A CONVERSATION WITH ZOLTAN P. DIENES

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LEADERS IN MATHEMATICAL THINKING AND LEARNING

A Conversation With Zoltan P. Dienes

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The name of Zoltan P. Dienes (1916–) stands with those of Jean Piaget, Jerome Bruner, Edward Begle, and Robert Davis as a legendary figure whose work left a lasting impression on the field of mathematics education. Dienes' name is synonymous with the multibase blocks that he invented for the teaching of place value. Among numerous other things, he also is the inventor of algebraic materials and logic blocks, which sowed the seeds of contemporary uses of manipulative materials in instruction. Dienes' place is unique in the field of mathematics education not only because of his theories on how mathematical structures can be effectively taught from the early grades onwards using manipulatives, games, stories, and dance (e.g., Dienes, 1973, 1987), but also because of his tireless attempts for over 50 years to inform school practice through his fieldwork in the United Kingdom, Italy, Australia, Brazil, Canada, Papua New Guinea, and the United States. Dienes' theories on the learning of mathematics have influenced many generations of mathematics education researchers, particularly those involved in the Rational Number Project (http://education.umn.edu/rationalnumberproject/), and more recently those working in the models and modeling area of research. Dienes championed the use of collaborative group work and concrete materials, as well as goals such as democratic access to the process of mathematical thinking, long before the words *constructivism*,

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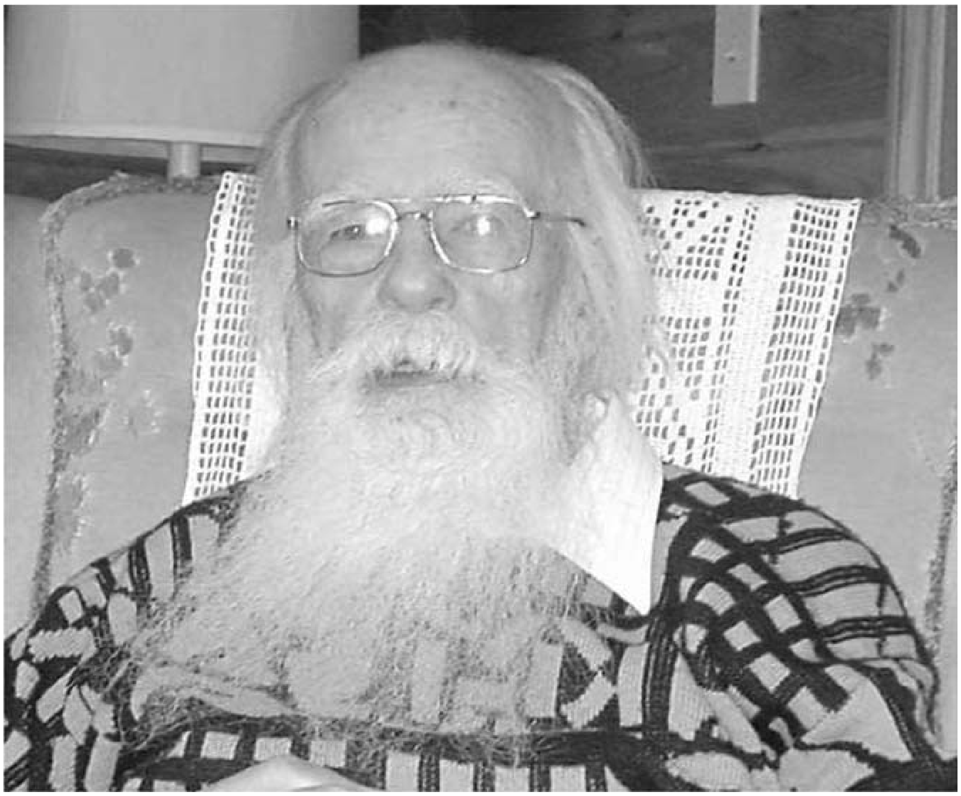

FIGURE 1 Zoltan P. Dienes, April 25, 2006, Wolfville, Nova Scotia.

equity, and *democratization* became fashionable. In this rare interview, Dienes (see Figure 1) reflects on his life, his work, the role of context, language, and technology in mathematics teaching and learning today, and on the nature of mathematics itself.

Sriraman: It is an honor to be able to talk with you. I really appreciate the invitation to visit.

Dienes: You have traveled so far…so I hope I am of some help. I can't have very much longer on this planet. So it's good you're here.

Sriraman: Your books have been very influential in my own work, many of your writings from the 60's [Dienes, 1960, 1963, 1964, 1965, 1971], and especially the one you wrote when you were in Adelaide.

Dienes: With Jeeves, yes [see Dienes & Jeeves, 1965].

Sriraman: Yes, particularly the innovative experiments you set up, which investigate reasoning about isomorphic structures such as groups. … Do you still believe this is the way to teach mathematics, especially knowing that mathematics has become more and more applied in today's world compared to the 50's and 60's?

Dienes: Well! It depends on what you think is important and what one is after. Mathematics is characterized by structures, there is no denying this fact and in my opinion it is important to expose students to these structures as early as possible. This does not mean we tell them directly what these structures are but use mathematical games and other materials to help them discover and understand these structures. You have read about my theory of the six stages of learning [see Dienes, 2000b]. And in this theory, the formalization stage comes at the very end.

Sriraman: As you know, there have been theorists who think that such topics are too difficult at earlier developmental stages—although your work indicates otherwise. Piaget, for instance thought this type of thinking (structural thinking) was only possible at the stage of formal operations.

Dienes: Children do not need to reach a certain developmental stage to experience the joy, or the thrill of thinking mathematically and experiencing the process of doing mathematics. We unfortunately do not give children the opportunities to engage in this type of thinking. One of the first things we should do in trying to teach a learner any mathematics is to think of different concrete situations with a common essence. (These situations) have just the properties of the mathematics chosen. Then … children will learn by acting on a situation. Introducing symbolic systems prematurely shocks the learner and impedes the learning of mathematics.

Sriraman: What are your thoughts on Piaget's theory?

Dienes: [Dienes gets up and retrieves a manuscript] I was working with Piaget's group of researchers at *Institut Rousseau* in Geneva. I did not hear one consistent answer when I asked them what it means to be "operational"? … You can look at this manuscript and read what I asked Piaget.

Sriraman: [reading from Dienes' manuscript] "Is it so Monsieur Piaget, that a pre-operational child can operate on states to get to other states, but is unable to operate on an operator to get another operator, whereas an operational child can also operate on an operator, without having to think of the intervening states?"

Dienes: Yes, and Piaget agreed with my definition [Laughing]. You can read about my conception of operationality in children yourself. It is a bit different from Piaget's.

Sriraman: You mentioned pre-mature use of symbolic systems in the teaching of mathematics earlier. I agree that notation is used too early without children completely understanding what it is they are being forced to represent and symbolize. I know you spent a year at Harvard with Jerome Bruner. Do you care to talk about this?

Dienes: My emphasis was on the use of mathematical games with appropriate learning aids (manipulatives), work, and communication in small groups with the teacher overseeing these groups. … I did have arguments with Bruner and his followers on this subject. I even invented a term "Symbol Shock" [Laughing] and there was disagreement with my approach from his camp.

Sriraman: What got you interested in the teaching and learning of mathematics? You come from the background of being a mathematician…

Dienes: I explained it to some extent in that book [Dienes, 2003]. I thought it was strange how people didn't understand mathematics. What makes it so hard? Then I thought of things like … the distributive law for instance. It is very hard to explain this law to somebody who is not a mathematician, but you can invent some games which work in exactly the same way, which you can play. I thought why not try and see if you can do something like that with kids and see if they buy it. And they did.

Sriraman: From the point of view of a university teacher educator who wishes to make a structural approach to learning more common, how much mathematics do you think prospective teachers and teacher educators need to know before they can truly appreciate mathematical structures?

Dienes: The answer to your question depends on what you mean by "how much mathematics?" There are several things that are important. One needs to be able think logically. How much mathematics one studies … depends…

Sriraman: I think what I am trying to ask you is whether or not you think studying a lot of mathematics is important before starting to teach it.

Dienes: It really depends on the person. Some are able to grasp the fundamental ideas very quickly. So, if one doesn't study a whole lot of mathematics formally, but understands the material they have studied…it doesn't matter. The real problem occurs when one doesn't understand what mathematics is about in the first place and then tries to teach it. It is a question of depth.

You can learn mathematics simply as a utility and learn how to use it. That happened during the 18th and 19th centuries, during the Industrial revolution. It became necessary for people to read instructions, to do simple number work, because it was economically necessary. (But), all you had to do was learn certain tricks. To add, to multiply, get percentages, a little bit of fractions and so on. But, the situation today is different economically than it was say 150 years ago. It was good enough then to know just how to do the tricks. But it is not good enough anymore for doing the work we do now in most jobs. So, we need to know a little more mathematics. Now as to what type of mathematics we need to know, I suppose it doesn't matter very much because most mathematics you learn, if you understand it, will teach you a way of thinking … structural thinking. Thinking in structures, how structures fit into one another. How do they relate to each other and so on. Now, whether you learn that in linear algebra or in infinite series or any other area…As long as you get the idea of what mathematical thinking is like, you can apply it to all sorts of other situations.

Sriraman: Recently, there have been initiatives by Richard Lesh, which are guided by your principles of learning. His research group uses *model-eliciting activities* and *model development sequences* in much the same way that you used *concrete embodiments* and *multiple representations*. But, his work focuses on simulations of "real life" situations more than on concrete manipulatives. Students often work in small groups, just as in your work; and, their work continues to focus on structure. What do you think of this approach?

Dienes: Well, it is good to hear that others are making use of my learning principles. I emphasized small group work long before it became popular.

Sriraman: Do you think that real-world contexts are relevant?

Dienes: Context is very important. In the work I've done in different parts of the world, I've always tried to put things in practical terms. It somewhat depends on the local culture in which you are operating. It wouldn't be the same in the United States as in India or China or New Guinea. … In New Guinea, I came across a tribe in whose language there was nothing for the concept of "either/or". … How do you teach logic if you don't know about "either/or." So I had to work out a way of making sure the kids understood. I did this by [tapping my arm] which meant it was correct; and, this [nodding head] meant no. So, with attribute blocks, a child would produce an answer to a question, and I would say [tapping my arm] that it was okay, and another child would produce another block, a different answer to the same question which was okay and I would again say [tapping my arm] that it was okay. That flabbergasted them. How could the answer be either this block or that block. … This was how I managed to teach the notion of "either or" in a culture where they had no words for such a concept. … In New Guinea, there is no "either/or" because the tribal system was so strict. You do THIS, and under these circumstances, you do THIS [under other circumstances]. And that's it. And God help you if you don't [Laughing].

Sriraman: Yeah, they had a uni-modal logic. That's interesting.

Dienes: Yes [laughing]. So we can't really lay down the law to what should be in a teacher education program. It depends on the local situation. A set of tools that one learns can become completely useless in a different situation, and this can happen very fast..

Sriraman: I agree that the context will have to be built according to the reality in which students are situated. For instance, I have been reading the work of a researcher in the Chicago area, who has taught in a Chicago public school in which the children are pre-dominantly from the community of Mexican immigrants. This researcher is inspired by the work of Paulo Freire, the Brazilian…I don't know if you know about his pedagogy for social justice. The whole point this researcher makes is that the mathematics taught to these particular children needs to be socially relevant and promote a critical awareness of the reality in which they are living.
[Bruce Dienes (BD), the youngest son of Zoltan Dienes enters].

BD: I made copies of these articles that Zoltan published in the New Zealand Mathematics Magazine [see list of references for these particular articles].

Sriraman: Thank you, I have been trying to get some of these for a long time. Please join us.

Sriraman: In this pedagogy for social justice, mathematics is used to make sense of their reality. They use projects with real world data like mortgage approval rates in bigger cities according to race; the misinformation or distortion of land mass given in older maps using the Mercator projection. Interestingly these things came out during the peak on colonization. Then there are other projects like using the cost of a B-2 bomber to compute how many poorer students in that community could be put through university. It seems that in this approach the goal is to impact social consciousness and larger issues. Do you have any thoughts on critical thinking in the mathematics classrooms?

Dienes: I do understand what you are saying about socially relevant mathematics. As long as the problem engages the students, allows them room for play, and getting through the representational stage with the experience of multiple embodiments, it doesn't matter what types of mathematics we are dealing with. I assume these children are older.

Sriraman: Yes, this researcher in Chicago was teaching at a middle school.

Dienes: What I have been doing for over 50 years is not so much outside social issues but critical thinking about what mathematics is and what it can be used for and to have it presented as fun, as play, and in this sense it can be self motivating because it is in itself a fun activity. I have critiqued mathematics being presented as a boring repetititious activity as opposed to a way to think. So it is not so much critical thinking of social issues but as a way to train the mind [e.g., Dienes & Golding, 1966], understand patterns and relationships, in ways that are playful and fun.

BD: What Zoltan used to do a lot when making presentations is to intentionally make mistakes, encouraging students to correct him, to challenge authority. To not assume that whatever you are told is correct. His goal was to build a social environment where students don't have to lean on the teacher or on the tricks being taught. So his goal has been to address a more fundamental learning process as opposed to using issues to motivate it.

Sriraman: A lot of the reform based curricula are data driven, and emphasize modeling activities. The data is usually taken from reality and the mathematics is motivated from the need to make sense of this data. The mathematics usually is the study of various families of functions which can be used to model the data. One of the stories I remember from your book is when you had to teach partial differential equations (PDE's) to a group of engineering students in England during the war (WWII). And you were trying to create examples which would make the mathematics interesting to these students.

Dienes: I remember that, having to teach partial differential equations to engineering students. This couldn't have been of any possible use to these engineers whatsoever because what they would be interested in would be to determine what the arbitrary functions are which are part of a solution, not so much solving the equation. There are so few PDE's that can actually be solved [laughing]. One of the examples I concocted was being chased by a bull and having to determine using PDE's, conditions that would allow the person to escape [laughing]. In this case, I did not see the point of teaching methods of solutions to engineers with cooked up nice equations….which they would never have to deal with in their profession. Engineering situations are usually more messy and not as nice as those prescribed in the books.

Sriraman: Do you think your being exposed to so many languages as a child has something to do with your fondness for mathematics and the proclivity for thinking in structures?

Dienes: You are asking whether being brought up multilingual has a connection with doing mathematics. Well I am only one example so one cannot deduce from one example. There are some conflicting research findings on multilingualism and the development of intelligence for example. I mean, I never thought anything about the priority of one language as opposed to another one. You know, being brought up multilingual you think of this as a minor problem. I don't know if you remember the bit about when I was quite young, we would go to Czechoslovakia for our holidays and we would climb up our tree house and listen to the mathematics talks without them knowing we were there. They were all discussing problems of the day including mathematics and philosophy. That is partly what got me interested in understanding the mathematics.

And when they were talking about philosophy, I thought aren't they clever, these guys, that just the process of thinking can be turned into fun [Laughing]. Just thinking about something can be fun. It's great…you don't really have to do anything. [Laughing]

Sriraman: I asked you the question about natural language because I noticed in the book, that at a very early age, you and your brother were able to create your own language. You came up with a list of nouns, a grammar and a syntax. Mathematics has some of these elements when one looks at if from a linguistic point of view.

Dienes: Have you looked at the Ruritania on my Web site?

Sriraman: Yes, I have. The adventures of Bruce and Alice. It is very clever, especially the creation of a new language.

Dienes: The vocabulary is made up. There is a certain logical relation between concepts which come out in the construction of the words. I don't explain that in the Web site. It is just there for somebody to discover. I have often thought that maybe this idea of constructing an artificial language could be a form of learning for the future. Like 50 years ago, the multibase blocks I brought in were regarded as absolute nonsense. Why did you do that? How could you possibly think of that as being of any use? Yet, now some people have finally understood. In fact those teachers that actually use it wouldn't want to go back. They've realized that it actually does teach children place value, the idea of the power as the exponent. I thought that something to do with expressing what you want to express in a language. You could think about what are the minimum conditions for a language to allow you to express what you want to express. You obviously have to have some kind of grammar, some kind of relationship between the words. What is the minimum core that you have to have. If you are able to construct a minimum core language like that, maybe it is something that will help you to understand different parts of the world, different cultures, and different languages. Now, I just think this may happen in about a 100 years time, that someone will finally dig up this Web site [laughing] and say oh, 100 years ago there was this guy called Dienes, who was already thinking about it. Now we think it is obvious. But, a 100 years ago nobody took any notice of it [laughing]. It is in there, in that Web site. Somebody one day will realize the power in it.

Sriraman: The reason I asked you about the connection between language and mathematics is because I personally have seen language as being very structural. Growing up in India, I picked up 4 different languages very early. After that, it was quite easy to learn languages like Urdu and Farsi because structurally they were the same to others I already knew. The same goes with German and knowing English makes many other Indo-European languages in that family quite accessible. Once one discovers the structure, it is merely a matter of filling in the vocabulary and other irregularities in syntax.

Dienes: Yes, that was pretty much the case with me as well. I see the point you're making.

BD: Like you said, and what Zoltan said, because of being forced to learn multiple languages, one is forced to be conscious of the structures and notice it in the next one and learn it. If people have learned only one language, it's like a fish in the water… you are completely unaware of the structure. Zoltan uses multiple embodiments as a learning strategy. When you do something with dance, you do it with blocks, and all of a sudden, as it is like for people like you and Zoltan with languages, all of a sudden multiple embodiments brings into consciousness the structures and patterns. I can see where you're coming from when you ask about the link to learning languages.

Dienes: The structural features one recognizes from these multiple embodiments - this brings out the essence of abstraction. Symbolism can be thrown in at this advanced stage, not earlier. The reason mathematics is boring in schools is because no real mathematics is taught in schools. [Laughing]

BD: When I first went to McGill, I did one year of pure mathematics and then decided to go off in a different direction. In that first year, it was like, that's what it is all about. It was as if there was a secret society which only got to know these things, these underlying structures. You had to be indoctrinated into honors courses in mathematics before they told you what mathematics was really all about. For 12 years until that point, I had really learned nothing. I didn't truly know what multiplication was until I took an honors course in Abstract Algebra. So, part of the reason, students get frustrated is because they aren't really being taught anything. The radical thing that Zoltan tried to do with young children is to try and teach them some mathematics, the process of thinking mathematically.… It worked, and everybody got up in arms…You can't do that! Because that would mean that the teachers and everyone else would really have to know it.

Dienes: [Laughing] …and the secret society would no longer be secret.

Sriraman: There is plenty of research now which indicates that mathematics is the key to opening up numerous professions for the children in schools, new opportunities. In order to get into the sciences or any applied fields, mathematics has historically served as the sieve or the gatekeeper to peter our people who supposedly don't fit into this secret society. It still is.

Dienes: You're absolutely right. I have seen this happening in different parts of the world.

Sriraman: If one were a cynic, I suppose you could think of mathematics as a means of keeping society stratified, and promoting the status quo and inequity in place.

Dienes: There are many such mechanisms, mathematics is one of them. You know reading used to be regarded as the preserve of the scribes. You didn't want the masses knowing how to read, that would be bad. It would disturb society.

Sriraman: It is clear from your memoir that you took to incorporating technology about as quickly as it was being invented. These days it seems that the younger generation is very adept at adapting to new technologies and becoming experts at using them. It is not uncommon to go to a public library in North America and see a 3 or 4 year old comfortably using the mouse to navigate their way through games etc. How do you think technology can be used to our advantage in the teaching and learning of mathematics?

Dienes: Like everything else, every major change will have an effect on just about everything else you do in life. I mean, just like running the house has changed because of technology, driving a car, everything has changed. So, it obviously has to have some kind of effect on the teaching of mathematics. … I don't know. I mean you are in the next generation, and I am already going out. I've done my bit so to speak and what I've done is probably not good enough. You need to develop it further.

Sriraman: The reason I asked you this was because there is always some form of controversy or another on the use of technology in a mathematics classroom. The purists versus the reformists, things like technology hindering the learning of mathematics.

Dienes: [Laughing] Well, you see it puts them out of business. If the calculator or other computer technology can now perform the calculations, they can no longer teach the tricks and hide behind it. The focus will now have to be on understanding the tricks.

BD: Creating structured materials was what Zoltan was doing with the logic blocks, etc. The nature of the materials determined the nature of play to some degree. So even if we gave them no instructions at all, they could learn some principles as a result of interacting with the materials. The computer is ideal for setting up structured environments. That is what programming is. What you need is a structured environment with principles built into it, but open enough that you can interact and change with it. This is why role playing games in computing environments are so popular. The question is to set up the right balance between interactivity and structure to whatever stage is appropriate for a learner. The biggest problem is that they are generally one-on-one because they've been designed for office use. It doesn't mean that is how they have to be used. I mean, why should a screen be this size by this size and placed the way it is? Why can't we have a table that is a screen so people can sit around and manipulate the environment together, interact with touch. We need to radicalize the interface between student and computer, not take an office machine and put it in the classroom and hope that it will work. Take the principles of computing technology, redesign things from scratch to make them suitable to teach.

Dienes: You'll be putting a lot of people out of business doing that [laughing].

Sriraman: What are your views on the nature of mathematics? Do you think mathematics is a human creation or do you think it is discovered?

Dienes: Well, I think this is a non-problem because we will never solve it. But we can talk about it [laughing]. Yes. Well, I have an idea that certain things are so whether we think them or not, so in that sense I am with Plato. I mean 2 + 2 is always going to be 4 in any system you are likely to design. There are certain core elements that must be so, whether we think so or not. When you discover, in quotes, mathematics. Are you creating the mathematics or are you discovering what is already there? I'd rather fancy you're discovering. Mathematics being the only branch of knowledge for which such a sentence can be said! If you take something like Physics, there are already certain assumptions you make about the nature of matter, which may be open to question before you can use mathematics to deal with a problem in physics. Like quantum computing, you have to first know something about quantum theory, assumptions about matter etc to even know what people are talking about. Do you read Scientific American? Yes…So you've read the recent arguments on this issue. One of my uncles used to be a research bio-chemist at Harvard. He gave me Scientific American for a year, 30 years ago or more, and at the end of the year I didn't think I could do without it and I've had it ever since [laughing]. So we leave the non problem. I really don't think it is a problem [laughing]. Playing with new forms of mathematics enables us to reframe how we look at the universe, and find things we may not have found had we not been able to reframe it that way. It is a bit of both.

BD: I spent some time visiting the classes in which Zoltan did his teaching. What I noticed was not so much the mathematics. You know his second degree was on the psychology of learning mathematics. So it was more about the learning than the mathematics. In fact in some of his classes in Quebec he would have a mixture of mathematics, language and art in the same classroom, with different learning stations where students could choose what they wanted to work on. Often these classes had multiple grades, so the older ones were teaching the younger ones. The other thing he did was he never ever set up competitive games. The games were always things that didn't work unless you worked together. His strategy was always to focus on the nature of learning. How do we create an environment in which people learn to cooperate, to have fun, and have choices and power over their own learning experience? And create an atmosphere where learning is empowering. It is pretty much the kinds of things you were talking about.

Sriraman: It seems to me, based on reading your works from the 60's, that there was certainly a Piagetian influence. Perhaps you remember the book Piaget wrote with Beth [see Beth & Piaget, 1966], where he claims there is a correspondence between the evolving cognitive structures within an individual's mind to those of the structures espoused by the Bourbaki, namely structures of order, algebraic structures and then topological structures. Piaget was never able to prove his claim. Do you have any thoughts on this?

Dienes: I know what you are talking about but I have never been very clear on that. The one time that Piaget came and listened to my teaching kids in Geneva. Well! Piaget himself and some of his students including Inhelder I think. All of them came and I showed them how to teach complex numbers with a story. Not with manipulatives mind you but by simply manipulating a story line and I remember that his students were flabbergasted and when they came out I remember them saying. C'est ne pas possible! C'est ne pas possible! You know as if this didn't really happen. And I asked Piaget. Et vous Monsier Piaget, Qu'est-ce que vous en pensez? His reply was Tiens, c'est très interéssant [laughing]. So that is the extent to which he commented on it. But the fact that you could have a story and get the learning going was not in his theory you see. So you see it was something neither he nor his students could handle. Piaget was a God you see and he can't be wrong. Yet, here was something his theory couldn't explain.

Sriraman: There is new evidence now [e.g., Sriraman & English, 2004] that many of the stages espoused by Piaget in this theory are not set in stone. Children are capable of doing things that could only occur in later stages according to his theory, especially if they are given child appropriate materials in contrast to the types of materials he used.

Dienes: I have always been more practical in my theorizing than people like Piaget or Bruner. Let's stick to the facts and see what is possible.

A lot of people that have seen me in the classroom with children have said that the things that I showed them were possible were things they never thought could be. When they saw actual learning taking place at my instigation with materials, stories etc.

Sriraman: Do you think it is possible to have some sort of a global theory of learning?

Dienes: I do not know.

Sriraman: Do you know why your theories of learning did not catch on as much as they should have?

Dienes: The answer to that is simple. They were politically unpalatable [laughing].

BD: I can give you an example of what he is saying based on his experiences in Brazil. You know they would always take him to the show schools, and he said, why don't we go down to the Barrio. They said, No, you don't want to go there. The children there are not so intelligent. In any case he went to these schools and did some work down there and of course they learned just as well and as quickly and he was never allowed to go back. This was politically incorrect information with the responsibility to provide these children the same kind of training and resources the privileged schools were getting.

Dienes: I have never believed in a curriculum for young children. What matters is that children learn how to think. So trying to convince principals and people and others in charge that this type of learning adapts to the needs of the students doesn't match their conception of how learning occurs. I have not been one to patronize the establishment you know.

I remember being taken into one of these really poor schools in Brazil. Yes, you went into the classroom and half the classroom was occupied with broken down tables and chairs and what not piled up in a pile. And each child has a piece of paper about this size [indicating the palm of his hand] and the one pencil and that was their material. I was taken in there and asked what can you do with these? I said this is a challenge [laughing]. I said, first thing we'll do is see what's in that corner. Let's take those broken chairs down and see what we can do with them. Oh! Shall we? Yes! I got the kids to sort out the mess in that corner. Any tables we could make into tables we made, with screws and nails. And we also made some actual mathematics materials with the stuff and started learning mathematics. All we needed was that thrown away stuff. Previously all they had thought about mathematics was this one little piece of paper and one pencil. [Laughing].

Dienes: The sad end of this story is that they brought us a box of expensive chocolates at the end and gave us as a present. And the first bit of information was that we musn't share them with the children. Oh no, we shouldn't do that. Goodness knows where they got the money to buy the chocolate. We weren't able to share them.

Sriraman: What are your thoughts on constructivism, which became popular when people in mathematics education started to rediscover Piaget and Vygotsky's writings in the 80's?

Dienes: My answer to that is simple really. These things were practiced in my classrooms long before people invented a word for it and I am sure there are people out there doing similar things for children. I do not care very much for isms, be it constructivism or behaviorism or any other ism. What really matters is that actual learning can take place with the proper use of materials, games, stories and such and that should be our focus. Ultimately, have they learned anything that is useful and made them think?

Sriraman: You seem to be getting tired now. I would be happy to stop here. I thank you again for your time.

Dienes: I hope I have been of some use. I'm not very young anymore as you can see. Let us eat some lunch.

COMMENTARY BY RICHARD LESH

I first met Zoltan Dienes in 1974, during the first in a series of annual research miniconferences that I used to hold at Northwestern University. These miniconferences later led directly to the founding of both PME/NA and the NCTM Research Presessions. At that meeting, one evening, I had the wonderful opportunity to sit around talking long into the night with Dienes and Bob Davis. It was an opportunity I'll never forget.

Like Davis, Dienes was educated as a mathematician. But, both Dienes and Davis were Renaissance men of the highest order—and visionaries whose expertise spanned a wide range of fields both practical and theoretical. Furthermore, both considered mathematics education to be a place for scientific inquiry—rather than a venue for spouting personal prejudices. So, both conducted extensive studies about the psychology of mathematics learning and problem solving; and, both hobnobbed regularly with people such as Jean Piaget, Jerome Bruner, and other giants whose thinking shaped modern theories of cognitive science.

Dienes, in particular, introduced many ideas that only now, 30 years later, are beginning to be appreciated for their power and beauty. Examples follow.

Like Piaget, Dienes emphasized the fact that mathematics is the study of structure—and that many of its most important concepts and processes have meanings that depend on thinking that is based on conceptual systems-as-a-whole. For instance, using a wide range of creative tasks, Piaget demonstrated the inherent systemic nature of (a) unit concepts whose meanings depend on invariance properties (with respect to a system), (b) relation concepts whose meanings depend on properties such as transitivity (with respect to a system), and (c) other properties such as those that depend on patterns or regularities (of a systems)—or on the maximization, minimization, or stabilization of properties with a system. He demonstrated that statements of belief often are emergent properties of systems of belief, that statements or principles often depend on systems of principles, and that these systems need to function as systems-as-a-whole before the relevant concepts, principles, and beliefs attain their intended meanings. Finally, he demonstrated what children's thinking is like before relevant conceptual systems-as-a-whole have begun to function as systemic wholes; and, he demonstrated some of the most important processes that influence development. But, one of the things that Piaget did not do, and that Dienes definitively has done, is to recognize that mathematics is not just about structure; but, even more important, it is about isomorphism, homomorphism, and more generally structural mappings among structures. Furthermore, relevant conceptual systems are molded and shaped by the external systems they are used to interpret, and that beyond entry-level mathematical systems, usually need to be expressed using some external media—or embodiment—if they are to function properly. Therefore, Dienes not only studied a phenomenon that later cognitive scientists have come to call embodied knowledge and situated cognition—but he also emphasized the *multiple embodiment principle* whereby students need to make predictions from one structured situation to another. And, he also emphasized the fact that, when conceptual systems are partly off-loaded from the mind using a variety of interacting representational systems (including not only spoken language written symbols, and diagrams, but also manipulatives and stories based on experience-based metaphors), every such model is, at best, a useful oversimplification of both the underlying conceptual systems being expressed and the external systems that are being described or explained.

Thus, Dienes' notion of *embodied knowledge* presaged other cognitive scientists who eventually came to recognize the importance of *embodied knowledge* and *situated cognition*—where knowledge and abilities are organized around experience as much as they are organized around abstractions (as Piaget, for example, would have led us to believe), and where knowledge is distributed across a variety of tools and *communities of practice*.

Dienes was an early pioneer in what is now coming to be called *sociocultural perspectives* on learning and problem solving. This can be seen in the fact that the learning activities that he designed nearly always involved groups of students; and, they were not just activities that were written for isolated individuals who happened to be approached as a group. Instead, Dienes' activities explicitly built in characteristics that demanded that the learner needed to be a group. Thus, for Dienes, and unlike what many modern sociocultural theorists suggest, the learning community was not simply an entity that allowed individuals to learn from others—or to learn by adopting cultural artifacts from the community as a whole. Instead, for Dienes, the learner often is a group; and, this fact is becoming increasingly important to notice in the knowledge economies of an information age—when learning organizations need to continually adapt to rapidly changing circumstances. Finally, Dienes' brand of situated cognition was not simply one in which knowledge development is considered to be task specific. Instead, Dienes emphasized the fact that, when conceptual tools are developed, they usually are designed to be sharable (with others) and reusable (beyond the initial situation in which they were needed).

As Sriraman mentions in this interview, Dienes' theories strongly influenced a number of the most productive research programs in mathematics education. One prime example is the series of projects that came to be known collectively as *The Rational Number Project* (Lesh et al., 1992; Post, Behr, Lesh, & Harel, 1992). Another is the work of Jim Kaput and others who focused on technology-based learning environments and on multiple-linked representational media (Lesh, Post, & Behr, 1987). And, more recently, extensions of Dienes' ideas play central roles in current research on *Models & Modeling Perspectives* on mathematical thinking and learning (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003) and on teacher development (Doerr & Lesh, 2003).

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