Safety and Reliability for Managing Risk – Guedes Soares & Zio (eds)
© 2006 Taylor & Francis Group, London, ISBN 0-415-41620-5

Analysis about extreme water levels along the Dutch north-sea using Grey models: Preliminary analysis

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ABSTRACT: In this paper an investigation about extreme water levels along the Dutch north-sea cost is presented. The paper is based on some earlier works using statistical and mixed methods for the estimation of probabilities of extreme water levels. In contrast to earlier work, this paper uses Grey system models according to Deng to verifying the results found. First, a simple Grey system model is introduced and applied to historical data sets of extreme water levels. This so-called exponential Grey model is used to investigate systematical changes inside the data provided. In addition, this model is also used to estimate low probability values of the extreme water levels. These achieved values can then be compared to an application of the exponential distribution. Since much work has been carried out to identify the type of the distribution of extreme water levels, this work should not be neglected. The logistic distribution based on the Verhulst differential equation has been applied successfully to the estimation of low probability values recently. Therefore in the next step the Grey Verhulst model is used. The Verhulst differential equation forms the basis for the Grey Verhulst model as well as for the logistic distribution. This should permit a comparison of the gained values.

1 INTRODUCTION

The ultimate prediction of future events regardless of their type seems to be, at least so far, impossible. This uncertainty is an essential part of the world known. Due to the mainly towards the future looking attitude of humans, they tend to deal with this uncertainty in different ways. So far there are many different mathematical theories dealing with uncertainty. Presumably the best known are statistic and probability theory. Nevertheless statistic and probability models have been extremely successfully applied, new models have been spreading over the last decades, such as fuzzy theory, rough theory, expert systems or neuronal networks. Grey models can be incorporated into this group of new uncertainty related theories. Introduced by Deng in 1982 in China (Deng 1988), Grey models have only been used very infrequent in Europe, whereas they have been intensively discussed in China. Examples of application range from the estimation of the waste water loads (Jun & Baoqing 1997), transportation problems (Bai, Mao & Lu 2004), energy consumption predictions (You-xin et al. 2004), maintenance planning (Guo & Dunne 2005) and flooding predictions (Yan, Zhou & Yan 2005).

Many of these topics dealt with in the publications mentioned are also of major concern in Europe. For example, flooding prediction is especially important for the Netherlands, since approximately 40% of the country is under sea level. Therefore many protecting structures had to be installed, such as dikes. The overall length of dikes in this country exceeds 3,000 km.

The required height of these dikes is still of interest, since they have to accomplish safety needs as well as economical requirements. For the design process extreme water level heights, usually with a probability of 10^{-4} per year are required.

An example for the consequences of dike failure is the 1953 disaster in the Netherlands, when during a storm surge several sea dikes broke and about 1,800 people lost their lives. Probably more people still have in mind the flooding of New Orleans 2005 caused by a hurricane or the heavy flooding in central Europe in the summer 2002 caused by extreme river water levels. These examples show the need for reliable dikes and other erosion control structures, which can only be designed using realistic estimations of extreme water levels.

2 EXTREME WATER LEVELS

For the estimation of extreme sea water levels the physical backgrounds of such events has to be considered first. Extreme water levels can be caused by a combination of astronomical, geological, meteorological and climatic causes. These causes have changed the mean sea level as well as extreme water levels over time. Because measured data of sea water levels is only available since several decades, in earlier investigations the value mostly has been considered time invariant. Only recently the change of sea level is increasingly considered in statistical investigations of extreme sea water levels. Sophisticated models nowadays consider many effects on sea water levels, such as lift and drop of land masses, temperature change of sea, change of weather conditions due to climate change etc. Still, having one population of sample data forms the basic assumption for the statistical estimation of extreme water levels. This is also be assumed to be valid for the investigation presented here.

Many methods have been applied to estimate extreme sea levels, such as annual maxima method, joint probability method, regional approach methods, spatial approach modeling, numerical tide and surge models, peaks over threshold or percentile values. The choice of the method and the usage of the data have been done carefully. Most data sets incorporate special spatial conditions, such as in European winter storms or in America Hurricanes. In addition data sets might include large outliers. Then it has to be decided if these samples are representative or not, as they affect analysis results considerably. For example a tsunami can be seen as an extreme water level or it can be seen as an outlier and is therefore excluded. In the following case it has been assumed that no outliers are in the data. Also special spatial conditions are first neglected.

3 DATA

3.1 *Introduction*

Data sets of peaks-over-threshold values of storm surge levels at five locations along the Dutch coast have been used for this investigation. The locations of the survey points are given in table 1 and visualized in figure 1. The data has been prepared by the Dutch Ministry of Transport Public Works and Water Management. The data sequence covers a time length between 53 and 104 years depending on the location. They consider the storm season from October 1 through March 15 per year. Pandey et al. 2003 have investigated the assessment of the optimal threshold value for extreme wind velocities. Such an investigation of the peak-over-threshold value is not part of the present investigation; instead the values have been used "as delivered". The adjustment of probability per storm to probability per year has been done using the simplified method presented by d'Angremond & van Roode 2004. Figure 2 shows the measured water levels over the number of measurements from the location Delfzijl as an example.

Table 1. General location of the measurement points.

	Survey location		Meas. time		
Setting	NB	OL	Begin	End	Number
Delfzijl	53°19′54″	6°55′05″	1882	1985	1767
Harlingen	53°10′14″	5°25′55′	1933	1985	936
Hoek van Holl.	51°59′	4°07′	1888	1985	1577
Vlissingen	51°27′	3°35′	1882	1985	1672
Den Helder	52°58′	4′46′	1933	1985	866

Figure 1. Location of the survey points.

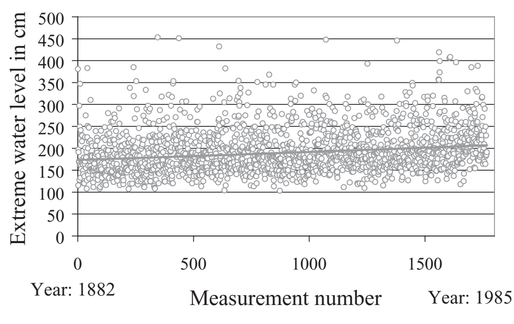

Figure 2. Peaks-over-threshold values for extreme water levels at Delfzijl including a linear trend fitting line.

Usually, the measured data can be used to identify a statistical distribution, which can then be applied for the extrapolation of extreme probabilities. The choice of the statistical distribution functions of extreme water levels differs and depends on the specific locations. Mostly extreme value distributions, such as Gumbel, Weibull or Generalized Extreme Value distribution have been used, because they can be theoretically justified. But other models such as Pearsons distribution, Generalized Pareto distribution, Exponential distribution, inverse Gaussian, Lognormal or Generalized Logistic distributions (Van Gelder & Neykov 1998) can be found in the literature as well. A nice overview about distributions commonly used in water sciences is presented by Bobée & Ashkar 1991. Also combinations of distributions are applied. Especially the recent recommended Logistic distribution is of interest here, since this distribution includes a change of curvature at the tail of the distribution and therefore yields to greater quantile values of extreme water levels compared to other distributions.

Unfortunately the adoption of such a curvature change is extremely uncertain since the data has to be largely extrapolated. In general, it has been stated, that the extrapolation time should not exceed four times the period of observation (Pugh 2004). For examples, with only one hundred years of measurement one could only estimate the value for a return period of 400 years. But for the design process of the dikes a value with a mean return period of 10,000 years is requested. Using five locations together with almost 500 years of data and assuming, that this data is taken from one population, one can estimate the extreme sea water level for a return period of 2,000 years. Still, this does not reach the value of 10,000 year, but the example shows the advantage of using the regional frequency analysis to increase the sample size. This idea will be applied in this investigation too. Using this procedure, the data has to be checked using a heterogeneity measure for assessing whether a proposed region is homogeneous and the data is comparable. This could be proven for the current data by van Gelder & Neykov 1998 already.

The crude data for the five locations is shown in figure 3 as decade logarithmic plot. The data has then be ranked using an appropriate ranking formulae. Either Bernard's median rank formulae or the modified Kaplan-Meier mean ranking formulae have been used in the investigation. Subsequently the data has to be normalized. Usually this is done over the mean value of the time series. In contrast here normalization over the first point of maximum curvature, over the second point of maximum curvature and over the mean value has been investigated. The authors assume that changing the center of normalization gives a better visual impression about differences of the data. From figure 3 it becomes clear, that three data sets (location Den Helder, Hoek van Holland and Vlissingen) show a close behavior (data set 1), whereas the datasets from Delfzijl and Harlingen show a different slope (data set 2). This can be interpreted as special spatial conditions.

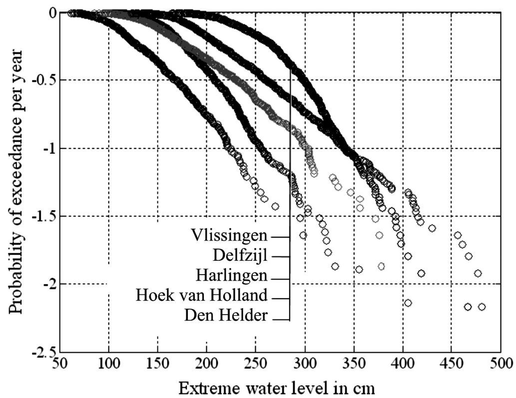

Figure 3. Decade logarithmic probability plot of extreme water level of the five locations considering sea level rise already.

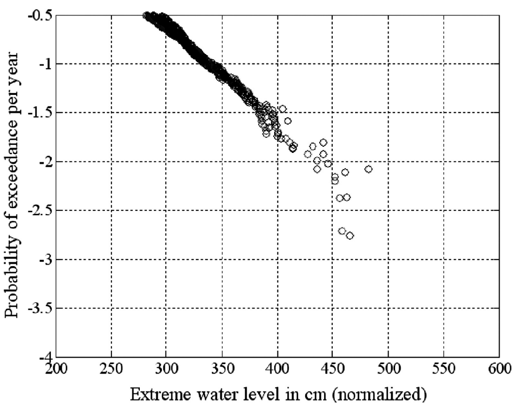

Figure 4. Decade logarithmic probability plot of extreme water level of the five locations after transformation (rotation and translation of the data to the normalization point).

In addition, a closer look at the data points with lowest probabilities reveals a possible change of population at the tail. If the extreme points are randomly, then they should indeed show a low clustering and high diffusion, but if one looks at the shape of the curvatures, it shows highly comparable properties (figures 3 and 4). In all cases there is a jump of water level without samples followed by a group of samples with a higher steepness than the curvature before. Only then, in most cases separated by another sample free area, the points are then seemed to be randomly distributed. It has been checked, whether the extreme points are measured at the same date (correlation). This is not valid. Also other transformations of the data did not disengage the mentioned properties (transformations investigated were Gumbel, Weibull, Logistic and Log-Logistic distribution).

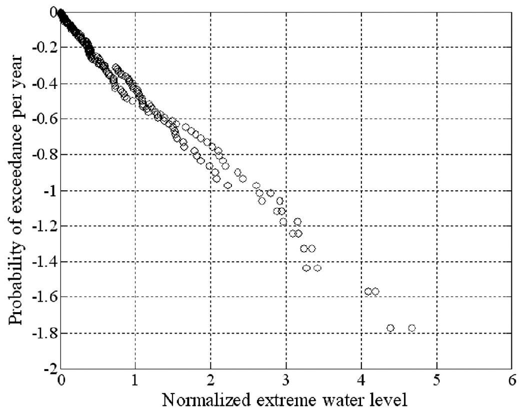

Figure 5. Example of sampled correlated Weibull distributed normalized extreme water levels.

Let us again consider the differences between the data set 1 and data set 2. This basis of the differences can be found in the properties of the locations, such as fetch length, water depth and angle and velocity of wind direction. If for such consideration a certain function exists, than the data set 2 can be transformed to meet the slope of data set 1. Indeed Voortman 2003 has investigated the extreme water levels as a function of wind properties. Therefore these properties can be justified.

Also the properties at the tails were more investigated. Tests considering the extreme water levels as Weibull distributed values have been carried out using Gumbel copulas to create correlated samples. An example is shown in figure 5. This was done to test, whether or not the mentioned artifacts can be found in correlated random data as well. The artifacts were not found, but this does not necessarily mean, they can not be random, but it seems to the authors, that the data includes systematical changes at extreme values.

In figure 4 the data is shown after shifting and rotating (correction factor 1.5 for Harlingen and 1.64 for Delfzijl for the decade logarithmic probability data). Due to the rotation of the data sets in the diagram the probability of some data points has been decreased. In a physical way that would mean that constant measures under harsh conditions give information about events with low probabilities under moderate conditions. Indeed in the field of civil engineering this is done for investigation of the duration of materials, such as brittle materials using strength-probability-time diagrams, see for example Munz & Fett 1989.

When considering, that the curves show different curvatures at low probabilities, this effect would yield to the assumption that such systematical effects might decrease or fade for extreme sea water values with extremely low probabilities. This would contradict the used assumption of a transformation procedure, either because such a transformation does not exist or the transformation is nonlinear and can not be handled in the described way. This discussion goes beyond this paper. The paper in the following focuses on the application of Grey model analysis for the estimation of low probability extreme sea water levels.

4 GREY MODEL

4.1 *Introduction*

Grey model theory is a mathematical description of uncertainty. This theory can be used alone or in connection with other mathematical theories dealing with uncertainty, such as fuzzy theory (Tsaur 2005).

Grey model theory is a theory consisting of many different fields, such as Grey theory controlling, Grey decision making or Grey model prediction. In general, the degree of Grey describes the information content of a number. The white number is perfectly known whereas a black number is not known at all. For Grey numbers rules of calculation exist which can be found, for example at Guo & Love 2005. It is not the focus of the paper to introduce the concept of Grey models in all details; instead this method should be applied for the mentioned tasks.

4.2 *Grey exponential model*

Grey model analysis permits the prediction of system behavior, such as extrapolation of data. The application of Grey models will be shown for the so-called Grey exponential model, sometimes called GM(1,1), which is the simplest model. The original data set is defined as:

$$X^{(0)} = \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(k) \end{pmatrix}$$

whereas the sum of the data is evaluated with:

$$X^{(1)}(m) = \sum_{n=1}^{m} x^{(0)}(n), \ m = 1 \dots k$$

Producing the sum is called the Accumulated Generating Operation (AGO). It yields to a continually growing series and smoothes the data. Smoothed data is considered as data with a higher information density and a decrease of random disturbances. The exponential model is based on the following differential equation:

$$\frac{dx^{(1)}}{dt} + a \cdot x^{(1)}(t) = b \, .$$

Considering the connection between the two data sets, one can assume:

$$\frac{dx^{(1)}}{dt} \to x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k+1)$$

and using a whitenisation process

$$x^{(1)}(t) \to z^{(1)}(k) = 0.5 \cdot x^{(1)}(k) + 0.5 \cdot x^{(1)}(k-1)$$

This yields to the equations:

$$x^{(0)}(2) = -a \cdot z^{(1)}(2) + b$$
$$x^{(0)}(3) = -a \cdot z^{(1)}(3) + b$$
$$x^{(0)}(4) = -a \cdot z^{(1)}(4) + b$$
...

Putting this into matrix form

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \end{bmatrix}, \; Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \end{bmatrix}$$

one gets:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T \cdot B)^{-1} \cdot B^T \cdot Y \,.$$

Now considering the solution of the aforementioned differential equation and using the starting information, one gets a general formula:

$$x^{(1)}(k) = C_1 \cdot e^{-a \cdot k} + \frac{b}{a} \,.$$

Considering boundary conditions given, one gets:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) \cdot e^{-a \cdot k} + \frac{b}{a}$$

and the estimator for the original data using the Inverse Accumulated Generating Operation data is:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \,.$$

The authors have carried out several tests using small amounts of data with random disturbances to test the capability of this procedure. In principal it can not be stated, that Grey models under all circumstances perform better than pure statistical methods, such as regression. Still this procedure can be applied successfully. According to the experience of the authors the success depends very much on the differential equation chosen, which indeed needs some assumption about the behavior of the data. In the last few years many models have been developed, such as UIRGM(1,1), GFM(1,1), GDM(2,2,1), GM(0,N), GM(2,2), DGDM(1,1,1) and DGDMMI(1,1,1) to mention only a few. There exists also a grey Verhulst. The Verhulst model forms in addition, at least to some extend the basis for the logistic distribution, which has recently recommended for the estimate of extreme water heights (Van Gelder & Neykov 1998).

4.3 *Grey Verhulst model*

The Grey Verhulst model is based on the following differential equation:

$$\frac{dx^{(1)}}{dt} + a \cdot x^{(1)}(t) = b \cdot \left(x^{(1)}(t) \right)^2$$

The application of the Grey theory in the preceding chapter yields to the following estimation formula:

$$\hat{x}^{(1)}(k) = \frac{\frac{a}{b}}{1 + \left(\frac{a}{b} \cdot \frac{1}{x_0(1)} - 1 \right) \cdot e^{a \cdot k}} \,.$$

The distribution based on the same differential equation is the logistic distribution:

$$P(x) = \frac{1}{1 + \left(\frac{1}{x^{(0)}(1)} - 1 \right) \cdot e^{\frac{x-m}{b}}}$$

5 APPLICATION OF GREY MODELS TO THE DATA

First the development of the extreme water levels over time will be investigated using standard regression models. Foremost a linear regression is used. Especially the long data sets show a lower increase of extreme sea water level rise over time than the short data sets. This statement might be based on the fact, that the increase of extreme sea water has recently accelerated and the linear regression is not able to consider this fact. The assumption could indeed be proven, since reducing the long data sets to short ones by excluding old data sets yields to an approach of the slope of linear regression for all data sets. Choosing a quadratic regression formulae considering this change of curvature fails in two cases (Data set Harlingen and Den Helder) because the curvature of the regression formulas are different compared to the other cases.

In a third step an exponential law is assumed. This assumption has been made considering an increase of

Table 2. Estimated coefficients for linear and exponential regression and Grey exponential model for systematical change over time of extreme water level heights.

	a (slope)		*a* (exponent)[1]	
Setting	LM[2]	LM[3]	EM[4]	GEM[5]
Delfzijl	0.0192	0.0340	0.00010	0.0001009
Harlingen	0.0277	0.0277	0.00020	0.0001651
Hoek v. Holl.	0.0151	0.0262	0.00010	0.0000973
Vlissingen	0.0193	0.0269	0.00008	0.0000783
Den Helder	0.0248	0.0248	0.00020	0.0001402
Standard dev.	0.0056	0.0036	0.0000	0.0000353
C.o.V.	0.243	0.127	0.433	0.304

[1] The exponent is negative defined in the model; therefore a minus is added to the factor.
[2] LM: Linear model.
[3] Long data sets have been shortened to homogenize the length and age of the data sets.
[4] Exponential model.
[5] Grey exponential model.

anthropogenic effects to the climate and to extreme water levels. In addition, the Grey exponential model has been used to estimate extreme water level rise over time as well. Table 2 shows, that the values of the data sets are reasonably close. It is noteworthy, that the C.o.V. of the exponent based on the Grey exponential model is 1/3 lower compared to the C.o.V. of the exponent of the exponential regression formulae over all data sets.

The results fit very well to the known increase of sea level by 1 to 2 mm per year value. But in contrast, Pugh 2004 has stated, that the sea rise level has not increased over the 20th century. In this investigation, an increase of sea rise level can be found.

To carry out further investigations about extreme water levels the data has been adopted considering the rise in sea level over time. The adapted data has then been used for application of grey models. This Grey model investigation about the estimation of extreme sea water levels yielded to the following coefficients of the Grey exponential equation (table 3).

The Grey exponential model consists of straight line in the logarithmic plot. In contrast the Verhulst model consists of a concave curvature and yields to comparable higher extreme water levels (Figure 6). It has to be added here, that the results of the Verhulst model depend very much on the threshold value and the transformation of the data. The highly sensible behavior of the Verhulst model for some initial values is known; see for example Hijmans 1995. Further research is needed to establish an optimal threshold value.

The application of the Grey model for the merged data sets gives an exponent of −0.0139, whereas traditionally goodness of fit gives an exponent value of

Table 3. Estimated coefficients for Grey exponential and Verhulst model (normalized data sets).

	Grey exponential		Grey Verhulst mode	
Location	*a*	*b*	*a*	*b*
Delfzijl	0.17486	0.17756	−0.38545	−0.39181
Harlingen	0.17110	0.17539	−0.38309	−0.38738
Hoek	0.26564	0.26901	−0.5649	−0.56977
Vlissingen	0.26789	0.27302	−0.60663	−0.61199
Den Helder	0.22932	0.23335	−0.48492	−0.48975

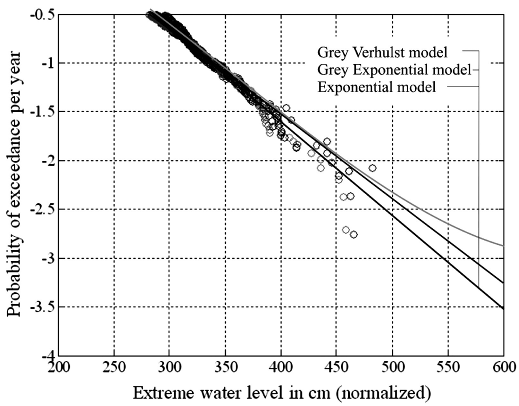

Figure 6. Principal decade logarithmic probability plot of extreme water level of the five locations using exponential law, grey exponential law and Verhulst model. Note: The behavior of the Verhulst model depends strongly on the threshold chosen.

−0.0152. The Verhulst model yields to −0.1656 but only shows a good fit at the tail of the data.

Based on the results the following can be stated:

(1) All data sets show a comparable increase in the extreme water height over time.
(2) Two data sets groups can be detected with Grey models very well.
(3) The estimation of lower percentiles values depends on the chosen function. This drawback is not only valid for the classical ways, such as regression or application of distribution types but also for Grey models. Therefore the consideration of subjective knowledge and additional information seems to be compulsory. Van Gelder has done that for example in 1996. Also in many other fields, such as climate research indirect measures are heavily used (Pfister 2001).
(4) If a procedure to transform the data can be found, the extrapolation of data points can be backed up and should be used.

Table 4. Estimated probability of exceedance of 10^{-4} per year of extreme water height levels for the different locations in cm in comparison to other publications. Basispeilen are the official used values.

Location	Basispeilen	Exponential	Grey exponential
Delfzijl	613	705	720
Harlingen	501	620	635
Hoek	500	470	550
Vlissingen	545	560	610
Den Helder	441	460	500

(5) Both, the Grey Verhulst model and the Grey exponential model experience difficulties to follow the exact shape of the data at the tail. Even more, the Grey Verhulst model was not able to fit the first part of the data well. Here further Grey models adapted to the special task have to be developed.
(6) The type of distribution can be evaluated with a Grey relation coefficient. This test yields to be done but should be carried out.

Finally the results for estimation of extreme sea water levels for the five locations are given in table 4 as probability of exceedance of 10^{-4} per year. These values can be directly compared to other publications, such as van Urk 1993.

All values calculated are higher then the published Basispeilen. This has two reasons: first the exponential distribution has been used heavily, which is known to overestimate the low probabilities, second the Verhulst model shows an increase of extreme water levels at low probabilities due to change of curvature. In addition a statistical investigation by van Gelder 1996 using historical storm surge levels showed an increasing water level for Hoek van Holland up to 545 cm using an exponential distribution, which would fit very well to the calculated value of 550 cm.

The investigation showed that Grey models can be applied for the prediction of extreme water levels. Nevertheless, a final answer can not be reached with this procedure as with others. But comparing the results with other investigations, it seems to be, that extreme water levels are underestimated currently.

REFERENCES

Deng, J. 1988. Essential topics on Grey Systems – Theory and Application. China Ocean Press, Bejing

Guo, R. & Love, C.E. 2005. Keynote lecture: Grey repairable system analysis. K. Kolowrocki (Edr.) Advances in Safety and Reliability – ESREL 2005, London: Taylor and Francis Group. page 753–765

Guo, R. & Dunne, T. 2005. Stochastic system age processes. K. Kolowrocki (Edr.) Advances in Safety and Reliability – ESREL 2005, London: Taylor and Francis Group. page 745–752

Chen, X.; Zhu, H. & Chen, C. 2005. Structural damage identification using test static data based on grey system theory. Journal of Zhejiang University Science, 2005 6A (8), page 790–796

Ng, D.K.W. 1995. Contrasting Grey System Theory to Probability and Fuzzy 1995. ACM SIGICE Bulletin, Volume 20, Number 3, January 1995, page 3–9

Jun, X. & Baoqing, H. 1997. A grey system model for predicting trend change of urban waste water load. Journal of Environmental Hydrology, Paper 2, Volume 5, page 1–10

Bai, G.; Mao, J. & Lu, G. 2004. Grey transportation problem. Kybernetes, Vol. 33, No. 2, page 219–224

You-xin, L.; Long-ting, Z.; An-hui, C. & Zhi-ming, H. 2004. Grey GM(1,1) model with function-transfer method and application to energy consuming prediction. Kybernetes, Vol. 33, No. 2, page 322–330

Tsaur, R.-C. 2005. Fuzzy grey GM(1,1) model under fuzzy systems. International Journal of Computer Mathematics, Volume 82, Number 2, February 2005, page 141–149

Yan, T.; Zhou, C. & Yan, L. 2005. Prognoses of environmental geoscientific problems by grey systems. Journal of Geosciences of China, Vol. 12, No. 1

Van Gelder, P.H.A.J.M. 1996. A Bayesian analysis of extreme water levels along the Dutch coast Using flood historical data, Stochastic Hydraulics, Vol. 7, page 243–251

Kauker, F. & Langenberg, H. 2000. Two models for the climate change related development of sea levels in the North Sea – a comparison. Climate Research, Vol. 15, page 61–67

Jensen, J.; Mudersbach, C.; Koyiar, C. & Bork, I. 2004. Modeling of extreme storm surge weather situations in the North Sea and its statistical analysis. Littoral, 20–24. September 2004, Aberdeen, U.K.

Tel, E.; Garcia, M.J. & Molinero, J. 2005. Statistical Study of extreme sea levels on the north Spanish Coast from 1943–2001, Instituto Español de Oceanografía, IEO

Van Gelder, P.H.J.M. & Neykov, N.M. 1998. Regional frequency analysis of extreme water levels along the dutch coast using L-moments: a preliminary study. In: Stochastic models of hydrological processes and their applications to problems of environmental preservation, page 14–20

Woodworth, P.L. Extreme Sea Levels, Presentation, www.pol.ac.uk

Munz, D. & Fett, T. 1989. Mechanisches Verhalten keramischer Werkstoffe. Berlin: Springer, 1989 (Werkstoff-Forschung und -Technik, Bd. 8)

Pandey, M.D.; van Gelder, P.H.A.J.M. & Vrijling, J.K. 2003. Bootstrap simulations for evaluating the uncertainty associated with peaks-over-threshold estimates of extreme wind velocity. January 2003, ENVIRONMETRICS, Vol. 14, page 27–43

Webbers, P.B.; Voortman, H.G.; van Gelder, P.H.A.J.M. & Vrijling, J.K. 2003. Synthesis of multivariate extreme sea climate. ICASP 9, Applications of Statistics and Probability in Civil Engineering, Der Kiureghian, Madanat & Pestana (eds.), Volume 1, 2003, Millpress, Rotterdam, page 889–896

d'Angremond, K. & van Roode, F.C. 2004. Breakwaters and Closure Dams. Spon Press, Delft, 2004

Pugh, C. 2004. Changing Sea Levels – Effects of Tides, Weather and Climate. Cambridge University Press, Cambridge 2004

Voortmann, H.G. 2003. Risk-based design of large-scale flood defense systems. PhD work, TU Delft, 2003

Bobee, B. & Ashkar, F. 1991. The Gamma family and derived distributions applied in hydrology. Littleton: Water resources publications
van Urk, A. 1993. De basispeilen langs de nederlandse kust. Ministerie van Verkeer en Waterstaat. Directoraat-Generaal Rijkswaterstaat, Dienst Getijdewateren. Rapport DGW-93.026, April 1993
Pfister, C. 2001. Klimawandel in der Geschichte Europas. Zur Entwicklung und zum Potenzial der Historischen Klimatologie. Österreichische Zeitschrift für Geschichtswissenschaften. 12. Jahrgang, Heft 2, page 7–43
Hijmans, J. 1995. Unpredictability in dynamic evolution models: Examples from population dynamics. Chance and Uncertainty, Vossiupers AUP, Amsterdam, page 39–59.