Introduction to kd-trees

- Dimension of data is k (but common to say k-d tree of dimension 3 instead of 3d-tree).
- kd-trees are binary trees
- Designed to handle spatial data in a simple way
- For n points, $O(n)$ space, $O(\log n)$ height (if balanced), supports range and nearest-neighbor queries.
- Node consists of
  - Two child pointers,
  - Satellite information (such as name).
  - A key: Either a single float representing a coordinate value, or a pair of floats (representing a dimension of a rectangle)

Basic Idea Behind kd-trees

Construct a binary tree

- At each step, choose one of the coordinate as a basis of dividing the rest of the points
- For example, at the root, choose x as the basis
  - Like binary search trees, all items to the left of root will have the x-coordinate less than that of the root
  - All items to the right of the root will have the x-coordinate greater than (or equal to) that of the root
- Choose y as the basis for discrimination for the root's children
- And choose x again for the root's grandchildren

Note: Equality (corresponding to right child) is significant

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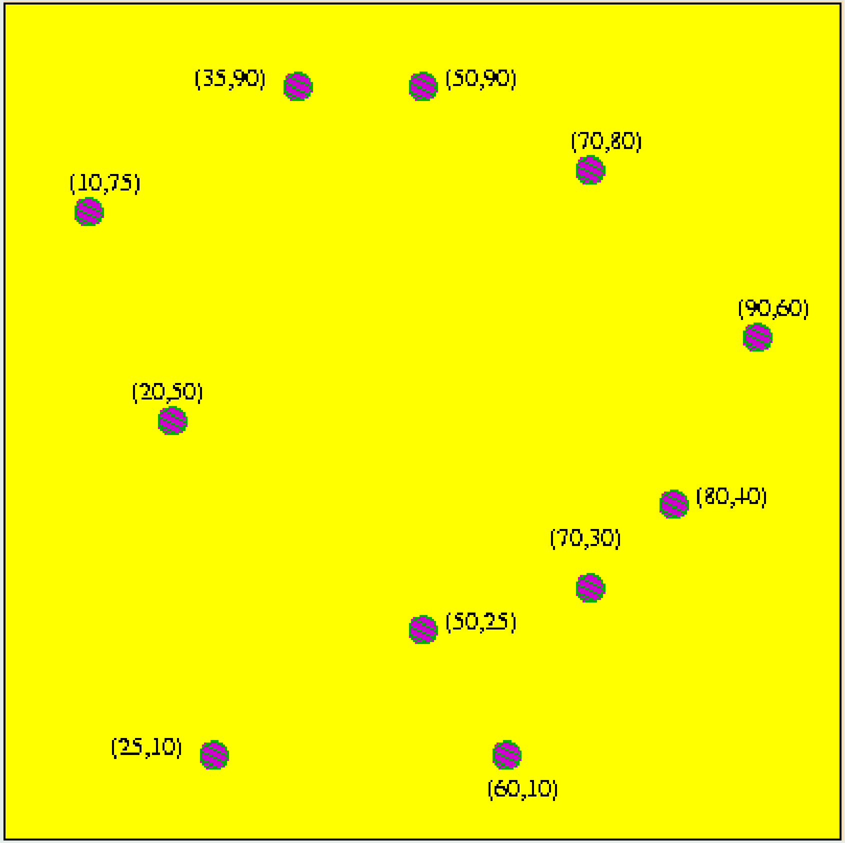
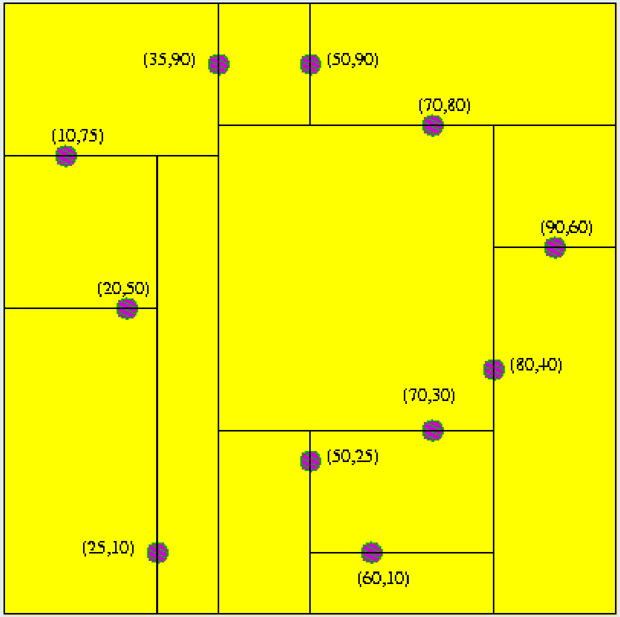
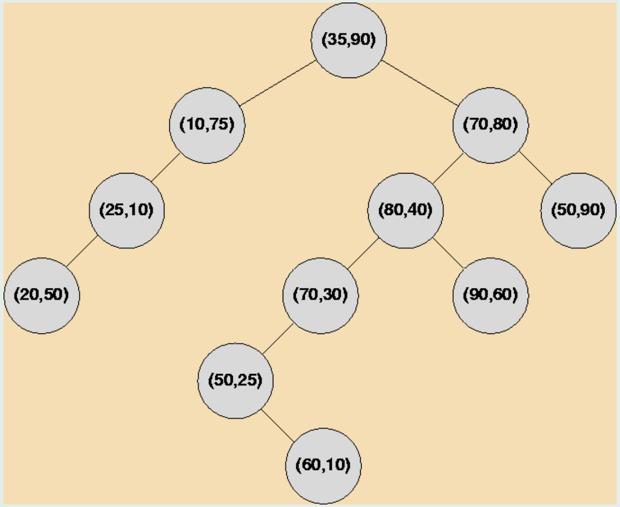
Example: Construct kd-tree Given Points

- Coordinates of points are $(35, 90)$, $(70, 80)$, $(10, 75)$ $(80, 40)$, $(50, 90)$, $(70, 30)$, $(90, 60)$, $(50, 25)$, $(25, 10)$, $(20, 50)$, and $(60, 10)$
- Points may be given one a time, or all at once.
- Data best visualized as shown below



Example: kdtree Insertion





Building: Dynamic Insertion

```
KDNode insert (point p, KDNode t, int cd) {
 if (t == null)  t = new KDNode (p);
 // sets up node.data.x and node.data.y
 else if (p == t.data)  ...  // duplicate
 else if (p.cd < t.data.cd)
   t.left = insert (p, t.left, cd+1);
 else t.right = insert (p, t.right, cd+1);
 return t;
}
```

- Initial call: `root = insert (p, root, 0);`
- Each node is associated with a rectangular region
- Tree is "balanced" if points are given in random order
- Or if all points are given in advance

Building: The Static Case

- Assume points are sorted on *both* x and y in a composite array S
- `S[x]` corresponds to a list of points sorted by x.

```
KDNode buildTree(SortedArray S, int cd) {
 if (S.empty()) return null
 else if S.singleton() return new KDNode(S[x][0], cd);
 else {
  m = median (S, cd) // median (cutting dimension)
  left = leftPoints(S, cd);    right = S - left;
  t = new KDNode(m);
  t.left = buildTree(left, cd+1);
  t.right = buildTree(right, cd+1);
  return t
 }
}
```

- $T(n) = kn + 2T(n/2)$, so the algorithm takes $O(n \log n)$ time.

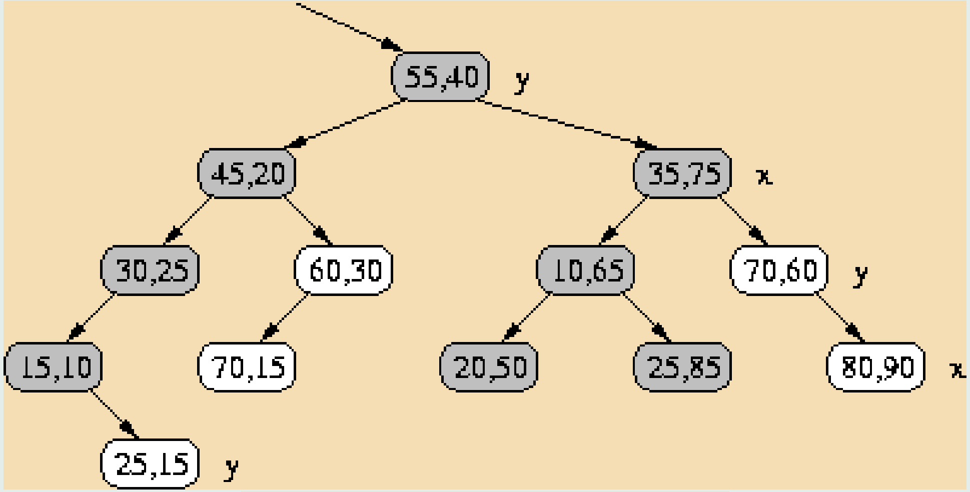
Remove Requires Finding Minimum

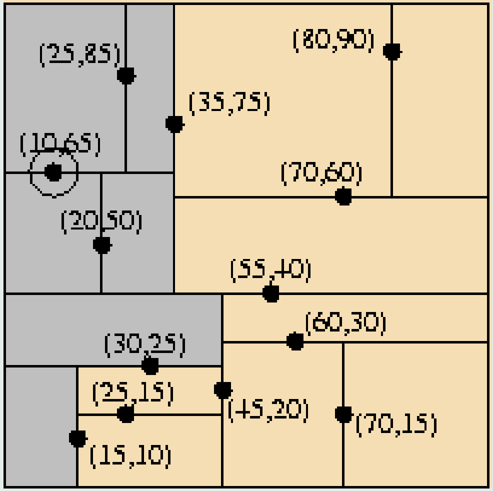
- Given a node, and a cutting dimension, find the node with minimum value (with respect to that cutting dimension)

```
Point findmin (KDNode t, int whichAxis, int cd) {
 if (t == null) return null;
 else if (whichAxis == cd)
   if (t.left == null) return t.data;
   else return findmin(t.left, whichAxis, cd+1)
 else  return
   minimum(t.data, findmin(t.left, whichAxis, cd+1),
     findmin(t.right, whichAxis, cd+1), i);
}
```

- If tree is balanced, `findmin (root)` takes no more than $O(\sqrt{n})$ time in the worst case.

Example: findmin(root, x, y)

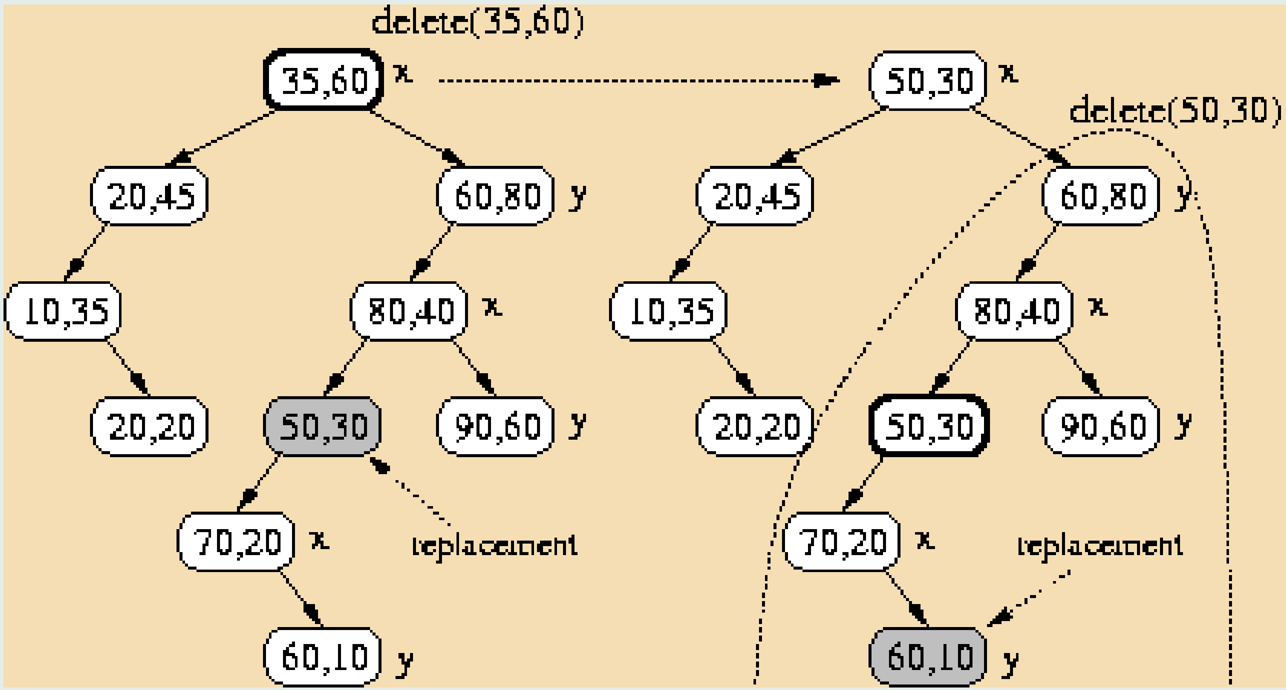




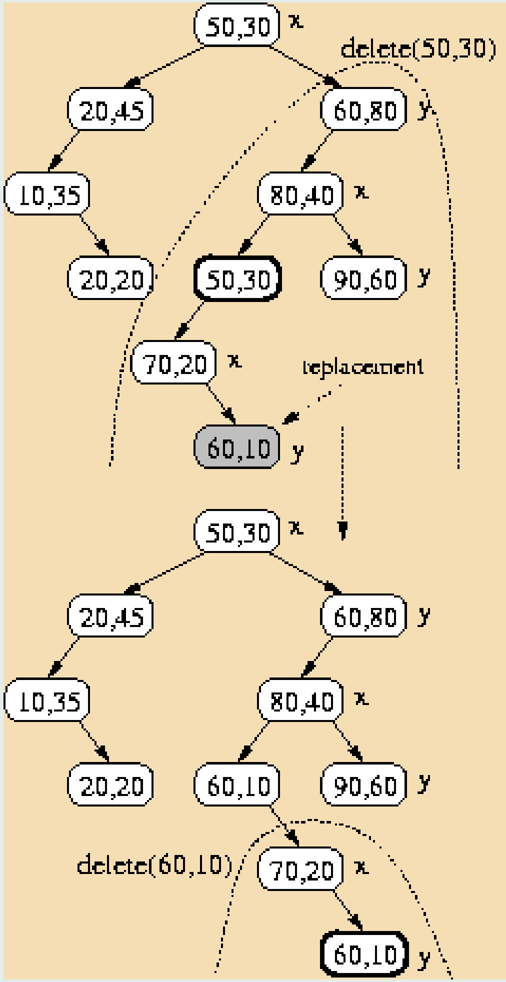
Basic Idea Behind Removing

- Want to remove point $p = (a, b)$
- First find node t which has this point
- Node t discriminates on x (say)
  - If t is a leaf node, replace it by null
  - Otherwise, find a replacement node r with coordinates (c, d)
  - Replace the data at t by (c, d). The kd-tree structure must not be violated
  - Recursively remove point $p = (c, d)$
- Finding the replacement
  - If t has a right child, use the inorder successor
  - Otherwise minimum value of the left child is appropriately used

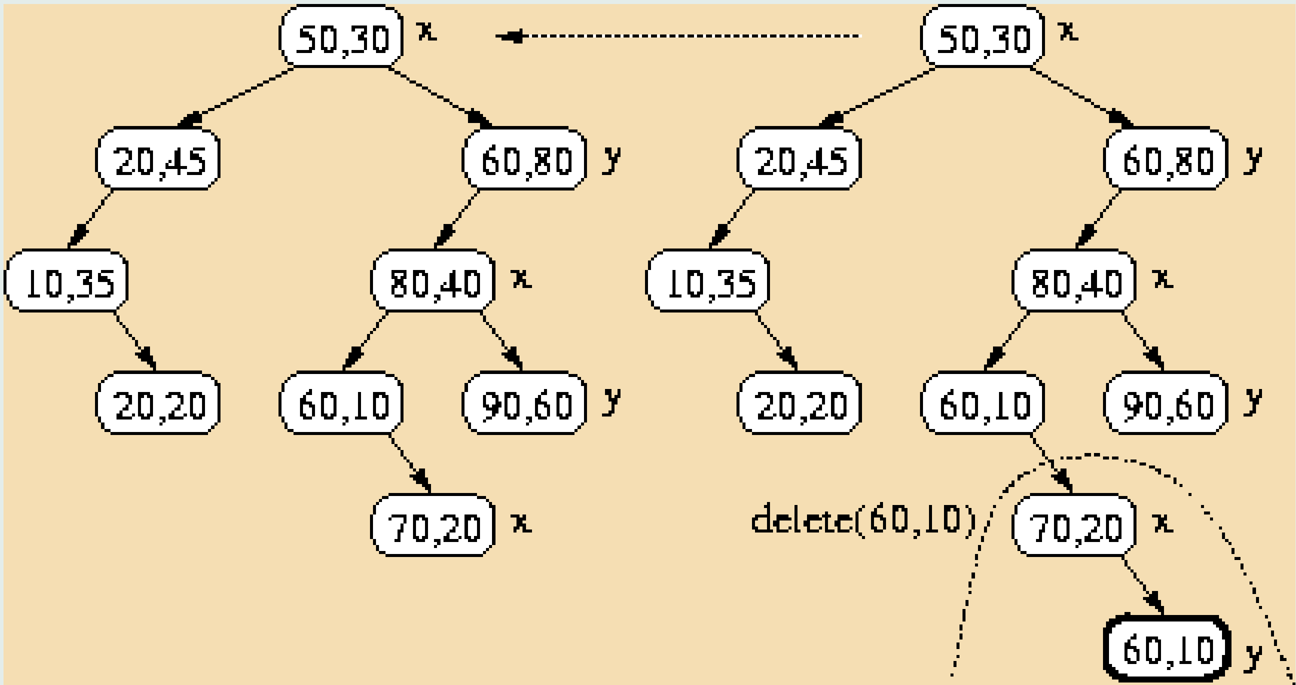
Remove Example: Delete Point At Root



Remove Example: Delete Point



Remove Example: Delete Point

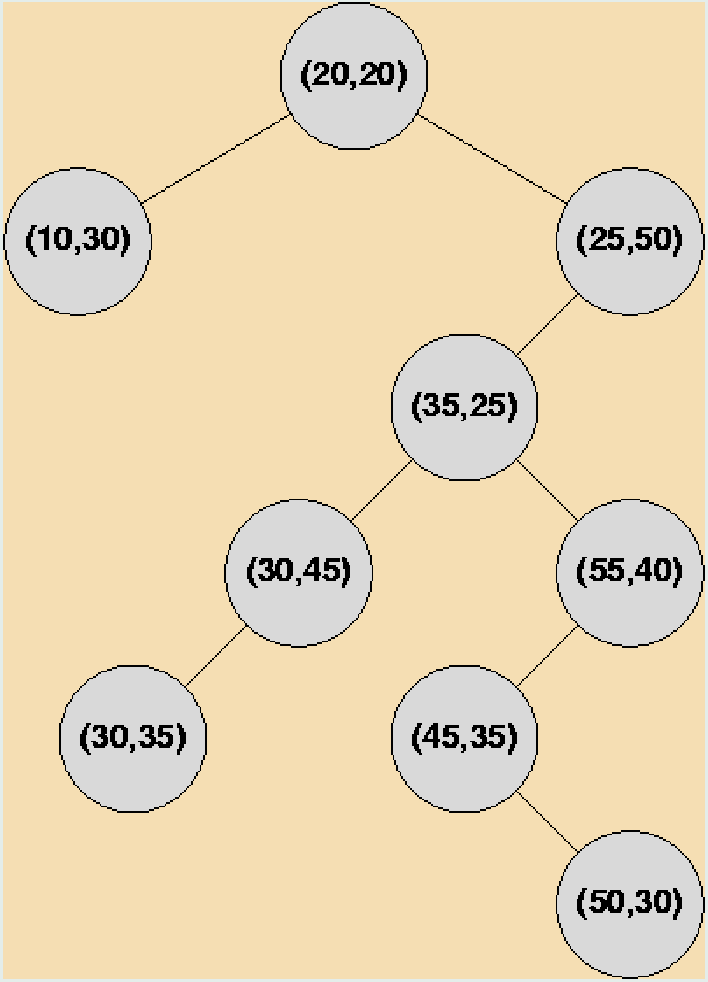


Remove Takes $O(\log n)$ Time

```
KDNode remove (KDNode t, Point p, int cd) {
 if(t == null) return null;
 else if(p.cd < t.data) t.left = remove(t.left, p, cd+1);
 else if(p.cd > t.data) t.right = remove(t.right, p, cd+1);
 else {
   if(t.right == null && t.left == null) return null;
   if(t.right != null)
        t.data = findmin(t.right, cd, cd+1);
   else {
        t.data = findmin(t.left, cd, cd+1);
        t.left = null;
   }
   t.right = remove(t.right, t.data, cd+1);
 return t;
 }}
```

We expect to delete nodes at leaf level. If tree is balanced, we expect `remove()` to take $O(\log n)$ time

Remove Example: Delete Point At Root



Remove: Solution