

Aero-propulsive modelling for climb and descent trajectory prediction of transport aircraft using genetic algorithms

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ABSTRACT

In this study, a new aero-propulsive model (APM) was derived from the flight manual data of a transport aircraft using Genetic Algorithms (GAs) to perform accurate trajectory predictions. This new GA-based APM provided several improvements to the existing models. The use of GAs enhanced the accuracy of both propulsive and aerodynamic modelling. The effect of compressible drag rise above the critical Mach number, which was not included in previous models, was considered along with the effects of compressibility and profile camber in the aerodynamic model. Consideration of the thrust dependency with respect to Mach number and the altitude in the propulsive model expression was observed to be a more practical approach. The proposed GA model successfully predicted the trajectory for the descent phase, as well, which was not possible in previous models. Close agreement was observed when comparing the time to climb and time to descent values obtained from the model with the flight manual data.

NOMENCLATURE

A	aspect ratio
a_p, b_i	propulsive model coefficients ($i = 1$ to 3)
	climb velocity model coefficients ($i = 10$)
	descent velocity model coefficients ($i = 12$)
a_p, b_p, c_i	aerodynamic model coefficients ($i = 4$ to 8)
$a_p, b_p, c_p, d_p, e_p, f_p, g_i$	climb altitude model coefficients ($i = 9$)
a_p, b_p, c_p, d_i	descent altitude model coefficients ($i = 11$)
C_D	drag coefficient
C_{Dmin}	minimum drag coefficient
C_{D0}	zero-lift drag coefficient
C_L	aircraft lift coefficient
C_{Lmin}	minimum aircraft lift coefficient
C_{lc}, C_{li}	aerofoil lift coefficient for compressible and incompressible flows
CF	climb factor
D	drag
DF	descent factor
e	Oswald efficiency factor
g	gravitational acceleration
h	flight altitude
K, K_i	induced drag coefficients ($i = 1$ to 4)
L	lift
M	Mach number
q	dynamic pressure
S	wing area
T	thrust
t	time
t/c	thickness-to-chord ratio
V	airspeed
W	aircraft weight
β	Prandtl-Glauert compressibility correction factor
β_t	throttle setting parameter
κ_a	aerofoil technology factor
Γ	compressibility parameter
Λ	sweep angle at quarter chord

1.0 INTRODUCTION

Accurate Aero-Propulsive Models (APMs) are essential in predicting trajectories for Air Traffic Management (ATM) applications, especially as decision support tools⁽¹⁾. Planning of air traffic flow, which is based on a precise APM, will provide seamless and effective management of the forecasted growth of air traffic, as well as increased safety and capacity⁽²⁾. There are notably few studies related to APMs for trajectory prediction in the literature. The most commonly known ones are Eurocontrol's Base of Aircraft Data (BADA) model and the Global Aircraft Modelling Environment (GAME) model. The accuracy of both models is very limited in achieving convenient trajectory estimations. For operational in-flight data production, aircraft manufacturers have

developed some computer programs and databases, such as INFLT/REPORT, Boeing's aircraft performance software. However, using aircraft manufacturers' models directly in trajectory prediction is not convenient, as mentioned by Suchkov *et al.* The possible problems include the dimensions of the database and the computational speed for obtaining the flight profiles. Above all, aircraft manufacturers are reluctant to provide their data to air traffic authorities and airline companies for trajectory prediction modelling and improvement of air traffic management systems. This reluctance means that usage of aircraft and engine manufacturers' data for trajectory prediction will create problems related to legal issues and Intellectual Property (IP) Rights. For this reason, the flight manual of an aircraft provides the best available resource for deriving an APM.

There is an APM that was proposed by Gong and Chan⁽³⁾ found in the literature and is used in the prediction of aircraft trajectories for the Center/TRACON Automation System (CTAS). INFLT was used in the derivation of this model. The application of this model was carried out on Boeing 737-400 and Learjet 60 to predict trajectories, but only for the climbing condition. Cavcar and Cavcar⁽¹⁾ proposed another APM derived using flight manual data and applied their model to obtain only time-to-climb values for Boeing 737-400. Compressibility and wing camber effects were included in both of these models, but because these models utilised a second degree drag polar, they are not applicable in considering the effects of compressible drag above the critical Mach number. This was also denoted by Gong and Chan and they stated that additional research into their aerodynamic model equation might offer improvements to their model^(1,3). Gong and Chan's propulsive model also needs improvement because it is not an empirical formula incorporating the effects of altitude and Mach number; only an engine scaling factor was used in their model⁽³⁾. Cavcar and Cavcar proposed an empirical propulsive model formula, but the propulsive model that simultaneously includes the effects of both altitude and Mach number and was derived using a genetic algorithm (GA) in this paper was observed to be more accurate and practical.

This paper describes a new GA based APM that can be derived from flight manual data and will enable more accurate climb as well as descent trajectory predictions. The use of a stochastic method, such as GAs, allowed a more accurate estimation of time to climb and time to descend values. The originality of this study lies in the fact that this is the first non-conventional trajectory prediction model in the literature and that trajectory predictions for the descent phase can also be obtained using this model, unlike previous models. Furthermore, as an improvement to the existing models, in this study, the aerodynamic modelling took into account the effects of compressible drag above the critical Mach number, which was not available in the previous models, as well as the effects of compressibility and profile camber. This newly proposed GA model will be useful in real world applications, especially decision support functions, such as conflict detection, direct routing and arrival metering. Additionally, congestion, delays, economic losses in air transportation and detrimental effects on the environment due to excessive fuel consumption will be minimised by the use of this accurate APM in the ATM system.

2.0 GENETIC ALGORITHMS

Because of their stochastic search technique, GAs provide one of the best approaches for deriving optimal models for highly non-linear, uncertain and multimodal spaces. The speed at which they can contrive a near-optimal solution and their ability to search the space for globally optimal solutions instead of getting stuck on local optima have caused GAs to be widely accepted among optimisation techniques. GAs were first introduced by John Holland in 1975⁽⁴⁾. GAs mimic the metaphor of natural biological selection and evolution to generate new solution alternatives. The creation of new sets of approximations is achieved at each generation through the selection of

individuals according to their fitness value in the domain and then breeding them together using genetic operators, which leads to the evolution of superior populations⁽⁵⁾.

An initial set of random solutions, called a population, constitutes the starting phase of genetic algorithms⁽⁶⁾. The building blocks of GAs are called genes⁽⁴⁾. Each individual in the population, which is called a chromosome (a sequence of genes), represents a potential solution to the problem being considered⁽⁷⁾. A chromosome is a string of symbols; it is usually, but not necessarily, a binary bit string⁽⁶⁾. By applying a combination of operators such as selection, crossover, mutation, etc. to the parent chromosomes at every generation, new offspring chromosomes are created⁽⁴⁾. Through the use of some measure of fitness, the chromosomes are evaluated during each generation⁽⁶⁾. The best possible solution obtained is not kept in standard GAs, which brings the risk of losing the best possible obtainable solution. However, by copying the best member of each generation into the next one, the elitism approach overcomes this problem⁽⁸⁾.

A common practice is to terminate the GA after a pre-determined number of generations (number of iterations) and then test the quality of the best members of the population against the objective function to reach a globally near-optimal fit for complicated non-linear models^(4,5).

3.0 CLIMB AND DESCENT TRAJECTORY PREDICTION

The rate of climb (or descent) of a turbofan aircraft can be expressed as follows:

$$\frac{dh}{dt} = \frac{T - C_D(C_L)qS}{W} \frac{V}{X} \quad \dots (1)$$

Here, C_D is a function of C_L . X can be CF or DF depending on the climb or descent flight phase. CF and DF are given as:

$$CF = 1 + \frac{V}{g} \frac{dV}{dh} \quad \dots (2(a))$$

$$DF = -1 + \frac{V}{g} \frac{dV}{dh} \quad \dots (2(b))$$

C_L can be written as:

$$C_L = \frac{W}{qS} \sqrt{1 - \left(\frac{1}{V} \frac{dh}{dt}\right)^2} \quad \dots (3)$$

For a known aircraft whose rate of climb data can be found in the flight manual but whose aerodynamic characteristics are unknown, Equation (1), (2(a)) and (3) can be utilised to estimate the $C_D(C_L)$ drag polar in combination with a proper propulsive model. Deriving a drag polar model that represents the aerodynamics of the aircraft can be used to determine the climb trajectory of the aircraft because the rate of climb is notably dependent on the aerodynamic drag polar. It should be noted that the $C_D(C_L)$ function is also dependent on the Mach number when compressibility effects should be taken into account at high Mach numbers. Likewise, for a known aircraft whose rate of descent data are obtained from the flight manual and $C_D(C_L)$ drag polar is derived from climb trajectory data, Equation (1), (2(b)) and (3) can be used with the propulsive model to predict the descent trajectory of the aircraft⁽¹⁾.

4.0 DEVELOPMENT OF GA-BASED APM

4.1 Propulsive model

The propulsive modelling in all of the previous APMs needs to be improved in terms of accuracy and the parameters included in the models. For instance, considering the user manual of BADA current version 3.11⁽⁹⁾, the variation of thrust with altitude and temperature are considered, but the effect of Mach number is not included. This corresponds to a single thrust value at a given altitude, which is far from providing an accurate propulsive modelling approach. Gong and Chan's propulsive model⁽³⁾ only used an engine scaling parameter to modify the thrust values of an existing engine model, which was also not adequate to achieve an accurate propulsive model. Although the propulsive model by Cavcar and Cavcar⁽¹⁾ is an empirical formula that incorporates the sea level static maximum take-off thrust of the engine, relative density, relative temperature, throttle setting and Mach number, it is not as accurate as the new GA propulsive model in this study. The new GA model developed in this paper simultaneously considers the engine thrust dependency with respect to Mach number and altitude in the same model expression. Using a stochastic method, such as GAs, prevents the model from getting stuck at local optima and provides more accurate results compared to Cavcar and Cavcar's propulsive model, which was derived using a classical method such as the least squares method. Combined with the aerodynamic modelling, the superiority of the GA propulsive model is clear from the obtained trajectory prediction results. Moreover, considering the number of parameters involved, this new propulsive model was observed to be more practical than Cavcar and Cavcar's model.

Thrust, altitude and Mach number data of the Pratt & Whitney JT9D-7A turbofan engine⁽¹⁰⁾ for the climb flight phase were utilised in deriving the GA propulsive model. The objective function for the propulsive model was defined as the mean squared error (MSE) between the actual thrust values of JT9D-7A and the thrust values predicted by the GA model. The coefficients in the model formulation were defined as chromosomes in the GA. Considering the precision of the engine data, the substring length was chosen as 20 bits for each coefficient so that chromosomes with 120 genes were utilised. Gray encodings, which enable GAs to converge to optimal solutions faster, were preferred over binary encodings. A population size of 20,000 was defined and an iteration number of 20,000 was chosen as the termination criterion for the GA. The model coefficients corresponding to the best fitness value were taken as the optimum solution after 50 runs of the algorithm. Tournament selection, two point crossover and elitism were applied during the GA. The crossover and mutation rates were chosen to be 0.85 and 0.008, i.e., 1/chromosome length, respectively.

The proposed propulsive model expression can be stated as follows:

For $i = 1$ to 3,

$$T = f_1 + f_2 M + f_3 M^2$$

$$\dots (4)$$

$$f_i = a_i + b_i h$$

The model coefficients with the best fitness value (minimum MSE) were found to be $a_1 = 0.3931$, $b_1 = -0.7426$, $a_2 = -0.3928$, $b_2 = 0.93$, $a_3 = 0.2489$ and $b_3 = -0.5873$. It should be noted that, here, T and h are in lb and ft, both of which were normalised by multiplying by 10^{-5} . The comparison of the thrust values calculated by the GA model, T_{model} , and the actual thrust values, T_{actual} , for the JT9D-7A turbofan engine is shown in Fig. 1. As seen from Fig. 1, there is good agreement between the actual and model-predicted thrust values.

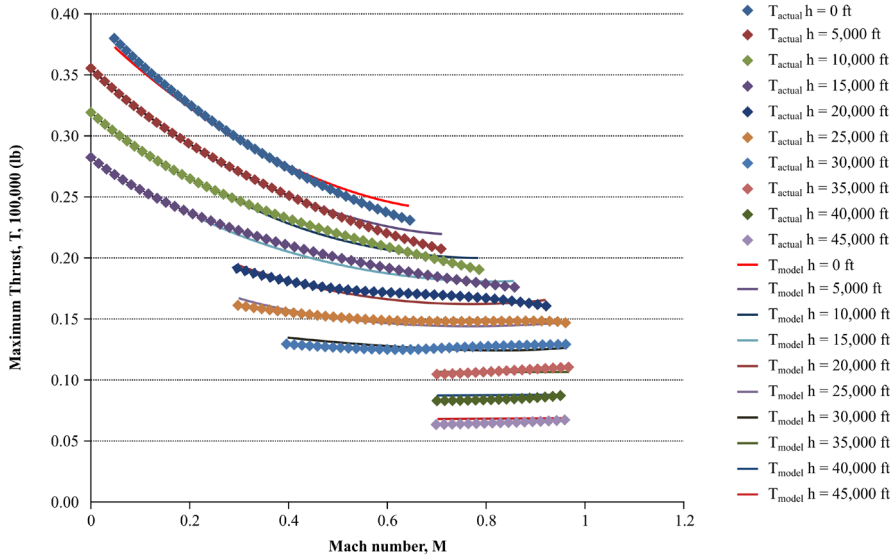


Figure 1. Actual and GAAPM thrust values for JT9D-7A.

4.2 Aerodynamic model

The aerodynamic modelling approaches applied in the previous APMs were not entirely adequate in incorporating the various effects, as discussed in this section. The quadratic drag polar model expression proposed in this study was observed to be more accurate and successful in trajectory prediction compared to previous ones.

For modern transport aircraft, a modified asymmetrical form of the drag polar approximation, called the simple cambered wing drag polar (SCDP), is often used to include the compressibility effect due to the high subsonic flying speeds and the profile camber effect resulting from cambered supercritical wings:

$$C_D = C_{D_{min}} + K(C_L - C_{L_{min}})^2 \quad \dots (5)$$

Because both $C_{D_{min}}$ and K are dependent on Mach number (along with Reynolds number and aircraft shape), a different curve is obtained for each Mach number.

The older versions of BADA (for instance, BADA 2.4) took into account the compressibility effects⁽¹⁾, but the current version BADA 3.11 ignores the compressibility and profile camber effects⁽⁹⁾.

The aerodynamic model derived by Gong and Chan⁽³⁾ considered both of these effects by incorporating the Prandtl-Glauert compressibility correction factor:

$$\beta = \sqrt{1 - M^2} \quad \dots (6)$$

into the simple cambered wing drag polar (SCDP)

$$K = 1/(\pi A e \beta) \quad \dots (7)$$

However, as stated by McCormick, the Prandtl-Glauert compressibility correction factor is only applicable for Mach numbers that are less than critical. Additionally, in some cases, the ratio Cl_c/Cl_i is overestimated or underestimated due to different aerofoil geometries. As discussed by Cavcar and Cavcar⁽¹⁾, accurate fits to the published aerodynamic data values cannot be obtained using SCDP with Prandtl-Glauert compressibility factor, especially for higher Mach numbers. Consequently, the aerodynamic model proposed by Gong and Chan cannot provide accurate results for higher Mach numbers.

The Cambered Wing Compressible Drag Polar (CCDP) given by Cavcar and Cavcar's aerodynamic model can be stated as

$$C_D = C_{D0} - K_1 C_L + K_2 C_L^2 \quad \dots (8)$$

where C_{D0} , K_1 and K_2 are functions of Mach number and polynomial functions of a compressibility parameter that is dependent on a reference Mach number at which the compressibility effects on the pressure coefficient become significant.

The aerodynamic model proposed by Cavcar and Cavcar considered both of the above mentioned effects. However, it should be emphasised that none of the existing models in the literature incorporated the effects of compressible drag above the critical Mach number except the new GA aerodynamic model developed in this study.

Because modern transport aircraft cruise in transonic flow regimes, generally close to the drag divergence Mach number, the effects of compressible drag above the critical Mach number should be included in the aerodynamic model and the resulting drag polar must be in the following form⁽¹¹⁾:

$$C_D = C_{D_{\min}} + K(C_L - C_{L_{\min}})^2 + \Delta C_{Dc} \quad \dots (9)$$

where the induced drag coefficient and compressible drag rise are dependent on Mach number. Incorporating Grasmeyer's modified version of Lock's equation for a transonic strut-braced wing concept, the drag rise expression can be given as:

$$\begin{aligned} \Delta C_{Dc} = & 20\Gamma^4 + (8\Gamma^3 / \cos^3 \Lambda) C_L \\ & + (1 \cdot 2\Gamma^2 / \cos^6 \Lambda) C_L^2 + (0 \cdot 08\Gamma / \cos^9 \Lambda) C_L^3 \\ & + (0 \cdot 002 / \cos^{12} \Lambda) C_L^4 \end{aligned} \quad \dots (10)$$

where:

$$\Gamma = M + 0 \cdot 1 - \kappa_a / \cos \Lambda + (t_i / c) \cos^2 \Lambda \quad \dots (11)$$

Thus, this drag rise expression leads to a quadratic drag polar for Mach numbers above critical⁽¹¹⁾:

$$C_D = C_{D0c} - K_{1c} C_L + K_{2c} C_L^2 + K_{3c} C_L^3 + K_{4c} C_L^4 \quad \dots (12)$$

To include the effects of compressible drag above the critical Mach number, a quadratic drag

polar (called the Cambered Wing Compressible Drag Rise Included Drag Polar, CCDRDP) of the following form was introduced in this study:

For $i = 4$ to 8,

$$C_D = f_4 + f_5 C_L + f_6 C_L^2 + f_7 C_L^3 + f_8 C_L^4 \dots (13)$$

$$f_i = a_i + b_i M + c_i M^2$$

Here, f_i are functions of the Mach number for a given aircraft design.

5.0 APPLICATION OF THE GA APM MODEL

The methodology of the GA APM model was applied to a transport category medium weight aircraft whose time-to-climb and time-to-descent data were obtained from its flight manual⁽¹²⁾. In deriving the APM and conducting trajectory predictions for the climb and descent flight phases, the actual and predicted time-to-climb and time-to-descent values were compared and an error analysis was performed. It should be noted that, as in the case of the propulsive model, all the other GA models used in developing this new APM, namely, aerodynamic, altitude and velocity models, were also performed for a population size of 20,000 with an iteration size of 20,000 in 50 runs. The specifications of the considered aircraft are given in Table 1⁽¹⁾.

Table 1
Specifications of the considered transport category medium weight aircraft⁽¹⁾

Aircraft Characteristics	
Wing area	91.04m ²
Wing sweepback at quarter-chord	25°
Average wing thickness/chord ratio	12.89%
Maximum take-off weight	68,038kg
Maximum zero-fuel weight	53,070kg
Engines	2 × CFM56-3-B1

The aircraft considered uses CFM56-3-B1⁽¹³⁾ high bypass ratio turbofan engines. Because data on the variation of thrust with respect to Mach number at various altitudes cannot be obtained from the engine manufacturer for this engine as discussed before, the propulsive model derived for the JT9D-7A turbofan engine was used by normalising a) the sea level maximum static take-off thrust with the CFM56-3-B1's sea level maximum static take-off thrust and b) the thrust value at 0.8 Mach number at an altitude of 35,000ft with the CFM56-3-B1's corresponding value.

5.1 Climb trajectory prediction

Time-to-climb data are provided for the recommended climb speed schedule, 250/280 KCAS/0.74M, in the flight manual. Considering the International Standard Atmosphere (ISA) conditions in the calculations, the aerodynamic model coefficients were computed by including all the climb data given for eleven initial aircraft weights.

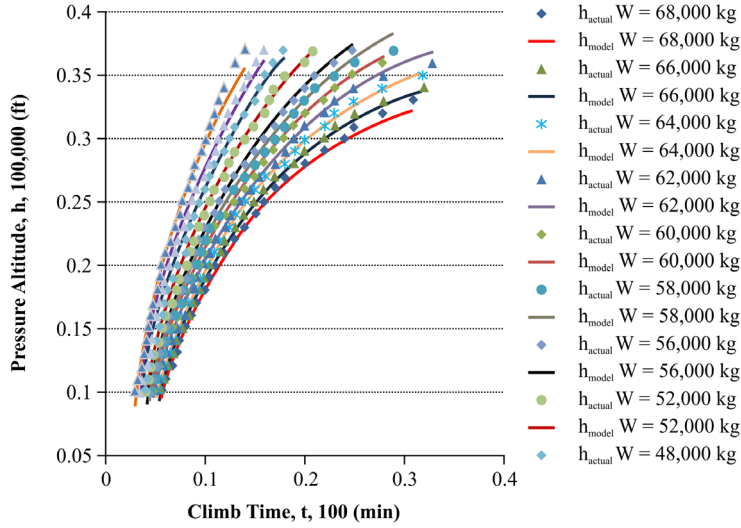


Figure 2. Actual and GAAPM altitude versus time-to-climb values.

Initially, an altitude model representing the climb trajectory was derived using the GA method. The expression for the altitude model is a function of aircraft weight and time-to-climb, which can be given in the following form:

$$h = a_g + b_g W + c_g t + d_g W t + e_g t^2 + f_g W t^2 + g_g \ln(t) \quad \dots (14)$$

where the model coefficients for the best fitness value were found as follows: $a_g = 0.7244$, $b_g = -0.3029$, $c_g = 0.7507$, $d_g = -0.9301$, $e_g = 0.0786$, $f_g = -1$ and $g_g = 0.1496$. Here, h , t and W are in ft, min and kg and are normalised by multiplying by the factors of 10^{-5} , 10^{-2} and 10^{-5} , respectively. The comparison of actual altitude values (h_{actual}) and the altitude values calculated by the model (h_{model}) versus time-to-climb is demonstrated in Fig. 2 for different initial aircraft weights.

The rate of climb model was achieved by taking the time derivative of the climb trajectory altitude model:

$$dh/dt = c_g + d_g W + 2e_g t + 2f_g W t + g_g (1/t) \quad \dots (15)$$

According to Equation (2(a)), a velocity (true airspeed (TAS)) GA model for the climb trajectory in this climb schedule was required to calculate CF. This velocity model was expressed in the following form:

$$V = a_{10} + b_{10} h \quad \dots (16)$$

Considering the best fitness value, the velocity GA model coefficients were achieved at two altitude ranges: $a_{10} = 0.1354$ and $b_{10} = 0.0956$ for $0.3048 < h < 0.8839$ and $a_{10} = 0.2537$ and $b_{10} = -0.0321$ for $0.9144 < h < 1.0973$. It should be noted that h and V are in m and ms^{-1} and are normalised by

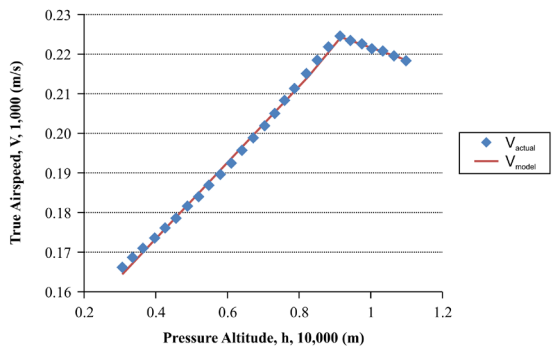


Figure 3. Actual and GAAPM velocity (TAS) versus altitude values.

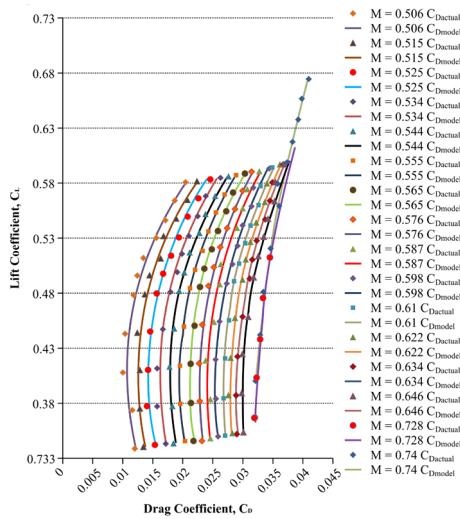


Figure 4. Calculated and GAAPM CD values.

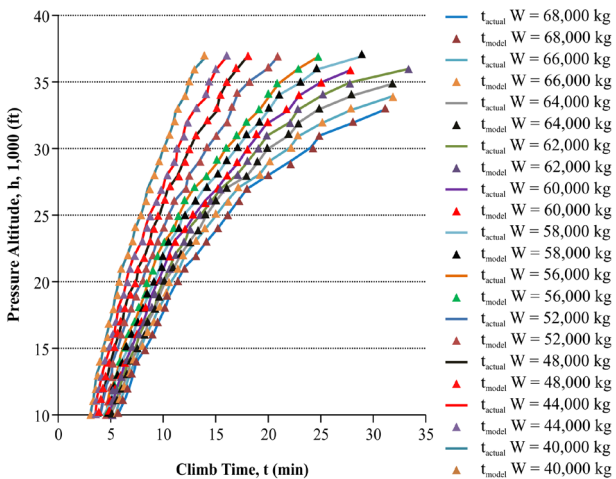


Figure 5. Actual and GAAPM climb trajectories.

multiplying by 10^{-5} and 10^{-3} , respectively. The actual velocity values and the values predicted by the GA model with regards to the considered climb schedule and trajectory were compared and are given in Fig. 3.

The C_L versus C_D variations were computed using Equation (1) using the rate of climb model values from Equation (15). Twenty-one different C_D (C_L) curves were obtained for Mach numbers ranging from 0.50 to 0.74. In developing the GA model expressed by Equation (13), the coefficients of CCDRDP were calculated as $a_4 = -0.0848$, $b_4 = 0.4790$, $c_4 = -0.4433$, $a_5 = -0.4759$, $b_5 = 0.7271$, $c_5 = -0.0299$, $a_6 = 0.0962$, $b_6 = -0.0913$, $c_6 = -0.3987$, $a_7 = 0.6011$, $b_7 = -0.6772$, $c_7 = 0.4258$, $a_8 = 0.1991$, $b_8 = -0.7880$ and $c_8 = 0.4110$. The comparison of actual and GA model C_D values is given in Fig. 4. As observed from this figure, good agreement between values was achieved.

The time-to-climb values defining the climb trajectory were then calculated using the derived GA aerodynamic model combined with the GA propulsive model normalised for CFM56-3-B1 turbofan engines. A comparison was performed between the time-to-climb values obtained from the GA APM and actual time-to-climb values. The results obtained for different initial aircraft weights are shown in Fig. 5.

5.2 Descent trajectory prediction

Time-to-descend data are provided for the recommended descent speed schedule, 0.74 M/250 KCAS and are independent of the aircraft weight in the flight manual. Aerodynamic model coefficients were computed for these data considering International Standard Atmosphere (ISA) conditions.

An altitude model representing the descent trajectory was derived using the GA method. The expression for the descent trajectory altitude model is a function of time-to-descend and has the following form:

$$h = a_{11} + b_{11}t + c_{11}t^2 + d_{11} \ln(t) \quad \dots (17)$$

Here, the altitude model coefficients for the minimum MSE were obtained as $a_{11} = 0.2093$, $b_{11} = 1$, $c_{11} = 1$ and $d_{11} = 0.0903$. It should be stated that h and t are in ft and min and are normalised by multiplying by the factors of 10^{-5} and 10^{-2} , respectively. The comparison of the actual altitude values

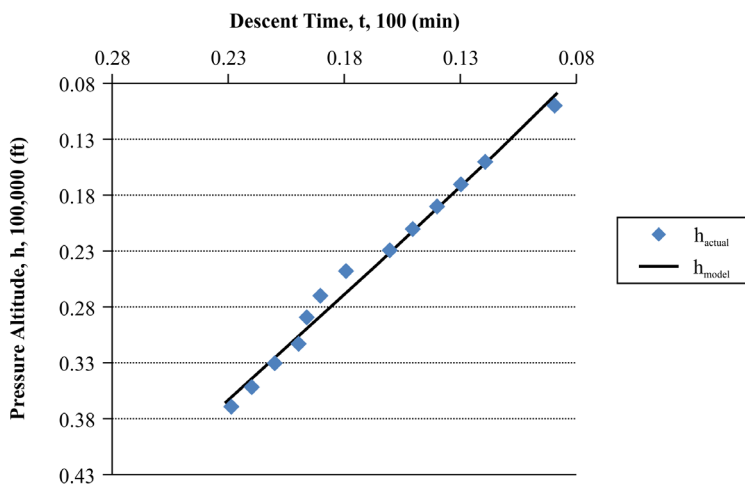


Figure 6. Actual and GAAPM altitude versus time-to-descend values.

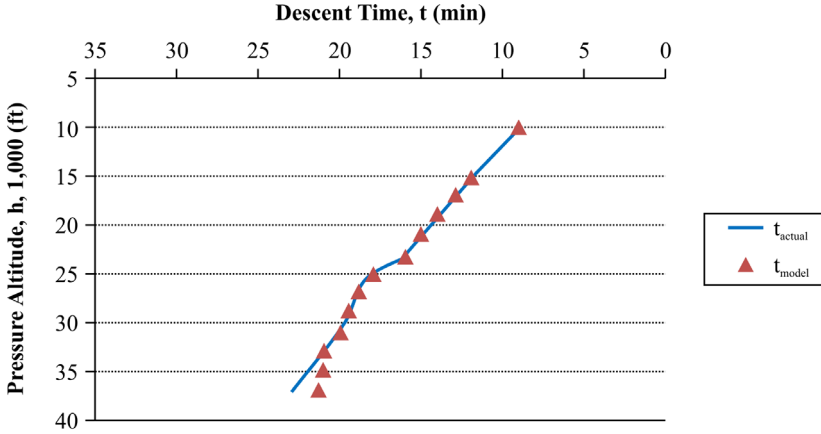


Figure 7. Actual and GAAPM descent trajectories.

(h_{actual}) and altitude values predicted by the model (h_{model}) versus time-to-descent is shown in Fig. 6. By taking the time derivative of the descent trajectory altitude model, the rate of descent model was found to be:

$$dh/dt = b_{11} + 2c_{11}t + d_{11}(1/t) \quad \dots (18)$$

A velocity (TAS) GA model for the descent trajectory was completed to calculate DF, as given in Equation (2(b)). The expression for this velocity model is the same given as in Equation (16):

$$V = a_{12} + b_{12}h \quad \dots (19)$$

Considering the best fitness obtained, the velocity GA model coefficients were determined for two altitude ranges: $a_{12} = 0.1178$ and $b_{12} = 0.0922$ for $0.3048 < h < 1.0058$ and $a_{12} = 0.2384$ and $b_{12} = -0.0178$ for $1.0668 < h < 1.1278$. Here, h and V are in m and ms^{-1} and are normalised by multiplication with 10^{-5} and 10^{-3} , respectively.

Time-to-descent values were computed using the normalised GA propulsive model combined with the GA aerodynamic model. When computing the climb trajectory, the throttle setting parameter was taken as $\beta_t = 1$ in the thrust model, whereas for the descent trajectory, descending β_t values were supposed as the altitude decreased. The comparison between the actual and APM time-to-descent values are demonstrated in Fig. 7.

6.0 ERROR ANALYSIS AND COMPARISON WITH PREVIOUS APMS

The errors between the actual values and the values obtained by the propulsive (T), aerodynamic (C_D), altitude (h) and velocity (V) models and the errors between the actual and output values obtained from the models for time-to-climb, time-to-descent and dh/dt were determined in terms of MSE. The corresponding results are shown in Table 2 along with the numbers and intervals of the data.

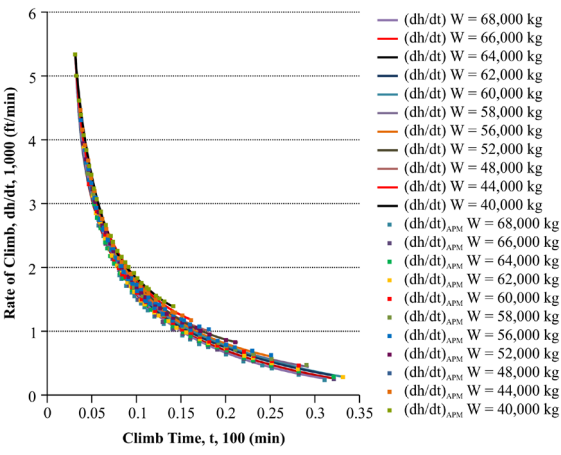


Figure 8. Actual and GAAPM rate of climb values.

Table 2
Errors, intervals and number of data points for models and outputs

Model/Output	MSE	Number of data	Interval of data
T	$122,674.958 \text{ lb}^2$	842	6,491-37,980 lb
C_D	2.133×10^{-8}	297	0.0104-0.0441
h (climb)	$222,545.065 \text{ ft}^2$	297	10,000-37,000 ft
V (climb)	$0.509 (\text{ms}^{-1})^2$	27	$166.055\text{-}224.343 \text{ ms}^{-1}$
h (descent)	$724,232.656 \text{ ft}^2$	13	10,000-37,000 ft
V (descent)	$1.451 (\text{ms}^{-1})^2$	13	$148.532\text{-}219.431 \text{ ms}^{-1}$
Time-to-climb	0.005 min^2	297	3-33 min
Time-to-descent	0.326 min^2	13	9-23 min
dh/dt (climb)	$87.884 (\text{ft/min})^2$	297	227-945-5346-043 ft/min
dh/dt (descent)	$5.014 (\text{ft/min})^2$	13	1849-912-2183.129 ft/min

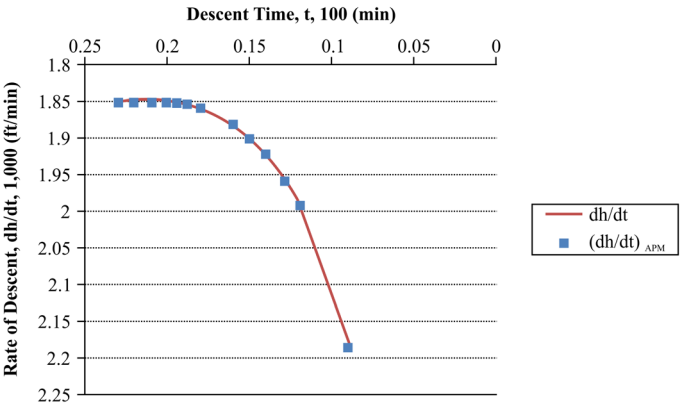


Figure 9. Actual and GAAPM rates of descent values.

The error in time-to-climb was found to be in the range of -0.995% and 0.975% , whereas the MSE was calculated as 0.005min^2 for the 297 data points of time values varying between 3 and 33min. The mean absolute percentage error in time-to-climb was found to be 0.41% , while the mean error obtained was 0.011min . The mean error obtained was -0.017min (corresponding to a mean absolute error of approximately 0.52%) for the climb trajectory prediction of a Learjet 60 using Gong and Chan's⁽³⁾ model. It should be noted that their time-to-climb prediction for a Boeing 737-400 was found to be more accurate because Boeing's INFLT data were used in that prediction; however, the use of manufacturers' data is generally impractical, as mentioned before. The time-to-climb error in the considered Boeing 737-400 climb trajectory was calculated in the range of 4.19% to -5.27% for CCDP and 3.44% to -5.51% for SCDP, according to Cavcar and Cavcar's⁽¹⁾ model. They also reported the erroneous climb trajectory resulting from the application of the BADA model. Another criterion used in evaluating the accuracy of APMs is the comparison between the actual rate of climb values used for generating the APM, dh/dt and the rate of climb values calculated from APM, $(dh/dt)_{\text{APM}}$. This comparison is shown in Fig. 8. The error in dh/dt was found to be in the range of -2.895% to 2.758% , while the MSE obtained was $87.884(\text{ft/min})^2$ for 297 dh/dt values ranging from 227.945 to 5346.043ft/min.

The error between the actual and model predicted time-to-descend values was found to be 7.85% and 4.4% above the crossover altitude, which is the altitude at which a given calibrated speed is equal to a given Mach number⁽¹⁴⁾, namely, for altitudes of 37,000ft and 35,000ft, respectively, for this descent trajectory, whereas the error range was found to be between -0.89% and 0.815% below the crossover altitude. The MSE for time-to-descend was found to be 0.326min^2 for the 13 data points of time values varying between 9 and 23mins. The mean absolute percentage error in time-to-descend was obtained as 1.212% , while the mean error was calculated to be 0.215min . It should be stated here that because previous APMs were not applicable in the descent trajectory prediction, it is not possible to make a comparison. The error between the actual rate of descent values, dh/dt and the rate of descent values obtained from the GA APM, $(dh/dt)_{\text{APM}}$, was found to be within the range of -0.272% and 0.144% , while the MSE was found to be $5.014(\text{ft/min})^2$ for the 13 data points for dh/dt ranging between 1,849.912 and 2,183.129ft/min. The comparison between dh/dt and $(dh/dt)_{\text{APM}}$ is given in Fig. 9. As a result, it is apparent that the proposed APM gives accurate and satisfactory results for the entire climb trajectory and above the crossover altitude in the descent trajectory.

7.0 CONCLUSIONS

The use of an accurate APM is vital for trajectory predictions of transport aircraft in an ATM system. In this study, a new APM was developed based on GAs to determine the climb and descent trajectories. The novelty of this study lies in several aspects. Firstly, this GA APM model is capable of accomplishing the descent trajectory in addition to the climb trajectory, which was not possible with previous APMs. Secondly, the aerodynamic modelling in this new APM includes the effects of compressible drag above the critical Mach number, which was not considered in the previous APMs, employing a quadratic aerodynamic drag polar (CCDRDP) instead of known parabolic drag polars. This approach enhanced the accuracy of the trajectory predictions obtained. Thirdly, incorporation of the engine thrust dependency with respect to two parameters, namely, Mach number and altitude, in the propulsive modelling expression was observed to be more practical and suitable for real world applications compared to the previous propulsive models. Fourthly, this model is the first non-conventional APM in the literature. The use of GAs improved the accuracy of the developed APM, rather than preferring conventional methods, as in the previous models.

In comparing the actual and predicted climb and descent trajectories, the percentage error in both time-to-climb and time-to-descent was detected to be less than 1% for all altitude values above 10,000ft in the climb phase and below the crossover altitude in the descent phase.

This new non-conventional APM will be beneficial in real world applications in ATM by aiding decision support functions, such as conflict detection, direct routing and arrival metering. The more accurate trajectory predictions obtained using this APM will enable the minimisation of time delays and economic losses in air transportation, as well as the reduction of fuel consumption and related detrimental environmental effects.

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