Choosing to Exchange Research Ideas

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Abstract

Why do academic economists tend to be secretive about their research ideas before the ideas are transformed into a written paper? More generally, why would agents not be willing to trade ideas that are used as an input into innovation? I develop a model to argue that the answer to this question is not obvious. In fact, under a broad set of conditions, such trade may be chosen by agents. I then identify two market conditions under which such exchange is limited or abandoned. The first condition is sufficiently intense competition in the market for the innovation for which ideas are an input. The second condition is sufficient asymmetry in the quality of ideas across agents.

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1 Introduction

As economists, the research questions we pursue, and proposed answer we explore to each question, are valuable commodities. These ideas, consisting of a research problem and a potential solution, are things to keep under wraps. Ideas are generally kept outside the public domain until a paper has been written. With the paper, we can claim priority over our idea, and then receive the requisite reward in the form of reputation within the profession. Why are we so protective of our ideas? Why are we not willing to trade ideas with other economists?

Economists protecting their research ideas is just one case of individuals or firms choosing not to share, or exchange, ideas that are inputs into a production process. Another example comes from the pharmaceutical industry. A drug that treats a symptom is an innovation. Each firm in the industry might know of a potential treatment for a set of symptoms. Each potential treatment may be successful, and a drug invented, with some probability less than one. Why do pharmaceutical firms choose not to exchange information regarding those potential treatments with one another?

This paper develops a model in order to understand why exchange of ideas between individuals or firms may not be chosen. In doing so, it is first argued that under reasonable and fairly general conditions, agents should in fact choose to exchange ideas. Thus, the reasons why exchange does not occur are not immediately obvious, as one might think. Then, conditions under which exchange breaks down are analyzed.

The setting used is now summarized and the main insights captured by the model described. Each decision-making unit is called a firm. Each firm exogenously possesses a mutually-exclusive set of non-substitutable intermediate goods. These intermediate goods will be called ideas. Each intermediate good can be used in a different production process. In particular, each idea may be used in production of an innovation. Innovations may generate reward for the firm in an innovation market. An innovation is produced using only one idea, and innovations produced using different ideas are non-substitutable.

A firm is defined by the set of ideas it possesses. Each firm also uses a set of labor as inputs into the production of innovation. Firms can choose how to allocate the labor input, across its ideas, to produce innovations. Each unit of labor is assumed to work independently from others on an idea; there is no positive externality associated with labor working in the same firm.

The distinctive feature of the innovation process as a production process is that ideas and labor are used to produce an innovation with some probability that will generally be less than one. Moreover, as any unit of labor within a firm is allocated more ideas to work on, there is a decrease in the probability that the unit of labor successfully turns any one idea into an innovation. This assumption captures labor's time constraint: when a unit of labor works on more ideas, less of that labor is allocated to any one idea.

In order to maximize expected profits, the firm's problem is to maximize the total expected reward from innovations. The labor cost of innovation is taken as given by the firm. Firms are assumed to know the relationship between the price of an innovation and the number of firms that have successfully produced the innovation. Thus, the firm's problem is to maximize the expected number of innovations for which reward is received, with each innovation weighted by its expected relative reward. Firms, given their original set of ideas, decide how to allocate labor across ideas in order to produce innovations.

Firms may also choose to exchange ideas. Firms then independently attempt to produce an innovation using each idea to which they have access and choose to use. Since innovations are non-substitutable, they will be sold in different markets. Firms compete in an innovation market only if they have both successfully produced the innovation.

What are the gains and losses of idea exchange between two firms? The gain is that there is a possible increase in the number of innovations each firm can achieve. One loss is that, because both firms will be working on at least some of the same ideas, if one firm produces a particular innovation, then the other may as well. Duplication of innovation will reduce the reward each firm receives. Another loss is that, as a firm works on more ideas, the number of ideas pursued by each unit of labor increases. The probability that any one innovation is produced by a unit of labor therefore decreases. It is not immediately obvious how these effects balance out. In addition to identifying these different effects, this paper studies how varying market conditions affects the balance.

This framework provides the following insights. In order to maximize the expected reward from innovations, firms may choose to exchange ideas if there is a priori symmetry in idea quality across firms, and if price competition in the innovation market is not too intense. This result is robust to asymmetry in firm size or productivity. However, if ideas vary in quality across firms, in a way to be made precise, then exchange will be limited. Also, if price competition is sufficiently intense, exchange will not be chosen.

These findings suggest that, as economists, at least one of two forces results in our choice to keep our ideas secret until they are successfully transformed into papers. First, the quality of ideas may vary sufficiently that it is not optimal for us to exchange ideas. We could end up sharing what turns out to be a great idea, and getting a bad idea in return. Second, competition for the reward of research, that comes in the form of reputation, may be sufficiently intense. Thus, if two or more researchers each produce a paper using the same idea, the reward to each is greatly diminished from that which would occur if only one paper using the idea had been produced.

The question examined here is related to that asked in work by Bhattacharya, Glazer and Sappington [1], [2] and d'Aspremont, Bhattacharya and Gérard-Varet [8]. They consider licensing arrangements, to divide reward, that may be used by firms to facilitate knowledge trade. In their frameworks, and other related ones in the literature, there is only one possible innovation, so that competition in innovation production is automatic. Knowledge exchange increases innovation quality. They assume asymmetry in firm knowledge quality. That asymmetry creates the need for licensing.

The result of this paper, that exchange is possible, is consistent with those of De Fraja [10] and Bhattacharya, Glazer and Sappington [1]. The result that price competition may reduce exchange is consistent with Severinov [14], who considers incentives for employee information exchange in one innovation setting. Cardon and Sasaki [5] find the opposite effect: reducing competition in duopoly will reduce the incentive to duplicate. In their model, an increase in competition provides incentive for preemptive duplication. Their structure is one of a single research problem and many possible solutions. Each firm may choose only one potential solution at a time, but if one firm is successful on one path, another firm may still receive reward in the future via a patent for success on another path.

In all of these models, and others related, there is an assumption of only one possible innovation. This paper's model deviates from that assumption in order to consider exchange of ideas that lead to non-substitutable innovations, but exchange generates the possibility of duplicating innovation. This model allows for the fact that, in a wide range of settings in which the research problems and potential solutions are an input in production of innovation, agents may work toward multiple innovations at one time.

2 A Model of Idea Exchange

A single sector model of innovation is developed. There are $n \geq 2$ symmetric research firms that produce innovations. The profit maximizing choice of firms, absent the possibility of exchange of ideas, is considered first. The possibility of exchange is introduced in Section 2.2.

2.1 Individual Research Firms

Each firm possesses a set of ideas $\check{H}^j$ known only to firm j. This set is assumed mutually-exclusive to that owned by any other firm. Recall, an idea is a well-defined technological problem with a potential solution. The number of researchers of firm j, $\check{L}_R^j$, is exogenously specified and fixed[1]. An idea and at least one researcher are necessary to produce an innovation.

Firms choose the number of ideas to work on, and how to allocate them across their research labor. Define $\beta^j \in [0,1]$ as the proportion of ideas $\check{H}^j$ that are chosen to be used by the research firm. Thus, $\beta^j \check{H}^j$ is the total number of ideas used. Define $\alpha^j \in [0,1]$ as the proportion of ideas allocated to each researcher in the research firm. Assuming symmetry across ideas and across researchers before the innovation process in each period, attention is restricted to symmetric allocations of ideas across researchers. Thus, $H^j = \alpha^j \beta^j \check{H}^j$ is the number of ideas used by each researcher. An implication of these definitions

[1] More complex models, in which the allocation of labor across firms and the reward from innovation are endogenous, yield similar results to those reported in this study. This is essentially because a firm will still be defined by its set of ideas, so one firm will not emerge.

and the symmetry assumptions is that the number of researchers per idea[2] is $L_R^j = \alpha^j \check{L}_R^j$. The assumption of idea symmetry is reconsidered in Section 4.

The probability that innovation of a single idea occurs at least once is given by

$$\left[1 - \left[1 - \exp\left(-H^j\right)\right]^{L_R^j}\right].$$

The probability of any one idea being successfully processed into an innovation by a researcher is given by the function $v\left(H^j\right) = \exp\left(-H^j\right)$. This probability is dependent on the number of ideas worked on by each researcher, H^j; specifically, $v\left(H^j\right) \in (0,1)$ is decreasing and convex in H^j. The relationship will be called the congestion effect; as more ideas are worked on by each researcher, the probability decreases that any one idea is processed into an innovation by that researcher. There are no spillovers between researchers; each researcher works independently.

Innovations $\dot{A}^j$ are assumed appropriable by some reward system, such as patenting. Each innovation produces a reward $\Re$. This reward can be interpreted as monopoly profits resulting from intellectual property protection[3]. Total labor costs $w\check{L}_R^j$ are taken as given by each firm, where w is the unit cost of labor. Labor is the only cost of innovation. In maximizing profits, firms are effectively innovation-maximizers. The firm's problem becomes one of how to organize the firm in order to maximize expected innovation.

The expected flow of innovations in firm j, for which reward is received, is

$$E\dot{A}^j = \beta^j \check{H}^j \left[1 - \left[1 - \exp\left(-H^j\right)\right]^{L_R^j}\right]. \tag{1}$$

The expectation sign is E.

The choice variable β^j enters expected innovation (1) in two places, with opposing effects: via $\beta^j \check{H}^j$ and $H^j = \alpha^j \beta^j \check{H}^j$. It can be verified analytically and numerically that a unique $\left(\alpha^j, \beta^j\right)$ combination exists that will maximize $E\dot{A}^j$, and the optimal $\beta^{j*} = 1$. All ideas are used. This is stated and demonstrated as Claim 1. In general, the results of the paper will include a combination of analytical and numerical results. Analytical results are used when possible, and numerical results will confirm and extend analysis.

Claim 1 With expected innovation given by (1), the innovation-maximizing $\left(\alpha^j, \beta^j\right)$ choice of firm j will yield an H^j that is invariant. Further, $\beta^{j*} = 1$.

Demonstration of Claim 1 It is clear from (1) that α^j has both a positive and negative effect on $E\dot{A}^j$, and it can be established numerically that a well-defined maximum of $E\dot{A}^j$ over the range of $\alpha^j \in (0,1]$ will exist.

[2] For example, if there are 4 researchers and 8 ideas, and each researcher works on 4 ideas, then there are 2 researchers working on each idea.

[3] Firms do not exchange ideas, and each firm has a distinct set of ideas, so there is no competition in innovation markets.

Taking the first derivative of (1) with respect to α^j and setting it to zero, the resulting expression is

$$\alpha^j \beta^j \check{H}^j \left[\frac{\exp\left(-\alpha^j \beta^j \check{H}^j\right)}{\left[1 - \exp\left(-\alpha^j \beta^j \check{H}^j\right)\right]} \right] = \ln\left[1 - \exp\left(-\alpha^j \beta^j \check{H}^j\right)\right].$$

Thus, $\alpha^j \beta^j \check{H}^{ij}$ equals the constant $c = .6931$ in innovation-maximizing equilibrium. Substituting for α^j in (1), numerical calculation demonstrates that $\beta^{j*} = 1$ is innovation-maximizing.■

It will also be the case that $\alpha^{j*} = \frac{c}{\beta^j \check{H}^j}$. The implied constant $c = .6931$, the number of ideas per researcher, is less than one. This implies that more than one researcher is working on each idea. Further, by $\alpha^{j*} < 1$, the firm allocates researchers across ideas such that each researcher works on a subset of all ideas used, $\beta^j \check{H}^j$.

The result that $\beta^{j*} = 1$ is dependent upon the choice of the variable α^j. It seems reasonable to include α^j as a variable, that is to allow research firms to choose how to allocate researchers across research projects. To specify α^j exogenously seems less tenable as an assumption. The result is, however, not specific to the exact form of $v\left(\alpha^j \beta^j \check{H}^j\right)$.

An increase in ideas, holding all else fixed, leads to a proportional increase in expected innovations. However, a decrease in the number of researchers per idea, holding the number of ideas fixed, leads to a less than proportional decrease in expected innovations. This less than proportional decrease is due to the possibility that two researchers in the same firm, and who work on the same idea, can duplicate innovation. Hence, it is always strictly optimal for a firm to work on the full range of ideas available to it. Firms will choose the optimal number of researchers per idea via α^j, which is implied by the optimal number of ideas per researcher $H^j = \alpha^j \beta^j \check{H}^j$.

2.2 Interfirm Organization

Now suppose that research firms can enter into binding agreements under which they exchange some or all of their research ideas. Again, ideas are excludable as trade secrets or otherwise. All research ideas continue to be considered *a priori* equal. It is assumed that each idea is unique.

Firms can write binding agreements solely for the purpose of exchanging ideas. The binding agreements assumption allows for clean demonstration of how exchange occurs.

Consider a two-firm industry[4], $n = 2$. Research firms are assumed to behave competitively except under the auspices of the exchange contract if one is

[4] For the case of $n > 2$, we could approach the exchange decision of firms as one taken two firms at a time and exclude the possibility of re-exchange of another firm's original ideas. This case would then be a direct extension of $n = 2$.

written[5]. Further, by the symmetry of research firms, it is natural to assume that bargaining power across research firms is equal. Only an equal exchange of ideas is considered as an outcome[6]. In the case of duplicated innovation, firms may compete in an innovation market. Since innovations are appropriable, the reward is randomly allocated to one or the other firm. The random allocation assumption eliminates the competitive effect of duplication, and will be reconsidered in Section 3.

It is assumed initially that firms treat exchanged (shared) ideas the same as unexchanged ideas in terms of research labor allocation. The assumption is strong, and unrealistic, since firms presumably can distinguish between ideas that are exchanged and those that are not. Nevertheless, the assumption is initially used to permit analytical results that are insightful despite the strong assumption. In Section 2.3, the assumption is relaxed, so that firms can allocate labor differently toward exchanged and unexchanged ideas. In that case, the possibility of full exchange is not eliminated.

As $\beta^{j*} = 1$ if ideas are not exchanged, the possibility that firms may choose to exchange ideas arises. The focus is on symmetric strategies across firms, so $\alpha^1 = \alpha^2 = \alpha$; $\check{L}_R^1 = \check{L}_R^2 = \check{L}_R$; $\check{H}^1 = \check{H}^2 = \check{H}$.

The proportion of each firm's ideas exchanged is defined as $\lambda \in [0, 1]$. Firms must agree upon λ jointly. Since exchange is taken to be equal between firms, the expected flow of innovations, for which reward is received, is

$$E\dot{A}^j = \lambda \check{H} \left[1 - \left[1 - \exp\left(-\alpha \left(1 + \lambda \right) \check{H} \right) \right]^{2\alpha \check{L}_R} \right] \tag{2}$$

$$+ \left(1 - \lambda \right) \check{H} \left[1 - \left[1 - \exp\left(-\alpha \left(1 + \lambda \right) \check{H} \right) \right]^{\alpha \check{L}_R} \right].$$

The expected innovation expression (2) allows for the possibility that no exchange is chosen. It is possible for there to be duplication, $\lambda^* > 0$, as described in Claim 2. The firm's objective is to maximize (2), that is to maximize the expected number of innovations for which reward is received, since the reward and the cost of innovation are fixed[7].

Claim 2 Each firm has a pool of ideas that are all *ex ante* of identical quality within and across pools. Allocation of labor by each firm, determined by α, is the same across exchanged and unexchanged ideas. If an innovation is duplicated, reward is allocated randomly to one of the innovating firms.

[5] The assumption that firms act cooperatively in R&D and then competitively in production has been used elsewhere (see Cabral [4], D'Aspremont and Jacquemin [9], Kamien, Muller and Zang [11], Petit and Tolwinski [12], and Suzumura [15]). In this paper's model, rather than assuming cooperation in one stage, the question being examined is whether firms competing in R&D nonetheless have incentive to cooperate in the idea generation stage of R&D.

[6] Consideration of symmetric strategies when firms are *ex ante* symmetric is used and justified by Bolton and Farrell [3], Cooper and John [6], and Crawford and Haller [7].

[7] One could equivalently assume that firms that duplicate innovation divide the reward $\Re$ in half. In that case the firm's objective would be interpreted as maximizing expected innovations, with each innovation weighted by its relative reward.

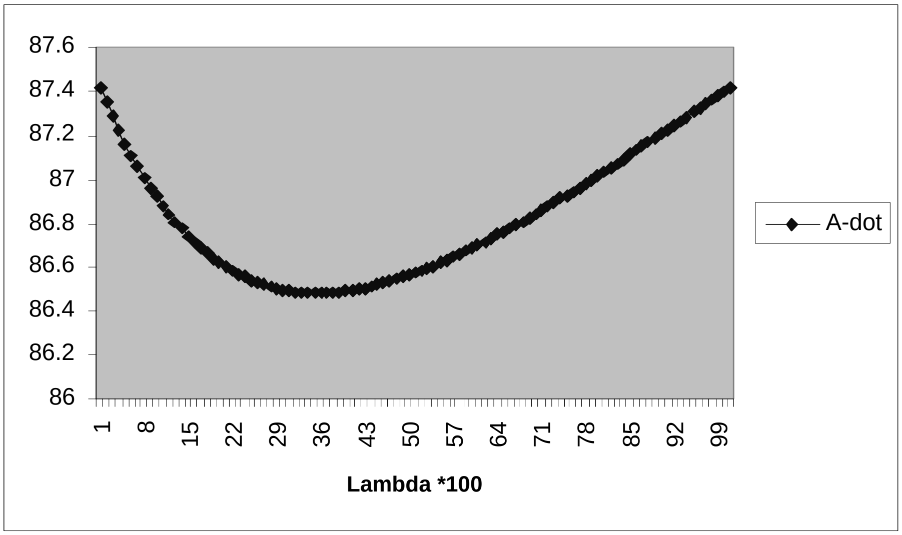

Figure 1: Choice of no or full duplication

Either no exchange $\lambda^* = 0$ or total exchange $\lambda^* = 1$ of ideas is innovation-maximizing for firms with symmetric strategies. Firms are indifferent between these two outcomes.

Demonstration of Claim 2 It is confirmed numerically that a well-defined maximum of $E\dot{A}^j$ exists over the possible values of α and λ. Taking the first derivative of $E\dot{A}^j$ with respect to α and setting to zero, the resulting expression is

$$\begin{aligned} & \alpha\left(1+\lambda\right)\check{H}\left[\frac{\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)}{\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]}\right] \\ = & -\ln\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]. \end{aligned} \tag{3}$$

From (3) it is clear that $\alpha\left(1+\lambda^j\right)\check{H}$, the number of ideas worked on per researcher, is constant in the expected innovation-maximizing outcome. Substituting this result into $E\dot{A}^j$, it can be shown numerically over the range $\lambda \in [0,1]$ that $\lambda^* = 0$ and 1 and that both yield the same value of $E\dot{A}^j$, and they are the innovation-maximizing choices of idea use.■

An example depicting this result is in Figure 1. The constancy of the number of ideas per researcher, $\alpha\left(1+\lambda\right)\check{H}$, and the indifference between no and total exchange of ideas remain under a more general structure of the probability that depends negatively on $\alpha\left(1+\lambda\right)\check{H}$. However, the ability for the firm to vary the intensity of effort via α is crucial.

There are three effects on expected innovation for each firm when the number of ideas exchanged increases, holding all else equal. First, the total number of ideas used by each firm increases; this effect is positive and linear. Second, the probability of successfully innovating any one idea decreases; this effect is negative. Third, the possibility of duplicated innovation arises for all exchanged ideas; this effect is negative. These three effects affect all firms.

Because firms will adjust $c = \alpha (1 + \lambda) \check{H}$ such that it remains constant, the second effect of exchanging ideas is eliminated. That is, firms adjust the number of ideas per researcher by their choice of α. The probability that any one idea is successfully innovated by any one researcher will remain constant.

Over low levels of λ, increased exchange lowers expected innovation as the third effect dominates. As the first effect is increasing linearly, after some threshold $\hat{\lambda}$ determined by parameter values, this dominance is reversed. At $\lambda = 1$ and $\lambda = 0$ the outcomes are the same because of the adjustment in $\alpha^{j*} = \frac{c}{(1+\lambda)\check{H}^j}$. That is, α^{j*} is chosen such that the number of ideas per researcher, c, is constant regardless of the level of exchange λ.

Claim 2 is dependent on the assumption that α is constant across all ideas for each firm. If sharing is only partial, the probability of reward from innovation is no longer constant across all ideas, shared and non-shared. Because α is constant across exchanged and unexchanged ideas, the expected reward differs across the two types of ideas. When either all ideas are exchanged or none are exchanged, then the restrictive assumption on α is not relevant, and so the level of rewarded expected innovation is higher. When the assumption is relaxed in Section 2.3, a set of innovation maximizing outcomes with $\lambda \in (0, 1)$ are possible, in addition to the two extremes, $\lambda = 0$, $\lambda = 1$.

The expected number of innovations for the industry is simply double that of each firm, because research is carried out independently across researchers. Overall innovation maximization is equivalent to firm innovation maximization. The expected number of innovations for the sector is the same as if there were just one merged venture where all ideas are exchanged and labor is pooled.

Assume that a rise in innovation in a sector leads to a rise in the consumed output. Also assume utility is an increasing function of output (and thus consumption). Welfare, measured as a function of total output, increases under the RJV in the benchmark model. Full idea exchange leads to increased innovation over partial idea exchange, and thus to increased welfare.

2.3 Variation of Intensity

Up to now it has been assumed that the intensity of effort for each idea, α, is the same for all ideas. This assumption is now relaxed. Firms are able to differentiate intensity for exchanged and unexchanged ideas. The implications for the level of exchange are analyzed. In short, the ability to differentiate intensity introduces the possibility that firms can maximize innovations at zero or full exchange, and at any level of exchange between.

The expression for expected innovation of each firm is determined as follows.

The only change in definitions is the replacement of α with the following: α_U is the proportion of unexchanged ideas allocated to each researcher, and α_E is the proportion of proportion of exchanged ideas allocated to each researcher. The number of exchanged ideas per researcher is $\alpha_E 2\lambda\check{H}$; the number of unexchanged ideas per researcher is $\alpha_U(1-\lambda)\check{H}$. Using the same reasoning as in Section 2.1, the number of researchers per unexchanged idea is $\alpha_U\check{L}_R$; the number of researchers per exchanged idea is $\alpha_E\check{L}_R$. The total number of ideas per researcher is thus $[\alpha_E 2\lambda + \alpha_U(1-\lambda)]\check{H} = \Theta\check{H}$.

Expected innovation for each firm is thus

$$\begin{aligned} E\dot{A}^j &= \lambda\check{H}\left[1-\left[1-\exp\left(-\Theta\check{H}\right)\right]^{2\alpha_E\check{L}_R}\right] \\ &+ (1-\lambda)\check{H}\left[1-\left[1-\exp\left(-\Theta\check{H}\right)\right]^{\alpha_U\check{L}_R}\right]. \end{aligned} \tag{4}$$

To determine the level of exchange chosen by firms, the values of α_U, α_E, and λ must be jointly determined. Analytical results are presented first, and are then confirmed and exemplified numerically.

The first order conditions[8] $\frac{\partial E\dot{A}^j}{\partial\alpha_E}=0$ and $\frac{\partial E\dot{A}^j}{\partial\alpha_U}=0$ can be written, respectively, as:

$$\begin{aligned} &\ln\left[1-\exp\left(-\Theta\check{H}\right)\right] + 2\alpha_E\lambda\check{H}\frac{\exp\left(-\Theta\check{H}\right)}{1-\exp\left(-\Theta\check{H}\right)} \\ = \quad & -(1-\lambda)\alpha_U\check{H}\left[1-\exp\left(-\Theta\check{H}\right)\right]^{(\alpha_U-2\alpha_E)\check{L}_R}\left[\frac{\exp\left(-\Theta\check{H}\right)}{1-\exp\left(-\Theta\check{H}\right)}\right] \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\ln\left[1-\exp\left(-\Theta\check{H}\right)\right] + \alpha_U(1-\lambda)\check{H}\frac{\exp\left(-\Theta\check{H}\right)}{1-\exp\left(-\Theta\check{H}\right)} \\ = \quad & -2\alpha_E\lambda\check{H}\left[1-\exp\left(-\Theta\check{H}\right)\right]^{(2\alpha_E-\alpha_U)\check{L}_R}\left[\frac{\exp\left(-\Theta\check{H}\right)}{1-\exp\left(-\Theta\check{H}\right)}\right]. \end{aligned} \tag{6}$$

By inspection of (5) and (6) it can be determined that these two first order conditions are consistent with one another if $\alpha_U = 2\alpha_E$ and $2\alpha_E\lambda = \alpha_U(1-\lambda)$. In turn, these two conditions imply that $\lambda = 0.5$. However, if one substitutes the condition $\alpha_U = 2\alpha_E$ into $E\dot{A}^j$, the dependence of $E\dot{A}^j$ on λ disappears. One is left with the expression

$$E\dot{A}^j = \check{H}\left[1-\left[1-\exp\left(-\alpha_U\check{H}\right)\right]^{\alpha_U\check{L}_R}\right]$$

[8] It can be determined numerically that a unique interior maximum for $E\dot{A}^j$ exists with respect to α_U if λ is not too large; otherwise $E\dot{A}^j$ is decreasing as α_U increases. Similarly, a unique interior maximum for $E\dot{A}^j$ exists with respect to α_E if λ is not too small; otherwise $E\dot{A}^j$ is decreasing as α_E increases. The value $\lambda = 0.5$ is not in these extreme ranges; thus, to proceed it is for now assumed that λ falls in this intermediate range where an interior maximum of $E\dot{A}^j$ exists for both α_U and α_E, and calculate the first order conditions that determine α_U^* and α_E^* jointly. It will turn out that this assumption on λ becomes irrelevant.

from which α_U^* can be determined, $\alpha_U^* = 2\alpha_E^*$, and the value of λ can be anything in the range $[0, 1]$.

This analysis is confirmed numerically. Using an example with $\check{H} = \check{L}_R = 5$, the expected innovation for firm j is maximized equally under three scenarios, summarized in the table below.

Scenario 1	Scenario 2	Scenario 3
$\lambda \in [0,1]$	$\lambda = 0$	$\lambda = 1$
$\alpha_U^* = 0.1386$	$\alpha_U^* = 0.1386$	$\alpha_U^* \in [0,1]$
$\alpha_E^* = 0.0693$	$\alpha_E^* \in [0,1]$	$\alpha_E^* = 0.0693$

In the first scenario, the intensity of exchanged ideas is twice that of unexchanged ideas, and the level of exchange can be any value. In Scenario 2, no ideas are exchanged, so it is obvious that the only relevant intensity level is α_U^*. In Scenario 3, all ideas are exchanged and so α_E^* becomes the only relevant intensity level.

This exercise highlights the effect of the choice of intensity, α, on the level of exchange. The variation of intensity between unexchanged and exchanged ideas in this setting allows any level of exchange to occur in an innovation-maximizing outcome, and in this case wipes out the effect of exchange on the expected innovation level entirely. This result is, however, dependent on the nature of competition between firms in the case of duplicated innovation. It is seen in the next section that when price competition is sufficiently intensified, the intensity choice can no longer permit different levels of exchange to occur in innovation-maximizing symmetric exchange between firms.

Many other variations on the basic model are possible. Those examined below in Sections 3 and 4 are of particular interest because of how the exchange outcomes change. Several other variations could be examined, but do not alter the results of Section 2 in important ways. These variations include: allowing the pool of ideas to be common to both firms and introducing dynamics into the model. The possibility of full exchange, described in Claim 2, is maintained with these variations.

It is also straightforward to reconsider the choice to exchange when firms are asymmetric. Two general types of asymmetry are: exogenous asymmetry in the size of research firms, $L_R^1 \neq L_R^2$, or asymmetry in the productivity of research labor, via the probability of innovation. In both sets of alterations to the original model, the possible outcomes are unchanged.

3 Intensified Price Competition

In Section 2 it is assumed that when both firms successfully turn the same idea into an innovation, then one firm receives the reward $\Re$ randomly. This was expressed as a 0.5 probability of having the innovation count as one's own in terms of profits. Under this assumption, the firm rewarded still is a monopolist. This assumption would be appropriate when, for instance, the same idea can be turned into innovations that differ in quality, and the higher-quality innovation wins the entire market, or when firms joint profit maximize in the case of duplicated innovation. The generality of such an assumption is open to question.

The competitive structure could vary from $p = \frac{1}{2}$ in many ways; another possibility is Bertrand competition[9]. In the case of Bertrand competition, reward from a duplicated innovation would be zero for both firms, $p = 0$.

Continuing with the case of two pools of ideas, firm symmetry, the possibility of interfirm exchange, α equal across ideas, and diminishing probability in ideas per researcher. The expected innovations $E\dot{A}_B^j$ that count toward a firm j's profits under Bertrand competition would then be written as:

$$E\dot{A}_B^j = 2\lambda\check{H}\left[1-\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha L_R}\right]\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha L_R} + \left(1-\lambda\right)\check{H}\left[1-\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha \check{L}_R}\right]. \tag{7}$$

Rewarded innovation from exchanged ideas occurs only if firm j successfully innovates, occurring with probability $\left[1-\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha L_R}\right]$, and the other firm does not, occurring with probability $\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha L_R}$. Taking the first derivative of $E\dot{A}_B^j$ with respect to α and setting it to zero[10], the number of ideas per researchers remains at a constant c as defined in Section 2.1. Substituting in for $\alpha = \frac{c}{(1+\lambda)\check{H}}$, it is established numerically that $E\dot{A}_B$ is maximized at $\lambda = 0$. That is, in the case of Bertrand competition, no duplication is chosen by the two firms. This result differs from earlier results, where exchange was a possible innovation-maximizing outcome for firms. The intuition is straightforward: there is a stronger negative effect of duplication in the Bertrand competition case that renders exchange of ideas profit-decreasing. If innovations are duplicated, the firms receive no profit, and the additional ideas do not outweigh that negative effect.

It is possible to verify numerically that this result holds for values of $p \in (0, 0.5)$, where p is the proportion of reward $\Re$ appropriated by each firm in the case of duplicated innovation. The result also holds if firms can vary the intensity of effort between exchanged and unexchanged ideas as in Section 2.3. In this case, expected innovation for each firm is:

$$\begin{aligned} E\dot{A}^j &= 2\lambda\check{H}\left[p\left[1-\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha_E L_R}\right]^2\right] \\ &\quad +2\lambda\check{H}\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha_E L_R}\left[1-\left[1-\exp\left(-\alpha\left(1+\lambda\right)\check{H}\right)\right]^{\alpha_E L_R}\right] \end{aligned}$$

[9] See Sah and Stiglitz [13] and Cardon and Sasaki [5].

[10] As for previous analysis, it is determined numerically that determining the optimal α and λ in this way is correct. An additional assumption is needed in this case:

$$\left[1-\exp(-\alpha\left(1+\lambda\right)\check{H}\right]^{\alpha\check{L}_R} > \frac{1}{2}.$$

This condition holds as long as $\check{L}_R$ is not much larger than $\check{H}$.

$$+(1-\lambda)\check{H}\left[1-\left[1-\exp\left(-\alpha(1+\lambda)\check{H}\right)\right]^{\alpha_U \check{L}_R}\right]. \tag{8}$$

The first term represents innovations from exchanged ideas that both firms successfully produce, yielding a proportion $p \in (0, 0.5)$ of the maximum reward $\Re$ to each firm. The second term represents innovations from exchanged ideas that firm j successfully produces but which the other firm does not. The third term represents innovations produced from unexchanged ideas. By comparing (4) and (8) it is straightforward to see that α_U^* is the same as in the case where $\lambda^* = 0$ and $p = 0.5$. Numerical examples for varying values of $\check{H}$ and $\check{L}_R$ confirm that for $p \in (0, 0.5)$, $\lambda^* = 0$.

Intensified price competition yields a gap between the profit-maximizing choice of firms and the welfare-maximizing outcome for a given level of total research labor. The welfare-maximizing outcome would still be zero or full exchange. However, expected innovation is still the same in the two cases, so the no exchange outcome in the Bertrand competition case is efficient.

4 Idea Asymmetry

Idea asymmetry is now considered. It is assumed exogenously specified. Asymmetry is defined by variation across ideas in the probability of using an idea to produce an innovation. A higher quality idea has, all else equal, a higher probability of being turned into an innovation. All assumptions from Section 2.2 are maintained, excetp as noted.

Two types of ideas are assumed, $s = 1, 2$. The probability of innovation of a type 1 idea is higher than that of a type 2 idea, all else equal. Firms rank ideas the same way. Firm 1 possesses all type 1 ideas, and firm 2 possesses all type 2 ideas. It is assumed that the number of each type of ideas is equal: $\check{H} = \check{H}_1 = \check{H}_2$.

Ideally, the analysis would continue with congestion in innovation via the probability v, where the probability of a research successfully producing an innovation from any one idea is decreasing in the number of ideas the researcher works on. Further, it would be appropriate to allow the intensity of effort to vary not only across exchanged and non-exchanged ideas, but also across the two types of ideas. This framework is not analytically tractable. Moreover, even if a numerical solution were to be found, it would be hard to interpret because of the large number of variables.

Therefore, a single simplifying assumption is made, in order to focus on and derive insight into the effect on exchange of introducing idea asymmetry. The assumption is to remove the congestion effect: associated with a type 1 idea is the constant probability of innovation v_1, and with a type 2 idea is the constant probability of innovation v_2. It is assumed that $v_1 > v_2$ for all firms $j = 1, 2$.

In order to illustrate the difference in outcomes when the probability of innovation includes the congestion effect relative to when the probabilities v_1 and v_2 are constant, the level of exchange that maximizes *total* expected innovation across both firms is considered first. The parameter β_1 is the proportion of type 1 ideas that are exchanged; β_2 is the proportion of type 2 ideas that are exchanged. Overall expected innovation is given by the sum of the two firms' innovation:

$$
\begin{aligned}
E\dot{A} = & (1-\beta_1)\check{H}\left[1-[1-v_1]^{\alpha\check{L}_R}\right] \\
& +\beta_1\check{H}\left[1-[1-v_1]^{2\alpha\check{L}_R}\right] \\
& +(1-\beta_2)\check{H}\left[1-[1-v_2]^{\alpha\check{L}_R}\right] \\
& +\beta_2\check{H}\left[1-[1-v_2]^{2\alpha\check{L}_R}\right]
\end{aligned}
$$

It is straightforward to show analytically that the constancy of v_1 and v_2 implies that overall innovation is maximized by: $\alpha^* = 1$ and $\beta_1^* = \beta_2^* = 1$. Expected innovation overall, and for each firm individually, is increasing in α.

Thus, the effect of removing the congestion effect on overall expected innovation is that full exchange yields the highest overall expected innovation. There is no longer equivalence between full exchange and no exchange. Without the congestion effect, it is optimal to have all researchers working on each idea, and to have as many ideas per researcher as possible.

Turning next to each individual firm, recall that firm 1 has all of the type 1 ideas, and firm 2 has all of the type 2 ideas. For firm 1, expected innovation is:

$$
\begin{aligned}
E\dot{A}^1 = & \check{H}(1-\beta_1)\left[1-[1-v_1]^{\check{L}_R}\right] \\
& +\check{H}\beta_1\frac{1}{2}\left[1-[1-v_1]^{2\check{L}_R}\right] \\
& +\check{H}\beta_2\frac{1}{2}\left[1-[1-v_2]^{2\check{L}_R}\right]
\end{aligned} \tag{9}
$$

The two firms must agree on the levels (β_1, β_2) jointly. To this end, it is useful to interpret β_2 as a function of β_1. Firm 1 is not willing to exchange type 1 ideas $\beta_1 > 0$ without sufficient type 2 ideas. It is straightforward to show that $\frac{\partial E\dot{A}^1}{\partial\beta_1} \geq 0$ if and only if

$$
\frac{\partial\beta_2}{\partial\beta_1} \geq \frac{\left[1-[1-v_1]^{\check{L}_R}\right]^2}{\left[1-[1-v_2]^{2\check{L}_R}\right]}. \tag{10}
$$

The derivative $\frac{\partial\beta_2}{\partial\beta_1}$ represents the exchange rate between type 1 and type 2 ideas: the number of type 2 ideas exchanged for each type 1 idea. The right hand side of the inequality (10) is greater than one - firm 1 must receive at least as many ideas as it gives - only if v_1 is sufficiently larger than v_2. It is also possible for this term to be less than one. As v_1 increases relative to v_2, there is an increase in the minimum number of type 2 ideas in exchange for each type 1 idea that firm 1 is willing to accept.

To determine whether exchange would take place it is necessary to examine whether the firms' acceptable ranges of $\frac{\partial \beta_2}{\partial \beta_1}$ are mutually consistent. Firm 2's expected innovation is:

$$\begin{aligned} E\dot{A}^2 &= \check{H}(1-\beta_2)\left[1-[1-v_2]^{L_R}\right] \\ &+\check{H}\beta_2\frac{1}{2}\left[1-[1-v_2]^{2L_R}\right] \\ &+\check{H}\beta_1\frac{1}{2}\left[1-[1-v_1]^{2L_R}\right]. \end{aligned}$$

Again interpreting β_2 to be a function of β_1, it is possible to show that $\frac{\partial E\dot{A}^2}{\partial \beta_2} \geq 0$ if and only if

$$\frac{\partial \beta_2}{\partial \beta_1} \leq \frac{\left[1-[1-v_1]^{2L_R}\right]}{\left[1-[1-v_2]^{L_R}\right]^2}. \tag{11}$$

The right hand side of the inequality (11) is greater or equal to 1 for $v_1 \geq v_2$. As v_1 increases relative to v_2, there is an increase in the maximum number of type 2 ideas that firm 2 is willing to exchange for each type 1 idea .

The inequalities in (10) and (11) are always mutually consistent. The range over which $\frac{\partial \beta_2}{\partial \beta_1}$ falls $\left[\frac{\left[1-[1-v_1]^{\check{L}_R}\right]^2}{\left[1-[1-v_2]^{2\check{L}_R}\right]}, \frac{\left[1-[1-v_1]^{2L_R}\right]}{\left[1-[1-v_2]^{L_R}\right]^2}\right]$ will lie strictly above 1 if v_1 is sufficiently large relative to v_2, as noted earlier. An example of this range and how it can vary is given in Figure 2, for $v_2 = 0.1$, and $\frac{v_1}{v_2}$ increasing. As L_R increases the size of the range decreases. The exchange of ideas agreeable to both firms will fall in this range and will yield at least one firm better off, and neither firm worse off. Any asymmetry in exchange, such that $\frac{\partial \beta_2}{\partial \beta_1} \neq 1$, will necessarily limit duplication, since at least one firm cannot be exchanging all of its ideas. The actual rate of exchange between type 1 and type 2 ideas will depend on the relative bargaining power of the two firms.

Up until now it has been assumed that firms' bargaining power was equal and so exchange would be symmetric. This cannot be the case in equilibrium if v_1 is sufficiently large relative to v_2 since firm 1 would never agree to such an exchange. Without needing to model the bargaining process explicitly, it is possible to say that exchange may be asymmetric; in this case there will not be full duplication of effort on research ideas in equilibrium.

Comparing this outcome to that which maximizes overall expected innovation, it is clear that the firm's agreed exchange will generally be less than $\beta_1 = \beta_2 = 1$. For v_1 is sufficiently large relative to v_2, equilibrium exchange is necessarily asymmetric and inefficient, there is too little exchange. In the particular case where v_1 is large relative to v_2, there is too little exchange of the higher quality type 1 ideas. Inefficiency due to too little exchange will exist more generally with asymmetric exchange. Thus, asymmetry in the quality of ideas can limit exchange of ideas.

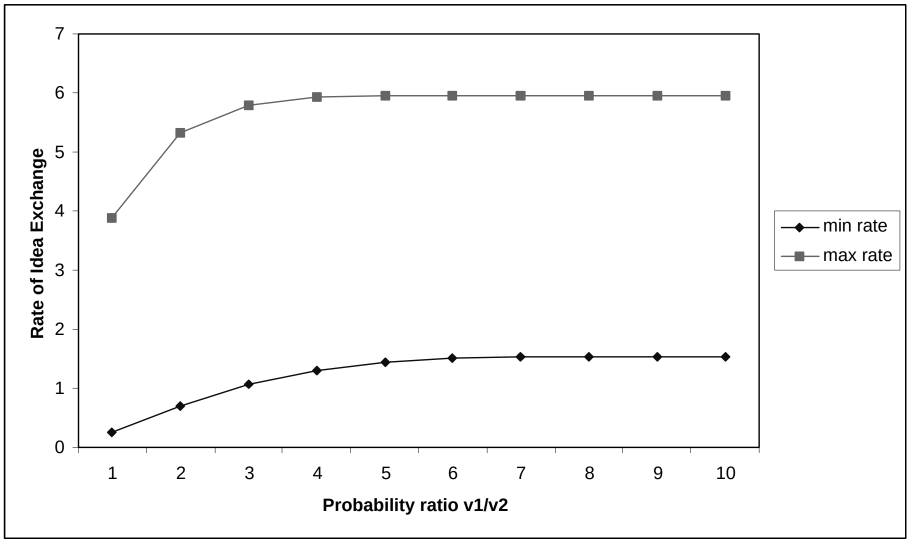

Figure 2: Ranges for rate of exchange

5 Conclusion

A framework has been presented for considering the exchange between firms of multiple private ideas, each of which is used as an input for a non-substitutable innovation by each firm. In this setting, firms choose to work on a set of the ideas, and also choose the intensity with which each idea is pursued via labor allocation. It is established that, when the intensity of effort can be chosen, that there are a set of innovation-maximizing levels of exchange for each firm and overall innovation, including full exchange. However, intense is price competition within an industry can eliminate idea exchange between firms. Also, if idea quality varies between firms, less exchange will take place than if idea quality is verifiably *ex ante* symmetric. This study suggests that the reluctance of agents to exchange ideas, as observed in many settings, is a phenomenon conditional on particular market conditions. Moreover, the lack of exchange is not particularly troubling from an efficiency point of view.

Being secretive about our research ideas may not promote collegiality. However, from the point of view of pushing the intellectual frontier of economics research forward, it does not appear detrimental.

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