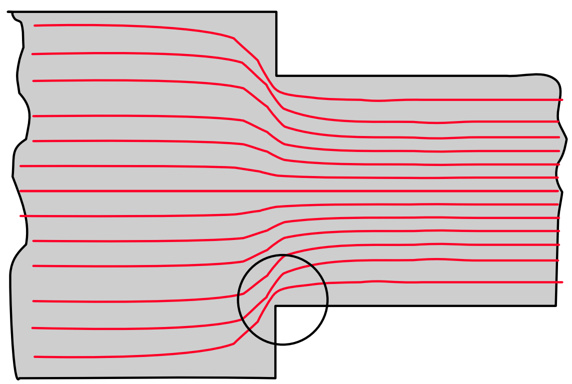
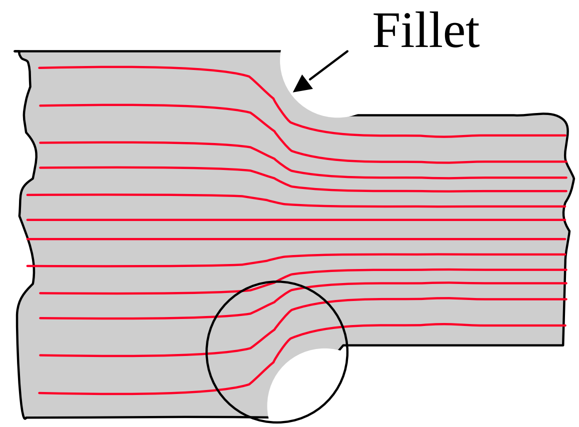
Stress concentration

- Transition of cross sections → High stresses
- More abrupt transition → Higher stresses

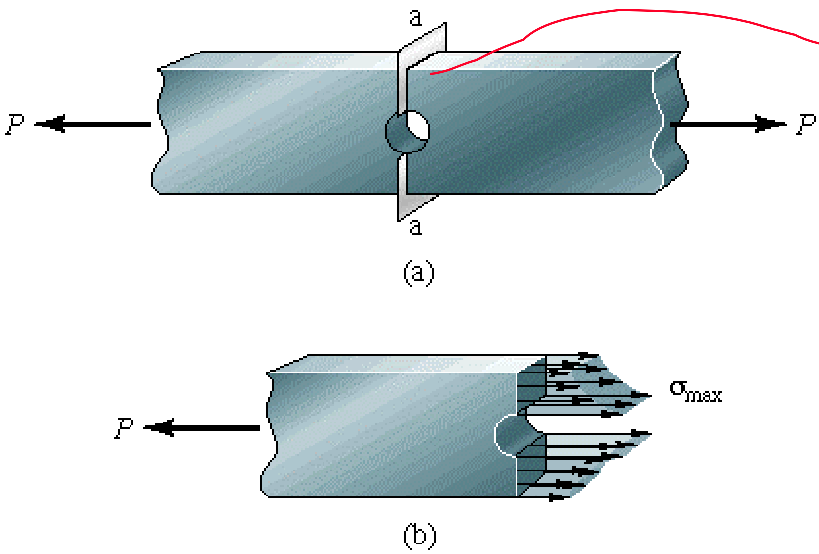


Abrupt change
Stress "flow lines" crowd
High stress concentration



Smoother change
"Flow lines" less crowded
Lower stress concentration

- Elementary stress equations don't apply in stress concentrations.



$$s_{avg} = \frac{P}{A} = \frac{P}{(b-d)h}$$

Figure 6.1 Rectangular plate with hole subjected to axial load.

(a) Plate with cross-sectional plane.

(b) Half of plate with stress distribution.

$$\boxed{s_{\max} = K_c s_{avg}}$$

Stress concentration factor
from charts

Text Reference: Figure 6.1, page 221

Stress concentration factor
K_c

- Obtained experimentally, analytically, etc
- Published in charts
- *Geometric* property
- Very important in ***brittle*** materials
- In ductile materials:
  - Important in ***fatigue*** calculation.
  - Important if safety is critical.
  - Localized yielding hardens material (strain hardening).
  - Redistributes stress concentration.

Stress Concentrations for Plate with Fillet

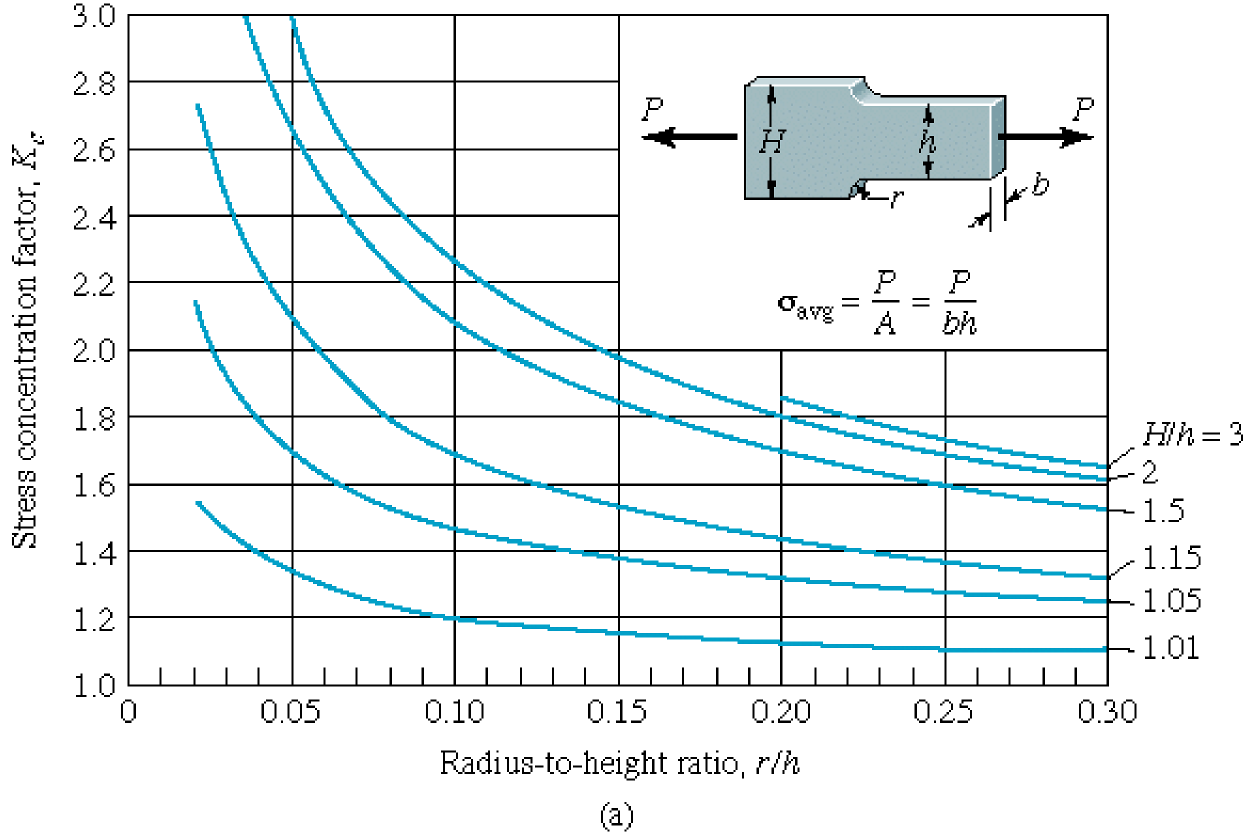
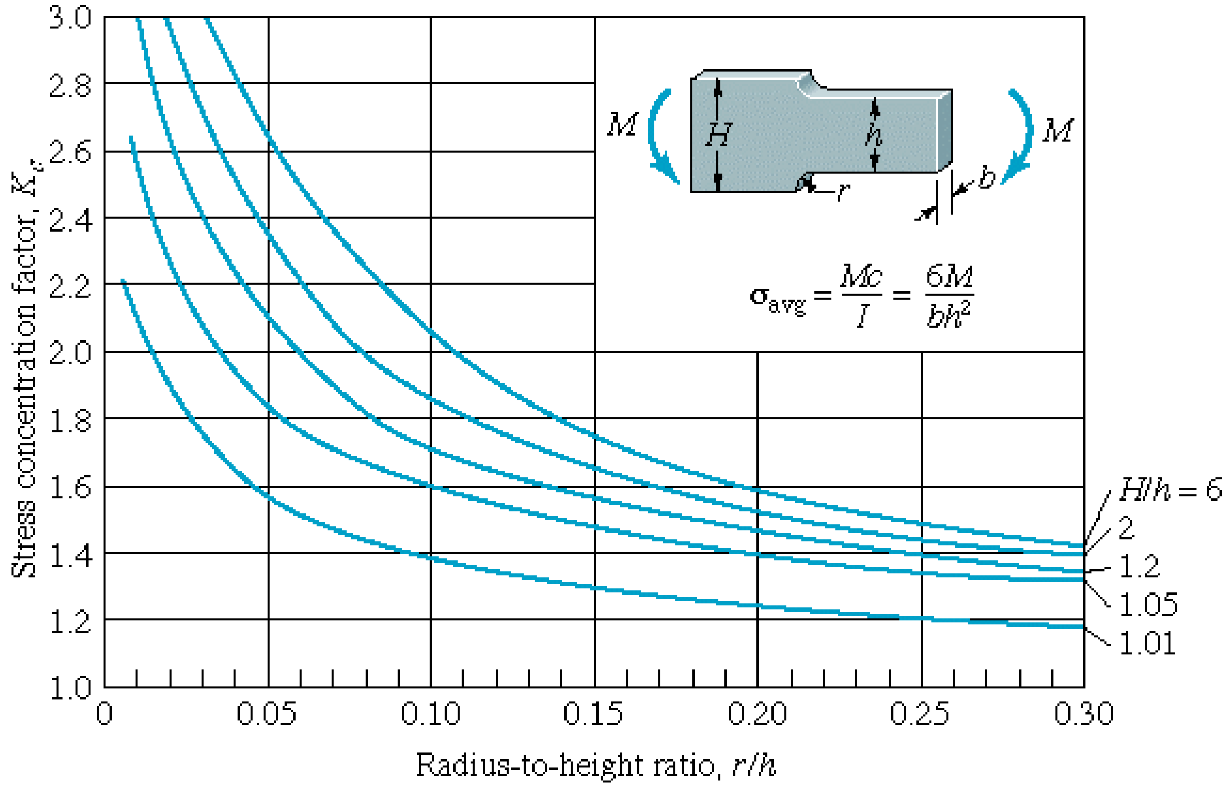


Figure 6.3 Stress concentration factor for rectangular plate with fillet. (a) Axial Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.3, page 223

Stress Concentrations for Plate with Fillet (cont.)

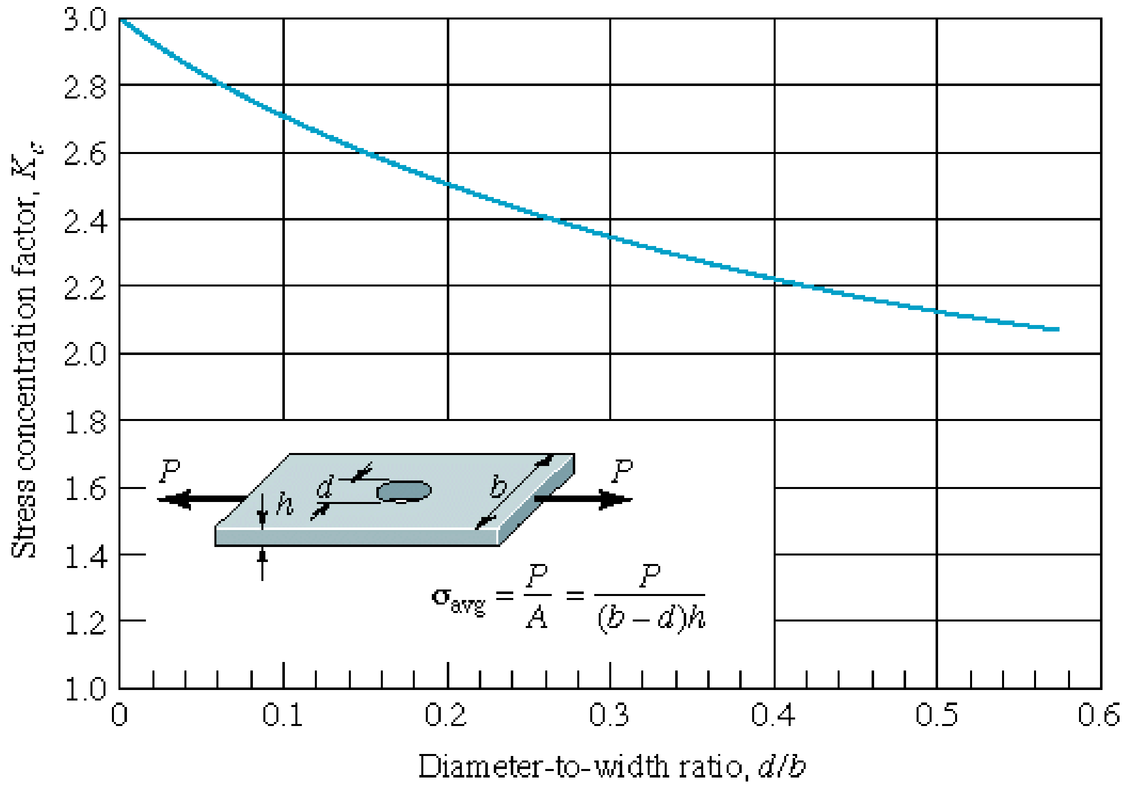


(b)

Figure 6.3 Stress concentration factor for rectangular plate with fillet. (b) Bending Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.3, page 223

Stress Concentrations for Plate with Hole

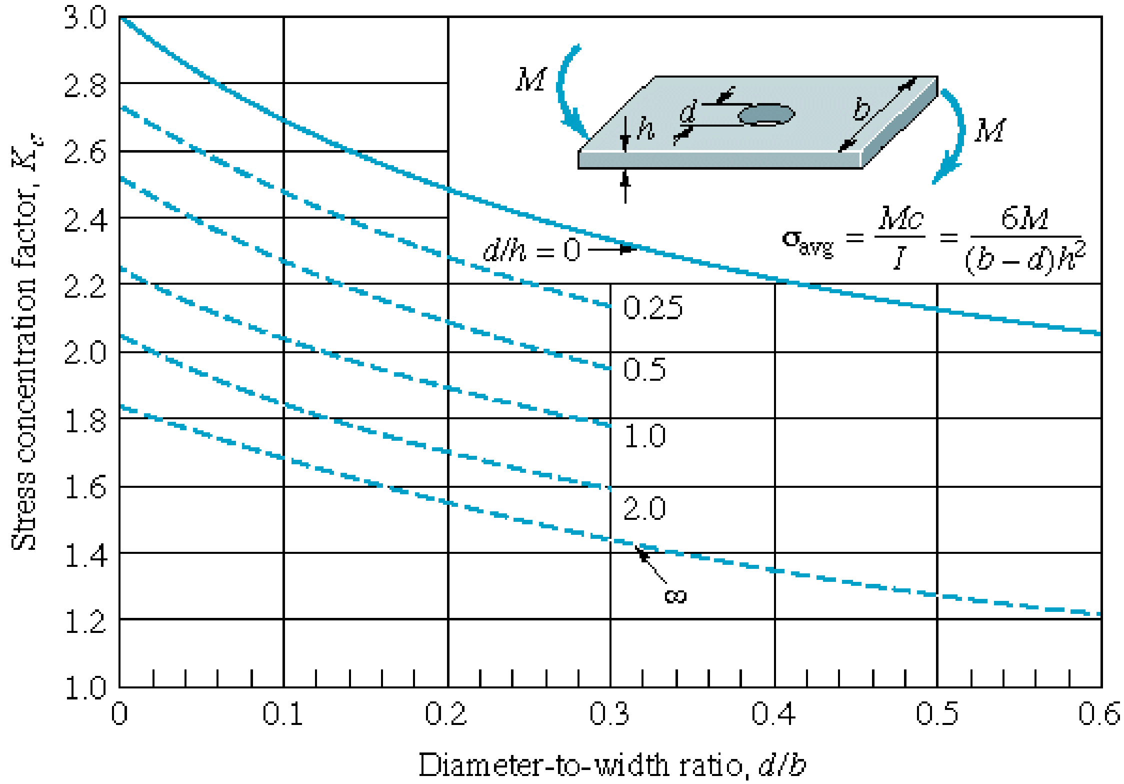


(a)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (a) Axial Load. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

Stress Concentrations for Plate with Hole (cont.)



(b)

Figure 6.2 Stress concentration factor for rectangular plate with central hole. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.2, page 222

Stress Concentrations for Bar with Fillet (cont.)

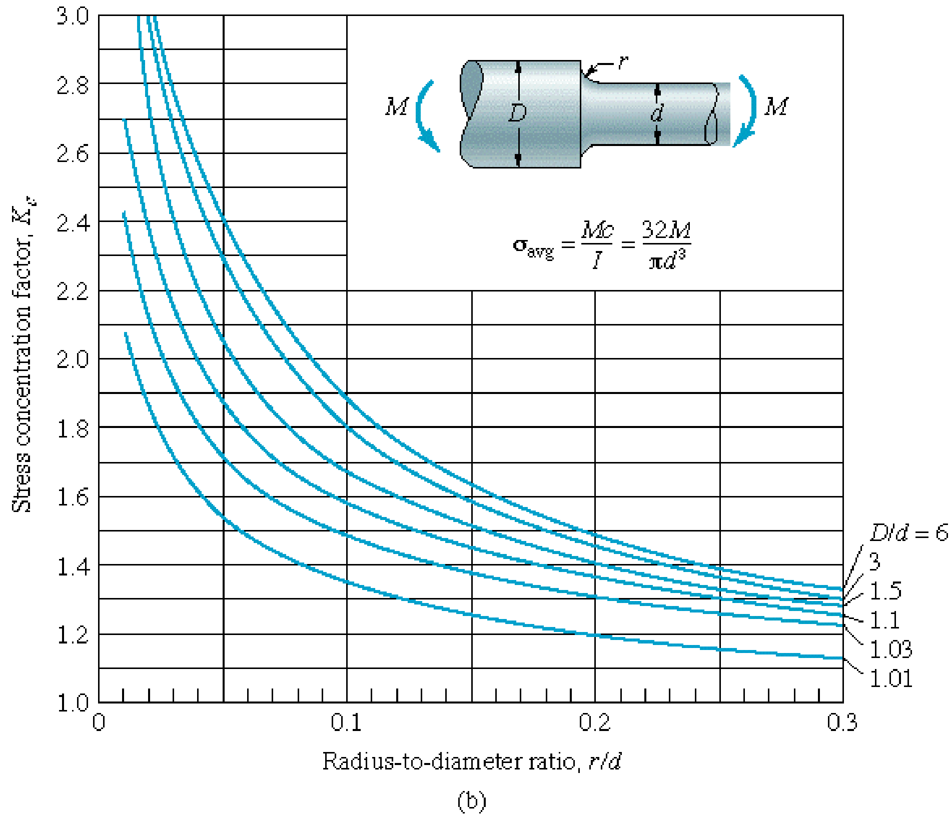


Figure 6.5 Stress concentration factor for round bar with fillet. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.5, page 225

Stress Concentrations for Bar with Fillet (cont.)

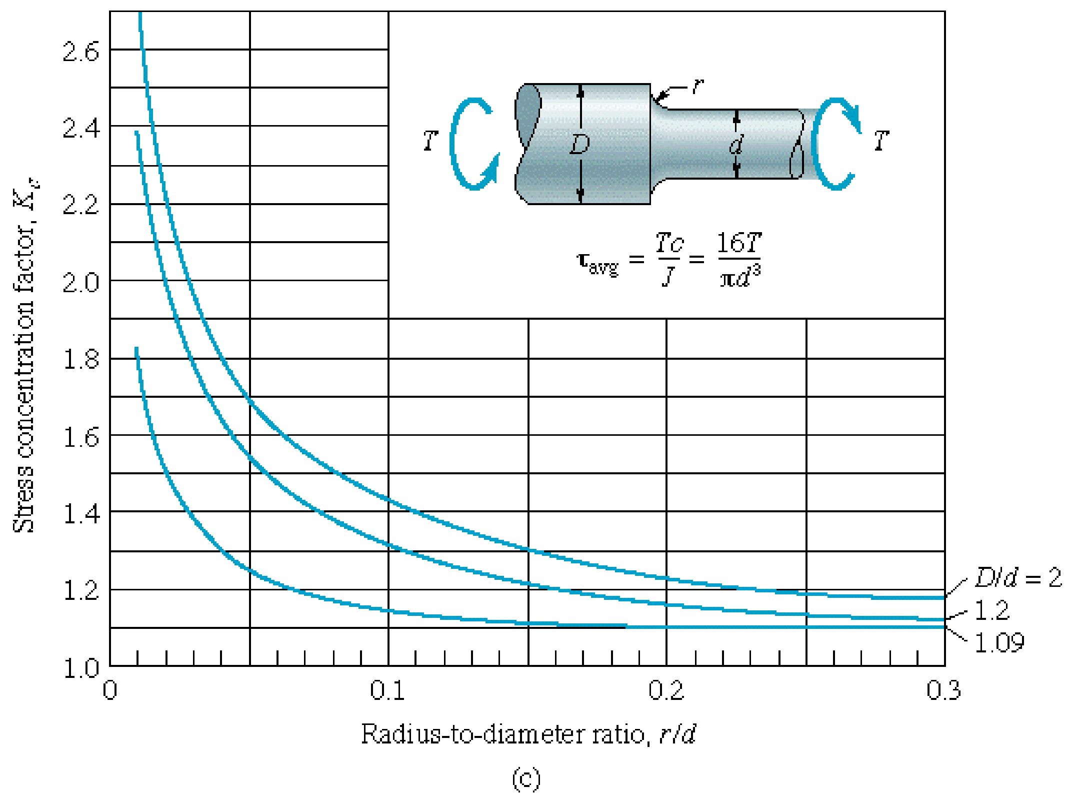


Figure 6.5 Stress concentration factor for round bar with fillet. (c) Torsion. [Adapted from Collins (1981).]

Text Reference: Figure 6.5, page 225

Stress Concentrations for Bar with Groove (cont.)

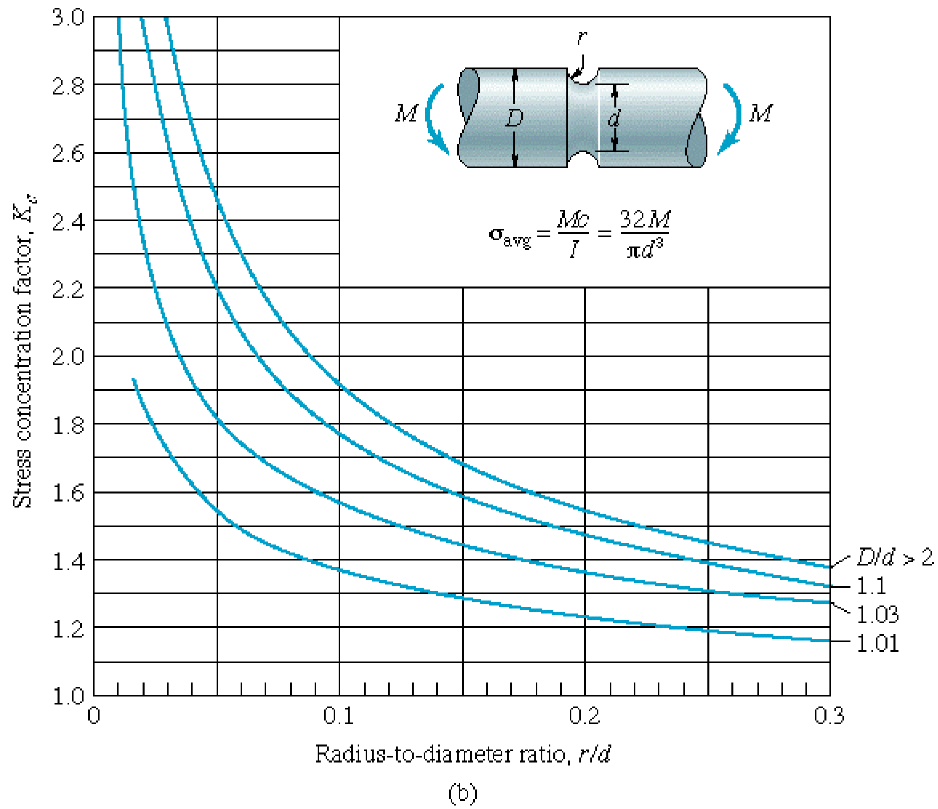


Figure 6.6 Stress concentration factor for round bar with groove. (b) Bending. [Adapted from Collins (1981).]

Text Reference: Figure 6.6, page 226

Stress Concentrations for Bar with Groove (cont.)

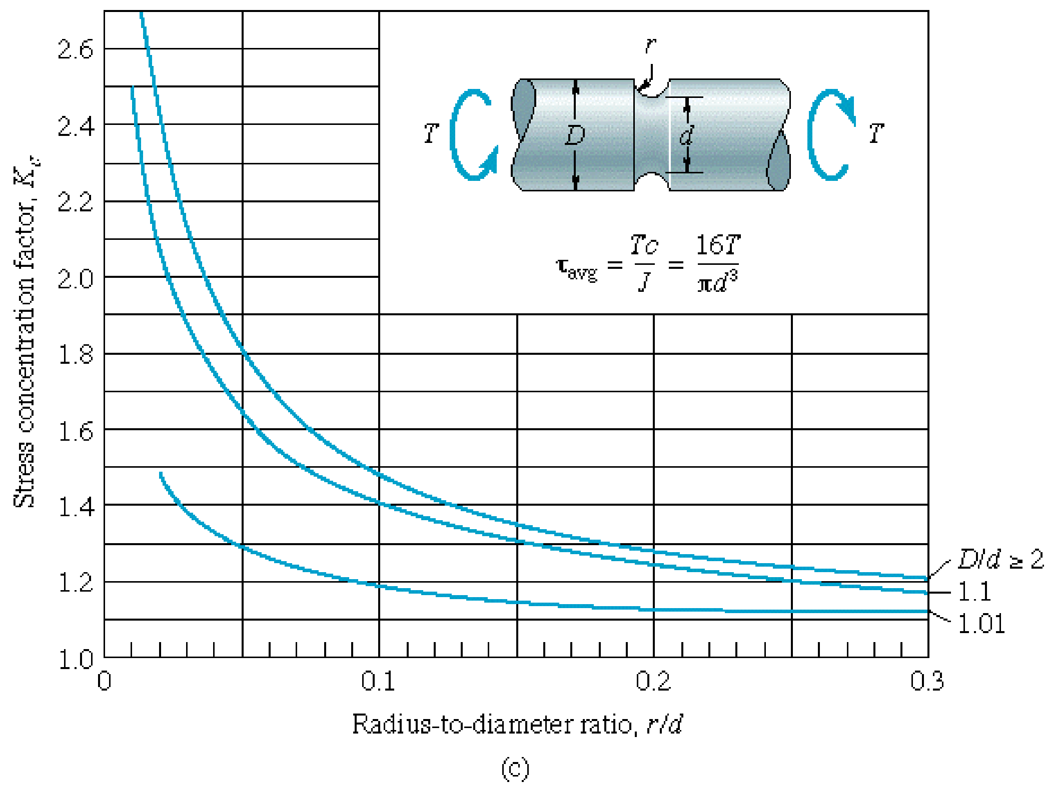


Figure 6.6 Stress concentration factor for round bar with groove. (c) Torsion. [Adapted from Collins (1981).]

Text Reference: Figure 6.6, page 226

Fracture mechanics

- All materials contain cracks.
- If crack is bigger than critical dimension, it propagates → Catastrophic failure.

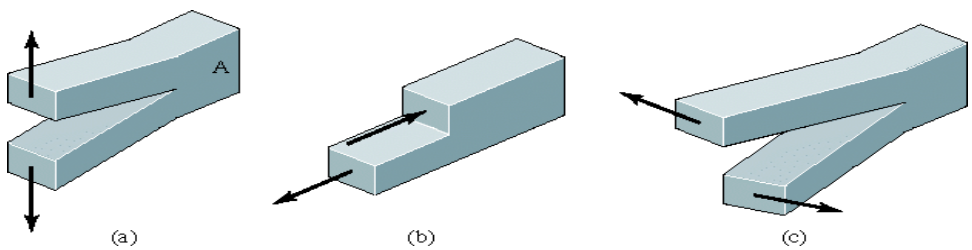


Figure 6.8 Three modes of crack displacement. (a) Mode I, opening; (b) mode II, sliding; (c) mode III, tearing.

- Fracture mechanics predict mode I crack propagation ***if crack size*** > 2 ***a***, where

$$a = \frac{1}{p}\left(\frac{K_{ci}}{Ys_{nom}}\right)^2$$

Fracture toughness, from tables

Geometric correction factor

Yield Stress and Fracture Toughness Data (@room Temperature)

Material	Yield Stress, S_y ksi	Yield Stress, S_y Mpa	Fracture Toughness, K_{ci} ksi in$^{1/2}$	Fracture Toughness, K_{ci} Mpa m$^{1/2}$
Metals				
Aluminum alloy 2024-T351	47	325	33	36
Aluminum alloy 7075-T651	73	505	26	29
Alloy steel 4340 tempered at 260℃	238	1640	45.8	50.0
Alloy steel 4340 tempered at 425℃	206	1420	80.0	87.4
Titanium alloy Ti-6Al-4V	130	910	40-60	44-66
Ceramics				
Aluminum oxide			2.7-4.8	3.0-5.3
Soda-lime glass			0.64-0.73	0.7-0.8
Concrete			0.18-1.27	0.2-1.4
Polymers				
Polymethyl methacrylate			0.9	1.0
Polystyrene			0.73-1.0	0.8-1.1

Text Reference: Table 6.1, page 232

Stress intensity factor

- Recall critical crack length = 2 a.

$$a = \frac{1}{p}\left(\frac{K_{ci}}{Ys_{nom}}\right)^2$$

K_{ci} = Fracture toughness, from tables

Y = Geometric correction factor

- So fracture toughness is $K_{ci} = Ys_{nom}\sqrt{pa}$
- If the actual crack length is 2x, the ***stress intensity factor*** is defined as $\boxed{K_i = Ys_{nom}\sqrt{px}}$
- Crack will propagate if $K_i > K_{ci}$, or 2x > 2a.
- Safety factor against crack propagation is K_{ci}/K_i.