Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Stars in other universes: stellar structure with different fundamental constants

Fred C Adams

Michigan Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA
E-mail: fca@umich.edu

Received 5 June 2008
Accepted 14 July 2008
Published 7 August 2008

Online at stacks.iop.org/JCAP/2008/i=08/a=010
doi:10.1088/1475-7516/2008/08/010

Abstract. Motivated by the possible existence of other universes, with possible variations in the laws of physics, this paper explores the parameter space of fundamental constants that allows for the existence of stars. To make this problem tractable, we develop a semi-analytical stellar structure model that allows for physical understanding of these stars with unconventional parameters, as well as a means to survey the relevant parameter space. In this work, the most important quantities that determine stellar properties—and are allowed to vary—are the gravitational constant G, the fine structure constant α and a composite parameter $\mathcal{C}$ that determines nuclear reaction rates. Working within this model, we delineate the portion of parameter space that allows for the existence of stars. Our main finding is that a sizable fraction of the parameter space (roughly one-fourth) provides the values necessary for stellar objects to operate through sustained nuclear fusion. As a result, the set of parameters necessary to support stars are not particularly rare. In addition, we briefly consider the possibility that unconventional stars (e.g. black holes, dark matter stars) play the role filled by stars in our universe and constrain the allowed parameter space.

Keywords: dark matter, massive stars, black holes, cosmology of theories beyond the SM

 1475-7516/08/08010+28$30.00

Contents

1. Introduction

The current picture of inflationary cosmology allows for, and even predicts, the existence of an infinite number of space–time regions sometimes called pocket universes [1]–[3]. In many scenarios, these separate universes could potentially have different versions of the laws of physics, e.g. different values for the fundamental constants of nature. Motivated by this possibility, this paper considers the question of whether or not these hypothetical universes can support stars, i.e. long-lived hydrostatically supported stellar bodies that generate energy through (generalized) nuclear processes. Toward this end, this paper develops a simplified stellar model that allows for an exploration of stellar structure with different values of the fundamental parameters that determine stellar properties. We then use this model to delineate the parameter space that allows for the existence of stars.

A great deal of previous work has considered the possibility of different values of the fundamental constants in alternate universes or, in a related context, why the values of the constants have their observed values in our universe (e.g. [4, 5]). More recent papers have identified a large number of possible constants that could, in principle, vary from universe to universe. Different authors generally consider differing numbers of constants, however, with representative cases including 31 parameters [6] and 20 parameters [7]. These papers generally adopt a global approach (see also [8]–[10]), in that they consider a wide variety of astronomical phenomena in these universes, including galaxy formation, star formation, stellar structure and biology. This paper adopts a different approach by focusing on the particular issue of stars and stellar structure in alternate universes; this strategy allows for the question of the existence of stars to be considered in greater depth.

Unlike many previous efforts, this paper constrains only the particular constants of nature that determine the characteristics of stars. Furthermore, as shown below, stellar structure depends on relatively few constants, some of them composite, rather than on large numbers of more fundamental parameters. More specifically, the most important quantities that directly determine stellar structure are the gravitational constant G, the fine structure constant α and a composite parameter $\mathcal{C}$ that determines nuclear reaction rates. This latter parameter thus depends in a complicated manner on the strong and weak nuclear forces, as well as the particle masses. We thus perform our analysis in terms of this $(\alpha, G, \mathcal{C})$ parameter space.

The goal of this work is thus relatively modest. Given the limited parameter space outlined above, this paper seeks to delineate the portions of it that allow for the existence of stars. In this context, stars are defined to be self-gravitating objects that are stable, long-lived and actively generate energy through nuclear processes. Within the scope of this paper, however, we construct a more detailed model of stellar structure than those used in previous studies of alternate universes. On the other hand, we want to retain a (mostly) analytic model. Toward this end, we take the physical structure of the stars to be polytropes. This approach allows for stellar models of reasonable accuracy; although it requires the numerical solution of the Lane–Emden equation, the numerically determined quantities can be written in terms of dimensionless parameters of order unity, so that one can obtain analytic expressions that show how the stellar properties depend on the input parameters of the problem. Given this stellar structure model, and the reduced $(\alpha, G, \mathcal{C})$ parameter space outlined above, finding the region of parameter space that allows for the existence of stars becomes a well-defined problem.

As is well known, and as we re-derive below, both the minimum stellar mass and the maximum stellar mass have the same dependence on fundamental constants that carry dimensions [11]. More specifically, both the minimum and maximum mass can be written in terms of the fundamental stellar mass scale M_0 defined according to

$$M_0 = \alpha_G^{-3/2} m_{\rm P} = \left(\frac{\hbar c}{G}\right)^{3/2} m_{\rm P}^{-2} \approx 3.7 \times 10^{33} g \approx 1.85 M_\odot, \tag{1}$$

where α_G is the gravitational fine structure constant:

$$\alpha_G = \frac{G m_{\rm P}^2}{\hbar c} \approx 6 \times 10^{-39}, \tag{2}$$

where $m_{\rm P}$ is the mass of the proton. As expected, the mass scale can be written as a dimensionless quantity $(\alpha_G^{-3/2})$ times the proton mass; the appropriate value of the exponent $(-3/2)$ in this relation is derived below. The mass scale M_0 determines the allowed range of masses in any universe.

In conventional star formation, our Galaxy (and others) produces stars with masses in the approximate range $0.08 \leq M_*/M_\odot \leq 100$, which corresponds to the range

JCAP08(2008)010

$0.04 \le M_*/M_0 \le 50$. One of the key questions of star formation theory is to understand, in detail, how and why galaxies produce a particular spectrum of stellar masses (the stellar initial mass function, or IMF) over this range [12]. Given the relative rarity of high-mass stars, the vast majority of the stellar population lies within a factor of $\sim$10 of the fundamental mass scale M_0. For completeness we note that the star formation process does not involve thermonuclear fusion, so that the mass scale of the hydrogen burning limit (at $0.08M_\odot$) does not enter into the process. As a result, many objects with somewhat smaller masses—brown dwarfs—are also produced. One of the objectives of this paper is to understand how the range of possible stellar masses changes with differing values of the fundamental constants of nature.

This paper is organized as follows. We construct a polytropic model for stellar structure in section 2 and identify the relevant input parameters that determine stellar characteristics. Working within this stellar model, we constrain the values of the stellar input parameters in section 3; in particular, we delineate the portion of parameter space that allows for the existence of stars. Even in universes that do not support conventional stars, i.e. those generating energy via nuclear fusion, it remains possible for unconventional stars to play the same role. These objects are briefly considered in section 4 and include black holes, dark matter stars and degenerate baryonic stars that generate energy via dark matter capture and annihilation. Finally, we conclude in section 5 with a summary of our results and a discussion of its limitations, including an outline for possible future work.

2. Stellar structure models

In general, the construction of stellar structure models requires the specification and solution of four coupled differential equations, i.e. force balance (hydrostatic equilibrium), conservation of mass, heat transport and energy generation. This set of equations is augmented by an equation of state, the form of the stellar opacity and the nuclear reaction rates. In this section we construct a polytropic model of stellar structure. The goal is to make the model detailed enough to capture the essential physics and simple enough to allow (mostly) analytic results, which in turn show how different values of the fundamental constants affect the results. Throughout this treatment, we will begin with standard results from stellar structure theory [11, 13, 14] and generalize to allow for different stellar input parameters.

2.1. Hydrostatic equilibrium structures

In this case, we will use a polytropic equation of state and thereby replace the force balance and mass conservation equations with the Lane–Emden equation. The equation of state thus takes the form

$$P = K\rho^\Gamma \qquad \text{where } \Gamma = 1 + \frac{1}{n}, \tag{3}$$

where the second equation defines the polytropic index n. Note that low-mass stars and degenerate stars have polytropic index $n = 3/2$, whereas high-mass stars, with substantial radiation pressure in their interiors, have index $n \to 3$. As a result, the index is slowly varying over the range of possible stellar masses. Following standard

JCAP08(2008)010

methods [15, 11, 13, 14], we define

$$\xi \equiv \frac{r}{R}, \qquad \rho = \rho_{\rm c} f^n, \qquad \text{and} \qquad R^2 = \frac{K\Gamma}{(\Gamma-1)4\pi G\rho_{\rm c}^{2-\Gamma}}, \tag{4}$$

so that the dimensionless equation for the hydrostatic structure of the star becomes

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2 \frac{\mathrm{d}f}{\mathrm{d}\xi}\right) + \xi^2 f^n = 0. \tag{5}$$

Here, the parameter $\rho_{\rm c}$ is the central density (in physical units) so that $f^n(\xi)$ is the dimensionless density distribution. For a given polytropic index n (or a given Γ), equation (5) thus specifies the density profile up to the constants $\rho_{\rm c}$ and R. Note that, once the density is determined, the pressure is specified via the equation of state (3). Further, in the stellar regime, the star obeys the ideal gas law so that the temperature is given by $T = P/(\mathcal{R}\rho)$, with $\mathcal{R} = k/\langle m\rangle$; the function $f(\xi)$ thus represents the dimensionless temperature profile of the star. Integration of equation (5) outwards, subject to the boundary conditions $f = 1$ and $\mathrm{d}f/\mathrm{d}\xi = 0$ at $\xi = 0$, then determines the position of the outer boundary of the star, i.e. the value ξ_* where $f(\xi_*) = 0$. As a result, the stellar radius is given by

$$R_* = R\xi_*. \tag{6}$$

The physical structure of the star is thus specified up to the constants $\rho_{\rm c}$ and R (see figure 1). These parameters are not independent for a given stellar mass; instead, they are related via the constraint

$$M_* = 4\pi R^3 \rho_{\rm c} \int_0^{\xi_*} \xi^2 f^n(\xi)\,\mathrm{d}\xi \equiv 4\pi R^3 \rho_{\rm c}\mu_0, \tag{7}$$

where the final equality defines the dimensionless quantity μ_0, which is of order unity and depends only on the polytropic index n.

2.2. Nuclear reactions

The next step is to estimate how the nuclear ignition temperature depends on more fundamental parameters of physics. Thermonuclear fusion generally depends on three physical variables: the temperature T, the Gamow energy $E_{\rm G}$ and the nuclear fusion factor $S(E)$. The Gamow energy is given by

$$E_{\rm G} = (\pi\alpha Z_1 Z_2)^2 \frac{2m_1 m_2}{m_1 + m_2} c^2 = (\pi\alpha Z_1 Z_2)^2 2 m_{\rm R} c^2, \tag{8}$$

where m_j are the masses of the nuclei, Z_j are their charge (in units of e) and where the second equality defines the reduced mass. For the case of two protons, $E_{\rm G} = 493$ keV. The parameter α is the usual (electromagnetic) fine structure constant:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}, \tag{9}$$

where the numerical value applies to our universe. Thus, the Gamow energy, which sets the degree of Coulomb barrier penetration, is determined by the strength of the electromagnetic force (through α). The strength of the strong and weak nuclear forces

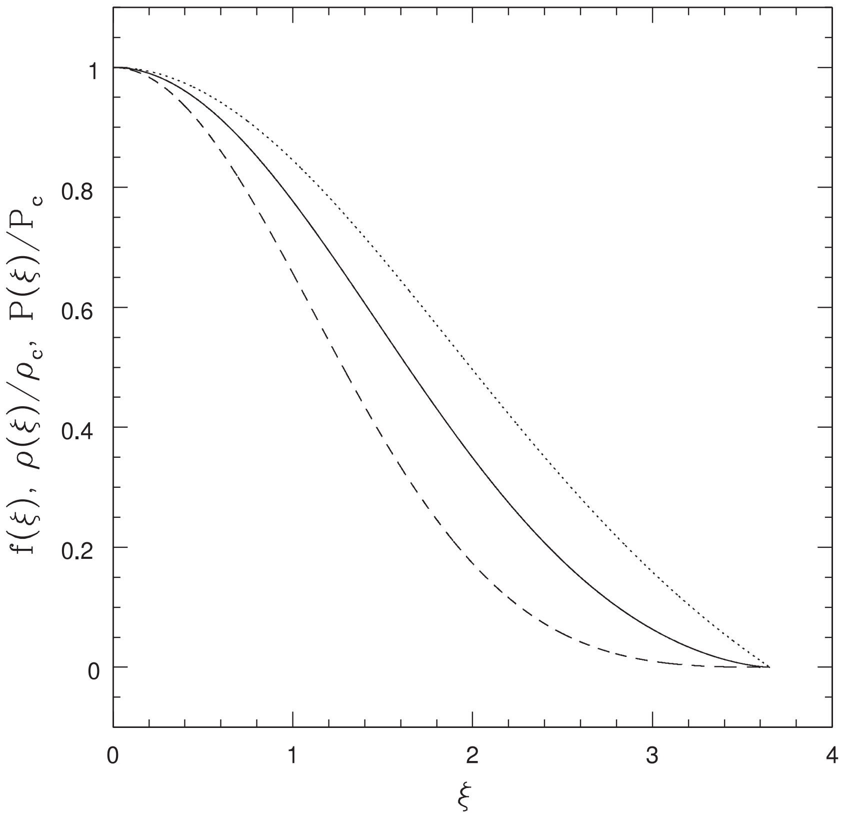

Figure 1. Density, pressure and temperature distributions for the $n = 3/2$ polytrope. The solid curve shows the density profile $\rho(\xi)/\rho_c$, the dashed curve shows the pressure profile $P(\xi)/P_c$ and the dotted curve shows the temperature profile $f(\xi) = T(\xi)/T_c$. For a polytrope, the variables are related through the expressions $P \propto \rho^{1+1/n}$ and $\rho \propto f^n$.

enters into the problem by setting the nuclear fusion factor $S(E)$ which in turn sets the interaction cross section according to

$$\sigma(E) = \frac{S(E)}{E} \exp\left[-\left(\frac{E_G}{E}\right)^{1/2}\right], \tag{10}$$

where E is the energy of the interacting nuclei. The temperature at the center of the star determines the distribution of E. Under most circumstances in ordinary stars, the cross section has the approximate dependence $\sigma \propto 1/E$ so that the nuclear fusion factor $S(E)$ is a slowly varying function of energy. This dependence arises when the cross section is proportional to the square of the de Broglie wavelength, so that $\sigma \sim \lambda^2 \sim (h/p)^2 \sim h^2/(2mE)$; this relation holds when the nuclei are in the realm of non-relativistic quantum mechanics.

The nuclei generally have a thermal distribution of energy so that

$$\langle \sigma v \rangle = \left(\frac{8}{\pi m_R}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty \sigma(E) \exp\left[-E/kT\right] E \, dE. \tag{11}$$

As a result, the effectiveness of nuclear reactions is controlled by an exponential factor $\exp[-\Phi]$, where the function Φ has contributions from the cross section and the thermal distribution, i.e.

$$\Phi = \frac{E}{kT} + \left(\frac{E_{\rm G}}{E}\right)^{1/2}. \tag{12}$$

The integral in equation (11) is dominated by energies near the minimum of Φ, where $E = E_0 = E_{\rm G}^{1/3}(kT/2)^{2/3}$, and where the function takes the value

$$\Phi_0 = 3\left(\frac{E_{\rm G}}{4kT}\right)^{1/3}. \tag{13}$$

If we approximate the integral using Laplace's method [16], the reaction rate R_{12} for two nuclear species with number densities n_1 and n_2 can be written in the form

$$R_{12} = n_1 n_2 \frac{8}{\sqrt{3}\pi\alpha Z_1 Z_2 m_{\rm R} c} S(E_0)\Theta^2 \exp[-3\Theta], \tag{14}$$

where we have defined

$$\Theta \equiv \left(\frac{E_{\rm G}}{4kT}\right)^{1/3}. \tag{15}$$

2.3. Stellar luminosity and energy transport

The luminosity of the star is determined through the equation

$$\frac{{\rm d}L}{{\rm d}r} = 4\pi r^2 \varepsilon(r), \tag{16}$$

where ε is the luminosity density, i.e. the power generated per unit volume. This quantity can be written in terms of the nuclear reaction rates via

$$\varepsilon(r) = \mathcal{C}\rho^2\Theta^2\exp[-3\Theta], \tag{17}$$

where Θ is defined above, and where

$$\mathcal{C} = \frac{\langle\Delta E\rangle R_{12}}{\rho^2\Theta^2}\exp[3\Theta] = \frac{8\langle\Delta E\rangle S(E_0)}{\sqrt{3}\pi\alpha m_1 m_2 Z_1 Z_2 m_{\rm R} c}, \tag{18}$$

where $\langle\Delta E\rangle$ is the mean energy generated per nuclear reaction. In our universe $\mathcal{C} \approx 2\times10^4$ cm^5 s^{-3} g^{-1} for proton–proton fusion under typical stellar conditions.

The total stellar luminosity is given by the integral

$$L_* = \mathcal{C}4\pi R^3\rho_{\rm c}^2\int_0^{\xi_*} f^{2n}\xi^2\Theta^2\exp[-3\Theta]\,{\rm d}\xi \equiv \mathcal{C}4\pi R^3\rho_{\rm c}^2 I(\Theta_{\rm c}), \tag{19}$$

where the second equality defines $I(\Theta_{\rm c})$, and where $\Theta_{\rm c} = \Theta(\xi=0) = (E_{\rm G}/4kT_{\rm c})^{1/3}$. Note that, for a given polytrope, the integral is specified up to the constant $\Theta_{\rm c}$: $T = T_{\rm c} f(\xi)$, $\Theta = \Theta_{\rm c} f^{-1/3}(\xi)$.

At this point, the definitions of equation (4), the mass integral constraint (7) and the luminosity integral (19) provide us with three equations for four unknowns: the radial scale R, the central density $\rho_{\rm c}$, the total luminosity L_* and the coefficient K in the equation of state. Notice that, if the star is degenerate, then the coefficient K is specified by quantum mechanics, $\Gamma = 5/3$, and one could solve the first two of these equations for R and ρ_c, thereby determining the physical structure of the star. Note that the quantum mechanical value of K represents the minimum possible value. If the star is not degenerate, but rather obeys the ideal gas law, then the central temperature is related to the central density through $\mathcal{R}T_c = K\rho_c^{1/n}$, so that T_c does not represent a new unknown, and the stellar luminosity L_* is the only new variable introduced by luminosity equation (19).

For ordinary stars, one needs to use the fourth equation of stellar structure to finish the calculation. In the case of radiative stars, the energy transport equation takes the form

$$T^3 \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\rho\kappa}{4ac}\frac{L(r)}{4\pi r^2}, \tag{20}$$

where κ is the opacity. In the spirit of this paper, we want to obtain a simplified set of stellar structure models to consider the effects of varying constants. As a result, we make the following approximation. The opacity κ generally follows Kramer's law so that $\kappa \sim \rho T^{-7/2}$. For the case of polytropic equations of state, we find that $\kappa\rho \sim \rho^{2-7/2n}$. For the particular case $n = 7/4$, the product $\kappa\rho$ is strictly constant. For other values of the polytropic index, the quantity $\kappa\rho$ is slowly varying. As a result, we assume $\kappa\rho = \kappa_0\rho_c = constant$ for purposes of solving the energy transport equation (20). This ansatz implies that

$$L_* \int_0^{\xi_*} \frac{\ell(\xi)}{\xi^2}\,\mathrm{d}\xi = aT_c^4 \frac{4\pi c}{3\rho_c\kappa_0} R, \tag{21}$$

where we have defined $\ell(\xi) \equiv L(\xi)/L_*$. The full expression for $\ell(\xi)$ is given by the integral in equation (19). For purposes of solving equation (21), however, we make a further simplification. We assume that the integrand of equation (19) is sharply peaked toward the center of the star, and that the nuclear reaction rates depend on a power-law function of temperature. Consistency then demands that the power-law index is Θ_c. Further, the temperature can be modeled as an exponentially decaying function near the center of the star so that $T \sim \exp[-\beta\xi]$. The expression for $\ell(\xi)$ then becomes

$$\ell(\xi) = \tfrac{1}{2}\int_0^{x_{\mathrm{end}}} x^2\,\mathrm{e}^{-x}\,\mathrm{d}x \qquad \text{where } x_{\mathrm{end}} = \beta\Theta_c\xi. \tag{22}$$

Using this expression for $\ell(\xi)$ in the integral of equation (21), we can write the luminosity in the form

$$L_* = aT_c^4 \frac{4\pi c}{3\rho_c\kappa_0}\frac{R}{\beta\Theta_c}. \tag{23}$$

2.4. Stellar structure solutions

With the solution (23) to the energy transport equation, we now have four equations and four unknowns. After some algebra, we obtain the following equation for the central temperature:

$$\Theta_c I(\Theta_c) T_c^3 = \frac{(4\pi)^3 ac}{3\beta\kappa_0\mathcal{C}}\left(\frac{M_*}{\mu_0}\right)^4 \left(\frac{G}{(n+1)\mathcal{R}}\right)^7, \tag{24}$$

or, alternatively,

$$I(\Theta_c)\Theta_c^{-8} = \frac{2^{12}\pi^5}{45} \frac{1}{\beta\kappa_0 \mathcal{C} E_G^3 \hbar^3 c^2} \left(\frac{M_*}{\mu_0}\right)^4 \left(\frac{G\langle m\rangle}{(n+1)}\right)^7. \tag{25}$$

The right-hand side of the equation is thus a dimensionless quantity. Further, the quantities μ_0 and β are dimensionless measures of the mass and luminosity integrals over the star, respectively; they are expected to be of order unity and to be roughly constant from star to star (and from universe to universe). The remaining constants are fundamental. Note that, for typical values of the parameters in our universe, the right-hand side of this equation is approximately 10^{-9}.

With the central temperature T_c, or equivalently Θ_c, determined through equation (25), we can find expressions for the remaining stellar parameters. The radius is given by

$$R_* = \frac{GM_*\langle m\rangle}{kT_c} \frac{\xi_*}{(n+1)\mu_0}, \tag{26}$$

and the luminosity is given by

$$L_* = \frac{16\pi^4}{15} \frac{1}{\hbar^3 c^2 \beta\kappa_0 \Theta_c} \left(\frac{M_*}{\mu_0}\right)^3 \left(\frac{G\langle m\rangle}{n+1}\right)^4. \tag{27}$$

The photospheric temperature is then determined from the usual outer boundary condition so that

$$T_* = \left(\frac{L_*}{4\pi R_*^2 \sigma}\right)^{1/4}. \tag{28}$$

For this simple polytropic stellar model, figures 2 and 3 show the H–R diagram and the corresponding luminosity versus mass relation for stars on the zero age main sequence (ZAMS). The three curves show different choices for the polytropic indices: the dashed curves show results for $n = 3/2$, the value appropriate for low-mass stars. The dotted curves show the results for $n = 3$, the value for high-mass stars. The solid line (marked by symbols) show the results for n varying smoothly between $n = 3/2$ in the limit $M_* \to 0$ and $n = 3$ in the limit $M_* \to \infty$. We take this latter case as our standard model (although the effects of changing the polytropic index n are small compared to the effects of changing the fundamental constants—see section 3).

One can compare these models with the results of more sophisticated stellar structure models [13, 14] or with observations of stars on the ZAMS. In both of these comparisons, this polytropic model provides a good prediction for the stellar temperature as a function of stellar mass. However, the luminosities of the highest-mass stars are somewhat low, mostly because the stellar radii from the models are correspondingly low; this discrepancy, in turn, results from our simplified treatment of nuclear reactions. Nonetheless, this polytropic model works rather well and produces the correct stellar characteristics (L_*, R_*, T_*), within a factor of $\sim$2, as a function of mass M_*, over a range in mass of $\sim$1000 and a range in luminosity of $\sim 10^9$. This degree of accuracy is sufficient for the purposes of this paper, and is quite good given the simplifying assumptions used in order to obtain analytic results. More sophisticated stellar models would include varying values of $\mathcal{C}$ to incorporate more complex nuclear reaction chains, detailed energy transport including

JCAP08(2008)010

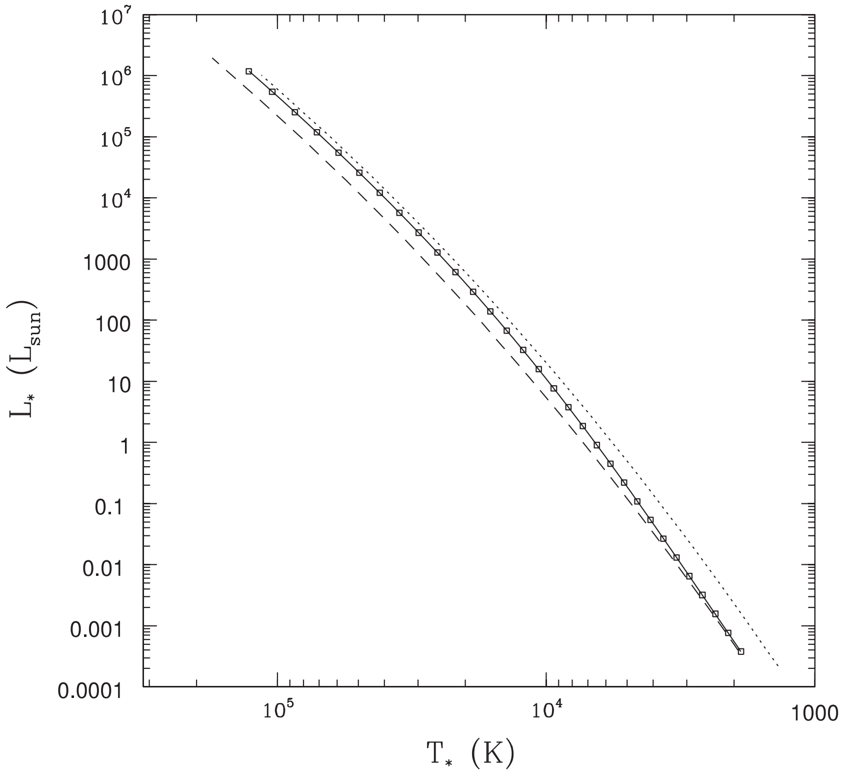

Figure 2. H–R diagram showing the main sequence for polytropic stellar model using standard values of the parameters, i.e. those in our universe. The three cases shown here correspond to the main sequence for an $n = 3/2$ polytrope (lower dashed curve), an $n = 3$ polytrope (upper dotted curve) and a model that smoothly varies from $n = 3/2$ at low masses to $n = 3$ at high masses (solid curve marked by symbols).

convection, a more refined treatment of opacity, and a fully self-consistent determination of the density and pressure profiles (i.e. the departures from our polytropic models). In particular, we can achieve even better agreement between this stellar structure model and observed stellar properties if we allow the nuclear reaction parameter $\mathcal{C}$ to increase with stellar mass (as it does in high-mass stars due to the CNO cycle). In the spirit of this work, however, we use a single value of $\mathcal{C}$, which corresponds to the case in which a single nuclear species is available for fusion (this scenario thus represents the simplest universes).

3. Constraints on the existence of stars

Using the stellar structure model developed in the previous section, we now explore the range of possible stellar masses in universes with varying values of the stellar parameters. First, we find the minimum stellar mass required for a star to overcome quantum mechanical degeneracy pressure (section 3.1) and then find the maximum stellar mass as limited by radiation pressure (section 3.2). These two limits are then combined to find the allowed range of stellar masses, which can vanish when the required nuclear burning temperatures become too high (section 3.3). Another constraint on stellar parameters

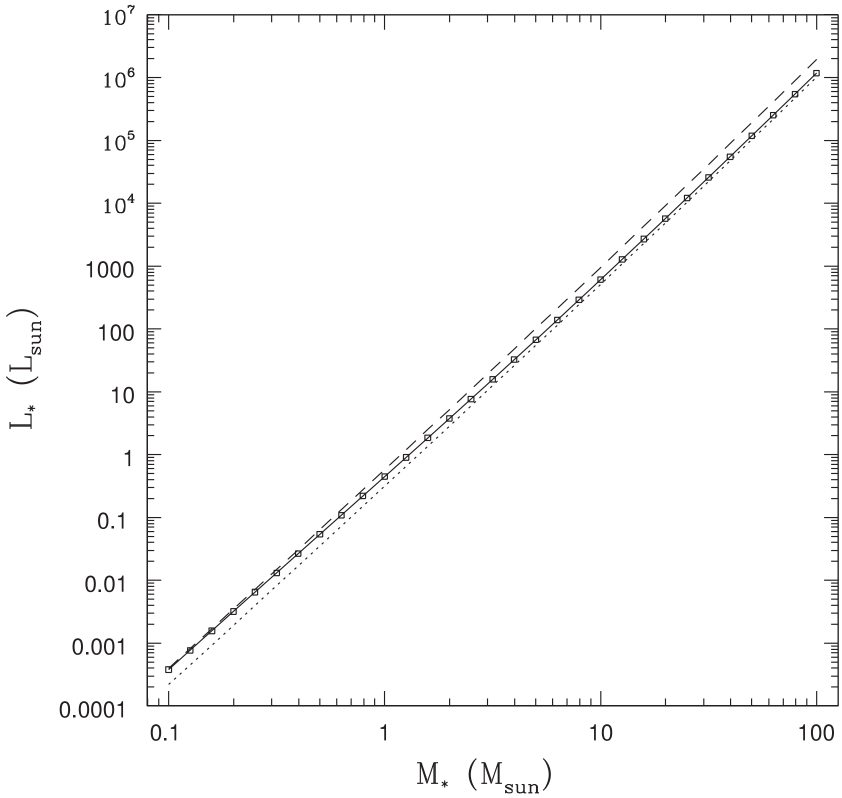

Figure 3. Stellar luminosity as a function of stellar mass for standard values of the parameters. The three curves shown here correspond to the L_*–M_* relation for an $n = 3/2$ polytrope (dashed curve), an $n = 3$ polytrope (dotted curve) and a model that smoothly varies from $n = 3/2$ at low masses to $n = 3$ at high masses (solid curve marked by symbols). All quantities are given in solar units.

JCAP08(2008)010

arises from the requirement that stable nuclear burning configurations exist (section 3.4). We delineate (in section 3.5) the range of parameters for which these two considerations provide the limiting constraints on stellar masses and then find the region of parameter space that allows the existence of stars. Finally, we consider the constraints implied by the Eddington luminosity (section 3.6) and show that they are comparable to those considered in the previous subsections.

3.1. Minimum stellar mass

The minimum mass of a star is determined by the onset of degeneracy pressure. Specifically, for stars with sufficiently small masses, degeneracy pressure enforces a maximum temperature which is below that required for nuclear fusion. The central pressure at the center of a star is given approximately by the expression

$$P_{\rm c} \approx \left(\frac{\pi}{36}\right)^{1/3} G M_*^{2/3} \rho_{\rm c}^{4/3}, \tag{29}$$

where the subscript denotes that the quantities are to be evaluated at the center of the star. This result follows directly from the requirement of hydrostatic equilibrium (e.g. [15]).

At the low-mass end of the range of possible stellar masses, the pressure is determined by contributions from the ideal gas law and from non-relativistic electron degeneracy pressure. As a result, the central pressure of the star must also satisfy the relation

$$P_c = \left(\frac{\rho_c}{m_{\text{ion}}}\right) kT_c + K_{\text{dp}} \left(\frac{\rho_c}{m_{\text{ion}}}\right)^{5/3}, \tag{30}$$

where m_{ion} is the mean mass of the ions (so that ρ_c/m_{ion} determines the number density of ions) and where the constant K_{dp} that determines degeneracy pressure is given by

$$K_{\text{dp}} = \frac{\hbar^2}{5m_e} \left(3\pi^2\right)^{2/3}, \tag{31}$$

where m_e is the electron mass. Notice that we have also assumed that the star has neutral charge so that the number density of electrons is equal to that of the ions, and that $m_e \ll m_{\text{ion}}$.

Combining the two expressions for the central pressure and solving for the central temperature, we obtain

$$kT_c = \left(\frac{\pi}{36}\right)^{1/3} GM_*^{2/3} m_{\text{ion}} \rho_c^{1/3} - K_{\text{dp}}(\rho_c/m_{\text{ion}})^{2/3}. \tag{32}$$

The above expression is a simple quadratic function of the variable $\rho_c^{1/3}$ and has a maximum for a particular value of the central density [11], i.e.

$$kT_{\max} = \left(\frac{\pi}{36}\right)^{2/3} \frac{G^2 M_*^{4/3} m_{\text{ion}}^{8/3}}{4K_{\text{dp}}}. \tag{33}$$

If we set this value of the central temperature equal to the minimum required ignition temperature for a star, T_{nuc}, we obtain the minimum stellar mass:

$$M_{*\min} = \left(\frac{36}{\pi}\right)^{1/2} \frac{(4K_{\text{dp}} kT_{\text{nuc}})^{3/4}}{G^{3/2} m_{\text{ion}}^2}. \tag{34}$$

After rewriting the equation of state parameter K_{dp} in terms of fundamental constants, this expression for the minimum stellar mass becomes

$$M_{*\min} = 6(3\pi)^{1/2} \left(\frac{4}{5}\right)^{3/4} \left(\frac{m_P}{m_{\text{ion}}}\right)^2 \left(\frac{kT_{\text{nuc}}}{m_e c^2}\right)^{3/4} M_0. \tag{35}$$

As expected, the minimum stellar mass is given by a dimensionless expression times the fundamental stellar mass scale defined in equation (1). Notice also that the gravitational constant G enters into this mass expression with an exponent of $-3/2$, as anticipated by equation (1).

3.2. Maximum stellar mass

A similar calculation gives the maximum possible stellar mass. In this case the central pressure also has two contributions, this time from the ideal gas law and from radiation pressure P_{R}, where

$$P_{\mathrm{R}} = \tfrac{1}{3} a T_{\mathrm{c}}^4, \tag{36}$$

where $a = \pi^2 k^4 / 15(\hbar c)^3$ is the radiation constant. Following standard convention [11], we define the parameter f_{g} to be the fraction of the central pressure provided by the ideal gas law. As a result, the radiation pressure contribution is given by $P_{\mathrm{R}} = (1 - f_{\mathrm{g}}) P_{\mathrm{c}}$. The central temperature can be eliminated in favor of f_{g} to obtain the expression

$$P_{\mathrm{c}} = \left(\frac{3}{a} \frac{(1 - f_{\mathrm{g}})}{f_{\mathrm{g}}^4} \right)^{1/3} \left(\frac{4\rho_{\mathrm{c}}}{\langle m \rangle} \right)^{4/3}, \tag{37}$$

where $\langle m \rangle$ is the mean mass per particle of a massive star. By demanding that the star is in hydrostatic equilibrium, we obtain the following expression for the maximum mass of a star:

$$M_{*\,\mathrm{max}} = \left(\frac{36}{\pi} \right)^{1/2} \left(\frac{3}{a} \frac{(1 - f_{\mathrm{g}})}{f_{\mathrm{g}}^4} \right)^{1/2} G^{-3/2} \left(\frac{k}{\langle m \rangle} \right)^2, \tag{38}$$

which can also be written in terms of the fundamental mass scale M_0, i.e.

$$M_{*\,\mathrm{max}} = \left(\frac{18\sqrt{5}}{\pi^{3/2}} \right) \left(\frac{1 - f_{\mathrm{g}}}{f_{\mathrm{g}}^4} \right)^{1/2} \left(\frac{m_{\mathrm{P}}}{\langle m \rangle} \right)^2 M_0, \tag{39}$$

where this expression must be evaluated at the maximum value of f_{g} for which the star can remain stable. Although the requirement of stability does not provide a perfectly well-defined threshold for f_{g}, the value $f_{\mathrm{g}} = 1/2$ is generally used [11] and predicts maximum stellar masses in reasonable agreement with observed stellar masses (for present-day stars in our universe). For this choice, the above expression becomes $M_{*\,\mathrm{max}} \approx 20 (m_{\mathrm{P}} / \langle m \rangle)^2 M_0$. Since massive stars are highly ionized, $\langle m \rangle \approx 0.6 m_{\mathrm{P}}$ under standard conditions, and hence $M_{*\,\mathrm{max}} \approx 56 M_0 \approx 100 M_\odot$ for our universe. As shown below, this constraint is nearly the same as that derived on the basis of the Eddington luminosity (section 3.6).

3.3. Constraints on the range of stellar masses: the maximum nuclear ignition temperature

As derived above, the minimum stellar mass can be written as a dimensionless coefficient times the fundamental stellar mass scale from equation (1). Further, the dimensionless coefficient depends on the ratio of the nuclear ignition temperature to the electron mass energy, i.e. $kT_{\mathrm{nuc}} / m_{\mathrm{e}} c^2$. The maximum stellar mass, also defined above, can be written as a second dimensionless coefficient times the mass scale M_0. This second coefficient depends on the maximum radiation pressure fraction f_{g} and (somewhat less sensitively) on the mean particle mass $\langle m \rangle$ of a high-mass star. For completeness, we note that the Chandrasekhar mass M_{ch} [15] can be written as yet another dimensionless coefficient times this fundamental mass scale, i.e.

$$M_{\mathrm{ch}} \approx \frac{1}{5} (2\pi)^{3/2} \left(\frac{Z}{A} \right)^2 M_0, \tag{40}$$

where Z/A specifies the number of electrons per nucleon in the star.

JCAP08(2008)010

These results thus show that, if the constants of the universe were different, or if they are different in other universes (or different in other parts of our universe), then the possible range of stellar masses would change accordingly. We see immediately that if the nuclear ignition temperature is too large, then the range of stellar masses could vanish. If all other constants are held fixed, then the requirement that the minimum stellar mass becomes as large as the maximum stellar mass is given by

$$\left(\frac{kT_{\rm nuc}}{m_{\rm e}c^2}\right) \geq \frac{5}{4}\left(\frac{360}{3\pi^4}\right)^{2/3}\left(\frac{\sqrt{1-f_{\rm g}}}{\sqrt{8}f_{\rm g}^2}\right)^{4/3}\left(\frac{m_{\rm ion}}{\langle m\rangle}\right)^{8/3} \approx 1.4\left(\frac{m_{\rm ion}}{\langle m\rangle}\right)^{8/3}, \quad (41)$$

where we have used $f_{\rm g} = 1/2$ to obtain the final equality. For high-mass stars in our universe, $\langle m\rangle/m_{\rm ion} = 0.6$, and the right-hand side of the equation is about 5.6. For the simplistic case where $\langle m\rangle = m = m_{\rm ion}$, the right-hand side is 1.4. In any case, this value is of order unity and is not expected to vary substantially from universe to universe. As a result, the condition for the nuclear burning temperature to be so high that no viable range of stellar masses exists takes the form $kT_{\rm nuc}/(m_{\rm e}c^2) \gtrsim 2$. For standard values of the other parameters, the nuclear ignition temperature (for hydrogen fusion) would have to exceed $T_{\rm nuc} \sim 10^{10}$ K. For comparison, the usual hydrogen burning temperature is about 10^7 K and the helium burning temperature is about 2×10^8 K. We stress that the hydrogen burning temperature in our universe is much smaller than the value required for no range of stellar masses to exist—in this sense, our universe is *not* fine-tuned to have special values of the constants to allow the existence of stars. The large value of nuclear ignition temperature required to suppress the existence of stars roughly corresponds to the temperature required for silicon burning in massive stars (again, for the standard values of the other parameters). Finally we note that the nuclear burning temperature $T_{\rm nuc}$ depends on the fundamental constants in a complicated manner; this issue is addressed below.

Equation (41) emphasizes several important issues. First, we note that the existence of a viable range of stellar masses—according to this constraint—does not depend on the gravitational constant G. The value of G determines the scale for the stellar mass range, and the scale is proportional to $G^{-3/2} \sim \alpha_G^{-3/2}$, but the coefficients that define both the minimum stellar mass and the maximum stellar mass are independent of G. The possible existence of stars in a given universe depends on having a low enough nuclear ignition temperature, which requires the strong nuclear force to be 'strong enough' and/or the electromagnetic force to be 'weak enough'. These requirements are taken up in section 3.5. Notice also that we have assumed $m_{\rm e} \ll m_{\rm P}$, so that electrons provide the degeneracy pressure, but the ions provide the mass.

3.4. Constraints on stable stellar configurations

In this section we combine the results derived above to determine the minimum temperature required for a star to operate through the burning of nuclear fuel (for given values of the constants). For a given minimum nuclear burning temperature $T_{\rm nuc}$, equation (35) defines the minimum mass necessary for fusion. Alternatively, the equation gives the maximum temperature that can be attained with a star of a given mass in the face of degeneracy pressure. On the other hand, equation (25) specifies the central temperature $T_{\rm c}$ necessary for a star to operate as a function of stellar mass. We also note

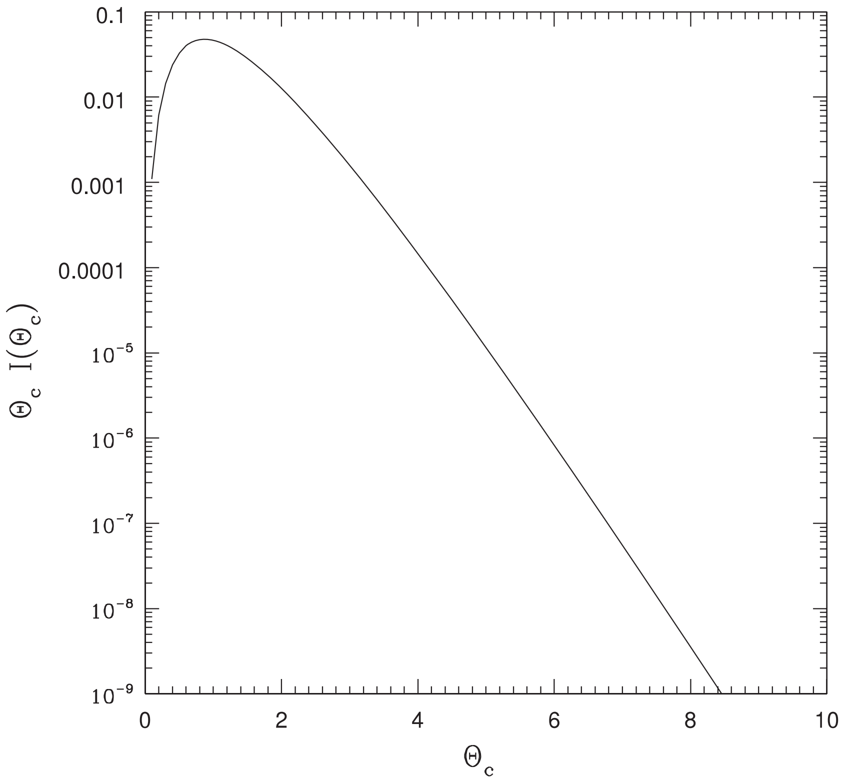

Figure 4. Profile of $\Theta_c I(\Theta_c)$ as a function of $\Theta_c = (E_G/4kT_c)^{1/3}$. The integral $I(\Theta_c)$ determines the stellar luminosity in dimensionless units and Θ_c defines the central stellar temperature. This profile has a well-defined maximum near $\Theta_c \approx 0.869$, where the peak of the profile defines a limit on the values of the fundamental constants required for nuclear burning, and where the location of the peak defines a maximum nuclear burning temperature (see text).

that the temperature T_c is an increasing function of stellar mass. By using the minimum mass from equation (35) to specify the mass in equation (25), we can eliminate the mass dependence and solve for the minimum value of the nuclear ignition temperature $T_{\rm nuc}$. The resulting temperature is given in terms of Θ_c, which is given by the solution to the following equation:

$$\Theta_c I(\Theta_c) = \left(\frac{2^{23}\pi^7 3^4}{5^{11}}\right)\left(\frac{\hbar^3}{c^2}\right)\left(\frac{1}{\beta\mu_0^4}\right)\left(\frac{1}{mm_e^3}\right)\left(\frac{G}{\kappa_0 \mathcal{C}}\right). \tag{42}$$

Note that the parameters on the right-hand side of the equation have been grouped to include numbers, constants that set units, dimensionless parameters of the polytropic solution, the relevant particle masses and the stellar parameters that depend on the fundamental forces. Within the treatment of this paper, these latter quantities could vary from universe to universe. Notice also that we have specialized to the case in which $\langle m\rangle = m_{\rm ion} = m$.

The left-hand side of equation (42) is determined for a given polytropic index. Here we use the value $n = 3/2$ corresponding to both low-mass conventional stars and degenerate stars. The resulting profile for $\Theta_c I(\Theta_c)$ is shown in figure 4. The right-hand side of equation (42) depends on the fundamental constants and is thus specified for a given universe. In order for nuclear burning to take place, equation (42) must have a solution—the left-hand side has a maximum value, which places an upper bound on the parameters of the right-hand side. Through numerical evaluation, we find that this maximum value is ~ 0.0478 and occurs at $\Theta_c \approx 0.869$. The maximum possible nuclear burning temperature thus takes the form

$$(kT)_{\max} \approx 0.38 E_G, \tag{43}$$

where E_G is the Gamow energy appropriate for the given universe. The corresponding constraint on the stellar parameters required for nuclear burning can then be written in the form

$$\frac{\hbar^3 G}{c^2 m m_e^3 \kappa_0 \mathcal{C}} \le \frac{5^{11}\beta\mu_0^4}{2^{23}\pi^7 3^4}[\Theta_c I(\Theta_c)]_{\max} \approx 2.6\times 10^{-5}, \tag{44}$$

where we have combined all dimensionless quantities on the right-hand side. For typical stellar parameters in our universe, the left-hand side of the above equation has the value $\sim 2.4\times 10^{-9}$, smaller than the maximum by a factor of $\sim$11 000. As a result, the combination of constants derived here can take on a wide range of values and still allow for the existence of nuclear burning stars. In this sense, the presence of stars in our universe does not require fine-tuning the constants.

Notice that, for combinations of the constants that allow for nuclear burning, equation (42) has two solutions. The relevant physical solution is the one with larger Θ_c, which corresponds to a lower temperature. The second, high temperature solution would lead to an unstable stellar configuration. As a consistency check, note that for the values of the constants in our universe, the solution to equation (42) implies that $\Theta_c \approx 5.38$, which corresponds to a temperature of about 9×10^6 K. This value is thus approximately correct: detailed stellar models show that the central temperature of the Sun is about 15×10^6 K and the lowest possible hydrogen burning temperature is a few million degrees [11, 13, 14].

3.5. Combining the constraints

Thus far, we have derived two constraints on the range of stellar structure parameters that allow for the existence of stars. The requirement of stable nuclear burning configuration places an upper limit on the nuclear burning temperature, which takes the approximate form $kT \lesssim 0.38 E_G$. In addition, the requirement that the minimum stellar mass (due to degeneracy pressure) not exceed the maximum stellar mass (due to radiation pressure) places a second upper limit on the nuclear burning temperature, $kT \lesssim 2 m_e c^2$. As a result, the reason for a universe failing to produce stars depends on the size of the dimensionless parameter

$$Q_F = 2\alpha^2 \frac{m}{m_e}, \tag{45}$$

where m is the mass of the nuclei that would experience reactions. Note that Q_F is proportional to the ratio of the Gamow energy to the rest mass energy of the electron and has the value $Q_F \approx 0.2$ in our universe. For $Q_F > 1$, stars can fail to exist due to the range of allowed stellar masses shrinking to zero, whereas for $Q_{\rm F} < 1$ stars can fail to exist due to the absence of stable nuclear burning configurations.

We can combine the constraints to delineate the portion of parameter space that allows for the existence of stars. For the sake of definiteness, we fix the values of the particle masses and specialize to the simplest case where the nuclear burning species has a single mass m. We also assume that the stellar opacity scales according to $\kappa_0 \propto \alpha^2$, as expected since $\kappa \sim \sigma_{\rm T}/m$ and $\sigma_{\rm T} \propto \alpha^2$. With these restrictions, the remaining stellar parameters that can be varied are the fine structure constant α, the gravitational constant G and the nuclear burning parameter $\mathcal{C}$. Note that α depends on the strength of the electromagnetic force, G depends on the strength of gravity and $\mathcal{C}$ depends on a combination of the weak and strong nuclear forces, which jointly determine the nuclear reaction properties for a given universe. Notice also that, since we are fixing particle masses, the gravitational constant G is proportional to the gravitational fine structure constant α_G (equation (2)).

Figure 5 shows the resulting allowed region of parameter space for the existence of stars. Here we are working in the (α, G) plane, where we scale the parameters by their values in our universe, and the results are presented on a logarithmic scale. For a given nuclear burning constant $\mathcal{C}$, figure 5 shows the portion of the plane that allows for stars to successfully achieve sustained nuclear reactions. Curves are given for three values of $\mathcal{C}$: the value for p–p burning in our universe (solid curve), 100 times larger than this value (dashed curve) and 100 times smaller (dotted curve). The region of the diagram that allows for the existence of stars is the area below the curves.

Figure 5 provides an assessment of how ‘fine-tuned’ the stellar parameters must be in order to support the existence of stars. First we note that our universe, with its location in this parameter space marked by the open triangle, does not lie near the boundary between universes with stars and those without. Specifically, the values of α, G and/or $\mathcal{C}$ can change by more than two orders of magnitude in any direction (and by larger factors in some directions) and still allow for stars to function. This finding can be stated another way: within the parameter space shown, which spans 10 orders of magnitude in both α and G, about one-fourth of the space supports the existence of stars.

Next we note that a relatively sharp boundary occurs in this parameter space for large values of the fine structure constant, where $\alpha \sim 200\alpha_0$, and this boundary is nearly independent of the nuclear burning constant $\mathcal{C}$. Strictly speaking, this well-defined boundary is the result of the required value of G becoming an exponentially decreasing function of α/α_0, as shown in section 3.7 below. For the given range of G and for values of α above this threshold, the Gamow energy is much larger than the rest mass energy of the electron, so that the maximum nuclear burning temperature becomes a fixed value (that given by equation (41)) and hence the nuclear reaction rates are exponentially suppressed by the electromagnetic barrier (section 2.2). On the other side of the graph, for values of α smaller than those in our universe, the range of allowed parameter space is limited due to the absence of stable nuclear burning configurations (section 3.4). In this regime, for sufficiently large G, the nuclear burning temperature becomes so large that the barrier disappears (and hence stability is no longer possible). Since the nuclear burning temperature $T_{\rm nuc}$ required to support stars against gravity increases as the gravitational constant G increases, and since $T_{\rm nuc}$ is bounded from above, there is a maximum value of G that can support stars (for a given value of $\mathcal{C}$). For the value of $\mathcal{C}$ appropriate for p–p burning in our universe, we thus find that $G/G_0 \lesssim 2 \times 10^5$. Finally, we note that ‘stellar’

JCAP08(2008)010

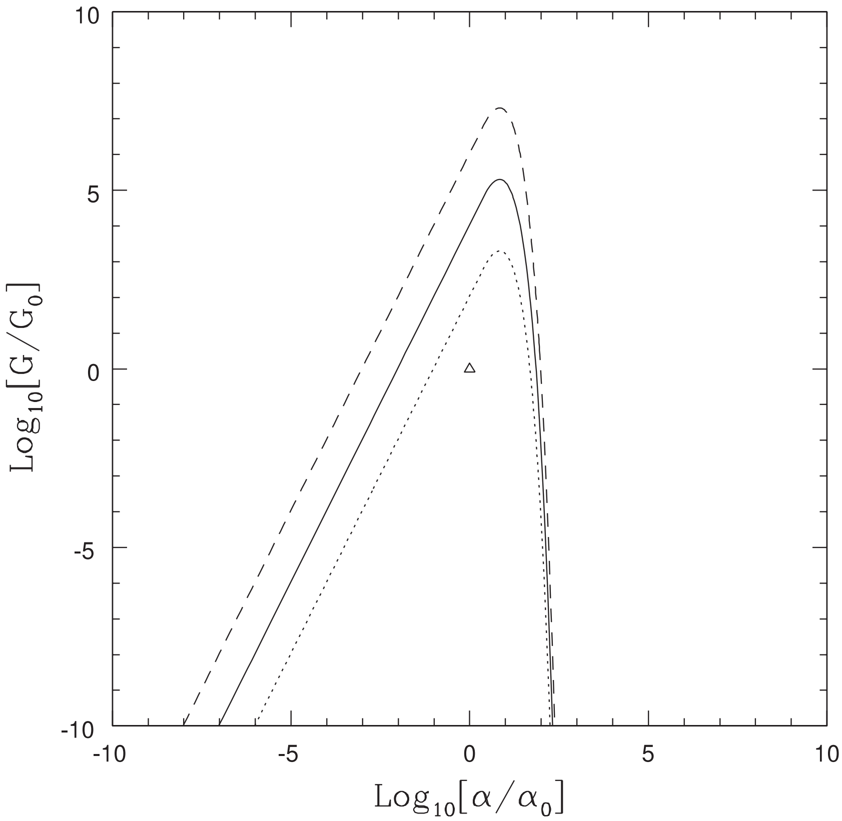

Figure 5. Allowed region of parameter space for the existence of stars. Here the parameter space is the plane of the gravitational constant $\log_{10}[G/G_0]$ versus the fine structure constant $\log_{10}[\alpha/\alpha_0]$, where both quantities are scaled relative to the values in our universe. The allowed region lies under the curves, which are plotted here for three different values of the nuclear burning constants $\mathcal{C}$: the standard value for p–p burning in our universe (solid curve), 100 times the standard value (dashed curve) and 0.01 times the standard value (dotted curve). The open triangular symbol marks the location of our universe in this parameter space.

bodies outside the range of allowed parameter space can exist, in principle, and can even generate energy, but they would not resemble the stable, long-lived nuclear burning stars of our universe.

3.6. The Eddington luminosity

For a star of given mass, the maximum rate at which it can generate energy is given by the Eddington luminosity. This luminosity defines a minimum lifetime for stars. The Eddington luminosity can be written in the form

$$L_{*\,\mathrm{max}} = 4\pi c G M_* / \kappa_{\mathrm{em}}, \tag{46}$$

where κ_{em} is the opacity in the stellar photosphere. For the sake of definiteness, we take κ_{em} to be the opacity appropriate for pure electron scattering, which is applicable to hot plasmas where the Eddington luminosity is relevant, i.e.

$$\kappa_{\rm em} = \frac{1+X_1}{2}\frac{\sigma_{\rm T}}{m_{\rm P}}, \tag{47}$$

where $\sigma_{\rm T}$ is the Thompson cross section and X_1 is the mass fraction of hydrogen. Since the maximum luminosity implies a minimum stellar lifetime, for a given efficiency ϵ of converting mass into energy, we obtain the following constraint on stellar lifetimes:

$$t_* > t_{*\,\rm min} = \epsilon\left(\frac{1+X_1}{3}\right)\frac{\alpha^2}{\alpha_G}\frac{\hbar m_{\rm P}}{(m_{\rm e}c)^2}. \tag{48}$$

Since atomic timescales are given (approximately) by

$$t_{\rm A} \approx \frac{\hbar}{\alpha^2 m_{\rm e}c^2}, \tag{49}$$

the ratio of stellar timescales to atomic timescales is given by the following expression:

$$\frac{t_{*\,\rm min}}{t_{\rm A}} = \epsilon\left(\frac{1+X_1}{3}\right)\frac{\alpha^4}{\alpha_G}\frac{m_{\rm P}}{m_{\rm e}}, \tag{50}$$

where the expression has a numerical value of $\sim 4\times 10^{30}$ for the parameters in our universe.

We can also use the Eddington luminosity to derive another upper limit on the allowed stellar mass. Within the context of our model, the stellar luminosity is given by equation (27). This luminosity must be less than the Eddington luminosity given by equation (46), which implies a constraint of the form

$$\frac{M_*}{M_0} \lesssim \frac{4}{\pi}\sqrt{60}\left(\frac{\beta\mu_0^3\kappa_0 m_{\rm P}\Theta_{\rm c}}{\sigma_{\rm T}}\right)^{1/2}, \tag{51}$$

where we have specialized to the case of polytropic index $n = 3$ (appropriate for high-mass stars with large admixtures of radiation pressure) and have taken $\langle m\rangle = m_{\rm P}$. Note that, since $\kappa_0 \sim \sigma_{\rm T}/m_{\rm P}$, and since β and μ_0 are given by the polytropic solution (and are of order unity), the right-hand side of the above equation is approximately $50\sqrt{\Theta_{\rm c}}$, as expected. In other words, the requirement that the stellar luminosity must be less than the Eddington limit (equation (46)) produces nearly the same bound on stellar masses as the requirement that the star not be dominated by radiation pressure (equation (39)).

Notice also that we expect $\kappa_0 \sim \sigma_{\rm T}/m_{\rm P}$ for other universes, so that the general constraint takes the approximate form $M_*/M_0 \lesssim 50\sqrt{\Theta_{\rm c}}$. In addition, as shown by figure 4, the parameter $\Theta_{\rm c}$ is confined to a narrow range—the function $\Theta_{\rm c}I(\Theta_{\rm c})$, and hence the left-hand side of equation (42), varies by 8 orders of magnitude for $1 \lesssim \sqrt{\Theta_{\rm c}} \lesssim 3$.

3.7. Limiting forms

For much of the allowed parameter space where stars can operate, the value of $\Theta_{\rm c}$ is large compared to its minimum value; specifically, this claim holds for the region of parameter space that is not near the upper left boundary in figure 5. In this case, we can derive an analytic asymptotic expression for the integral function $I(\Theta_{\rm c})$, which takes the form

$$I(\Theta_{\rm c}) \sim 3\Theta_{\rm c}\,{\rm e}^{-3\Theta_{\rm c}-1}\left(\frac{3\pi}{\Theta_{\rm c}+4/3}\right)^{1/2} \to (3/{\rm e})\sqrt{3\pi\Theta_{\rm c}}\,{\rm e}^{-3\Theta_{\rm c}}. \tag{52}$$

Comparing this asymptotic expression to the numerically determined values, we find that equation (52) provides an estimate that is within a factor of 2 of the correct result over the range $1 \leq \Theta_c \leq 100$, where $I(\Theta_c)$ varies by $\sim$128 orders of magnitude.

With this asymptotic expression in hand, we can find the relationship between the gravitational constant and the fine structure constant on the boundary of parameter space (as shown in figure 5). We find that

$$G \sim G_0 \exp\left[-\frac{3}{2}\left(\frac{\alpha}{\alpha_0}\right)^{2/3}\right]. \tag{53}$$

At the edge of the allowed stellar parameter space, G is thus an exponentially decreasing function α, which results in the nearly vertical boundary shown in figure 5.

4. Unconventional stars

The results of the previous section show that stars can exist in a relatively large fraction of the parameter space. On the other hand, in order for stars to exist at all, the nuclear burning parameter $\mathcal{C}$ must be nonzero; otherwise, stars, as objects powered by nuclear reactions, cannot exist. In situations where $\mathcal{C} = 0$, or where the values of the other parameters conspire to disallow stars (see figure 5), other types of stellar objects could, in principle, fill the role played by stars in our universe. This section briefly explores this possibility with three examples: black holes that generate energy through Hawking evaporation (section 4.1), degenerate dark matter stars that generate energy via annihilation (section 4.2) and degenerate baryonic matter stars that generate energy by capturing dark matter particles which then annihilate (section 4.3). We note that a host of other possibilities exist (e.g. astrophysical objects powered by proton decay), but a proper treatment of such cases is beyond the scope of this present work.

4.1. Black holes

Black holes can exist in any universe with gravity and will generate energy (at *some* rate) through Hawking evaporation (e.g. [17]). Further, the stellar structure of these objects depends only on the gravitational constant G. In order to consider black holes playing the role of stars, however, we must invoke additional constraints. For the purposes of illustration, this section finds the values of the fundamental constants for which black holes can serve as stellar bodies to support 'life'. Specifically, in order for black holes to fill the role played by stars in our universe, two constraints must be satisfied: first, the black holes must live long enough to allow for life to develop. Second, the black holes must provide enough power to run a biosphere. The first constraint implies that black holes must be sufficiently massive, whereas the second constraint implies that the black holes must be sufficiently small. The compromise between these two requirements provides an overall constraint that must be met in order for black holes to play the role of stars.

The lifetime of a black hole with mass $M_{\rm bh}$ is given by

$$\tau_{\rm bh} = \frac{2650\pi}{g_* \hbar c^4} G^2 M_{\rm bh}^3, \tag{54}$$

where g_* is the total number of effective degrees of freedom in the radiation field produced through the Hawking effect. This lifetime should be compared with the typical atomic timescale τ_A given by equation (49). We thus have a constraint of the form

$$\frac{\tau_{bh}}{\tau_A} = \frac{2560\pi}{g_*(\hbar c)^2} \alpha^2 G^2 M_{bh}^3 m_e \geq N_{bio}, \tag{55}$$

where N_{bio} is the number of atomic timescales required for life to evolve. In our solar system, the number $N_{bio} \approx 10^{34}$, which is also the number of atomic timescales in the life of a solar-type star. Although the minimum value of N_{bio} remains uncertain, we expect it to be within a few orders of magnitude of this value. For the sake of definiteness, we take the (somewhat optimistic) value of $N_{bio} = 10^{33}$ for this analysis.

The second constraint is that the black hole must provide enough power to run a biosphere. In our solar system, the Earth intercepts about 100 quadrillion watts of power from the Sun. We thus expect that the black hole must have a minimum luminosity and obey a constraint of the form

$$L_{bh} = \frac{g_* \hbar c^6}{7680\pi} (G M_{bh})^{-2} \geq L_{min}, \tag{56}$$

where L_{min} is the minimum luminosity of a stellar object required to support life. In general, this minimum value of luminosity will vary with the values of the fundamental constants. In the absence of a definitive theory, we adopt the following simple scaling law: the energy levels E_A of atoms vary according to $E_A \propto \alpha^2$, and the atomic timescale varies as $t_A \propto \alpha^{-2}$. In order for the luminosity to provide the same number of atomic reactions over the total lifetime of the system, the luminosity should scale with the fine structure constant as

$$L_{min} = L_{min0} (\alpha/\alpha_0)^4, \tag{57}$$

where L_{min0} is the minimum necessary luminosity in our universe. Although the value of this latter quantity is uncertain, we adopt $L_{min0} \approx 10^{17}$ erg s^{-1} as a representative value. The scaling law of equation (57) is also not definitive, but rather illustrative.

Combining the two constraints allows for the elimination of the mass, and thereby provides an overall constraint of the from

$$\frac{N_{bio} \hbar^{1/2} (G/\alpha)^2}{m_e} \leq \frac{c^7}{96 (15\pi)^{1/2} {L_{min}}^{3/2}}. \tag{58}$$

If we scale this constraint using measured values of the constants, we obtain the relation

$$\left(\frac{G}{G_0}\right) \left(\frac{\alpha}{\alpha_0}\right)^4 \leq 24 \left(\frac{N_{bio}}{10^{33}}\right)^{-1} \left(\frac{L_{min0}}{10^{17} \text{ erg s}^{-1}}\right)^{-3/2}. \tag{59}$$

In our universe, black holes must have masses greater than about 6×10^{13} g in order to last for $N_{bio} = 10^{33}$ atomic timescales, and must have masses less than about 2×10^{14} g in order to produce enough power (L_{min}). As a result, a biosphere could be powered by a black hole, although we have adopted somewhat optimistic requirements, e.g. the required luminosity is only L_{min}, which is much less than a solar luminosity. The largest obstacle, however, is the production of black holes with this mass scale.

Figure 6 shows the region of parameter space for which black holes can play the role of stars. To construct this diagram, we assume that black holes must live $N_{bio} = 10^{33}$ atomic timescales and produce enough luminosity. For this latter requirement, we use

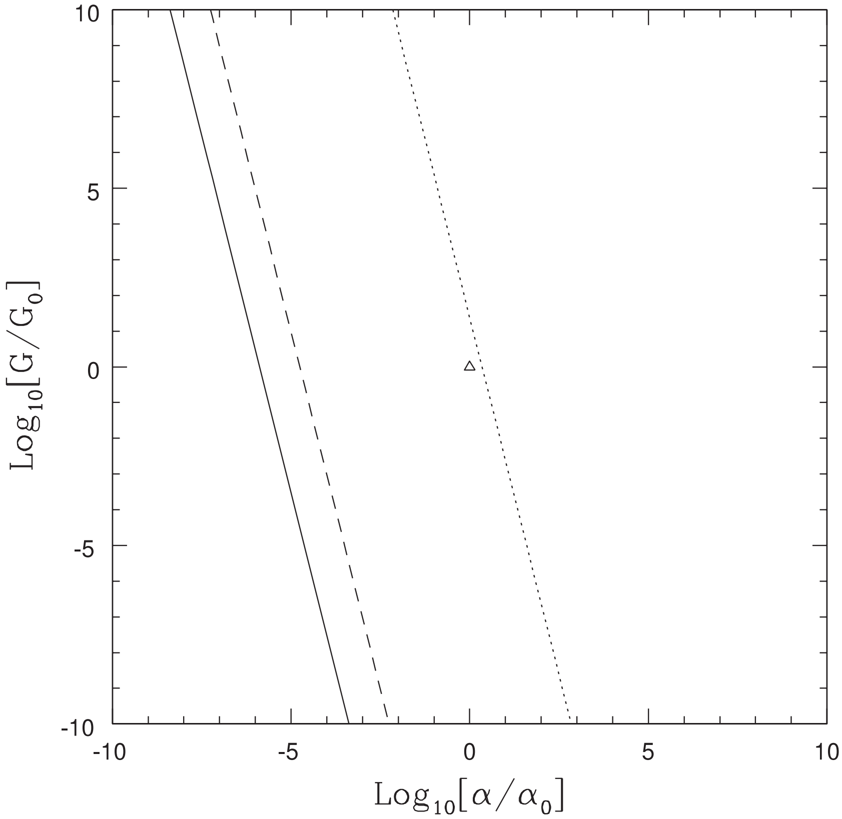

Figure 6. Allowed region of parameter space for the existence of black holes that can play the role of stars. The parameter space is the plane of the gravitational constant $\log_{10}[G/G_0]$ versus the fine structure constant $\log_{10}[\alpha/\alpha_0]$, where both quantities are scaled relative to the values in our universe. The allowed region lies under the curves, which are plotted here for three cases: the black hole luminosity is required to be greater than that of the Sun (solid curve), a low-mass star (dashed curve) and the solar luminosity intercepted by the Earth (dotted curve). The open triangle marks the location of our universe in this parameter space.

the power intercepted from the Sun by the Earth (as a minimum value; dotted curve), the luminosity of a low-mass star ($L \sim 10^{-3} L_\odot$; dashed curve) and $1.0 L_\odot$ (solid curve), all scaled according to equation (57). If the black hole is required to have luminosity in the stellar range, then the allowed region of parameter space is highly constrained, in that the parameters (α, G) must have values quite far from those in our universe. In particular, the gravitational constant must be small (so that the luminosity is large), and the fine structure constant must also be small (so that atomic energy levels are low). If the necessary luminosity is determined by $L_{\mathrm{min}0} = 10^{17}$ erg s^{-1}, however, black holes can play the role of stars over a much wider range of parameter space.

4.2. Degenerate dark matter stars

In principle, alternate universes can produce degenerate stars made of dark matter particles. Such stars could exist in our universe as well, although their formation is expected to be so highly suppressed that they play no significant role. This section considers the structure of these hypothetical objects in possible other universes.

A degenerate star has the structure of an $n = 3/2$ polytrope, with the constant K in the equation of state given by

$$K = (3\pi^2)^{2/3} \frac{\hbar^2}{5 m_\mathrm{d}^{8/3}}, \tag{60}$$

where m_d is the mass of the dark matter particle. Since the constant K is specified, we can solve directly for the stellar properties. The mass–radius relation is given by

$$M_* R_*^3 = \xi_*^3 \mu_0 \frac{9\pi^2}{128} \hbar^6 m_\mathrm{d}^{-8} G^{-3}, \tag{61}$$

and the central density is given by

$$\rho_\mathrm{c} = \frac{32}{9\pi^2 \mu_0^2} \frac{G^3 m_\mathrm{d}^8 M_*^2}{\hbar^6}. \tag{62}$$

For completeness, we note that the Chandrasekhar mass for these dark matter stars is given approximately by the expression

$$M_\mathrm{ch} = \mu_0 \frac{(3\pi)^{1/2}}{2} \left(\frac{\hbar c}{G m_\mathrm{d}^2} \right)^{3/2} m_\mathrm{d}, \tag{63}$$

where $\mu_0 \approx 2.714$ for an $n = 3/2$ polytrope; this expression does not include general relativistic corrections (e.g. [18]). For reference, note that a typical expected value for the dark matter particle mass, $m_\mathrm{d} = 100 m_\mathrm{P}$, implies that this mass scale $M_\mathrm{ch} \approx 0.0007 M_\odot$.

For these stars, the luminosity is provided by annihilation of the dark matter particles. The annihilation rate per particle Γ_1 is given by

$$\Gamma_1 = n \langle \sigma_\mathrm{d} v \rangle \approx \sigma_\mathrm{d} \hbar n^{4/3} / m_\mathrm{d}, \tag{64}$$

where σ_d is the cross section. To find the stellar luminosity due to dark matter annihilation, we must integrate over the star to find the total annihilation rate Γ_T:

$$\Gamma_\mathrm{T} = \frac{M_* \sigma_\mathrm{d} \hbar}{\mu_0 m_\mathrm{d}^2} n_\mathrm{c}^{4/3} \gamma_0, \qquad \text{where } \gamma_0 \equiv \int_0^{\xi_*} \xi^2 f^{7/2} \, \mathrm{d}\xi. \tag{65}$$

For this $n = 3/2$ polytrope, $\gamma_0 \approx 0.913$. As a result, the total annihilation rate is given by $\Gamma_\mathrm{T} \approx N_\mathrm{T} \Gamma_1 / 3$, where N_T is the total number of particles in the star and Γ_1 is evaluated at the stellar center. The corresponding stellar luminosity is then given by

$$L_* = \left(\frac{32}{9\pi^2} \right)^{4/3} \frac{\gamma_0}{\mu_0^{11/3}} \frac{c^2}{\hbar^7} \sigma_\mathrm{d} G^4 M_*^{11/3} m_\mathrm{d}^{25/3}. \tag{66}$$

If the mass of the degenerate star were close to the Chandrasekhar mass, the luminosity would be enormous and its lifetime would be short (see below). To put this in perspective, if we use reasonable values of the dark matter properties for our universe ($m_\mathrm{d} = 100 m_\mathrm{P}$ and $\sigma_\mathrm{d} = 10^{-38}$ cm^2), then the mass required to produce $L_* = 1.0 L_\odot$ is only about $M_* \sim 10^{-13} M_\odot \sim 10^{20}$ g (about the mass of a large asteroid). As a result, for the range of parameter space for which these objects play the role of stars, the masses are far below the Chandrasekhar mass.

If the dark matter star starts its evolution with initial mass M_0 and later has a mass $M(t) \ll M_0$, then its age $\Delta t(M)$ is related to its current mass through the expression

$$\Delta t(M) = \frac{3}{8}\left(\frac{9\pi^2}{32}\right)^{4/3} \frac{\mu_0^{11/3}}{\gamma_0} \frac{\hbar^7}{\sigma_{\rm d}} G^{-4} M^{-8/3} m_{\rm d}^{-25/3} = \frac{3}{8}\frac{Mc^2}{L_*}, \tag{67}$$

where L_* is the luminosity of the star when it has mass M. For example, if $M = 10^{20}$ g (the mass scale that generates $L_* = 1.0 L_\odot$), the timescale from equation (67) is only about 100 d. In order for the timescale to be 1 Gyr, say, the mass scale must be about 3×10^{16} g and the corresponding luminosity is only $\sim 10^{-13} L_\odot = 4 \times 10^{20}$ erg s^{-1}, i.e. still substantially larger than the expected value of $L_{\rm min0}$.

When the masses are well below the Chandrasekhar mass (see above), the star must satisfy two constraints. The first requirement is that the star is sufficiently luminous, which implies that

$$L_* = B \frac{c^2 \sigma_{\rm d} G^4 m_{\rm d}^{25/3}}{\hbar^7} M^{11/3} \geq L_{\rm min0} (\alpha/\alpha_0)^4, \tag{68}$$

where we have defined a dimensionless constant B:

$$B = \left(\frac{32}{9\pi^2}\right)^{4/3} \frac{\gamma_0}{\mu_0^{11/3}} \approx 0.0060. \tag{69}$$

Next we require that the stellar lifetime is sufficiently long. In rough terms, this constraint can be written in the form

$$\Delta t(M) = \frac{3}{8B} \frac{\hbar^7}{\sigma_{\rm d} G^4 m_{\rm d}^{25/3}} M_*^{-8/3} \geq \frac{\hbar N_{\rm bio}}{m_{\rm e} c^2 \alpha^2}, \tag{70}$$

where we have not made the distinction between M and M_* in using equation (67). The first constraint puts a lower limit on the mass and the second constraint puts an upper limit on the mass. By requiring that both constraints be met simultaneously, the mass can be eliminated and a global constraint can be derived:

$$\left(\frac{\alpha}{\alpha_0}\right)^{21/8} \left(\frac{G m_{\rm d}^2}{\hbar c}\right)^{3/2} \leq C_B \frac{m_{\rm e} c^2}{L_{\rm min0}} \left(\frac{m_{\rm e}^3 c^{10}}{\sigma_{\rm d}^3 m_{\rm d} \hbar^2}\right)^{1/8} \left(\frac{\alpha_0^2}{N_{\rm bio}}\right)^{11/8}, \tag{71}$$

where the constant $C_B = (3/8)^{11/8} B^{-3/8} \approx 1.75$. This result defines the parameters necessary for dark matter stars to play the role of ordinary stars (keep in mind that the formation of these bodies remains a formidable obstacle). The luminosity is determined by the dark matter annihilation cross section, which is independent of the constants that determine the physical structure of the star. As a result, the parameter space of constants (α, G) considered here always contains a region where these stars can operate. For fixed properties of the dark matter ($m_{\rm d}$ and $\sigma_{\rm d}$), equation (71) delineates the portion of the (α, G) plane that allows these degenerate dark matter objects to act as stars. On the other hand, one can use equation (71) to constrain the allowed dark matter properties for given values of α and G.

JCAP08(2008)010

4.3. Other possibilities for unconventional stars

If the nuclear burning constant $\mathcal{C} = 0$, then baryonic objects can still, in principle, generate energy in a variety of ways. In the absence of nuclear reactions, stellar bodies will often tend to form degenerate configurations, analogous to white dwarfs in our universe (provided that their mass is below the relevant Chandrasekhar mass scale). These degenerate objects can generate energy through several channels, including residual heat left over from formation, proton decay and dark matter capture and annihilation.

In the latter case, dark matter particles are captured by scattering off nuclei (which could be simply protons in a universe with no nuclear reactions). After a scattering event, the recoil energy of the dark matter particle can be less than the escape speed of the star and the particle can be captured. After capture, the dark matter particles sink to the stellar center, where they collect until their population is dense enough for annihilation to balance the incoming supply of particles. The star thus reaches a steady state, where the luminosity is given by the total capture rate. This process has been discussed previously in a variety of contexts, including as a solution to the solar neutrino problem [19] and as a means to keep white dwarfs hot beyond their cooling times [20].

The capture rate of dark matter particles is given by

$$\Gamma = n_{\rm dm}\sigma_{*{\rm dm}}v_{\rm rel}, \tag{72}$$

where $n_{\rm dm}$ is the number density of dark matter particles, $\sigma_{*{\rm dm}}$ is the total cross section for capture subtended by the star and $v_{\rm rel}$ is the relative velocity. These quantities depend on dynamical structure (distributions of density, velocity, angular momentum) of the background halo of dark matter [21]. In our universe, for example, the capture rate of dark matter particles by white dwarfs is of order $\Gamma \sim 10^{25}$ s^{-1} [20]. With the capture rate specified, the corresponding luminosity is given by

$$L_* = f_\nu m_{\rm d}\Gamma, \tag{73}$$

where $m_{\rm d}$ is the mass of the dark matter particles and where the efficiency factor f_ν takes into account energy loss from the star due to some fraction of the annihilation products being neutrinos.

In this scenario, the luminosity depends on the number density of dark matter particles in the background (in the galactic halo in the context of white dwarfs in our universe). This density is independent of stellar properties. In a similar vein, the timescale over which the luminosity can be maintained depends on the overall supply of dark matter particles; this quantity is also independent of stellar properties. Thus, for any values of the constants (α, G), considered here as the relevant parameters that specify stellar properties, a universe *can* have the proper values of dark matter densities and cross sections so that degenerate stars can serve in place of nuclear burning stars. The specification of the allowed parameter space depends on more global properties of the universe, however, and is beyond the scope of this paper.

5. Conclusion

In this paper, we have developed a simple stellar structure model (section 2) to explore the possibility that stars can exist in universes with different values for the fundamental parameters that determine stellar properties. This paper focuses on the parameter space given by the variables $(G, \alpha, \mathcal{C})$, i.e. the gravitational constant, the fine structure constant and a composite parameter that determines nuclear fusion rates. The main result of this work is a determination of the region of this parameter space for which bona fide stars can exist (section 3). Roughly one-fourth of this parameter space allows for the existence of 'ordinary' stars (see figure 5). In this sense, we conclude that universes with stars are not especially rare (contrary to previous claims), even if the fundamental constants can vary substantially in other regions of space–time (e.g. other pocket universes in the multiverse). Another way to view this result is to note that the variables $(G, \alpha, \mathcal{C})$ can vary by orders of magnitude from their measured values and still allow for the existence of stars.

For universes where no nuclear reactions are possible, we have shown that unconventional stellar objects can fill the role played by stars in our universe, i.e. the role of generating energy (section 4). For example, if the gravitational constant G and the fine structure constant α are smaller than their usual values, black holes can provide viable energy sources (figure 6). In fact, all universes can support the existence of stars, provided that the definition of a star is interpreted broadly. For example, degenerate stellar objects, such as white dwarfs and neutron stars, are supported by degeneracy pressure, which requires only that quantum mechanics is operational. Although such stars do not experience thermonuclear fusion, they often have energy sources, including dark matter capture and annihilation, residual cooling, pycnonuclear reactions and proton decay. Dark matter particles can also (in principle) form degenerate stellar objects (see section 4).

In order to assess the suitability of non-nuclear power sources, one must specify how much power is required, and for how long. In this work we have used the power that Earth intercepts from the Sun as the minimum benchmark value L_{min0}, and scaled the necessary power according to equation (57) to account for variations in the fine structure constant; similarly, the required amount of time is taken to $\sim$1 Gyr, scaled by the atomic time of equation (49). These choices are not definitive and hence alternative scalings can be explored.

The issue of alternative values for the fundamental constants, as considered herein, is related to the issue of time variations in the constants in our universe. However, current experiments place rather strong limits on smooth time variations, with timescales exceeding the current age of the universe (see the review of [22]). Another possibility is for the constants to have different values at other spatial locations within our universe, although this scenario is also highly constrained [23].

This paper has focused on stellar structure properties. An important related question (beyond the scope of this work) is whether or not stellar bodies can be readily made in universes with varying values of the constants. Even if the laws of physics allow for stellar objects to exist and actively burn nuclear fuel, there is no guarantee that such bodies will be produced in significant numbers. In our universe, for example, there is a moderate mismatch between the mass range of possible stars and the distribution of masses of stellar bodies produced by the star formation process. At the present cosmological epoch, star formation produces objects over the entire possible range of stellar masses, with additional bodies produced in the substellar range (brown dwarfs). The matching is relatively good, in that the fraction of bodies in the brown dwarf range is small, only about 1 out of 5 [24]. Since the masses of these objects are small, the fraction of the total mass locked up in

JCAP08(2008)010

the smallest bodies is even smaller, less than 5%. On the other hand, nearly all of the stars in our universe have small masses. As one benchmark, only about 3 or 4 out of a thousand stars are larger than the $\sim 8M_{\odot}$ threshold required for stars to experience a supernova explosion, whereas stellar masses can extend up to $\sim 100M_{\odot}$. The high-mass end of the possible mass range is thus sparsely populated. The corresponding match between the range of allowed stellar masses and the mass range of objects produced can be quite different in other universes.

In future work, another issue to be considered is coupling the effects of alternative values of the fundamental constants to the cosmic expansion, big bang nucleosynthesis and structure formation. Each of these issues should be explored in the same level of detail as stellar structure is studied in this work. With the resulting understanding of these processes, the coupling between them should then be determined.

Finally, we note that this paper has focused on the question of whether or not stars can exist in universe with alternative values of the relevant parameters. An important and more global question is whether or not these universes could also support life of some kind. Of course, such questions are made difficult by our current lack of an *a priori* theory of life. Nonetheless, some basic requirements can be identified (with reasonable certainty). In addition to energy sources (provided by stars), there will be additional constraints to provide the right mix of chemical elements (e.g. carbon in our universe) and a universal solvent (e.g. water). These additional requirements will place additional constraints on the allowed region(s) of parameter space.

Acknowledgments

We thank Greg Laughlin for useful discussions. This work was supported by the Foundational Questions Institute through grant RFP1-06-1 and by the Michigan Center for Theoretical Physics.

References

[1] Guth A H, 2000 *Phys. Rep.* **333** 555 [SPIRES]
[2] Rees M J, 1997 *Before the Beginning* (Reading, MA: Perseus)
[3] Vilenkin A, 1998 *Phys. Rev. Lett.* **81** 5501 [SPIRES]
[4] Carr J B and Rees M J, 1979 *Nature* **278** 611 [SPIRES]
[5] Barrow J D and Tipler F, 1986 *The Anthropic Cosmological Principle* (Oxford: Clarendon)
[6] Tegmark M, Aguirre A, Rees M J and Wilczek F, 2006 *Phys. Rev.* D **73** 3505 [SPIRES]
[7] Hogan C J, 2000 *Rev. Mod. Phys.* **72** 1149 [SPIRES]
[8] Wilczek F, 2007 *Preprint* 0708.4361 [physics.gen-ph]
[9] Tegmark M, 2005 *J. Cosmol. Astropart. Phys.* JCAP04(2005)001 [SPIRES]
[10] Aguirre A and Tegmark M, 2005 *J. Cosmol. Astropart. Phys.* JCAP01(2005)003 [SPIRES]
[11] Phillips A C, 1994 *The Physics of Stars* (Chichester: Wiley)
[12] Shu F H, Adams F C and Lizano S, 1987 *Ann. Rev. Astron. Astrophys.* **25** 23 [SPIRES]
[13] Kippenhahn R and Weigert A, 1990 *Stellar Structure and Evolution* (Berlin: Springer)
[14] Hansen C J and Kawaler S D, 1995 *Stellar Interiors: Physical Principles Structure and Evolution* (New York: Springer)
[15] Chandrasekar S, 1939 *An Introduction to the Study of Stellar Structure* (Chicago, IL: University of Chicago Press)
[16] Bleistein N and Handelsman R A, 1975 *Asymptotic Expansions of Integrals* (New York: Dover)
[17] Hawking S W, 1975 *Commun. Math. Phys.* **43** 199 [SPIRES]
[18] Shapiro S L and Teukolsky S A, 1983 *Black Holes White Dwarfs and Neutron Stars: The Physics of Compact Objects* (New York: Wiley)
[19] Press W H and Spergel D N, 1985 *Astrophys. J.* **296** 679 [SPIRES]

[20] Adams F C and Laughlin G, 1997 *Rev. Mod. Phys.* **69** 337 [SPIRES]
[21] Binney J and Tremaine S, 1987 *Galactic Dynamics* (Princeton, NJ: Princeton University Press)
[22] Uzan J-P, 2003 *Rev. Mod. Phys.* **75** 403 [SPIRES]
[23] Barrow J D, 2005 *Phil. Trans. R. Soc.* A **363** 2139
[24] Luhman K L, 2007 *Astrophys. J. Suppl.* **173** 104