Adaptive Expectation and its weaknesses:

Here our intention is to get some idea about adaptive expectation and its weaknesses by pivoting our discussion around a model that was proposed by Cagan. We do not intend to go deep into the model. However, we'll mention some of the features of the model that would suffice to highlight the weaknesses of the adaptive expectation hypothesis.

Given that the Cagan's model is model of inflation, it will come as no surprise to you to learn that its only two central variables are the price level and the money shock. What may come as a surprise is the implied suggestion that the satisfactory analysis can be conducted using only these two variables. The reason this is possible is that the model is intended only for the analysis of truly severe inflations; precisely because of their severity during hyperinflationary periods. During these periods the movements in the price level and money stock are so large that they swamp movements in real variables such as output or the real rate of interest. In such circumstances, consequently, movements in the real variables can be neglected.

To be more specific, suppose that an economy's aggregate money demand function is of the form described as

$$m_t^d - p_t = \gamma + \alpha\Pi_t + u_t \qquad (1)$$

As we saw in the class, m_t^d and p_t are in log form. Π_t is the expected inflation rate. To be more precise, Π_t is the expected value of $\Delta p_{t+1} = (p_{t+1} - p_t)$, that is the value of $p_{t+1} - p_t$ anticipated by economy's individuals as of period t. One of the main objectives of Cagan's study was to present evidence bearing the proposition that money demand behavior is orderly and well behaved, rather than erratic or irrational. One way of doing this was to estimate, statistically, a demand function of the form (1). If it fit the data well even under the extreme conditions of a hyper-inflation and exhibited the expected properties, then this would be evidence supporting the notion that money demand is well behaved.

A major difficulty in conducting this type of exercise is the nonexistence of data on Π_t. Since the latter is the expected inflation rate, not the actual rate, Cagan had no observations or official data pertaining to this variable. Accordingly, he was forced to devise a model of expectation formation to represent Π_t in terms of the variables that could be observed and measured. The expectation model that Cagan developed eventually became well known as the model of "adaptive expectations". In terms of our notation for the formula in hand, the adaptive expectation formula for the unobserved Π_t can be expressed as

$$\Pi_t - \Pi_{t-1} = \lambda(\Delta p_t - \Pi_{t-1}); \quad 0 \leq \lambda \leq 1 \tag{2}$$

Here the idea is that the expected inflation rate is adjusted upward, relative to its previous value, when the most recent actual inflation rate (Δp_t) exceeds its own previously expected value (Π_{t-1}). Correspondingly, if Δp_t were smaller than Π_{t-1}, the value of Π_t would be lowered relative to Π_{t-1} (remember the mid-term example I gave in the class). Here the extent of the adjustment is indicated by the given parameter λ.

Now, we can re-write equation (2) as

$$\Pi_t = \lambda\Delta p_t + (1-\lambda)\Pi_{t-1} \tag{3}$$

But this form of relation implies $\Pi_{t-1} = \lambda\Delta p_{t-1} + (1-\lambda)\Pi_{t-2}$, which can be substituted back into equation (3) to give,

$$\Pi_t = \lambda\Delta p_t + (1-\lambda)[\lambda\Delta p_{t-1} + (1-\lambda)\mathrm{B}\Pi_{t-2}] \tag{4}$$

Similarly, $\Pi_{t-2} = \lambda\Delta p_{t-2} + (1-\lambda)\Pi_{t-3}$ could be used in (4) to eliminate Π_{t-2}, and so on. Repeating this substitution infinitely leads to an expression of the form

$$\Pi_t = \lambda\Delta p_t + \lambda(1-\lambda)\Delta p_{t-1} + \lambda(1-\lambda)^2 \Delta p_{t-2} + \dots \dots \tag{5}$$

since the term $(1-\lambda)^n$ approaches to zero as $n \to \infty$. From equation (5), then, we see that the expected inflation rate Π_t can be expressed (under the adaptive expectation formula) as a weighted average of all current and past actual inflation rates (Please refer to the explanation given in the class about what it means by the term ‘weighted average’). Here, more weight is attached to recent as opposed to distant values of inflation.

We are now in a position to understand the logical weakness underlying the error correction behavior expressed in equation (2). In particular, the formula implies the possibility of systematic expectational errors. To see this possibility exists, consider a hypothetical case in which, because of constantly increasing money growth rates, inflation regularly increases each period, always exceeding its previous values. From equation (5) we know that the value of Π_t given by the given by the adaptive expectation formula is a weighted average of current and past Δp_t values. Also, $\Delta p_t > \Delta p_{t-1} > \Delta p_{t-2}.....$. Therefore Π_t in all periods will be smaller than Δp_t. Then, given $\Delta p_{t+1} > \Delta p_t$, we have $\Delta p_{t+1} > \Pi_t$. Thus the expected inflation rate, Π_t, will in all periods will be smaller than the actual inflation rate, Δp_{t+1}, that it is intended to forecast. There will be period after period, repeated expectational errors of the same kind.

But since economic actions are in part based on expectations, expectational errors are costly to the individuals who make them. Consequently, purposeful economic agents – utility maximizing individuals and profit maximizing firms – will seek to avoid expectational errors. They cannot be entirely successful in this endeavor, of course, because no one can foresee the future. But they can reduce and virtually eliminate systematic sources of error by appropriate reactions. That the adaptive expectations formula can be sub-optimal in such an obvious manner is a very telling criticism of that formula. Although, during 1956-75, this formula was very popular, few macroeconomists rely on it today.

Towards a theory of Rational Expectation:

The above discussion points to the rationale underlying the endeavor of economic agents to eliminate systematic expectational errors. To put it a bit explicitly, here agents’ objective is to find a way of forming expectation in such a way that (1) the average expetational error will be zero and (2) there is no systematic relationship between the expectational error and any information available at the time when expectations are formed. In its absence, the errors would be predictable.

But, how the absence of systematic expetational error expressed analytically? Is there a formula, perhaps more complex than that for adaptive expectations, that will yield this condition? In this regard it is important to recognize that the absence of systematic expetational error cannot generally be represented by any algebraic formula comparable to the adaptive expectation formula. It might be possible to write a formula expressing Π_t in terms of Δp_t, $\Delta p_{t-1}, \ldots$ that would avoid errors in the particular case of ever-increasing inflation, but the coefficients in the formula would then be wrong if different inflationary pattern – say repeated cycles – was generated by monetary authority.

The message of the foregoing is that to express analytically the hypothesis that agents avoid systematic expectational error, we want not a formula but instead an anlytic condition that rules out such error. To see what the appropriate condition is, let us consider an agent forming her expectation at time t of p_{t+1} - the next period's (log) price level – let us denote the expectation by p_{t+1}^e. Then the expectational error that will occur, when period $t+1$ come to pass, is $p_{t+1} - p_{t+1}^e$. And the condition that we want to adopt is that this error, $p_{t+1} - p_{t+1}^e$, not be systematically related to any information possessed by the agent in period t (when the expectation was formed).

The way to achieve this condition, it turns out, is to assume that expectations subjectively held (i.e. believed) by agents are equal to the mean of the probability distribution of the variable being forecast, given available information. For example, p_{t+1} is from the vantage point of period t is a random variable. Its means from that vantage point $E(p_{t+1}|\Omega_t)$ is the mathematical expectation (i.e. mean) of the probability distribution of p_{t+1}, given the information set, Ω_t, available to the agent at time t. Thus the expectational hypothesis that we are seeking to can be adopted by assuming that, for any variable p_{t+j} and any period t,

$$p_{t+j}^e = E(p_{t+j}|\Omega_t) \qquad (6)$$

In words, this condition requires that the subjective expectation (forecast) of p_{t+1} held by agents in t be equal to the objective (mathematical) expectation of p_{t+1} conditional on Ω_t (i.e., the mean of the actual conditional probability distribution of p_{t+1} given information available in t).

To this point we have argued in favor of an expectational hypothesis that rules out systematic errors and have asserted that condition (6) will do so. Let us now prove this assertion is true. First we calculate what the average expectational error will be over a large number of periods if (6) is utilized. To do that we find the mean of the distribution of $p_{t+1} - p_{t+1}^e$ values. That is, we compute

$$E(p_{t+1} - p_{t+1}^e) = E[p_{t+1} - E(p_{t+1}|\Omega_t)] = E(p_{t+1}) - E[E(p_{t+1}|\Omega_t)] = E(p_{t+1}) - E(p_{t+1}) = 0 \tag{7}$$

Showing that the average error is zero. In this calculation the only tricky step is the next-to-last one (See explanation in the class).

Thus we see that the average expectational error under hypothesis (6) will be zero. To complete the demonstration that there will be mo systematic relation between $p_{t+1} - p_{t+1}^e$ and any information available in t, let x_t denote any variable whose value is known to agents at t. Thus, x_t is an element of Ω_t. Then consider the covariance of $p_{t+1} - p_{t+1}^e$ and x_t. Since $E\,(\,p_{t+1} - p_{t+1}^e)$ is zero, this covariance will be the mean of the distribution of the product $(\,p_{t+1} - p_{t+1}^e)\,x_t$. We evaluate this covariance as follows:

$$E[(p_{t+1} - p_{t+1}^e)x_t] = E[(p_{t+1} - E(p_{t+1}|\Omega_t))x_t] = E(p_{t+1}x_t) - E[E(p_{t+1}|\Omega_t)x_t] \tag{8}$$

But because x_t is an element of Ω_t, it is true that $x_t E(p_{t+1}|\Omega_t) = E(x_t\, p_{t+1}|\Omega_t)$. Then using the law of iterated expectations, we see that the final term in (8) equals $E(x_t p_{t+1})$. That shows, then, that the covariance is zero:

$$E[(p_{t+1} - p_{t+1}^e)x_t] = 0 \tag{9}$$

Thus we have shown that the adoption of assumption (6) will, in fact, imply that systematic expectational errors will be absent.

The hypothesis concerning expectation that we have outlined here was first put forward by John F. Muth (1961). Because the type of expectational behavior postulated is purposeful, and the absence of avoidable errors is necessary for optimality on the part of the agents in the modeled economy, Muth chose the term rational expectation to describe this hypothesis. As it happens, Muth's ideas were not immediately embraced by the economics profession, in part because this paper was difficult to understand and some aspects were not clearly spelled out. The profession's appreciation of the theory was greatly enhanced in early 1970s by a number of path-breaking papers by Robert E. Lucas, Jr., in which the rational expectations notion was extended and also applied to important issues in macroeconomics.