Summary of Atomic Quantum Numbers

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For Hydrogen Atoms

When we ignored the spin of the electron, we came up with three quantum numbers to describe the energy eignestates of a hydrogen atom: n, l, m_l.

Quant. #	Name	Possible Values	What it Represents	Formula
n	principle q. n.	$n = 1, 2, 3, ...$	energy, total number of "nodes"	$E = E_1/n^2$
l	orbital q. n.	$l = 0, 1, 2, ..., n-1$	orbital angular momentum, angular "nodes"	$L = \hbar\sqrt{l(l+1)}$
m_l	magnetic q. n.	$m_l = 0, \pm1, \pm2, ..., \pm l$	z-component of orb. ang. mom., "nodes" in ϕ	$L_z = \hbar m_l$

When we realized that electrons had a spin described by the quantum number $s = 1/2$, we realized that we needed one more quantum number to describe the z-component of the electron's spin angular momentum: m_s

Quant. #	Possible Values	What it Represents	Formula
m_s	$m_s = \pm 1/2$	z-component of spin angular momentum	$S_z = \hbar m_s$

When we realized that the spin of the electron would interact with the orbital angular momentum, we decided that m_l and m_s were no longer good quantum numbers to describe the energy eigenstates. So we came up with two new ones to replace them: j and m_j.

Quant. #	Possible Values	What it Represents	Formula
j	$j = \lvert l-s\rvert, \lvert l-s\rvert+1, ..., l+s$	total angular momentum	$J = \hbar\sqrt{j(j+1)}$
m_j	$m_j = 0, \pm1, \pm2, ..., \pm j$	z-component of total angular momentum	$J_z = \hbar m_j$

For Multi-Electron Atoms

For multi-electron atoms we first added together the spin angular momentum of all of the electrons to get a total spin quantum number **S**. Then we added together the orbital angular momentum for each electron to get a total orbital quantum number **L**. Then we put those two together to get a total angular momentum quantum number **J**. Of course each of these has an associated z-component quantum number: $\mathbf{M_S}$, $\mathbf{M_L}$, and $\mathbf{M_J}$.

Quant. #	Possible Values	Formula
S	Integer steps from $\mathbf{S_{min}} = 1/2-1/2+1/2-1/2+...$ to $\mathbf{S_{max}} = 1/2+1/2+1/2+1/2+...$ with as many 1/2s as there are unpaired electrons.	$S = \hbar\sqrt{\mathbf{S}(\mathbf{S}+1)}$
L	Integer steps from $\mathbf{L_{min}} = l_1 \pm l_2 \pm l_3 \pm ...$ to $\mathbf{L_{max}} = l_1 + l_2 + l_3 + ...$with as many ls as there are unpaired electrons.	$L = \hbar\sqrt{\mathbf{L}(\mathbf{L}+1)}$
J	$\mathbf{J} = \lvert\mathbf{L}-\mathbf{S}\rvert, \lvert\mathbf{L}-\mathbf{S}\rvert+1, ..., \mathbf{L}+\mathbf{S}$	$J = \hbar\sqrt{\mathbf{J}(\mathbf{J}+1)}$
$\mathbf{M_J}$	$\mathbf{M_J} = 0, \pm1, \pm2, ..., \pm\mathbf{J}$	$J_z = \hbar\mathbf{M_J}$