COLLECTIVE ACTION AS AN AGREEABLE n-PRISONERS' DILEMMA

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The problem of collective action to produce a group collective good is analyzed as the game of Individual vs. Collective and then as an n-person game to show that, under the constraints of Mancur Olson's analysis, it is an n-prisoners' dilemma in the cases of latent and intermediate groups. The usual analysis according to which noncooperation is considered the rational strategy for classical 2-prisoners' dilemma is logically similar to Olson's analysis, which suggests that rational members of a latent group should not contribute toward the purchase of the group collective good. However, in the game analysis it is clear that the latent and intermediate groups are not logically different, but rather are distinguishable only statistically. Some prisoners' dilemma experimental results are used to suggest how the difference might arise and how the vast prisoners' dilemma literature can be related to the problem of collective action.

The game of collective action is then analyzed not from the view of strategies but of outcomes. There is presented a theorem which states that the outcome in which all player-members of a group pay and all benefit is a Condorcet choice from the set of realizable outcomes for the game. Hence the cooperative outcome in such a game would prevail in election against all other outcomes.

In *The Logic of Collective Action*, Mancur Olson (1968) has proposed a mathematical explanation for the notable failure of the memberships of large interest groups to work together to provide themselves with their mutually desired collective goods. He concludes that the success of a group in providing itself with a collective good depends on the logical structure of the group.

> In a small group in which a member gets such a large fraction of the total benefit that he would be better off if he paid the entire cost himself, rather than go without the good, there is some presumption that the collective good will be provided. In a group in which no member got such a large benefit from the collective good that he had an interest in providing it even if he had to pay all the cost, but in which the individual was still so important in terms of the whole group that his contribution or lack of contribution to the group objective had a noticeable effect on the costs or benefits of others in the group, the result is indeterminate. By contrast, in a large group in which no single individual's contribution makes a perceptible difference to the group as a whole . . . it is certain that a collective good will *not* be provided unless there is coercion or some outside inducements. . . . (p. 44)

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These three sorts of group can be distinguished as the privileged group (i.e., the group in which at least one member could justify his full payment for the provision of the good on the basis of his sufficiently great return), the intermediate group, and the latent group.

Common sense and experience seem to confirm Olson's conclusions, although they seem to suggest a logic counter to our expectations. They suggest that "rational, self-interested individuals will not act to achieve their common or group interests" (Olson, 1968, p. 2). To clarify the logic of collective action, therefore, Olson gives a mathematical demonstration, which can be easily summarized.

The advantage (A_i) which accrues to an individual member (i) of a group as the result of his contribution to the purchase of the group collective good is given by:

$$A_i = V_i - C,$$

where V_i is the value to i of his share of the total collective good provided to the group at cost C to i. Clearly, if A_i is to be positive, then V_i must be greater than C. But this implies that i will contribute toward the

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purchase of the group collective good on his own rational incentive only if his share of that part of the good purchased at his cost is worth more to him than it cost him (Olson, 1968, pp. 22–25). Hence, the collective good will be provided in a privileged group, where this condition is met, but not in a latent group, where it is not met.

COLLECTIVE ACTION AND PRISONERS' DILEMMA

As with the prisoners' dilemma, we have for the latent group a result that tells us that individual effort to achieve individual interests will preclude their achievement, because if the collective good is not provided, the individual member fails to receive a benefit that would have exceeded his cost in helping purchase that good for the whole group. It would be useful to perform a game theory analysis of collective action to demonstrate that the logic underlying it is the same as that of the prisoners' dilemma. First, however, since Olson's analysis was accomplished from the perspective of an individual in the group, let us consider a particular instance of collective action in the game of Individual vs. Collective.

Individual vs. collective

Let us construct a game matrix in which the row entries will be the payoffs for Individual, and the column entries will be the per capita payoffs for Collective, where Collective will be the group less Individual. The payoffs will be calculated by the prescription for rational behavior: that is, the payoffs will be benefits less costs. The group will comprise ten members whose common interest is the provision of a collective good of value twice its cost. There are two possible results of having one member of the group decline to pay his share: either the total benefit will be proportionately reduced, or the costs to the members of the group will be proportionately increased. Let us assume the former, but either choice would yield the same analysis. For the sake of simplicity, assume also that there are no initial costs in providing the collective good and no differential costs as payments and resultant benefits rise, that is, assume exactly two units of the collective good will be provided for each unit paid by any member of the group.[2]

MATRIX 1

INDIVIDUAL VS. COLLECTIVE

		Collective: Pay	Collective: Not Pay
Individual	Pay	1, 1	−0, 8, 0.2
	Not Pay	1.8, 0.8	0, 0

If all members of the group pay 1 unit (for a total cost of 10 units), the benefit to each member will be 2 units (for a collective good of 20). The individual payoffs will be benefit less cost, or 1 unit. In the matrix, the first row gives the payoffs to Individual if he contributes his share; the first column gives the per capita payoffs to the remaining members of the group, i.e., to Collective, if they pay. The second row gives the payoffs to Individual if he does not pay, and the second column gives those for Collective if it does not pay. The various payoffs are readily calculated, e.g., if Individual does not pay but Collective does, the total cost will be 9 units, the total benefit will be 18 units, and the per capita benefit will be 1.8 units (for Individual cannot be excluded from the provision of the collective good); consequently, Individual's payoff for this condition will be his benefit less his cost for a pleasant 1.8 units. From the payoffs for the game in Matrix 1, one can see that it is evidently in Individual's advantage to choose the strategy of not paying toward the purchase of the collective good.

Since it is individuals who decide on actions, and since each member of the group sees the game matrix from the vantage point of Individual, we can assume that Collective's

[2] Within a broad range, this assumption entails only that the payoffs in the upper right and lower left cells in Matrix 1 will contain payoffs only slightly higher or lower than might have been the case for a real world problem. Consequently, the logical dynamics of the game are unaffected by the assumption.

strategy will finally be whatever Individual's strategy is, irrespective of what Collective's payoffs suggest. The dynamic under which Individual performs is clearly the same as that for the prisoners' dilemma: his strategy of not paying dominates his strategy of paying. For no matter what Collective does, Individual's payoff is greater if he does not pay. This can be seen more clearly perhaps in Matrix 1a, which displays only the payoffs to Individual for each of his choices. As in prisoners' dilemma, not paying is invariably more lucrative than paying.

The payoffs to the two players in a game of prisoners' dilemma are shown in Matrix 2, and Row's payoffs only are shown in Matrix 2a. In this classic game, the delight of game theoreticians, Row and Column will both profit (1 unit each) if both cooperate, and both will lose (1 unit each) if both defect. But as is clear in Matrix 2a, Row is wise to defect no matter what Column does. The Matrices 1a and 2a are strategically equivalent; the preference orderings of the payoffs to Individual and to Row are identical as shown by the arrows in Fig. 1.

n-prisoners' dilemma

For the theorist of n-person games, a more cogent analysis of the problem of collective action defined by the game of individual vs. Collective would require a 10-

MATRIX 1a

INDIVIDUAL'S PAYOFFS

Individual	Pay	1	−0.8
	Not Pay	1.8	0

MATRIX 2

PRISONERS' DILEMMA

		Column	
		Cooperate	Defect
Row	Cooperate	1, 1	−2, 2
	Defect	2, −2	−1, −1

MATRIX 2a

ROW'S PAYOFFS

Row	Cooperate	2	−2
	Defect	1	−1

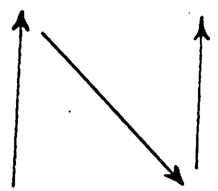

FIG. 1. Preference Ordering for Row and Individual.

dimensional matrix pitting the payoffs of each individual against all others. The payoffs can easily be calculated. The cell defined by all players paying would be (1,1,1, 1,1,1,1,1,1,1), and that defined by all players not paying would be (0, ⋯, 0). Every other cell would have payoffs whose sum would be equal to the number (m) of players paying in that cell; each player would receive a payoff of $2m/10 - 1$ if he paid, or $2m/10$ if he did not pay. Anyone able to visualize a 10-dimensional matrix can readily see that each player's dominant strategy is not to pay, because it yields the best payoff for whatever the other players do. The rest of us can easily enough calculate that whereas the payoff to player i is $2m/10 - 1$ with m players including himself paying, the payoff to i with $m - 1$ players not including himself paying would be the preferred $2(m - 1)/10$ (in the latter case i's payoff is 0.8 units greater than in the former). Hence, for each player i, the strategy of not paying dominates the strategy of paying. But playing dominant strategies yields all players the poor payoff (0, ⋯, 0), and this solution is the only equilibrium for the game.

The game now defined is simply the 10-prisoners' dilemma, to which any solution algorithm generalized from the 2-prisoners' dilemma can be applied. To generalize the game further, n prisoners can be substituted for 10, and a ratio r of benefits to costs (with cost being 1 unit to each player) for the ratio of 2 assumed in Individual vs. Collective. The result is analogous, with the choice of not paying always yielding a payoff $(n - r)/n$ units higher than the choice of

paying (the bonus increases as n increases); and if all pay, all receive payoffs of $(r - 1)$. Olson's privileged group would be the case in which r is greater than n in some player's perception (if costs are a matter of binary choice between paying a fixed sum for all players who pay, or paying nothing).

In this game there is only one (strongly stable) equilibrium (at the payoff of zero to every player, i.e., all players not paying); but this equilibrium solution is not Pareto-optimal. Moving from the equilibrium to the payoff of r to every player (i.e., all players paying) would improve the payoff to every player. Among the 78 strategically nonequivalent 2 × 2 games in the scheme of Rapoport and Guyer (1966), prisoners' dilemma is unique in its class. It is the only game defined by the condition that it has a single strongly stable equilibrium which, however, is Pareto-nonoptimal. Hence, the generalized game of collective action defined above is logically similar to prisoners' dilemma. (It should be clear that the reason for the equivalence of prisoners' dilemma and the game of collective action for a large, i.e., latent, group is precisely the condition that in such a group a player's contribution to the purchase of the collective good is of only marginal utility to himself. Hence, his payoff is increased by almost the amount he does not pay when he does not pay.)

Empirical consequences

The significance of this result is that any analysis which prescribes a solution for prisoners' dilemma must prescribe a similar solution for the game of collective action. That means that the vast body of experimental and theoretical work on prisoners' dilemma is relevant to the study of collective action in general (and conversely that the growing body of work on collective action can be applied to the study of the prisoners' dilemma). In particular, any analysis of prisoners' dilemma which yielded the conclusion that the mutual loss payoff was not rational would, by implication, contravene Olson's (1968, p. 44) claim that, for logical reasons, in a latent group "it is certain that a collective good will *not* be provided unless there is coercion or some outside inducements." Considering the fact that there are arguments that the rational solution to prisoners' dilemma is the payoff which results from mutual cooperation, before turning to the rationale of group success, we should perhaps reconsider why it might be that, empirically, latent groups do generally seem to fail. Let us view the 10-prisoners' dilemma defined above in the light of some 2-prisoners' dilemma experimental results.

Some experimental data suggest that about one-half of bona fide players cooperate with and one-half exploit a noncontingent, 100 percent cooperative adversary-partner in 2-prisoners' dilemma (Rapoport, 1968). In the 10-prisoners' dilemma described above, let us assume that this result would mean that 5 of the players would not pay even if the other 5 did pay. In this circumstance, the benefit to each player would be 1 unit, and the cost to each of the 5 payers would be 1 unit: hence, the payoff to the payers would be zero. Consequently, even an analysis which prescribed cooperation, or paying, as the rational strategy under the assumption of all players rational would allow nonpayment as a rational strategy to players in a real world game in which habitual nonpayers drained off any positive payoff to the payers. Assuming the validity of the generalization from the prisoners' dilemma experimental data, in real world games in which the law of large numbers applies and in which the perceived benefits of the collective good are not more than twice the costs, one can expect no provision of the collective good for reasons different from Olson's logic. In the intermediate group (where the statistics of large numbers do not apply), even with benefits considerably less than twice the costs, there is some statistical chance that a collective good will be provided. In either case, the propects for success decline as the ratio (or perceived ratio) of benefits to costs decreases, and as the differential perception of that ratio increases while the average perceived ratio remains constant.

As Olson (1968, p. 24) notes, the issue is not so much what an adversary-member's payoff will be, but rather whether anyone will choose to play the game at all. In the 10-prisoners' dilemma analysis here, by a different logic, it follows that one of the basic tenets of game theory is in one sense not useful in real world application. Ordinarily, in game theoretical analyses the actual values of payoffs are not important; the only consideration is the rank ordering of payoffs. But clearly, the normal inducement to play a real world game is the expectation of positive payoffs. Hence, a rational player in the game of collective action does not refuse to pay merely because his strategy of not paying is dominant and yields a higher payoff; rather he refuses to pay because enough others in the group do not pay that he would suffer a net cost if he did. Consequently, it would be irrational for him to play the game, and not playing means not paying. (However, this reasoning cannot be considered to give a proof that collective action will fail. That remains an empirical matter.)

THE CONDORCET CHOICE SOLUTION OF THE GAME OF COLLECTIVE ACTION

The usual analysis of prisoners' dilemma prescribes a strategy: the dominating strategy which in the 10-person game of collective action discussed above would be not to pay. Because the general employment of that strategy produces an undesirable outcome, and because many (roughly half) of the subjects in some 2-prisoners' dilemma experiments have not employed that strategy, it would be useful to analyze the outcomes (as opposed to the strategies) of the larger game of collective action. The matrix for the 10-person binary choice game has 2^{10} or 1024 cells, each of which is a uniquely defined potential outcome of the game. Instead of considering the strategies of the players, let us view the game as though the 10 players were collectively choosing among the 1024 outcomes. With a simple notation these 1024 outcomes can be represented as 20 classes of outcomes. We can readily ascertain which among these classes are realizable outcomes, and can determine whether any among the realizable outcomes is a Condorcet choice. It will be a simple matter to demonstrate that in any game of collective action with n players and a ratio r, $r > 1$, of benefits to payments there is a Condorcet choice among the realizable outcomes, and it is the outcome defined by all players paying and all receiving payoffs of $r - 1$ units.

2n classes of outcomes

In the 10-person game of collective action, the possible outcomes in the view of an Individual in the game are as in Matrix 3. The entries in the top row are Individual's payoffs when all ten players pay, nine players including Individual pay, etc. Those in the bottom row are Individual's payoffs when he does not pay, ranging from the case in which all nine other players pay to the case in which no one pays. The upper left payoff results from only one outcome of the game: all pay. The upper row payoff of 0, however, results from 126 different outcomes of the game: all the possible combinations in which Individual and four other players pay while five players do not pay.

It will be useful to represent these classes of outcomes more generally. Let N_k represent any outcome in which exactly k players, including Individual, do not pay. And let P_k represent any outcome in which exactly k players, including Individual, pay. Matrix 3 can be rewritten as Matrix 3a. It is now a simple matter to rank order the outcomes according to Individual's preference; the position of an outcome P or N is determined by the payoff to Individual associated with it. Table 1 presents Individual's preference ordering and gives the total number of outcomes in the full 10-dimensional game matrix associated with each payoff class.

MATRIX 3

Pay	1.0	0.8	0.6	0.4	0.2	0	−0.2	−0.4	−0.6	−0.8
Not Pay	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0

MATRIX 3a

Pay	P_{10}	P_9	P_8	P_7	P_6	P_5	P_4	P_3	P_2	P_1
Not Pay	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}

TABLE 1

Payoff Class	Number of Outcomes in this Class
N_1	1
N_2	9
N_3	36
N_4	84
P_{10}, N_5	1,126
P_9, N_6	9,126
P_8, N_7	36,84
P_7, N_8	84,36
P_6, N_9	126,9
P_5, N_{10}	126,1
P_4	84
P_3	36
P_2	9
P_1	1
Total	1024

Clearly, Individual can guarantee himself his minimax payoff (N_{10} at the lower right in Matrix 3a). Those outcomes P_4, P_3, P_2, and P_1 which fall below the minimax line in Table 1, therefore, are outcomes which he can unilaterally prevent by not paying. Similarly, however, every other player in the game can prevent his own P_4, P_3, P_2, and P_1 outcomes, so that the complementary outcomes N_6, N_7, N_8, and N_9 of opposing players will be prevented (for instance, an N_9 can occur only if some player is willing to pay when no one else does, thus putting himself into a P_1 outcome). Hence, none of these outcomes is realizable, i.e., they would require that some player willingly recline below his minimax, as few of us are wont to do.[3] The only outcomes which can obtain in a play of the game are those of Table 2. It is from this set of realizable outcomes that the players must seek an agreeable outcome. If one of these outcomes is a Condorcet choice for the set, it is the prominently rational outcome of the game.

TABLE 2

N_1
N_2
N_3
N_4
P_{10}, N_5
P_9
P_8
P_7
P_6
P_5, N_{10}

Condorcet choice

We can define strong and weak Condorcet choices.[4] Let C be the collective (i.e., the group) of n members choosing among outcomes in the matrix of an n-person collective action game, and let j and k be outcomes from the set M of realizable outcomes in the game matrix (in the 10-prisoners' dilemma matrix there are 1024 cells, of which 639 are realizable outcomes). Let c_{jk} be the number of those in C who prefer outcome j to outcome k, and let c'_{jk} be the number of those in C who are indifferent to whether outcome j or outcome k obtains. Clearly, $c_{jk} + c_{kj} + c'_{jk} = n$.

Definition: j is a strong Condorcet choice if it is preferred by a majority in C to every k ($\neq j$) in M. Reduced to symbolic brevity, this condition is

$$c_{jk} > n/2 \text{ for all } k \neq j.$$

Definition: j is a weak Condorcet choice if it is not a strong Condorcet choice but if, for each $k \neq j$, more of those in C prefer j to k than k to j. This condition is simply

$$c_{jk} > c_{kj} \text{ for all } k \neq j.$$

It should be clear that there can be at most one Condorcet choice.

[3] The use of this term conforms with Howard (1967, p. 24), in whose metagama theory an outcome is metarational for all players if every player's payoff in that outcome is at least equal to his minimax payoff. Hence, the only realizable outcomes are those which are metarational for all players.

[4] Named for the eighteenth century French economist and intellectual in general, the Marquis de Condorcet, who studied the problem of electoral majorities, believed in man's capacity for unlimited progress, and chose to poison himself rather than meet the guillotine during the Terror.

From the definition of the game of collective action for n players the following theorem for the existence of a Condorcet choice among the set of realizable outcomes can be derived.

Theorem: For an n-person game of collective action, P_n is a Condorcet choice from the set of realizable outcomes for the game; it is a strong Condorcet choice except in a game in which n is even and $r = 2$, in which case P_n is a weak Condorcet choice from the set of realizable outcomes.

The proof of this theorem, which is not difficult but is tedious, is left to the appendix. However, it will be instructive to see that it holds for the case of 10-prisoners' dilemma. We need only to compare the outcome P_{10} to each of the other realizable outcomes listed in Table 2 to show that P_{10} is preferred to each of these others. Given an outcome of the class N_1, nine players will prefer P_{10}. Similarly, given an outcome of the class N_2, N_3, or N_4, eight, seven, or six players, respectively, will prefer P_{10}. And nine, eight, seven, or six players will prefer P_{10} to any outcome of the class P_9, P_8, P_7, or P_6, respectively. Finally, given an outcome of the class N_5, the five players whose outcomes are of the class P_5 will prefer outcome P_{10}; and the five players whose outcomes are of the class N_5 will be indifferent to the choice between N_5 and P_{10}. Consequently, a clear majority of the players will prefer P_{10} to any outcome except N_5, in which case all of those with a preference will prefer P_{10}. It follows that P_{10} is a weak Condorcet choice. It is weak because the game has an even number of players and a ratio of benefits to contributions of 2.[5]

Degeneracy—back to the prisoners' dilemma

At the limits of the preceding analysis there occur several classes of degenerate games of collective action. These result when $r = 1$ or $n = 2$.

In the degenerate case of $r = 1$, the realizable outcomes are P_n and N_n, and all players are indifferent as to which of these obtains. For all cases of $r < 1$, the only realizable outcome is N_n. The game will not be played.

In the degenerate case of $n = 2$ there are five possibilities: $r < 1$, $r = 1$, $1 < r < 2$, $r = 2$, $r > 2$. The first two of these are degenerate in r. In the case of $r = 2$, all outcomes are realizable and the outcome of both pay is a weak Condorcet choice. If $r > 2$, each player's return from his own contribution is greater than his contribution, so presumably both will pay and reap appropriate benefits (recall that in general $r > n$ implies that the group is a privileged group in Olson's terms). The interesting cases remain. They are those for $1 < r < 2$. They are represented in Matrix 4.

The payoffs in the games of Matrix 4 are related according to the preference ordering (if $1 < r < 2$):

$$r/2 > (r - 1) > 0 > (r/2 - 1).$$

This condition meets the definition of the symmetric 2-prisoners' dilemma game. For example, Rapoport and Chammah (1965, pp. 33–34) define the symmetric prisoners' dilemma by the condition that the payoffs (as given in Matrix 5) satisfy the relation:

$$T > R > P > S,$$

in which the letters didactically stand for Temptation (to defect), Reward (for cooperating), Punishment (for defecting), and Sucker's payoff (for cooperating). Note that the preference ordering for row is as in Figure 1. From the preference ordering and Matrix 4, it can be seen that only the outcomes $(r - 1, r - 1)$ and $(0, 0)$ are realizable, and that of these $(r - 1, r - 1)$ is a strong Condorcet choice.

MATRIX 4

$r - 1, r - 1$	$(r/2 - 1), r/2$
$r/2, (r/2 - 1)$	$0, 0$

MATRIX 5

R, R	S, T
T, S	P, P

CONCLUSION

It has been shown that the problem of collective action can be represented as a game with a strategic structure similar to that of prisoners' dilemma. The logic which prescribes that a member of a group should not contribute toward the purchase of his group collective interest is the same as that which prescribes that a player in a game of prisoners' dilemma should defect (i.e., should not cooperate). However, from the set of all realizable outcomes in a game of collective action in which the ratio of benefits to contributions exceeds 1, the outcome in which all contribute is a Condorcet choice. The existence of a Condorcet choice, which is by definition unique, implies that a real world group could decide in favor of the Condorcet choice over every other realizable outcome. Consequently, it is rational in a world in which distrust seems endemic to use sanctions to enforce all members of an interest group to contribute toward the purchase of the group interest (Olson, 1968, p. 51). In a world not quite Hobbesian a threat of all against all might, ironically, help overcome distrust.

However, the threat of all against all is not a logical necessity; rather, it is only a potentially useful device, given human psychology. For, there is debate in the literature on the prisoners' dilemma as to whether the cooperative or the noncooperative outcome is rational or logically determinate. Therefore, it can hardly be granted that, as Olson contends, in the absence of sanctions in a latent group "it is certain that a collective good will *not* be provided," whereas in an intermediate group the result is merely indeterminate. The clarity of the analogy between the logic of collective action and the strategic structure of the prisoners' dilemma game makes it seem likely (as suggested above) that the differences in the statistics of success for the intermediate and latent groups is a function of statistics on, for example, the social distribution of distrust; but in any case it is not a derivation from the logic inherent in the group interactions.

APPENDIX: PROOF OF THEOREM

Assume a group of m player-members in a game of collective action as defined with a ratio r of benefits to payments. When an outcome of class P_k obtains for k players, its complementary outcome of class N_{m-k} obtains for the other players. Let (P_k, N_{m-k}) represent the k outcome set for the game: it is the set of all outcomes which are of class P_k for k players and of class N_{m-k} for $(m - k)$ players. (The total number of outcomes represented by this set is $m!/k!(m - k)!$. For instance, when $k = m$, all players are in the single outcome of class P_m.) Finally, let c_{mk} represent the number of players who prefer outcome P_m to an outcome of the set k.

In order to demonstrate that P_m is a Condorcet choice among the set of realizable outcomes of the m-person game of collective action, we need only show that, for each $k < m$,

(1a) $c_{mk} > m/2$, or

(1b) $c_{mk} > c_{km}$, or

(1c) the outcomes in set k are not realizable.

Let us note two general conditions before proving the theorem. The condition which renders an outcome not realizable is that in that outcome some player receives a payoff less than his minimax, i.e., less than zero. If p_k represents the payoff to a player in an outcome of class P_k, and n_{m-k} the payoff to a

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[5] It was noted above that prisoners' dilemma is the only one of the Rapoport-Guyer games with a strongly stable equilibrium that is not Pareto-optimal. This statement can be made stronger. Every outcome in the 2-prisoners' dilemma is Pareto-optimal except the outcome of mutual loss. In a game of collective action this stronger statement also usually holds. However, if r divides n, then any outcome defined by $N_{n/r}$ for n/r of the players is not Pareto-optimal. (This is because $n - n/r$ of the players would benefit in a shift from this outcome to P_n, and the other n/r players would be indifferent to the shift.) All other outcomes in any game of collective action are Pareto-optimal except the single dismal solution N_n.

player in the complementary outcome of class N_{m-k}, then

$$p_k = kr/m - 1, \text{ and} \tag{2}$$

$$n_{m-k} = kr/m. \tag{3}$$

By definition it follows that:

Condition 1. The outcomes of the k outcome set are not realizable if $p_k < 0$.

The payoff at P_m is $(r - 1)$. At P_k, $k < m$, the payoff is $p_k < (r - 1)$. It follows that:

Condition 2. The k players in an outcome of class P_k, $k < m$, prefer outcome P_m to P_k.

Proof of the theorem

To prove the theorem, we must show that requirement (1) is met for three possible values of k: (I) $k > m/2$; (II) $k < m/2$; and (III) $k = m/2$.

Region (I)

$$k > m/2$$

By Condition 2, P_m is preferred to P_k by k players, so that $c_{mk} > m/2$. Requirement (1a) is met.

Region (II)

$$k < m/2 \tag{4}$$

There are three regions in the value of the payoff to the players not paying: (a) $kr/m < (r - 1)$; (b) $kr/m = (r - 1)$; and (c) $kr/m > (r - 1)$. We must show that requirement (1) is met in each of these regions.

$$kr/m < (r - 1) \tag{a}$$

In this region, it is clear from (2) and (3) that all players prefer P_m to the set (P_k, N_{m-k}). Hence, requirement (1a) is met.

$$kr/m = (r - 1) \tag{b}$$

It follows that

$$kr = mr - m, \text{ or}$$

$$(m - k)r = m. \tag{5}$$

But from (4), we have

$$m > 2k \tag{6}$$

From (5) and (6) we have

$$(2k - k)r < m, \text{ or}$$

$$kr/m < 1. \tag{7}$$

But the payoff to those who pay is, according to (2),

$$p_k = kr/m - 1.$$

From (7) it follows that

$$p_k < 1 - 1, \text{ or}$$

$$p_k < 0.$$

From Condition 1 it follows that the outcomes of the k outcome set are not realizable. Hence, requirement (1c) is met.

$$kr/m > (r - 1). \tag{c}$$

By an argument almost identical to that for the condition of (b) above, we have

$$kr > mr - m, \text{ or}$$

$$(m - k)r < m.$$

From (6) it follows that

$$kr < m, \text{ or}$$

$$kr/m < 1.$$

This is the same as (7); from the argument above it follows that requirement (1c) is met.

Region (III)

$$k = m/2.$$

It follows from (2) and (3) that

$$p_k = r/2 - 1, \text{ and} \tag{8}$$

$$n_{m-k} = r/2. \tag{9}$$

As in (II), there are three possibilities: (a) $r/2 < r - 1$; (b) $r/2 = r - 1$; and (c) $r/2 > r - 1$.

$$r/2 < r - 1. \tag{a}$$

In this region, it is clear from (8) and (9) that all players prefer outcome P_m to the set (P_k, N_{m-k}). Hence, requirement (1a) is met.

(10) $$r/2 = r - 1. \tag{b}$$

From Condition 2 it follows that $m/2$ players prefer P_m to the set (P_k, N_{m-k}); and from (9) it follows that the other $m/2$ players are indifferent in the choice between P_m and this set. Hence, $c_{mk} = m/2$, $c_{km} = 0$, so that $c_{mk} > c_{km}$. Requirement (1b) is met.

$$r/2 > r - 1. \tag{c}$$

It follows that

$$r/2 < 1.$$

Hence, from (8) we have

$$p_k < 1 - 1, \text{ or}$$

$$p_k < 0.$$

From Condition 1 it follows that the outcomes of the k outcome set are not realizable. Hence, requirement (1c) is met.

Requirement (1) is met for all values of r and m, so that there exists a Condorcet choice among the set of realizable outcomes in a game of collective action. Moreover, in almost every case, either (a) or (c) of requirement (1) is met; for all these cases, P_m is therefore a strong Condorcet choice. The only exception to this is case (IIIb), in which requirement (1b) is met; in this case, m is divisible by 2, and from (10) it can be seen that $r = 2$. Consequently, P_m is a weak Condorcet choice among the realizable outcomes in a game of collective action in which there is an even number of players and $r = 2$. The theorem is proved.

REFERENCES

Howard, N. A Method for metagame analysis of political problems. Mimeographed working paper, Management Science Center, University of Pennsylvania, 1967.

Olson, M., Jr. *The Logic of collective action.* New York: Schocken, 1968 (first published, 1965).

Rapoport, A. Editorial comments. *J. conflict Resolut.*, 1968, 12, 222–23.

Rapoport, A., & Chammah, A. M. *Prisoner's dilemma.* Ann Arbor, Mich.: Univ. of Michigan Press, 1965.

Rapoport, A., & Guyer, M. J. A taxonomy of 2 × 2 games. *General Systems*, 1966, 11, 203–14.

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There's always something
one's ignorant of
About anyone, however well
one knows them;
And that may be something of
the greatest importance.
T. S. ELIOT