Paul Erdős: The Master of Collaboration

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Over a span of more than 60 years, Paul Erdős has taken the art of collaborative research in mathematics to heights never before achieved. In this brief look at his collaborative efforts, we will explore the breadth of Paul's interests, the company he has kept, and the influence of his collaboration in the mathematical community. Rather than focusing on the mathematical content of his work or the man himself, we will see what conclusions can be drawn by looking mainly at publication lists. Thus our approach will be mostly bibliographical, rather than either mathematical or biographical. The data come mainly from the bibliography in this present volume and records kept by *Mathematical Reviews* (*MR*) [13]. Additional useful sources of information include *The Hypertext Bibliography Project* (a database of articles in theoretical computer science) [11], *Zentralblatt* [16], the *Jahrbuch* [10], various necrological articles too numerous list, and personal communications. Previous articles on these topics can be found in [3,4,7,8,14].

Paul has certainly become a legend, whose fame (as well as his genius and eccentricity) has spread beyond the circles of research mathematicians. We find a popular videotape about him [2], articles in general circulation magazines [9,15] (as well as in mathematical publications—see [1] for a wonderful example), and graffiti on the Internet (e.g., his quotation that a mathematician is a device for turning coffee into theorems, on a World Wide Web page designed as a sample of the use of the html language [12]). But even within the academic (and corporate research) community, his style and output have created a lot of folklore.

The reader is probably familiar with the concept of *Erdős number*, defined inductively as follows. Paul has Erdős number 0. For each $n \geq 0$, a person not yet assigned an Erdős number who has written a joint mathematical paper with a person having Erdős number n has Erdős number $n+1$. Anyone who is not assigned an Erdős number by this process is said to have Erdős number ∞. Thus a person's Erdős number is just the distance from that person to Paul Erdős in the *collaboration graph* C (in which two authors are joined by an edge if they have published joint research—of course one need not restrict the field to mathematics). For example, Albert Einstein has Erdős number 2, since he did not collaborate with Paul Erdős, but he did publish joint research with Ernst Straus, who was one of Paul's major collaborators. Purists can argue over how to count papers with more than two authors, but here we will adopt the liberal attitude that each of the $\binom{k}{2}$ pairs of authors in k-author paper are adjacent in C.

A common variant is to give a person who has written $p > 0$ papers with Paul the Erdős number $1/p$. András Sárközy, with $\frac{1}{57}$, and András Hajnal, with $\frac{1}{54}$, seem to have the smallest positive Erdős numbers under this definition, followed in order by Faudree, Schelp, Sós, Rényi, Rousseau, Szemerédi, Turán, Graham, Burr, Spencer, Simonovits, Pomerance, Straus, Nathanson, Rado, Nicolas, Pach, Milner, Bollobás, Piranian, F. Chung, Hall, Selfridge, and Reddy, who all have a value under 0.1.

The 458 people currently known to have Erdős number 1 are listed in Figure 1. The entry "x y: n" indicates that x first published with Erdős in year 19y and to date has n joint papers with him (with or without other coauthors); n is omitted when it equals 1. About 60% of the coauthors have just one joint paper. The mean number of papers per coauthor is 3.2, with a standard deviation of 6.1.

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Figure 1. Coauthors of Paul Erdős.

The author maintains electronic lists of coauthors of Paul Erdős and coauthors of these coauthors (i.e., all people with Erdős number not exceeding 2) and updates these lists annually. He intends to make them available indefinitely via anonymous ftp [6] and on the World Wide Web [5]. (Difficulties in author identification, among other problems, surely make the data less than 100% accurate, but we believe that the number of errors is not large.)

As was pointed out in [7], the average number of authors per research article in the mathematical sciences has increased dramatically over Paul Erdős's lifetime. (One can speculate whether his existence is part of the reason for this.) Specifically, the fraction of all authored items reviewed in *MR* having two or more authors has increased, as a function of time. While over 90% of all papers 56 years ago (when *MR* began) were the work of just one mathematician, today scarcely more than half of them are solo works. In the same period, the fraction of two-author papers has risen from under 10% to about one third. Also, in 1940 there were virtually no papers with three authors, let alone four or more; now about 10% of all papers in the mathematical sciences have three or more authors, including about 2% with four or more.

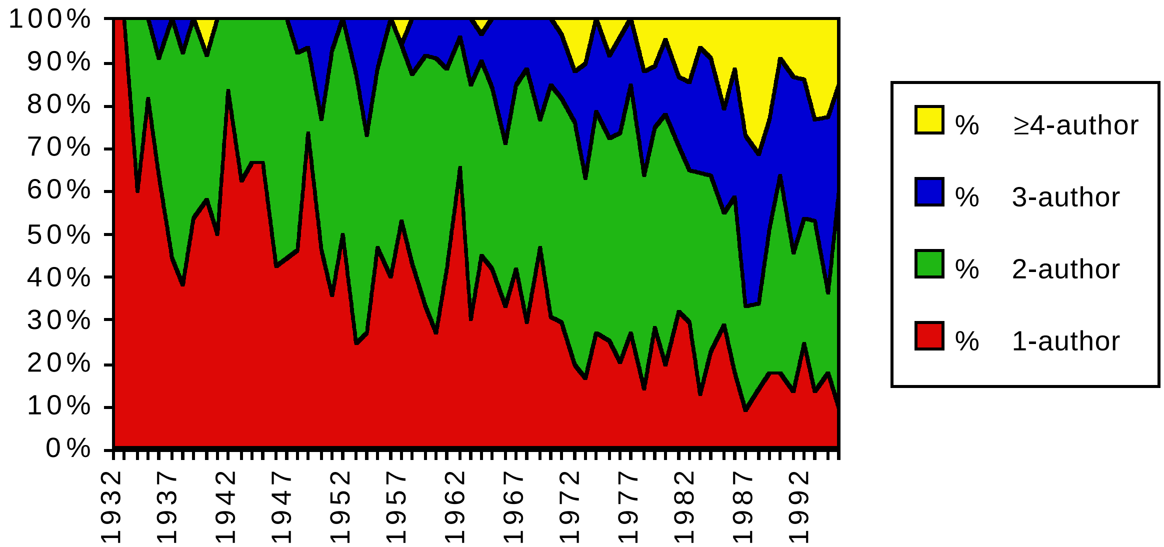

Figure 2. The number of coauthors on Paul Erdős's papers over the years.

The same trend can be seen in Paul's work, but with an even greater amount of collaboration. The graph in Figure 2 shows the fraction of 1-, 2-, 3-, and ≥4-author items in Paul's publication list, year by year. (Almost all of these are research papers. The rest are books, articles about people, or other writings.) For reference, Figure 3 shows the absolute sizes we are talking about—the number of publications year by year. (The last two figures—for 1994 and 1995—are probably too low, due to incomplete publication data.) Cumulatively (Figure 4), fewer than one third of Paul's 1400 works are solo ventures. In fact, the mean number of authors (including Erdős) is almost exactly two.

Paul's current mode of operation (dating from his departure from a permanent position at Notre Dame around 1954) is unique among mathematicians. Rather than staying at a home institution (research institute or academic department), he is constantly on the move, visiting mathematicians at conferences and research centers around the world. He often spends some summer months in Budapest, where he is a member of the Hungarian Academy of Sciences and where he can work with several of his most prolific collaborators. Some of his favorite haunts are Memphis, Tennessee, the New York City area, and

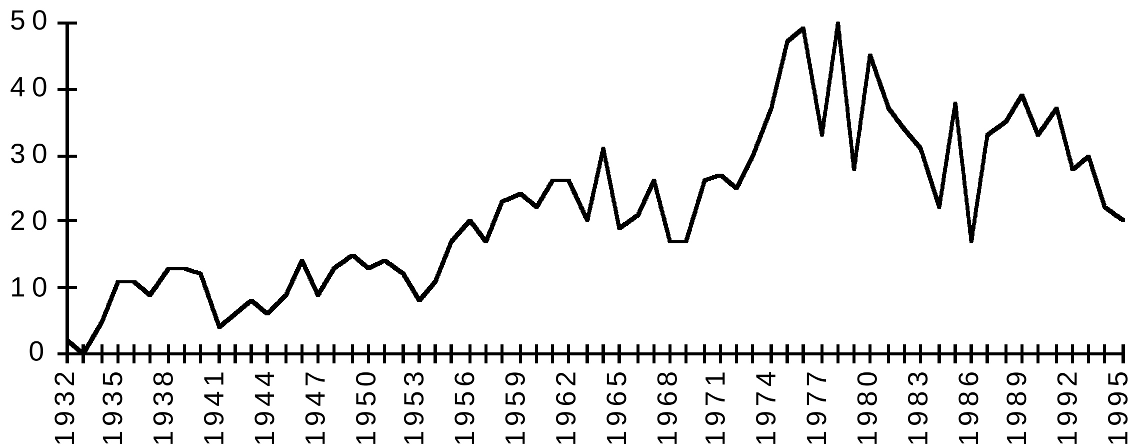

Figure 3. The number of papers by Paul Erdős over the years.

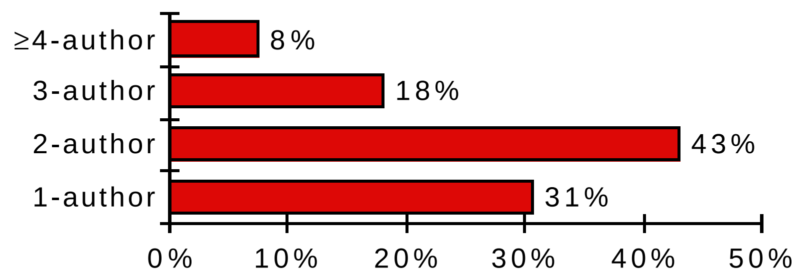

Figure 4. The fraction of Paul Erdős's papers with different numbers of authors.

other places too numerous to list. He is a permanent fixture at the annual Southeastern Combinatorics Conference (in Boca Raton, Florida, or Baton Rouge, Louisiana) and other regular meetings in his various fields. For example, in the few months around the time this article is being written, Paul reports having been (or planning to go) at least to Atlanta, Memphis, three cities in Texas, New Jersey, New Haven, Baton Rouge, Colorado, France, Germany, Kalamazoo, and Pennsylvania, in that order.

As he has met and worked with ever-increasing numbers of people on his travels, it is not surprising that Paul has added new coauthors every year since 1936. (Only two—George Szekeres and Pál Turán—published with him before that, in 1934.) Figure 5 shows how the cumulative number of coauthors has increased, while Figure 6 shows the discrete time derivative of this function. Paul usually leaves the actual writing up of the papers to his collaborators—partly, he says, because he does not type.

As the present collection shows, Paul Erdős's papers span many branches of mathematics and exploit relationships among them. (One fine example of the latter is his application of probability to combinatorics.) *Mathematical Reviews* currently has about 60 broad subject classifications, ranging from "Mathematical Logic and Foundations" through "Information and Communication, Circuits" (plus a section on history and biography), one of which it assigns to every item it records as its primary subject area. This list has varied

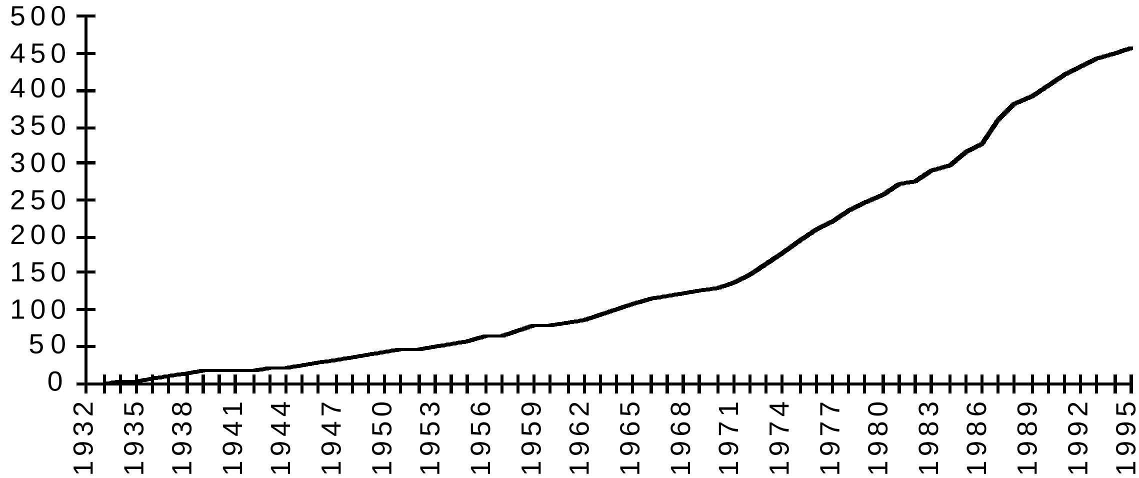

Figure 5. The cumulative number of Paul Erdős's coauthors as a function of time.

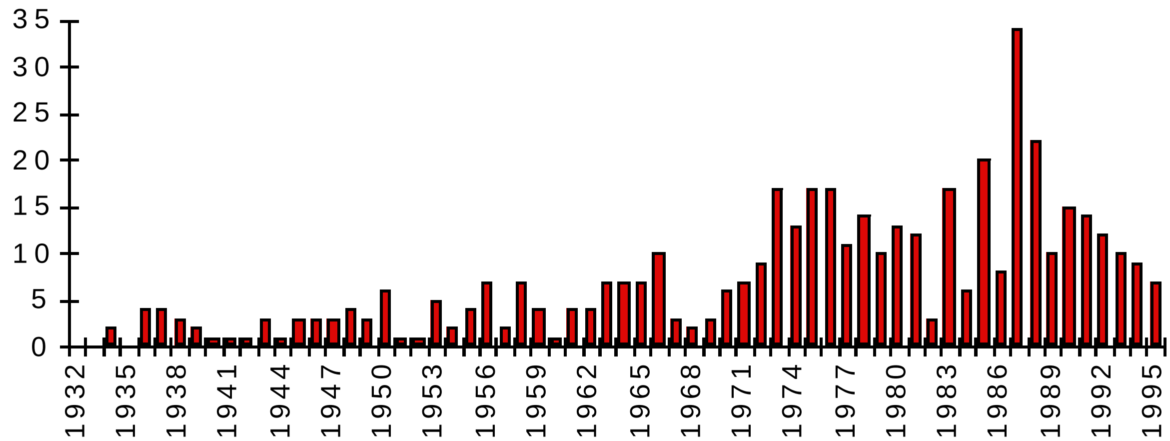

Figure 6. The number of new coauthors added each year.

slightly over time, with some categories being added or discarded as *MR* tries to keep up with current trends. Paul's works, although often spanning two or more subject areas and therefore difficult to pinpoint into one category, have been given primary classification in about 40% of these subject areas or their equivalent predecessors. They include not only the two main areas of number theory and combinatorics, and substantial work in approximation theory, geometry, set theory, and probability theory, but also papers in mathematical logic and foundations, lattices and ordered algebraic structures, linear algebra, group theory, topological groups, polynomials, measure and integration, functions of a complex variable, finite differences and functional equations, sequences and series, Fourier analysis, functional analysis, general and algebraic topology, statistics, numerical analysis, computer science, and information theory.

The chart in Figure 7 shows the fractions of Paul's publications reviewed in *MR* since

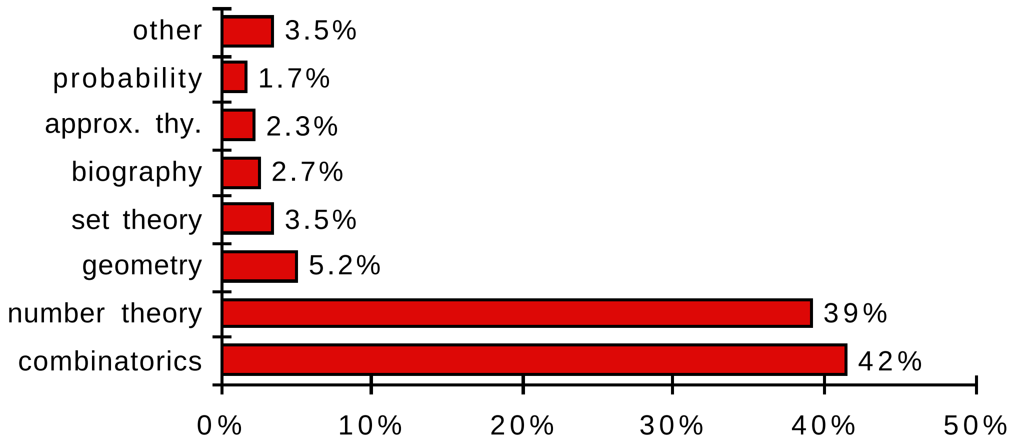

Figure 7. Paul Erdős's papers since 1979 by broad category.

1980 in the various categories. Such a tabulation is easy to do, since the *MR* review number includes the category code as the first two digits following the colon. Figure 8 is a less accurate pie chart covering all years. It can be seen that there has been a slight trend toward increased work in combinatorics (including graph theory, of course) in the past 15 years, with a comparable decline in the output in number theory. Indeed, nearly all his early papers were in number theory (61 of the first 64, by one count, covering the period 1932–1939).

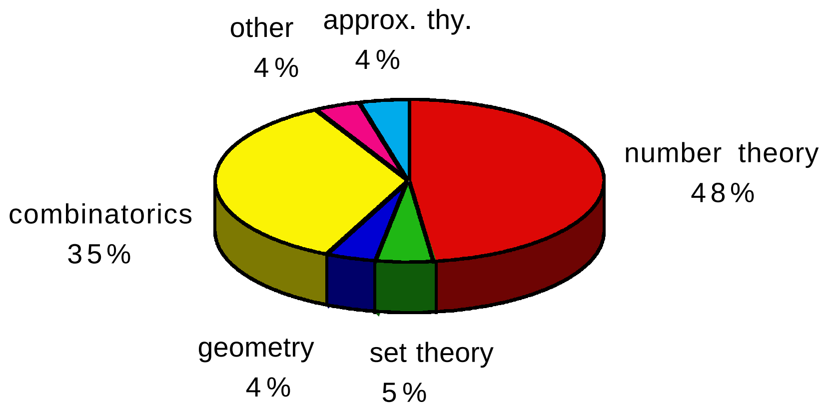

Figure 8. Paul Erdős's papers by broad category (approximate).

Since Erdős's coauthors work in such varied fields, one would expect the set of people with Erdős number 2, 3, or a little higher to range over essentially all of mathematics. Indeed, this is the case. All Fields Medalists over the past three cycles (1986–1994) are in the Erdős component of the collaboration graph, with Erdős number at most 9. This group includes people working in theoretical physics; for instance, there is the path Edward

Witten—Chiara Nappi—Robert Israel—Robert Phelps—David Preiss—Paul Erdős. Thus one can conjecture that many (if not most) physicists are also in the Erdős component, as are, therefore, many (or most) scientists in general. The large number of applications of graph theory and statistics to the social sciences might also lead one to suspect that many researchers in other academic areas are included as well.

It is interesting to explore the publication lists (or at least the coauthor lists) of Erdős's coauthors, to see how much collaboration goes on after Paul has left town. Let E_1 be the subgraph of C induced by people with Erdős number 1. According to data collected through 1995, E_1 contains 458 vertices and 1218 edges; thus an average Erdős coauthor collaborated with over 5 other Erdős coauthors. (The median, as opposed to the mean, of this statistic is only 3, however. Its standard deviation is about 6, and it takes values over 30 in four cases—Ron Graham, Frank Harary, Vojtěch Rödl, and Joel Spencer). There are only 40 isolated vertices in E_1 (less than 9%), and three components with two vertices each. The remaining 412 vertices in E_1 induce a connected subgraph. Paul's style seems to rub off.

Looking at it more broadly, we find that people with Erdős number 1 have a mean of 20 other collaborators (median 15, standard deviation 22), and only six of them collaborated with no one except Erdős. Five of them have over 100 coauthors (Frank Harary, Saharon Shelah, Ron Graham, Noga Alon, and Dan Kleitman).

Another 4546 people have felt Paul's influence second-hand—by doing joint research with one of the honored 458. Three quarters of the people with Erdős number 2 have only one coauthor with Erdős number 1 (i.e., each such person has a unique path of length 2 to Erdős in C). However, their mean number of Erdős number 1 coauthors is 1.5 (standard deviation 1.2), and the count ranges as high as 13 (for Dwight Duffus and Linda Lesniak).

The accompanying bibliography lists about 1400 papers, and it is probably incomplete, especially with regard to recent works. Most of the papers published since 1939 appear in the *Mathematical Reviews* database. Reviews of Paul Erdős's papers appear in every volume that *MR* has published, including a review of a joint paper with Tibor Gallai on page 1 of volume 1, written by George Pólya. It is interesting to note that the second most prolific writer in the *MR* database is Leonard Carlitz, with about 735 items. Carlitz has Erdős number 2 (via seven different coauthors) and has written the *MR* review of several of Paul's papers. In all, nearly 500 different people have reviewed Erdős's papers for *MR*. Paul writes for *MR* as well and to date has over 700 reviews to his credit.

Readers with additions or corrections to any of the information in this article, the accompanying bibliography, or the current coauthor lists are urged to communicate with the author.

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