Philip Green Wright, the Identification Problem in Econometrics, and Its Solution

A celebration in honor of the 150th anniversary of the birth of Philip Green Wright, Tufts 1884

October 3, 2011

James H. Stock
Department of Economics
Harvard University

I. A Modern Textbook Treatment of the Identification Problem and Instrumental Variables Regression

A. The Identification Problem

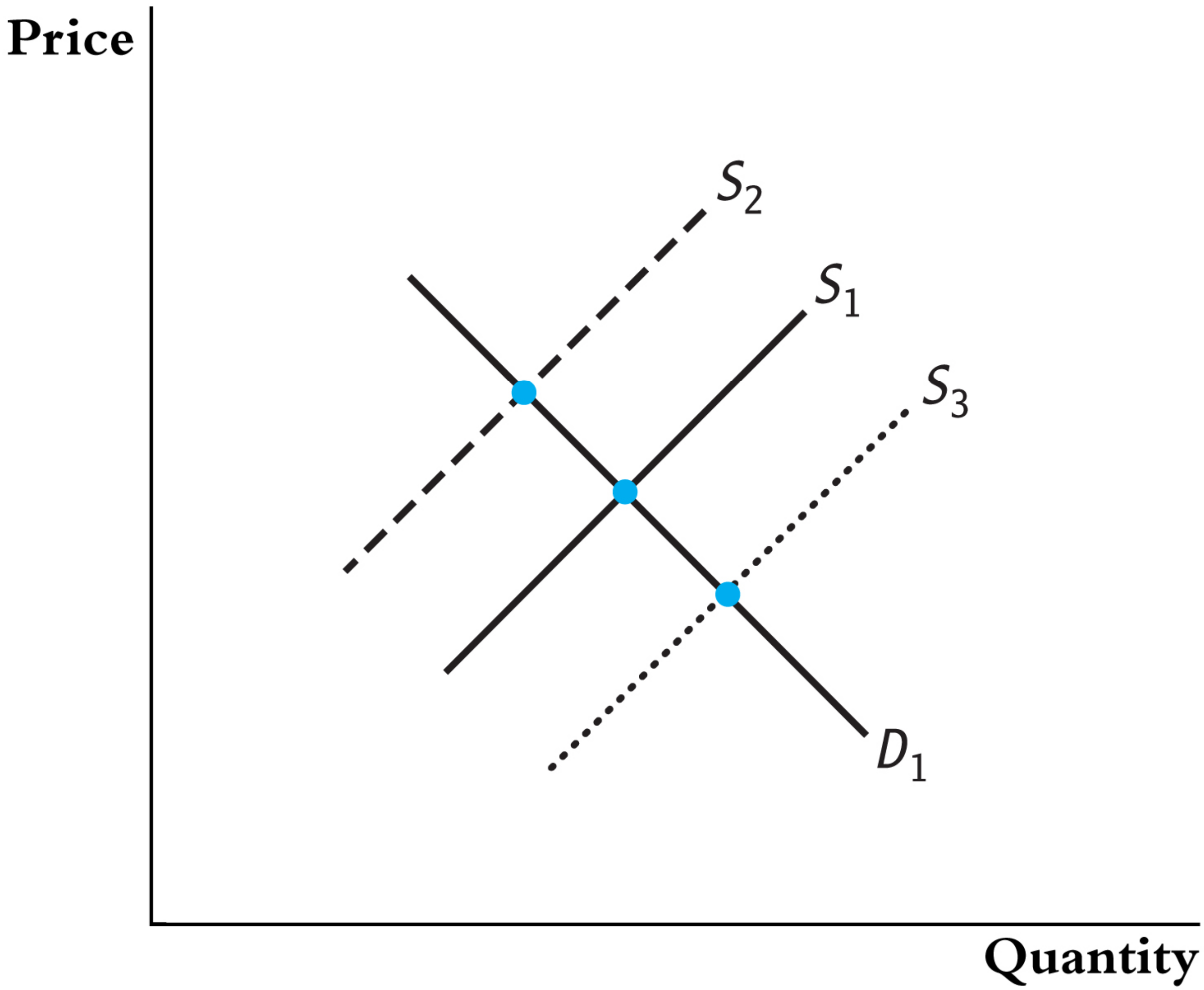
Suppose you are interested in estimating the elasticity of demand:

$$q = \beta_1 p + u^D \qquad \text{(demand)}$$

where $q = \ln(Q)$ and $p = \ln(P)$, deviated from means, so

$$\beta_1 = \frac{d\ln Q}{d\ln P} = \frac{dQ/Q}{dP/P} = \frac{\%\text{ change in } Q}{\%\text{ change in } P}$$

Data: observations on (p, q) (time series or cross-section)



(c) Equilibrium price and quantity when only the supply curve shifts

The identification problem:

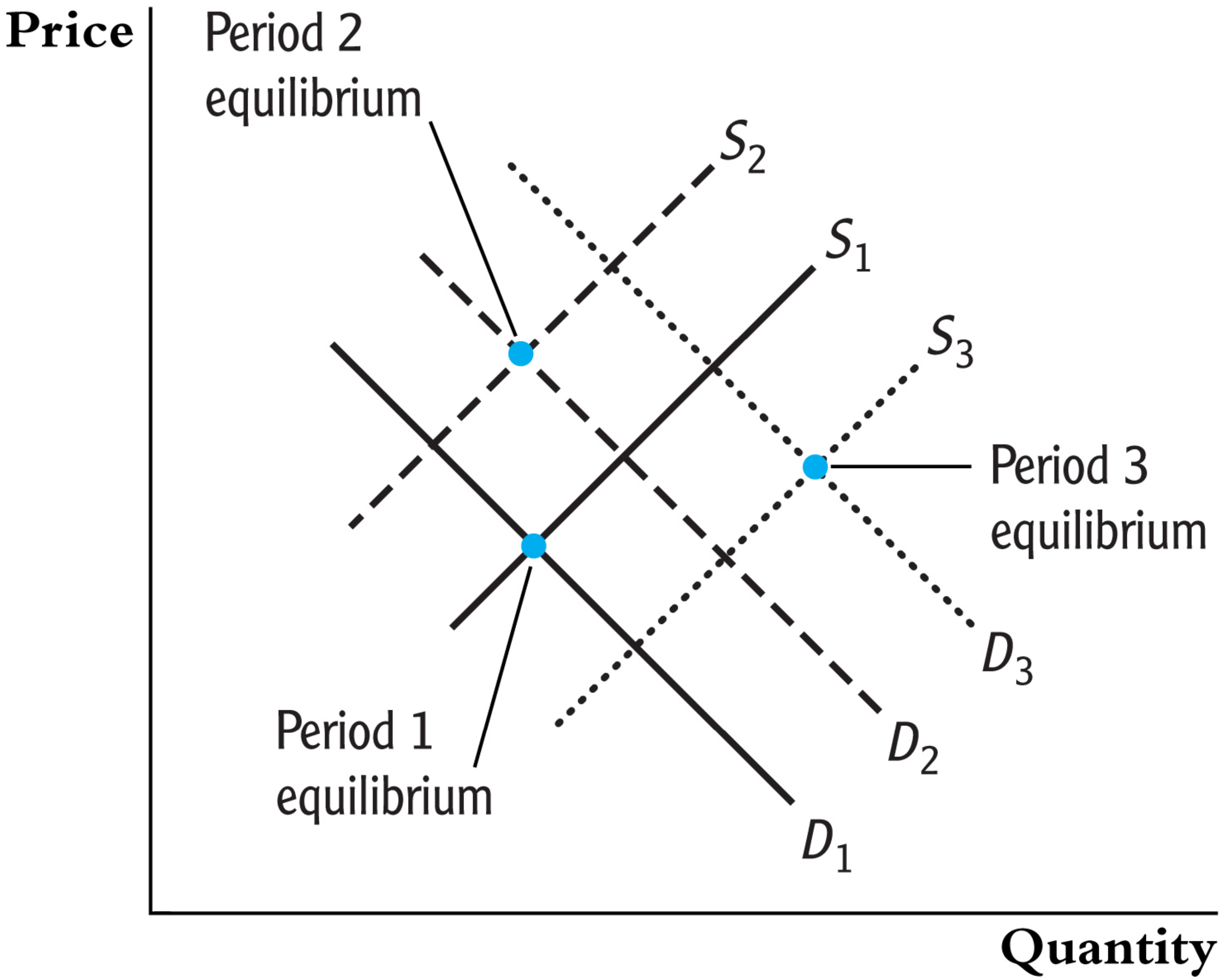
$$q = \beta_1 p + u^D \qquad \text{(demand)}$$

$$q = \beta_2 p + u^S \qquad \text{(supply)}$$

As before $p = \ln(P)$ and $q = \ln(Q)$, deviated from their means.

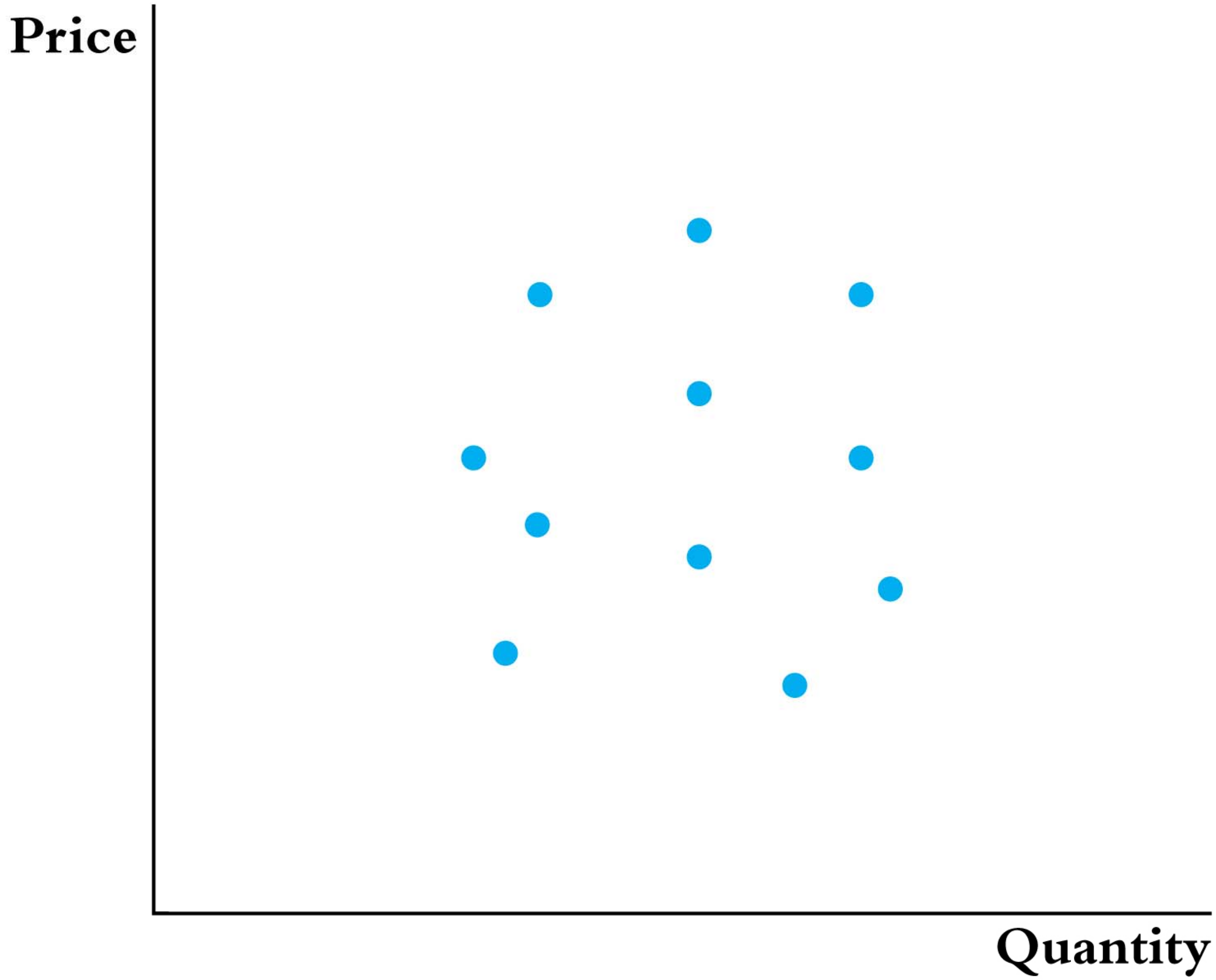
p and q are both endogenous: determined within the system

Simultaneous causality bias in the OLS regression of q on p arises because price and quantity are determined by the interaction of demand *and* supply:



(a) Demand and supply in three time periods

This interaction of demand and supply produces data like…



(b) Equilibrium price and quantity for 11 time periods

The algebra of the identification problem:

$$q = \beta_1 p + u^D \qquad \text{(demand)}$$
$$q = \beta_2 p + u^S \qquad \text{(supply)}$$

Solve for q and p:

$$q = (\beta_2 - \beta_1)^{-1}\beta_2 u^D - (\beta_2 - \beta_1)^{-1}\beta_1 u^S$$
$$p = (\beta_2 - \beta_1)^{-1} u^D - (\beta_2 - \beta_1)^{-1} u^S$$

$\mathrm{corr}(p, u^D) \neq 0$, that is, $E(pu^D) \neq 0$, so OLS is biased and inconsistent.

B. Instrumental variables regression

$$q = \beta_1 p + u^D \qquad\qquad \text{(demand)}$$

Suppose there is an instrument Z such that:

1. $E(Z,p) \neq 0$ (*instrument relevance*)
2. $E(Zu^D) = 0$ (*Instrument exogeneity*)

Then
$$\begin{aligned} \mathrm{E}(Zq) &= \mathrm{E}[Z(\beta_1 p + u^D)] \\ &= \beta_1 \mathrm{E}(Zp) + \mathrm{E}(Zu^D) \\ &= \beta_1 \mathrm{E}(Zp) + 0 \text{ by instrument exogeneity} \end{aligned}$$

so
$$\beta_1 = \frac{E(Zq)}{E(Zp)} \quad \text{and} \quad \hat{\beta}_1^{IV} = \frac{\sum_{i=1}^{n} Z_i q_i}{\sum_{i=1}^{n} Z_i p_i}$$

What constitutes a valid instrument?

$$q = \beta_1 p + u^D \qquad \text{(demand)}$$

- A valid instrument Z is correlated with the endogenous regressor (p) but uncorrelated with u^D.
- An example in the supply-demand context:

  $$q = \beta_1 p + u^D \qquad \text{(demand)}$$

  $$q = \beta_2 p + \beta_3 Z + u^S \qquad \text{(supply)}$$

  A valid instrument shifts supply, but not demand.

  In an agricultural market, Z might be rainfall
- Violations of the two conditions lead to:
  - Relevance: weak instruments
  - Exogeneity: inconsistency

II. Backdrop:
Some Economics and Statistics from the Late 19th Century

Economics: demand elasticities

Jevons (1871)

Marshall (1885, 1890)

Statistics: regression analysis

Legendre (1805), Gauss (1809) (OLS)

Galton (1877) (heredity)

Galton (1890) (correlation)

Pearson (1896) (correlation)

Persons (*JASA*, 1910)

Persons (*JASA*, 1910) (concluding para.)

The various illustrations which have been cited show the importance of questions of correlation in economics. The ordinary graphic method of measuring correlation is inadequate. The coefficient of correlation is simple and yet is sensitive to small changes. It has been used in many fields of statistics by Galton, Pearson, Yule, Hooker, Elderton and others. The experience of these writers warrants the adoption of the coefficient of correlation by economists as one of their standard averages.

III. Demand Analysis, 1900-1914

Hooker (1905) (multiple regression and R^2)

Mackeprang (1906)

Persons (1910)

Moore (1914)

A COMPARISON OF ELASTICITIES OF DEMAND OBTAINED BY DIFFERENT METHODS[1]

By Henry Schultz

A summary of this paper was read before the joint meeting of the Econometric Society with the American Statistical Association, Washington, D. C., December 29, 1931.

I. INTRODUCTION

I believe that inductions with regard to the elasticity of demand, and deductions based on them, have a great part to play in economic science.—Alfred Marshall, "On the Graphic Method in Statistics," *Jubilee Volume of the Roy. Statist. Soc.* (1885), p. 260.

Although these dicta of the great teacher were written nearly fifty years ago, and although the great Cournot, Marshall's main source of inspiration, had expressed a similar conviction over half a century before,[2] it was not until 1914 that the first definitive attack on the problem of deriving the elasticity of demand from statistics was made. In that year Professor Henry L. Moore published his *Economic Cycles: Their Law and Cause*, in which he obtained equations expressing the relations between the demands for, and the prices of, corn, hay, oats, and potatoes; determined the precision of these equations as formulas for estimating prices; and measured the elasticity of demand for each crop. True, we now know that Moore's attempt to derive the numerical

Moore (1914, p. 82)

tion, we are able, by means of the laws of demand for the several commodities, to measure their respective degrees of elasticity of demand. It will be recalled that, in the form in which the laws of demand have been presented in preceding pages, the variable x has been taken to represent the relative change in the quantity of the commodity, and the variable y, the corresponding relative change in the price. The coefficient of the elasticity of demand, therefore, is equal to $\frac{dx}{dy}$ when x is zero. All that is needed to obtain the measure of the degree of elasticity of demand is to differentiate y with respect to x in the equation to the law of demand, place $x =$ zero, and then take the reciprocal of the result.

Moore (1914, p. 82), ctd.

The method may be illustrated in case of the four representative commodities, corn, hay, oats, and potatoes. The law of demand for corn—see Figure 17—is

$$y = .94 - 1.0899x + .02391x^2 - .000234x^3$$

Therefore, $\frac{dy}{dx} = -1.0899 + 2(.02391)x - 3(.000234)x^2$

When $x = 0$, $\frac{dy}{dx} = -1.0899$, $\frac{dx}{dy} = -\frac{1}{1.0899} = -.92$

and consequently the coefficient of the elasticity of demand for corn is —.92. Since the law of demand for

Moore (1914, p. 13)

To determine the value of a_1, multiply throughout by cos kt and integrate between limits o and T.

$$\int_0^T f(t)\cos kt\,dt = A_0\int_0^T \cos kt\,dt + a_1\int_0^T \cos^2 kt\,dt + b_1\int_0^T \sin kt\cos kt\,dt + \ldots$$

Or $\int_0^T f(t)\cos kt\,dt = a_1\int_0^T \cos^2 kt\,dt$, since $\int_0^T \cos kt\,dt$ and $\int_0^T \sin kt\cos kt\,dt$ are both equal to zero and all the other terms on the right-hand side of the equation, according to our lemma, disappear. But

$$\int_0^T \cos^2 kt\,dt = \int_0^T \frac{1+\cos 2kt}{2}\,dt = \tfrac{1}{2}\left[t + \frac{\sin 2kt}{2k}\right]_0^T = \frac{T}{2}$$

and as a result, we have

$$a_1\frac{T}{2} = \int_0^T f(t)\cos kt\,dt, \text{ or } a_1 = 2\,\frac{\int_0^T f(t)\cos kt\,dt}{T}.$$

Therefore a_1 is equal to twice the mean value of the product $f(t)\cos kt$.

Moore (1914, p. 110 and 114):

A New Type of Demand Curve

…

The graph of the law of demand for pig-iron is given in Figure 24. The correlation between the percentage change in the product and the percentage change in the price is $r = .537$. The equation to the law of demand is $y = .5211x - 4.58$, the origin being at (o, o). Our representative crops and representative producers' good exemplify types of demand curves of contrary character. In the one case, as the product increases or decreases the price falls or rises, while, in the other case, the price rises with an increase of the product and falls with its decrease.

IV. The Statement of the Identification Problem

Lenoir (1913)

Wright (May, 1915)

Lehfeldt (Sept., 1915)

….

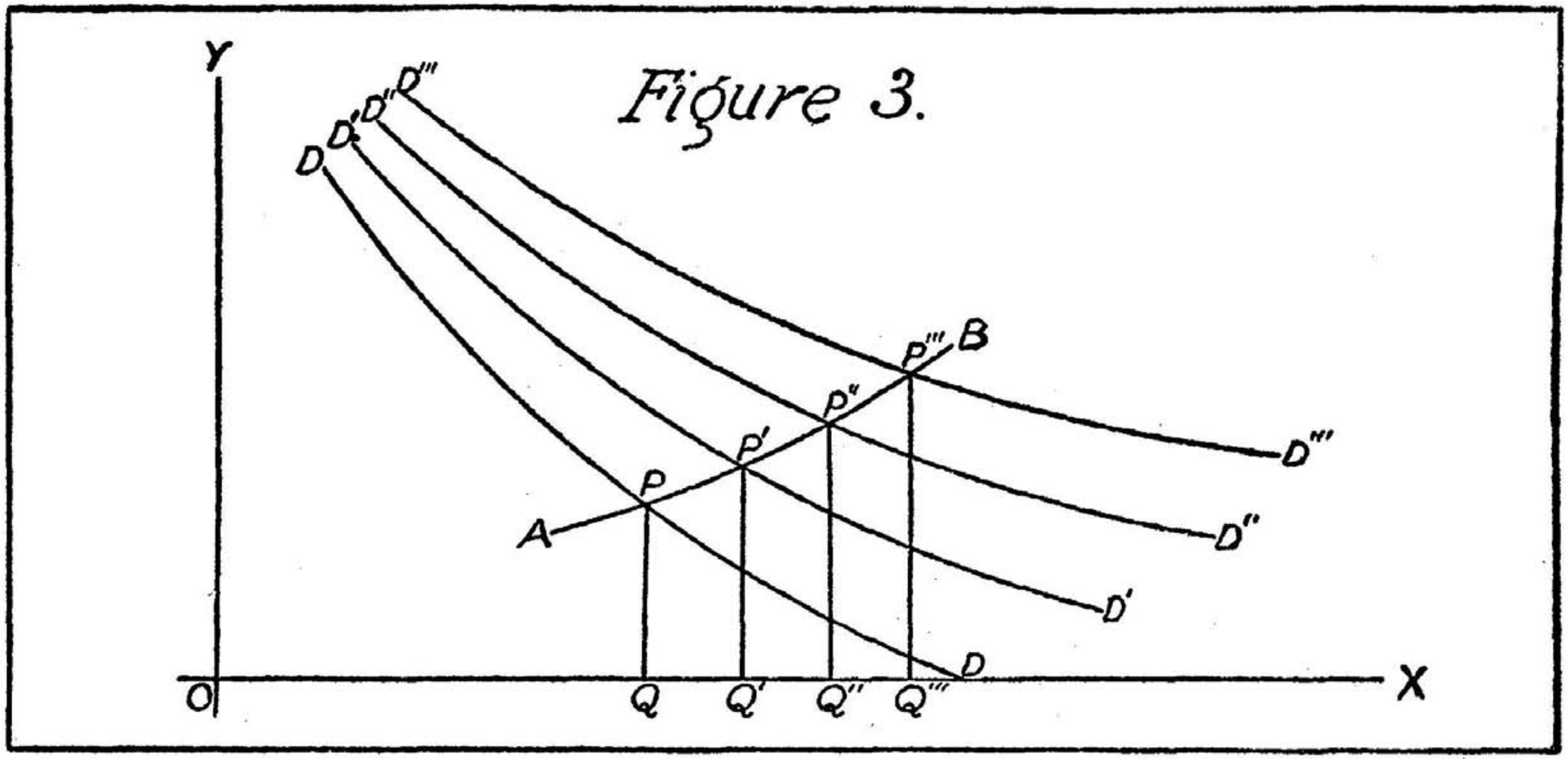
Wright (1928, Appendix B)

Working (*QJE* 1927)

Wright (*QJE*, May 1915, p. 638)

Professor Moore's studies in demand curves illustrate the principle that the need of checking statistical inductions by abstract reasoning is quite as great as that of verifying abstract reasoning by statistics. The demand curves for crops harmonize perfectly with theory: the conditions of demand remain approximately constant; there is an increased output of crops (very probably due to heavier rainfall); with the diminishing utility due to this increased supply, the marginal utility and hence the price falls. But how about the "new type," the ascending demand curve for pig iron, is it so hopelessly irreconcilable with theory? Not at all. The conditions of demand are changed (very probably by improved business conditions) in the direction of a rapid and continuous increase. This would be indicated, conformably to theory, by shifting the entire demand curve progressively to the right. The ordinates to this shifting curve, corresponding with the lagging supply, will yield Professor Moore's "new type." Thus (see Figure 3):

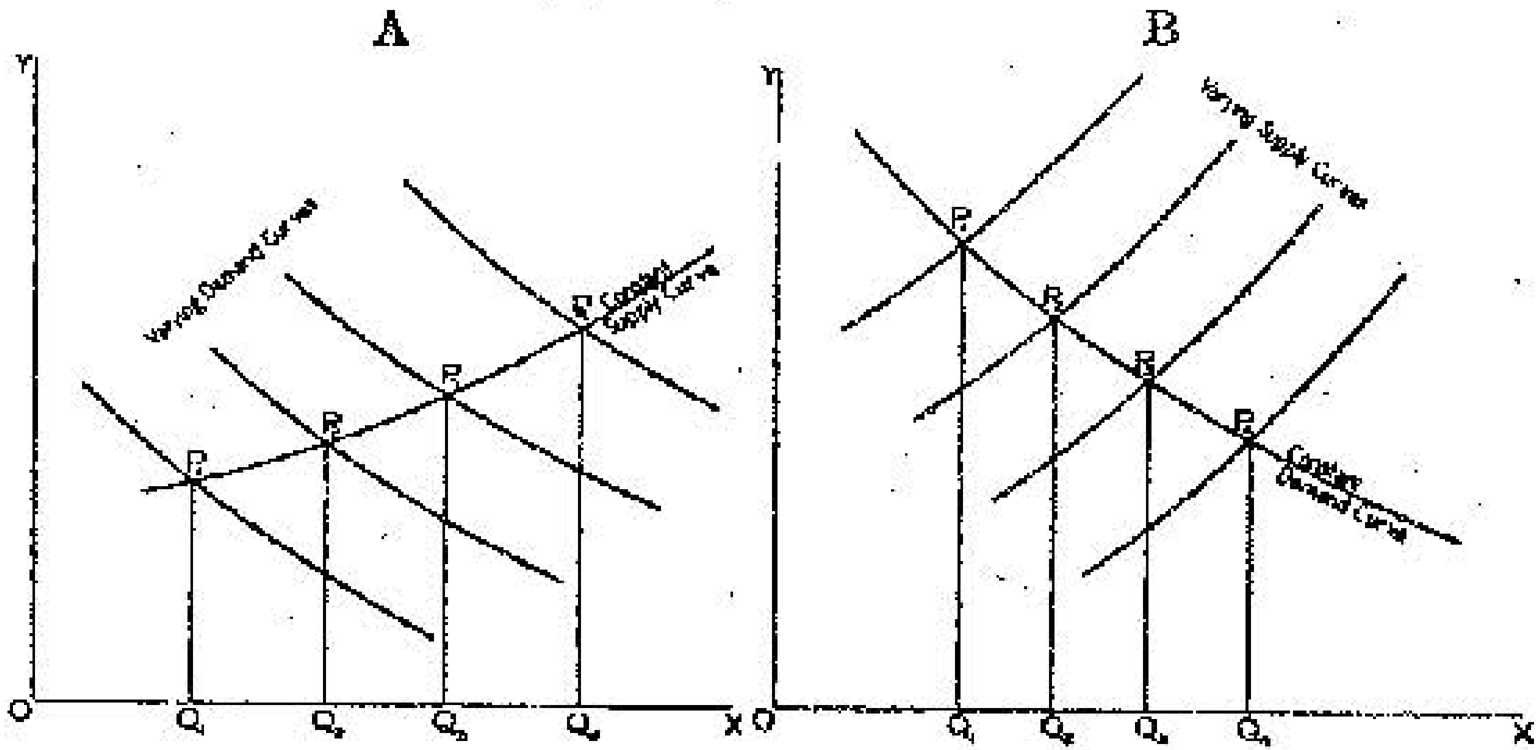
Wright (*QJE*, May 1915, p. 638), ctd



D, D', D'', etc., represent the conditions of increasing demand. OQ, OQ', OQ'', etc., the corresponding lagging supply. PQ, $P'Q'$, $P''Q''$, etc., the marginal utilities (and hence prices) corresponding with these supplies, and AB the "new type" of demand curve.

Wright (1928, Appendix B)

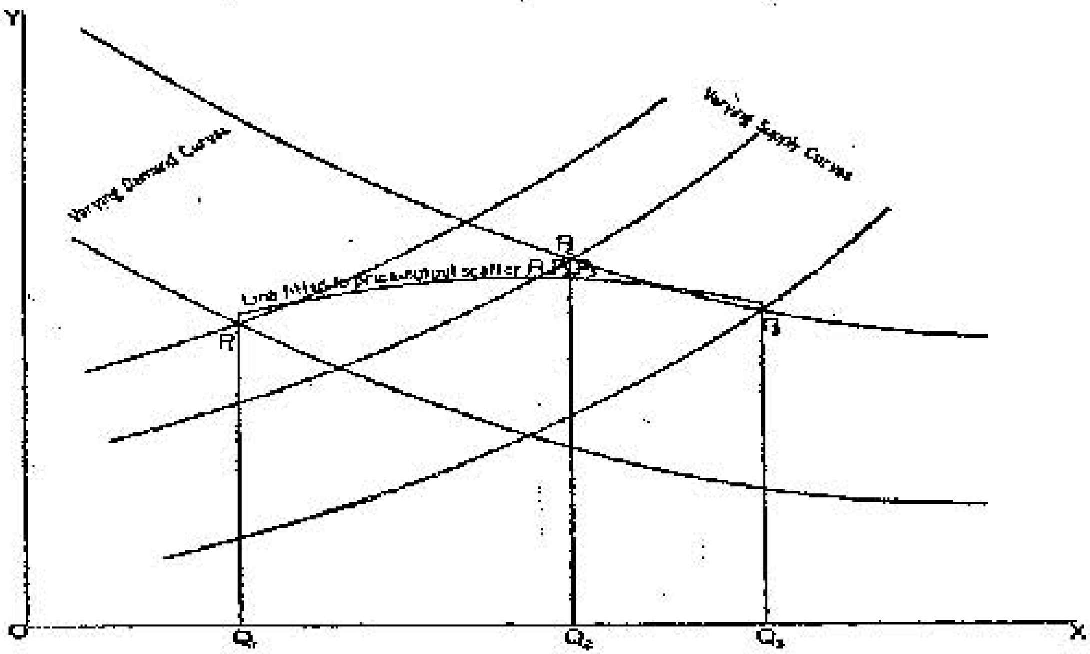
FIGURE 3. PRICE-OUTPUT DATA REVEAL—
(A) SUPPLY CURVE
(B) DEMAND CURVE



Wright (1928, Appendix B)

If both supply and demand conditions change, price-output data yield no direct information as to either curve. (Figure 4.)

FIGURE 4. PRICE-OUTPUT DATA FAIL TO REVEAL EITHER SUPPLY OR DEMAND CURVE.



Unfortunately for our problem, the case represented by Figure 4 is the more common, and even if either curve does remain fixed during the period covered by the observations there is no certain way of knowing this fact in advance.[5]

Lehfeldt (*Economic Journal*, Sept. 1915, p. 410)

There are very few workers in this field as yet, which makes it all the more of a pity that one of them should lay himself open to accusations of unsoundness. Research along these lines is not easy, for it requires a thorough grasp of the mathematical theory of statistics, patience to do the lengthy arithmetic involved, and ceaseless and acute criticism of the mathematical processes in the light of common sense and everything that is known about the subject-matter. Prof. Moore is weak in this last arm; he has, let us say, the artillery of mathematics and the plodding infantry of numbers, but not the aerial corps to save him from making attacks in wrong directions.

Lehfeldt (*Economic Journal*, 1915, p. 410)

The author thinks he has discovered a new type of demand curve, sloping the opposite way to the usual kind! But the curve (for pig-iron) on p. 115 is not a demand curve at all, but much more nearly a supply curve. It is arrived at by the intersection of irregularly fluctuating demands on supply conditions that, on the whole, may be regarded as steady but for a secular growth, whereas the whole line of argument with regard to crops is based on considering the intersection of irregular supply (due to fluctuations of weather) with a steady demand.

V. The 1920s: Solutions to the Identification Problem

V. The 1920s: Solutions to the Identification Problem

1. Ignore it.

    Schultz (1928; *Econometrica*, 1933)

    Sewall Wright (1925) (Hog/corn correlations)

    P.G. Wright (*JASA* 1929) review of Schultz (1928)

Wright (*JASA*, 1929)

CHART I

SUPPLY CONDITIONS STATIONARY; DEMAND CONDITIONS CHANGE; DATA REVEAL SUPPLY CURVE

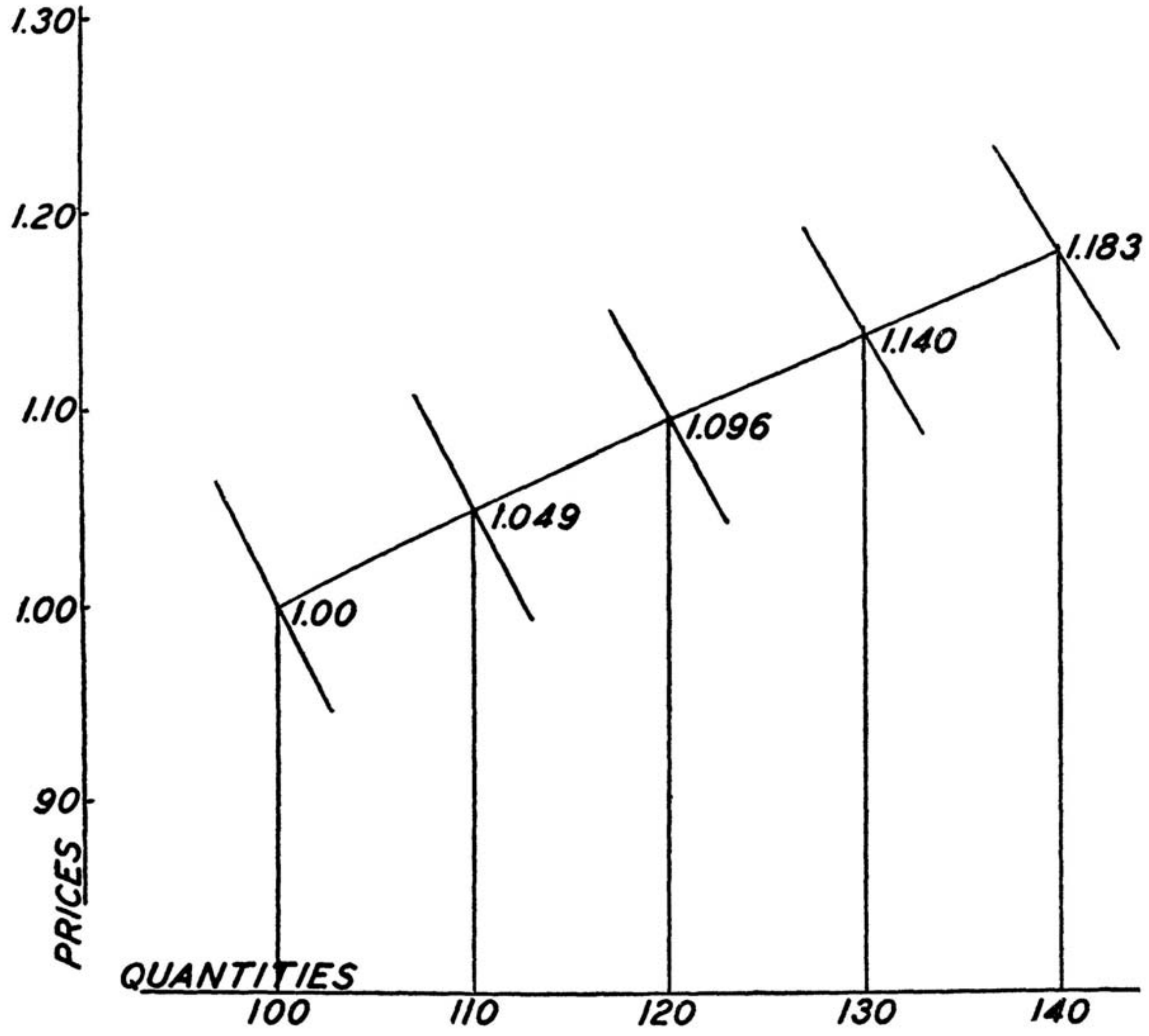


CHART II

DEMAND CONDITIONS STATIONARY; SUPPLY CONDITIONS CHANGE; DATA REVEAL DEMAND CURVE

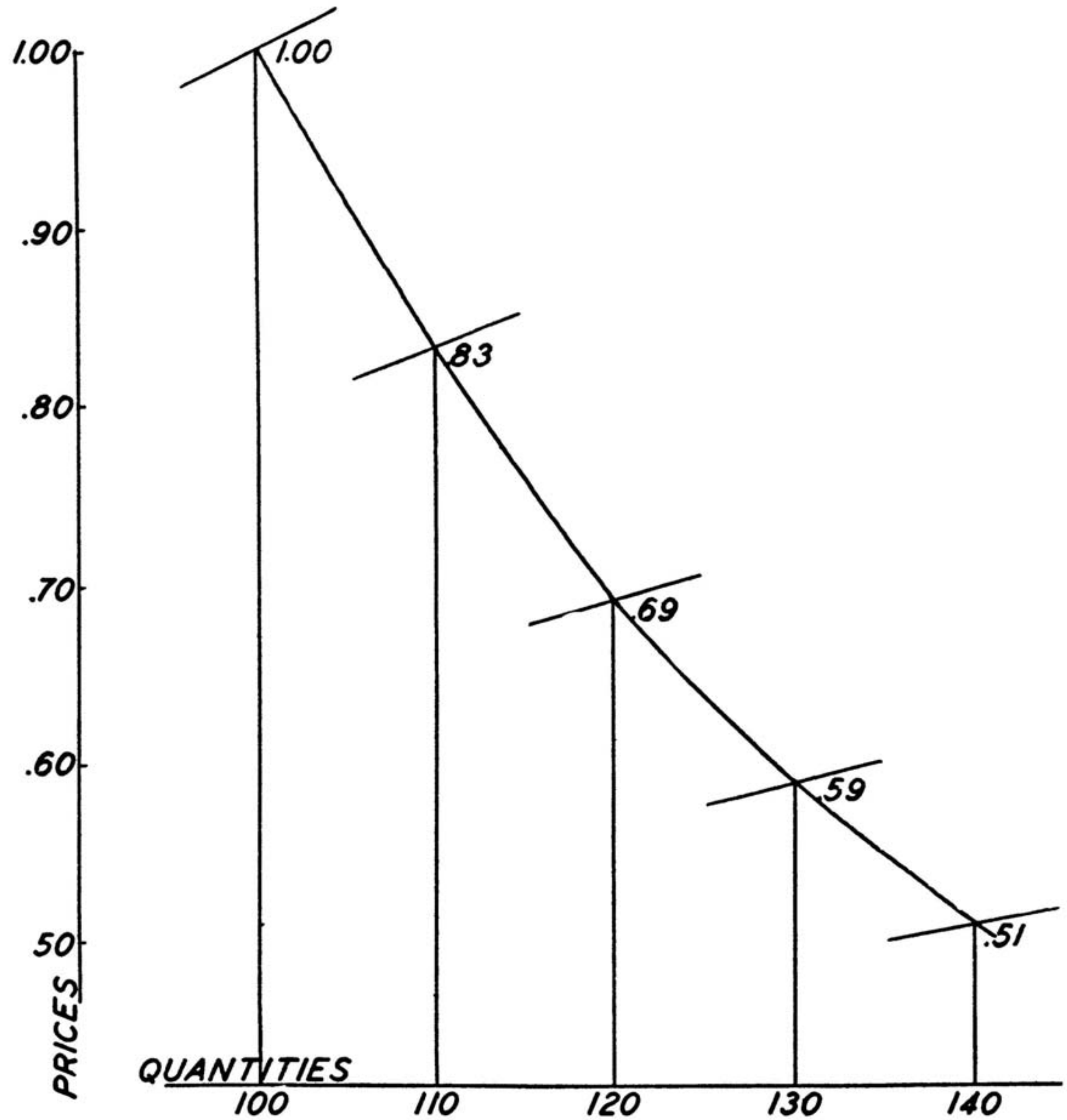
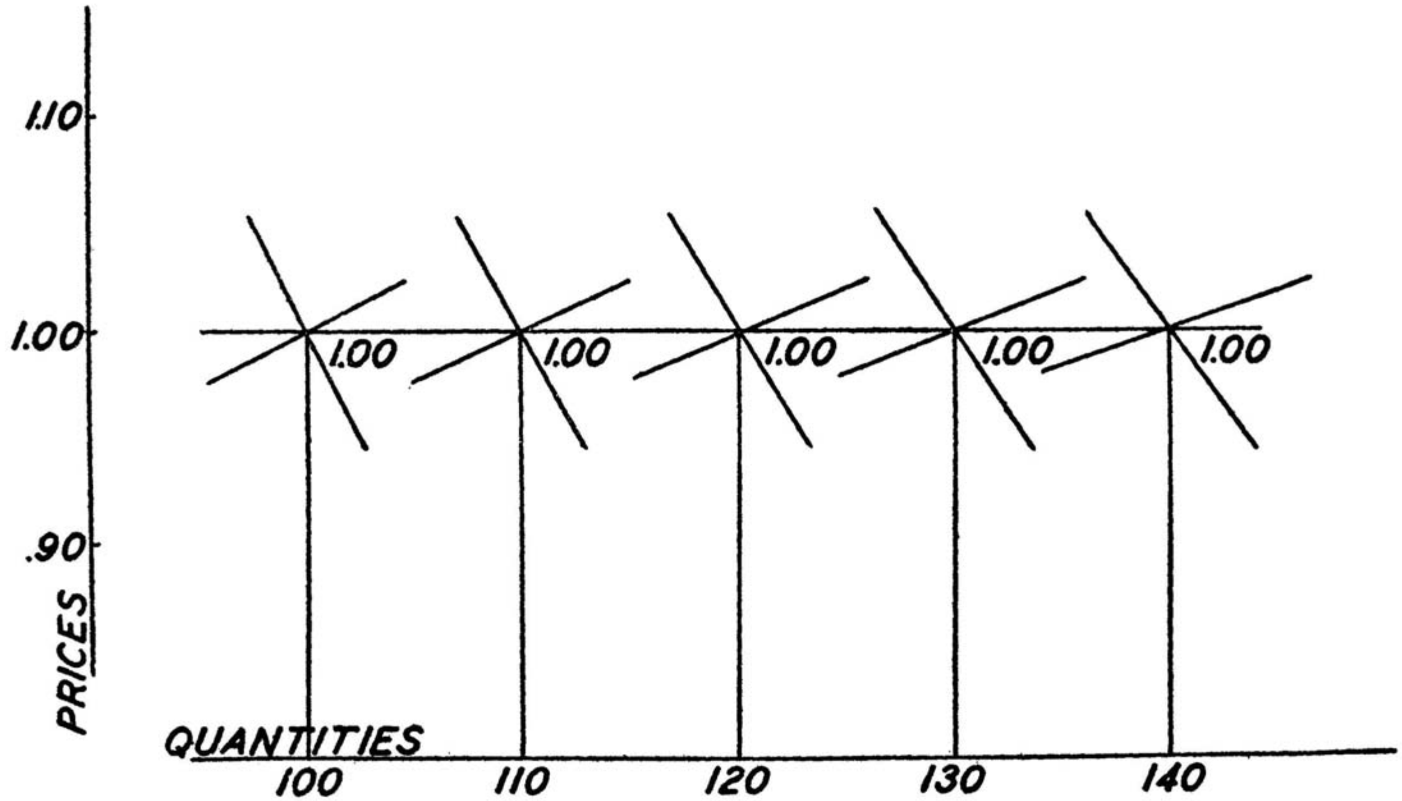


CHART III

DEMAND AND SUPPLY CONDITIONS BOTH CHANGE; DATA AFFORD NO EVIDENCE OF EITHER SUPPLY OR DEMAND CURVE



Wright (*JASA* 1929)

But because Dr. Schultz's methods will not educe from data the neo-classical demand and supply curves, it does not follow that no importance is to be attached to his work.[1] For certain purposes it is of very great importance. What all of

…

tion to the truth, the higher the correlation the closer the approximation. And to the extent that it may be assumed that the dynamic forces will continue to operate thereafter in the same manner as they have been operating during that period, the methods may be used with some confidence for forecasting.

V. The 1920s: Solutions to the Identification Problem

1. Ignore it.
2. Use micro data.

    [Pigou (1910)]
    Ragnar Frisch (1926)
    Irving Fisher (1927)
    René Roy (1930)
    Jacob Marschak (1931)

    Microeconometrics began in earnest as a response to Wright (1915) and Lehfeldt (1915)!

V. The 1920s: Solutions to the Identification Problem

1. Ignore it.
2. Use micro data.
3. Adopt a recursive system. Moore (1925):

$$p_t = \beta_1^{-1} q_t + u^D \qquad \text{(demand)}$$
$$q_t = \beta_2 p_{t-1} + u^S \qquad \text{(supply)}$$

- This recursive structure makes p_{t-1} exogenous in the supply equation and, if u^S is uncorrelated with u^D (no common shifters), then q_t is exogenous in the demand equation with p_t on the left hand side
- This is the cobweb model, introduced in response to Wright (1915) and Lehfeldt (1915)!

V. The 1920s: \Solutions to the Identification Problem

1. Ignore it.
2. Use micro data.
3. Adopt a recursive system.
4. Hold demand constant ("*ceteris paribus*").
   - (a) adjustments to the data based on "intimate knowledge"
   - (b) multiple regression adjustments

The "hold demand constant" approach

Suppose that quantity is measured with error ε_t

$$q_t^* = \beta_1 p_t + \gamma W_t \qquad \text{(true/latent demand)}$$

$$q_t = q_t^* + \varepsilon_t \qquad \text{(observed quantity)}$$

where W_t represents <u>*all*</u> determinants of demand and ε_t is pure independent measurement error. Solve for observed demand:

$$q_t = \beta_1 p_t + \gamma W_t + \varepsilon_t \qquad \text{(observed demand)}$$

where $E(p_t\varepsilon_t) = 0$.

(b) Multiple regression approach: OLS is consistent

(a) "Intimate knowledge" approach: if you "know" γ and W_t, then you can adjust q and OLS is consistent in the regression,

$$q_t - \gamma W_t = q_t^{adjusted} = \beta_1 p_t + \varepsilon_t \quad \text{(adjusted demand)}$$

Example: Frisch-Waugh (*Econometrica*, 1933)…

PARTIAL TIME REGRESSIONS AS COMPARED WITH INDIVIDUAL TRENDS

BY RAGNAR FRISCH AND FREDERICK V. WAUGH[1]

I. INTRODUCTION

THERE are in common use two methods of handling linear trend in correlation analysis of time series data, first, to base the analysis on *deviations* from trends fitted separately to each original series, and, second to base the analysis on the original series without trend elimination, but instead to introduce *time* itself as one of the variables in a multiple correlation analysis. The first method may be called the individual trend method and the latter the partial time regression method.

There are certain misconceptions about the relative value of the two methods and about the kinds of statistical results that are obtained by the two methods. The following simple example illustrates the situation. Suppose we are studying the relation of sugar consumption to sugar prices. We have data representing total annual consumption and price for several consecutive years. There may be a strong upward trend in consumption due to population increase or to some other factor that changes or shifts the demand curve. If we want to study only the relation of consumption to price, naturally we must eliminate the trend from the consumption before using the data to measure demand elasticity.

V. The 1920s: Solutions to the Identification Problem

1. Ignore it.
2. Use micro data.
3. Adopt a recursive system.
4. Hold demand constant.
5. Instrumental variables regression.

Wright (1928, Appendix B)

VI. IV Regression in the 1930s and 1940s

Tinbergen (1930)? (reduced-form solution/ILS – maybe)

Reiersøl (1941) (Instrumental variables, 1 instrument)

Extension to multiple instruments

- LIML: Anderson-Rubin (1949)
- 2SLS: Theil (1953, 1954), Basman (1957), Sargan (1958)

Sargan (*Econometrica*, 1958)

Econometrica **26** (1958) 393-415 *North-Holland Publishing Co. Amsterdam*

THE ESTIMATION OF ECONOMIC RELATIONSHIPS USING INSTRUMENTAL VARIABLES

BY J. D. SARGAN

1. INTRODUCTION

THE USE OF INSTRUMENTAL variables was first suggested by Reiersøl [**13, 14**] for the case in which economic variables subject to exact relationships are affected by random disturbances or measurement errors. It has since been discussed for the same purpose by several authors, notably by Geary [**9**] and Durbin [**7**]. In this article the method is applied to a more general case in

Stock and Watson, *Introduction to Econometrics* 3e, p. 423

Who Invented Instrumental Variables Regression?

. . .

degree in economics from Harvard University in 1887, and he taught mathematics and economics (as well as literature and physical education) at a small college in Illinois. In a book review [Wright (1915)], he used a figure like Figures 12.1a and 12.1b to show how a regression of quantity on price will not, in general, estimate a demand curve, but instead estimates a combination of the supply and demand curves. In the early 1920s, Sewall Wright (1889–1988) was researching the statistical analysis of multiple equations with multiple

Philip G. Wright

Sewall Wright