimensional space).

The research on quaternion linear algebra and matrices with quaternions has been active in recent years, with a considerable number of research papers published in various journals each year. A monograph that brings together all important results in the area in one place is needed and meaningful. Rodman did the job, and did it in a good way.

The book consists of two parts, almost equal in pages. The first part has 7 chapters (1 Introduction, 2 The algebra of quaternions, 3 Vector spaces and matrices: Basic theory, 4 Symmetric matrices and congruence, 5 Invariant subspaces and Jordan form, 6 Invariant neutral and semidefinite subspaces, 7 Smith form and Kronecker canonical form). Basic properties and results of quaternions, quaternion matrices, and quaternion linear algebra are introduced and studied in part one, while the second part of the book, also containing 7 chapters, deals with various advanced topics (8 Pencils of hermitian matrices, 9 Skewhermitian and mixed pencils, 10 Indefinite inner products: Conjugation, 11 Matrix pencils with symmetries: Nonstandard involution, 12 Mixed matrix pencils: Nonstandard involutions, 13 Indefinite inner products: Nonstandard involution, 14 Matrix equations). There is an appendix (Chapter 15) which recaps the canonical forms of real and complex matrices (and their pencils).

The first part begins with (Chapter 1) detailed descriptions for all chapters in the book, then deals with preliminary material on quaternions, i.e., the algebra of quaternions, then it moves on by introducing the structures of the quaternion vector spaces and quaternion matrices and developing the representations of quaternion matrices in terms of real and complex matrices, which are key for the study of many quaternion matrices. Many aspects of quaternion matrices are explored, including determinants, numerical ranges, symmetric matrices and congruence, and Smith form and Kronecker canonical forms.

The second part starts with the canonical forms of quaternion matrices, with great emphasis on matrix pencils (hermitian matrix pencils, skewhermitian matrices pencils, matrix pencils with symmetries, and mixed matrix pencils). The approaches to obtaining the results are both algebraic and analytic. Applications are given to systems of linear differential equations with symmetries and to matrix equations.

Notes sections following each chapter provide discussions and references for the chapter, guiding the reader to read and find more about the results shown in the chapter. This definitely helps students or even researchers to do further studies. The book provides a systematic, accessible, and self-contained exposition of quaternion linear algebra and matrices of quaternions, and treats all the material in the area carefully and comprehensively. Some results are not previously

seen and are given complete proofs here; many open problems and exercises at various difficulty levels are provided for discussion and/or as research projects.

The book is of course an invaluable reference tool for mathematicians and researchers in any field who have interest in quaternions, quaternion linear algebra, and matrices of quaternions.

(As we were writing this book review, we learned that Leiba had just passed away. Leiba: Rest in peace; you will be remembered.)

Introduction to Matrix Analysis and Its Applications,
by Fumio Hiai and Dénez Petz

Universitext. Springer, Cham; Hindustan Book Agency, New Delhi, 2014,
ISBN 978-3-319-04149-0, viii+332 pages.

Reviewed by Martin Argerami, University of Regina, argerami@uregina.ca

This is not your typical matrix analysis book, as can already be seen from the names of the first three chapters: Fundamentals of Operators and Matrices, Mappings and Algebras, Functional Calculus and Derivation. The material is developed from a point of view strongly influenced by operator algebras. This leads to terminology that is unusual in mainstream linear algebra, as with the use of “positive” for positive semidefinite matrices, for example. The content also departs from typical matrix analysis in a variety of ways, such as in the considerable attention paid to complete positivity. The second half of the book is definitely more specialized towards the use of matrices and their techniques in quantum information, with an emphasis on the authors’ speciality of majorization, matrix means, and matrix inequalities. The last chapter is devoted concretely to more or less recent applications to physics and quantum information theory.

The list of exercises at the end of each chapter is extensive. Exercises include a lot of new information, including some mainstream matrix analysis results, like Schur’s theorem on the Hadamard product, and the Schur decomposition of a matrix.

Hiai and Petz have an extensive background in operator algebras and mathematical physics, and it shows in their presentation and notation. For instance, the inner product on \mathbb{C}^n is denoted with the common $\langle x, y \rangle$, but there are instances where notation suddenly goes to bra-ket notation, $\langle x|y \rangle$. The rank-one operators are systematically written as $x\rangle\langle y$. While there are certain benefits to this notation (as most physicists will gladly attest), it can easily lead to unclear mathematics: On page 8, in an example destined to advertise bra-ket notation, Hiai-Petz compute $\text{Tr}(E_{kj}XE_{jk}Y) = X_{jj}Y_{kk}$, where the E_{kj} are the canonical matrix units, denoted by $E(ij)$ in the book, by

$$\text{Tr}(|e_i\rangle\langle e_j|X|e_j\rangle\langle e_i|Y) = \langle e_j|X|e_j\rangle\langle e_i|Y|e_i\rangle.$$

The physicist will not think twice about cycling the leftmost vector to the right, as that is the kind of thing you do inside the trace. But notation alone is not math, and the step needs to be justified, losing all possible advantage of using such notation. Actually, the “physicist’s argument” is wrong, because after we pass the $|e_i\rangle$ to the right, there is nothing to take the trace of anymore: just the product of two numbers. What is happening here is simply the easily proven fact $\text{Tr}(xy^*) = \langle x, y \rangle$. At the other end of the spectrum, Hiai and Petz’s proof of the uniqueness of the Kraus decomposition of a completely positive map (Theorem 2.56, via Lemma 1.24) is a nice application of the bra-ket notation.

I think this book will make an interesting contribution to the linear algebraist’s library, by providing a point of view that is far from the main avenues of linear algebra.

JOURNAL ANNOUNCEMENT

***LAA* Special Issue in Honor of Rajendra Bhatia**

Linear Algebra and Its Applications (*LAA*) is pleased to announce a special issue in honor of Professor Rajendra Bhatia in recognition of his many important contributions in research and books to matrix analysis and on the occasion of his 65th birthday in 2017. *LAA* solicits papers for the special issue within the entire scope of *LAA*, with a special emphasis on research topics related to the work of Rajendra Bhatia. The deadline for submissions of papers is March 1, 2016. All submissions will be subject to normal refereeing procedures and the usual standards of *LAA* will be applied. They should be submitted via the Elsevier Editorial System EES (<http://ees.elsevier.com/laa/>) by choosing the special issue called “In Honor of Rajendra Bhatia” and the responsible editor-in-chief, Peter Šemrl.

Authors will have the opportunity to suggest one of the following special editors to handle their submissions: Ravindra B. Bapat, Shmuel Friedland, John Holbrook, Roger Horn, Fuad Kittaneh.

UPCOMING CONFERENCES AND WORKSHOPS

Rocky Mountain–Great Plains Graduate Research Workshop in Combinatorics 2015 (GRWC 2015) Iowa State University, USA, June 1–12, 2015

GRWC 2015 is a 2-week collaborative research workshop for experienced graduate students from all areas of combinatorics, broadly interpreted. Students will work in collaborative groups with faculty and postdocs on research problems from across the discipline. The workshop will also host a variety of professional development workshops to prepare students and postdocs for the job hunt and for their transition to careers in academia, government, or industry.

The organizing institutions for GRWC are Iowa State University, University of Colorado Denver, University of Denver, University of Nebraska–Lincoln, and University of Wyoming. The workshop is funded in part by these institutions, the Institute for Mathematics and its Applications (IMA), Elsevier, the International Linear Algebra Society (ILAS), and we have applied for NSF funding.

Please direct all inquiries to grwc2015@gmail.com. More information is available at the workshop website: <http://sites.google.com/site/rmgpgrwc>.

Gene Golub SIAM Summer School 2015 Delphi, Greece, June 15–26, 2015

The 2015 Gene Golub SIAM Summer School (G2S3) on RandNLA: Randomization in Numerical Linear Algebra will take place June 15–26, 2015, in Delphi, Greece. The summer school, hosted by the University of Patras, will be held at the European Cultural Centre of Delphi (ECCD).

Information about the G2S3 program, including the composition of the G2S3 committee, can be found at http://www.siam.org/about/com_golub.php.

8th Slovenian International Conference on Graph Theory Kranjska Gora, Slovenia, June 21–27, 2015

The 8th Slovenian International Conference on Graph Theory will be held in Kranjska Gora, Slovenia, June 21–27, 2015.

In addition to the general session, the conference will sport a number of mini-symposia that might be of interest to researchers working in both linear algebra and graph theory.

Further information about the conference can be found at <http://kg15.imfm.si>.

Summer School on “Exploiting Hidden Structure in Matrix Computations: Algorithms and Applications” Cetraro, Italy, June 22–26, 2015

The C.I.M.E. Foundation Summer School on “Exploiting Hidden Structure in Matrix Computations: Algorithms and Applications” will take place June 22–26, 2015 in Cetraro, Italy. The Lecturers are: Michele Benzi (Emory University, USA), Dario Bini (Università di Pisa, Italy), Daniel Kressner (EPFL, Lausanne, Switzerland), Hans Munthe-Kaas (University of Bergen, Norway) and Charles Van Loan (Cornell University, USA).

For further details, please visit <http://web.math.unifi.it/users/cime/>.

2015 International Workshop on Matrix Inequalities and Matrix Equations Shanghai University, China, June 28–30, 2015

The purpose of the workshop is to stimulate research and foster interaction of researchers interested in matrix inequalities, matrix equations, and their applications.

The organizers are: Chi-Kwong Li (College of William & Mary, Shanghai University), Tin-Yau Tam (Auburn University), Qingwen Wang (Shanghai University), and Changqing Xu (Suzhou University of Science and Technology).

For further details, please visit <http://cklixx.people.wm.edu/mime2015.html>.

**Advanced Course on Combinatorial Matrix Theory
Barcelona, Spain, June 29–July 3, 2015**

This Advanced Course is oriented towards Ph.D. students and young postdocs, but it is also open to more senior researchers. The summer school will take place at the Centre de Recerca Matemàtica (CRM) in Bellaterra, Barcelona, Spain, from June 29 to July 3, 2015.

The lecturers and their topics are: Richard A. Brualdi, University of Wisconsin-Madison, *Combinatorial Matrix Theory*; Angeles Carmona Mejías, Universitat Politècnica de Catalunya–BarcelonaTech, *Boundary Value Problems on Finite Networks*; Stephen J. Kirkland, University of Manitoba, *The Group Inverse for the Laplacian Matrix of a Graph*; Dragan Stevanovic, Serbian Academy of Sciences and Arts (SANU), *Spectral Radius of Graphs*; and Pauline van den Driessche, University of Victoria, *Sign Pattern Matrices*.

Further details are posted at the web site of the school http://www.crm.cat/en/Activities/Curs_2014-2015/Pages/Combinatorial-Matrix-Theory.aspx.

**Summer Research Workshop on Quantum Information Science
Tsinghua Sanya International Mathematics Forum, China, July 13–17, 2015**

The purpose of the summer workshop is to promote collaborative research on the rapidly developing interdisciplinary area of quantum information science. The focus will be on the mathematical aspects of the subject. Invited participants will present their recent results and open problems on these topics. They will share their experiences and insights to solve open problems.

The organizers are: Shiu-Yuen Cheng (Tsinghua University, China), Man-Duen Choi (University of Toronto, Canada), Jianlian Cui (Tsinghua University), Jinchuan Hou (Taiyuan University of Technology), Chi-Kwong Li (College of William & Mary, USA), and Guilu Long (Tsinghua University).

For further details, please visit <http://cklixx.people.wm.edu/qc2015.html>.

**Graduate Student Modeling Workshop (IMSM 2015)
North Carolina State University, USA, July 12–22, 2015**

The 21st Industrial Mathematical & Statistical Modeling (IMSM) Workshop for Graduate Students will take place at North Carolina State University, July 12–22, 2015. The workshop is sponsored by the Statistical and Applied Mathematical Science Institute (SAMSI) together with the Center for Research in Scientific Computation (CRSC) and the Department of Mathematics at North Carolina State University.

The IMSM workshop exposes graduate students in mathematics, engineering, and statistics to exciting real-world problems from industry and government. The workshop provides students with experience in a research team environment and exposure to possible career opportunities. On the first day, a Software Carpentry boot camp will bring students up-to-date on their programming skills in Python/Matlab and R, and introduce them to version control systems and software repositories.

Further information is available at <http://www.samsi.info/IMSM15> and questions can be directed to grad@samsi.info.

**Second International Symposium on “Riordan Arrays and Related Topics”
Lecco, Italy, July 14–16, 2015**

The first meeting on “Riordan Arrays and Related Topics” was held in Seoul as one of eight invited minisymposia at ILAS 2014. The Second International Symposium on “Riordan Arrays and Related Topics” will be held in Lecco (on Como lake), Italy, July 14–16, 2015.

The meeting is organized in honor of Lou Shapiro and Renzo Sprugnoli, in consideration of all their important contributions on the topic and will include invited lectures and contributed talks.

For further details please visit the website: <http://www.mate.polimi.it/RART2015/>.

2015 Workshop on Matrices and Operators
Shaanxi Normal University, China, July 19–21, 2015

The purpose of the workshop is to stimulate research and foster interaction of researchers interested in matrix theory, operator theory, and their applications.

The organizers are: Huai-Xin Cao (Shaanxi Normal University), Xiao-Hong Cao (Shaanxi Normal University), Man-Duen Choi (University of Toronto), Hong-Ke Du (Shaanxi Normal University), Zhi-Wah Guo (Shaanxi Normal University), Guo-Xing Ji (Shaanxi Normal University), Chi-Kwong Li (College of William & Mary), Bao-Wei Wu (Shaanxi Normal University), and Jian-Hua Zhang (Shaanxi Normal University).

For further details, please visit <http://cklix.people.wm.edu/mao2015.html>.

4th International Conference on Matrix Methods in Mathematics and Computations
Moscow, Russia, August 24–28, 2015

The 4th International Conference on Matrix Methods in Mathematics and Computations (previous title: “Matrix Methods in Mathematics and Applications”) will be held in Moscow, Russia, August 24–28, 2015. Arrival is on Sunday, August 23; departure is on Saturday, August 29.

The organizing institutes are: Marchuk Institute of Numerical Mathematics (INM), Lomonosov Moscow State University (MSU), and SkolkovoTech University (SkolTech).

The Organizing Committee consists of: Dario Bini (University of Pisa), Alexander Guterman (MSU), Ivan Oseledets (SkolTech), Denis Zorin (Courant Institute of Mathematics), and Eugene Tyrtyshnikov (INM-MSU, chair). We plan to have plenary talks and three (maybe parallel) sessions: (1) Matrices and Algebra; (2) Matrices and Algorithms; and (3) Matrices and Applications.

A modest fee will be established of about 200€ covering everything (book of abstracts, coffee-tea breaks, welcome party) except for accommodations and meals. Please let us know about your intention concerning this conference (even if only preliminary) at your earliest convenience. For correspondence please use the conference email: matrix.methods.2015@gmail.com.

For further details, please visit the conference webpage: <http://matrix.inm.ras.ru/mmma-2015/>.

Mat-Triad’2015 – Conference on Matrix Analysis and its Applications
Coimbra, Portugal, September 7–11, 2015

The purpose of the conference is to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications, special emphasis being given to applications in other areas of science. One of the main goals is to highlight recent achievements in these mathematical domains. The program will cover different aspects with emphasis on: recent developments in matrix and operator theory, direct and inverse spectral problems, matrices and graphs, applications of linear algebra in statistics, matrix models in industry and the sciences, linear systems and control theory, quantum computation, and combinatorial matrix theory.

Researchers and graduate students interested in the scope of the conference are particularly encouraged to attend. The conference will provide a friendly atmosphere for the discussion and exchange of ideas, which hopefully will lead to new scientific links among participants. The format of the meeting will involve plenary sessions and sessions with contributed talks. The list of Invited Speakers includes winners of Young Scientists Awards of MatTriad’2013. We are also planning two short courses delivered by leading experts. Thematic workshops are welcome. The work of young scientists will receive special consideration in MatTriad’2015. The best poster as well as the best talk of graduate students or scientists having recently completed a Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference. The conference is endorsed by ILAS. Up-to-date information is available at <http://www.matriad.ipt.pt>.

The Lecturers are Peter Šemrl (Slovenia) and Ludwig Elsner (Germany). Confirmed Invited Speakers include Peter Benner (Germany), Froilán M. Dopico (Spain) and Dietrich von Rosen (Sweden). The winners of the Young Scientists Awards of Mat Triad’2013: Maja Nedović (Serbia), and Jaroslav Horáček (Czech Republic).

The Scientific Committee consists of Tomasz Szulc (Poland) – Chair, Natália Bebiano (Portugal), Ljiljana Cvetković (Serbia), Heike Faßbender (Germany) and Simo Puntanen (Finland). The Organizing Committee are Natália Bebiano (Portugal) – Chair, Francisco Carvalho (Portugal), Susana Furtado (Portugal), Celeste Gouveia (Portugal), Rute Lemos (Portugal) and Ana Nata (Portugal).

For further information, please contact Natália Bebiano (matriad@mat.uc.pt) or Francisco Carvalho (fpcarvalho@ipt.pt).

Workshop on Matrix Equations and Tensor Techniques Bologna, Italy, September 21–22, 2015

The fifth Workshop on “Matrix Equations and Tensor Techniques” will be held from September 21–22, 2015 at the Mathematics Department, Alma Mater Studiorum Università di Bologna, Italy.

As in the previous meetings, the focus will be on the latest developments in the theory, computation and applications of linear and nonlinear matrix equations and tensor equations. We will extend our focus to sparsity and low rank structures, and to decay properties of matrices and tensors associated with these problems.

Additional details and registration information can be found at <http://mett15.dm.unibo.it/>.

Organizers: Peter Benner (Max-Planck Institut, Magdeburg, Germany), Heike Faßbender (Technische Universität Braunschweig, Germany), Lars Grasedyck (Aachen University, Germany), and Daniel Kressner (École polytechnique fédérale de Lausanne, Switzerland).

Local Organizer: Valeria Simoncini (Alma Mater Studiorum Università di Bologna, Italy).

SIAM Conference on Applied Linear Algebra (LA15) Hyatt Regency Atlanta, Georgia, USA, October 26–30, 2015

The Organizing Committee co-chairs are Chen Greif (University of British Columbia, Canada) and James Nagy (Emory University, USA).

Organizing Committee: Zhaojun Bai (University of California, Davis, USA); Zhong-Zhi Bai (Chinese Academy of Sciences, China); Petros Drineas (Rensselaer Polytechnic Institute, USA); David Gleich (Purdue University, USA); Bruce Hendrickson, (Sandia National Laboratories, USA); Beatrice Meini (Università di Pisa, Italy); Alison Ramag, (University of Strathclyde, UK); Zdeněk Strakoš (Charles University of Prague, Czech Republic); Françoise Tisseur (University of Manchester, UK); Sabine Van Huffel (KU Leuven, Belgium).

Invited Plenary Speakers: Haim Avron (IBM T. J. Watson Research Center, USA); Raymond Chan (Chinese University of Hong Kong, Hong Kong); Geir Dahl* (University of Oslo, Norway); Zlatko Drmac (University of Zagreb, Croatia); Howard Elman (University of Maryland, USA); Maryam Fazel (University of Washington, USA); Melina Freitag (University of Bath, UK); Xiaoye Sherry Li (Lawrence Berkeley National Laboratory, USA); Volker Mehrmann (TU Berlin, Germany); Michael Overton* (New York University, USA); Haesun Park (Georgia Institute of Technology, USA); Eugene Tyrtshnikov (Russian Academy of Sciences, Russia).

* These speakers are supported in cooperation with ILAS.

The conference is sponsored by the SIAM Activity Group on Linear Algebra (SIAG/LA). For further information, check the conference website: <http://www.siam.org/meetings/la15/>.

5th International Conference on Matrix Analysis and Applications (ICMA) Nova Southeastern University, Fort Lauderdale, USA, December 18–20, 2015

The 5th International Conference on Matrix Analysis and Applications (ICMA) will be held at Nova Southeastern University, Fort Lauderdale, Florida, USA, December 18–20, 2015. This meeting aims to stimulate research and interaction of mathematicians in all aspects of linear and multilinear algebra, matrix analysis, graph theory, and their applications, and to provide an opportunity for researchers to exchange ideas and developments on these subjects. The previous conferences were held in China (Beijing, Hangzhou), the United States (Nova Southeastern University), and Turkey (Konya). Former keynote speakers are Roger Horn, Richard Brualdi, Chi-Kwong Li, Steve Kirkland and Alexander A. Klyachko (ILAS guest speaker). The keynote speaker of the 5th ICMA is Professor Shmuel Friedland, University of Illinois, Chicago, USA.

A special issue of the electronic research journal *Special Matrices* published by De Gruyter (<http://www.degruyter.com/view/j/spma>) will be devoted to the conference.

The Scientific Organizing Committee of the 5th conference of the series consists of Shaun Fallat (University of Regina, Canada), Peter Šemrl (University of Ljubljana, Slovenia), Tin-Yau Tam (Auburn University, USA), Qingwen Wang (Shanghai University, Shanghai, China) and Fuzhen Zhang (Nova Southeastern University, USA, Chair). Local Organizing Committee members are Shahla Nasserar [Chair], Vehbi Paksoy, and mathematics faculty and students of Nova Southeastern University. The conference is sponsored by Nova Southeastern University (USA) and Shanghai University (China).

For more information and updates, please check the conference website: <https://www.fcas.nova.edu/about/matrix/>.

Tensor Decompositions and Blind Signal Separation Leuven, Belgium, January 15–16, 2016

This course is intended as a short introduction to tensor decompositions and their applications. The course will consist of 4 lectures (2 hours each; lecturer: Lieven De Lathauwer) and 4 exercise sessions (also 2 hours each).

The course will be followed by the Workshop on Tensor Decompositions and Applications (TDA 2016).

For more information, please visit <http://homes.esat.kuleuven.be/~sistawww/winterschool16/index.php>.

Third Workshop on Tensor Decompositions and Applications (TDA 2016) Leuven, Belgium, January 18–22, 2016

Higher-order tensor methods are intensively studied in many disciplines nowadays. The developments gradually allow us to move from classical vector and matrix based methods in applied mathematics and mathematical engineering to methods that involve tensors of arbitrary order. This step from linear transformations and quadratic and bilinear forms to polynomials and multilinear forms is relevant for the most diverse applications. Furthermore, tensor methods have firm roots in multilinear algebra, algebraic geometry, numerical mathematics and optimization.

This workshop will bring together researchers investigating tensor decompositions and their applications. It will feature a series of invited talks by leading experts and contributed presentations on specific problems. TDA 2016 is held in cooperation with the Society for Industrial and Applied Mathematics (SIAM) and has been endorsed by the International Linear Algebra Society (ILAS). Part of the workshop is supported by the ERC Advanced Grant BioTensors.

For more information, please visit: <http://www.esat.kuleuven.be/stadius/TDA2016/index.php>.

The Householder Symposium XX on Numerical Linear Algebra Virginia Tech, Blacksburg, Virginia, USA, June 18–23, 2017

The Householder Symposium XX on Numerical Linear Algebra will be held at Virginia Tech in Blacksburg, Virginia, USA, June 18–23, 2017. This symposium is the twentieth in a series, previously called the Gatlinburg Symposia, and will be hosted by the Virginia Polytechnic Institute and State University (Virginia Tech) in cooperation with the Society for Industrial and Applied Mathematics (SIAM) and the SIAM Activity Group on Linear Algebra. Details are available at: <http://www.math.vt.edu/HHXX>.

ILAS NEWS

ILAS Member Becomes SIAM Fellow



H. Wolkowicz

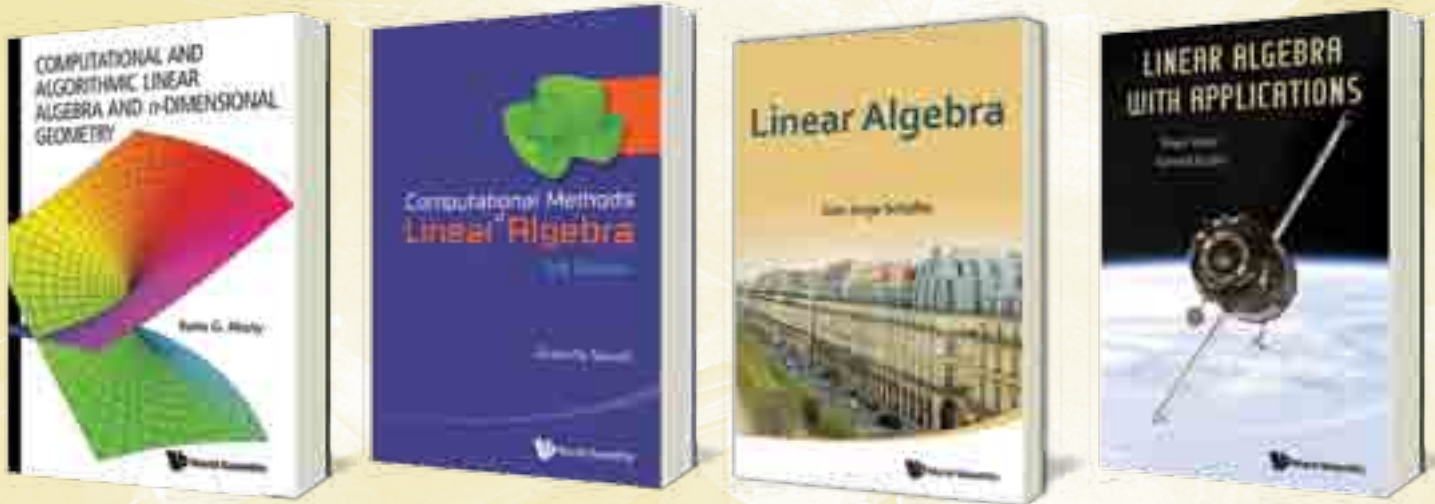
The Society for Industrial and Applied Mathematics (SIAM) has recently announced its SIAM Fellows for 2015 for individuals who have made outstanding contributions to the fields of applied mathematics and computational science. Dr. Henry Wolkowicz, who has just joined the ILAS board, has been named a SIAM Fellow.

As noted at *SIAM Connect*: “Henry Wolkowicz, a professor in the department of combinatorics & optimization at the University of Waterloo is being conferred Fellowship for contributions to convex optimization and matrix theory. His areas of research span optimization, mathematical programming, scheduling problems, quadratic assignment problem, numerical analysis, convex analysis, matrix theory, and generalized inverses. Wolkowicz has previously served on the SIAM Council and as Chair of the SIAM Activity Group on Optimization.”

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Computational and Algorithmic Linear Algebra and n -Dimensional Geometry

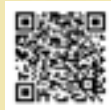
by **Katta G Murty** (*University of Michigan, Ann Arbor, USA*)

This undergraduate textbook on Linear Algebra and n -Dimensional Geometry has a unique focus on developing the mathematical modeling as well as computational and algorithmic skills in students at this level. The explanations in this book are detailed, lucid, and supported with numerous well-constructed examples to capture the interest and encourage the student to master the material.

Readership: Undergraduate students in linear algebra and n -dimensional geometry.

480pp
978-981-4366-62-5
978-981-4366-63-2(pbk)

Sep 2014
US\$128 £84
US\$68 £45



Computational Methods of Linear Algebra (3rd Edition)

by **Granville Sewell** (*University of Texas El Paso, USA*)

This book presents methods for the computational solution of some important problems of linear algebra: linear systems, linear least squares problems, eigenvalue problems, and linear programming problems. The book also includes a chapter on the fast Fourier transform and a very practical introduction to the solution of linear algebra problems on modern supercomputers.

Readership: Undergraduate and graduate students, researchers in computational mathematics and linear algebra.

328pp
978-981-4603-85-0
978-981-4603-86-7(pbk)

Sep 2014
US\$88 £58
US\$48 £32



Linear Algebra

by **Juan Jorge Schäffer** (*Carnegie Mellon University, USA*)

This volume is a careful exposition of the concepts and processes of Linear Algebra. It stresses cautious and clear explanations, avoiding reliance on co-ordinates as much as possible, and with special, but not exclusive, attention to the finite-dimensional situation. It is intended to also serve as a conceptual and technical background for use in geometry and analysis as well as other applications.

Readership: Undergraduate and graduate students in mathematics; Mathematicians in algebra.

140pp
978-981-4623-49-0

Aug 2014
US\$48 £32



Linear Algebra with Applications

by **Roger Baker & Kenneth Kuttler**
(*Brigham Young University, USA*)

This book gives a self-contained treatment of linear algebra with many of its most important applications. It is very unusual if not unique in being an elementary book which does not neglect arbitrary fields of scalars and the proofs of the theorems. It will be useful for beginning students and also as a reference for graduate students and others who need an easy to read explanation of the important theorems of this subject.

Readership: Undergraduates in linear algebra.

332pp
978-981-4590-53-2

Apr 2014
US\$76 £50



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ILAS President/Vice President Annual Report: 4 April 2015

Respectfully submitted by Peter Šemrl, ILAS President, peter.semrl@fmf.uni-lj.si
and Bryan Shader, ILAS Vice-President, bshader@uwyo.edu

1. Board approved actions since the last report include:

- An amendment to ILAS By-laws to have the President and Vice-President reports at the ILAS Business meetings be a combined report from 2014 on.
- A more selective process for ILAS Lecturers coupled with a commitment to more substantial support for those selected.
- Starting in 2015, the ILAS print-*IMAGE* membership fee was raised to US\$60 per year (the e-*IMAGE* membership remains at US\$40 per year). A member 60 or older and who has been a dues paying member for at least 5 years (not necessarily consecutive or immediately prior to life membership) can secure lifetime e-*IMAGE* membership in ILAS by payment of US\$200. A member 63 or older, or 60 or older and retired, and who has been a dues paying member for at least 5 years (not necessarily consecutive or immediately prior to life membership) can secure lifetime e-*IMAGE* membership in ILAS by payment of US\$100.

2. ILAS elections ran from November 17, 2014 to January 16, 2015 and proceeded via electronic voting. The following were elected to offices with three-year terms that began on March 1, 2015:

- Secretary/Treasurer: Leslie Hogben
- Board of Directors: Beatrice Meini and Henry Wolkowicz.

The following continue in the ILAS offices to which they were previously elected:

- President: Peter Šemrl (term ends February 28, 2017)
- Vice President: Bryan Shader (term ends February 29, 2016)
- Board of Directors: Avi Berman (term ends February 29, 2016), Froilán Dopico (term ends February 28, 2017), Volker Mehrmann (term ends February 29, 2016), and Michael Overton (term ends February 28, 2017).

Dale Olesky and Eugene Tyrtysnikov completed their terms on the ILAS Board of Directors on February 28, 2014. We thank them for their valuable contributions as Board members; their service to ILAS is most appreciated. We also thank the members of the Nominating Committee—Dario Bini, Shaun Fallat (chair), Marko Huhtanen, Rachel Quinlan, and Xingzhi Zhan—for their work on behalf of ILAS, and also extend gratitude to all candidates that agreed to have their names stand for the elections.

Based on recommendations of the ILAS Program Review Committee, the ILAS Board appointed Steve Kirkland as Second Vice President (for ILAS conferences) with term from January 1, 2015 to February 28, 2017. The Second Vice President is a member of the Executive Committee and a non-voting member of the ILAS Board of Directors, and serves as a connection between organizers of ILAS Conferences and the ILAS Executive Committee.

3. The following ILAS-endorsed meetings have taken place since our last report:

- Graph Theory, Matrix Theory and Interactions Conference (A conference to celebrate the mathematics of David A. Gregory), Queen's University, Kingston, ON, Canada, June 20–21, 2014.
- The 2014 International Conference on Tensors and Matrices and their Application, Suzhou University of Science and Technology, Suzhou, China, December 17–19, 2014.
- International Conference on Linear Algebra and its Applications (A conference in honor of Professor Ravindra B. Bapat on his 60th birthday), Manipal University, Manipal, India, December 18–20, 2014.

The first of these featured Shaun Fallat as the first Hans Schneider Lecturer. The Hans Schneider ILAS Lecture Fund was established with a generous donation from ILAS' founding President, Hans Schneider. Additional contributions to the Fund are welcome. The Fund supports one speaker per year to give a lecture on linear algebra (and possibly its applications to, or connections with, a related discipline) at a non-ILAS meeting. The Hans Schneider ILAS Lecture is one of ILAS' signature programmes. Consequently, the Hans Schneider ILAS Lecture will be given by a researcher with strong expository skills whose work represents linear algebra at its highest level. Each year, the ILAS Board of Directors evaluates proposals for ILAS Lectures at non-ILAS meetings. The top ranked proposal may be identified as the Hans Schneider ILAS Lecture.

4. ILAS has endorsed the following conferences of interest to ILAS members:

- 24th International Workshop on Matrices and Statistics, Haikou, China, May 25–28, 2015. Karl Gustafson will be the ILAS Lecturer.
- Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics, Ames, Iowa, USA, June 1–12, 2015. Franklin Kenter will be the ILAS Lecturer.
- MatTriad - International Conference on Matrix Analysis and its Applications, Coimbra, Portugal, September 7–11, 2015.
- SIAM Conference on Applied Linear Algebra, Atlanta, Georgia, USA, October 26–30, 2015, <http://www.siam.org/meetings/la15/>. Geir Dahl and Michael Overton will be the ILAS Lecturers.
- Workshop on Tensor Decompositions and Applications (TDA 2016), Leuven, Belgium, January 18–22, 2016.

5. The following ILAS conferences are scheduled:

- 20th ILAS Conference, July 11–15, 2016, Leuven, Belgium. The chair of the organizing committee is Raf Vandebril. For more information see <http://ilas2016.cs.kuleuven.be/>.
- 21st ILAS Conference: Connections, July 24–28, 2017, Ames, Iowa, USA. The chair of the organizing committee is Leslie Hogben. For more information see <http://www.math.iastate.edu/Events/2017ILAS/index.html>.

6. The 2016 Hans Schneider Prize in Linear Algebra Committee was appointed by ILAS President, Peter Šemrl, and consists of Richard Brualdi (chair), Shmuel Friedland, Ilse Ipsen, Thomas Laffey, Peter Šemrl (ex-officio member) and Xingzhi Zhan.

The committee will solicit nominations, and make a recommendation to the ILAS Executive Board, for this Prize to be awarded at the ILAS conference in Leuven, Belgium, July 11–15, 2016. Nominations are open until December 1, 2015 and should be sent, preferably by email, to the Chair, Richard Brualdi (brualdi@math.wisc.edu).

7. The *Electronic Journal of Linear Algebra (ELA)* is now in its 30th volume. The Editor-in-Chief is Bryan Shader. In 2014, *ELA* published 61 papers totaling 972 pages. There are currently two Special Volumes of *ELA* open. The first is the Proceedings of Graph Theory, Matrix Theory and Interactions Conference, Queen's University. Special editors for the volume are: Sebastian Cioabă, Ram Murty, Claude Tardif, Kevin Vander Meulen and David Wehlau. The second is the Proceedings of the International Conference on Linear Algebra and its Applications Conference. Special editors for this volume are: Rajendra Bhatia, Steve Kirkland, K. Manjunatha Prasad and Simo Puntanen. In August 2015, *ELA* moved to new platform and new site (<http://repository.uwo.edu/ela>), which provides a modern web portal, online submission capabilities, as well as better tracking and discoverability.

8. *IMAGE* is the semi-annual bulletin for ILAS. The Editor-in-Chief is Kevin N. Vander Meulen, who is supported by contributing editors Minerva Catral, Michael Cavers, Douglas Farenick, Carlos Fonseca, Bojan Kuzma, Naomi Shaked-Monderer, David Strong, and Amy Wehe, as well as Louis Deaett in the new position of managing editor. The Spring 2015 edition is *IMAGE*'s 54th issue. Recent additions (e.g. featured interviews, linear algebra education sections) have been well received.

9. ILAS-NET is a moderated newsletter for mathematicians worldwide, with a focus on linear algebra; it is managed by Sarah Carnochan Naqvi. As of April 5, 2015 there are 681 subscribers to ILAS-NET. An archive of ILAS-NET messages is available at <http://www.ilasic.org/ilas-net/>. To send a message to ILAS-NET, please send the message (preferably in text format) in an email to ilasic@uregina.ca indicating that you would like it to be posted on ILAS-NET. If the message is approved, it will be posted soon afterwards. To subscribe to ILAS-NET, please complete the form at <http://www.ilasic.org/ilas-net/subform.html>.

ILAS' website, known as the ILAS Information Centre (IIC), is located at <http://www.ilasic.org> and provides general information about ILAS (e.g. ILAS officers, By-laws, Special Lecturers), as well as links to pages of interest to the ILAS community.

Respectfully submitted,

Peter Šemrl, ILAS President (peter.semrl@fmf.uni-lj.si); and
 Bryan Shader, ILAS Vice-President (bshader@uwo.edu).

ILAS 2014-2015 Treasurer's Report

April 1, 2014 – March 31, 2015

By Leslie Hogben

Net Account Balance on March 31, 2014

Vanguard (ST Fed Bond Fund Admiral 7714.044 Shares)	\$ 82,694.55
Checking Account - Great Western	\$ 53,866.79
Certificate of Deposit	\$ 20,497.31
Account's Payable	\$ (700.00)

\$ 156,358.65

General Fund	\$ 78,503.01
Conference Fund	\$ 9,879.01
ILAS/LAA Fund	\$ 12,840.57
Olga Taussky Todd/John Todd Fund	\$ 11,749.14
Frank Uhlig Education Fund	\$ 4,996.34
Hans Schneider Lecture Fund	\$ 15,000.00
Hans Schneider Prize Fund	\$ 23,390.57

\$ 156,358.64

INCOME:

Dues	\$ 8,880.00
Elsevier Sponsorship (Flow Through)	\$ 2,500.00
General Fund	\$ 3,340.00
Royalty Income	\$ 219.92
ELA	\$ 51.00
Conference Fund	\$ -
LAMA for Speaker	\$ -
Taussky-Todd Fund	\$ 101.00
Uhlig Education Fund	\$ 9.43
Schneider Prize Fund	\$ 5,411.00
Hans Schneider Lecture Fund	\$ 401.00
Interest - Great Western	\$ 61.96
Interest on Great Western Certificate of Deposit (#1)	\$ 15.67
Interest on Great Western Certificate of Deposit (#2)	\$ 395.75
Vanguard - Dividend Income	\$ 439.65
- Short Term Capital Gains	\$ 49.70
- Long Term Capital Gains	\$ -
Variances in the Market	\$ 632.52
Misc Income	\$ -
Total Income	<u>\$ 22,508.60</u>

EXPENSES:

First Data - Credit Card Processing Fees	\$ 627.90
Speaker Fees	\$ 3,100.00
Hans Schneider Lecture	\$ 1,000.00
Treasurer's Assistant	\$ 140.00
Conference Expenses	\$ 650.00
2014 ILAS Conference - Elsevier Flowthru	\$ 2,500.00
Business License	\$ -
IMAGE Costs	\$ 1,159.74
Web Hosting & Online Membership Forms	\$ 298.90
Misc Expenses	\$ 203.23
Ballot Costs	\$ 304.50
Total Expenses	<u>\$ 9,984.27</u>

Net Account Balance on March 31, 2015

Vanguard (7776.346 @ 10.80/share)	\$ 83,984.54
Checking Account - Great Western	\$ 40,570.77
Certificate of Deposit #1 (<i>cashd in and deposited</i>)	\$ -
Certificate of Deposit #2 (<i>purchased</i>)	\$ 45,395.75
Accounts Payable	\$ (700.00)

\$ 169,251.06

General Fund	\$ 86,872.00
Conference Fund	\$ 9,429.01
ILAS/LAA Fund	\$ 12,840.57
ELA Fund	\$ 51.00
Olga Taussky Todd/John Todd Fund	\$ 11,850.14
Frank Uhlig Education Fund	\$ 5,005.77
Hans Schneider Lecture Fund	\$ 14,401.00
Hans Schneider Prize Fund	\$ 28,801.57

\$ 169,251.06



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IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to *IMAGE* Problems 52-3, 52-5 and to all problems in issue 53 of *IMAGE* except Problem 53-1, for which we still await a solution. Six new problems are on the last page; solutions are invited.

Problem 52-3: Removing Zero From Numerical Range

Proposed by Janko BRAČIČ, *University of Ljubljana, IMFM, Slovenia*, janko.bracic@fmf.uni-lj.si
and Cristina DIOGO, *Departamento de Matemática, Lisbon University Institute, Portugal*, cristina.diogo@iscte.pt

Let $A \in M_n(\mathbb{C})$. Show that 0 is not in the convex hull of the spectrum $\sigma(A)$ if and only if there exists a positive definite $P \in M_n(\mathbb{C})$ such that 0 is not in the numerical range of PA .

Solution 52-3.3 by JOHANNES DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let $F(A)$, $F'(A)$ and $\text{Conv}(\sigma(A))$ denote, respectively, the numerical range of A , the angular numerical range of A and the convex hull of $\sigma(A)$, where $\sigma(A)$ is the spectrum of A . It suffices to consider A nonsingular, because $0 \in \sigma(A)$ implies $0 \in \sigma(PA) \subseteq F(PA)$ for every positive definite $P \in \mathbb{C}^{n \times n}$ ($P > 0$).

If $0 \notin F(PA)$ for some $P > 0$, then $0 \notin F'(PA)$ because $0 \notin F(PA)$ if and only if $0 \notin F'(PA)$. Hence there is an open angular sector of \mathbb{C} anchored at the origin (denoted by Γ) with angle less than 180 degrees so that $F'(PA) \subseteq \Gamma$, which implies, according to [1, Theorem 1.7.12], $\sigma(A) \subseteq \Gamma$, and as Γ is convex, we conclude that $0 \notin \text{Conv}(\sigma(A)) \subseteq \Gamma$.

Conversely, if $0 \notin \text{Conv}(\sigma(A))$, then there is some open angular sector Γ anchored at the origin with angle less than 180 degrees so that $\sigma(A) \subseteq \text{Conv}(\sigma(A)) \subseteq \Gamma$. According to [2, Theorem 3.1.13], there is for each $\epsilon > 0$ a nonsingular $S_\epsilon \in \mathbb{C}^{n \times n}$ so that $S_\epsilon AS_\epsilon^{-1}$ is in modified Jordan canonical form: In place of every off-diagonal 1 that occurs in the Jordan canonical form of A is ϵ . Then, for sufficiently small ϵ , $F(S_\epsilon AS_\epsilon^{-1}) \subseteq \Gamma$, hence $0 \notin F(S_\epsilon AS_\epsilon^{-1}) \subseteq \Gamma$, which implies $0 \notin F'(S_\epsilon AS_\epsilon^{-1}) = F'(S_\epsilon^* S_\epsilon AS_\epsilon^{-1} S_\epsilon) = F'(PA)$, with $S_\epsilon^* S_\epsilon = P$, and therefore $0 \notin F(PA)$. \square

References

- [1] R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis*, first edition, Cambridge University Press, Cambridge, 1991.
- [2] R. A. Horn and C. R. Johnson, *Matrix Analysis*, second edition, Cambridge University Press, Cambridge, 2012.

Editorial note: Two solutions to IMAGE problem 52.3 were published in issue 53 of IMAGE. The present solution differs from them and was received too late to be included in issue 53.

Problem 52-5: Matrix Hadamard Power

Proposed by Roger A. HORN, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu
and Fuzhen ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA*, zhang@nova.edu

For a positive integer r and any $n \times n$ complex matrix $X = (x_{ij})$, let $X^{[r]} = (x_{ij}^r)$ be the Hadamard power of X and let $|X| = (|x_{ij}|)$ be the entrywise absolute value matrix of X . It is known that if A is positive semidefinite, then A^r (regular power) and $A^{[r]}$ are also positive semidefinite for any positive integer r . Moreover, $|A|^r$ and $|A|^{[r]}$ are positive semidefinite for each *even* positive integer r . Prove or disprove that $|A|^r$ and $|A|^{[r]}$ are positive semidefinite for $r = 3$.

Solution 52-5.1 by the proposers Roger A. HORN and Fuzhen ZHANG

It is known [1, 7.5.P.4 on p. 482] that if $n \leq 3$ then $A \geq 0$ implies $|A| \geq 0$. (Here $X \geq 0$ means that X is positive semidefinite.) It follows that for $n = 1, 2, 3$, both $|A|^r$ and $|A|^{[r]}$ are positive semidefinite for any positive integer r , in particular for $r = 3$. If r is even, then $|A|^r \geq 0$ because $|A|$ is real symmetric, and $|A|^{[r]}$ is positive semidefinite too due to the Hadamard (Schur) product theorem. There exists an example for $n = 4$ showing that $|A| \not\geq 0$ even though $A \geq 0$ (see [1, 7.5.P.6 on p. 482]). For such a matrix, $|A|^r \not\geq 0$ for any odd r . Here is an example for $n = 6$ and $r = 3$:

$$A = \begin{pmatrix} 1.0000 & 0.8660 & 0.5000 & 0.0000 & -0.5000 & -0.8660 \\ 0.8660 & 1.0000 & 0.8660 & 0.5000 & 0.0000 & -0.5000 \\ 0.5000 & 0.8660 & 1.0000 & 0.8660 & 0.5000 & 0.0000 \\ 0.0000 & 0.5000 & 0.8660 & 1.0000 & 0.8660 & 0.5000 \\ -0.5000 & 0.0000 & 0.5000 & 0.8660 & 1.0000 & 0.8660 \\ -0.8660 & -0.5000 & 0.0000 & 0.5000 & 0.8660 & 1.0000 \end{pmatrix}.$$

A is positive semidefinite, but $|A|^3$ is not; it has a negative eigenvalue $-0.0490\dots$. $|A|^{[3]}$ is not positive semidefinite either; it has a negative eigenvalue $-0.0489\dots$ \square

Reference

[1] R. A. Horn and C. R. Johnson, *Matrix Analysis*, second edition, Cambridge University Press, Cambridge, 2012.

Problem 53-2: AGM Inequality for Determinants

Proposed by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Let $X_1, \dots, X_n \in \mathbb{R}^{m \times m}$ be positive definite. Give a non-analytic proof that $\det\left(\frac{1}{n} \sum_{k=1}^n X_k\right) \geq \left(\prod_{k=1}^n \det X_k\right)^{1/n}$.

Solution 53-2.1 by the proposer Eugene A. HERMAN

More generally, let p_1, \dots, p_n be positive real numbers summing to 1. We show

$$\det\left(\sum_{k=1}^n p_k X_k\right) \geq \prod_{k=1}^n (\det X_k)^{p_k} \quad (*)$$

by using induction on n . When $n = 1$, inequality $(*)$ is an obvious equality. When $n = 2$, we factor out $X_1^{1/2}$ on the left and on the right from $p_1 X_1 + p_2 X_2$ to obtain

$$\det(p_1 X_1 + p_2 X_2) = (\det X_1) \det(p_1 I + p_2 X_1^{-1/2} X_2 X_1^{-1/2}).$$

Since $X_1^{-1/2} X_2 X_1^{-1/2}$ is positive definite, it has a full set of eigenvalues, $\lambda_1, \dots, \lambda_n$, all positive. Hence, by the weighted arithmetic-geometric mean,

$$\det(p_1 I + p_2 X_1^{-1/2} X_2 X_1^{-1/2}) = \prod_{k=1}^n (p_1 + p_2 \lambda_k) \geq \prod_{k=1}^n \lambda_k^{p_2} = \left(\det(X_1^{-1/2} X_2 X_1^{-1/2})\right)^{p_2} = (\det X_1)^{-p_2} (\det X_2)^{p_2}.$$

Therefore $\det(p_1 X_1 + p_2 X_2) \geq (\det X_1) (\det X_1)^{-p_2} (\det X_2)^{p_2} = (\det X_1)^{p_1} (\det X_2)^{p_2}$. Now assume $(*)$ holds for some $n \geq 2$. Then

$$\begin{aligned} \det\left(\sum_{k=1}^{n+1} p_k X_k\right) &= \det\left((1 - p_{n+1}) \left(\sum_{k=1}^n \frac{p_k}{1 - p_{n+1}} X_k\right) + p_{n+1} X_{n+1}\right) \geq \det\left(\sum_{k=1}^n \frac{p_k}{1 - p_{n+1}} X_k\right)^{1-p_{n+1}} (\det X_{n+1})^{p_{n+1}} \\ &\geq \left(\prod_{k=1}^n (\det X_k)^{p_k/(1-p_{n+1})}\right)^{1-p_{n+1}} (\det X_{n+1})^{p_{n+1}} = \prod_{k=1}^{n+1} (\det X_k)^{p_k}. \quad \square \end{aligned}$$

Editorial note: We were informed by Rajendra BHATIA that the AGM inequality also follows from concavity of the function $X \mapsto \log \det(X)$ see, e.g., F.2.c, p. 686 in the book [A. W. Marshall, I. Olkin, and B. C. Arnold, Inequalities: Theory of Majorization and its Applications, second edition, Springer, 2011.]

Problem 53-3: Normal and Symmetric Matrices

Proposed by Roger A. HORN, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu

Let $A = VBW^* \in \mathbb{C}^{n \times n}$ be normal, in which V and W are unitary and $B \in \mathbb{C}^{n \times n}$. Let $\mathcal{A} = V^* A V$.

First suppose that $B = B_1 \oplus \dots \oplus B_k$ is block diagonal, and partition $\mathcal{A} = [\mathcal{A}_{ij}]_{i,j=1}^k$ conformally to B . Prove the following three statements:

- If some block B_j has distinct singular values, then the corresponding block \mathcal{A}_{jj} is normal.
- If some block B_j is real and has distinct singular values, then corresponding block \mathcal{A}_{jj} is normal and symmetric.
- If each block B_i has only one singular value σ_i and if $\sigma_i \neq \sigma_j$ ($i \neq j$), then $\mathcal{A} = \sigma_1 Z_1 \oplus \dots \oplus \sigma_k Z_k$, with Z_i unitary.

Now suppose that B is real; it need not be block diagonal. Prove the following three statements:

- $\mathcal{A} = Q(\sigma_1 Z_1 \oplus \dots \oplus \sigma_d Z_d)Q^T$ for some orthogonal $Q \in \mathbb{R}^{n \times n}$ and unitary Z_1, \dots, Z_d , in which $\sigma_1 > \dots > \sigma_d \geq 0$.
- If A has distinct singular values, then \mathcal{A} is symmetric.
- If the eigenvalues $\lambda_1, \dots, \lambda_n$ of A have the property that $|\lambda_i| \neq |\lambda_j|$ whenever $\lambda_i \neq \lambda_j$, then \mathcal{A} is symmetric.

Solution 53-3.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Let $U = W^*V$. Then $\mathcal{A} = V^*VBW^*V = BU$, and $UBB^* = W^*VBW^*WB^*V^*V = W^*AA^*V = W^*A^*AV = W^*WB^*V^*VBW^*V = B^*BU$. In summary,

$$\mathcal{A} = BU, \quad UBB^* = B^*BU \quad \text{with } U \text{ unitary.} \quad (1)$$

If $B = B_1 \oplus \cdots \oplus B_k$ and $U = [U_{ij}]_{i,j}^n$ is the corresponding partition of U , then (1) implies

$$\mathcal{A}_{jj} = B_j U_{jj}, \quad U_{ji} B_i B_i^* = B_j^* B_j U_{ji}, \quad j, i = 1, \dots, k. \quad (2)$$

(a) We are given that $B_j = XSY^*$, where X, Y are unitary and Σ is diagonal with distinct nonnegative diagonal entries. From the second identity in (2) we obtain $Y^*U_{jj}X\Sigma^2 = \Sigma^2Y^*U_{jj}X$. Thus, $Y^*U_{jj}X$ commutes with a diagonal matrix whose diagonal entries are all different, and so it is itself diagonal. That is, $U_{jj} = YDX^*$, where D is diagonal. Hence, by the first identity in (2), we have $\mathcal{A}_{jj}\mathcal{A}_{jj}^* = B_j U_{jj} U_{jj}^* B_j^* = XSY^*YDX^*XD^*Y^*Y\Sigma X^* = X\Sigma D D^* \Sigma X^* = XD^*\Sigma^2 DX^* = XD^*Y^*Y\Sigma X^* XSY^*YDX^* = U_{jj}^* B_j^* B_j U_{jj} = \mathcal{A}_{jj}^* \mathcal{A}_{jj}$. Thus, \mathcal{A}_{jj} is normal.

(b) Since B_j is real, we may choose the matrices X, Y in the SV decomposition $B_j = XSY^*$ to be real and orthogonal. Hence, from $\mathcal{A}_{jj} = B_j U_{jj}$ in (2) and the formula $U_{jj} = YDX^*$ with D diagonal from part (a), we have $\mathcal{A}_{jj}^T = U_{jj}^T B_j^T = XDY^T Y \Sigma X^T = X D \Sigma X^T = X \Sigma D X^T = X \Sigma Y^T Y D X^T = B_j U_{jj} = \mathcal{A}_{jj}$. Thus, \mathcal{A}_{jj} is also symmetric.

(c) For $1 \leq i \leq k$, write $B_i = X_i \Sigma_i Y_i^*$, where X_i, Y_i are unitary and $\Sigma_i = \sigma_i I$. Hence $B_i = \sigma_i X_i Y_i^*$. Also, $B_i B_i^* X_i = X_i \Sigma_i^2 = \sigma_i^2 X_i$, and so $B_i B_i^* = \sigma_i^2 I$. Similarly, $B_i^* B_i = \sigma_i^2 I$. Hence, by the second identity in (2), we have $U_{ij} \sigma_j^2 I = \sigma_i^2 I U_{ij}$, $1 \leq i, j \leq k$. When $i \neq j$, we are given $\sigma_i \neq \sigma_j$ and so $U_{ij} = 0$. Hence, since U is unitary, so are U_{11}, \dots, U_{kk} . Therefore, by the first identity in (1), we have $\mathcal{A} = BU = B_1 U_{11} \oplus \cdots \oplus B_k U_{kk} = \sigma_1 X_1 Y_1^* U_{11} \oplus \cdots \oplus \sigma_k X_k Y_k^* U_{kk} = \sigma_1 Z_1 \oplus \cdots \oplus \sigma_k Z_k$, where each $Z_i = X_i Y_i^* U_{ii}$ is unitary.

(d) Since B is real, we may write $B = XSY^T$, where X, Y are real and orthogonal and where $\Sigma = \sigma_1 I_1 \oplus \cdots \oplus \sigma_d I_d$ is diagonal with $\sigma_1 > \cdots > \sigma_d \geq 0$. Partition $Z = Y^T U X = [Z_{ij}]_{i,j=1}^d$ conformally with Σ ; here U is the unitary matrix from (1) so Z is unitary. The second identity in (1) gives $Z\Sigma^2 = Y^T U X S Y^T Y \Sigma X^T X = Y^T U B B^T X = Y^T B^T B U X = Y^T Y \Sigma X^T X S Y^T U X = \Sigma^2 Z$. Hence, Z and Σ^2 commute and so $Z_{ij} = 0$ whenever $i \neq j$. It follows that Z_{11}, \dots, Z_{dd} are unitary. Therefore, by the first identity in (1), $\mathcal{A} = BU = XSY^T U X X^T = X \Sigma Z X^T = X(\sigma_1 Z_{11} \oplus \cdots \oplus \sigma_d Z_{dd}) X^T$.

(e) Since $A^*A = WB^*V^*VBW^* = WB^*BW^*$, A^*A and B^*B are unitarily similar and so A and B have the same singular values. As in (d), we write $B = XSY^T$ and $Z = Y^T U X$. Since Z and Σ^2 commute and the diagonal entries of the diagonal matrix Σ^2 are all different, Z is also diagonal. Therefore $X^T \mathcal{A}^T X = X^T U^T B^T X = X^T U^T Y \Sigma X^T X = Z^T \Sigma = Z \Sigma = \Sigma Z = X^T X S Y^T U X = X^T B U X = X^T \mathcal{A} X$, and so \mathcal{A} is symmetric.

(f) The proof is given only under the more restrictive assumption that $|\lambda_i| \neq |\lambda_j|$ whenever $i \neq j$. Since A is normal, its singular values are then the distinct numbers $|\lambda_1|, \dots, |\lambda_n|$. By part (e), \mathcal{A} is symmetric. \square

Solution 53-3.2 by the proposer Roger A. HORN

Let $\mathcal{A} = PU$ be a polar decomposition, in which P is Hermitian positive semidefinite and U is unitary. Partition $U = [U_{ij}]_{i,j=1}^k$ conformally to B and observe that $P = P_1 \oplus \cdots \oplus P_k$, in which each $P_i = (B_i B_i^*)^{1/2}$, which is real if B_i is real. Normality of \mathcal{A} implies that P commutes with U , so each P_i commutes with U_{ii} ; see [1, 7.3.P35]. Let each $P_i = X_i \Sigma_i X_i^*$, in which X_i is unitary and Σ_i is nonnegative diagonal. Then $P_i U_{ii} = U_{ii} P_i$ implies

$$\Sigma_i (X_i^* U_{ii} X_i) = (X_i^* U_{ii} X_i) \Sigma_i. \quad (*)$$

(a) If the diagonal entries of Σ_j are distinct, then by (*), $X_j^* U_{jj} X_j = D_j$ is diagonal, so $U_{jj} = X_j D_j X_j^*$ and $\mathcal{A}_{jj} = X_j (\Sigma_j D_j) X_j^*$ is normal; see [1, 2.4.4.3].

(b) If B_j is real, then P_j is real and X_j in (a) can be chosen to be real orthogonal, so $U_{jj} = X_j D_j X_j^T$ and $\mathcal{A}_{jj} = X_j (\Sigma_j D_j) X_j^T$ is normal and symmetric.

(c) If each $P_i = \sigma_i I_{n_i}$ and if $\sigma_i \neq \sigma_j$ whenever $i \neq j$, then $PU = UP$ implies $(\sigma_i - \sigma_j) U_{ij} = 0$ and so $U_{ij} = 0$ whenever $i \neq j$. Consequently, each U_{ii} is unitary and each $\mathcal{A}_{ii} = P_i U_{ii} = \sigma_i U_{ii}$.

(d) If B is real then P is also real. Let $\sigma_1 > \cdots > \sigma_d \geq 0$ be the distinct singular values of A . There is a real orthogonal Q such that $P = Q \Sigma Q^T = Q(\sigma_1 I_{n_1} \oplus \cdots \oplus \sigma_d I_{n_d}) Q^T$. Since $PU = UP$ we have $\Sigma(Q^T U Q) = (Q^T U Q) \Sigma$, so $Q^T U Q = Z_1 \oplus \cdots \oplus Z_d$ is block diagonal and unitary. Consequently, $\mathcal{A} = PU = Q(\sigma_1 Z_1 \oplus \cdots \oplus \sigma_d Z_d) Q^T$.

(e) Take $d = n$ in (d), so $\mathcal{A} = Q(\sigma_1 z_1 \oplus \cdots \oplus \sigma_n z_n) Q^T$, in which each $|z_i| = 1$.

(f) Since A is normal, its singular values are $|\lambda_1|, \dots, |\lambda_n|$. The hypothesis ensures that if, in (d), $\sigma_j > 0$ then the corresponding unitary Z_j has only a single eigenvalue, so $Z_j = e^{i\theta_j} I_{n_j}$. Thus, $\sigma_1 Z_1 \oplus \dots \oplus \sigma_d Z_d$ is symmetric. \square

Reference

[1] R. A. Horn and C. R. Johnson, *Matrix Analysis*, second edition, Cambridge University Press, Cambridge, 2012.

Problem 53-4: A Positive Semidefinite Matrix

Proposed by Minghua LIN, *University of Victoria, BC, Canada*, mlin87@gmail.com

Let $x, y \in \mathbb{C}^n$. Show that the $(2n)$ -by- $(2n)$ matrix $\begin{pmatrix} 2(x^*x)I_n - xx^* & 2(y^*x)I_n - xy^* \\ 2(x^*y)I_n - yx^* & 2(y^*y)I_n - yy^* \end{pmatrix}$ is positive semidefinite.

Solution 53-4.1 by the proposer Minghua LIN

For a Hermitian matrix X , we write $X \geq 0$ to mean X is positive semidefinite. Note that $\begin{pmatrix} A & B \\ B^* & C \end{pmatrix} \geq 0$ if both $C \geq 0$ and the Schur complement $A - BC^{-1}B^* \geq 0$ (see [1, Theorem 7.14]). Hence, we may assume x and y are unit vectors and it suffices to show the Schur complement $2I - xx^* - (2(y^*x)I - xy^*)(2I - yy^*)^{-1}(2(x^*y)I - yx^*) \geq 0$, i.e.,

$$2I - xx^* - \frac{1}{2} \left(2(y^*x)I - xy^* \right) (I + yy^*) \left(2(x^*y)I - yx^* \right) \geq 0.$$

After expanding, this is equivalent to

$$I - xx^* - |y^*x|^2 I + (yy^*xx^* + xx^*yy^*) - |y^*x|^2 yy^* \geq 0.$$

We may write $x = \alpha y + \beta w$, where w is a unit vector such that $y^*w = 0$, and $|\alpha|^2 + |\beta|^2 = 1$. Now $xx^* = (\alpha y + \beta w)(\alpha y + \beta w)^* = |\alpha|^2 yy^* + \alpha \bar{\beta} y w^* + \beta \bar{\alpha} w y^* + |\beta|^2 ww^*$. A little calculation gives

$$I - xx^* - |y^*x|^2 I + (yy^*xx^* + xx^*yy^*) - |y^*x|^2 yy^* = (1 - |\alpha|^2)I - |\beta|^2 ww^* = |\beta|^2(I - ww^*) \geq 0. \quad \square$$

Reference

[1] F. Zhang, *Matrix theory: Basic results and techniques*, second edition, Universitext, Springer-Verlag, New York, 2011.

Solution 53-4.2 by Denis SERRE, *École Normale Supérieure de Lyon, France*, denis.serre@ens-lyon.fr

A more general result actually holds true: Let $x_1, \dots, x_d \in \mathbb{C}^n$ be given. Then the following Hermitian matrix $A \in \mathbf{H}_{dn}$ is positive semidefinite,

$$A = \left((mx_j^* x_i I_n - x_i x_j^*) \right)_{1 \leq i, j \leq n} \quad \text{with } m = \min(n, d).$$

It suffices to prove the inequality

$$\xi^* A \xi = m \sum_{i,j} (x_j^* x_i) (\xi_i^* \xi_j) - \sum_{i,j} (\xi_i^* x_i) (x_j^* \xi_j) \geq 0, \quad \forall \xi_1, \dots, \xi_d \in \mathbb{C}^n.$$

Defining a matrix $F = \sum_i \xi_i x_i^*$, this is equivalent to

$$m \operatorname{Tr}(FF^*) \geq |\operatorname{Tr}(F)|^2,$$

which, due to $\operatorname{rank} F \leq m$, follows once the inequality $|\operatorname{Tr}(M)|^2 \leq \operatorname{rank} M \cdot \operatorname{Tr}(MM^*)$ is proved for $M \in \mathbf{M}_n(\mathbb{C})$. But this follows from the Cauchy–Schwarz inequality for the Frobenius norm since with orthogonal projection P onto the range of M one has $|\operatorname{Tr}(M)|^2 = |\operatorname{Tr}(PM)|^2 \leq \operatorname{Tr}(P^*P) \cdot \operatorname{Tr}(M^*M) = \operatorname{Tr}(P) \operatorname{Tr}(M^*M) = \operatorname{rank}(P) \operatorname{Tr}(M^*M) = \operatorname{rank}(M) \operatorname{Tr}(M^*M)$. \square

Solution 53-4.3 by Henry WOLKOWICZ, *University of Waterloo, ON, Canada*, hwolkowicz@uwaterloo.ca

We may assume that $x, y \neq 0$. The result can easily be verified for $n = 1$ by checking minors. Therefore, we assume that $n \geq 2$. We solve the problem by finding $2n$ nonnegative eigenvalues.

First we note that 0 is an eigenvalue of

$$A := \begin{pmatrix} 2(x^*x)I_n - xx^* & 2(y^*x)I_n - xy^* \\ 2(x^*y)I_n - yx^* & 2(y^*y)I_n - yy^* \end{pmatrix},$$

i.e., we can solve $A \begin{pmatrix} v \\ w \end{pmatrix} = 0$ with nonzero $\begin{pmatrix} v \\ w \end{pmatrix}$ by solving the equivalent equation

$$\begin{pmatrix} 2(x^*x)I_n & 2(y^*x)I_n \\ 2(x^*y)I_n & 2(y^*y)I_n \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} = (x^*v + y^*w) \begin{pmatrix} x \\ y \end{pmatrix}$$

whose solution is $v = (y^*y)x - (y^*x)y$ and $w = -(x^*y)x + (x^*x)y$.

Next we note that we can solve for two eigenvalues using Gaussian elimination, i.e., when subtracting αI_n from the diagonal of A and multiplying the equivalent equation by $P_\alpha = \begin{pmatrix} I_n & 0 \\ \frac{-2x^*y}{2x^*x-\alpha} I_n & I_n \end{pmatrix}$ we obtain

$$\begin{pmatrix} (2x^*x - \alpha)I_n & 2(y^*x)I_n \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} = (x^*v + y^*w) \begin{pmatrix} x \\ y - \frac{2x^*y}{2x^*x-\alpha}x \end{pmatrix}$$

provided that $2y^*y - \alpha - \frac{(2y^*x)(2x^*y)}{2x^*x-\alpha} = 0$ or equivalently, that the quadratic equation $\alpha^2 - 2(x^*x + y^*y)\alpha + 4(y^*yx^*x - y^*xx^*y) = 0$. The two real (nonnegative radical) roots

$$\lambda_i = (x^*x + y^*y) \pm \sqrt{(x^*x + y^*y)^2 - 4(y^*yx^*x - y^*xx^*y)}, \quad i = 1, 2$$

provide two eigenvalues with corresponding eigenvectors $\begin{pmatrix} v \\ w \end{pmatrix}$ orthogonal to $\begin{pmatrix} x \\ y \end{pmatrix}$ and satisfying $(2x^*x - \lambda_i)v + (2y^*x)w = 0$. Hence, for each of λ_1, λ_2 we may choose $n - 1$ linearly independent eigenvectors, i.e., the multiplicity of each is at least $n - 1$. We now add these eigenvalues (plus the zero eigenvalue) and subtract from the trace to show that the last eigenvalue is positive, i.e. we see that the last eigenvalue is $\lambda = (2n - 1)(x^*x + y^*y) - 2(n - 1)(x^*x + y^*y) > 0$. \square

Also solved by Eugene A. HERMAN.

Problem 53-5: Symmetric Matrices and Triangles in the Plane

Proposed by Denis SERRE, *École Normale Supérieure de Lyon, France*, denis.serre@ens-lyon.fr

Assume $M \in \mathbf{Sym}_3(\mathbb{R})$ satisfies $m_{jj} = 1$ and $|m_{ij}| \leq 1$; $1 \leq i < j \leq 3$. Thus, if $i < j$ then $m_{ij} = \cos \theta_{ij}$.

- (i) Prove that $\det M = 0$ if, and only if, there exist signs such that $\pm \theta_{12} \pm \theta_{13} \pm \theta_{23} \in 2\pi\mathbb{Z}$.
- (ii) Let $x = (x_1, x_2, x_3)^t \in \mathbb{R}^3 \setminus \{0\}$. Prove that the matrix M with the above properties and with $Mx = 0$ exists if and only if $|x_i| \leq |x_k| + |x_j|$ for every pairwise distinct i, j, k .

Hint: For sufficiency reduce the problem to the case where $x_i \geq 0$ and form a triangle with edge lengths x_1, x_2, x_3 .

Solution 53-5.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

(i) Using the identities $2 \sin u \sin v = \cos(u - v) - \cos(u + v)$, $2 \cos u \cos v = \cos(u - v) + \cos(u + v)$, and $\cos 2u = 2 \cos^2 u - 1$, we have

$$\begin{aligned} & 4 \sin \left(\frac{\theta_{12} + \theta_{13} + \theta_{23}}{2} \right) \sin \left(\frac{\theta_{12} + \theta_{13} - \theta_{23}}{2} \right) \sin \left(\frac{\theta_{12} + \theta_{23} - \theta_{13}}{2} \right) \sin \left(\frac{\theta_{13} + \theta_{23} - \theta_{12}}{2} \right) \\ &= (\cos \theta_{23} - \cos(\theta_{12} + \theta_{13})) (\cos(\theta_{12} - \theta_{13}) - \cos \theta_{23}) \\ &= -\cos^2 \theta_{23} + \cos \theta_{23} (\cos(\theta_{12} - \theta_{13}) + \cos(\theta_{12} + \theta_{13})) - \cos(\theta_{12} + \theta_{13}) \cos(\theta_{12} - \theta_{13}) \\ &= -\cos^2 \theta_{23} + 2 \cos \theta_{23} \cos \theta_{12} \cos \theta_{13} - \frac{1}{2} (\cos 2\theta_{12} + \cos 2\theta_{13}) \\ &= 1 - \cos^2 \theta_{12} - \cos^2 \theta_{13} - \cos^2 \theta_{23} + 2 \cos \theta_{12} \cos \theta_{13} \cos \theta_{23} = \det M. \end{aligned}$$

Since $\sin u/2 = 0$ if and only if $u \in 2\pi\mathbb{Z}$, part (i) has been proven.

(ii) The equation $Mx = 0$ may be written as

$$\begin{aligned} x_1 + x_2 \cos \theta_{12} + x_3 \cos \theta_{13} &= 0 \\ x_1 \cos \theta_{12} + x_2 + x_3 \cos \theta_{23} &= 0 \\ x_1 \cos \theta_{13} + x_2 \cos \theta_{23} + x_3 &= 0. \end{aligned} \tag{*}$$

Suppose M does exist and that $Mx = 0$. Then, from the first of equations in (*), we have $|x_1| = |x_2 \cos \theta_{12} + x_3 \cos \theta_{13}| \leq |x_2| + |x_3|$. The other two inequalities of this type follow from the remaining two equations in (*). Now suppose the

three inequalities of the form $|x_i| \leq |x_k| + |x_j|$ hold. Since $x \neq 0$, it follows that at most one of the components of x is zero. In that case, by symmetry we may assume $x_1 = 0$ and so $x_2 = \pm x_3$. Let $s = x_3/x_2 = \pm 1$ and choose angles $\theta_{12}, \theta_{13}, \theta_{23}$ so that $\cos \theta_{23} = -s$ and $\cos \theta_{12} = -s \cos \theta_{13}$. Then equations (*) hold. For the case in which all components of x are nonzero, note that $Mx = 0$ if and only if $M(-x) = 0$, and so we may assume that either all components of x are positive or exactly two are positive. For the former possibility, consider the triangle with sides x_1, x_2, x_3 (the three given inequalities ensure that such a triangle exists). Let α, β, γ denote the interior angles opposite sides x_3, x_2, x_1 , respectively. For the matrix M , choose $\theta_{12} = \alpha + \pi, \theta_{13} = \beta + \pi, \theta_{23} = \gamma + \pi$. Then $\frac{x_1}{\sin \gamma} = \frac{x_2}{\sin \beta} = \frac{x_3}{\sin \alpha}$, and so

$$\begin{aligned} x_1 + x_2 \cos \theta_{12} + x_3 \cos \theta_{13} &= x_1 - x_1 \cos \alpha \frac{\sin \beta}{\sin \gamma} - x_1 \cos \beta \frac{\sin \alpha}{\sin \gamma} = (\sin \gamma - \sin(\alpha + \beta)) \frac{x_1}{\sin \gamma} \\ &= (\sin(\pi - \alpha - \beta) - \sin(\alpha + \beta)) \frac{x_1}{\sin \gamma} = 0. \end{aligned}$$

The other two equations in (*) are proved similarly. Finally suppose, as we may, that $x_1 < 0$ and $x_2, x_3 > 0$. Then there exists M as in (i) such that $M(-x_1, x_2, x_3)^T = 0$. Obtain N from M by changing the angle θ_{12} to $\theta_{12} - \pi$ and angle θ_{13} to $\theta_{13} - \pi$; then $Nx = 0$. \square

Solution 53-5.2 by the proposer Denis SERRE

Let a, b, c be the off-diagonal entries of M . We have $\det M = 1 + 2abc - a^2 - b^2 - c^2 = (1 - a^2)(1 - b^2) - (c - ab)^2$. In terms of the angles, this gives

$$\begin{aligned} \det M &= \sin^2 \theta_1 \sin^2 \theta_2 - (\cos \theta_3 - \cos \theta_1 \cos \theta_2)^2 \\ &= (\sin \theta_1 \sin \theta_2 + \cos \theta_3 - \cos \theta_1 \cos \theta_2)(\sin \theta_1 \sin \theta_2 - \cos \theta_3 + \cos \theta_1 \cos \theta_2) \\ &= (\cos \theta_3 - \cos(\theta_1 + \theta_2))(\cos(\theta_1 - \theta_2) - \cos \theta_3). \end{aligned}$$

This vanishes if and only if either $\theta_1 + \theta_2 = \pm \theta_3$ or $\theta_1 - \theta_2 = \pm \theta_3$, modulo 2π .

Suppose now that for such a matrix M there exists $x \neq 0$ in \mathbb{R}^3 such that $Mx = 0$. Remark that if $D = \text{diag}(\pm 1, \pm 1, \pm 1)$, we have $DMDDx = 0$, and $M' = DMD$ is still as above. Choosing D so that Dx has nonnegative entries, we may assume that $x \geq 0$. Then $x_i + m_{ij}x_j + m_{ik}x_k = 0$ and the triangle inequality gives $x_i \leq x_j + x_k$.

Conversely, let $x \geq 0$ be given such that $x \neq 0$ and $x_i \leq x_j + x_k$ whenever $\{i, j, k\} = \{1, 2, 3\}$. Then there exists a triangle T whose sides have length x_1, x_2, x_3 . Let $\alpha_j \in [0, \pi]$ be the angle opposite to the side of length x_j . One has easily $x_j \cos \alpha_k + x_k \cos \alpha_j = x_i$, with i, j, k as above. Define

$$M = \begin{pmatrix} 1 & -\cos \alpha_3 & -\cos \alpha_2 \\ -\cos \alpha_3 & 1 & -\cos \alpha_1 \\ -\cos \alpha_2 & -\cos \alpha_1 & 1 \end{pmatrix}.$$

Then $Mx = 0$, so M is singular and hence $\det M = 0$. \square

Problem 53-6: Matrix Equations Under the Ordinary Product and Kronecker Product of Matrices

Proposed by Yongge TIAN, *Central University of Finance and Economics, Beijing, China*, yongge.tian@gmail.com

Let $A_i \in \mathbb{C}^{m_i \times n_i}$, $B_i \in \mathbb{C}^{p_i \times q_i}$, and $C_i \in \mathbb{C}^{m_i \times q_i}$ be nonzero matrices, $i = 1, 2$. Show that there exists a matrix X such that $(A_1 \otimes A_2)X(B_1 \otimes B_2) = C_1 \otimes C_2$ if and only if there exist two matrices X_1 and X_2 such that $A_1 X_1 B_1 = C_1$ and $A_2 X_2 B_2 = C_2$; here \otimes denotes the Kronecker product of matrices.

Solution 53-6.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

If $A_1 X_1 B_1 = C_1$ and $A_2 X_2 B_2 = C_2$, then $(A_1 \otimes A_2)(X_1 \otimes X_2)(B_1 \otimes B_2) = (A_1 X_1 B_1) \otimes (A_2 X_2 B_2) = C_1 \otimes C_2$; that is, we may choose $X = X_1 \otimes X_2$. For the converse, write $A_1 = [a_{ij}]$, $B_1 = [b_{ij}]$, $C_1 = [c_{ij}]$. Thus $A_1 \otimes A_2$ has an $m_1 \times n_1$ block structure, in which each block, $a_{ij} A_2$, has size $m_2 \times n_2$; similarly for $B_1 \otimes B_2$ and $C_1 \otimes C_2$. Partition X into a compatible $n_1 \times p_1$ block structure in which each block, X_{ij} , has size $n_2 \times p_2$. Thus $(A_1 \otimes A_2)X(B_1 \otimes B_2) = C_1 \otimes C_2$ yields the following equality among blocks:

$$c_{ij} C_2 = \sum_{k,l} a_{ik} A_2 X_{kl} b_{lj} B_2 = A_2 \left(\sum_{k,l} a_{ik} X_{kl} b_{lj} \right) B_2, \quad 1 \leq i \leq m_1, 1 \leq j \leq q_1.$$

Since C_1 is not zero, some c_{ij} is nonzero. For such an i and j , we have $C_2 = A_2 X_2 B_2$, where

$$X_2 = \frac{1}{c_{ij}} \sum_{k,l} a_{ik} X_{kl} b_{lj}.$$

Existence of X_1 follows by applying this result to the equation $(A_1 \otimes A_2)X(B_1 \otimes B_2) = C_1 \otimes C_2$ with all the Kronecker products reversed. We accomplish this by using the perfect shuffle permutation matrices $S_{mn} = \sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} E_{ij}^T \otimes E_{ij}$ (where $E_{ij} \in \mathbb{C}^{m \times n}$ are the standard unit matrices):

$$\begin{aligned} C_2 \otimes C_1 &= S_{m_1 m_2} (C_1 \otimes C_2) S_{q_1 q_2}^T = S_{m_1 m_2} (A_1 \otimes A_2) X (B_1 \otimes B_2) S_{q_1 q_2}^T \\ &= S_{m_1 m_2} (A_1 \otimes A_2) S_{n_1 n_2}^T S_{n_1 n_2} X S_{p_1 p_2}^T S_{p_1 p_2} (B_1 \otimes B_2) S_{q_1 q_2}^T = (A_2 \otimes A_1) Y (B_2 \otimes B_1) \end{aligned}$$

where $Y = S_{n_1 n_2} X S_{p_1 p_2}^T$. □

Solution 53-6.2 by the proposer Yongge TIAN

We need the following three lemmas:

Lemma 1. (Penrose 1955) There exists a matrix X such that $AXB = C$ if and only if $\text{Im}(C) \subseteq \text{Im}(A)$ and $\text{Im}(C^*) \subseteq \text{Im}(B^*)$ or equivalently, $AA^\dagger CB^\dagger B = C$, where $(\cdot)^\dagger$ denotes the Moore-Penrose inverse.

Lemma 2. Let A and C be nonzero $m \times n$ matrices, and let B and D be nonzero $p \times q$ matrices, respectively, and assume that they satisfy $A \otimes B = C \otimes D$. Then $A = \lambda C$ and $B = \lambda^{-1} D$ holds for some nonzero scalar λ .

Proof of Lemma 2. $A \otimes B = C \otimes D$ implies that $B \otimes A = D \otimes C$ by the well-known permutation equality for Kronecker products, $P_{mp}(A \otimes B)P_{nq}^T = B \otimes A$. Since both $A = (a_{ij}) \neq 0$ and $B = (b_{st}) \neq 0$, both $A \otimes B = C \otimes D$ and $B \otimes A = D \otimes C$ imply that $a_{ij} B = c_{ij} D$ for some nonzero a_{ij} , and $b_{st} A = d_{st} C$ for some nonzero b_{st} , that is, $A = b_{st}^{-1} d_{st} C$ and $B = a_{ij}^{-1} c_{ij} D$. Substituting them into $A \otimes B = C \otimes D$ and comparing both sides leads to $(b_{st}^{-1} d_{st})(a_{ij}^{-1} c_{ij}) = 1$. Letting $\lambda = b_{st}^{-1} d_{st}$ yields the conclusion in the lemma. □

Lemma 3. Let A and C be nonzero $m \times n$ matrices, and let B and D be nonzero $p \times q$ matrices, respectively. Then $\text{Im}(A \otimes B) \subseteq \text{Im}(C \otimes D)$ holds if and only if both $\text{Im}(A) \subseteq \text{Im}(C)$ and $\text{Im}(B) \subseteq \text{Im}(D)$ hold.

Proof of Lemma 3. Let $\text{Im}(A \otimes B) \subseteq \text{Im}(C \otimes D)$. It follows from Lemma 1 and a known formula $(C \otimes D)^\dagger = C^\dagger \otimes D^\dagger$ that

$$A \otimes B = (C \otimes D)(C \otimes D)^\dagger (A \otimes B) = (C \otimes D)(C^\dagger \otimes D^\dagger)(A \otimes B) = CC^\dagger A \otimes DD^\dagger B,$$

which by Lemma 2 implies $CC^\dagger A = \lambda A$ and $DD^\dagger B = \lambda^{-1} B$ for some λ . These two equalities obviously imply that both $\text{Im}(A) \subseteq \text{Im}(C)$ and $\text{Im}(B) \subseteq \text{Im}(D)$ hold. Conversely, if $\text{Im}(A) \subseteq \text{Im}(C)$ and $\text{Im}(B) \subseteq \text{Im}(D)$, then there exist two matrices U and V such that $A = CU$ and $B = DV$ by Lemma 1. In this case,

$$A \otimes B = CU \otimes DV = (C \otimes D)(U \otimes V),$$

so that $\text{Im}(A \otimes B) \subseteq \text{Im}(C \otimes D)$. □

Proof of the main result. From Lemma 1, $(A_1 \otimes A_2)X(B_1 \otimes B_2) = C_1 \otimes C_2$ is solvable for X if and only if $\text{Im}(C_1 \otimes C_2) \subseteq \text{Im}(A_1 \otimes A_2)$, and $\text{Im}(C_1^* \otimes C_2^*) = \text{Im}(C_1 \otimes C_2)^* \subseteq \text{Im}(B_1 \otimes B_2)^* = \text{Im}(B_1^* \otimes B_2^*)$. Under the assumptions that A_i, B_i and C_i are all nonzero matrices, it can be seen from Lemma 3 that both range inequalities are equivalent to the four range inequalities

$$\begin{aligned} \text{Im}(C_1) \subseteq \text{Im}(A_1), \quad \text{Im}(C_2) \subseteq \text{Im}(A_2), \\ \text{Im}(C_1^*) \subseteq \text{Im}(B_1^*), \quad \text{Im}(C_2^*) \subseteq \text{Im}(B_2^*), \end{aligned}$$

which, in turn, are equivalent to the fact that both $A_1 X_1 B_1 = C_1$ and $A_2 X_2 B_2 = C_2$ are solvable, respectively. □

Reference

[1] R. Penrose, A generalized inverse for matrices, *Proc. Cambridge Philos. Soc.* 51 (1955) 406–413.

IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce 6 new problems in this issue and invite readers to submit solutions for publication in *IMAGE*. **Solutions:** We present solutions to all problems in the previous issue [*IMAGE* 53 (Fall 2014), p. 52] except problem 53.1 for which we still seek solutions. **Submissions:** Please submit proposed problems and solutions in macro-free L^AT_EX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Bojan Kuzma (bojan.kuzma@famnit.upr.si). The working team of the Problem Corner consists of Steve Butler, Gregor Dolinar, Shaun Fallat, Alexander Guterman, Rajesh Pereira, and Nung-Sing Sze.

NEW PROBLEMS:

Problem 54-1: EP Product of EP Matrices

Proposed by JOHANNES DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

A complex matrix X is EP or range-Hermitian if $\text{Im}(X) = \text{Im}(X^*)$. Let A , B and AB be EP matrices. Show that BA is an EP matrix if and only if $\text{Im}(AB) = \text{Im}(BA)$.

Problem 54-2: An Identity Involving the Inverse of LCL^T

Proposed by RAVINDRA BAPAT, *Indian Statistical Institute, New Delhi, India*, rbb@isid.ac.in and ALOKE DEY, *Indian Statistical Institute, New Delhi, India*, aloke.dey@gmail.com

Let C be a $v \times v$ real symmetric matrix of rank $v - 1$ and let L be a $(v - 1) \times v$ matrix of rank $v - 1$ such that $\mathcal{R}(L) \subseteq \mathcal{R}(C)$ where \mathcal{R} denotes the row space. Show that (i) LCL^T is nonsingular, and (ii) $L = LL^T(LCL^T)^{-1}LC$.

Problem 54-3: When Is the Adjugate a One-to-One Operator?

Proposed by DENNIS S. BERNSTEIN, *University of Michigan, Ann Arbor, MI, USA*, dsbaero@umich.edu and KHALED ALJANAIDEH, *University of Michigan, Ann Arbor, MI, USA*, khaledfj@umich.edu

Determine the values of n for which the adjugate operator is one-to-one on the set of $n \times n$ nonsingular real matrices.

Problem 54-4: When k -Cycles Imply Fixed Points

Proposed by EUGENE A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

What conditions on the integers k, n and the field \mathbb{F} ensure that each affine transformation T on an n -dimensional \mathbb{F} -vector space V has a fixed point in V if it induces a k -cycle (i.e., if there exists $k \geq 1$ and $v \in V$ such that $T^{(k)}(v) = v$)?

Problem 54-5: Inertia Formula for a Partitioned Hermitian Matrix

Proposed by YONGGE TIAN, *Central University of Finance and Economics, Beijing, China*, yongge.tian@gmail.com

Let A and C be two positive semidefinite Hermitian matrices of orders m and n , respectively, and let $B \in \mathbb{C}^{m \times n}$. Show that the inertia of $T = \begin{pmatrix} A & B \\ B^* & -C \end{pmatrix}$ satisfies

$$i_+(T) = \text{rank}(A|B), \quad i_-(T) = \text{rank}(B^*|C), \quad i_0(T) = m + n - \text{rank}(A|B) - \text{rank}(B^*|C).$$

Problem 54-6: An Inequality for a Contraction Matrix

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Let $n \geq 2$ and let $C = (c_{ij})$ be an $n \times n$ contraction matrix (i.e., the largest singular value of C is no more than 1). Show that

$$\sum_{i,j} |c_{ij}|^2 + 1 \leq |\det C|^2 + n.$$

When does equality occur?

Solutions to Problems 53-2 through 53-6 are on pages 37-43.