Constant Perimeter, Varying Area: A Case Study of Teaching and Learning Mathematics to Design a Fish Rack

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This case study examines a sixth-grade teacher and her students in an urban school district in Alaska, engaging in an activity from a module that is part of the *Math in a Cultural Context* (MCC) series. By analyzing the module, the teacher's practice, classroom discourse, and students' work, the case shows that the teacher and the MCC module supported students in developing substantive reasoning and understanding about the mathematical relationship between constant perimeter and varying area in rectangles. Comparison of students' scores on pre- and post-tests show that the class as a whole outperformed the control group. Moreover, Alaska Native students, comprising slightly over one-fourth of the class, outperformed the control group, had gains in achievement commensurate with the entire class, and outperformed their Alaska Native peers in the control group by a wide margin. The case shows that the MCC module and the teacher's practice support improved mathematics achievement through interwoven connections between content, pedagogy, and culture.

Introduction

> A society that has tended to view mathematical ability as possessed by only a few select individuals will need to promote and support forms of instruction that help all to acquire high levels of quantitative literacy, skill in using mathematics, and appreciation of its nature and importance (Kilpatrick & Silver, 2000, p. 226).

High expectations and achievement in mathematics for all students is a central goal for schools. However, in *Principles and Standards for School Mathematics,* the National Council of Teachers of Mathematics [NCTM] (2000) emphasizes that higher expectations by themselves will not provide equal opportunities or higher achievement for all students. Rather, NCTM argues that students' differences, needs, and strengths must be addressed for all students to have equal opportunities to learn mathematics. This should include the mathematics program that is provided for students:

> All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests (NCTM, 2000, p. 13).

Curriculum materials, which can contribute to such a mathematics program, are one important avenue for addressing equity in school mathematics. Tate (1996) notes that introducing "multicultural elements" (e.g., diverse names, cultural holidays, national flags) into mathematics textbooks is a common approach used to connect mathematics to diverse learners in school mathematics curriculum materials. However, Sleeter (1997) argues that such efforts have not resulted in higher mathematics achievement for all students and do little to address the learning needs of minority students.

In contrast to sprinkling multicultural elements throughout traditional school mathematics textbooks, curriculum materials that seek to implement the vision of the NCTM (2000) *Principles and Standards* show promise in providing school mathematics programs that are equitable and that raise the mathematics achievement of all students. Features of such "standards-based" mathematics curricula generally include problem-centered, inquiry-oriented mathematical tasks through which students engage with important mathematical ideas and concepts. Students explore mathematics with their teacher who provides guidance by asking and answering questions, drawing students' attention to important features of the mathematics, and helping students develop, express, refine, and generalize their ideas into appropriate understandings about mathematical concepts and procedures. Growing evidence suggests that standards-based curriculum materials support higher achievement for all students in mathematics (Reys, Reys, Lapan, Holliday, & Wasman, 2003) and standards-based curricula have been shown in some studies to produce greater gains in achievement for minority students than nonminority students (Rivette, Grant, Ludema, & Rickard, 2003). Other inquiry has shown how a standards-based curriculum can support improved mathematics achievement for minority students but can also be uneven in how and to what extent it explicitly addresses multicultural education (Legaspi & Rickard, 2005).

Math in a Cultural Context: Lessons Learned from Yup'ik Eskimo Elders is a series of modules that are intended to supplement a complete K-6 mathematics curriculum and explicitly connect important mathematics with the culture and knowledge of the Yup'ik people, one of the major groups of Alaska Natives. The modules in the MCC series are aligned with the NCTM (2000) *Principles and Standards* and incorporate substantive, inquiry-oriented activities that engage students in learning about mathematical content and processes by exploring rich mathematical problems. The MCC modules provide support to teachers to teach the mathematics consistent with constructivist practice, including guiding students through mathematical explorations rather than demonstrating rote procedures to students (e.g., how to plug numbers into a formula like "1/2 x base x height" to find the area of a triangle) and then monitoring how they imitate them (i.e., do they get the right answer). In this way, the MCC modules parallel standards-based curriculum materials in how mathematics content is aligned with the NCTM (2000) *Principles and Standards* and teaching inquiry-oriented, problem-centered mathematics (e.g., can students explain where the area formula for triangles comes from and what area means). The MCC modules also put standards-based content and pedagogy in the context of Yup'ik culture. All MCC modules use traditional Yup'ik knowledge and practices to motivate and inform students in the elementary and middle school grades. For example, using the traditional base 20 Yup'ik counting system to help students learn about place value and the base 10 system (Lipka, 2003). Therefore, all MCC modules weave reform mathematics content and pedagogy (i.e., alignment with the NCTM *Principles and Standards*) together with Yup'ik culture.

Fish Racks, Perimeter, and Area

One of the MCC modules is *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area* (Adams & Lipka, 2003), intended for sixth grade students. This summary describes how the module brings together the content and pedagogy of reform mathematics with Yup'ik culture and the knowledge of elders:

> The hands-on activities related to building a fish rack for the harvest of salmon form the basis upon which formal mathematics develops in this module. Students engage in activities that simulate the way Yup'ik elders might go about building a fish rack for drying salmon. In the process, they consider a number of factors: ease of access, durability, strength, and capacity to hold a large amount of fish. For example, students in one activity learn to maximize the area of a rectangular drying rack, given a fixed perimeter. This exercise applies directly to the real-life situation in which materials such as wood are often limited, and Yup'ik fishermen thus optimize the drying rack with the few resources they have. In many exercises students increase their understanding of both Yup'ik culture and Western mathematics by learning cultural constructs, such as *sufficient* and *adequate* instead of *maximum* and best (Adams & Lipka, 2003, p. 3, emphasis in original).

In addition to addressing mathematics content by investigating perimeter and area, the authors emphasize that the module incorporates the NCTM process standards (i.e., communication, reasoning and proof, representation, connections, problem solving). For example, students learn about proof by engaging in hands-on activities, often using diagrams or models, to justify their reasoning. Students also learn about representation in mathematics because they will, "… represent their solutions verbally, numerically, graphically, geometrically, and symbolically" (Adams & Lipka, 2003, p. 3).

The *Building a Fish Rack* module begins by giving the teacher and students an overview of the geography and ecology of southwestern Alaska, particularly the subsistence lifestyle of the Yup'ik people who live along the coasts and rivers of the region. These people construct fish racks from available materials to dry harvested salmon. Building fish racks becomes the overall mathematical context for the module and yields rich opportunities for learning mathematics. For example, students learn how to construct the base of the fish rack using traditional Yup'ik techniques and then develop their understanding of formal (i.e., Western) mathematics further by verifying that the base is a rectangle and by determining the length of the diagonals. A central problem in the module is to determine what the dimensions of a rectangular fish rack should be if, for a fixed amount of material available to build the fish rack, you want to be able to dry the most fish. Mathematically, this means determining the dimensions of the rectangle that will have the largest area for a fixed, or constant, perimeter. Understanding the dynamic relationship between constant perimeter and varying area in rectangles is not trivial and is frequently misunderstood by both students and adults. For example, a common misconception is that perimeter determines area, so that if two rectangles have the same perimeter they must also have the same area, but for example a 2x6 rectangle and a 1x7 rectangle have the same perimeters but different areas. The focus of this case study is the work of a sixth-grade teacher and her students in an urban Alaska school district as they engage with the problem of constant perimeter and varying area in designing rectangular fish racks.

Case Study Methodology and Background

Methodology

Data for this case study of a teacher and her sixth-grade class using the *Building a Fish Rack* module were collected from a variety of sources. The author analyzed transcripts of interviews conducted with the teacher in which she was asked about her approach to and beliefs about teaching mathematics. An informational questionnaire completed by the teacher was also studied, along with demographic information about her students. Detailed classroom observation records of the teacher's teaching were used to gain further insight into the teacher's practice. Videos of the teacher teaching the *Building a Fish Rack* module were studied carefully; the central activity used in this case study was analyzed multiple times, and portions of the video were transcribed. Mathematical discourse between the teacher and students was analyzed, and samples of students' work (e.g., solutions to problems, entries in math journals) were reviewed. The interview, questionnaire, and class observation data were collected between spring 2002 and spring 2004; the teacher used the *Building a Fish Rack* module and was videotaped during the spring of 2004.

In addition to the qualitative measures of students' learning (e.g., discourse, work samples, math journal entries), students completed pre- and post-tests that focused on perimeter and area concepts and relationships before and after completing the *Building a Fish Rack* unit. Students in the control group were also sixth-graders and studied perimeter and area using mainly commercial textbooks and completed the same pre- and post-tests. Comparing students' pre- and post-test scores between the treatment group (i.e., students learning from the *Fish Rack* module) and control group (i.e., students who did not use the *Fish Rack* module, but used typical textbooks) allowed us to compare students' understanding of perimeter and area and provides quantitative information about the overall impact of the *Building a Fish Rack* module. The pre- and post-tests were completed by both the treatment and control group students during the spring of 2004. All of these sources of qualitative and quantitative data were used in developing the final case study.

The activity from the *Building a Fish Rack* module that is the focus of this case study is Activity 12: Investigating the Relationship of Perimeter and Area of Rectangles. This activity was chosen because it is a watershed investigation in the module. Activity 12 explores the mathematics of how the area of rectangles can vary when perimeter is held constant and provides key results for students to apply in designing their own fish rack. Another reason to focus on Activity 12 in this case study is that, after we reviewed videos of the teacher teaching other activities from *Building a Fish Rack,* we determined that Activity 12 provides a representative example of how the teacher taught the entire module (and other modules in the MCC series). Finally, comparison across the interviews with the teacher, information from the questionnaire, and detailed observations of her teaching using *Building a Fish Rack* and other MCC modules shows that Activity 12 exhibits the characteristics of the teacher's practice that consistently emerge from all the data sources.

<u>Background</u>

Janet Speed (her real name, used with her permission) teaches sixth grade in the Fairbanks North Star Borough School District, an urban school district in interior Alaska. The district enrolls about 14,000 students in grades K-12 and encompasses a large geographic area with a total population of about 86,000. Ms. Speed has been teaching for 19 years, including 15 years teaching at her current K-6 school. Ms. Speed is Caucasian and does not speak Yup'ik. Her class is diverse and includes 22 students: six Alaska Native, four African American, and 12 Caucasian. One of the Alaska Native students is Yup'ik, with the other five students from other Alaska Native groups. In this case study, we specifically note when students referred to in the case are Alaska Native; where not specifically noted, students should be assumed to be non-Alaska Native. Ms. Speed has earned her master's degree and is very comfortable with mathematics and with teaching mathematics. Ms. Speed's formal study of mathematics includes meeting general education requirements for her undergraduate degree and completing mathematics for elementary teachers courses for teaching certification.

Ms. Speed finds mathematics interesting, and although she says that she seldom tries to incorporate students' experiences in her teaching, she does value their ideas when they occur in the classroom and encourages students to share them. While Ms. Speed says she does not typically connect mathematics to other disciplines, she does try to illustrate mathematical concepts with practical examples (e.g., food, money). Teaching mathematics daily for about 75 minutes, Ms. Speed says that she frequently has her students work with each other and routinely has them write explanations of their work and justify their reasoning. She occasionally engages her students with problems that take more than one day to complete but almost always uses worksheets and seatwork in her mathematics teaching and frequently makes use of the textbook. Ms. Speed says that she almost always uses tasks or problems to introduce new math topics to her students and also makes use of counterexamples. She sometimes works with her students to understand mathematical formulas and why they work. Ms. Speed says that she is "always looking for new and better ways to teach math." As a participant in this study examining the use and effectiveness of the MCC materials (including the *Building a Fish Rack* module), Ms. Speed completed professional development workshops on use of the MCC materials before she began to teach with them.

Janet Speed originally worked with the MCC project as a control group teacher: i.e., she and her students participated in MCC but used their traditional math textbook and supporting materials, rather than MCC modules. As part of the control group, Ms. Speed's students attained high achievement scores on MCC module post-tests relative to other control-group classes. Ms. Speed was subsequently invited to participate in MCC as a treatment group teacher (i.e., using MCC modules in her classroom) partly because her students' performance in the control group stood out to MCC researchers. Another factor leading to Ms. Speed joining the treatment group is that her classes have been typically more diverse, with a higher population of Alaska Native students, than most other teachers in the urban control group. A final reason for Ms. Speed's participation in the treatment group is that she teaches in an urban school district. Most research on mathematics achievement of Alaska Native and other American Indian students occurs in rural settings. However, in Alaska the Alaska Native population is becoming increasingly urbanized. While demographic data on rural and urban residency of Alaska Natives is sketchy (there are no uniform classifications of "rural" or "urban" for many Alaska communities), recent census data shows that about 20% of the Alaska Native population lives in Anchorage (the largest city in Alaska, with a population of over 277,000) with clear demographic trends of Alaska Natives relocating from small, rural villages to larger Alaska communities (Goldsmith, Howe, & Leask, 2005). The significant and increasing urbanization of Alaska's Native population provides additional motivation to study the implementation of MCC modules in urban classrooms like Janet Speed's, to learn how to best meet the needs of Alaska's increasingly urban K-12 Alaska Native student population.

Constant Perimeter, Varying Area: Learning Mathematics to Design a Fish Rack

Janet Speed's students sit at their desks, which are pushed together into clusters. At her direction, students clear their desks, except for their math journals, and then focus their attention on her as she stands at the front of the room next to the overhead projector. Ms. Speed then asks her students to put their pencils down and listen carefully to her conjecture:

> Alright—here's my conjecture—is that [reads from text on the overhead transparency], 'The perimeters of different rectangles are the same, so they will hold the same number of fish—meaning that they will have the same area.' So, my conjecture is that if the perimeter [of rectangles] is the same then the area is also the same. (Pause.) So, what I want you to do is just take a minute and talk with somebody just across from you or right at your table and see if you agree with me. If you agree with me you need to be able to prove it and say why you agree. Or, disprove it—and if you're going to disprove it you need to give me a counterexample, an example that shows that that's not true. OK—does everyone understand what I'm saying here?

With nods from her students that they understand the task, Ms. Speed then reminds her students of an earlier activity in the *Building a Fish Rack* module where they found all the possible rectangles (with edges that have whole number lengths) that have a perimeter of 28. Ms. Speed then puts another transparency on the overhead, which is a reproduced blackline master from Activity 10: Exploring Perimeter of Rectangles, in the *Building a Fish Rack* module (Adams & Lipka, 2003, p. 132):

Length (cm)	Width (cm)	Perimeter (cm)
1	13	28
2	12	28
3	11	28
4	10	28
5	9	28
6	8	28
7	7	28

Using the table of their earlier findings to further illustrate her conjecture, Ms. Speed says, "My conjecture is that since they [rectangles in the table] all have a perimeter of 28, they'll all have the same area too. OK?" Satisfied that her students understand the task, Ms. Speed directs her students to begin talking at their tables to decide whether they agree or disagree with the conjecture and how to prove or disprove it.

This introduction to Activity 12 provides a clear example of beginning an inquiry-oriented lesson. Ms. Speed presented a conjecture that is relevant to the context of the module (i.e., building a fish rack) and made sure that her students understood the conjecture. Further, she provided clear guidance on criteria for an appropriate answer (i.e., providing proof or a counterexample) and connected the conjecture back to earlier mathematics the students had learned to help them begin to formulate their responses. All of these features of Ms. Speed's practice reflect mathematics teaching reforms described in the NCTM *Principles and Standards*. It is also notable that Ms. Speed closely follows the directions provided in the *Building a Fish Rack* module to begin Activity 12 (Adams & Lipka, 2003, p. 142).

As her students begin discussing the conjecture with peers, Ms. Speed circulates around the room, making sure to visit each cluster of students' desks. At one group of four students, the two boys disagree with the conjecture and the two girls agree (one of the girls is Alaska Native). Ms. Speed reminds these students that they not only need to decide whether they agree or disagree but also explain how to prove or disprove the conjecture. "Your group needs to be able to explain what you did," she says. As Ms. Speed moves away, one of the boys flips through his math journal and pulls out sketches on grid paper of all the rectangles with perimeter of 28 in the table Ms. Speed put on the overhead (i.e., 1x13 rectangle, 2x12 rectangle, etc.). All four students begin looking at the sketched rectangles and the other boy says, "So area has to be the same on all of them?" as the other students nod. One of the boys says, "The area is all the squares, right?" and the other boy replies, "Yeah." At this point, the students begin counting squares in the sketches of rectangles with a perimeter of 28. After a moment, one of the boys says, "You were right—they're not the same." The other boy, obviously pleased, shows everyone in the group how one rectangle has an area of 24 (i.e., the 2x12) and another has an area of 13 (the 1x13 rectangle). One of the girls (an Alaska Native) counts up the area of two rectangles once more and verifies that they are different. Both of the girls now agree with the boys that the conjecture is not true. A few moments later, Ms. Speed, who has been circulating around the room, returns to the group and asks how they are doing. One of the girls (non-Alaska Native) replies, "We agreed [gesturing to herself and the other girl in the group], they disagreed [gesturing to the two boys], and then they showed us how that it was wrong, so now we disagree." One of the boys then shows Ms. Speed that they all disagree with the conjecture because some of the rectangles with perimeter 28 have different areas.

This example of students working in small groups is typical of Ms. Speed's classroom. Students engaged in thoughtful reasoning about the validity of the conjecture and disproved it using a counterexample (e.g., the areas of the 1x13 and 2x12 rectangles are not the same, even though they both have a perimeter of 28). The group's thinking about the conjecture was initially split, but by comparing it to the results of their prior work, all the students in the group came to a valid and mathematically supported conclusion. An additional point about this example is that the Alaska Native student in the group did not participate in the verbal discussion as much as the other students. However, she did sketch some rectangles during the work her group did together, comparing the areas of the different rectangles with a perimeter of 28, and she verified the counterexample by finding areas not the same. Also, when the group was trying to decide how to find the area of the different rectangles, she reminded them with the sketches how to count up the grid squares in the rectangles to determine the area. Therefore, like her peers, the Alaska Native student did participate in the group work, but in a more nonverbal way.

After checking with each group, Ms. Speed brought the class back together for a discussion to summarize the groups' findings. The class was unanimous in disagreeing with her conjecture. Other groups volunteered valid counterexamples to disprove the conjecture (e.g., the 10x4 and 7x7 rectangles have perimeter of 28 but different areas). As Ms. Speed recorded her students' counterexamples at the overhead projector, one student described his group's counterexample as showing that "the inside is different, but the outside would be the same." This student is distinguishing that while the areas are different (the "inside"), the perimeters of the rectangles are the same (the "outside"). Mathematically, this shows that for rectangles with constant perimeter, the area can vary. Next, Ms. Speed asked her students to find the area of each of the seven rectangles in the table shown earlier. Most students had already found the area of most or all of the rectangles, but Ms. Speed asked them to be sure that they had sketches of each. Students worked individually for several minutes, sketching the rectangles and then finding and recording the area of each by either counting the grid squares within the rectangles or by multiplying the length by the width (applying the formula that the area of a rectangle is the product of its length and width). Continuing to follow the module very closely, Ms. Speed then brought the class back together and used an overhead transparency from the *Building a Fish Rack* module to summarize the areas at the overhead projector (see Adams & Lipka, 2003, p. 146). The transparency shows sketches of the rectangles as fish racks, and Ms. Speed mentioned at the beginning of the summary that the rectangles were pictured, "as if they were fish racks." With her students' input, Ms. Speed filled in the area of each.

Next, Ms. Speed asked her students to complete a table, provided in the module, listing the width, length, and area of all the rectangles, with whole-number side measures, that have a perimeter of 28 (see Adams & Lipka, 2003, p. 150). Before handing out a copy of the table to each student, Ms. Speed puts a transparency of the table on the overhead and notes that the perimeter column is given and directs her students to "fill in the width, and the length, and then the area." She then adds, "I want you to start with one by thirteen and then go to two by twelve, and go to three by eleven (pause)—see what I'm doing here? And keep going." The purpose of this part of Activity 12 is to help students identify patterns among rectangles that have constant perimeter and varying area. Ms. Speed's direction to list the rectangles in order (i.e., 1x13, 2x12, 3x11, …) is a pedagogical strategy so that patterns will be more apparent to students when they study the completed table. Ms. Speed's students completed their individual tables in about four minutes. Ms. Speed then brought the class back together and put a completed table on the overhead (Adams & Lipka, 2003, p. 147):

Perimeter (cm)	Width (cm)	Length (cm)	Area (cm^2)
28	1	13	13
28	2	12	24
28	3	11	33
28	4	10	40
28	5	9	45
28	6	8	48
28	7	7	49
28	8	6	48
28	9	5	45
28	10	4	40
28	11	3	33
28	12	2	24
28	13	1	13

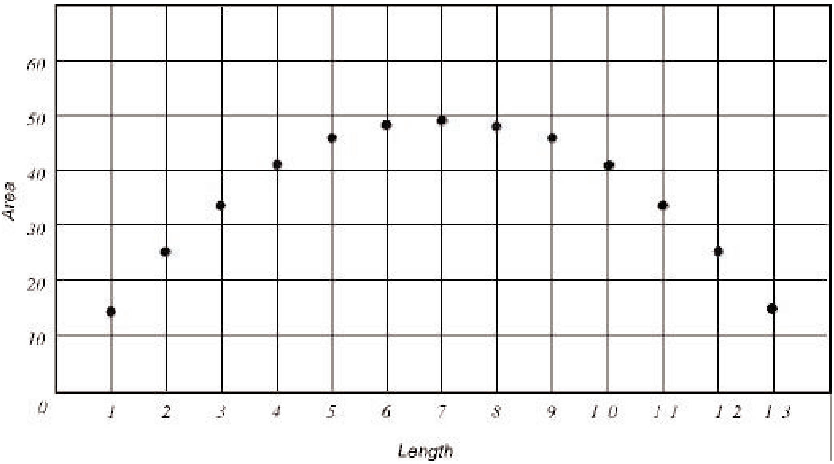
Bringing the class back together, Ms. Speed says, "OK. Here's my chart that, hopefully, matches your chart [she pauses and gets nods from the class]. I want you just to look at your charts for a minute and then tell me what patterns you notice." A number of students quickly raise their hands. The first student Ms. Speed calls on (an Alaska Native girl) says, "Length times width equals area." The next student Ms. Speed calls on (an Alaska Native boy) says that he notices how the "length and width are the same [pause] they go backwards," and then adds that "the width goes from one to thirteen and the length goes from thirteen to one." Another student (a non-Alaska Native boy) observes that, "The area goes odd, even, odd, even" and a final student (a non-Alaska Native girl) points out that "after the area gets to 49, which is, like, in the middle, then it starts going back down again." Ms. Speed extends this observation, noting that, in the table, as the width gets larger the area initially gets larger also, but then peaks at 49—as the student noted—and then starts decreasing. An Alaska Native student in the class adds to this observation that it shows how the rectangles in the table "start over again." All the students' comments demonstrate how they are identifying patterns in the table, reasoning about what the patterns mean, and connecting them back to the rectangles that the data represent.

Drawing attention back to the completed table on the overhead, Ms. Speed asks her students to use the table to think about this question and asks, "So, if I were going to build my fish rack, and I can only make it—I only have enough materials to make it with a perimeter of 28, but I need it as big as possible, what shape do you suggest?" One student (a non-Alaska Native boy) quickly raises his hand and volunteers, "Seven by seven." When Ms. Speed asks him why seven by seven, he replies, "It's a square!" When Ms. Speed pushes the student to explain his reasoning further about what's special about the square, the same student responds, "Uh (pause) seven by seven has 49." Praising the student, Ms. Speed then calls on another student who notes that, "It [the 7x7 square] has the biggest area." In this segment of her instruction, Ms. Speed is closely following the *Building a Fish Rack* module and has, through questioning her students and pushing them to explain their reasoning, helped them formalize that for rectangles that have constant perimeter and varying area, the square is the rectangle that will have the greatest area. Ms. Speed has also connected this mathematical finding to the central problem of the unit, which is how to design a fish rack, in this case how to make it as large as possible given a fixed amount of construction material.

As a follow-up question, Ms. Speed asks her students what rectangular fish rack would hold the most fish if only enough material were available for a perimeter of 25. Several students volunteer that a 5x5 fish rack would be the biggest because, as one student says, "Doubles are bigger," referring back to the finding that the rectangle with the largest area is a square (which has the same width and length, i.e., "doubles"). However, when Ms. Speed attempts to sketch a square with perimeter 25, she realizes that such a square can't have sides with whole-number lengths. Ms. Speed acknowledges to the class that she's made an error and that she was thinking about the rectangle having an *area* of 25, not a *perimeter* of 25. To clarify this, Ms. Speed sketches a 5x5 square at the overhead and says, "That's a *perimeter* of 20 [pause] a five by five—is a perimeter of 20. Do you agree? It has an *area* of 25." All students in the class nod in agreement. Ms. Speed then asks the class to generate all the possible rectangles with a perimeter of 25 that have whole number side lengths. Most students seem initially puzzled, but after a moment, several students note from their math journals an earlier finding (from Activity 10 of the *Building a Fish Rack* module) that such a rectangle does not exist because, as one student says, "You have to split the perimeter in half to get the length and width, and 25 doesn't have even halves." This student's comment gets at a mathematical property of rectangles. In particular, since the sum of the length and width of a rectangle has to be half of the perimeter, if half the perimeter is not a whole number (i.e., "doesn't have even halves"), then either the length or the width (or both) cannot be a whole number (e.g., if the sum of two numbers is 12.5, you cannot get this sum by adding two whole numbers). This part of Ms. Speed's lesson shows how she used an error as an impromptu exploration to think more about constant perimeter and varying area in rectangles with students connecting the problem to prior findings to address it. Ms. Speed is also modeling how to share, test, and revise her own mathematical thinking, demonstrating what she expects her students to do. Modeling problem solving and cognition is a well-documented approach to helping students develop problem-solving skills, including the metacognitive processes that support problem solving (Schoenfeld, 1985).

For additional practice, Ms. Speed asked her students to help her find all the rectangles with a perimeter of 20 (with whole number side lengths). She records her students' responses at the overhead and quickly generates a complete and correct list (e.g., 1x9, 2x8, 3x7, …) as possible rectangles. Ms. Speed notes the same finding as with their work with the rectangles with a perimeter of 28, namely that the 5x5 rectangle, or square, has the largest area for rectangles with a perimeter of 20. To check her students' understanding further, Ms. Speed asks the class, "Suppose I am going to [pause] I've got a really good fishing season and I'm going to make a huge fish rack and it's going to have a perimeter of 100. What should the dimensions be so it can be as big as possible?" About one-third of the class immediately raises their hands and the first student called on volunteers, "25 by 25!" Other students in the class nod in agreement. The class also agrees with another student (Alaska Native) who says that the area will be 625. Ms. Speed then asks her students if this answer is consistent, asking, "Does that fit [pause] our pattern? Is 25 by 25 a square?" All students nod in agreement. This segment of Ms. Speed's teaching shows how she has verified that her students understand that for rectangles with a fixed perimeter, a square will have the largest area. She also, once again, connected this mathematical relationship to the task of designing a fish rack.

For the next 10 minutes of class, Ms. Speed had her students individually complete a graph from the *Building a Fish Rack* module (Adams & Lipka, 2003, p. 151) as she circulated around the room monitoring their progress. The students are asked to make a graph plotting the length and area of the rectangles with a perimeter of 28 from the table completed earlier. The completed graph shows how the area first increases as the length increases but then the area decreases as the length continues to increase. Here is the completed graph (Adams & Lipka, 2003, p. 149):



All of Ms. Speed's students were successful in sketching the graph. As Ms. Speed circulated around the room, asking students about their graphs and what the graph means, students made comments that included, "It looks like a hill" and "In the middle of it there's a square." These comments illustrate how students made the connection between the graph and the data in their tables that as the length of rectangles with fixed perimeter increases, the area first increases, peaks when the rectangle becomes a 7x7 square, and then decreases. The graph and the numerical data in the table are different mathematical representations of this property of rectangles with constant perimeter and varying area.

To summarize the lesson, Ms. Speed takes a loop of string at the overhead and makes rectangles, starting with long, flat rectangles and gradually making it look more like a square. She asks the class to tell her when the area is the biggest. The class correctly stops her when the rectangle is a square. The class also correctly observes that, as Ms. Speed continues to change the rectangle into tall, skinny rectangles, the perimeter is unchanged but the area is now decreasing. This is a dynamic representation that shows the same mathematical relationship as the table and graph. For the remaining nine minutes of this 75-minute lesson to complete Activity 12 in the *Building a Fish Rack* module, Ms. Speed directs the class, "I want you to write in your *Fish Racks* journal what you learned today [pause] … about the relationship of length and width of a rectangle to area." Ms. Speed monitors her students as they write in their journals. Here are examples of what students wrote: "I learned how to find the biggest possible area if the perimeter is like, 100, 20, or 25 by using doubles." Another student wrote, "What I learned today was when the length goes between 1-7 the area gets bigger then when it goes from 7-13 the area gets smaller because the numbers are repeating." And an Alaska Native student wrote "a square has the largest area." These examples of students' journal entries suggest that Ms. Speed's students can explain the mathematics of constant perimeter and varying area with rectangles in their own words, although no explicit connections are made to designing a fish rack.

Student Achievement

We measured the achievement and changes in achievement of Janet Speed's students on perimeter, area, and proof by comparing her students' results on pre- and post-tests designed to assess these concepts. We compared student achievement between Ms. Speed's Alaska Native students and her non-Alaska Native students. Ms. Speed's students' achievement was also compared with a group of control students who also studied perimeter, area, and proof in urban school settings in Alaska, but used commercially available textbooks as the primary curriculum resource rather than the *Building a Fish Rack* module. This comparison was made as part of a larger study of the MCC modules and their effects on student achievement. We investigated changes in achievement between the Alaska Native students in Ms. Speed's and other urban treatment classes (i.e., the other teachers in urban districts who also taught *Building a Fish Rack* and other MCC modules) and the Alaska Native students in the urban control group. All of the scores are averages and are expressed as raw scores in terms of percentage correct. Teachers in the treatment group received professional development in using the MCC materials before teaching with them. Teachers who did not use MCC materials (control group classrooms) also received comparable professional development in using the curriculum materials adopted by their school district before beginning participation in the MCC study. In general, students in the MCC study treatment and control groups showed similar levels of achievement on the pre-test on perimeter, area, and proof concepts but significantly different levels of achievement on the post-test.

The table below shows changes in achievement between Ms. Speed's entire class (including her Alaska Native students) and the subgroup of her Alaska Native students; recall that there are 22 students in Ms. Speed's class, six of whom are Alaska Native (the data in this table and all following tables are from spring 2004 when Ms Speed taught the *Building a Fish Rack* module):

	Ms. Speed's whole class (% correct); N = 22	Ms. Speed's Alaska Native students only (% correct); N = 6
Pre-Test	42.91%	38.29%
Post-Test	72.41%	68.0 %

This data shows that Ms. Speed's entire class made substantial gains in achievement in perimeter, area, and proof after completing the *Building a Fish Rack* module. Moreover, the achievement gains of Ms. Speed's Alaska Native students are commensurate with the class as a whole.

The table below shows how changes in achievement of Ms. Speed's entire class and her Alaska Native students compare to the changes in achievement of all the students in the urban control group (urban students who did not use the *Building a Fish Rack* module):

	Ms. Speed's whole class (% correct); N = 22	Ms. Speed's Alaska Native students only (% correct); N = 6	Urban control students (% correct); N = 30
Pre-Test	42.91%	38.29%	37.83%
Post-Test	72.41%	68.0%	40.20%

The table shows that Ms. Speed's entire class and her Alaska Native students outperformed the urban control group students by a wide margin. Taken together, these results suggest that the *Building a Fish Rack* module made a significant difference in achievement for Ms. Speed's Alaska Native and non-Alaska Native students. Moreover, the gains in achievement of Ms. Speed's students go beyond statistical significance to be of practical significance. In particular, the gains are substantial enough to have positive implications for students' verbal and written work and for students' grades in school and scores on standardized mathematics tests.

The next table compares the performance of all Alaska Native students in urban classrooms who completed the *Building a Fish Rack* module (including Ms. Speed's students), with the Alaska Native students in the urban control classrooms. The sample size for both treatment and control groups is small but potentially informative:

	Alaska Native students who studied *Fish Rack* (% correct); N = 14	Alaska Native students who did not study *Fish Rack* (% correct); N = 3
Pre-Test	40.86%	51.5%
Post-Test	61.21%	48.5%

Due to the small sample sizes, particularly with the control group (N = 3), it is not possible to make generalizations from the results in the above table. However, the changes in achievement between the groups do suggest that the *Building a Fish Rack* module, in addition to making a difference in the achievement of all urban students, may play a particularly strong role in improving the achievement of urban Alaska Native students. Given the increasing urbanization of the Alaska Native population (Goldsmith, Howe, & Leask, 2005), this finding is promising and warrants more extensive investigation in future research.

This case study and all of the above data about student achievement focus on students in urban settings. However, the table below shows the pre- and post-test data on perimeter, area, and proof concepts for all of the students, both urban and rural, in the study who were taught with the *Building a Fish Rack* module (treatment group) and those taught the concepts primarily with conventional, commercially available textbooks (control group):

	All students in treatment group (% correct); N = 99	All students in control group (% correct); N = 47
Pre-Test	38.60%	41.26%
Post-Test	53.33%	42.04%

Because students and teachers in rural schools in Alaska have much different circumstances than their peers in urban schools (e.g., multiage classrooms, students who are predominantly Alaska Native), it can be argued that aggregating the rural and urban student groups is problematic. However, the data do suggest, and the larger sample sizes reinforce, that across a variety of school settings in Alaska, the *Building a Fish Rack* module makes a significant and positive impact on the mathematics achievement of all students. Because students in Alaska's rural schools are predominantly from Alaska Native groups, including Yup'ik Eskimo, the above data underscore that the *Building a Fish Rack* module supports increased mathematics achievement in Alaska Native students that significantly exceeds that of typical textbooks.

Analysis: Content, Pedagogy, and Culture

This case study suggests that students in Ms. Speed's class developed substantive understandings about perimeter, area, and proof. Evidence for this claim is both qualitative and quantitative. The case study shows Ms. Speed's students engaging in thoughtful mathematical discourse and expressing, testing, and revising their ideas about mathematics, particularly about the relationship between constant perimeter and varying area in rectangles. The achievement data shows that Ms. Speed's students outperformed their peers who did not learn from the *Building a Fish Rack* module, based on the results of comparable pre-test scores but significantly higher post-test scores. Taken together, these findings demonstrate how the *Building a Fish Rack* module provided support to Ms. Speed and her students in addressing key mathematics content (e.g., measurement concepts like perimeter and area) and processes (e.g., reasoning and proof, communication, representation) identified in the NCTM *Principles and Standards for School Mathematics* (NCTM, 2000).

The case shows how the problem-centered, inquiry-oriented structure of the *Building a Fish Rack* module is central to engaging students with the mathematics content. Students are not asked to quickly find answers to computational exercises by plugging numbers into thinly understood formulas, but, for example, to decide whether a conjecture is true or false and provide evidence for their claim. Such problem-centeredness of the *Building a Fish Rack* module supports inquiry because the problems are substantive and require investigation, discussion, and possibly revision by the students. Therefore, the content of what students are learning in Activity 12 in the *Building a Fish Rack* module in Ms. Speed's class is not only about relationships between perimeter and area of rectangles but also about how to actually do mathematics. Learning about how to do mathematics by testing conjectures (e.g., rectangles that have the same perimeter also have the same area) and investigating relationships between mathematical concepts (e.g., constant perimeter and varying area in rectangles), as demonstrated in this case, supports students in learning, understanding, and applying mathematics content (NCTM, 2000; Reys et al., 2003). The fact that Ms. Speed's and other students who used the *Building a Fish Rack* module significantly outperformed the control group students who did not learn from problem-centered, inquiry-oriented materials strengthens this claim. Studies of student achievement using problem-centered curricula other than MCC show similar results (Rivette et al., 2003).

While the *Building a Fish Rack* module plays an important role in understanding students' achievement in this case, how Ms. Speed taught Activity 12 (and all of the module) is also central. In particular, what a teacher does with a mathematics curriculum in her classroom shapes not only what students learn, but what they have opportunities to learn (Remillard & Bryans, 2004; Rickard, 2005). This case study shows how Ms. Speed followed the *Building a Fish Rack* module closely, but also consistently used a pedagogical approach of open mathematical discourse. Ms. Speed used the *Building a Fish Rack* module to provide mathematically rich problems for her students to investigate and then guided them by pushing them to share their ideas, explain their reasoning, and providing opportunities for revision. Supporting this climate of open mathematical discourse is another feature of Ms. Speed's practice: she consistently called only upon volunteers (students who had raised their hand) as opposed to nominating students (calling on students whether or not they are comfortable or prepared to share their ideas). Calling on volunteers seemed an effective strategy for Ms. Speed in encouraging participation from her students, particularly her Alaska Native students. Ms. Speed also modeled problem solving for her students, including sharing a solution, testing it, realizing it does not fit the conditions of the problem, and then revising the solution so that it is valid (i.e., trying to find the largest rectangle with a perimeter of 25 instead of 20). Ms. Speed was also consistent in her persistence in completing investigations of complex mathematics. For example, she made sure that all of her students completed the various tables and graphs in Activity 12 of the *Building a Fish Rack* module and, even when she asked students to find the largest rectangle with a perimeter of 25 (instead of 20), she followed through with the problem, uncovering and correcting an error in the process. In this way, Ms. Speed maintained high expectations of her students to engage in mathematical discourse and tasks in the *Building a Fish Rack* module and modeled these expectations herself.

The mathematics content in the *Building a Fish Rack* module is developed within a cultural context. Properties of shapes, perimeter, area, and proof are investigated in the context of designing a fish rack based on traditional Yup'ik practices and knowledge. Because she followed Activity 12 in the *Building a Fish Rack* module closely, Ms. Speed made multiple connections to designing a fish rack in her teaching. For example, the case shows that the conjecture Ms. Speed posed to her students at the beginning of the lesson is connected to designing a fish rack, as are other follow-up questions she asked students (e.g., what are the dimensions of the largest rectangular fish rack made with a certain amount of material). Of the 18 activities in the *Building a Fish Rack* module, several address Yup'ik culture and fish racks more extensively than Activity 12. For example, the first four activities focus on the cultural and ecological background of fishing in southwestern Alaska so that students understand how designing and building fish racks fits within Yup'ik culture; the culminating activity of the module is for students to build a model of a fish rack (Adams & Lipka, 2003). Therefore, by closely following the module in her teaching, Ms. Speed addresses Yup'ik culture specifically, and Alaska Native culture generally, more extensively over the entire module than is suggested by Activity 12 alone. Ms. Speed's close following of the module, as demonstrated in this case study, implies that the mathematics content of the module is connected explicitly to Yup'ik culture because the module makes these connections.

By carefully following the module as part of her pedagogy, Ms. Speed made both mathematical connections and cultural connections explicitly intended by the curriculum developers. What is different about how Ms. Speed addressed content and culture in her pedagogy is that she went beyond the module in teaching the mathematics, but stopped at the module in making connections to and learning about Yup'ik culture. Ms. Speed went beyond the module in teaching mathematics content by consistently and persistently fostering rich mathematical discourse in her classroom. However, in Activity 12 she connected her students' study of rectangles to fish racks in several instances, but, for example, did not ask her students, particularly her six Alaska Native students, about their own experiences with designing or building fish racks. Making such connections could potentially benefit all students, but particularly Alaska Native students, because a major goal of multicultural education is to help make academic content more meaningful and relevant by connecting it to students' lives and experiences (Tate, 1996). In Activity 12, for example, this could take the form of asking Alaska Native students, some of whom may have been to a traditional fish camp where fish racks are constructed and used, to describe how the size of a fish rack is determined given the material available. The class could then compare such practice at fish camp with the mathematical relationship of constant perimeter and varying area in rectangles. Even if few or none of the Alaska Native students in a particular class have had direct experience with building a fish rack—and this may be the case in an urban classroom like Janet Speed's—these students may have family members who have or have heard stories about fish camp. In these cases, connections to traditional Yup'ik knowledge and practices in *Building a Fish Rack* and other MCC modules could still be meaningful to the lives and cultural backgrounds of Alaska Native students and all students who live in Alaska. Research on the relationship between rich cultural connections in problem-centered, inquiry-oriented mathematics curricula and students' mathematics achievement suggests that even curriculum that provide very limited cultural connections support gains in achievement for minority students that match or outpace all students as a group (Legaspi & Rickard, 2005; Rivette et al., 2003). However, research also suggests that connecting mathematics to students' own lives, cultures, and circumstances is critical in achieving equity in school mathematics for all students (Sleeter, 1997).

Conclusions and Implications

Janet Speed effectively taught the MCC *Building a Fish Rack* module to her class of 22 sixth graders, including six Alaska Native students. Her effectiveness is demonstrated in the case study by how she maintained a stimulating classroom environment for investigating rich mathematical situations in the module and how she supported students' investigations with consistent mathematical discourse to share, test, and revise their ideas about perimeter, area, and proof. Ms. Speed also made connections to fish racks in Activity 12, addressing Yup'ik culture and the Alaskan context in her pedagogy as well as content. Going beyond the mathematics content of the curriculum in her teaching, but stopping at the curriculum in addressing culture, is consistent with her beliefs about her own teaching. In particular, Ms. Speed describes herself as comfortable with mathematics and always asking students to explain their thinking, but acknowledges that she seldom connects mathematics to other disciplines or her students' own experiences. In addition to the qualitative evidence from the case study about her effectiveness in teaching the *Building a Fish Rack* module, Ms. Speed's students, both her Alaska Native students and her class as a whole, outperformed the control group on post-tests of achievement in perimeter, area, and proof, providing quantitative evidence of her effectiveness teaching the module. All of this evidence also suggests that the MCC *Building a Fish Rack* module is effective in helping students, particularly Alaska Native students, to attain higher mathematics achievement in perimeter, area, and proof concepts than students using traditional textbooks to learn the same concepts. The role of MCC modules in enhancing students' mathematics achievement has been replicated in other qualitative and quantitative studies of other MCC modules that address other mathematics concepts (Adams & Lipka, 2003; Lipka, 2003).

One implication of this case study is that problem-centered, inquiry-oriented mathematics curricula that embed mathematics in a non-Western cultural context, in this case Yup'ik culture and the Alaska context, can support the mathematics achievement of all students. Perhaps more than any other subject, mathematics has been taught in K-12 schools from a Western perspective that tends to gloss over or completely ignore the contributions of non-Western cultures and may perpetuate a belief that only some people can learn mathematics (Kilpatrick & Silver, 2000; NCTM, 2000; Sleeter, 1997). This case study shows that Alaska Native students and non- Native students can demonstrate strong mathematics achievement that exceed that of their peers when they use problem-centered, inquiry-oriented curricula that are culturally based.

This case study also raises the issue of what roles content and culture play in students' learning of mathematics. For example, to what extent was students' learning shaped by the problem-centered, inquiry-oriented structure of the *Building a Fish Rack* module, in tandem with Ms. Speed's pedagogy that fostered and maintained rich mathematical discourse? Similarly, to what extent was students' achievement shaped by the Yup'ik and Alaska cultural context in which the mathematics in the *Building a Fish Rack* module is embedded and to which Ms. Speed made connections? Perhaps the structure of the *Building a Fish Rack* module, combined with Ms. Speed's pedagogy supporting discourse, accommodated more learning styles and enhanced her students' mathematics achievement relative to the control group students. Perhaps the Yup'ik cultural context of the *Building a Fish Rack* module allowed more students, both Alaska Native students and non- Native students who know about or may take part in harvesting salmon, to connect mathematics to their own lives and circumstances, thereby supporting their learning. Both of these perspectives are supported by data on Ms. Speed's students' achievement—all of her students, including the subgroup of her Alaska Native students, outperformed the control group students on the post-tests on perimeter, area, and proof concepts. Both perspectives are also supported by other research (Rivette et al., 2003; Sleeter, 1997; Tate, 1996). This case study, therefore, builds on prior research that implies that learning more about the roles of content, pedagogy, and culture in using problem-centered, inquiry-oriented mathematics curricula may lead to more specific recommendations for teacher development, pedagogy, and curriculum (Lipka & Adams, 2004). For example, if Ms. Speed had asked her students to share experiences they may have had at fish camp as part of the mathematical discourse in her classroom while teaching *Building a Fish Rack*, would that have further enhanced the mathematics achievement of all her students, especially her Alaska Native students? Research that could answer this kind of question could inform both the development of mathematics curriculum and its implementation by teachers.

Finally, this case study demonstrates how content, pedagogy, and culture are entwined in Ms. Speed's classroom while studying the *Building a Fish Rack* module. The Yup'ik cultural context of the module provides a coherent purpose, meaning, and motivation to learning the mathematics content through investigations. Both content and culture are supported by Ms. Speed's pedagogy, through which she pushes students to formulate, express, test, and revise their ideas about the mathematics and connect it to fish racks and other aspects of Yup'ik culture. The entwined nature of content, pedagogy, and culture implies that all three need to be addressed, not only in mathematics curriculum like *Building a Fish Rack* and other MCC modules, but also in teachers' professional development so that more teachers can orchestrate such curricula like Ms. Speed and, hopefully, achieve similar results in their students' mathematics achievement.

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