Experiments with hydrogen - discovery of the Lamb shift

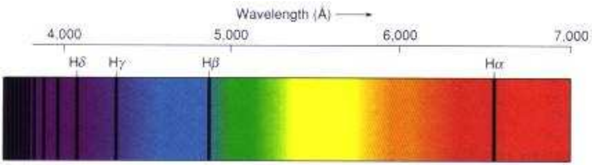
Haris Đapo

Relativistic heavy ion seminar, October 26, 2006

Outline

The beginning

- why hydrogen?
- "simple" object, only tho bodies: proton and electron
- easy to test theories $\Rightarrow$ established and ruled out many



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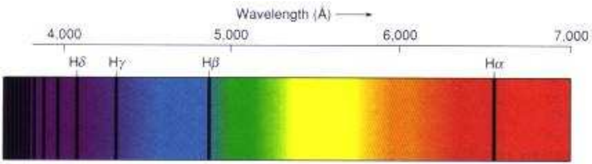
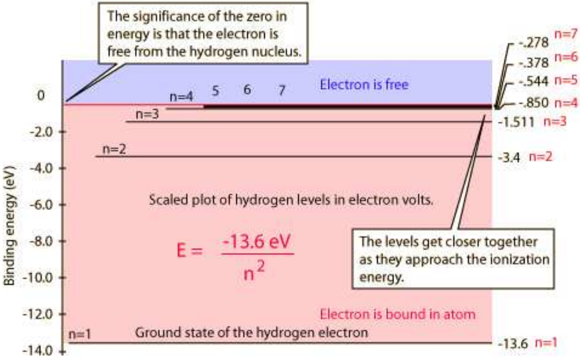


- 1885 Balmer's simple equation for fourteen lines of hydrogen
- 1887 fine structure of the lines, Michelson and Morley
- 1900 Planck's quantum theory

Bohr's formula

- 1913 Bohr derived Balmer's formula
- point-like character and quantization lead to:

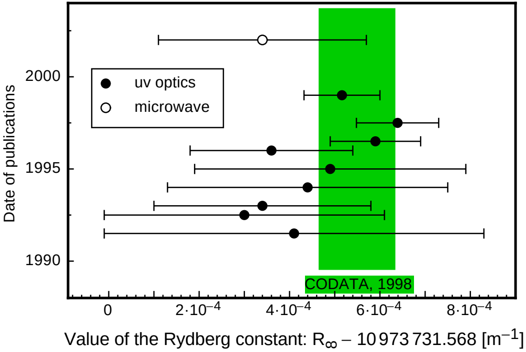


$$E_n = -\frac{Z^2 hcRy}{n^2}$$

- Ry is Rydberg wave number

Rydberg constant

- two-photon Doppler-free spectroscopy of hydrogen and deuterium
- measurement of two or more transitions



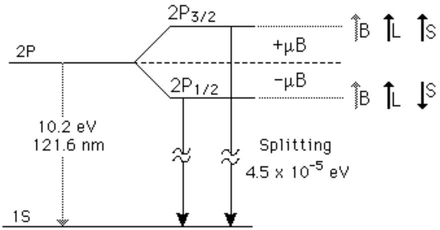
$$Ry(1998) = 10\,973\,731.568\,549(83)\ m^{-1}$$

Fine structure

- 1916 Sommerfeld: fine structure is the result of relativistic effects
- "fine structure", relativistic hydrogen, dependence of energy on eccentricity
- for $n = 2$ circular ($l = 1$) and elliptic orbit ($l = 0$) differ by

$$\Delta E_2 = \frac{1}{16}\alpha^2 Z^4 hcRy$$

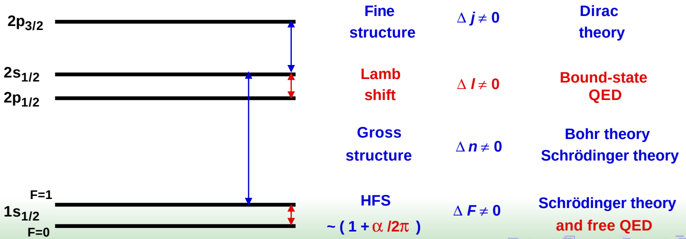
- fine structure constant is $\alpha^{-1} =$ 137.035 999 11(46)



Dirac's equation

- 1924 De Broglie attributed wave properties to particles
- quantum mechanics of hydrogen atom emerge
- 1925 spin and magnetic moment
- 1928 Dirac's equation

$$E = m_0 c^2 \left[1 + \left(\frac{\alpha Z}{n - j - 1/2 + \sqrt{(j + 1/2)^2 - \alpha^2 Z^2}} \right)^2 \right]^{-1/2}$$



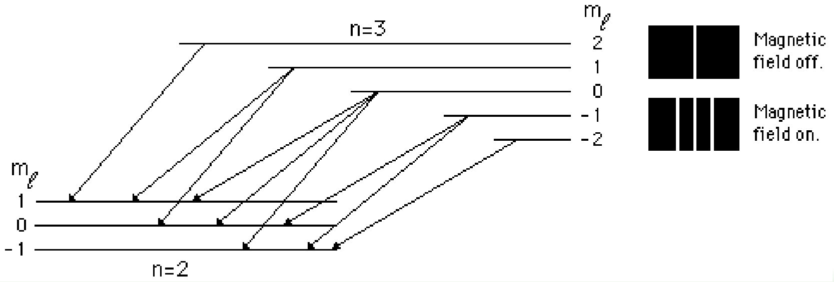
Zeeman effect and HFS

- hyperfine structure: interaction of nuclear spin and the angular momentum of orbiting particle

Zeeman effect and HFS

- hyperfine structure: interaction of nuclear spin and the angular momentum of orbiting particle
- external magnetic field

Without spin: $\Delta E = m_l \frac{e\hbar}{2m} B$; With spin: $\Delta E = g \frac{e\hbar}{2m} m_j B$



Deviations form Dirac theory

- 1938 deviations form Dirac theory for H_α observed by Houston and Williams
- Pasternak suggested that these results could be interpreted as 0.03 cm^{-1} higher $2S_{1/2}$ relative to $2P_{1/2}$
- not enough attention, discrepancies attributed to impurity of the source

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Shelter Island

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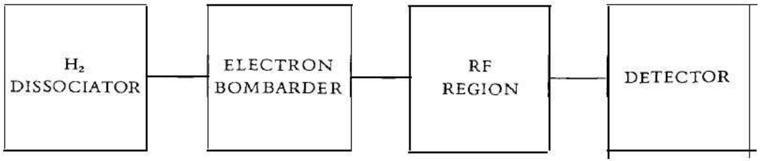
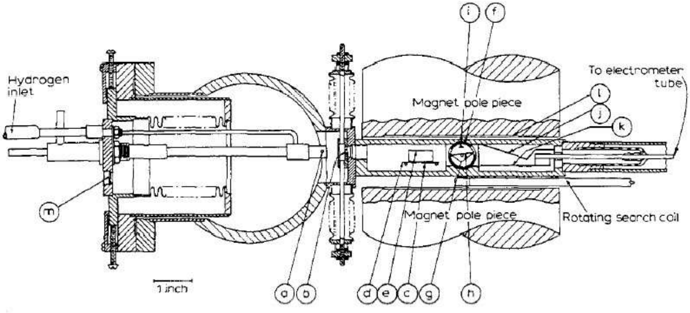


Fig. 2. Modified schematic block diagram of apparatus.

- molecular hydrogen is thermally dissociated
- jet of atoms is cross-bombarded by an electron stream
- one part in a hundred million atoms is excited to $2S_{1/2}$

Lamb shift experiment

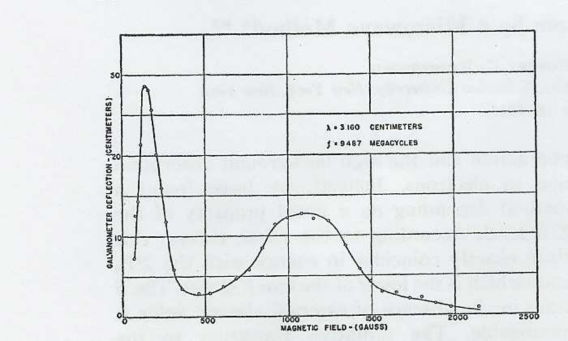


- detected by electron ejection from metal target
- between bombardier and detector the atoms are exposed to radio waves

RF region

- good reasons for process being carried out in magnetic field
- energy levels are subject to Zeeman splitting
- frequencies of possible transition depend on magnetic field
- $2S_{1/2}$ lifetime $1/7$ seconds
- $2P_{1/2}$ lifetime 10^{-8} seconds
- any perturbation leads to mixing with the $2P_{1/2}$ thus reducing the lifetime

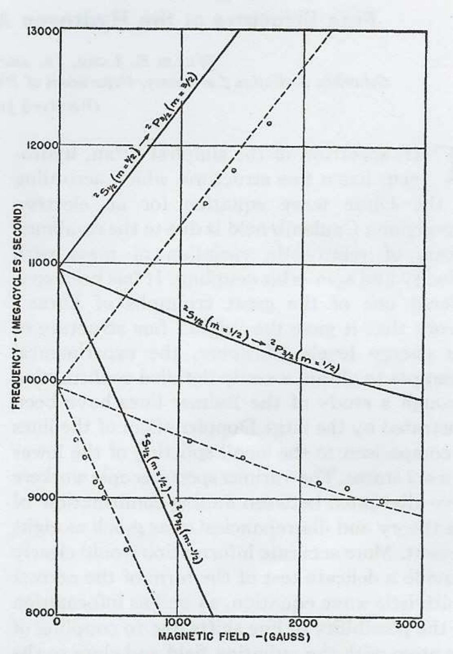
Beam decay



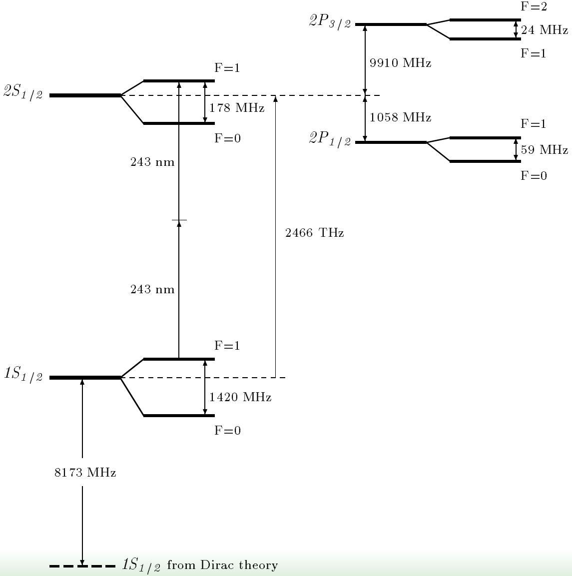
- metastable atoms in the beam are quenched
- atoms in the ground state are unable to eject electrons from the detector
- transition $2S \rightarrow 2P$ induced by RF radiation
- $2P$ decays while moving trough a very small distance

Result

- experimental results are shown by circles
- resonant magnetic fields
- solid curves are theoretical predictions
- dashed are solid $-$ 1000 Mc/sec
- not a "best fit"



Hydrogen levels



Bethe’s calculation

- Bethe H.A., Phys. Rev. 72, 339 (1947)
- interaction of electron with radiation field
- "However, it is possible to identify the most strongly (linearly) divergent term in the level shift with an electromagnetic *mass* effect which must exist for a bound as well as for a free electron."
- subtract it from theoretical expression
- only logarithmic divergence remains in non-relativistic theory
- a relativistic theory should converge

Self-energy

Self-energy of electron in quantum state m

$$W' = W - W_0 = \frac{2e^2}{3\pi\hbar c^3}\sum_n |v_{mn}|^2 (E_n - E_m)\ln\frac{K}{|E_n - E_m|}$$

- logarithm is very large, independent of n

For S state

$$W'_{ns} = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3 Ry\frac{Z^4}{n^3} ln\frac{K}{< E_n - E_m >_{Av}} = 1040 megacycles$$

- $< E_n - E_m >_{Av}$ is average excitation energy
- $K \approx mc^2$ is the natural cut-off from relativity theory

Bethe's calculation

Conclusions

- level shift due to interaction with radiation is a real effect
- effect of infinite electromagnetic mass can be eliminated
- accurate investigation may establish relativistic effects
- a first mayor success of the "renormalization idea"

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Consequences

Nobel pize acceptance speech (1955): "It is very important that this problem should receive further experimental and theoretical attention. When an accuracy of comparison of 0.1 Mc/sec has been reached, it will mean that the energy separation of the $2S$ and $2P$ states of hydrogen agree with theory to a precision of few parts in 10^9 of their binding energy or that the exponent in Coulomb law of force is two with comparable accuracy."

Last results for H and D

- hydrogen, hydrogen-like atoms, positronium, muonic atoms
- microwave spectroscopy has been surpassed by optical measurements in the last decade
- accuracy of the radiofrequency measurements is limited by width of the $2P$ state (about 100 MHz)
- direct measurements and fine structure
- optical,two photon Doppler free spectroscopy

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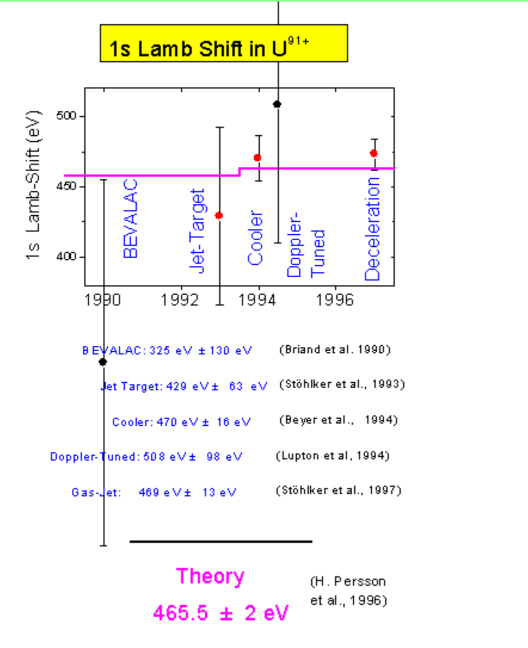
- last experimental result *H* (1999): 1057.845(3) MHz
- last theoretical result *H* (1999): 1057.841(5) MHz
- last experimental result deuterium: 1059.2337(29) MHz
- *He* exp: 14 041.13(17) MHz
- *He* th: 14 041.18(13) MHz

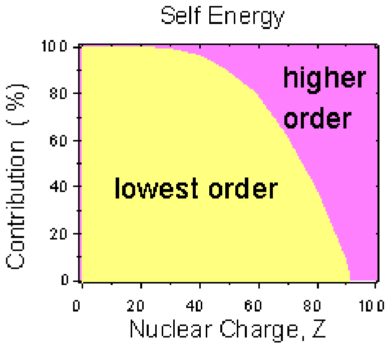
U Lamb shift

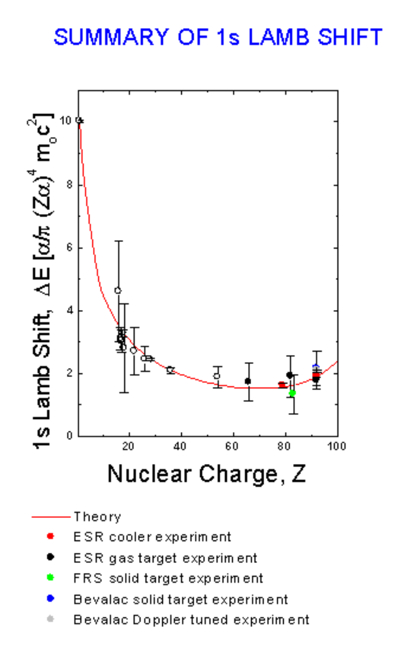
- 1*S* (ground state) Lamb shift is difference between what Dirac equation predicts and real value (experiment or QED)
- most highly charged ion available in laboratory
- highly relativistic, approaches $Z = 137$



Lamb shift overview

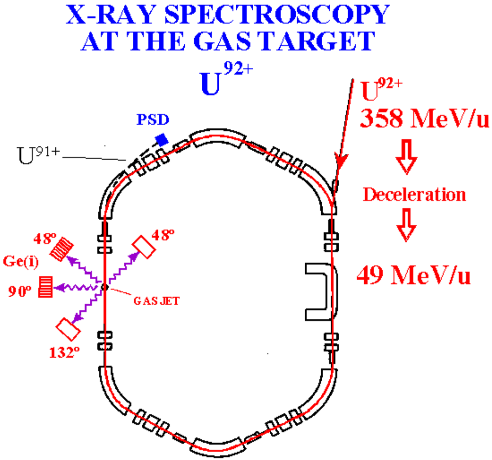
- perturbative treatment no longer applicable, all orders must be considered





ESR at GSI

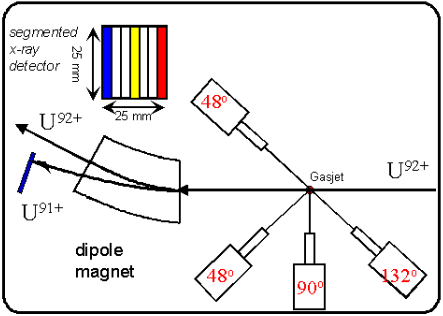
- ions injected at 358 MeV/u
- electron cooler is used to cool ions
- up to 10^8 ions
- ions decelerated to final beam energy of 68 and 49 MeV/u



ESR at GSI

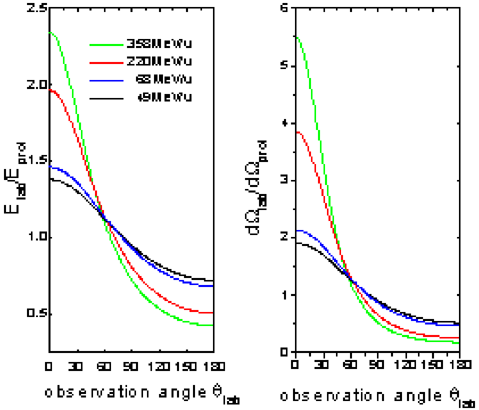
- gas jet target
- electron capture on bare ions
- four *Ge* detector
- forward left/right symmetry and forward backward symmetry
- fast plastic scintillator
- 469 ± 13 eV

Geometry of the experimental set-up



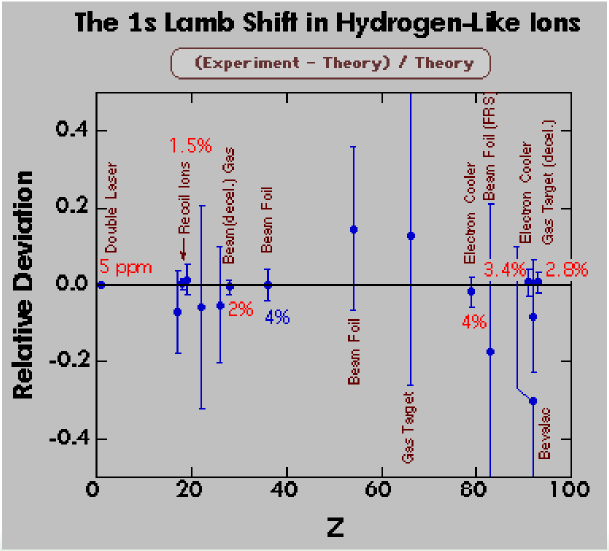
Doppler shift

- main limitation: uncertainty in correction for the Doppler shift
- ions striped at high energy (360 MeV$/u$)
- decelareting ions after striping is improvement



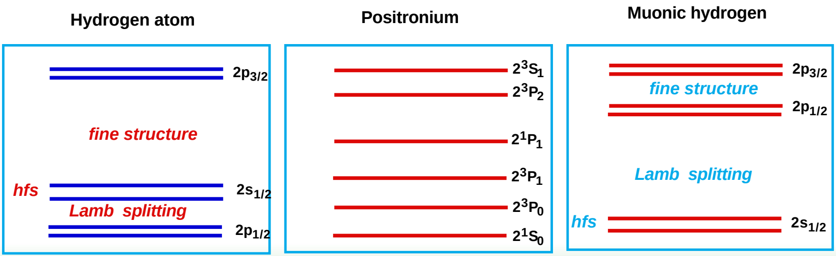
$$E_{proj} = \gamma(1-\beta\cos\theta)E_{lab}$$
$$\Delta\Omega_{proj} = [\gamma(1-\beta\cos\theta)]^2\Delta\Omega_{lab}$$

Theory/experiment



Positronium and muonium

- muonium is bound system of muon and electron
- lifetime $2.2 \cdot 10^{-6}$ sec
- exp:2 455 528 941.0(9.8)MHz,th:2 455 528 934.0(0.3)MHz
- positronium is bound system of positron and electron
- lifetime para $1.25 \cdot 10^{-10}$ sec, ortho $1.4 \cdot 10^{-7}$ sec
- exp:1 233 607 216.4(3.2)MHz,th:1 233 607 222.2(6)MHz



Theory

- gross structure: $E_n = -\frac{(Z\alpha)^2 mc^2}{2n^2}$
- reduced mass correction: $m \rightarrow \frac{Mm}{M+m}$
- relativistic corrections, hyperfine structure, recoil correction, nuclear-structure correction
- QED corrections: self energy, radiative with, vacuum polarization, etc.

Various contributions

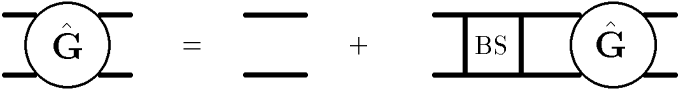
In units of $(Z\alpha)^2 mc^2$

Contribution	Hydrogen-like electronic atom	Positronium	Hydrogen-like muonic atom
Schrödinger eq.			
- with $M = \infty$	1	1	1
- with m_R (corr.)	m/M	1	m/M
Relativistic corr.			
- Dirac equation	$(Z\alpha)^2$	α^2	$(Z\alpha)^2$
- Two-body effects	$(Z\alpha)^2 m/M$	α^2	$(Z\alpha)^2 m/M$
QED			
- Self energy	$\alpha(Z\alpha)^2 \ln(Z\alpha)$	$\alpha^3 \ln\alpha$	$\alpha(Z\alpha)^2 \ln(Z\alpha)$
- Radiative width	$\alpha(Z\alpha)^2$	α^3	$\alpha(Z\alpha)^2$
- Vacuum pol.	$\alpha(Z\alpha)^2$	α^3	$\alpha \ln(Z\alpha m/m_e)$
- Annihilation			
– virtual	–	α^2	–
– real	–	α^3	–
Nuclear effects			
- Magnetic moment	$(Z\alpha)^2 m/M$ or $\alpha(Z\alpha)m/m_p$	α^2	$(Z\alpha)^2 m/M$ or $\alpha(Z\alpha)m/m_p$
- Charge distribution	$(Z\alpha mcR_N/\hbar)^2$	–	$(Z\alpha mcR_N/\hbar)^2$

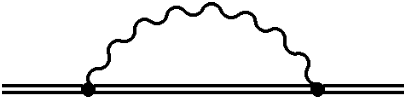
Bethe-Salpeter equation

- the theory agrees with experiment very well
- to find energy levels of any composite system
- positions of the poles of respective Greens functions

$$\hat{G} = S_0 + S_0 K_{BS} \hat{G}$$



For Lamb shift:



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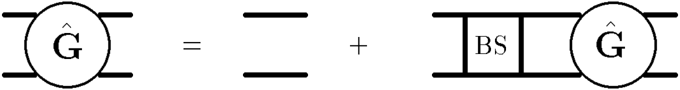

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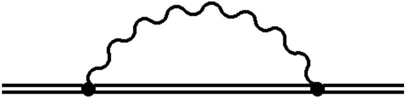
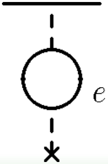
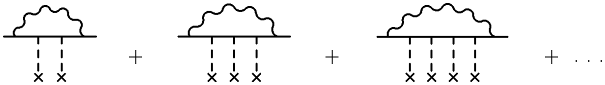




Lamb shift from QED



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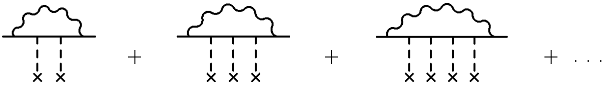
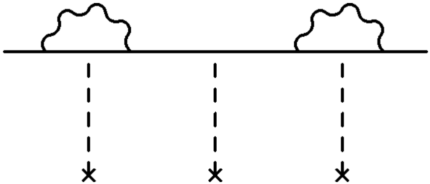


$$\Delta E = \left\{ \left[\frac{1}{3} \ln \frac{m(Z\alpha)^{-2}}{m_r} + \frac{11}{72} \right] \delta_{l0} - \frac{1}{3} \ln k_0(n,l) \right\} \frac{4\alpha(Z\alpha)^4}{\pi n^3} \left(\frac{m_r}{m} \right)^3 m$$

- m_r is reduced mass
- $ln k_0(n,l)$ is Bethe logarithm, can be calculated to arbitrary accuracy
- normalized infinite sum of matrix elements of the coordinate operator over the Schrödinger-Coulomb wave function

Higher order contributions

- one-loop and two loop self-energy
- highest order terms important for the comparison of theory and experiment are $\alpha(Z\alpha)^7$ and $\alpha^2(Z\alpha)^6$



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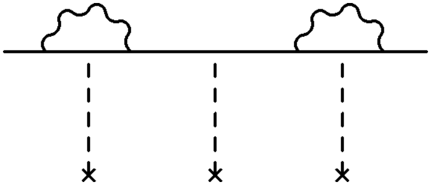

$$\Delta E = -6.862(1)\frac{\alpha^2(Z\alpha)^5}{\pi n^3}\left(\frac{m_r}{m}\right)^3 m\,\delta_{l0} = -296.92\,(4)\ \mathrm{kHz}_{|n=1}$$
$$= -37.115\,(5)\ \mathrm{kHz}_{|n=2}$$

Higher order contributions

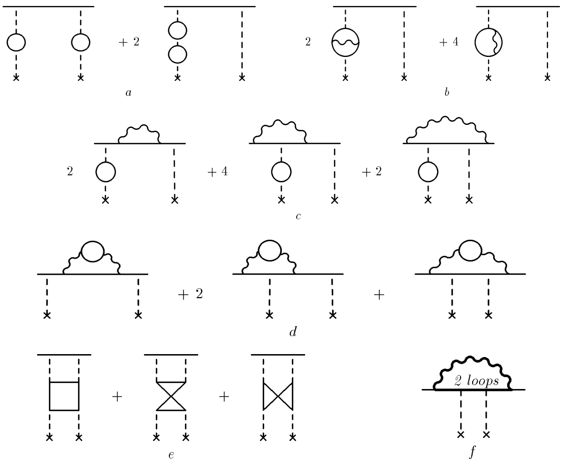

Figure: Six gauge invariant sets of diagrams for corrections of order $\alpha^2(Z\alpha)^5 m$

Future

Experimental future

- the agreement between theory and experiment is extraordinary
- direct microwave techniques reached theirs peak, room for improvement: fine structure measurements
- the real future are two photon Doppler shift experiments
- GSI future, Lamb shift studies at the ESR electron cooler

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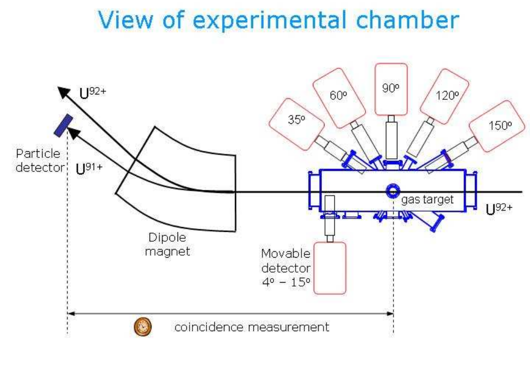
Theoretical future

- obviously even higher corrections $\alpha(Z\alpha)^8$, $\alpha(Z\alpha)^9$, etc.
- however progress is mostly limited by the need to use experimental values for fundamental constants

Future at GSI

- a micro-strip *Ge* detector, detector 48 vertical strips, 128 horizontal strips
- FOcusing Compensated Asymmetric Laue (FOCAL) geometry



Summary

- spectrum of hydrogen has stimulated development of physics in general and quantum mechanics in particular
- many cornerstones of physics
- Lambs shift discovery revitalized development of QED
- today hydrogen and hydrogen like spectroscopy serves as a extreme precision test of QED, with unprecedented accuracy
- similar measurement still being performed
- Lamb shift remais at the forefront of physics all this time
- there are still thing that can be learned

Bethe's calculation

Self-energy of electron in quantum state m

$$W = -\frac{2e^2}{3\pi\hbar c^3}\int_0^K kdk \sum_n \frac{|v_{mn}|^2}{E_n - E_m + k}$$

- where $k = \hbar\omega$ is energy of the quantum
- $v = p/m = (\hbar/im)\Delta$ is velocity

Self-energy of free electron

$$W_0 = -\frac{2e^2}{3\pi\hbar c^3}\int kdk \frac{v^2}{k}$$