EXPLORING A COMPLEX SPACE-FILLING SHAPE

Part One: Building a Three-Dimensional 12-pointed Star

[45 minutes]

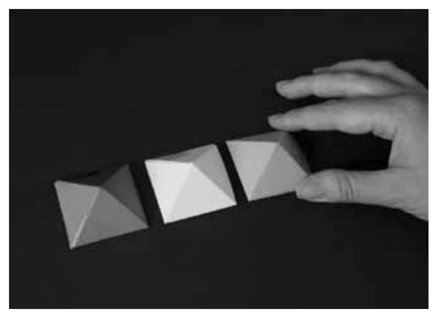
Materials (per person):

- Three Pyramid Patterns (included)—copied onto card stock, if possible. (Note: You will need at least 24 pyramid patterns to make one 12-pointed star.)
- Scissors
- Transparent tape

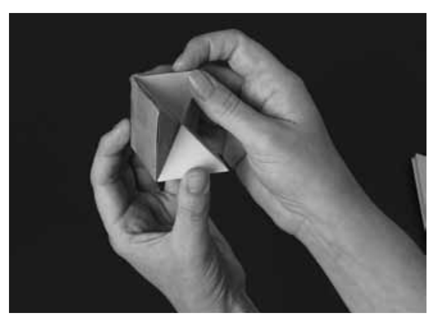
Try This:

Step 1 Each person, cut out three pyramid patterns.

Step 2 Fold along all the lines. All folds should be "mountain folds." (A mountain fold has both sides folded away from you.)

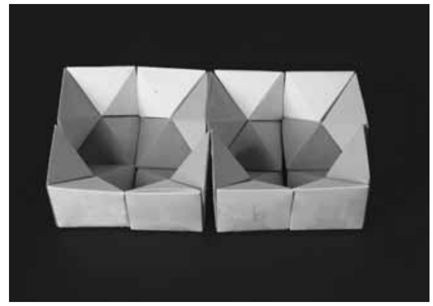


Step 3 Assemble each pyramid by taping the sides together. Write your name on the base of each pyramid.

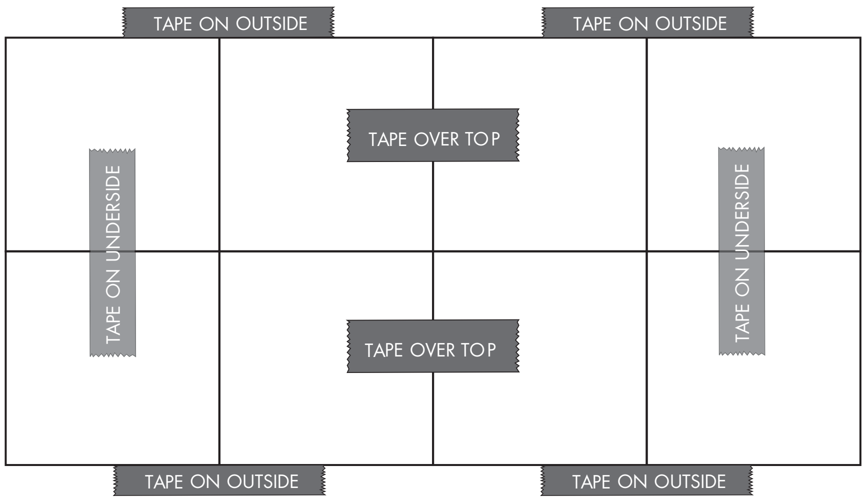


Step 4 Tape your three pyramids together as shown. Let's call this new shape a "tri-pyramid."

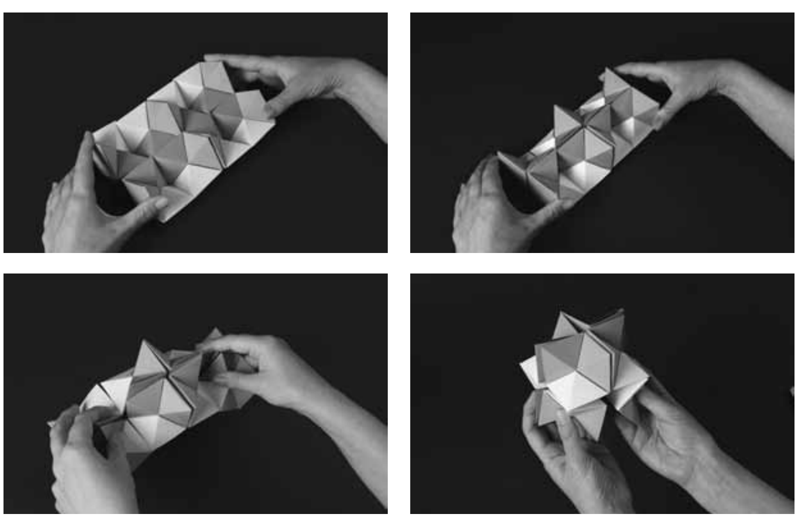
Step 5 Arrange eight tri-pyramids (yours plus seven others) to form two “cups”, side by side.



In the sketch below each square represents one tri-pyramid. Tape the eight tri-pyramids together, as shown.

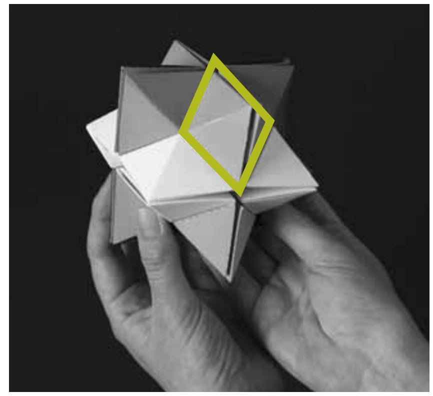


When you are done taping, you can manipulate the pattern to form a stellated rhombic dodecahedron.



Each tri-pyramid is what fraction of the volume of the whole shape?

Step 6 Notice the spiky shapes sticking out. These spiky shapes are called *stellations*. Look at the shape of the base of one stellation. Do you see a diamond shape? This shape is called a *rhombus*, which is an equilateral four-sided figure. How many of these stellations can you find on this shape? (There are 12.) A figure with 12 faces is called a *dodecahedron*. Can you explain the name of this figure (stellated rhombic dodecahedron)?



Step 7 We know that cubes will *tessellate* three-dimensional space, that is, fill the space with no gaps in between. Do you think these stellated rhombic dodecahedrons will tessellate three-dimensional space? Build several of them and try to fit them together.

Part Two: Exploring the Volume of a Stellated Rhombic Dodecahedron

[30 minutes]

Materials:

- Paper models from Part One
- One Cube Pattern (included) for every two students
- Scissors
- Transparent tape

Try This:

Step 1 Carefully cut the stellated rhombic dodecahedron into eight tri-pyramids. Find the tri-pyramid with your name on it.

Step 2 Cut out the cube pattern and assemble it into an open box.

Step 3 Can you and your partner fit your two tri-pyramids into the open box?

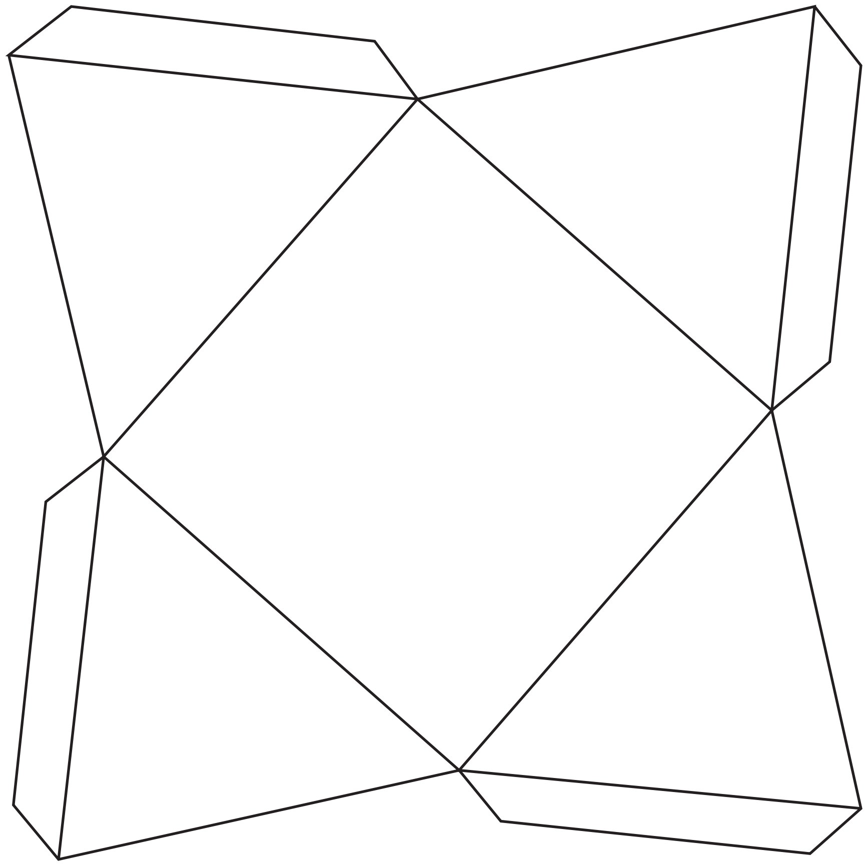
Step 4 If the length of one side of the cube is equal to 1, what is the volume of the cube? (By cubing the length of the side, we find that the volume of the cube is one times one times one, or $1^3 = 1$ cubic unit.)

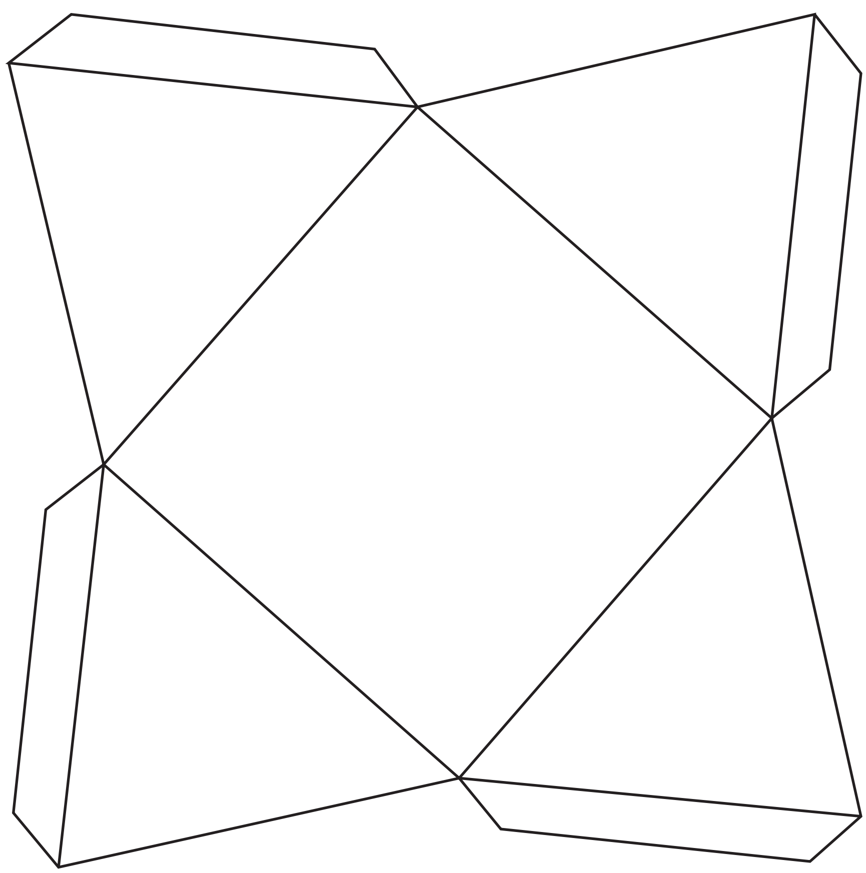
If the cube's volume is one, what is the volume of the tri-pyramid? (Because two tri-pyramids just fit into the cube, we know the volume of each tri-pyramid is ½ cubic unit.)

Step 5 Recall that it took eight tri-pyramids to make one stellated rhombic dodecahedron (SRD). If you combined eight smaller cubes (side = 1), you could make a larger cube (side = 2) that would be big enough (if it were hollow) for an SRD to fit inside. What would be the volume of this cube? How does the volume of this cube compare to the volume of the SRD?

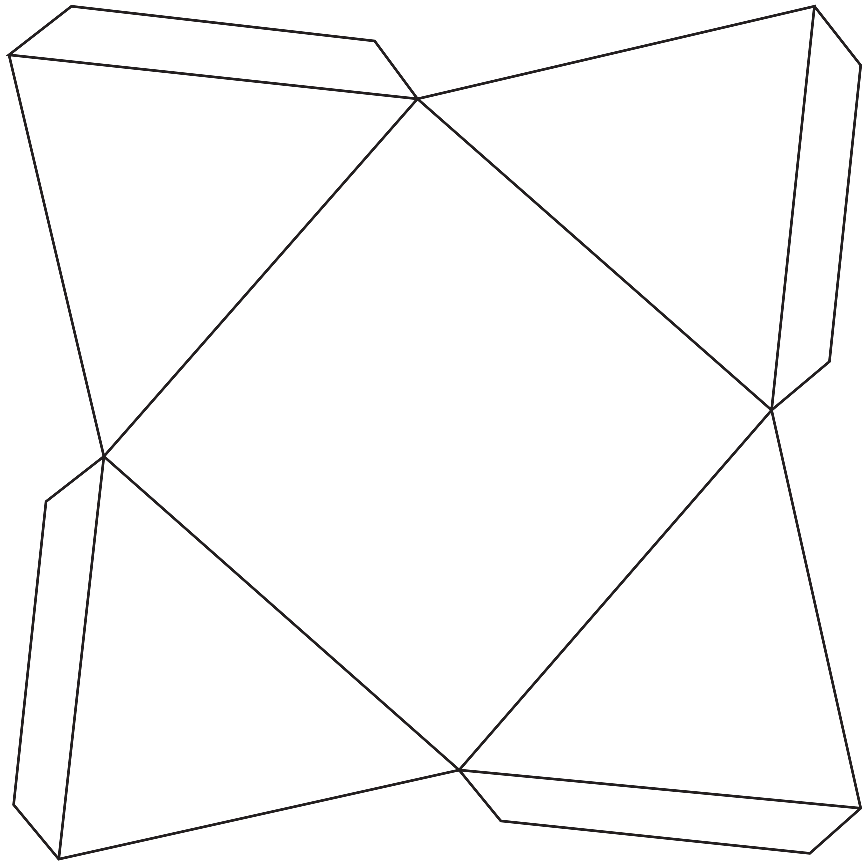
The volume of the cube that would contain the SRD would be eight cubic units. The volume of the SRD is half of that, or four cubic units. You can confirm this because it took eight tri-pyramids to make one SRD. Each tri-pyramid has a volume of ½, so 8 x ½ = 4 cubic units.

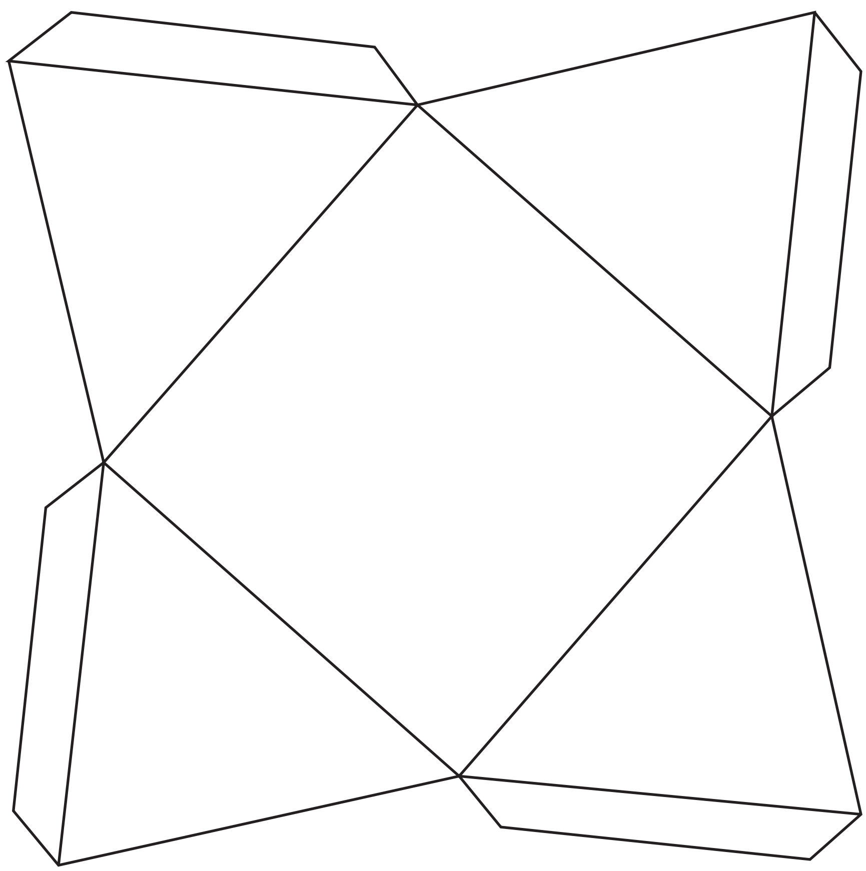
Pyramid Pattern





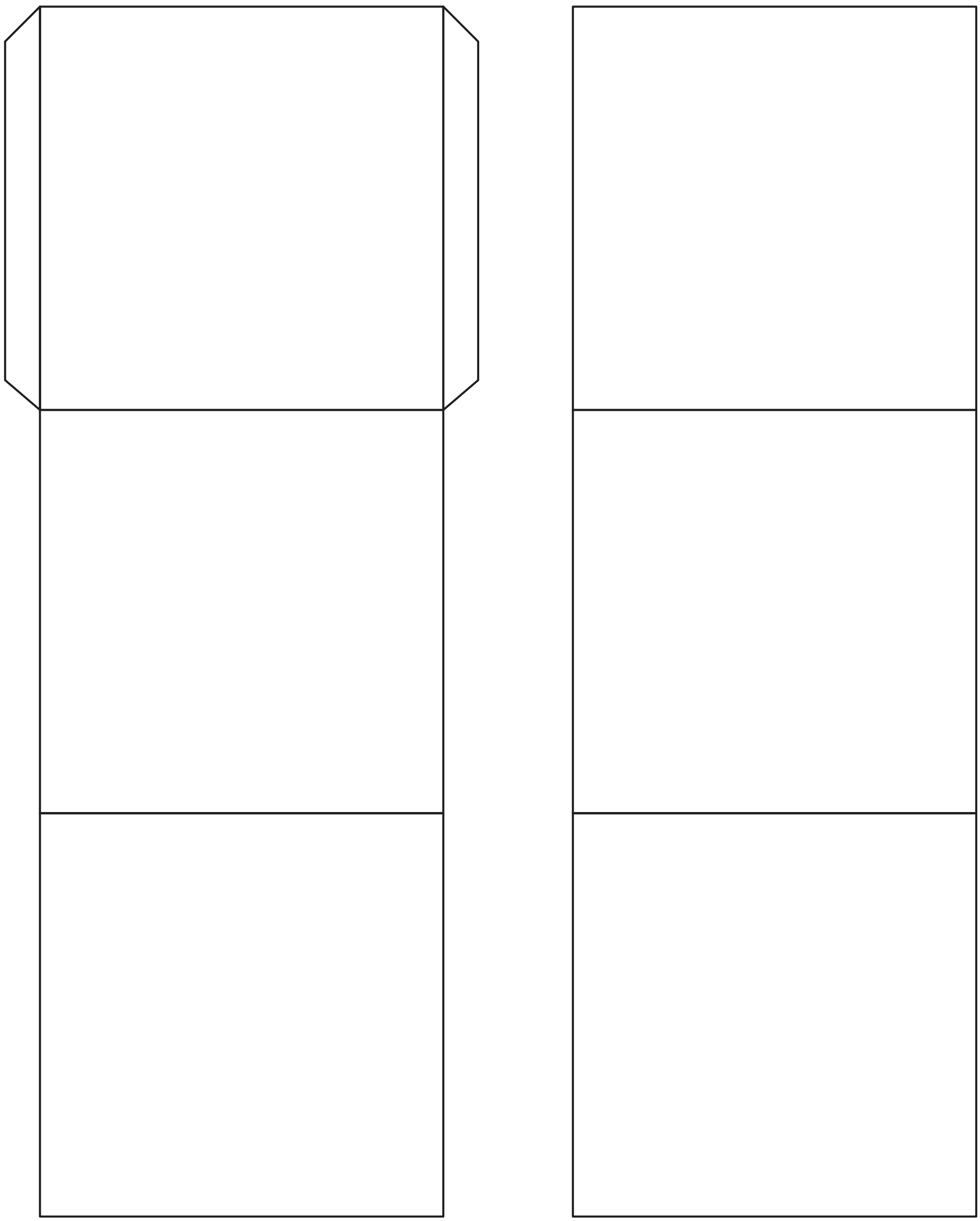
Pyramid Pattern





Cube Pattern

Place the center squares on top of each other to form a a "+".
Tape them together and fold up the sides to create an open box.



National Education Standards | Grades 6–8

From the National Council of Teachers of Mathematics (NCTM)

EXPLORING A COMPLEX SPACE-FILLING SHAPE

Use visualization, spatial reasoning, and geometric modeling to solve problems:

- Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

Apply appropriate techniques, tools, and formulas to determine measurements:

- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders.