Stochastic Macro-equilibrium and Microfoundations for the Keynesian Economics

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Abstract

This paper begins with pointing out limitations of the standard labor search theory. It then presents an alternative concept of stochastic macro-equilibrium based on the principle of statistical physics. This concept of equilibrium is motivated by the presence of all kinds of unspecifiable micro shocks in the macroeconomy. They make the use of optimization exercises on representative agent assumptions dim. We present a model which mimics the empirically observed distribution of labor productivity. The distribution of productivity depends crucially on aggregate demand. When aggregate demand rises, more workers are employed by firms with higher productivity. At the same time, the unemployment rate declines. The result provides a proper micro-foundation for Keynes' principle of effective demand.

General Equilibrium (GE) Theory from Walras to Arrow and Debrew (1954) Sees the Economy as a System of Simultaneous Equations.

GE Theory Is a Grand and Well-established Theory. But It Cannot be More Different from the Real Economy.

GE Theory Specifies Preferences and Technologies of All the Consumers and Firms,

and Defines the Equilibrium In Which Micro-Behavior of All the Economic Agents are Precisely Determined.

It is Just as One Analyzes Object such as Gas Comprising Many Particles by Determining the Equations of Motion for All the Particles.

Two Challenges to GE

(1) Keynes' *General Theory* (1936)

(2) Diamond-Mortensen-Pisarides

Search Theory

The Heyday of Keynesian Economics was the 1950's and 60's.

Beginning the 1970's, Macroecnomics has turned to the neoclassical doctrine.

"The most interesting recent developments in macroeconomic theory seem to me describable as the reincorporation of aggregative problems such as inflation and the business cycle within the general framework of 'microeconomic' theory. If these developments succeed, the term 'macroeconomic' will simply disappear from use and the modifier 'micro' will become superfluous."

Robert Lucas (1987)

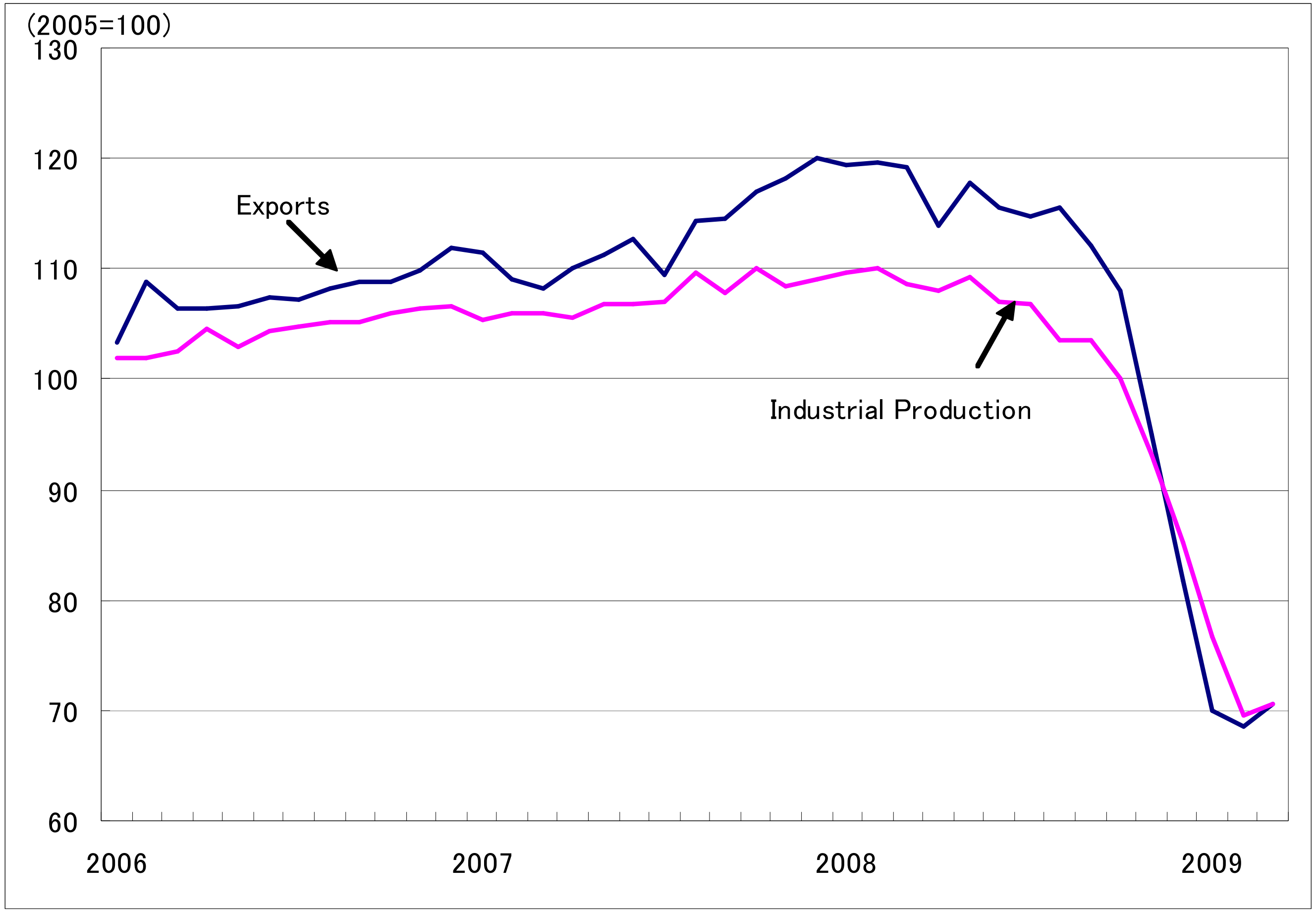
"Macroeconomics was born as a distinct field in the 1940's, as a part of the intellectual response to the Great Depression. The term then referred to the body of knowledge and expertise that economic disaster. My thesis in this lecture is that macroeconomics in this original sense has succeeded: Its central problem of depression prevention has been solved, for all practical purposes, and has in fact been solved for many decades."

Robert Lucas, Nobel Laureate

Presidential Address to the 2003 American Economic Association

The 2008-09 Financial Crisis in Japan

Demonstrates the Principle of Effective Demand Once Again!



Source: METI, Cabinet Office

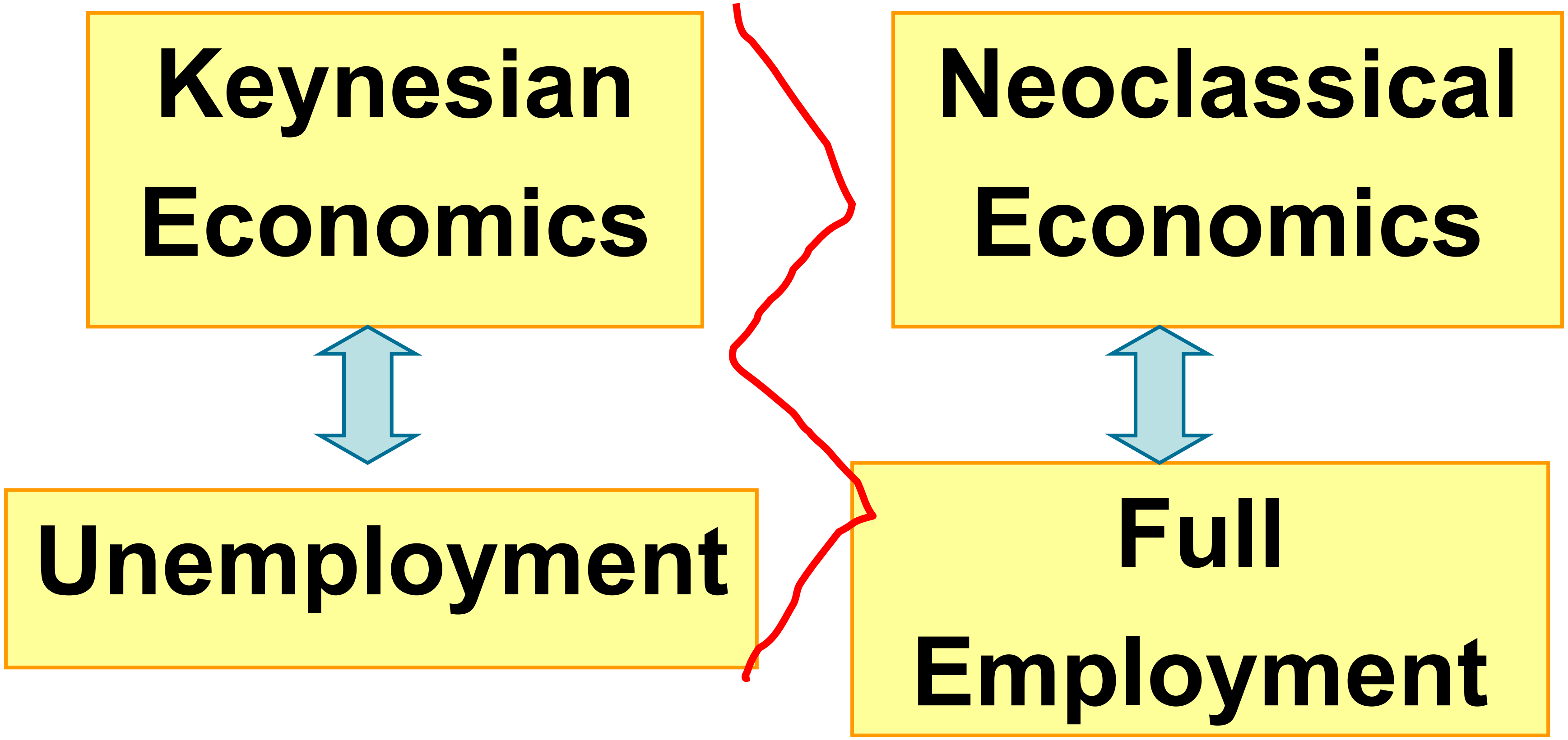
The Turning Point of Macroeconomics?

" Most macroeconomics of the past 30 years was spectacularly useless at best, and positively harmful at worst."

Paul Krugman、June 2009

at Lionel Robbins Lectures

in London School of Economics


Keynesian Economics
Unemployment
Neoclassical Economics
Full Employment

Microeconomic Foundations for Keynesian Economics

Standard Micro-Foundations for Keynesian Economics

Inflexible Price / Wages

↓

Involuntary Unemployment

New Micro-Foundations for Keynesian Economics

Not based on optimization of micro agents

But on the methods of statistical physics.

New Micro-Foundations for Keynesian Economics

We can not explain macro troubles such as congestions on turnpikes by way of optimizing behaviors of micro agents.


JAPAN-US CENTER UFJ BANK MONOGRAPHS ON INTERNATIONAL FINANCIAL MARKETS
Praise for Reconstructing Macroeconomics
"Thoughtful macroeconomists are uncomfortably aware that consumers, firms, and workers vary widely in their local environments, perceptions, and beliefs. Ignoring this heterogeneity, as 'modern macro' does, is a likely source of systematic error. Aoki and Yoshikawa propose to repair this failure by modeling the macroeconomy explicitly as a cloud of interacting particles. The goal is to deduce the distributions of economic characteristics that describe the system as a whole. This puts more emphasis on statistical properties and less on the internal decision making of each agent. There are already some surprising beginning results, including a novel treatment of aggregate demand, and one can expect more when their approach is combined with standard economic reasoning. This is the start, not the finish, of a potentially far-reaching research program. It should excite the curiosity of all those thoughtful macroeconomists."
– Robert M. Solow, Nobel Laureate, Massachusetts Institute of Technology
"This book is a bold and daring challenge to the growing influence of neoclassical equilibrium theory in the field of modern macroeconomics. Not simply an approach to traditional Keynesian theory that attempts to refine it and make it more accurate, the treatment makes use of a new methodology in statistical physics and combinatorial stochastic processes to mount a direct challenge to real business cycle theory and rational expectations theory. This technique makes it possible to analyze the interactions of a large number of fluctuating micro agents. Professor Aoki has made important contributions to the application of statistical physics to economics, and Professor Yoshikawa is a leading Japanese economist who has done outstanding work in the fields of both theoretical and empirical economics. This book is the superb product of the optimum combination of these two scholars' different talents."
– Ryuzo Sato, New York University and University of Tokyo
"Masanao Aoki and Hiroshi Yoshikawa have written no less than the foundation of a new approach (and I believe the right one) to the core problem of macroeconomics, which is to aggregate behaviors by stressing the importance of the heterogeneity and variability of real economic agents. Getting inspiration from and adapting the concepts and tools of statistical physics, they masterfully derive important and novel insights on the most crucial open problems of the field: the principle of effective demand, role of uncertainty, sticky prices/wages, and the endogenous business cycle. By systematically discussing and comparing their theory with empirical data and real economic situations, this book is perhaps the first successful effort to develop macroeconomics as a real science on par with physics, with falsifiable hypotheses underpinned by sound micro-principles and testable predictions."
– Didier Sornette, Swiss Federal Institute of Technology, Zurich
"This book shows the impossibility of efficient equilibria in economies with market clearing mainstream hypotheses when such an economy is populated by a large number of heterogeneous agents. In such a case, Aoki and Yoshikawa show that, through combinatorial stochastic processes, a new approach to macroeconomics is not only possible: it is real and this book shows how to reach it."
– Mauro Gallegati, Università Politecnica delle Marche
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Aoki Yoshikawa
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COMBINATORIAL STOCHASTIC PROCESSES
MASANAO AOKI AND HIROSHI YOSHIKAWA
CAMBRIDGE

Solow’s comment

“Thoughtful macroeconomists are uncomfortably aware that consumers, firms, and workers vary widely in their local environments, perceptions, and beliefs. Ignoring this heterogeneity, as ‘modern macro’ does, is a likely source of systematic error. Aoki and Yoshikawa propose to repair this failure by modeling the macroeconomy explicitly as a cloud of interacting particles. The goal is to deduce the distributions of economic characteristics that describe the system as a whole. This puts more emphasis on statistical properties and less on the internal decision making of each agent. There are already some surprising beginning results, including a novel treatment of aggregate demand, and one can expect more when their approach is combined with standard economic reasoning. This is the start, not the finish, of a potentially far-reaching research program. It should excite the curiosity of all those thoughtful macroeconomists.” Robert M. Solow (2007)

Typical Objection to the Method

This method has been time and again successful in natural sciences when we analyze object comprising many micro elements. Economists might be still skeptical of the validity of the method in economics saying that inorganic atoms and molecules comprising gas are essentially different from optimizing economic agents. Every student of economics knows that behavior of dynamically optimizing economic agent such as the Ramey consumer is described by the Euler equation for a problem of calculus of variation. On the surface, such a sophisticated economic behavior must look remote from “mechanical” movements of an inorganic particle which only satisfy the law of motion.

Objection Unfounded

However, every student of physics knows that the Newtonian law of motion is actually nothing but the Euler equation for a certain variational problem. It is called the principle of least action: see Chapter 19 of Feynmann (1964)'s *Lectures on Physics*, Vol. II. Therefore, behavior of dynamically optimatizing economic agent and motions of inorganic particle are on a par to the extent that they both satisfy the Euler equations for respective variational problems. The method of statistical physics can be usefully applied not because motions of micro units are "mechanical," but because object under investigation comprises many micro units individual movements of which we are unable to know.

What Matters Is Not

Employment or Unemployment
0 or 1

But

Many Levels of Productivity

namely

Distribution of Productivity

The Principle of Statistical Physics

⇩

Boltzmann or Exponential Distribution in stationary state

Yoshikawa, H. (2003) “The Role of Demand in Macroeconomics,” *Japanese Economic Review*, Vol.54, No.1, 1-27.

The Basic Model

Suppose that n_k workers belong to firms whose productivity is c_k. There are K levels of productivity in the economy. The total number of workers is N.

$$\sum_{k=1}^{K} n_k = N$$

The probability that a particular allocation $n=(n_1, n_2, \ldots, n_k)$ is obtained is

$$P_n = \frac{W_n}{K^N} = \frac{1}{K^N} \frac{N!}{\prod_{k=1}^{K} n_k!}$$

The maximization of p_n is equivalent to that of the following entropy S.

$$S = -\sum_{k=1}^{K} p_k \ln p_k$$

$$(p_k = \frac{n_k}{N})$$

Macro Constraints

$$\sum_{k=1}^{K} c_k n_k = D$$

The State to be Realized

$$\frac{n_k}{N} = \frac{e^{-\frac{Nc_k}{D}}}{\sum_{k=1}^{K} e^{-\frac{Nc_k}{D}}}$$

Distribution Changes as Aggregate Demand Changes

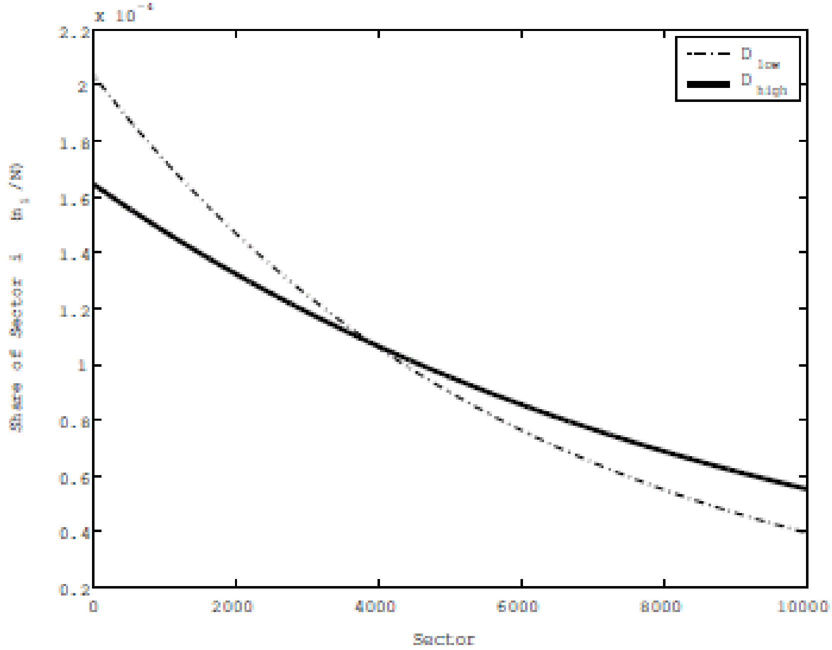


Figure 3.2. Distribution of Production Factors across Sectors with Different Productivity: The Boltzmann-Gibbs Distribution

Note: $c_1 < c_2 < c_3 < ...,\quad c_i \propto 0.1 + 0.9 \times i/S,$

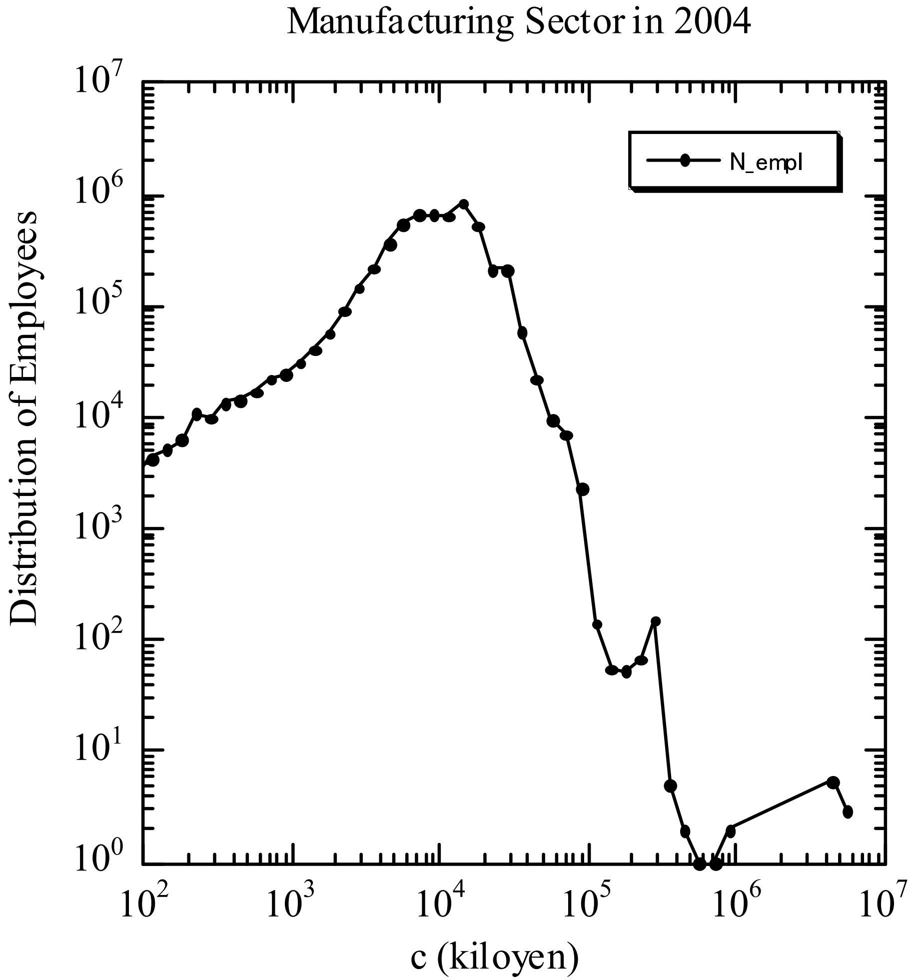
$S = 10^4$, $N = 10^8$, $D_{low} = 10^4$, $D_{high} = D_{low} \times 1.5$,

$$\frac{n_i}{N} = \frac{e^{-Nc_i/D}}{\sum_{i=1}^{S} e^{-Nc_i/D}}$$

Source : Aoki M. and H. Yoshikawa, (2007) *Reconstructing Macroeconomics: A Perspective from Statistical Physics and Combinatorial Stochastic Processes,* Cambridge University Press

Distribution of Labor Productivity in Japan

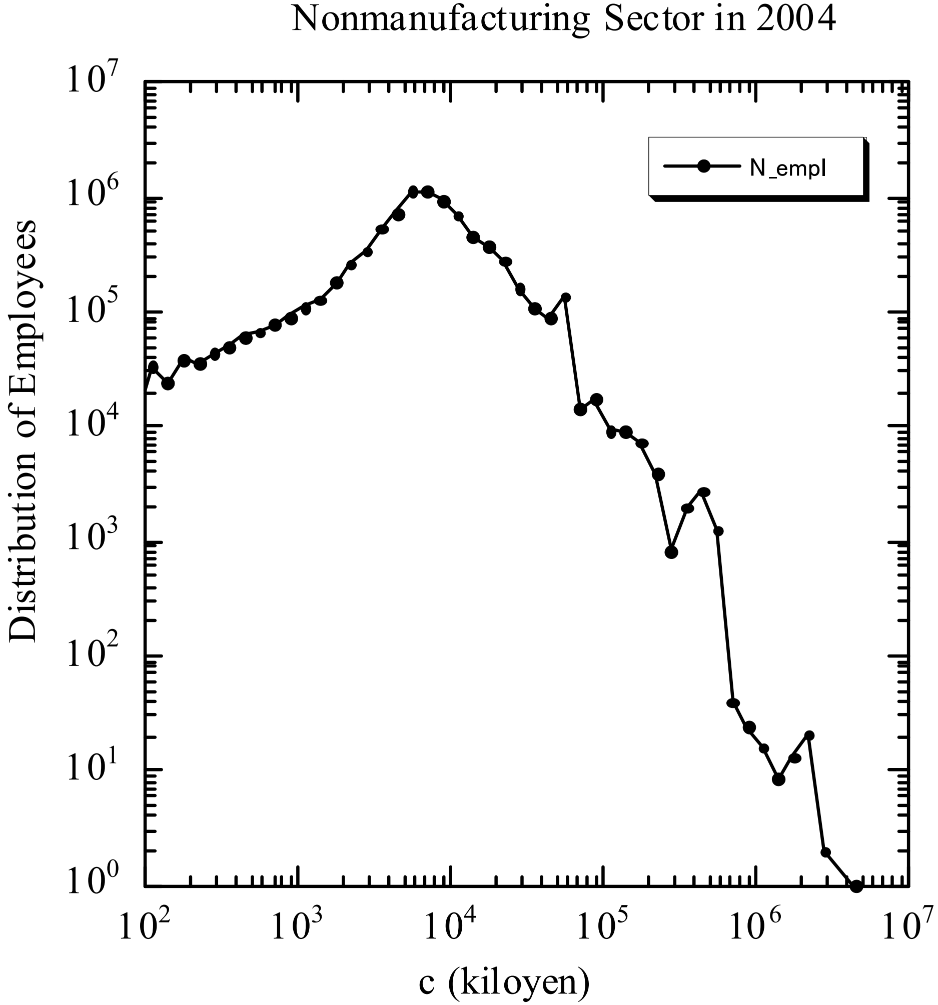
(a) Manufacturing Sector in 2004



Source: Iyetomi (2012)

Distribution of Labor Productivity in Japan

(b) Non-manufacturing Sector in 2004



Source: Iyetomi (2012)

The Purpose of the Present Study Is to Explain this Empirical Distribution of Productivity

And

To Provide a Micro-Foundation for Keynes' Principle of Effective Demand

To explain the left-hand side upward sloping distribution,

Iyetomi (2012) introduced the

Negative Temperature.

The Nature of Micro Shocks

Self-Averaging

⇕

Non Self-Averaging

See : Sornette, D (2000), *Critical Phenomena in Natural Sciences*, Springer.

The Nature of Micro Shocks

In Standard Models,
"Micro Shocks" are Assumed to Wash Out

⇩

We Can Understand
"Macro" by the Average

Self-Averaging

Non Self-Averaging

We Cannot Understand "Macro" by the Average

⇕

Standard Microeconomic Foundations Based on Representative Agent Do not Make sense

See : Aoki M. and H. Yoshikawa, (2011) "Non-Self-Averaging in Macroeconomic Models: A Criticism of Modern Micro-founded Macroeconomics", *Journal of Economic Interaction and Coordination*, Vol.7, 1, pp1-22 .

Diamond-Mortensen-Pissarides Equilibrium Search Theory

The Theory is a curious hybrid.

(1) Heterogeneous agents due to various market frictions and matching costs.

(2) Optimization exercises on the representative agent assumption.

To get closer to the actual markets, it was necessary to introduce the Matching Function ---

a macro black box NOT explicitly derived from micro optimization exercises.

For the method to make sense, the difference between inorganic “particles” and brainy economic agents is inessential.

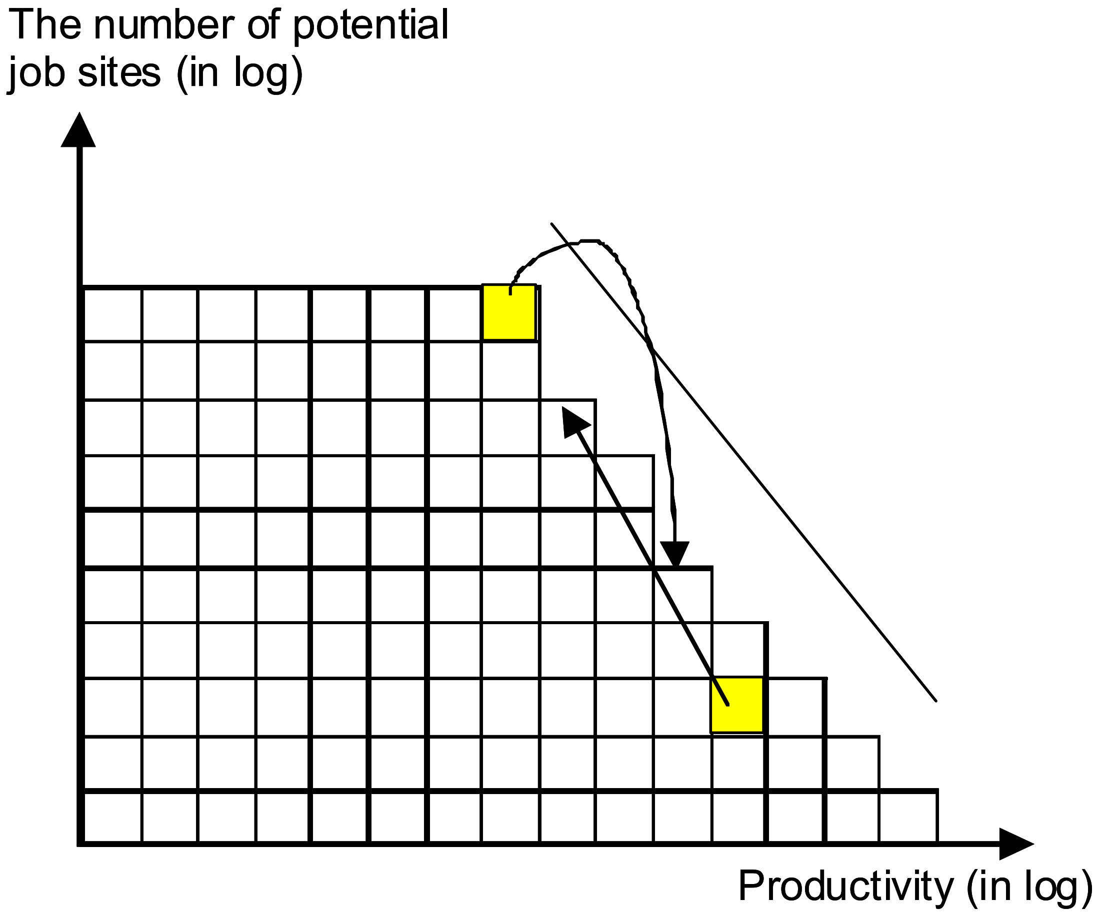
The point is that the system consists of a large number of micro units.

The number of household $\sim 10^7$

The number of firms $\sim 10^6$

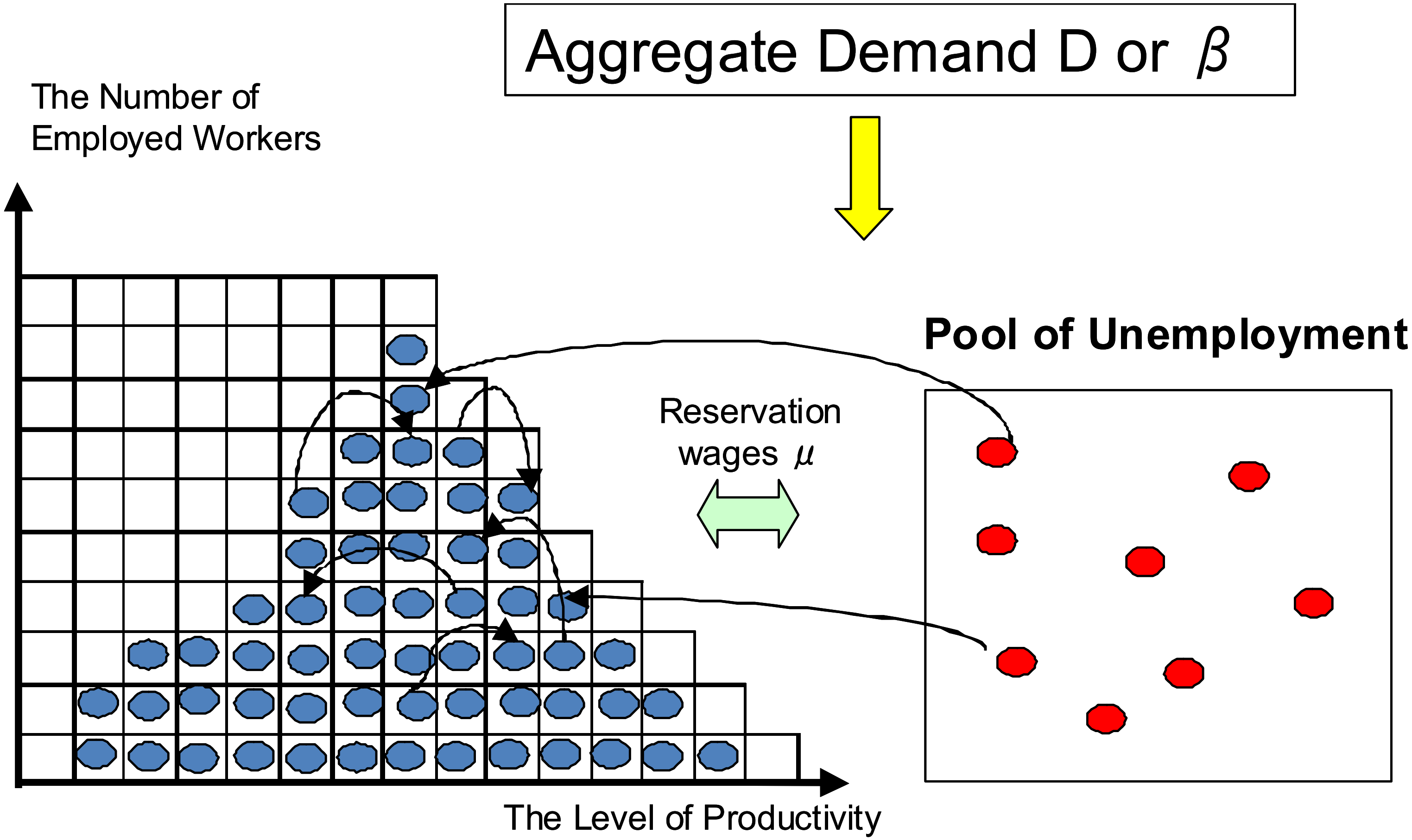
Model of Stochastic Macro-equilibrium

(a) Dynamics of Creation and destruction of Potential Jobs



Model of Stochastic Macro-equilibrium

(b) Stochastic Macro-equilibrium



The Model

$$N = \sum_{j=1}^{K} n_j$$

$$Y = \sum c_j n_j$$

$$U = L - N$$

$$n_j \in \{0, 1, \cdots, f_j\} \quad (j = 1, 2, \cdots K)$$

Partition Function

$$Z = \sum_i e^{-\beta Y_i} \qquad g(Y) = \frac{e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}}$$

$$-\frac{d \log Z}{d\beta} = -\frac{d}{d\beta}\log(\sum_i e^{-\beta Y_i}) = -\frac{\sum_i (-Y_i) e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}}$$

$$= \sum_i Y_i \left(\frac{e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}}\right) = \sum_i Y_i g(Y_i) = E(Y_i)$$

Grand Canonical Partition Function (1)

$$g(Y) = \frac{e^{-\beta Y_i}}{\sum_i e^{-\beta Y_i}} \qquad Z = \sum_i e^{-\beta Y_i}$$

$$Z_N = \sum_i e^{-\beta Y_i(N)} = \sum_{\{n_i\}} \exp(-\beta \sum_{i=1}^{K} c_i n_i)$$

$$\Phi = \sum_{N=0}^{\infty} z^N Z_N$$ where $$z = e^{\beta\mu}$$

Grand Canonical Partition Function (2)

$$\Phi = \sum_{N=0}^{\infty} z^N \sum_{nj} \exp\{-\beta \sum_j n_j c_j\}$$ where $$z = e^{\beta\mu}$$

$$\Phi = \sum_{N=0}^{\infty} e^{\beta\mu(n_1+\cdots+n_K)} \sum_{n_j} \exp\{-\beta \sum_j n_j c_j\} = \prod_{j=1}^{K} \sum_{nj} \exp[\beta(\mu - c_j)n_j]$$

$$\Phi = \prod_{j=1}^{K} [1 + e^{\beta(\mu-c)} + \cdots e^{f\beta(\mu-c)}] = \prod_{j=1}^{K} [\frac{1 - e^{(f_j+1)\beta(\mu-c)}}{1 - e^{\beta(\mu-c)}}]$$

$$\frac{1}{\beta}[\frac{\partial}{\partial\mu}\log\Phi] = \frac{1}{\beta}[\frac{\partial}{\partial\mu}\log(\sum_{N=0}^{\infty} e^{\beta\mu N} Z_N)] = \frac{1}{\beta}[\frac{\beta\sum_{N-0}^{\infty} N e^{\beta\mu N} Z_N}{\sum_{N=0}^{\infty} e^{\beta\mu N} Z_N}] =< N >$$

Distribution of Productivity

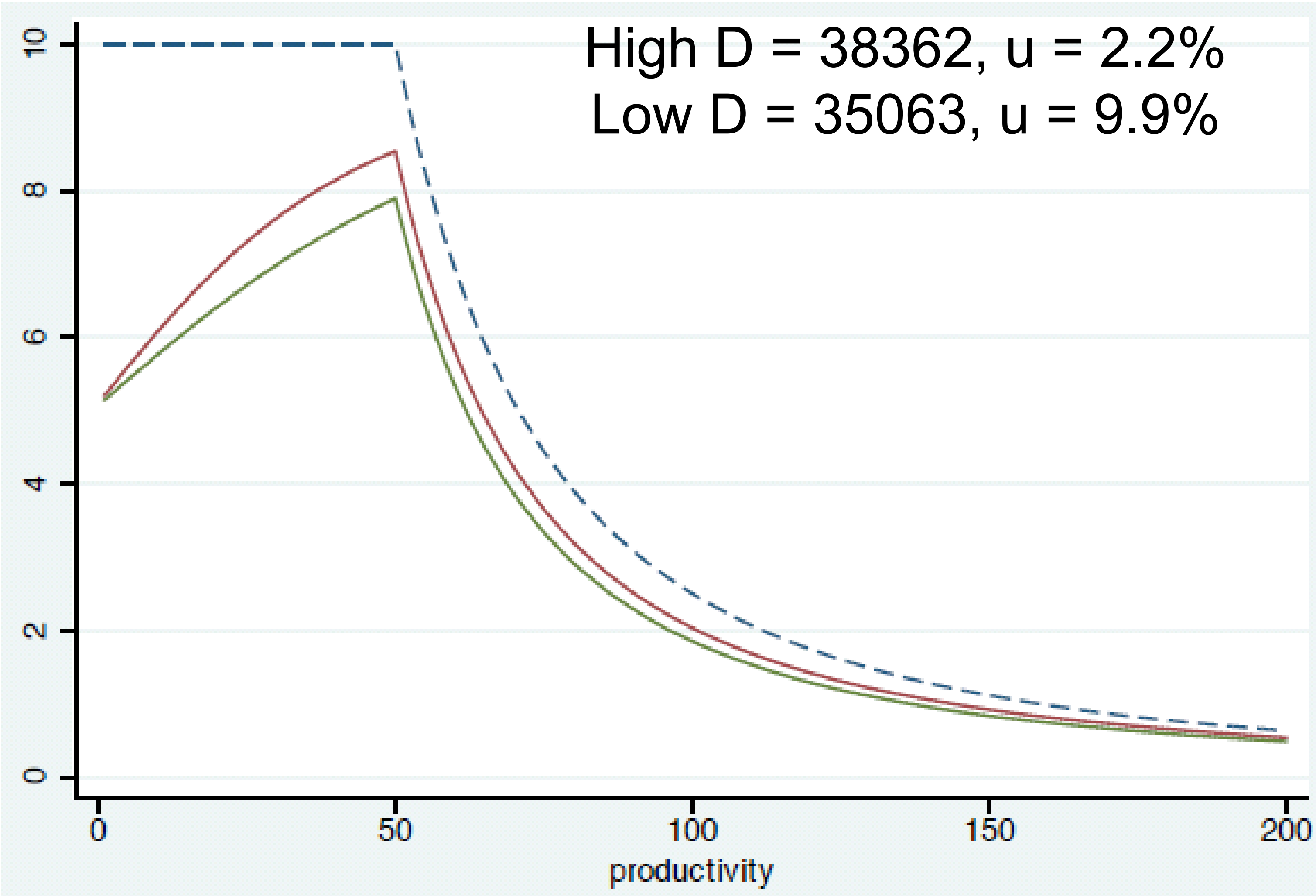
$$\left\langle N \right\rangle = \frac{1}{\beta}[\frac{\partial}{\partial \mu}\log\Phi]$$

$$= \frac{1}{\beta}\sum_{j=1}^{K}\frac{\partial}{\partial \mu}\{\log(1-e^{(f_j+1)\beta(\mu-c_j)}) - \log(1-e^{\beta(\mu-c_j)})\}$$

$$= \sum_{j=1}^{K}[\frac{(f_j+1)e^{(f_j+1)\beta(\mu-c_j)}}{e^{(f_j+1)\beta(\mu-c_j)}-1} - \frac{e^{\beta(\mu-c_j)}}{e^{\beta(\mu-c_j)}-1}]$$

$$\left\langle n_j \right\rangle = \frac{(f_j+1)e^{(f_j+1)\beta(\mu-c_j)}}{e^{(f_j+1)\beta(\mu-c_j)}-1} - \frac{e^{\beta(\mu-c_j)}}{e^{\beta(\mu-c_j)}-1}$$

Simulated Distribution of Labor Productivity



Percentage of Potential Job Sites Occupied by Employed Workers